WHY MIGHT A COUNTRY WANT TO DEVELOP ITS COMPARATIVE DISADVANTAGE INDUSTRIES? A GENERAL EQUILIBRIUM ANALYSIS

Wenli Cheng and Dingsheng Zhang

ABSTRACT

This paper develops a general equilibrium 2x2 Ricardian model that demonstrates the possibility of immiserizing growth as a result of a productivity improvement in a country’s export industry. The model also shows that immiserizing growth can be avoided by improving the productivity of the country’s comparative disadvantage industry. However this strategy may inflict harm on its trading partner. In comparison, a balanced growth strategy can improve welfare of the growing country without hurting its trading partner.

Key words: 2x2 Ricardian model, immiserizing growth, balanced growth

JEL classification: F10, F11
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1. INTRODUCTION

Standard theory of international trade predicts that if each country specialises (partially or completely) according to its comparative advantage, and trades with one another, all will gain from trade. One might be tempted to infer from this result that a country should focus on developing the industries that it has a natural comparative advantage in. However, this inference is not necessarily correct partly because productivity progress in a country’s export industry (which the country has a comparative advantage in) can cause the country’s terms of trade to deteriorate, which may lead to a net reduction of the country’s welfare. This possibility is referred to in the literature as “immiserizing growth”.

According to Melvin (1969), the possibility of “immiserizing growth” was probably first discovered by Edgeworth (1894) who referred to it as economic damnification. Bhagwati (1958) coined the term “immiserizing growth” and provided a modern treatment of this phenomenon in a series of papers (Bhagwati, 1958, 1968, 1969). Other early analyses of immiserizing growth include Johnson (1967) and Melvin (1969). More recently, Samuelson (2004) provided an example of immiserizing growth where an invention can reduce the welfare of the inventor if the industry that the invention affects faces an inelastic demand.

Knowing the possibility of immiserizing growth, how can a country avoid it? Bhagwati (1968) contends that there are two generic types of immiserizing growth, one is caused by a welfare-reducing distortion in the economy (such as monopoly power), and the other is not. The first type of immiserizing growth can be avoided by introducing optimal (from the growing country’s perspective) policy interventions such as imposing an optimal tariff to eliminate the welfare reducing effects of the distortion. However Bhagwati (1968) does not offer strategies for avoiding immiserizing growth of the second type where no distortion is involved. Indeed to our knowledge nobody has – the focus of the literature appears to have been on showing that immiserizing growth can occur without giving too much thought to how it can be avoided.

This paper follows Samuelson (2004) to model immiserizing growth in a general equilibrium Ricardian model with a CES utility function. However our model considers all possible trade structures and explicitly defines the conditions under which immiserizing growth can occur in equilibrium. In addition, our model shows that the growing country can avoid immiserizing growth and increase welfare by improving the productivity of its comparative disadvantage (i.e., import
substituting) industry. This result suggests that for developing countries that mainly export goods with low demand elasticity (e.g., agricultural products), it may be important that they develop their capacities in their comparative disadvantage industries in order to obtain more gains from trade.

The remainder of the paper is structured as follows. Section 2 presents the model. Section 3 demonstrates the possibility of immiserizing growth, and section 4 considers possible strategies to avoid it. The concluding section discusses the limitations of the model and possible areas of further research.

2. THE MODEL
Consider a world economy with two countries, country 1 and country 2, each endowed with a workforce \( L_i \) \((i = 1, 2)\) which can be used to produce two consumer goods X and Y. We assume that the consumer goods can be freely traded between the countries, but labour is not mobile across countries.

2.1. Consumer decision
Following Samuelson (2004), we assume that the representative consumer maximizes utility that takes a simple CES form:

\[
\max_{x, y} U_i = \left( \frac{1}{2} x_i^{-1} + \frac{1}{2} y_i^{-1} \right)^{-1}
\]

s.t. \( p_x x_i + p_y y_i = w_i \) \( (1) \)

where \( x_i \) and \( y_i \) are quantities of goods X and Y, respectively; \( p_x \) is the price of good X, \( p_y \) is the price of good Y which is set to be 1; and \( w_i \) is the wage rate in country \( i \).

