Information Transmission through Influence Activities

Chongwoo Choe† and In-Uck Park

Abstract
This paper studies the information transmission aspect of influence activities within an organization where privately informed division managers strategically communicate divisional information to headquarters to influence its capital allocation decisions. Although costly, influence activities can play a role in transmitting valuable information to headquarters. We define influence activities to be informative if they improve headquarters’s inference and detrimental if they hamper it. We find that influence activities are more likely to be informative in organizations that are less averse to risk taking, that rely more on higher-power incentives, and that encourage competition in the form of contest. We also find that competition over scarce resources increases the overall level of influence activities.

Keywords: Influence activities, information transmission

JEL Classification Number: C72, D23, D82, L22

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‡ The corresponding author. Department of Economics, Monash University, PO Box 197, Caulfield East, VIC 3145, Australia. (Email) chongwoo.choe@monash.edu.

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1 Introduction

Influence activities in an organization are those directed to influence decision making within the organization to the benefit of the party that engages in influence activities. Milgrom and Roberts (1988) highlight informational asymmetries as central to influence activities: the informed party optimally chooses influence activities in an attempt to influence the uninformed decision maker. The costs of influence activities include the resources that are devoted to affecting the distribution of benefits rather than to creating value, the value that is lost when influence results in suboptimal decisions, and the degradation in organizational performance that comes from altering policies, decision processes, or organizational structure to limit influence activities or their effects (Meyer, Milgrom and Roberts, 1992). On the other hand, influence activities can bring benefits in the form of information transmission that can improve decision making. Limiting influence activities could reduce influence costs but also stifle information transmission at the cost that valuable information is not made available to support decision making (Milgrom and Roberts, 1988, p S157).

Existing studies on influence activities either do not focus on the information transmission aspect or use the standard signal jamming approach where there is no sense in which valuable information is transmitted. Thus there is a gap between the notion of influence activities originally put forward by Milgrom and Roberts (1988) and the literature that was developed subsequently. The purpose of our paper is to fill this gap. The basic feature of our model is noisy communication between a privately informed party and an uninformed party. The informed party manipulates private information at some cost, which is called an influence activity, before communicating it to the uninformed party. The uninformed party receives information with a further noise, makes inference on the informed party’s private information, and takes an action that affects both parties’ payoffs. In anticipation of the uninformed party’s action choice, the informed party chooses the optimal level of influence activity.

To fix ideas, we consider the capital allocation problem in a firm that consists of two parties, an informed division manager and uninformed headquarters. The return to capital depends on divisional state, which is either ‘good’ or ‘bad’. The manager privately observes his divisional state although headquarters observes only a noisy signal of the state, based on which to determine capital allocation. The manager’s utility increases in the amount of capital allocated to his division and, therefore, he has incentives to distort the signal observed by headquarters. Such efforts, referred to as influence activities,
result in a parallel shift of the distribution of the signal observed by headquarters. In inferring the underlying state from the observed signal, headquarters takes into account the equilibrium level of influence activities that may differ depending on the divisional state. We define influence activities to be informative if they improve headquarters’s inference on the underlying state, neutral if they do not affect headquarters’s inference, and detrimental if they hamper headquarters’s inference.

We investigate how equilibrium influence activities hinge on the underlying environment, in particular, on the ways in which the manager is motivated, on the properties of the firm’s overall performance indicator, and on different types of competition between divisions. Our basic model abstracts away competition and considers a single division where the manager is motivated by private benefits proportional to the amount of allocated capital. We show that the effect of influence activities differs depending on the properties of the firm’s performance indicator, and provide sufficient conditions under which they are informative, neutral, or detrimental. In a nutshell, equilibrium influence activities are informative when marginal capital allocation is larger when the inferred state is good, and detrimental otherwise. We interpret the former to be more likely when the firm is less averse to risk taking as reflected in its performance indicator, and the latter more likely when the firm is more conservative in its capital allocation.

In the second environment, we continue to abstract away competition but assume that the manager is motivated by explicit compensation proportional to the realized output. It is shown that such explicit incentives induce informative influence activities for a larger set of performance indicators than when the manager is motivated through implicit private benefits. This is due to our (natural) assumption that marginal return to capital is higher in a good state, which does not affect the manager’s motivation in our basic model. Therefore explicit incentives tend to enlarge the gap in the level of influence activities between the two states, enhancing the information content of the signals received by headquarters. This implies that influence activities tend to be more informative in an organization that relies on higher-powered incentives than otherwise identical organizations.

Lastly, we extend our basic model to study the effect of competition on influence activities. We consider two types of competition. First, we consider a firm with two divisions where headquarters runs a contest between them in which only the division with a higher value of realized signal is allocated capital. We show that such competition induces informative influence activities for a larger set of performance indicators than in the absence of competition. It is because influence activities lead to a larger increase in the expected capital allocation conditional on winning the contest when the division is in a good state than when it is in a bad state. Second, we consider the case where multiple divisions compete for a limited amount of capital that can be allocated across
divisions based on the signals received by headquarters. We show that overall influence activities increase as competition toughens in the sense that the number of divisions increases. It is because competition imposes constraints only when the available resources are insufficient to optimally fund all divisions independently. Since the optimal capital allocation equates the marginal returns to capital across divisions, competition under the resource constraint enhances the manager’s marginal benefit from extra influence activity.

The remainder of this paper is organized as follows. Section 2 provides a review of related studies. Section 3 describes a baseline model in which a single manager is motivated by implicit private benefits proportional to the size of allocated capital, which is analyzed in Section 4. Section 5 extends our baseline model to the three environments described above. Section 6 contains some concluding remarks and the appendix contains all the proofs.

2 Related Literature

There are several ways the existing literature models influence activities in the firm. First, Milgrom (1988) studies influence activities in the firm’s employment decisions although information transmission is not the main focus. In his model, an employee can spend time either on productive activity or on influence activity where the latter affects the probability that management’s discretion will lead to a transfer of rent to the employee. Second, several studies model influence activities in the form of rent seeking (Bagwell and Zechner, 1993; Edlin and Stiglitz, 1995; Scharfstein and Stein, 2000).

Influence activities in these models are interpreted as an entrenchment strategy or an attempt to raise bargaining power. Once again, information transmission is not the main focus in these studies. One could argue that these approaches are not true to the original spirit of Milgrom and Roberts (1988) in that they ignore the potential benefit from influence activities in the form of information transmission.

Influence over information received by headquarters is central in Milgrom and Roberts (1988), Meyer, Milgrom and Roberts (1992), and Wulf (2002). Information in the first two studies is for the firm’s employment decisions while, in the third study, it is used for capital allocation within the firm. These authors use the modelling approach à la Fudenberg and Tirole (1986) and Holmström (1999), which we call the standard signal jamming model. The main difference between our model and the standard signal jamming model is that, in the latter, signal jamming is typically under symmetric information. As such, it results only in neutral influence activities in our language since it

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3 Although rent seeking has the same objective as influence activities of affecting decision making within the organization, a crucial difference is that influence activities can transmit valuable information. Rent seeking, on the other hand, serves no function other than to transfer rents (e.g., Krueger, 1974).
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does not transmit valuable information: the receiver of information can perfectly back out any influence in equilibrium. As noted above, influence activities can be useful given the information asymmetry that pervades the organization. In our model, the manager engages in influence activities after observing the divisional state. Thus, unlike in the standard signal jamming model, some influence can transmit valuable information that can improve headquarters’s inference.

Influence over information transmission is also central in the accounting literature on disclosure in general, and earnings management in particular. In this literature, the firm’s manager issues a report to the market either before or after receiving a private signal about firm value. The market updates its belief on firm value, which is impounded on the firm’s stock price, which in turn affects the manager’s payoff. Stein (1989) uses the standard signal jamming approach in that the earnings report is issued before the manager receives the signal. Therefore earnings management does not change the market’s posterior. Others typically use the model of insider trading as in Kyle (1985). For example, Fischer and Verrecchia (2000), and Fischer and Stocken (2004) study earnings management after the manager receives the private signal. But they assume all relevant random variables are independent and normally distributed, and focus on linear reporting strategies. As a result, the equilibrium stock price is linear in report, which makes the equilibrium report independent of the private signal. Therefore, earnings management in these models does not have additional bite compared to the standard signal jamming model. In addition, the manager in these models issues a report after random noise is realized, hence there is little sense in which the manager’s report conveys private information. In our model, the manager does so before the realization of random noise, which makes it clear when the manager’s influence can be informative or detrimental.

Finally, our model is different from standard signalling models since headquarters observes only a noisy signal of the underlying state. It also departs from the cheap-talk literature since there are exogenous costs to influence activities. Moreover, the signal received by headquarters has an additional noise with full support, which renders even partial sorting impossible.

3 The Baseline Model

The firm comprises two parties: headquarters (HQ) and a division manager. Our focus is on how HQ allocates capital to the division based on information that can be influenced by the division manager. The output from the division, denoted by \( g(k, \theta) \), is determined by the amount of capital \( k \) allocated to the division and the state of the division \( \theta \). We treat \( \theta \) as the realized value of a random variable \( \tilde{\theta} \) which has a binary support.

