The Arm’s Length Principle, Transfer Pricing and Foreclosure under Imperfect Competition

Wenli Cheng¹ and Dingsheng Zhang²

Abstract:
This paper studies a multinational firm’s transfer price decisions in imperfectly competitive market settings. It investigates whether the firm’s optimal transfer price coincides with the arm’s length price and examines how the firm might respond if it is compelled to follow the arm’s length principle. The main findings are: (1) in the absence of tax transfer incentives, the firm’s optimal transfer price does not coincide with the arm’s length price. If the firm is compelled to follow the arm’s length principle, it has an incentive to circumvent the arm’s length principle by keeping two sets of books, one for internal management, and another for tax reporting purposes; (2) the arm’s length principle can affect the MNF’s decision on whether or not to foreclose its competitor. Absent profit shifting incentives, the firm will foreclose its downstream competitor. Imposing the arm’s length principle induces the firm to supply its competitor, but the firm can revert to its foreclosure decision by keeping two sets of books. If the firm’s upstream and downstream divisions face different tax rates, the firm’s foreclosure decision will be reversed if the arm’s length principle is enforced.

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1. Introduction

Within a multidivisional firm, an upstream division routinely transfers some intermediate goods to a downstream one. How should the multidivisional firm set prices for such internal transactions? The standard literature tells us that i) if there is no external market for the intermediate good, the internal transfer should be conducted at marginal cost; ii) if the intermediate good market is perfectly competitive, the transfer price should be the market price which, in equilibrium, is also equal to marginal cost (Hirshleifer, 1956; Milgrom & Roberts, 1992).

For multidivisional firms operating in different tax jurisdictions, the determination of transfer prices is complicated by tax considerations. In general a multinational firm (MNF) would want to shift profits from a high-tax-rate country to a low-tax-rate country so as to minimize the tax liability for the firm as a whole. This means that the MNF has an incentive to charge a higher transfer price for the purchasing division operating in a high tax rate country. Aware of this, tax authorities have developed detailed rules for transfer pricing in order to defend what they consider to be their legitimate tax bases. In addition, tax authorities of different countries have made considerable progress in coordinating their policies regarding transfer pricing for tax purposes. The most significant example of the coordinated outcome is probably the endorsement of the OECD’s *Transfer Pricing Guidelines for Multinational Enterprises and Tax Administrations* (OECD, 2001) by its member countries.

The OECD Guidelines provide that transfer prices should be set in accordance to the arm’s length principle, that is, transfer prices should be same as the prices that would have been charged if the two divisions were independent companies. The adoption of the arm’s length price principle was based on the belief that it is “sound in theory since it provides the closest approximation of the working of the open market where goods and services are transferred between associated enterprises” (OECD, 2001, p. 16). At least partly due to the influence of the OECD Guidelines, accounting professional are advising firms to consider market based transfer pricing if the internal transactions are comparable to any transactions with third parties (Amerkhail, 2006). In fact many MNFs are adopting the arm’s length principle in their internal transactions. Studies show that between 30% and 45% of firms surveyed say that they determine transfer prices on a market basis (Baldenius & Reichelstein, 2006).
While the reliance on the arm’s length principle is no doubt supported by the standard literature given the assumption of perfect competition in the intermediate good market, it is not straightforward that such reliance can be readily extended to situations of imperfect competition in the intermediate good and/or final good market.

The purpose of this paper is

1) to find out whether a MNF’s optimal transfer price coincides with the arm’s length price when competition in the intermediate market and/or the final good market is imperfect; and

2) to examine how the MNF’s decisions, including the decision to foreclose its downstream rivals, would be affected if it is compelled to adopt arm’s length transfer pricing for tax purposes.

There is a large literature that studies various instances where a firm’s preferred transfer prices are not the same as the arm’s length prices, and where the firm can set transfer prices strategically in competition with its rivals. A nice summary of the strategic implications of transfer pricing can be found in Gox & Schiller (2007). More recent contributions in this area include Arya & Mittendorf (2008) and Arya, Mittendorf, & Yoon (2008). In regard to the implications of the arm’s length restriction, Baldeinius et al. (2004) and Baldeinius and Reichelstein (2006) investigate possible inefficiencies associated with market based internal prices and how the design of intra-company discount policies can improve firm profitability.

This paper draws insights from the existing literature and makes two new discoveries. First, if the firm’s is compelled to follow the arm’s length principle, it can at least partially circumvent the restriction by keeping two sets of books. Secondly, the arm’s length restriction can alter the firm’s foreclosure decisions. If the MNF’s downstream and upstream divisions face the same tax rate so that there is no profit shifting incentives, the MNF will foreclose its downstream competitor. Imposing the arm’s length principle induces the firm to supply its competitor, but the firm can revert to its foreclosure decision by keeping two sets of books. If the MNF’s downstream and upstream divisions face different tax rates, imposing the arm’s length principle can reverse the MNF’s foreclosure decision.