Solving the consumer’s decision problem, we obtain the following demand functions for goods X and Y

\[
x_i^d = \frac{w_i}{p_x + \sqrt{p_x}}, \quad y_i^d = \frac{\sqrt{p_x} w_i}{p_x + \sqrt{p_x}}
\]

(2)

2.2. Firm decision
We assume that the production functions for X and Y in country \( i \) are:

\[
x_i = a_{ix} L_i, \quad y_i = a_{iy} L_i
\]

(3)
where $L_{ix}, L_{iy}$ are labor devoted to the production of good $X$ and $Y$, respectively; $a_{ij} (i = 1, 2; j = x, y)$ is the labor productivity coefficient. Since $a_{ij}$ is country specific, it captures productivity differences between the two countries.

Constrained by the production technology, the representative firm producing $X$ in country $i$ maximizes its profit, i.e.,

$$\max_{L_{ix}} \pi_{ix} = p_x a_{ix} L_{ix} - w_i L_{ix}$$

$s.t. \quad L_{ix} \geq 0$

The Lagrangian function for the above optimisation problem is

$$Z_{ix} = p_x a_{ix} L_{ix} - w_i L_{ix} + \lambda_{ix} L_{ix}$$

The Kuhn-Tucker conditions are

$$\frac{\partial Z_{ix}}{\partial L_{ix}} = p_x a_{ix} - w_i + \lambda_{ix} = 0 \quad (4)$$

$$L_{ix} \geq 0, \lambda_{ix} \geq 0, \lambda_{ix} L_{ix} = 0 \quad (5)$$

Similarly, the representative firm producing $Y$ in country $i$ maximises its profit:

$$\max_{L_{iy}} \pi_{iy} = p_y a_{iy} L_{iy} - w_i L_{iy}$$

$s.t. \quad L_{iy} \geq 0$

Define a Lagrangian function for above optimisation problem as

$$Z_{iy} = p_y a_{iy} L_{iy} - w_i L_{iy} + \lambda_{iy} L_{iy}$$

The Kuhn-Tucker conditions are

$$\frac{\partial Z_{iy}}{\partial L_{iy}} = p_y a_{iy} - w_i + \lambda_{iy} = 0 \quad (6)$$

$$L_{iy} \geq 0, \lambda_{iy} \geq 0, \lambda_{iy} L_{iy} = 0 \quad (7)$$

Since $L_{ix}, L_{iy}$ can be positive or zero, there are 32 (=24) possible production structures. However, only 7 of these production structures are feasible if the two countries stay in autarky or trade with each other. The 7 feasible production structures are listed in Table 1. As shown in Table 1, the feasible structures consist (1) 1 structure where both countries produce both goods; (2) 4 structures where 1 country produces one good, and the other produces two goods; (3) 2 structures where each country produces 1 good.
Table 1: Economic Structures