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4See Verrecchia (2001) for a comprehensive survey.
\{\theta_H, \theta_L\} \subset \mathbb{R} and \Delta \theta := \theta_H - \theta_L > 0. The commonly held prior probability of \(\theta_H\) is \(p_H\). HQ cannot observe \(\theta\) but instead observes a signal \(s\) generated from a random variable \(\tilde{s}\) that is correlated with \(\theta\). The manager can further influence HQ's observation of \(s\): he can choose \(i \in \mathbb{R}_+\), which increases the observed value of signal by \(i\). In choosing \(i\), the manager incurs private cost \(c(i)\), which is twice-differentiable with \(c' > 0\) and \(c'' > 0\).

Given \(\theta\) and \(i\), the signal observed by HQ is generated from \(\tilde{s} = \theta + i + \varepsilon\) where \(\varepsilon\) is a white noise with full support and density function denoted by \(f(\varepsilon)\). With slight abuse of notation, we use \(f(\cdot|\theta + i)\) to denote the probability density function of \(\tilde{s} = \theta + i + \varepsilon\). Thus the final report received by HQ is further modified by additional noise that is beyond the manager's control. The additional noise may represent frictions in the communication channel or influence activities by other division managers as in Wulf (2002).

As for the timing of the manager’s influence activities, there are two possibilities. If the manager chooses \(i\) before observing \(\theta\), then we have the standard signal jamming model. In this case, HQ can correctly infer the level of influence in equilibrium. Thus influence activities do not lead to distortion in capital allocation. The key aspect of capital allocation within the firm is the informational asymmetry between HQ and the division manager: the division manager is better informed about the divisional state than HQ and tries to influence HQ's capital allocation decision. In addition, there is ample empirical evidence on the distortion of capital allocation through internal capital markets.\(^5\) Thus we assume that the manager chooses \(i\) after observing \(\theta\).

We also assume that the signal observed by HQ is soft information, hence HQ cannot commit to a capital allocation rule. This is often assumed in the literature on internal capital markets. For example, Inderst and Laux (2005) argue that HQ's inability to pre-commit to a state-contingent capital allocation rule resembles the difficulty of relinquishing formal authority inside an organization; such a pre-commitment can also create a hold-up problem and stifle incentives. In addition, influence activities are meaningful when HQ has discretionary authority over decisions (Milgrom, 1988). In this case, the manager chooses \(i\) after observing \(\theta\), based on his conjecture of HQ’s capital allocation. After observing \(s\), HQ chooses capital allocation, which is consistent with the manager’s conjecture in equilibrium.

Let \(i_H\) (\(i_L\), resp.) denote the manager’s choice of \(i\) under the state \(\theta_H\) (\(\theta_L\), resp.). Then, upon observing a signal \(s\), HQ forms a posterior belief that \(\theta = \theta_H\), denoted by \(\pi_H(s|i_H, i_L)\), by Bayes rule:

\[
\pi_H(s|i_H, i_L) = \frac{p_H f(s|\theta_H + i_H)}{p_H f(s|\theta_H + i_H) + (1 - p_H) f(s|\theta_L + i_L)}.
\]

(1)

Notice that, if the manager chooses \(i\) before observing \(\theta\) so that \(i_H = i_L\), then the

\(^5\)See, for example, Stein (2003) and the references therein.
posterior belief in (1) would the one as in the standard signal jamming model. Thus
a key difference between our model and the signal jamming model is the possibility of
$i_H \neq i_L$ in equilibrium.

For each $s$, HQ forms the posterior as in (1) based on its conjecture of $(i_H, i_L)$, and
chooses $k$ to maximize its objective function

$$
\pi_H(s|i_H, i_L)y(k, \theta_H) + (1 - \pi_H(s|i_H, i_L))y(k, \theta_L) - k
$$

where we have normalized the cost of capital to one. The solution to HQ’s problem is
an allocation strategy contingent on $s$, which we denote by $k^*(s)$.

To capture the aspect of managerial preference for larger capital budget, we simply
assume that the manager derives positive private benefits proportional to capital $k$ allo-
cated to his division, denoted by $\alpha k$ with $\alpha > 0$. It is clear that adding a fixed salary
for the manager would not alter our analysis. Given the allocation strategy $k^*(s)$, the
manager’s problem is thus to choose $i$ for each $\theta \in \{\theta_H, \theta_L\}$ to maximize

$$
\int_{-\infty}^{\infty} \alpha k^*(s)f(s|\theta + i)ds - c(i).
$$

Throughout the rest of the paper, we keep the following assumptions.

**Assumption 1**: For each $\theta \in \{\theta_H, \theta_L\}$, $y(k, \theta)$ is smooth, strictly increasing and strictly
concave in $k$.

**Assumption 2**: $dy(k, \theta_H)/dk > dy(k, \theta_L)/dk$ for all $k$.

**Assumption 3**: Prior belief is symmetric: $p_H = 1/2$.

**Assumption 4**: $f(\cdot)$ is strictly positive, differentiable, symmetric around 0 with $f'(e) > 0$
for all $e < 0$ and $f'(e) < 0$ for all $e > 0$.

**Assumption 5**: $f(\cdot|\theta + i)$ satisfies the monotone likelihood ratio property (MLRP),
i.e., $f(s|\theta + i)/f(s|\theta' + i')$ is an increasing function of $s$ for all $\theta + i > \theta' + i'$.

Assumption 1 is standard with additional smoothness to facilitate characterization
of equilibrium. Assumption 2 implies that the marginal return to capital is higher in a
more favorable state. This condition is quite natural in our context. If $dy(k, \theta_H)/dk =

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Non-contractible private benefits are a simple, albeit *ad hoc*, artefact to introduce agency conflicts.
See, for example, Scharfstein and Stein (2000).
\[ dy(k, \theta_L)/dk, \text{ then } \theta \text{ is irrelevant in capital allocation decisions. If } dy(k, \theta_H)/dk < dy(k, \theta_L)/dk, \text{ then we can simply relabel } \theta \text{'s so that Assumption 2 holds. Assumptions 3 and 4 are mainly to simplify our analysis. Clearly many known density functions satisfy Assumption 4. Assumption 5 is also standard and implies that the posterior belief } \pi_H(s|i_H, i_L) \text{ in (1) is increasing in } s \text{ if and only if } \theta_H + i_H > \theta_L + i_L. \]

In equilibrium, HQ updates its posterior using Bayes rule given its correct conjecture of the manager’s equilibrium strategy and optimally allocates capital for each signal based on the posterior, and the manager chooses his best response to HQ’s equilibrium strategy.

**Definition 1**: The manager’s influence activities \((i_H^*, i_L^*)\), HQ’s capital allocation rule \(k^*(s)\) and posterior belief \(\pi_H^*(s|i_H, i_L)\) constitute an equilibrium if (a) given \((i_H^*, i_L^*)\), \(\pi_H^*\) satisfies (1), (b) given \((i_H^*, i_L^*)\) and \(\pi_H^*, k^*(s)\) maximizes (2) for every \(s\), and (c) given \(k^*(s), i_H^* (i_L^*, \text{ resp.})\) maximizes (3) for \(\theta = \theta_H (\theta = \theta_L, \text{ resp.})\).

Before we move on to the next section, we offer some discussions on our modelling choice. First, we do not consider explicit incentive contracts for the manager; the manager in our baseline model is motivated only through non-contractible private benefits. While this is to narrow down the focus of our analysis on the aspect of information transmission, one can take our model to be applicable to situations where managerial compensation is largely based on fixed salary or performance signal is highly noisy. For example, managerial behavior in non-profit organizations or bureaucracies could fit our model. Nonetheless, we discuss the case of explicit incentives in Section 5. Second, HQ in our model does not have access to any information other than the noisy signal that can be influenced. Thus our model can be interpreted to describe situations where the firm is venturing into new markets, rather than investing in its existing business lines. While this is also for the sake of simplicity, it should be clear as we go on that our qualitative results are robust to adding additional information that cannot be influenced.

## 4 Characterization of Equilibrium

Since our main focus is on when influence activities can transmit valuable information, we start by formalizing the notion of influence activities being informative. Given signal observation, HQ’s inference problem is to distinguish between \(\theta_H\) and \(\theta_L\). Relative to the

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7 Non-profit organizations are typically bound by a ‘nondistribution constraint’, which prohibits them from distributing profits to their managers (Ballou and Weisbrod, 2003). Studies in bureaucracies (e.g., Niskanen, 1971; Tullock, 1965) have long argued bureaucrats’ preference for larger budgets. Khalil et al. (2011) provide various reasons for low-powered incentives for bureaucrats. They also discuss the political science literature on why funding authorities may have little control over a bureaucratic agency other than being able to fix its budget.
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case of no influence, HQ’s inference can be improved if influence increases HQ’s posterior on $\theta_H$ ($\theta_L$, resp.) when the true state is $\theta_H$ ($\theta_L$, resp.). To formalize this, let us define

$$G_L(q) := \text{Prob}[\pi_H(s|i_H, i_L) < q | \theta = \theta_L] \text{ for some } q \in [0, 1],$$

$$G_H(r) := \text{Prob}[\pi_H(s|i_H, i_L) > r | \theta = \theta_H] \text{ for some } r \in [0, 1].$$

Thus $G_L(q)$ is the probability that HQ’s posterior is less than $q$ given the manager’s influence ($i_H, i_L$) conditional on the true state being $\theta_L$. Likewise $G_H(r)$ is the probability that HQ’s posterior is greater than $r$ given the manager’s influence ($i_H, i_L$) conditional on the true state being $\theta_H$. Define $G^0_L(q)$ and $G^0_H(r)$ similarly when there is no influence, i.e., $i_H = i_L = 0$. Then it is natural to define influence to be informative if $G_L(q) > G^0_L(q)$ for all $q$ less than the prior probability of $\theta_L$, and $G_H(r) > G^0_H(r)$ for all $r$ larger than the prior probability of $\theta_H$. Otherwise, influence can be at best neutral.