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3 Many firms (including Microsoft and Hewlett-Packard) keep separate books for internal management and tax reporting purposes (Springsteel, 1999). Apart from the effect of circumventing regulatory restrictions that this paper investigates, another advantage for having two books is to have two instruments to coordinate different policies in tax accounting and cost accounting (Choe & Hyde, 2007; Hyde & Choe, 2005).
The rest of the paper is organized as follows. Section 2 studies the MNF’s decisions with and without the arm’s length restriction in a model where the MNF is a downstream monopoly provider (Structure One). Section 3 extends the model to allow for downstream Cournot competition (Structure Two). Section 4 concludes.

[Insert Figure 1 here]

2. Structure One: Monopoly Downstream Markets

Consider an MNF consisting of two divisions. The upstream division is located in country 1 and produces an intermediate good; the downstream division is located in country 2 and uses the intermediate good to make a final good sold in country 2. In country 1, there is another downstream firm (“independent firm”) which buys the intermediate good from the MNF to produce a final good sold in country 1. Assuming that the market for the final good is not integrated internationally due to high transaction costs (such as transportation costs and trade barriers), the MNF is a downstream monopoly in country 2, and the independent firm is a monopoly in country 1.

Case 1: MNF not constrained by the arm’s length principle

In this case, the internal transfer price \( m \) and the external price \( m^* \) charged to the independent firm may be different. The decision process is as follows. The MNF’s headquarters moves first. It chooses both \( m \) and \( m^* \) to maximize the MNF’s profit as a whole. Taking \( m \) and \( m^* \) as given, the downstream division as a profit centre and the independent firm then choose their outputs to maximize their after-tax profits, respectively.

The firms’ decision problems have to be solved backwards, i.e., the downstream division and the independent firm’s output are solved before internal and external prices for the intermediate good are determined.

Assuming that the production technology is such that one unit of the final good requires one unit of the intermediate good and one unit of labor, the downstream division’s after-tax profit function is

\[
\pi_d = (1 - t_z)(p_2 - m - w_2)x
\]  

(1)
where $t_2$ is the tax rate in country 2; $p_2 = p_2(x)$ is the inverse demand function for the final good sold in country 2; $w_2$ is the unit wage cost in country 2; and $x$ is the quantity of the final good produced (which is the same as the quantity of the intermediate good bought) by the downstream division.

Similarly the independent firm’s after-tax profit function is

$$\pi_j = (1 - t_j)(p_j - m^* - w_j)y$$

(2)

where $w_j$ is the unit wage cost in country 1; $p_j = p_j(y)$ is the inverse demand function for the final good in country 1; and $y$ is the quantity of the final good produced (which is the same as the quantity of the intermediate good bought) by the independent firm.

At the analytical first stage, profit maximization by the MNF’s downstream division and by the independent firm yields the following first-order conditions:

$$p_2 + xp_2' - m - w_2 = 0$$

(3)

$$p_1 + yp_1' - m^* - w_1 = 0$$

(4)

which implicitly give the optimal quantities of the final good as functions of the prices of the intermediate good:

$$x(m), \ y(m^*)$$

(5)

Total differentiation of equations (3) and (4) yields

$$x_m = \frac{\partial x}{\partial m} < 0, \ y_m = \frac{\partial y}{\partial m} < 0$$

(6)

This is the familiar result that an increase in input prices leads to a fall in the quantities of the final good.

At the analytical second stage, the MNF’s headquarters chooses the internal transfer price ($m$) and external market price for the intermediate good ($m^*$) to maximize the MNF’s profit as a whole, which is

$$\pi = \pi_u + \pi_d$$

(7)

where $\pi_u$ is the profit of the upstream division, and

$$\pi_u = (1 - t_j)[mx + m^*y - c(x + y)]$$

(8)

where $c(\cdot)$ is the cost function, with $c' > 0$ and $c'' \geq 0$. 
Substituting solution (5) into equation (7) and maximizing equation (7) with respect to \( m \) and \( m^* \), we obtain the following first order conditions

\[
(1-t_1)[x + (m - c_x)x_m] + (1-t_2)(-x) = 0 \quad (9)
\]

\[
y + (m^* - c_y)y_m^* = 0 \quad (10)
\]

Equations (9) and (10) implicitly determine the MNF’s optimal internal transfer price (\( m \)) and external price for the intermediate good (\( m^* \)).

If we know the cost function and inverse demand function, we can obtain explicit solutions for the prices, quantities and profits in our model. As an illustration, we solve the system of equations assuming linear cost and demand functions:

\[
c(x) = cx, \quad p(x) = a - bx, \quad a > 0, b > 0, c > 0
\]

The solutions are presented in Table 1 and Table 2. In Table 1 and Table 2, subscript \( i \) (\( i=1, 2, 3 \)) on prices, quantities and profits denote these values in case \( i \).