<table>
<thead>
<tr>
<th>Sequence Number</th>
<th>Characters of Economic Structures</th>
<th>Structures</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$L_{1x} &gt; 0, L_{4y} &gt; 0, L_{2x} &gt; 0, L_{2y} &gt; 0$</td>
<td>(XY)(XY)</td>
</tr>
<tr>
<td>2</td>
<td>$L_{1x} &gt; 0, L_{4y} &gt; 0L_{2x} &gt; 0, L_{2y} = 0$</td>
<td>(XY)(X)</td>
</tr>
<tr>
<td>3</td>
<td>$L_{1x} &gt; 0, L_{4y} &gt; 0, L_{2x} &gt; 0, L_{2y} &gt; 0$</td>
<td>(XY)(Y)</td>
</tr>
<tr>
<td>4</td>
<td>$L_{1x} &gt; 0, L_{4y} = 0, L_{2x} &gt; 0, L_{2y} &gt; 0$</td>
<td>(X)(XY)</td>
</tr>
<tr>
<td>5</td>
<td>$L_{1x} = 0, L_{4y} &gt; 0, L_{2x} &gt; 0, L_{2y} &gt; 0$</td>
<td>(Y)(XY)</td>
</tr>
<tr>
<td>6</td>
<td>$L_{1x} &gt; 0, L_{4y} = 0, L_{2x} = 0, L_{2y} &gt; 0$</td>
<td>(X)(Y)</td>
</tr>
<tr>
<td>7</td>
<td>$L_{1x} = 0, L_{4y} &gt; 0L_{2x} &gt; 0, L_{2y} = 0$</td>
<td>(Y)(X)</td>
</tr>
</tbody>
</table>

2.3. Market clearing conditions

In equilibrium, consumers’ utility and firms’ profit are maximised, and the markets for goods and for labor clear. The market clearing conditions include the following:

\[ x_1^d + x_2^d = x_1^s + x_2^s \]  
\[ y_1^d + y_2^d = y_1^s + y_2^s \]  
\[ L_{1x} + L_{4y} = L_1 \]  
\[ L_{2x} + L_{2y} = L_2 \]

It should be noted that, according to Walras’ law, only three of the above four equations are independent.

2.4. General equilibrium structures

Solving the consumer’s and firms’ decision problems, and applying the market clearing conditions, we can obtain the equilibrium prices and utility levels. Moreover, the Kuhn-Tucker conditions for
the firms’ decision problems and the market clearing conditions also define the conditions under which a specific structure emerges in equilibrium.

To illustrate, consider structure (X)X(XY). As is clear from Table 1, this structure requires $L_{1x} > 0, L_{1y} = 0, L_{2x} > 0, L_{2y} > 0$. From the Kuhn-Tucker conditions which are equations (4)–(7), we obtain the equilibrium relative price and wage rates

$$p_x = \frac{w_1}{a_{1x}} = \frac{a_{2y}}{a_{2x}}$$

$$w_1 = \frac{a_{2y}}{a_{2x}} a_{1x} > a_{1y}, \quad w_2 = a_{2y}$$

Equation (13) implies

$$\frac{a_{1x}}{a_{2x}} > \frac{a_{1y}}{a_{2y}}$$

which defines one condition under which structure (X)X(XY)Y emerges in equilibrium.

In addition, using market clear conditions (8)-(11), we have

$$\frac{w_1 L_1 + w_2 L_2}{p_x + \sqrt{p_x}} = a_{1x} L_1 + a_{2x} L_2$$

(14)

From equation (14) and the condition $L_{2x} > 0$, we get another condition for this structure to emerge in equilibrium, which is

$$\left(\frac{L_2}{L_1}\right)^2 > \frac{a_{1x}^2}{a_{2x} a_{2y}}$$

Therefore, for structure (X)X(XY)Y to emerge in equilibrium, the parameter must satisfy

$$\frac{a_{1x}}{a_{2x}} > \frac{a_{1y}}{a_{2y}}, \quad \left(\frac{L_2}{L_1}\right)^2 > \frac{a_{1x}^2}{a_{2x} a_{2y}}$$

(15)
Under these conditions, the equilibrium utility levels for an individual in each country (which we refer to as per capital real income hereafter) can be solved, and the solutions are:

\[
u_1 = \frac{2a_{1x} \frac{a_{2y}}{a_{2x}}}{(1 + \sqrt{\frac{a_{2y}}{a_{2x}}})^2}, \quad u_2 = \frac{2a_{2y}}{(1 + \sqrt{\frac{a_{2y}}{a_{2x}}})^2}\]

Similarly we can solve the equilibrium prices, wages for other structures. The solutions are presented in Table 2.