Since we assumed symmetric prior, we have the following definition.

**Definition 2**: Influence activities ($i_H, i_L$) are informative if $G_L(q) > G^0_L(q)$ for all $q \in [0, 1/2]$ and $G_H(r) > G^0_H(r)$ for all $r \in [1/2, 1]$, neutral if equality holds in both cases, and detrimental if the inequality is reversed in both cases.

The following lemma shows that influence activities are informative if the manager chooses more influence in $\theta_H$ than in $\theta_L$. The intuition is as follows. Since the conditional density of signal satisfies the MLRP by Assumption 5, a larger value of observed signal is more indicative of the signal having been generated in $\theta_H$. While the manager’s influence can affect the value of observed signal, it is the difference in the level of influence activities that matters for HQ’s inference. For example, suppose the level of influence activities is the same in both states. Then HQ can discount the value of observed signal by the same amount of influence and, hence, influence has no effect on HQ’s inference as in the signal jamming model. This implies that influence can have an effect on HQ’s inference when the level of influence activities is different in the two states. In this case, the MLRP implies that influence is informative when it is done more in $\theta_H$ and detrimental when it is done more in $\theta_L$. Moreover we have $\theta_H + i_H > \theta_L + i_L$ in equilibrium, as we show in Lemma 2 below. Thus our characterization of influence activities is for the case $\theta_H + i_H > \theta_L + i_L$.

**Lemma 1**: Provided that $\theta_H + i_H > \theta_L + i_L$, influence activities are (a) informative if and only if $i_H > i_L$, (b) neutral if and only if $i_H = i_L$, and (c) detrimental if and only if $i_H < i_L$. 

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Proof: See the appendix.

We now turn to HQ’s capital allocation problem. Recall that \( \pi_H(s|i_H, i_L) \) is HQ’s posterior given its conjecture of \((i_H, i_L)\). When there is no confusion, we will simply denote \( \pi_H(s|i_H, i_L) \) by \( \pi_H(s) \). Since \( y \) is strictly concave in \( k \), the first-order necessary and sufficient conditions for \( k^*(s) \) to be the solution to HQ’s problem are

\[
\pi_H(s)y'(k^*(s), \theta_H) + (1 - \pi_H(s))y'(k^*(s), \theta_L) = 1 \quad \text{for all } s.
\]  

(4)

The following lemma shows that HQ allocates more capital when the observed value of signal is higher. Moreover equilibrium influence activities are such that the mean value of signal under \( \theta_H \) is higher than that under \( \theta_L \). Both follow directly from the MLRP, the manager’s preference for more capital, and the assumption that the marginal return to capital is higher in \( \theta_H \).

Lemma 2: In equilibrium, \( \theta_H + i^*_H > \theta_L + i^*_L \) and \( \frac{dk^*(s)}{ds} > 0 \).

Proof: See the appendix.

Next we show how HQ’s capital allocation depends on its posterior. Although HQ allocates capital based on the posterior endogenously obtained from the observed signal, it proves useful to consider the optimal capital allocation treating the posterior as an independent variable. Specifically, it shows clearly how equilibrium influence activities are related to HQ’s posterior. If \( \pi_H \) is HQ’s posterior belief that \( \theta = \theta_H \), then the optimal capital allocation problem is

\[
\max_{k \geq 0} \pi_H y(k, \theta_H) + (1 - \pi_H)y(k, \theta_L) - k.
\]

Thus the solution denoted by \( \kappa(\pi_H) \) satisfies the first-order condition:

\[
\pi_H y'(\kappa(\pi_H), \theta_H) + (1 - \pi_H)y'(\kappa(\pi_H), \theta_L) = 1 \quad \text{for all } \pi_H \in [0,1].
\]  

(5)

Comparing (4) and (5), we have \( k^*(s) = \kappa(\pi_H(s|i_H, i_L)) \) for all \( s \).

The next lemma shows that \( \kappa(\pi_H) \) is an increasing function of \( \pi_H \) and that its curvature depends on \( y(k, \theta) \). For this, let us denote \( \Delta y' := y'(k, \theta_H) - y'(k, \theta_L) \), \( \Delta y'' := y''(k, \theta_H) - y''(k, \theta_L) \), \( E y'' := \pi_H y''(k, \theta_H) + (1 - \pi_H)y''(k, \theta_L) \), and \( E y''' := \pi_H y'''(k, \theta_H) + (1 - \pi_H)y'''(k, \theta_L) \).

Lemma 3: (a) \( \frac{d\kappa}{d\pi_H} > 0 \) for all \( \pi_H \in [0,1] \); (b) \( \frac{d^2\kappa}{d\pi_H^2} > 0 \) if and only if \( \frac{2\Delta y''}{\Delta y'} \geq \frac{E y'''}{E y''} \).
**Proof:** See the appendix.

Lemma 3 is useful in characterizing equilibrium influence activities. Nonetheless the condition in the lemma involves third derivatives and one might question its intuitive appeal. To provide intuition, suppose \( y(k, \theta) \) is multiplicatively separable in \( k \) and \( \theta \): 

\[
y(k, \theta) = \theta y(k).\]

Then \( Ey'' = [\pi_H \theta_H + (1 - \pi_H) \theta_L]y'' \) and \( Ey''' = [\pi_H \theta_H + (1 - \pi_H) \theta_L]y''' \). Thus the condition in Lemma 3 boils down to \( y'y'' > 2(y'')^2 \). This condition roughly implies that, as \( k \) increases, the marginal return to capital decreases at an increasingly slower rate, or \( y \) becomes less concave.\(^8\) In this case, HQ would increase capital allocation in an increasing rate as its posterior belief in \( \theta_H \) improves. As a result, \( \kappa \) is a convex function of \( \pi_H \).

We offer examples from three familiar classes of strictly concave functions. Suppose first \( y(k) = ak^n \) where \( a > 0 \) and \( n < 1 \). Then we have \( y'y'' > 2(y'')^2 \), hence \( \kappa \) is a convex function of \( \pi_H \). Our next example is \( y(k) = b \ln(k+1) \) where \( b > 0 \). In this case, \( y'y'' = 2(y'')^2 \). Therefore \( \kappa \) is an increasing, affine function of \( \pi_H \). Finally if \( y(k) = A - Be^{-pk} \) where \( \rho, A, B > 0 \), we have \( y'y'' < 2(y'')^2 \). Thus \( \kappa \) is a concave function of \( \pi_H \). Plotting these three functions, we can see that the rate of decrease in the second derivative is the largest for the negative exponential function and the smallest for the power function.

By Lemma 2, the manager has incentives to increase the value of signal. Lemma 3 then shows how such incentives depend on how \( \kappa \) responds to changes in HQ’s posterior. Combining these two, we can conjecture that, if \( \kappa \) is convex in \( \pi_H \), then the manager has larger incentives to increase the value of signal when the signal is generated from \( \theta_H \) than when it is generated from \( \theta_L \). This is because \( \theta_H > \theta_L \), hence the expected marginal increase in capital is higher at higher values of signal if \( \kappa \) is convex in \( \pi_H \). In this case, we expect equilibrium influence to be informative. If \( \kappa \) is affine in \( \pi_H \), then the expected marginal increase in capital is the same at all values of signal. Thus we expect equilibrium influence to be neutral. Similar reasoning leads us to conjecture equilibrium influence to be detrimental when \( \kappa \) is concave in \( \pi_H \). Below we prove more general results that verify that this conjecture is indeed correct.

Given \( \kappa \) and HQ’s equilibrium posterior \( \pi_H^*(s) = \pi_H(s|i_H^*, i_L^*) \), the manager’s optimization problem after observing \( \theta \) is to choose \( i \) to maximize (3) where \( k^*(s) \) is replaced by \( \kappa(\pi_H^*(s)) \). By change of variables, (3) is equivalent to

\[
\int_{-\infty}^{\infty} \alpha \kappa(\pi_H^*(s+i))f(s|\theta)ds - c(i).
\]

\(^8\)We can also interpret this condition in a way similar to how risk aversion is measured. Analogous to the Arrow-Pratt measure of absolute risk aversion, define \( A(k) := -y''(k)/y'(k) \). Then \( A'(k) \geq 0 \) if and only if \( (y'')^2 \geq y'y''' \). Thus \( y'y'' > 2(y'')^2 \) implies \( A'(k) < 0 \), or \( y \) becomes less concave.
Then the first-order conditions for \((i^*_H, i^*_L)\) to be the solution to the manager’s problem are
\[
\int_{-\infty}^{\infty} \kappa'(\pi^*_H(s + i^*_j))\pi'_H(s + i^*_j)f(s|\theta_j)ds = c'(i^*_j)/\alpha, \quad j = H, L.
\]
By change of variables once more, these conditions are equivalent to
\[
\int_{-\infty}^{\infty} \kappa'(\pi^*_H(s))\pi'_H(s)f(s|\theta_j + i^*_j)ds = c'(i^*_j)/\alpha, \quad j = H, L. \tag{6}
\]
Thus the manager’s optimal influence activities are chosen where the marginal cost of influence is equated to the marginal benefit, the latter being \(\alpha\) times the expected increase in capital allocation through changes in HQ’s posterior.