[Insert Table 1 and Table 2 here]

From Table 2, it is clear that if \( t_1 = t_2 \) and \( a - w > c \) (which is required for positive output), then \( m_1 = c \) and \( m^*_1 > c = m_1^* \). That is, in the absence of profit shifting incentives, the MNF will set its internal transfer price at marginal cost which is lower than the arm’s length price.

**Case 2: MNF constrained by the arm’s length principle**

If the MNF is constrained by the arm’s principle, that is, it has to set \( m = m^* \), then the firms’ decisions will be the same as in Case 1 except that the headquarters chooses one price (instead of two prices) for both internal and external sales of the intermediate good, and accordingly the upstream division’s profit function (equation (8) above) becomes

\[
\pi_u = (1-t_1)[(m - c)(x + y)]
\]

(8’)

Assuming the same linear demand and cost functions as in Case 1, we can solve for the optimal prices, quantities and profits. The solutions are presented in Table 1 and Table 2. Notably, if \( t = t^* \), and \( a - w > c \), we have \( m > c \). In other words, in the absence of profit shifting incentives, the MNF constrained by the arm’s length principle will set both the
internal transfer price and the external price at a level higher than marginal cost. As a result, the MNF’s downstream output will be lower \((x_2 < x_1)\) and so will be its profit \((\pi_2 < \pi_1)\).

**Case 3: MNF keeping two books in response to the arm’s length principle requirement**

The comparison between case 1 and case 2 shows that using the arm’s length principle in its internal pricing creates some distortions in the MNF’s decisions, which lowers its profit. The MNF therefore has an incentive to mitigate such distortions. One way of doing that is to keep two sets of books, one for internal management and another for tax reporting. This means that the MNF will choose an internal transfer price \((m)\) and a different external price \((m^*)\), but it will report \(m^*\) as the internal transfer price to the tax authorities. Accordingly, the MNF’s downstream division’s after-tax profit function changes from equation (1) to:

\[
\pi_d = (p_2 - m - w_2)x - t_2(p_2 - m^* - w_2)x
\]  
\[ (1') \]

The upstream branch’s after-tax profit function changes from equation (8) to:

\[
\pi_u = [mx + m^*y - c(x + y)] - t_i[(m^* - c)(x + y)]
\]  
\[ (8'') \]

Following the same procedure as in case 1 and case 2, we solve the firms’ decision problems and present the solutions in Table 1 and Table 2.

The results in Table 2 show that if \(t = t^*, a - w_2 > c\), then \(c < m < m^*\), which means that the MNF’s transfer price will be higher than marginal cost but lower than the arm’s length price. Moreover, it is easy to see that \(x_1 > x_3 > x_2\) and \(\pi_1 > \pi_3 > \pi_2\), that is, relative to case 2 where the MNF follows the arm’s length principle, the MNF can increase its final good output and its profit by keeping two books, one for internal management and another for tax reporting. However keeping two books does not restore the MNF’s profit to the case where it is not constrained by the arm’s length principle (case 1).

Summarizing the results in this section, we have

**Proposition 1:** In Structure One (where the MNF is a monopoly in the intermediate good market and does not compete with its customer in the final good market), the MNF will, in the absence of profit shifting incentives, set its internal transfer price at a lower level than the arm’s length price. Compelling the MNF to following the arm’s length principle will create a distortion in the MNF’s decisions which leads to lower profits. The MNF can mitigate, but not eliminate the distortion by keeping two books, one for internal management, and another for tax reporting purposes.
3. Structure Two: Cournot Duopoly Competition in Downstream Markets

This section looks at a different market structure (Structure Two) where duopoly competition is introduced in the downstream market. In Structure Two, the MNF’s upstream division located in country 1 supplies the intermediate good to its own downstream division and an independent firm. Since both the MNF’s downstream division and the independent downstream firm are located in country 2, they compete with each other in the final good market. We capture this competition with the well-known Cournot model. As in section 2, we consider the firms’ decisions in three different cases.

Case 1: MNF not constrained by the arm’s length principle

Consider the downstream market first. The MNF’s downstream division’s after-tax profit function can be written as

$$\pi_d = (1-t_2)(p_2 - m - w_2)x$$

(11)

where $p_2 = p_2(x + y)$ is the inverse demand function for the final good sold in country 2.

The independent firm has a similar after-tax profit function

$$\pi_j = (1-t_2)(p_2 - m^* - w_2)y$$

(12)

The MNF’s downstream division and the independent firm choose output quantities simultaneously to maximize their respective profits, yielding the following first-order conditions:

$$p_2 + xp'_2 - m - w_2 = 0$$

(13)

$$p_2 + yp'_2 - m^* - w_2 = 0$$

(14)

Solving equations (13) and (14) simultaneously gives the optimal quantities of the final goods produced by the MNF’s downstream division and the independent firm, respectively

$$x(m, m^*) , y(m, m^*)$$

(15)

Total differentiation of equations (13) and (14) yields

$$x_m = \frac{\partial x}{\partial m} < 0, x_m = \frac{\partial x}{\partial m} > 0, y_m = \frac{\partial y}{\partial m} > 0, y_m = \frac{\partial y}{\partial m} < 0$$

(16)

These suggest that an increase in the internal transfer price ($m$) reduces the MNF’s downstream division’s output and increases the output of the independent firm. Conversely,
an increase in the arm’s length price \((m^*)\) reduces the independent firm’s output and increases the output of the MNF’s downstream division.