Following a similar approach as illustrated above, we can also define the conditions under which a specific structure emerges in equilibrium. It can be shown that structure (XY)(XY) occurs in equilibrium only when \((a_{1x} / a_{1y}) = (a_{2x} / a_{2y})\), that is, no comparative advantage exists. This is a very special case which we do not focus on in this paper. The remaining 6 structures fall into two symmetric categories with 3 structures emerging.

### Table 2: Structures and General equilibria

<table>
<thead>
<tr>
<th>Structures</th>
<th>Equilibria</th>
</tr>
</thead>
<tbody>
<tr>
<td>(XY)(XY)</td>
<td>(p_x = \frac{a_{1y}}{a_{1x}} = \frac{a_{2y}}{a_{2x}}, w_1 = a_{1y}, w_2 = a_{2y})</td>
</tr>
<tr>
<td>(XY)(X)</td>
<td>(p_x = \frac{a_{1y}}{a_{1x}} = \frac{w_1}{a_{2x}}, w_1 = a_{1y}, w_2 = \frac{a_{1x}}{a_{2x}} a_{2x} &gt; a_{2y})</td>
</tr>
<tr>
<td>(XY)(Y)</td>
<td>(p_x = \frac{a_{1y}}{a_{1x}} &lt; \frac{a_{2y}}{a_{2x}}, w_1 = a_{1y}, w_2 = a_{2y})</td>
</tr>
<tr>
<td>(X)(XY)</td>
<td>(p_x = \frac{w_1}{a_{1x}} = \frac{a_{2y}}{a_{2x}}, w_1 = \frac{a_{2y}}{a_{2x}} a_{1x} &gt; a_{1y}, w_2 = a_{2y})</td>
</tr>
<tr>
<td>(Y)(XY)</td>
<td>(p_x = \frac{a_{2y}}{a_{2x}} &lt; \frac{a_{1y}}{a_{1x}}, w_1 = a_{1y}, w_2 = a_{2y})</td>
</tr>
<tr>
<td>(X)(Y)</td>
<td>(p_x = \frac{w_1}{a_{1x}} &lt; \frac{a_{2y}}{a_{2x}}, w_1 = \left(\frac{a_{2y}}{a_{1x}} \frac{L_2}{L_1}\right)^2 a_{1x} &gt; a_{1y}, w_2 = a_{2y})</td>
</tr>
<tr>
<td>(Y)(X)</td>
<td>(p_x = \frac{w_2}{a_{2x}} &lt; \frac{a_{1y}}{a_{1x}}, w_1 = a_{1y}, w_2 = \left(\frac{a_{1y}}{a_{2x}} \frac{L_1}{L_2}\right)^2 a_{2x} &gt; a_{2y})</td>
</tr>
</tbody>
</table>
if \( \frac{a_{1x}}{a_{1y}} > \frac{a_{2x}}{a_{2y}} \), and the other 3 emerging if \( \frac{a_{1x}}{a_{1y}} < \frac{a_{2x}}{a_{2y}} \). Because the equilibrium structures, corresponding conditions for existence of general equilibrium, and the equilibrium utility levels are symmetric, we only present the results assuming \( \frac{a_{1x}}{a_{1y}} > \frac{a_{2x}}{a_{2y}} \), which means country 1 has a comparative advantage in producing good X. The results are presented in Table 3.