Based on (6), we can identify conditions under which equilibrium influence activities can be informative, neutral, or detrimental. In choosing the level of influence activities, what matters to the manager is marginal capital allocation \(\kappa'(\pi_H)\) relative to \(\kappa'(1 - \pi_H)\).

If \(\kappa'(\pi_H) = \kappa'(1 - \pi_H)\) for all \(\pi_H \in (1/2, 1)\), the HQ increases capital allocation at the same rate as the posterior increases on both sides of \(\pi_H = 1/2\). We call such a capital allocation rule symmetric. If \(\kappa'(\pi_H) > \kappa'(1 - \pi_H)\) for all \(\pi_H \in (1/2, 1)\), then we call the capital allocation rule aggressive in that marginal increase in capital is higher when HQ’s posterior is biased above \(\theta_H = 1/2\) than when it is biased below by the same amount. If the oppositive inequality holds, then we call the capital allocation rule conservative. This leads to the following definition.

**Definition 3:** HQ’s capital allocation rule \(\kappa(\pi_H)\) is aggressive if \(\kappa'(\pi_H) > \kappa'(1 - \pi_H)\) \(\forall \pi_H \in (1/2, 1)\), conservative if \(\kappa'(\pi_H) < \kappa'(1 - \pi_H)\) \(\forall \pi_H \in (1/2, 1)\), and symmetric if \(\kappa'(\pi_H) = \kappa'(1 - \pi_H)\) \(\forall \pi_H \in (1/2, 1)\).

We provide the main result of this section below.

**Proposition 1:** Equilibrium influence activities are informative (detrimental, neutral, resp.) if HQ’s capital allocation rule is aggressive (conservative, symmetric, resp.).

**Proof:** See the appendix.

Lemma 1 has shown that influence is informative if more is done in \(\theta_H\) than in \(\theta_L\). Lemma 3 has related the capital allocation rule \(\kappa\) to the curvature of output function \(y\). Since \(\kappa\) is aggressive (conservative, symmetric, resp.) if \(\kappa\) is strictly convex (strictly concave, linear, resp.), we have
Corollary 1: Equilibrium influence activities are informative if \( 2\Delta y''/\Delta y' > E\gamma'''/E\gamma'' \), detrimental if the inequality is reversed, and neutral if \( 2\Delta y''/\Delta y' = E\gamma'''/E\gamma'' \).

We offer some discussions of Proposition 1 and its corollary. Suppose we interpret \( y \) more generally as an organization’s performance indicator that also reflects its internal culture. When \( y \) is less concave, we may say the organization is less averse to risk taking: it responds more aggressively when the good prospect improves even further than when the bad prospect turns better. Such a culture encourages informative communication and discourages detrimental signal jamming. On the other hand, a more conservative organization with more concave \( y \) does not respond as aggressively to an improvement in the good prospect. In this case, detrimental communication to cover up bad news is more likely. Returning to our examples of \( y(k, \theta) = \theta y(k) \), the above result shows that equilibrium influence activities are informative if \( y(k) \) is a concave power function, neutral if it is a logarithmic function, and detrimental if it is a negative exponential function.

5 Extensions

So far we have assumed that the manager’s incentives are based entirely on non-contractible private benefits. Since these private benefits are derived from the amount of capital and HQ’s capital allocation decision depends on the output function \( y(k, \theta) \), our main result relates equilibrium influence activities to the properties of the output function. In this section, we consider three alternative environments. First, we analyze the case where the division manager is motivated by explicit compensation that increases in divisional output. We show that such explicit compensation induces informative influence activities for a larger set of output functions compared to when the manager is motivated through private benefits. Second, we consider the case where there are two symmetric divisions and HQ uses a contest between the two divisions in allocating capital. Such a contest is also shown to induce informative influence activities for a larger set of output functions compared to the case with a single division. Third, we consider a more general type of competition among multiple divisions where HQ has limited resources to fund all divisions. We show that overall influence activities increase as competition toughens in the sense that the number of divisions increases.

5.1 Explicit linear incentives

Suppose the manager is given explicit incentives based on output. In this case, the eventual effect on the manager’s utility of boosted signals due to influence activities can be understood in two stages. First, they induce increased capital allocation by HQ, which
will then increase the divisional output. Since the increase in output due to increased capital is larger in $\theta_H$ by Assumption 2, the manager benefits more from influence activities in $\theta_H$ than in $\theta_L$. Consequently, equilibrium influence activities are more likely to be informative than when the manager is motivated through private benefits. In this section we verify this intuition for the simple case when the explicit incentives are linear in output.

Suppose the manager’s compensation is given by $b \cdot y$ where $b \in (0, 1)$. Then HQ’s objective function in (2) changes with $y$ replaced by $(1-b)y$ while the manager’s objective function in (3) changes with $\alpha k^*(s)$ replaced by $by(k^*(s), \theta)$. It is straightforward to verify that Lemma 2 continues to hold since $y$ is increasing in $k$ by Assumption 1. HQ’s first-order condition (5) is modified with the right hand side replaced by $1/(1-b)$. Thus Lemma 3 also holds for the solution to this modified first-order condition, denoted by $\hat{\kappa}(\pi_H)$. Note from $y'(\hat{\kappa}(0), \theta_L) = y'(\hat{\kappa}(1), \theta_H) = 1/(1-b)$ that

$$y'(\hat{\kappa}(\pi), \theta_H) > 1/(1-b) > y'(\hat{\kappa}(\pi), \theta_L) \quad \text{for all} \quad \pi \in (0, 1). \quad (7)$$

Let $i^* = (i^*_L, i^*_H)$ denote the equilibrium influence activity levels. Then from the manager’s objective function, we have

$$\frac{d}{dL} \int by(\hat{\kappa}(\pi_H(s|i^*)), \theta_L)f(s|\theta_L + i_L)ds \bigg|_{i_H = i_H^*} = \int by'(\hat{\kappa}(\pi_H(s|i^*)), \theta_L)\hat{\kappa}'(\pi_H(s|i^*))\pi_H'(s|i^*)f(s|\theta_H + i_H)ds \quad (8)$$

and

$$\frac{d}{dL} \int by(\hat{\kappa}(\pi_H(s|i^*)), \theta_L)f(s|\theta_L + i_L)ds \bigg|_{i_L = i_L^*} = \int by'(\hat{\kappa}(\pi_H(s|i^*)), \theta_L)\hat{\kappa}'(\pi_H(s|i^*))\pi_H'(s|i^*)f(s|\theta_H + i_H)ds. \quad (9)$$

Note that $\theta_H + i_H^* > \theta_L + i_L^*$ by Lemma 2, $\pi(s|i^*) = 1/2$ where $\hat{s} = (\theta_H + i_H^* + \theta_L + i_L^*)/2$, $\pi'_H(s|i^*)$ is symmetric around $\hat{s}$, and $f(s|\theta_H + i_H^* + i_L^*)$ and $f(s|\theta_L + i_H^* + i_L^*)$ are mirror images of each other around $\hat{s}$. Therefore we deduce that (8) is greater than (9) if

$$y'(\hat{\kappa}(\pi_H), \theta_H)\hat{\kappa}'(\pi_H) > y'(\hat{\kappa}(1-\pi_H), \theta_L)\hat{\kappa}'(1-\pi_H) \quad \forall \pi_H \in (1/2, 1). \quad (10)$$

This leads us to the following proposition.

**Proposition 2**: If (10) holds, then equilibrium influence activities are informative under the explicit linear incentives.

**Proof**: See the appendix.
Observe that (10) is satisfied when the modified capital allocation rule $\hat{\kappa}$ is aggressive or symmetric. Since Lemma 3 holds when $\kappa$ is replaced by $\hat{\kappa}$, Proposition 2 implies that equilibrium influence activities are informative under explicit linear incentives if $2\Delta y''/\Delta y' \geq Ey'''/Ey''$.

**Corollary 2:** Under explicit linear incentives, equilibrium influence activities are informative if $2\Delta y''/\Delta y' \geq Ey'''/Ey''$.

With private benefits only, equilibrium influence activities are informative if $2\Delta y''/\Delta y' > Ey'''/Ey''$, as shown in Corollary 1. With explicit linear incentives, they are also informative if $2\Delta y''/\Delta y' = Ey'''/Ey''$. Thus explicit linear incentives are more likely to induce informative influence activities for a larger set of output functions.

### 5.2 Contest

The aim of this section is to study how competition affects influence activities. Specifically, we consider a firm with two divisions and HQ runs a contest between the two divisions where only the division with a higher value of realized signal is funded. This is the simplest way competition can be introduced. Intuitively such competition should induce more informative influence activities than in the absence of competition. It is because influence activities lead to a larger increase in the expected capital allocation conditional on winning the contest when the division is in a good state than when it is in a bad state. We formalize this intuition below.