In the upstream market, the MNF’s upstream division is the monopoly supplier of the intermediate good. Its after-tax profit function is

\[
\pi_u = (1-t_1)[mx + m^*y - c(x + y)]
\]  

(17)

where \(c(\cdot)\) is the cost function, with \(c' > 0\) and \(c'' \geq 0\).

The MNF headquarters chooses the internal transfer price and the external price to maximize the profit of the MNF as a whole

\[
\pi = \pi_u + \pi_d
\]  

(18)

Substituting solutions (15) into equation (18) and maximizing equation (18) with respect to \(m\) and \(m^*\), we obtain the following first-order conditions

\[
(1-t_1)[x + (m - c_x)x_m + (m^* - c_y)y_m^*] + (1-t_2)[x(p_y^t y_m - 1)] = 0
\]  

(19)

\[
(1-t_1)[y + (m - c_x)x_m + (m^* - c_y)y_m^*] + (1-t_2)xp_y^t y_m^* = 0
\]  

(20)

Conditions (19) and (20) implicitly determine the MNF’s optimal internal transfer price and the external price it charges the independent firm.

As an illustration, we assume the following linear cost and demand functions to solve for the prices, quantities and profits in the above model.

\[
c(x) = cx, \quad p_y(x) = ax - bx, \quad a > 0, b > 0, c > 0.
\]

The solutions are presented in Table 3 and Table 4.

[Insert Table 3 and Table 4 here]

From Table 4, we see that if \(t_1 = t_2\) and \(a - w > c\), then \(m = c < m^*\). That is, in the absence of profit shifting incentives, the MNF will set its internal transfer price at marginal cost, and charge the independent firm a higher price. However, in this case \(m^*\) is only a “shadow” arm’s length price because at \(t_1 = t_2\), \(y(m, m^*) = 0\). In other words, if country 1 has the same tax rate as country 2, the MNF will foreclose the independent downstream firm and become a
downstream monopoly\(^4\). Foreclosure does not occur (i.e., \(y(m, m^*) > 0\)) if and only if \(a - w_2 > c\) and \(t_1 < t_2\), or \(a - w_2 > c\) and \(t_2 > (1 + 2t_2)/3\).

**Case 2: MNF constrained by the arm’s length principle**

If the MNF has to follow the arm’s length principle and charge the same price internally and externally for the intermediate good, then its upstream division’s profit function (equation (17)) becomes \(\pi_u = (1 - t_1)[(m - c)(x + y)]\) and the headquarters choose only one price (m) for the intermediate good. The model can be solved in the same way as in case 1 above. The solutions are presented in Table 3 and Table 4.

The result in Table 4 shows that if \(t_1 = t_2\), \(a - w > c\), we have \(m = (a + 6c - w)/7 > c\). In other words, in the absence of profit shifting incentives, the MNF will set the price for intermediate good at a level higher than marginal cost. Interestingly, when \(t_1 = t_2\), the MNF does not foreclose the independent firm as it occurs in case 1. The condition for non-foreclosure is \(a - w > c\) and \(t_2 > (9t_1 - 7)/2\). However, the MNF’s profit is lower (\(\pi_2 < \pi_1\)).

**Case 3: MNF keeping two books in response to the arm’s length principle requirement**

If the MNF keeps two books to circumvent the arm’s length principle requirement, its downstream division’s after-tax profit function changes from equation (11) to:

\[
\pi_d = (p_2 - m - w_2)x - t_1(p_2 - m^* - w_2)x \quad (11')
\]

The upstream branch’s after-tax profit function changes from equation (17) to:

\[
\pi_u = [mx + m^*y - c(x + y)] - t_1[(m^* - c)(x + y)] \quad (17')
\]

Following the same procedure as in case 1 and case 2, we solve the firms’ decision problems and present the solutions in Table 3 and Table 4. The results in Table 4 show if that \(t_1 = t_2\) and \(a - w > c\), then \(y_3 = 0\) and \(\pi_3 = \pi_1 > \pi_2\). These mean that the MNF will revert to foreclosing the independent firm and earn the same profit as in case 1 where it is not constrained by the arm’s length principle.

Summarizing the results from the 3 cases, we have:

**Proposition 2:** In Structure Two (where the MNF is a monopoly in the intermediate good market and competes with its customer in the final good market), absent profit shifting

\(^4\) This result is the same as that in Zhao (2000).
incentives the MNF will foreclose its competitor in the final good market. Compelling the MNF to following the arm’s length principle will induce the MNF to supply its competitor and earn a lower profit. However, the MNF can revert to foreclosing its competitor by keeping two books, one for internal management, and another for tax reporting purposes.