Table 3: General equilibrium conditions, structures and per capita real income

<table>
<thead>
<tr>
<th>Existence conditions</th>
<th>Structures</th>
<th>Per capita real income</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{L_2}{L_1} ) &amp; ( \frac{a_{1x}a_{1y}}{a_{2x}a_{2y}} ) &amp; ( (XY)<em>X(Y)<em>Y ) &amp; ( u_1 = \frac{2a</em>{1y}}{1 + \left( \frac{a</em>{1y}}{a_{1x}} \right)^2}, u_2 = \frac{2a_{2y}}{1 + \left( \frac{a_{2y}}{a_{2x}} \right)^2} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{a_{1x}a_{1y}}{a_{2x}a_{2y}} ) &amp; ( \frac{L_2^2}{L_1^2} &lt; \frac{a_{1x}a_{1y}}{a_{2x}a_{2y}} ) &amp; ( (X)X(Y)Y ) &amp; ( u_1 = \frac{2a_{1y}}{1 + \left( \frac{a_{1y}}{a_{1x}} \right)^2}, u_2 = \frac{2a_{2y}}{1 + \left( \frac{a_{2y}}{a_{2x}} \right)^2} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{L_2^2}{L_1} ) &amp; ( \frac{a_{1x}a_{1y}}{a_{2x}a_{2y}} ) &amp; ( (X)X(Y)<em>Y ) &amp; ( u_1 = \frac{2a</em>{1x}a_{2y}}{1 + \left( \frac{a_{2y}}{a_{2x}} \right)^2}, u_2 = \frac{2a_{2x}a_{1y}}{1 + \left( \frac{a_{1y}}{a_{1x}} \right)^2} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In Table 3, the subscripts denote the good exported. For example, \( (XY)_X(Y)_Y \) means that country 1 produce goods X and Y and exports good X, and country 2 produces good Y and exports good Y.

3. THE POSSIBILITY OF IMMISERIZING GROWTH

From Table 3, we obtain the comparative statics of general equilibrium per capita real income with respect to country 2’s productivity in good X (\( a_{2x} \)) and good Y (\( a_{2y} \)). The results are presented in Table 4.
Table 4: Comparative statics of equilibrium per capita real income

<table>
<thead>
<tr>
<th>Structure</th>
<th>Comparative statics</th>
</tr>
</thead>
<tbody>
<tr>
<td>(XY)X(Y)</td>
<td>( \frac{\partial u_1}{\partial a_{2y}} = 0, \frac{\partial u_1}{\partial a_{2x}} = 0, \frac{\partial u_2}{\partial a_{2y}} = \frac{2}{(1 + \frac{a_{2y}}{a_{1x}})^2} &gt; 0, \frac{\partial u_2}{\partial a_{2x}} = 0 )</td>
</tr>
<tr>
<td>(X)X(Y)</td>
<td>( \frac{\partial u_1}{\partial a_{2y}} = \frac{4(a_{2y} L_2)}{a_{1x} L_1} &gt; 0, \frac{\partial u_1}{\partial a_{2x}} = 0, \frac{\partial u_2}{\partial a_{2y}} = \frac{2(1 - \frac{a_{2y} L_2}{a_{1x} L_1})}{(1 + \frac{a_{2y} L_2}{a_{1x} L_1})^3} &gt; 0, \frac{\partial u_2}{\partial a_{2x}} = 0 )</td>
</tr>
<tr>
<td>(X)X(Y)</td>
<td>( \frac{\partial u_1}{\partial a_{2y}} = \frac{2 a_{1x}}{(1 + \frac{a_{2y}}{a_{1x}})^3} &gt; 0, \frac{\partial u_1}{\partial a_{2x}} = \frac{-2 a_{1x} a_{2y}}{(1 + \frac{a_{2y}}{a_{1x}})^3} &lt; 0, \frac{\partial u_2}{\partial a_{2y}} = \frac{2}{(1 + \frac{a_{2y}}{a_{1x}})^3} &gt; 0, \frac{\partial u_2}{\partial a_{2x}} = \frac{a_{2y}^2}{a_{1x}} )</td>
</tr>
</tbody>
</table>

The results in Table 4 indicate that within structure (X)X(Y)Y, \( \frac{\partial U}{\partial a_{2y}} < 0 \) if \( \frac{a_{2y} L_2}{a_{1x} L_1} > 1 \). In other words, there exists the possibility of immiserizing growth. For instance, if we adopt the same initial parameter values as assumed in Samuelson (2004), namely,

Population parameters: \( L_1 = 100, L_2 = 1000 \).