Suppose there are two symmetric divisions, each being exactly the same as the single division in our basic model.\(^9\) For simplicity, we assume divisional states are uncorrelated. For $d = 1, 2$, let $(i_{Hd}, i_{Ld})$ be the level of influence activities chosen by the manager of division $d$, to be called manager $d$, and $s_d$ be the corresponding signal observed by HQ. Since the two divisions are symmetric, we focus on manager 1’s problem and symmetric equilibrium where $i_{t1} = i_{t2}$ for $t = H, L$. Division 1 is funded if and only if $s_1 > s_2$, hence with probability  

\[
\Phi(s_1) := \frac{1}{2} \int_{-\infty}^{s_1} \sum_{t=H,L} f(s_2|\theta_t + i_{t2})ds_2. \tag{11}
\]

Clearly $\Phi'(s_1) > 0$ and, due to Assumption 4, we have $\Phi'(s_1) = \Phi'(2s - s_1)$ where $\hat{s} = (\theta_H + i_{H1} + \theta_L + i_{L1})/2 = (\theta_H + i_{H2} + \theta_L + i_{L2})/2$.

Conditional on $s_1 > s_2$, HQ’s optimal capital allocation to division 1 is exactly the same as in our basic model. Thus manager 1’s problem is to choose $i$ for each $\theta$ to

\(^9\)An extension to $N$ symmetric divisions is straightforward. The only change is in the probability of winning the contest, which, given symmetry and independence, is $\Phi(s_1)^N$.\(^{-1}\).
maximize
\[ \int_{-\infty}^{\infty} \alpha \Phi(s_1) k^*(s_1) f(s_1 | \theta + i) ds_1 - c(i). \] (12)

As before, we use change of variables to derive the first-order conditions for \((i_{H1}^*, i_{L1}^*)\) to be the solution to manager 1’s problem:
\[ \int_{-\infty}^{\infty} \left[ \Phi'(s_1) \kappa(\pi_H^*(s_1)) + \Phi(s_1) \kappa'(\pi_H^*(s_1)) \pi_H^*(s_1) \right] f(s_1 | \theta_j + i_{j1}^*) ds_1 = c'(i_{j1}^*) / \alpha, \quad j = H, L. \] (13)

The marginal benefit from influence activities on the left-hand side of (13) is different from that in (6) in two ways. First, more influence activities increase the probability of winning the contest and, therefore, the amount of expected capital allocation as captured by the term \(\Phi'(s_1) \kappa(\pi_H^*(s_1))\). Second, the increase in capital allocation through changes in HQ’s posterior is conditional upon winning the contest, as captured by the second term in the integrand on the left-hand side. To compare the integrals in (13), we use the following condition:
\[ \frac{\Phi(s_1)}{\Phi(2\hat{s} - s_1)} \geq \frac{\kappa'(1 - \pi_H(s_1))}{\kappa'(\pi_H(s_1))} \forall s_1 \in (\hat{s}, \infty) \text{ where } \hat{s} = \frac{\theta_H + i_{H1} + \theta_L + i_{L1}}{2}. \] (14)

Given (14), we can show the following.

**Proposition 3**: If (14) holds, then equilibrium influence activities are informative.

**Proof**: See the appendix.

Observe that \(\Phi(s_1) > \Phi(2\hat{s} - s_1)\). Therefore if the capital allocation rule is aggressive or symmetric, i.e., \(\kappa'(1 - \pi_H(s_1)) \leq \kappa'(\pi_H(s_1))\), then (14) clearly holds. This leads to

**Corollary 3**: Competition in the form of contest induces informative influence activities if \(2\Delta y'' / \Delta y' \geq E y''' / E y''\).

Once again, a sufficient condition for informative influence activities is weaker than when there is only a single division. Moreover even when the capital allocation rule is conservative, (14) may still hold unless \(\kappa'(1 - \pi_H(s_1))\) is too large relative to \(\kappa'(\pi_H(s_1))\). Thus even for conservative capital allocation rules, equilibrium influence activities can be informative. Based on this, we may conclude that competition in the form of contest is more likely to induce informative influence activities for a larger set of output functions.
5.3 Competition among multiple divisions

In this section we consider the case where multiple divisions within the firm compete for limited amount of capital that can be allocated across divisions based on the signals received by HQ. To simplify analysis, we assume divisional states are not correlated and each division’s output function is multiplicatively separable: \( y(k, \theta) = \theta y(k) \). As in Section 3, we also assume that managers are motivated by non-contractible private benefits that are linear in the amount of capital allocated to their own division. If available capital is unlimited and divisional states are not correlated, it is easy to see that capital will be allocated optimally to each division independently of other divisions, in the manner explained in Section 4. By the same token, existence of other divisions has no impact on the amount of capital allocated to each division if there is sufficient amount of capital to fund all divisions when all divisions are known to be in a good state.

To consider the effect of competition in a meaningful way, we therefore focus on the case where divisional states inferred by HQ are too good for all of them to be funded optimally. Only in such contingencies, the effect of each division’s influence activity would be different from the case of a single division. In this case, HQ’s optimal capital allocation will satisfy the condition that the marginal returns to capital are equalized across all divisions. Therefore, when the inferred state of a division improves due to extra influence activity from that division, resources are moved away from other divisions into that division to equalize the marginal returns for the changed profile of inferred states. This implies that the marginal benefit from influence activity is larger when there is divisional competition. Consequently, we expect division managers to have more incentives to invest in influence activities when there are other divisions that compete for limited resources.

In what follows, we formalize the above intuition when there are two divisions. Let \( K \) be total amount of capital available for allocation by HQ. We assume \( K < 2\kappa(1) \) so that \( K \) is not enough to fund both divisions optimally when both are in good states. First, we examine the marginal benefit from influence activity for each division manager in this setting. Then we consider an \( N \)-replica firm that consists of \( N \) “copies” of each division, hence \( 2N \) divisions in total. This will allow us to examine how divisional influence activities change when \( N \) increases.

First, let us consider the case with two divisions. Let \( \theta_d \in \{\theta_H, \theta_L\} \) be the realized state of division \( d \) and \( s_d \in \mathbb{R} \) be the realized signal of division \( d \), \( d = 1, 2 \). Since the two divisions are symmetric, we focus on manager 1’s problem in symmetric equilibrium where \( i_{1t} = i_{2t} = i_t \) for \( t = H, L \). Given the putative levels of influence activities \((i_H, i_L)\) and realized signals \((s_1, s_2)\), the solution to HQ’s optimal capital allocation problem
satisfies
\[ k_1 + k_2 \leq K, \quad \text{and} \]
\[ (\theta_L + \pi_H(s_1|i_H,i_L)\Delta \theta') y'(k_1) = (\theta_L + \pi_H(s_2|i_H,i_L)\Delta \theta') y'(k_2) \geq 1 \]
where \( \Delta \theta = \theta_H - \theta_L \). Note that (15) is satisfied as equality if the inequality in (16) is strict. Let \( k_d(s_1,s_2|i_H,i_L) \) \((d = 1,2)\) denote the solution to HQ’s problem. Given the putative levels \( (i_H,i_L) \), division manager 1’s total benefit from investing \( i = i_L + \delta \) in influence activities conditional on \( \theta_1 = \theta_L \) is
\[ \frac{\alpha}{2} \sum_{t=H,L} \int_{s_1} \int_{s_2} k_1(s_1 + \delta, s_2|i_H,i_L)f(s_2|\theta_t + i_t)f(s_1|\theta_L + i_L)ds_2ds_1. \]

Therefore, division manager 1’s marginal benefit from influence activities at \( i = i_L \) is
\[ \frac{\alpha}{2} \sum_{t=H,L} \int_{s_1} \int_{s_2} \frac{\partial k_1(s_1 + \delta, s_2|i_H,i_L)}{\partial \delta} f(s_2|\theta_t + i_t)f(s_1|\theta_L + i_L)ds_2ds_1. \]

Division manager 1’s marginal benefit from influence activities at \( i = i_H \) can be written in a similar way.

Next, consider a firm consisting of \( N \) copies of each division described above in the following sense: every copy of division 1 shares a common realized state \( \theta_1 \) and a common realized signal net of influence activity levels. For the \( n \)th copy of division 1, denote the realized signal by \( s^n_1 \) and the level of influence activity by \( i^n_1 \). Then we have \( s^n_1 - s^m_1 = i^n_1 - i^m_1 \) for all \( n,m \in \{1,2,\cdots,N\} \). The same holds for every copy of division 2. Given the putative levels \( (i_H,i_L) \) common to all copies and a profile of realized signals \( (s^n_1,s^n_2)_{n=1,2,\cdots,N} \), HQ allocates total available capital \( N \cdot K \) optimally based on the posteriors \( \pi_H(s^n_1|i_H,i_L), d = 1,2 \). Thus the optimal allocation levels denoted by \( k^n_d\{s^n_d\}|i_H,i_L) \) \((d = 1,2, n \in \{1,\cdots,N\}\) solve
\[ \sum_{n=1}^{N} (k^n_1 + k^n_2) \leq N \cdot K, \quad \text{and} \]
\[ (\theta_L + \pi_H(s^n_1|i_H,i_L)\Delta \theta') y'(k_1) = (\theta_L + \pi_H(s^n_2|i_H,i_L)\Delta \theta') y'(k_2) \geq 1, \ \forall n,m. \]

Note that (19) is satisfied as equality if the inequality in (20) is strict. Given \( (i_H,i_L) \), the total benefit for the manager of the \( n \)th copy of division 1 from investing \( i = i_L + \delta \) conditional on \( \theta_1 = \theta_L \) is
\[ \frac{\alpha}{2} \sum_{t=H,L} \int_{s_1} \int_{s_2} k^n_1(s_1 + \delta, s_1,s_2|i_H,i_L)f(s_2|\theta_t + i_t)f(s_1|\theta_L + i_L)ds_2ds_1 \]
where \( k^n_i(s_1 + \delta, s_1, s_2|i_H, i_L) = k^n_i(s^n_d)|i_H, i_L) \) when \( s^n_1 = s_1 + \delta, s^n_m = s_1 \) for all \( m \in \{1, 2, \cdots, N\} \) and \( s^n_2 = s_2 \) for all \( m \in \{1, 2, \cdots, N\} \). Therefore, the manager’s marginal benefit from influence activities at \( i = i_L \) is

\[
\frac{\alpha}{2} \sum_{t=H,L} \int_{s_1} \int_{s_2} \frac{\partial k^n_i(s_1 + \delta, s_1, s_2|i_H, i_L)}{\partial \delta} \bigg|_{\delta=0} f(s_2|\theta_t + i_t)f(s_1|\theta_L + i_L)ds_2ds_1.
\]

As before, the manager’s marginal benefit from influence activities at \( i = i_H \) can be written in a similar way.

