MONASH University

School of Mathematics

2020 / 2021 SUMMER VACATION RESEARCH PROJECTS

Summer Vacation Period **07 December 2020 - 19 February 2021**

Applications are open for the 2020 / 2021 AMSI and Monash summer vacation scholarships

Please contact the supervisor for more details, prerequisites and to obtain the ***required*** formal letter of support, ***before*** applying.

How to apply

What does this AMSI/Monash scholarships offer me?

- the chance to work on a real research project for six weeks
- travel and accommodation to attend the *AMSICONNECT* student conference in Melbourne
- a six-week award of $350/week (total $2,100)

Eligibility

These scholarships are open to students who:

- are currently enrolled at an AMSI Member university
- are in their third year doing a major in the mathematical sciences (outstanding second year students with the support of their department may apply)
- have a strong academic record
- intend to go on to honours and/or postgraduate study in the mathematical sciences, this includes students doing joint degrees that include mathematics and statistics.

To apply visit: http:vrs.amsi.org.au/

Applications close: 30 September 2020 (AMSI) and 02 October, 2020 (Monash)

IMPORTANT application information for students applying through AMSI website

There are two steps to this application process, namely that you are required to **submit *two applications*** for a scholarship:

1. First application to AMSI at http:vrs.amsi.org.au/ .
2. Second application to Monash University at https://www.monash.edu/students/scholarships/current/research-projects (Monash applications can be made online from Monday 7 Sep - Friday 2 Oct 2020)

If you are unsuccessful with the AMSI application, then you will still be considered for the Monash University scholarship, which gives you a second chance. Scholarships offered, by *either* AMSI or Monash, are administered by Monash University, including your weekly payments.

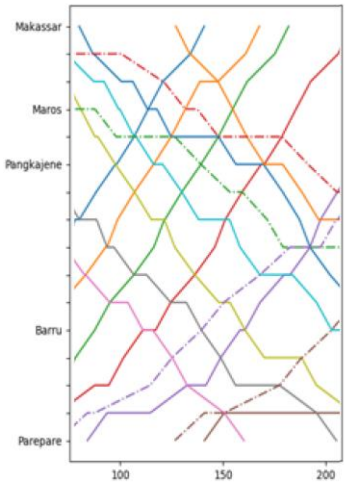
AMSI and Monash summer vacation scholarship offers

Successful AMSI scholarship applicants will receive an offer directly from AMSI, and then a subsequent offer from Monash for the same project (not a second scholarship). **You must accept *both* offers.**

Projects

1. *Taking the fast train to Makassar*

Project title: Single track scheduling for the Makassar railway line.
Supervisor/s: Prof Andreas Ernst & Dr Simon Bowly



Prerequisites: Some programming skills required. At least one of MTH3330 or MTH3170.

This project will look at the railway line between Makassar & Pare-Pare in the province of South Sulawesi, Indonesia. This railway line is currently being built to carry both slow freight trains and fast (150km/hr) passenger trains. However, many parts of the railway are just single track with passing loops at occasional stations. This means both the overtaking of trains in the same direction and the passing of trains in opposite directions has to be scheduled very carefully. This project will develop optimisation models and scheduling algorithms to try to maximise the number of trains that can be scheduled on this new railway. This project combines discrete mathematics (directed graphs), computational mathematics (optimisation) and a practical application focus.

2. *Optimal classification trees in machine learning*

Project title: Optimal classification trees in machine learning
Supervisor/s: Prof Andreas Ernst

Most forms of machine learning involves solving an optimisation problem to find the best fit between a mathematical model and a collection of training data. Typically the optimisation problem is solved heuristically (that is in a somewhat ad-hoc manner). A recent paper showed that for a class of models called classification trees, the fitting of the model to the data can be optimally using integer programming approaches. The advantage is with a better fitting, models of the same complexity tend to produce a more accurate classification. However, the integer programs that arise in this application are challenging to solve, particularly for big data sets. This project will look at improving the performance through better integer programming formulations and smarter algorithms.

Prerequisites: Solid programming skills required. At least one of MTH3330, MTH3310 or MTH3170.

3. *Data-driven PDE-constrained Methods for Subsurface Identification*

Project title: Data-driven PDE-constrained Methods for Subsurface Identification
Supervisor/s: Prof Santiago Badia and Dr Tiangang Cui

Prerequisites: Basic knowledge of numerical methods for PDEs and numerical linear algebra, e.g., to have completed MTH2051/MTH3051, MTH3320, or similar units. Some programming experience (in Julia, Python, Matlatb, etc) is an asset.