The above result shows that if \( t_1 = t_2 \), the arm’s length principle can induce the MNF not to foreclose its competitor but the MNF can circumvent the principle by keeping two books. We next investigate the impact of the arm’s length principle on the MNF’s foreclosure decision when \( t_1 \neq t_2 \). From Table 4, we can derive the conditions for non-closure for the three cases (i.e., \( y_i > 0, i = 1, 2, 3 \)) given that that the MNF produces the final good (i.e., \( x_i > 0, i = 1, 2, 3 \)).

1. In case 1, \( x_1 > 0 \) requires that \( a - w_2 > c \) and \( t_1 < (1 + 2t_2)/3 \) or \( a - w_2 < c \) and \( t_1 < (1 + 2t_2)/3 \). Under either set of conditions, \( y_1 > 0 \) iff \( t_1 < t_2 \).

2. In case 2, \( x_2 > 0 \) requires that \( a - w_2 > c \) and \( t_2 > (9t_1 - 7)/2 \) or \( a - w_2 < c \) and \( t_2 < (9t_1 - 7)/2 \). Under either set of conditions, \( y_2 = x_2 > 0 \).

3. In case 3, \( x_3 > 0 \) requires that \( a - w_2 > c \) and \( 4(1-t_1)(1-t_2) > (t_1 - t_2)^2 \) or \( a - w_2 < c \) and \( 4(1-t_1)(1-t_2) < (t_1 - t_2)^2 \). Under either set of conditions, \( y_3 > 0 \) iff \( t_1 > t_2 \).

Note that MNF’s foreclosure decision when its unconstrained (case 1) is exactly opposite to that if it is constrained (case 3). If the MNF is unconstrained by the arm’s length principle, it will choose not to foreclose if upstream tax rate is lower. On the contrary, if it is constrained, and chooses to keep two books, it will foreclose if upstream tax is lower.

Suppose that the MNF faces a lower tax rate upstream \((t_1 < t_2)\). Starting with case 1 where the MNF is unconstrained, the MNF will not foreclose its rival. This is because when the upstream tax rate is lower, the MNF has an incentive charge a high internal transfer price for tax purposes. To encourage purchase at the high internal price, the MNF finds it in its interests to have more competition downstream by not foreclosing its rival. If the MNF is constrained to follow the arm’s length principle, it has an incentive to keep two books. This is because when the MNF keeps two books, it still has the option of charging the same price internal and externally, thus logically keeping two books should lead to no lower profits for the MNF as compared to the case where it is constrained by the arm’s length principle (i.e.,
However when \( t_1 < t_2 \), keeping two sets of books will induce the MNF to foreclose its rival. Thus starting from non-foreclosure with no arm’s length constraint, imposing the arm’s length principle may provide incentives for the MNF to keep two books and also reverse the foreclosure decision.

If the MNF faces a higher tax rate upstream (\( t_1 > t_2 \)), then it has an incentive to charge lower internal price. And to discourage excessive purchase at the lower price, it will choose to foreclose its rival. Imposing the arm’s length principle may induce the firm to keep two books, and also choose to supply its rival.

The above analysis can be summarized as

**Proposition 3:** In Structure Two (where the MNF is a monopoly in the intermediate good market and competes with its customer in the final good market), when the tax rate is lower upstream (i.e., \( t_1 < t_2 \)), the MNF will not foreclose its competitor if it is not constrained by the arm’s length principle. Compelling the MNF to follow the arm’s principle may induce the MNF to keep two books and foreclose its competitor. Conversely, when the tax rate is higher upstream, the MNF will foreclose its competitor if it is not constrained by the arm’s length principle. The arm’s length principle will induce the MNF to keep two books and supply its competitor.

4. Conclusion

In this paper we have investigated transfer pricing in situations where the markets for both the intermediate good and the final good are not perfectly competitive. Our main findings are as follows.

First, in the absence of profit shifting incentives, an MNF’s optimal transfer price does not coincide with the arm’s length price. If the MNF is compelled to follow the arm’s length principle, its profit will be reduced, and the MNF has an incentive to circumvent the arm’s length principle by keeping two sets of books, one for internal management, and another for tax reporting purposes. The practice of keeping two books can reduce the profit loss in

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5 This result is not so straightforward by comparing \( \pi_2 \) and \( \pi_3 \) in Table 3 as the solutions are very complex. To illustrate this point, we run some simulations to compare \( \pi_2 \) and \( \pi_3 \). The results are presented in Table 5. It is clear from Table 5 that \( \pi_3 \geq \pi_2 \) in all simulations.
Structure One where the MNF is a monopoly in the final good market, and avoid the profit loss in Structure Two where the MNF has a rival in the final good market.