Comparing (18) and (22) for different values of \( N \), we obtain the following proposition. It confirms our intuition that, as competition for scarce resources toughens, managers invest more in influence activities since the resource constraint increases the marginal benefit from influence activities.

**Proposition 4**: As \( N \) increases, equilibrium influence activities increase in both states.

**Proof**: See the appendix.

### 6 Conclusion

This paper has focused on the information transmission aspect of influence activities. Although influence activities are costly since they are not directly productive activities, they nonetheless play a role in transmitting valuable local information to the central decision maker. They are informative if they improve the central decision maker’s inference and detrimental if they hamper it. We have considered four different environments and identified when influence activities can be informative or detrimental. We find that influence activities are more likely to be informative in organizations that are less averse to risk taking, that rely more on higher-power incentives, and that encourage competition in the form of contest. We also find that competition over scarce resources increases the overall level of influence activities.

Our findings offer implications for optimal organization design. Particularly relevant is the issue of centralization versus decentralization. Although centralized organizations may be better at coordinating decisions,\(^{11}\) they are more susceptible to influence activities since the center retains much of discretionary authority and communication tends to

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\(^{10}\)Note that we assume that HQ does not infer anything from the fact that the signal from one division slightly differs from those from \( N - 1 \) divisions that are identical. This can be interpreted as the limit of the case where each division’s signal has a vanishing idiosyncratic unbiased error (on top of \( s^n_d \) described above).

\(^{11}\)Alonso et al. (2008) and Rantakari (2008) challenge this by showing that cheap talk communication between division managers can achieve coordination, the benefits of which improve with decentralization.
be vertical. Our findings suggest that centralized organizations can improve their vertical communication channels by using higher-powered incentives, introducing competition among divisions, and relying on performance indicators that encourage risk taking. If it is not possible to incorporate these elements in organization design for whatever reasons, then more decentralization can reduce detrimental influence activities. Given the difficulty in relying on high-powered incentives in bureaucracies, our findings suggest that more bureaucratic organizations are likely to benefit more from decentralization. An additional implication is that firms in growth industries where performance indicators tend to be more volatile than those in mature industries are likely to benefit more from informative influence activities. However, our model is not rich enough to formalize these implications in a rigorous way, which we leave for future work.

Appendix

Proof of Lemma 1:

We will prove (a) first, followed by (b). We omit the proof of (c) since the argument used to prove (a) can be replicated to prove it with the only changes being in the direction of relevant inequalities.

Let \( s', t' \) be such that \( \pi_H(s'|i_H, i_L) = \pi_H(t'|0, 0) \). Since

\[
\pi_H(s'|i_H, i_L) = \frac{f(s'|\theta_H + i_H)}{f(s'|\theta_H + i_H) + f(s'|\theta_L + i_L)} = \frac{f(s' - i_H|\theta_H)}{f(s' - i_H|\theta_H) + f(s'- i_L|\theta_L)}
\]

and

\[
\pi_H(t'|0, 0) = \frac{f(t'|\theta_H)}{f(t'|\theta_H) + f(t'|\theta_L)}
\]

we have

\[
f(t'|\theta_H)f(s'- i_L|\theta_L) - f(t'|\theta_L)f(s' - i_H|\theta_H) = 0. \tag{A1}
\]

We will show (a) using (A1).

Suppose first \( \pi_H(s'|i_H, i_L) = \pi_H(t'|0, 0) = q \leq 1/2 \). Since \( G_L(q) = \int_{-\infty}^{s'-i_L} f(s|\theta_L) ds, G_L^0(q) = \int_{-\infty}^{t'} f(s|\theta_L) ds \), and \( f \) is strictly positive, we have \( G_L(q) > G_L^0(q) \) if and only if \( s' - i_L > t' \). Thus it suffices to show \( s' - i_L > t' \) if and only if \( i_H > i_L \).

Suppose \( i_H > i_L \) but \( s' - i_L \leq t' \). Then we have \( f(s' - i_H|\theta_H) < f(s' - i_L|\theta_H) \leq f(t'|\theta_H) \) since \( q \leq 1/2 \) implies \( t' \leq (\theta_H + \theta_L)/2 < \theta_H \) and \( f(s|\theta_H) \) is strictly increasing...
for all $s \leq \theta_H$ by Assumption 4. Thus we have

$$f(t'|\theta_H)f(s' - i_L|\theta_L) - f(t'|\theta_L)f(s' - i_H|\theta_H)$$

$$> f(t'|\theta_H)f(s' - i_L|\theta_L) - f(t'|\theta_L)f(s' - i_L|\theta_H)$$

$$\geq 0$$

where the first inequality is due to $f(s' - i_H|\theta_H) < f(s' - i_L|\theta_H)$ and the second inequality follows from the MLRP since it was assumed $s' - i_L \leq t'$. This contradicts (A1). Suppose next $s' - i_L > t'$ but $i_H \leq i_L$. Then we have $t' < s' - i_L \leq s' - i_H$. Moreover, $\theta_H + i_H > \theta_L + i_L$ and $q \leq 1/2$ imply $s' \leq (\theta_H + \theta_L + i_H + i_L)/2 \leq \theta_H + i_H$. Since $f(s|\theta_H + i_H) = f(s - i_H|\theta_H)$ is strictly increasing for all $s$ such that $s \leq \theta_H + i_H$, we have $f(t'|\theta_H) < f(s' - i_L|\theta_H) \leq f(s' - i_H|\theta_H)$. Then we can apply the same argument as before to reach a contradiction to (A1). Thus we have shown $G_L(q) > G_H^0(r)$ if and only if $i_H > i_L$.

We now turn to the comparison of $G_H(r)$ and $G_H^0(r)$ for $r \geq 1/2$. Our argument is analogous to the previous case except that we now focus on the conditional density on $\theta_L$. Let $s', t'$ be such that $\pi_H(s'|i_H, i_L) = \pi_H(t'|0, 0) = r \geq 1/2$. Then we have $s' \geq (\theta_H + \theta_L + i_H + i_L)/2$ and $t' \geq (\theta_H + \theta_L)/2$. Notice also that $f(s|\theta_L)$ is strictly decreasing for all $s \geq \theta_L$ by Assumption 4. Since $G_H(r) = \int_{s' - i_H}^{\infty} f(s|\theta_H)ds, G_H^0(r) = \int_{s' - i_L}^{\infty} f(s|\theta_H)ds$, and $f$ is strictly positive, we have $G_H(r) > G_H^0(r)$ if and only if $s' - i_H < t'$. Thus if suffices to show $s' - i_H < t'$ if and only if $i_H > i_L$. Suppose first $i_H > i_L$ but $s' - i_H \geq t'$. Then we have $s' - i_L > s' - i_H \geq t'$. Since $f(s|\theta_L)$ is strictly decreasing for all $s \geq \theta_L$ and $t' \geq (\theta_H + \theta_L)/2 > \theta_L$, we have $f(s' - i_L|\theta_L) < f(s' - i_H|\theta_L) \leq f(t'|\theta_L)$. As before, this leads to a contradiction to (A1). Suppose next $s' - i_H < t'$ but $i_H \leq i_L$. Given $\theta_H + i_H > \theta_L + i_L$ and $r \geq 1/2$, we again reach a contradiction to (A1). This shows $G_H(r) > G_H^0(r)$ if and only if $i_H > i_L$.

Next we prove (b). Consider first the case $\pi_H(s'|i_H, i_L) = \pi_H(t'|0, 0) = q \leq 1/2$. Then, from (A1), it is immediate to see that $i_H = i_L = i$ implies $s' - i = t'$. Suppose next $s' - i_L = t'$. Then (A1) becomes $f(t'|\theta_H) = f(s' - i_H|\theta_H)$. Note that $q \leq 1/2$ and $\theta_H + i_H > \theta_L + i_L$ imply $t' \leq \theta_H$ and $s' - i_H \leq \theta_H$. Since $f(s|\theta_H)$ is strictly increasing for all $s \leq \theta_H$, we have $t' = s' - i_H$, hence $i_H = i_L$. This proves $G_L(q) = G_H^0(r)$ if and only if $i_H = i_L$. For the other case $\pi_H(s'|i_H, i_L) = \pi_H(t'|0, 0) = r \geq 1/2$, a similar argument shows $s - i_H = t'$, hence $G_H(r) = G_H^0(r)$, if and only if $i_H = i_L$.