Initial technology parameters: \( a_{1x} = 2, a_{1y} = \frac{1}{2}, a_{2x} = \frac{1}{20}, a_{2y} = \frac{2}{10} \).

Since these parameters satisfy \( \frac{a_{1x}}{a_{2x}} > \frac{a_{1y}}{a_{2y}} \), and \( \frac{a_{1x} a_{1y}}{a_{2x} a_{2y}} < \left( \frac{L_2}{L_1} \right)^2 < \frac{a_{1x}^2}{a_{2x} a_{2y}} \), the equilibrium trade structure is (X)X(Y)Y. From Table 4, we can calculate real per capital income for country 1 and country 2, which are 1 and 0.1, respectively. Now suppose country 2’s productivity in good Y improves from \( a_{2y} = \frac{2}{10} \) to \( a_{2y}' = \frac{3}{10} \), while other parameters remain the same. Following the productivity improvement, the parameters still satisfy \( \frac{a_{1x}}{a_{2x}} > \frac{a_{1y}}{a_{2y}} \) and \( \frac{a_{1x} a_{1y}}{a_{2x} a_{2y}} < \left( \frac{L_2}{L_1} \right)^2 < \frac{a_{1x}^2}{a_{2x} a_{2y}} \), and the equilibrium structure is still (X)X(Y)Y. The new equilibrium per capita real income levels become 1.44 and 0.096 for country 1 and country 2 respectively. In this example, a 50%
productivity growth in its export good Y has led country 2’s per capita income to fall by 4% from 0.1 to 0.096.

If country 2’s productivity growth in good Y is so large that the equilibrium structure moves away from structure (X)(Y), country 2 may still suffer from immiserizing growth. For instance, in the above example, suppose country 2’s productivity in good Y improves from \( a_{2y} = \frac{2}{10} \) to \( a'_{2y} = \frac{8}{10} \), while other parameters remain unchanged. Since the new parameters satisfy \( \frac{L_2}{L_1} \geq \frac{a_{1x}^2}{a_{2y}a_{2y}} \), the equilibrium trade structure will be structure (X)(XY), and the per capita real income for country 1 and country 2 will become 2.56 and 0.064, respectively. In other words, following a four-fold increase in productivity of its export good, country 2’s per capita real income has fallen 36% from 0.1 to 0.064. This is the same result as in Samuelson (2004).

To illustrate how changes in country 2’s productivity in the Y industry affect the two countries’ per capita real income, we specify values of other parameters as follows:

\[
L_1 = 100, \quad L_2 = 1000, \quad a_{1x} = 2, a_{1y} = \frac{1}{2}, \quad a_{2x} = \frac{1}{20}.
\]

Given these parameters, we can express the equilibrium per capita real income in the two countries as functions of \( a_{2y} \) as follows.

\[
u_1 = \begin{cases} 
\frac{4}{9}, & 1 < a_{2y} \leq \frac{1}{10} \\
\frac{100a_{2y}^2}{(1+5a_{2y})^2}, & \frac{1}{10} < a_{2y} < \frac{8}{10} \\
\frac{80a_{2y}}{(1+\sqrt{20a_{2y}})^2}, & a_{2y} \geq \frac{8}{10}
\end{cases}
\]

\[
u_2 = \begin{cases} 
\frac{8}{9}a_{2y}, & 1 < a_{2y} \leq \frac{1}{10} \\
\frac{2a_{2y}}{(1+5a_{2y})^2}, & \frac{1}{10} < a_{2y} < \frac{8}{10} \\
\frac{2a_{2y}}{(1+\sqrt{20a_{2y}})^2}, & a_{2y} \geq \frac{8}{10}
\end{cases}
\]

* Samuelson (2004) does not develop a model that allows different structures, but his example implies a change of trading structure as a result of productivity improvement.
The per capita real income levels in country 1 and country 2 are depicted in Figure 1 and Figure 2, respectively.