Second, the arm’s length principle can affect the MNF’s decision on whether or not to foreclose its competitor. If the MNF faces the same tax rates upstream and downstream, it forecloses its competitor when it is unconstrained by the arm’s length principle. Imposing the arm’s length principle can induce the MNF to supply its competitors. However the MNF can choose to keep two books and revert to the foreclosure decision. If the MNF faces different tax rates upstream and downstream, then imposing the arm’s length constraint will reverse the MNF’s foreclosure decision.

Our findings suggest that where there is imperfect competition, arm’s length transfer pricing is not a “neutral” tax policy because it leads to deviations from the firms’ optimal decisions in the absence of profit shifting incentives, with the consequence of lower profits. Naturally the firms have an incentive to mitigate the adverse effects of the tax policy; keeping two books is one way of doing it. Apart from affecting the firms’ marginal decisions (on output and price), the arm’s length principle can alter the firms’ infra-marginal decisions on foreclosure.

While the arm’s length principle may be a good description of competitive market practices, this paper has shown that its enforcement may have complex ramifications in an imperfectly competitive market setting. These need to be taken into account by policy makers. To better inform policy making, future research may investigate more complex market structures. For instance, oligopolistic competition may be introduced in the upstream market as well as the downstream market.
Figure 1: Two Structures

Structure One

Structure Two
Table 1. Structure One: Prices, Quantities and Profits

<table>
<thead>
<tr>
<th>Case</th>
<th>MNF not constrained by the arm’s length principle</th>
<th>MNF constrained by the arm’s length principle</th>
<th>MNF keeping two books in response to the arm’s length principle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>$m_1 = \frac{(a-w_2)(t_2-t_1)+(1-t_2)c}{2(1-t_1)-(1-t_2)}$, $m_1^* = \frac{a+c-w_1}{2}$</td>
<td>$x_1 = \frac{(1-t_1)(a-c-w_2)}{2b[2(1-t_1)-(1-t_2)]}$, $y_1 = \frac{a-c-w_1}{4b}$</td>
<td>$\pi_1 = \frac{(1-t_1)^2(a-c-w_2)^2}{4b[2(1-t_1)-(1-t_2)]} + \frac{(1-t_1)(a-c-w_1)^2}{8b}$, $\pi_{11} = \frac{(1-t_1)(a-c-w_1)^2}{16b}$</td>
</tr>
<tr>
<td>Case 2</td>
<td>$m_2 = \frac{(1-t_1)(2a-w_2-w_1+2c)-(1-t_2)(a-w_2)}{4(1-t_1)-(1-t_2)}$</td>
<td>$x_2 = \frac{(1-t_1)(a-c-w_2)}{b[4(1-t_1)-(1-t_2)]}$, $y_2 = \frac{(1-t_1)(a-c-w_2)}{b[4(1-t_1)-(1-t_2)]}$</td>
<td>$\pi_2 = \frac{2(1-t_1)^2(a-c-w_2)^2}{b[4(1-t_1)-(1-t_2)]^2}$, $\pi_{12} = \frac{(1-t_1)^2(a-c-w_2)^2}{b[4(1-t_1)-(1-t_2)]^2}$</td>
</tr>
<tr>
<td>Case 3</td>
<td>$m_3 = \frac{t_1(1-t_1)(1-t_2)(a-w_1)+t_1(1-t_2)(a-w_2)+(1-t_1)(2-t_1-2t_2+2t_2t_2-t_2^2)c}{2(1-t_1)(1-t_2)-(t_1-t_2)^2}$</td>
<td>$m_3^* = \frac{(1-t_1)(1-t_2)(a-w_1)−(t_1-t_2)(1-t_2)(a-w_2)+(1-t_1)(1-t_1-2t_2)c}{2(1-t_1)(1-t_2)-(t_1-t_2)^2}$</td>
<td>$x_3 = \frac{(1-t_1)(2-t_1-t_2)(a-c-w_2)}{2b[2(1-t_1)(1-t_2)-(t_1-t_2)^2]}$, $y_3 = \frac{(1-t_1)(1+t_1-2t_2)(a-c-w_2)}{2b[2(1-t_1)(1-t_2)-(t_1-t_2)^2]}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\pi_3 = \frac{(1-t_1)^2<a href="a-c-w_2">5(1-t_1)(1-t_2)^2-2(1-t_2)(t_1-t_2)(1+t_1-2t_2)</a>^2}{4b[2(1-t_1)(1-t_2)-(t_1-t_2)^2]^3}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\pi_{13} = \frac{(1-t_1)^2(1+t_1-2t_2)^2(a-c-w_2)^2}{4b[2(1-t_1)(1-t_2)-(t_1-t_2)^2]^3}$</td>
</tr>
</tbody>
</table>
Table 2. Structure One: Prices, Quantities and Profits if $t_1 = t_2 = t$ and $w_1 = w_2 = w$