The direct current resistivity (DCR) method is one of the most widely used nonintrusive technologies in identifying and assessing valuable natural resources in the subsurface. In a DCR campaign, the electrodes are placed in many different positions, then a large number of data (in the form of electrical potential fields) measured at the electrodes are used for identifying subsurface properties. At the heart of the DCR, there is a mathematical challenge: one needs to integrate all these measurements with a forward PDE simulator of the DCR process (which maps subspace properties to electrical potential fields) and apply inversion techniques to extract meaningful information about the physical properties of the subsurface.

In this project, the student will collaborate in the development of a new computational framework to address this challenge. The framework will take DCR measurements as the input, blend the measurements together with forward PDE solvers under the Bayesian inference framework, and then issue subsurface identifications and quantify the uncertainty of the results. We will investigate new computational linear algebra methods and machine learning tools to accomplish this goal, having in mind the reduction of computational cost and parallelization of computations. The framework should also be generalisable to other statistical learning tasks governed by PDE models.

The computational framework will rely on a set of Bayesian inference algorithms developed by Dr Cui and a PDE solver provided by Prof. Badia [https://github.com/gridap/Gridap.jl].

4. *Embedded virtual element methods for numerical PDEs*

Project title: Embedded virtual element methods for numerical PDEs
Supervisors: Prof Santiago Badia

One of the main bottlenecks in computational science and engineering when dealing with the numerical approximation of partial differential equations (PDEs) in complex geometries is the generation of suitable body-fitted meshes. The meshing step is poorly automatized (requires intensive human intervention) and scales poorly on distributed memory supercomputers. The human intervention is particularly problematic when dealing with stochastic geometry simulations, e.g., in multilevel Monte Carlo methods, since thousands of samples must be generated on different geometries at different mesh resolutions. In this project, the student will explore the use of unfitted discretization methods that consider non-body-fitted meshes, i.e., meshes that are not aligned with the geometry. This approach drastically simplifies the meshing step, but poses new challenges to the discretization part, e.g., integration on cut cells and ill-conditioning issues.

In this project, the student will learn and implement virtual element methods, a quite recent approach to deal with arbitrary cell topologies, which will be combined with level set methods to implicitly describe surfaces. Next, the student will apply the methods to approximate elliptic PDEs (e.g, linear elasticity) and analyze the robustness of the algorithms with respect to surface/background mesh intersection (e.g., bounds for condition numbers). The student will learn basic Julia programming and use/develop Gridap [https://github.com/gridap/Gridap.jl], a framework for the grid-based approximation of PDEs.

Prerequisites: Basic knowledge of numerical methods, e.g., to have completed MTH2051/MTH3051 or similar unit. Some programming experience (in Julia, Python, Matlatb, etc) is an asset.

5. *Topics in geometric partial differential equations*

Project title: Topics in geometric partial differential equations.
Supervisor: Dr Julie Clutterbuck

Geometric shapes that arise in nature (such as liquid droplets, the shape of a hanging thread, or biomembranes) can be understood as minimisers of an energy, and as such often are solutions to partial differential equations. In this project we will examine some physical energies and the geometries they give rise to.

Prerequisites: Real analysis is essential.

6. *The fluid dynamics of forming ores in magmas*

Project title: The fluid dynamics of forming ores in magmas
Supervisor: Dr Anja Slim

Over 50% of the world's nickel and 96% of the world's platinum are found in magmatic sulphide deposits, which form from droplets of sulphide-rich liquid that develop in magma and concentrate in particular locations. Finding new resources for these valuable metals requires understanding the fluid dynamical processes involved in the evolution, transport and deposition of these droplets in a crystallising magma. There are a number of interesting problems that could be explored. Please contact Anja for further information.

Prerequisites: MTH3360 Fluid dynamics or equivalent

7. *Volcanoes, thunderstorms, ocean overflows and oil spills*

Project title: Volcanoes, thunderstorms, ocean overflows and oil spills.
Supervisor: Dr Anja Slim

An important and common flow is an intrusion, in which fluid flows predominantly horizontally because of a difference in density between it and the surrounding fluid. Examples in nature include volcanic ashclouds, thunderstorm outflows and ocean overflows. The Deepwater Horizon disaster a decade ago involved a similar flow that dispersed oil extensively deep in the Gulf of Mexico. A number of features of these flows are not fully understood and several different aspects, either theoretical or numerical could be explored in a summer project. Please contact Anja for further information.

Prerequisites: MTH3360 Fluid dynamics or MTH3011 or equivalent; or proficiency with coding and scripting

8. *Geometry and knot theory*

Project title: Geometry and knot theory
Supervisor: A/Prof Jessica Purcell

We will investigate problems in the overlap of knot theory and geometry. For example, the complement of a knot often can be given a metric, and it obtains metric information such as volume. There are many open questions about the relationship of the volume of the knot to other invariants. We will study a question along these lines for families of knots or links.