**Figure 1** Per capita real income in country 1 as a function of $a_{2y}$

![Graph of per capita real income in country 1 as a function of $a_{2y}$](image1)

From Figure 1, it is clear that any increase in $a_{2y}$ beyond the level $a_{2y} = 1/10$ will benefit country 1.

**Figure 2** Per capita real income in country 2 as a function of $a_{2y}$

![Graph of per capita real income in country 2 as a function of $a_{2y}$](image2)

From Figure 2, we can see that country 2’s per capita real income is maximised at $a_{2y} = 2/10$. Further improvement in $a_{2y}$ alone will lead to immiserizing growth; and even when immiserizing
growth stops if \( a_{2y} > 8/10 \), continuing improvement in \( a_{2y} \) alone can at best lead country 2’s per capita real income back to the maximum level achieved at \( a_{2y} = 2/10 \).

Why would country 2 suffer from its own productivity improvement? The traditional explanation is that because the demand for country 2’s export good Y is inelastic, an improvement in productivity in Y lowers country 2’s terms of trade (the price of Y relative to X), which causes a loss that outweighs the productivity gain. While the statement is intuitively appealing, it does not shed light on the conditions under which the loss from terms of trade deterioration may be outweighed by the productivity gain.

Clearly a fall in a country’s terms of trade does not necessarily lead to a fall in the country’s real income. For instance, in our model, given structure (X)X(Y)Y, a increase in \( a_{2y} \) will worsen country 2’s terms of trade \( (1/p_x) \) as \( \partial (1/p_x) / \partial a_{2y} < 0 \), but will increase country 2’s per capita income as \( \partial a_2 / \partial a_{2y} > 0 \). Therefore to determine the impact of productivity on real income, we need to know more than how the terms of trade have changed. Indeed, in a general equilibrium model such as ours, the terms of trade and income levels are simultaneously determined, thus we can and should examine directly how productivity growth affects both countries’ income levels from the equilibrium utility functions instead of indirectly through the terms of trade effect.

As noted earlier, the condition for immiserizing growth to occur in structure (X)X(Y)Y is \( a_{2y} L_2 > a_{1x} L_1 \). Combining this condition with the condition under which structure (X)X(Y)Y occurs in equilibrium,

\[
\frac{a_{1x} a_{1y}}{a_{2y}^2} < \left( \frac{L_2}{L_1} \right)^2 < \frac{a_{1x}^2}{a_{2x} a_{2y}},
\]

we can define the conditions for immiserizing growth to occur in our model, which is:

\[
\frac{a_{1x}^2}{a_{2x} a_{2y}} > \left( \frac{L_2}{L_1} \right)^2 > \max\left[\left( \frac{a_{1x}}{a_{2y}} \right)^2, \frac{a_{1x} a_{1y}}{a_{2y}^2}\right]
\]  

(17)

4. HOW TO AVOID IMMISERIZING GROWTH?

Since immiserizing growth can occur in our model only if condition (17) is satisfied, logically if country 2 can change the parameters in its control so that the parameter values do not satisfy condition (17), immiserizing growth will be avoided. For simplicity we assume that all the parameters related to country 1 and population in country 2 (\( L_2 \)) are fixed, so that country 2 can only change the two productivity parameters \( a_{2x} \) and \( a_{2y} \). Condition (17) suggests that an
increase in $a_{2x}$ will make the first inequality of condition (17) less likely to be satisfied, and have no impact on the second inequality. In comparison, an increase in $a_{2y}$ will make the first inequality of condition (17) less likely to be satisfied, and make the second inequality more likely to be satisfied. This suggests that a strategy for country 2 to avoid immiserizing productivity growth in its export industry is to improve productivity of its import substituting or comparative disadvantage industry (i.e., to increase $a_{2x}$).