<table>
<thead>
<tr>
<th>Case</th>
<th>MNF not constrained by the arm’s length principle</th>
<th>Case</th>
<th>MNF constrained by the arm’s length principle</th>
<th>Case</th>
<th>MNF keeping two books in response to the arm’s length principle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$m_1 = c, m_1^* = \frac{a+c-w}{2}$</td>
<td></td>
<td>$m_2 = \frac{2(a-w+c)}{3}$</td>
<td></td>
<td>$m_3 = \frac{t(a-w) + (2-t)c}{2}, m_3^* = \frac{a+w+c}{2}$</td>
</tr>
<tr>
<td></td>
<td>$x_1 = \frac{a-c-w}{2b}, y_1 = \frac{a-c-w}{4b}$</td>
<td></td>
<td>$x_2 = \frac{a-c-w}{3b}, y_2 = \frac{a-c-w}{3b}$</td>
<td></td>
<td>$x_3 = \frac{a-c-w}{2b}, y_3 = \frac{a-c-w}{2b}$</td>
</tr>
<tr>
<td></td>
<td>$\pi_1 = \frac{3(1-t)(a-c-w)^2}{8b}, \pi_{11} = \frac{(1-t)(a-c-w)^2}{16b}$</td>
<td></td>
<td>$\pi_2 = \frac{2(1-t)(a-c-w)^2}{9b}, \pi_{12} = \frac{(1-t)(a-c-w)^2}{9b}$</td>
<td></td>
<td>$\pi_3 = \frac{5(1-t)(a-c-w)^2}{16b}, \pi_{13} = \frac{(1-t)(a-c-w)^2}{16b}$</td>
</tr>
</tbody>
</table>
Table 3. Structure Two: Prices, Quantities and Profits

<table>
<thead>
<tr>
<th>Case 1: MNF not constrained by the arm’s length principle</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1 = \frac{3(t_2 - t_1)(a - w_3) + (2 + t_2 - 3t_1)c}{6(1-t_1) - 4(1-t_2)}$, $m_1^* = \frac{a + c - w_3}{2}$</td>
<td></td>
</tr>
<tr>
<td>$x_1 = \frac{(1-t_1)(a - c - w_2)}{b[6(1-t_1) - 4(1-t_2)]}$, $y_1 = \frac{(t_2 - t_1)(a - c - w_2)}{b[6(1-t_1) - 4(1-t_2)]}$</td>
<td></td>
</tr>
<tr>
<td>$\pi_1 = \frac{[12(1-t_1)^2(t_2 - t_1) + 2(1-t_2)(1-t_1)(1-2t_2 + t_1)](a - c - w_2)^2}{2b[6(1-t_1) - 4(1-t_2)]^2}$</td>
<td></td>
</tr>
<tr>
<td>$\pi_{11} = \frac{(1-t_2)(t_2 - t_1)^2(a - c - w_2)^2}{b[6(1-t_1) - 4(1-t_2)]^2}$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 2: MNF constrained by the arm’s length principle</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_2 = \frac{3(1-t_1)(a + 2c - w_3) - 2(1-t_2)(a - w_3)}{9(1-t_1) - 2(1-t_2)}$</td>
<td></td>
</tr>
<tr>
<td>$x_2 = \frac{2(1-t_1)(a - c - w_2)}{b[9(1-t_1) - 2(1-t_2)]}$, $y_2 = \frac{2(1-t_2)(a - c - w_2)}{b[9(1-t_1) - 2(1-t_2)]}$</td>
<td></td>
</tr>
<tr>
<td>$\pi_2 = \frac{4(1-t_1)^2[3(1-t_1) - (1-t_2)](a - c - w_2)^2}{b[9(1-t_1) - 2(1-t_2)]^2}$</td>
<td></td>
</tr>
<tr>
<td>$\pi_{12} = \frac{4(1-t_2)(1-t_1)^2(a - c - w_2)^2}{b[9(1-t_1) - 2(1-t_2)]^2}$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 3: MNF keeping two books in response to the arm’s length principle requirement</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_3 = \frac{(1-t_2)(t_1 + t_2 - 2t_1^2)(a - w_3) + (1-t_1)(4-t_1 - 5t_2 + 2t_2^2)c}{4(1-t_1)(1-t_2) - (t_1 - t_2)^2}$</td>
<td></td>
</tr>
<tr>
<td>$m_3^* = \frac{(1-t_2)(2 + t_2 - 3t_1)(a - w_3) + (1-t_1)(2 - 3t_2 + t_1)c}{4(1-t_1)(1-t_2) - (t_1 - t_2)^2}$</td>
<td></td>
</tr>
<tr>
<td>$x_3 = \frac{(1-t_1)(2 - t_1 - t_2)(a - c - w_2)}{b[4(1-t_1)(1-t_2) - (t_1 - t_2)^2]}$, $y_3 = \frac{(1-t_1)(t_1 - t_2)(a - c - w_2)}{b[4(1-t_1)(1-t_2) - (t_1 - t_2)^2]}$</td>
<td></td>
</tr>
<tr>
<td>$\pi_3 = \frac{(1-t_2)(1-t_1)^2[4(1-t_1)(1-t_2) - (t_1 - t_2)^2](a - c - w_2)^2}{b[4(1-t_1)(1-t_2) - (t_1 - t_2)^2]^2}$</td>
<td></td>
</tr>
<tr>
<td>$\pi_{13} = \frac{(1-t_2)(1-t_1)^2(t_2 - t_1)^2(a - c - w_2)^2}{b[4(1-t_1)(1-t_2) - (t_1 - t_2)^2]^2}$</td>
<td></td>
</tr>
</tbody>
</table>
Table 4. Structure Two: Prices, Quantities and Profits if \( t_1 = t_2 = t \) and \( w_1 = w_2 = w \)