Prerequisites: This project is open to students who have taken a second year mathematics unit with a strong foundation in proofs, for example analysis or algebra.

9. *Hadamard matrices and perfect sequences*

Project title: Hadamard matrices and perfect sequences
Supervisor: Dr Santiago Barrera Acevedo

Hadamard matrices and perfect sequences are combinatorial objects with applications in cryptography and information security. Recently an interesting connection between Hadamard matrices and perfect sequences over quaternions was established. In this project we aim to understand and use this connection to construct Hadamard matrices from families of perfect sequences over quaternions and vice versa.

Prerequisites: The project is open to students who have completed at least two years of their undergraduate degree and are interested in pursuing honours in Pure Mathematics. Algebra and Number Theory I is essential. Some experience with computer programming would be desirable, but not essential.

10. *The percolation on cellular automata*

Project title: The percolation on cellular automata
Supervisor/s: Dr Andrea Collevecchio & A/Prof Kais Hamza

Abstract. Consider the following self-organized system on $Z^{+}\times Z^{+}$. The vertices are labelled using the familiar euclidean coordinates. To each vertex we assign a value in {-1,1} according to the following rule. The value at vertex (i,j) is the product of the values at $(I, j - 1)$ and $(i -1, j)$. The system is well defined once we provide the boundary conditions, which are the following. The vertices with coordinates $(1, j)$, with $j \geq 1$ have all the same values. The values of the vertices of the form $(i, 1)$, with $i \geq 1$ are generated at random according to a certain rule. This project investigates the main features of this system.

Prerequisites: A strong background in probability and stochastic processes is necessary for this project. You must have completed at least the unit MTH3241 (or equivalent).

Duration and period: 4 to 6 weeks in January and/or February.

11. *Nash Equilibria in random games*

Project title: Nash Equilibria in random games
Supervisor/s: Dr Andrea Collevecchio

Abstract: we study the geometry of Nash Equilibria in games where payoffs are generated at random. We use methods from Probability, Game Theory, Statistical Mechanics.

Prerequisites: A strong background in probability and stochastic processes is necessary for this project. You must have completed at least the unit MTH3241 (or equivalent).

12. *Topics in discrete probability and statistical mechanics*

Project title: Topics in discrete probability and statistical mechanics
Supervisor/s: A/Prof Tim Garoni

Additional details: Please contact Tim Garoni for project details.

13. *Structural properties of distance-regular graphs*

Project title: Structural properties of distance-regular graphs
Supervisor/s: Dr Greg Markowsky

The aim of this project is to study a number of properties of distance-regular graphs, which are a family of highly-structured graphs which are prominent in computer science and other fields. This family of graphs is highly amenable to study by linear algebra techniques, and includes many famous examples, such as the Peterson graph, Hamming graphs, Johnson graphs, and Moore graphs.

14. *Planar Brownian motion and complex analysis*

Project title: Planar Brownian motion and complex analysis
Supervisor/s: Dr Greg Markowsky

The image of a planar Brownian motion under a non-constant analytic function is again a planar Brownian motion. This fact has a large number of interesting consequences. The purpose of this project is to study this connection in depth.

Prerequisites: MTH3020 or equivalent, familiarity with Brownian motion.

15. *Intersection local times of Brownian motion and related processes*

Project title: Intersection local times of Brownian motion and related processes
Supervisor/s: Dr Greg Markowsky

Intersection local times are a measure placed on the sets of self-intersections of a stochastic process. Their study is largely an application of Fourier analysis, and there are a number of interesting open problems in the field. In this project, we will look at some of the basic facts and techniques.

Prerequisites: Familiarity with Brownian motion and stochastic processes in general.

16. *Topics on unique continuation theorems*

Project title: Topics on unique continuation theorems
Supervisor/s: Dr Wenhui Shi

Classical unique continuation theorem says that a holomorphic function on a domain vanishes identically if it vanishes in a nonempty open set. In this project we will investigate several generalizations of this result to harmonic functions and explore their applications.

Prerequisites: MTH3011 (or MTH3020) and MTH3140 are recommended

17. *Algebra and geometry of knot complements*

Project title: Algebra and geometry of knot complements
Supervisor/s: Dr Dan Mathews

Details: We will investigate some questions about the algebra, geometry and topology of knots and their complements. For example, any triangulation of a knot complement gives rise to a matrix called the Neumann-Zagier matrix, which has very interesting properties and about which there are numerous open questions.

Prerequisites: At least one second or third year pure mathematics unit, including real analysis, or algebra and number theory.

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