To illustrate how changes in $a_{2x}$ affect both countries’ per capita real income levels, we set $L_1 = 100$, $L_2 = 1000$, $a_{1x} = 2, a_{1y} = \frac{1}{2}, a_{2y} = \frac{2}{10}$. Given these parameters, we can rewrite the equilibrium utility functions in the two countries as follows.

$$
u_1 = \begin{cases} 
1, & a_{2x} \leq \frac{2}{10} \\
\frac{0.8a_{2x}^{-1}}{(1 + \sqrt{0.2a_{2x}^{-1}})^2}, & \frac{2}{10} < a_{2x} < \frac{8}{10}
\end{cases}$$

$$
u_2 = \begin{cases} 
0.1, & a_{2x} \leq \frac{2}{10} \\
\frac{0.4}{(1 + \sqrt{0.2a_{2x}^{-1}})^2}, & \frac{2}{10} < a_{2x} < \frac{8}{10}
\end{cases}$$

The per capita real income levels in both countries are depicted in Figure 3.

**Figure 3:** Per capita real income levels in both countries as a function $a_{2x}$

![Figure 3: Per capita real income levels in both countries as a function $a_{2x}$](image-url)
It is clear from Figure 3 that, before $a_{2x}$ reaches 0.2, there is no impact on either country’s per capita real income (because X is not produced in country 2). If $a_{2x}$ increases beyond 0.2 where other parameters remain unchanged, country 2’s per capita real income will increase, whereas country 1’s will fall. In other words, while country 2 can benefit from improving productivity of its comparative disadvantage industry, this strategy will hurt country 1 and may invite retaliation. What strategy should country 2 adopt so that it can benefit from growth without inflicting losses on its trading partner? Our model suggests that a good strategy is to pursue balanced growth, that is, to improve productivity in both its comparative advantage industry (the Y industry) and comparative disadvantage industry (the X industry). In particular, refer to the equilibrium utility functions for structure $(X)X(XY)Y$ in Table 3. If $a_{2x}$ and $a_{2y}$ increase, the conditions for structure $(X)X(XY)Y$ to occur in equilibrium still hold. Moreover, in the new equilibrium, country 1’s per capita real income will not change if $a_{2x}/a_{2y}$ remains constant, whereas country 2’s per capita real income will increase even if $a_{2x}/a_{2y}$ remains constant as long as $a_{2y}$ increases. This means that, starting from an equilibrium structure of $(X)X(XY)Y$, if country 2 improves productivity in both industries by the same proportion so that $a_{2x}/a_{2y}$ remain constant, then the productivity growth will benefit country 2 without causing welfare losses in country 2.

5. CONCLUDING REMARKS

In this paper, we have developed a 2x2 Ricardian model with CES utility functions which demonstrates the possibility of immiserizing growth and defines the conditions under which it occurs. Our model also suggests that a country may avoid immiserizing growth by developing its comparative disadvantage industry. However developing a country’s comparative disadvantage industry alone may hurt the country’s trading partner; a more sustainable growth strategy that benefits the growing country without causing losses of its trading partner would be to pursue balanced growth in both the country’s comparative advantage and comparative disadvantage industries.

There are some caveats of the model that should be noted. Firstly, the model focuses on one country’s strategy to gain for its own productivity growth. A nationally optimal strategy may not be globally optimal. Secondly, since productivity growth is exogenous in our model, the model does not consider the source of growth. Thirdly, while recognising that country 2’s growth strategy may invite retaliation if hurts country 1, the model takes the parameters relating to country 1 as given, thus does not explicitly model country 1’s reactions to country 2’s growth strategy. These caveats point to possible areas of further research. The contribution of this paper is limited to that it models the conditions for immiserizing growth and suggesting a sustainable strategy to avoid it.
REFERENCES