<table>
<thead>
<tr>
<th>Case 1: MNF not constrained by the arm’s length principle</th>
<th>( m_1 = c ), ( m^*_1 = \frac{a + c - w}{2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 = \frac{a - c - w}{2b} ), ( y_1 = 0 )</td>
<td>( \pi_1 = \frac{(1-t)(a-c-w)^2}{4b} ), ( \pi_{11} = 0 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 2: MNF constrained by the arm’s length principle</th>
<th>( m_2 = \frac{a - w + 6c}{7} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_2 = \frac{2(a-c-w)}{7b} ), ( y_2 = \frac{2(a-c-w)}{7b} )</td>
<td>( \pi_2 = \frac{8(1-t)(a-c-w)^2}{49b} ), ( \pi_{12} = \frac{4(1-t)(a-c-w)^2}{49b} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 3: MNF keeping two books in response to the arm’s length principle requirement</th>
<th>( m_3 = \frac{t(a-w) + (2-t)c}{2} ), ( m^*_3 = \frac{a + c - w}{2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_3 = \frac{a - c - w}{2b} ), ( y_3 = 0 )</td>
<td>( \pi_3 = \frac{(1-t)(a-c-w)^2}{4b} ), ( \pi_{13} = 0 )</td>
</tr>
</tbody>
</table>
Table 5. Simulation Results

<table>
<thead>
<tr>
<th>Tax rates</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1 = 0.3$, $t_2 = 0.2$</td>
<td>$\pi_2 = 0.115B$, $\pi_3 = 0.392B$,</td>
</tr>
<tr>
<td>$t_1 = 0.3$, $t_2 = 0.22$</td>
<td>$\pi_2 = 0.115B$, $\pi_3 = 0.382B$</td>
</tr>
<tr>
<td>$t_1 = 0.3$, $t_2 = 0.24$</td>
<td>$\pi_2 = 0.115B$, $\pi_3 = 0.372B$</td>
</tr>
<tr>
<td>$t_1 = 0.28$, $t_2 = 0.24$</td>
<td>$\pi_2 = 0.118B$, $\pi_3 = 0.394B$</td>
</tr>
<tr>
<td>$t_1 = 0.26$, $t_2 = 0.24$</td>
<td>$\pi_2 = 0.121B$, $\pi_3 = 0.416B$</td>
</tr>
<tr>
<td>$t_1 = 0.32$, $t_2 = 0.24$</td>
<td>$\pi_2 = 0.112B$, $\pi_3 = 0.351B$</td>
</tr>
<tr>
<td>$t_1 = 0.34$, $t_2 = 0.24$</td>
<td>$\pi_2 = 0.109B$, $\pi_3 = 0.331B$</td>
</tr>
<tr>
<td>$t_1 = 0.2$, $t_2 = 0.3$</td>
<td>$\pi_2 = 0.1294B$, $\pi_3 = 0.1778B$</td>
</tr>
<tr>
<td>$t_1 = 0.22$, $t_2 = 0.3$</td>
<td>$\pi_2 = 0.1264B$, $\pi_3 = 0.1769B$</td>
</tr>
<tr>
<td>$t_1 = 0.24$, $t_2 = 0.3$</td>
<td>$\pi_2 = 0.1234B$, $\pi_3 = 0.1761B$</td>
</tr>
<tr>
<td>$t_1 = 0.24$, $t_2 = 0.28$</td>
<td>$\pi_2 = 0.1236B$, $\pi_3 = 0.1805B$</td>
</tr>
<tr>
<td>$t_1 = 0.24$, $t_2 = 0.26$</td>
<td>$\pi_2 = 0.1238B$, $\pi_3 = 0.1851B$</td>
</tr>
<tr>
<td>$t_1 = 0.24$, $t_2 = 0.32$</td>
<td>$\pi_2 = 0.1231B$, $\pi_3 = 0.1719B$</td>
</tr>
<tr>
<td>$t_1 = 0.24$, $t_2 = 0.34$</td>
<td>$\pi_2 = 0.1228B$, $\pi_3 = 0.1679B$</td>
</tr>
</tbody>
</table>

Where $B = \frac{(a-c-w_2)^2}{b}$
References