**Proof of Lemma 2:**

Differentiating (4) with respect to $s$, we obtain

$$\frac{dk^*(s)}{ds} = -\frac{\pi_H(s)(y'(k^*(s), \theta_H) - y'(k^*(s), \theta_L))}{\pi_H(s)y''(k^*(s), \theta_H) + (1 - \pi_H(s))y''(k^*(s), \theta_L)}.$$
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Since \( y'' < 0 \) by Assumption 1 and \( y'(k^*(s), \theta_H) > y'(k^*(s), \theta_L) \) by Assumption 2, \( dk^*(s)/ds \) has the same sign as \( \pi'_H(s) \). If \( \theta_H + i^*_H > \theta_L + i^*_L \), then \( \pi'_H(s) > 0 \) by the MLRP, hence we have \( \frac{dk^*(s)}{ds} > 0 \).

It remains to show that \( \theta_H + i^*_H \leq \theta_L + i^*_L \) is not possible in equilibrium. If it is, then the MLRP implies \( \pi'_H(s) \leq 0 \) and, consequently, \( \frac{dk^*(s)}{ds} \leq 0 \). Then a higher \( i \) increases cost for the manager without increasing capital allocation. This would imply \( i^*_H = i^*_L = 0 \), contradicting \( \theta_H + i^*_H \leq \theta_L + i^*_L \).

**Proof of Lemma 3:**

(a) Differentiating (5) with respect to \( \pi_H \), we obtain

\[
\frac{dk}{d\pi_H} = -\frac{\Delta y'}{Ey''} > 0 \tag{A2}
\]

since \( \Delta y' > 0 \) by Assumption 2 and \( Ey'' < 0 \) by Assumption 1.

(b) Differentiating (A2) once more leads to

\[
\frac{d^2k}{d\pi_H^2} = \frac{1}{(Ey'')^2} \left[ \Delta y' \frac{dEy''}{d\pi_H} - Ey'' \frac{d\Delta y'}{d\pi_H} \right] \tag{A3}
\]

Using (A2), it is easy to see \( dEy''/d\pi_H = \Delta y'' - \Delta y'(Ey''/Ey'') \) and \( d\Delta y'/d\pi_H = -(\Delta y'\Delta y'')/Ey'' \). Substituting these into (A3) and arranging terms, we obtain

\[
\frac{d^2k}{d\pi_H^2} = \frac{\Delta y'}{(Ey'')^2} \left[ 2\Delta y'' - \Delta y' \left( \frac{Ey''}{Ey''} \right) \right].
\]

From the above follows (b).

**Proof of Proposition 1:**

We start by showing that the derivative of \( \pi_H(s) \) is symmetric given Assumptions 3 and 4. Denote \( \bar{\pi} := (\theta_H + \theta_L + i_H + i_L)/2 \). Then we claim

\[
\pi_H(s) = 1 - \pi_H(2\bar{\pi} - s) \quad \text{and} \quad \frac{d\pi_H(s)}{ds} = \frac{d\pi_H(2\bar{\pi} - s)}{ds} \quad \text{for all } s. \tag{A4}
\]

To show (A4), note that we have, by Assumption 4, \( f(s|\theta_H + i_H) = f(2\bar{\pi} - s|\theta_L + i_L) \) and \( f(s|\theta_L + i_L) = f(2\bar{\pi} - s|\theta_H + i_H) \). Since \( p_H = 1/2 \) by Assumption 3, we have

\[
\pi_H(s) = \frac{f(s|\theta_H + i_H)}{f(s|\theta_H + i_H) + f(s|\theta_L + i_L)} = \frac{f(2\bar{\pi} - s|\theta_L + i_L)}{f(2\bar{\pi} - s|\theta_L + i_L) + f(2\bar{\pi} - s|\theta_H + i_H)} = \pi_L(2\bar{\pi} - s) = 1 - \pi_H(2\bar{\pi} - s).
\]
Thus $d\pi_H(s)/ds = -(d\pi_H(2\pi - s)/ds)(d(2\pi - s)/ds) = d\pi_H(2\pi - s)/ds$, proving (A4).

Since $\theta_H + \hat{i}_H^* \leq \theta_L + \hat{i}_L^*$ is not possible by Lemma 2, we only consider the case $\theta_H + \hat{i}_H^* > \theta_L + \hat{i}_L^*$. Let $\hat{s} := (\theta_H + \hat{i}_H^* + \theta_L + \hat{i}_L^*)/2$ and $\Delta f(s) := f(s|\theta_H + \hat{i}_H^*) - f(s|\theta_L + \hat{i}_L^*)$.

Comparing the manager’s first-order conditions in (6), we have

$$
\int_{-\infty}^{\hat{s}} \kappa'(\pi_H^*(s))\pi_H'(s)f(s|\theta_H + \hat{i}_H^*)ds - \int_{-\infty}^{\hat{s}} \kappa'(\pi_H^*(s))\pi_H'(s)f(s|\theta_L + \hat{i}_L^*)ds
= \int_{-\infty}^{\hat{s}} \kappa'(\pi_H^*(s))\pi_H'(s)\Delta f(s)ds + \int_{-\infty}^{\hat{s}} \kappa'(\pi_H^*(s))\pi_H'(s)\Delta f(s)ds
= \int_{-\infty}^{\hat{s}} \kappa'(\pi_H^*(s))\pi_H'(s)\Delta f(s)ds
= \int_{\hat{s}}^{\infty} \kappa'(\pi_H^*(s))\pi_H'(s)\Delta f(s)ds
$$

where, for the second equality, we have used change of variables, (A4) and Assumption 4. Note that $\pi_H^*(s) > 0$ due to the MLRP and $\theta_H + \hat{i}_H^* > \theta_L + \hat{i}_L^*$, and $\Delta f(s) = f(s|\theta_H + \hat{i}_H^*) - f(s|\theta_L + \hat{i}_L^*) > 0 \forall s \in (\hat{s}, \infty)$. Thus the value of the last integral is positive, negative, or 0, respectively, if $\kappa$ is aggressive, conservative, or symmetric. Given $c'' > 0$ and (6), this proves Proposition 1.

**Proof of Proposition 2:**

Denote the integrand of (8) by $X$, that of (9) by $Y$, and $\pi_H^* := \pi_H(s|\hat{s}^*)$. Define

$$
T := \{s \in (\hat{s}, \infty) \mid y'(\hat{\kappa}(1 - \pi_H^*), \theta_H)\hat{\kappa}'(1 - \pi_H^*) < y'(\hat{\kappa}(\pi_H^*), \theta_L)\hat{\kappa}'(\pi_H^*)\} \tag{A5}
$$

and $\tilde{T} := \{s \in (-\infty, \hat{s}) \mid s + s' = 2\hat{s} \text{ for some } s' \in T\}$. Let $T^c = (\hat{s}, \infty) \setminus T$ and $\tilde{T}^c = (-\infty, \hat{s}) \setminus \tilde{T}$. We need to show that (10) implies

$$
\int_{T^c} X ds + \int_{\tilde{T}^c} X ds + \int_{T \cup \tilde{T}} X ds > \int_{T^c} Y ds + \int_{\tilde{T}^c} Y ds + \int_{T \cup \tilde{T}} Y ds. \tag{A6}
$$

Observe that (i) $\theta_H + \hat{i}_H^* > \theta_L + \hat{i}_L^*$ by Lemma 2, (ii) $\pi_H(s|\hat{s}^*) = 1/2$ where $\hat{s} = (\theta_H + \hat{i}_H^* + \theta_L + \hat{i}_L^*)/2$, (iii) $\pi_H^*(s|\hat{s}^*)$ is symmetric around $\hat{s}$, and (iv) $f(s|\theta_H + \hat{i}_H^*)$ and $f(s|\theta_L + \hat{i}_L^*)$ are mirror images of each other around $\hat{s}$. Then we have

$$
\int_{T^c} X ds > \int_{T^c} Y ds \tag{A7}
$$
due to (10), (iii) and (iv). We also have

$$
\int_{T^c} X ds > \int_{T^c} Y ds \tag{A8}
$$
by (iii), (iv), and the fact that the inequality in (A5) is reversed for \( s \in T^c \). Finally, to evaluate \( \int_{T \cup \hat{T}} (X - Y) ds \), note that
\[
\int_{T} X ds - \int_{\hat{T}} Y ds = b \int_{T} \left[ y'(\hat{\kappa}(\pi^*_H), \theta_H)\hat{\kappa}'(\pi^*_H) \right.
\]
\[
- y'(\hat{\kappa}(1 - \pi^*_H), \theta_L)\hat{\kappa}'(1 - \pi^*_H)] \pi^*_H(s|i^*) f(s|\theta_H + i^*_H) ds
\]
which is positive due to (10), and
\[
\int_{T} Y ds - \int_{\hat{T}} X ds = b \int_{T} \left[ y'(\hat{\kappa}(\pi^*_H), \theta_L)\hat{\kappa}'(\pi^*_H) \right.
\]
\[
- y'(\hat{\kappa}(1 - \pi^*_H), \theta_H)\hat{\kappa}'(1 - \pi^*_H)] \pi^*_H(s|i^*) f(s|\theta_L + i^*_L) ds.
\]
Observe that \( y'(\hat{\kappa}(\pi^*_H), \theta_H)\hat{\kappa}'(\pi^*_H) > y'(\hat{\kappa}(\pi^*_H), \theta_L)\hat{\kappa}'(\pi^*_H) \) and \( y'(\hat{\kappa}(1 - \pi^*_H), \theta_H)\hat{\kappa}'(1 - \pi^*_H) > y'(\hat{\kappa}(1 - \pi^*_H), \theta_L)\hat{\kappa}'(1 - \pi^*_H) \) by (7). Furthermore, \( f(s|\theta_H + i^*_H) > f(s|\theta_L + i^*_L) \) for all \( s \in T \). Therefore, it follows that
\[
\int_{T \cup \hat{T}} (X - Y) ds = \int_{T} X ds - \int_{\hat{T}} Y ds - \left( \int_{T} Y ds - \int_{\hat{T}} X ds \right) > 0.
\]
Together with (A7) and (A8), we have shown that (10) implies (A6).

**Proof of Proposition 3:**

Since \( \theta_H + i^*_{H1} \leq \theta_L + i^*_{L1} \) is not possible by Lemma 2, we only consider the case \( \theta_H + i^*_{H1} > \theta_L + i^*_{L1} \). Let \( \hat{s} := (\theta_H + i^*_{H1} + \theta_L + i^*_{L1})/2 \) and \( \Delta f(s_1) := f(s_1|\theta_H + i^*_{H1}) - f(s_1|\theta_L + i^*_{L1}) \). Comparing manager 1’s first-order conditions in (13), we have
\[
\int_{\hat{s}}^{\infty} \left[ \Phi'(s_1)\kappa(\pi_H'(s_1)) + \Phi(s_1)\kappa'(\pi_H'(s_1))\pi_H''(s_1) \right] \Delta f(s_1) ds_1
\]
\[
= \int_{\hat{s}}^{\infty} \left[ \Phi'(s_1)\kappa(\pi_H'(s_1)) + \Phi(s_1)\kappa'(\pi_H'(s_1))\pi_H''(s_1) \right] \Delta f(s_1) ds_1
\]
\[
+ \int_{-\infty}^{\hat{s}} \left[ \Phi'(s_1)\kappa(\pi_H'(s_1)) + \Phi(s_1)\kappa'(\pi_H'(s_1))\pi_H''(s_1) \right] \Delta f(s_1) ds_1
\]
\[
= \int_{\hat{s}}^{\infty} \left[ \Phi'(s_1)\kappa(\pi_H'(s_1)) + \Phi(s_1)\kappa'(\pi_H'(s_1))\pi_H''(s_1) \right] \Delta f(s_1) ds_1
\]
\[
- \int_{\hat{s}}^{\infty} \left[ \Phi'(2\hat{s} - s_1)\kappa(1 - \pi^*_H(s_1)) + \Phi(2\hat{s} - s_1)\kappa'(1 - \pi^*_H(s_1))\pi_H''(s_1) \right] \Delta f(s_1) ds_1
\]
\[
= \int_{\hat{s}}^{\infty} \Phi'(s_1)[\kappa(\pi_H'(s_1)) - \kappa(1 - \pi^*_H(s_1))]\Delta f(s_1) ds_1
\]
\[
+ \int_{\hat{s}}^{\infty} \left[ \Phi(s_1)\kappa'(\pi_H'(s_1)) - \Phi(2\hat{s} - s_1)\kappa'(1 - \pi^*_H(s_1))\pi_H''(s_1) \right] \Delta f(s_1) ds_1.
\]
For the second equality, we have used change of variables, (A4) in the proof of Proposition 1, and Assumption 4. The third equality is due to \( \Phi'(s_1) = \Phi'(2\hat{s} - s_1) \) since
\[ \dot{s} = (\theta_H + \dot{i}_{H2} + \theta_L + \dot{i}_{L2})/2 \]

in symmetric equilibrium. The first integral is positive since \( \Phi' > 0 \), \( \kappa(\pi_H^*(s_1)) > \kappa(1 - \pi_H^*(s_1)) \), and \( \Delta f(s_1) > 0 \). The second integral is positive given (14).

\[ \quad \blacksquare \]

**Proof of Proposition 4:**

First, observe that if the inequality in (16) or (20) binds while the budget constraint (15) or (19) is lax, then

\[ \frac{\partial k_1(s_1 + \delta, s_2|i_H, i_L)}{\partial \delta} \bigg|_{\delta=0} = \frac{\partial k_1^n(s_1 + \delta, s_2|i_H, i_L)}{\partial \delta} \bigg|_{\delta=0} = \kappa'(\pi_H(s_1|i_H, i_L)). \]

Furthermore, conditional on \((i_H, i_L)\), this case (i.e., the inequality in (16) or (20) binds while the budget constraint is lax) holds for the same set of realized signals \((s_1, s_2)\) regardless of \(N\). Second, observe that conditional on \((i_H, i_L)\), the inequality in (16) or (20) and the budget constraint both bind only for a measure zero set of \((s_1, s_2)\).

Consider now the remaining case where the inequality in (16) or (20) is strict, in which case the budget constraint has to be binding. Note that this case takes place for identical sets of \((s_1, s_2)\) for all \(N\). Start with \(N = 1\). Denoting \( \frac{\partial k_1(s_1, s_2)}{\partial \delta} = \frac{\partial k_1(s_1 + \delta, s_2|i_H, i_L)}{\partial \delta} \bigg|_{\delta=0} \) for notational brevity, by differentiating both sides of (16) with respect to \(\delta\), we get

\[ \pi'_H(s_1|i_H, i_L)\Delta \theta y'(k_1) + (\theta_L + \pi_H(s_1|i_H, i_L)\Delta \theta) y''(k_1) \frac{\partial k_1(s_1, s_2)}{\partial \delta} = (\theta_L + \pi_H(s_2|i_H, i_L)\Delta \theta) y''(k_2) \frac{\partial k_2(s_1, s_2)}{\partial \delta}. \]

Since the budget constraint binds in this case, we have \( \frac{\partial k_1(s_1, s_2)}{\partial \delta} + \frac{\partial k_2(s_1, s_2)}{\partial \delta} = 0 \). Thus

\[ \frac{\partial k_1(s_1, s_2)}{\partial \delta} = -\pi'_H(s_1|i_H, i_L)\Delta \theta y'(k_1) \frac{\partial k_1^n(s_1, s_2)}{\partial \delta}. \]

Next, for \(N > 1\), denote \( \frac{\partial k_2(s_1, s_2)}{\partial \delta} = \frac{\partial k_2(s_1 + \delta, s_2|i_H, i_L)}{\partial \delta} \bigg|_{\delta=0} \) for notational brevity. By differentiating both sides of (20) for \(n = m\) with respect to \(\delta\), we get

\[ \pi'_H(s_1|i_H, i_L)\Delta \theta y'(k_1^n) + (\theta_L + \pi_H(s_1|i_H, i_L)\Delta \theta) y''(k_1^n) \frac{\partial k_1^n(s_1, s_2)}{\partial \delta} = (\theta_L + \pi_H(s_2|i_H, i_L)\Delta \theta) y''(k_2^n) \frac{\partial k_2^n(s_1, s_2)}{\partial \delta}. \]

Since \( \frac{\partial k_1^n(s_1, s_2)}{\partial \delta} \) is the same for all \(m \in \{1, 2, \cdots, N\} \setminus \{n\} \) and so is \( \frac{\partial k_2^n(s_1, s_2)}{\partial \delta} \) for all \(m \in \{1, \cdots, N\} \), the budget constraint implies

\[ \frac{\partial k_1^n(s_1, s_2)}{\partial \delta} + (N - 1) \frac{\partial k_1^n(s_1, s_2)}{\partial \delta} + N \frac{\partial k_2^n(s_1, s_2)}{\partial \delta} = 0. \]
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for any \( m \neq n \). Thus differentiating both sides of (20), with \( n \) and \( m \) swapped, with respect to \( \delta \) leads to

\[
(\theta_L + \pi_H(s_1|i_H, i_L)\Delta \theta) y''(k^m_1) \frac{\partial k^m_1(s_1, s_2)}{\partial \delta} = (\theta_L + \pi_H(s_2|i_H, i_L)\Delta \theta) y''(k^m_2) \frac{\partial k^m_2(s_1, s_2)}{\partial \delta}.
\]

Rearranging terms, we have

\[
\frac{\partial k^m_1(s_1, s_2)}{\partial \delta} = \frac{\theta_L + \pi_H(s_1|i_H, i_L)\Delta \theta}{-\pi_H(s_1|i_H, i_L)\Delta \theta y''(k^m_1)} \times \left[ y''(k^m_1) + \frac{(\theta_L + \pi_H(s_2|i_H, i_L)\Delta \theta)y''(k^m_2)y''(k^m_1)}{(N-1)(\theta_L + \pi_H(s_2|i_H, i_L)\Delta \theta)y''(k^m_1) + N(\theta_L + \pi_H(s_1|i_H, i_L)\Delta \theta)y''(k^m_1)}\right]^{-1},
\]

which increases as \( N \) increases. This result straightforwardly extends to the case when \( \theta_1 = \theta_H \). Therefore, given the putative levels \((i_H, i_L)\), the marginal benefit for the manager of division 1 (or its copy) at \( i = i_t \) conditional on \( \theta_1 = \theta_t, t = H, L \), strictly increases as \( N \) increases.

References


