Estimating Local Climate Change and Variability in Australia

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Abstract:

General Circulation Models predict increasing global and regional temperatures with increasing variability. Although mitigation of climate change is global, adaptation must be local. As a complement to General Circulation Models, we apply a new method in time-series analysis to estimate climate change and climate variability from local weather data. There are challenges in using weather data. We do our best to overcome these challenges by using data from long-running weather stations on the coast of Australia. Unlike inland weather stations, some stations along the coast have changed very little over more than a century. We find that local temperatures have increased in accord with the global and regional projections, but local variability has decreased. Our research continues and we are scrutinising the results. We note, however, that General Circulation Models are not estimated from data and cannot calculate errors or measure variability. This must be done statistically. Our method for estimating climate change and variability is statistically sophisticated and our result that climate variability is decreasing along the coast of Australia may prove to be true.

Introduction

The science of climate change is settled. Our world is warming and weather is becoming more variable with more extreme events. We see the effects every day on the news. Hurricanes batter the coasts. Infernos raze the forests.

As scientists, however, we know that science is never settled. Science is always probing, always asking the next question. Some types of science are purely theoretical, some are applied. For most types of science, however, the scientific method requires a theory with testable hypotheses and testing of hypotheses against the data. In this way our understanding evolves.

The science of climate change has evolved from early beginnings in the 19th century to create General Circulation Models (Le Truet, *et. al*, 2007). The strengths of General Circulation models are in attributing the causes of climate change, projecting global and regional changes into the future, and informing policies which seek to govern the global commons and mitigate climate change (IPCC, 2014). There are limitations, however.

First, General Circulation Models simulate using finite element methods first developed by engineers to predict the internal responses of bridges, airplanes and buildings to forcings on their boundaries. Finite element methods are interpolation methods. Conditions can be imposed on the boundaries of the model, but not internally to the calculations. Parameters of the model must be calibrated rather than estimated from data. Consequently, General Circulation Models are deterministic with no formal statistical method for calculating errors, estimating variability or testing hypotheses.

Second, General Circulation Models simulate global and regional climate changes but adaptation is local. To inform adaptation, global and regional projections must be downscaled to local areas. This is a demanding task that the climate science community has left largely to decision-makers (AARC, 2014; CSIRO and Bureau of Meteorology, 2015).

In this study, we complement General Circulation Models by applying a new statistical method to time-series of weather data. The strengths of this method are formal hypothesis tests, local forecasts of climate change and variability and better information for adaptation. Just as there are limitations to General Circulation Models, however, there are limitations to a statistical approach.

First, weather data can say very little about global warming. Statistical estimation can detect climate change and variability in a local area but may not help in attributing the causes of climate change nor in designing mitigation policies.

Second, weather stations change. In Australia, rainfall and temperature were collected sporadically following European settlement and more systematically since the 1850s (Bureau of Meteorology, 2017a). The weather station at the Melbourne Regional Office began collecting data in 1855 but methods were undocumented prior to 1908 and the station was closed in 2015. The station in Sydney at Observatory Hill opened in 1858, was never moved but is now encircled by the Cahill Expressway on-ramp to the Sydney Harbour Bridge. Weather stations in other capital cities, a few regional centres and rural towns have collected data since the Australian Bureau of Meteorology was established in 1908 and methods were standardised in 1910. Some weather stations have been poorly tended, stations once at Post Offices were moved to airports and, unfortunately, documentation prior to 1998 is incomplete (Bureau of Meteorology, 2017b). In summary, weather data may measure changes in weather stations rather than changes in climate.

To adjust for changes, weather bureaus around the world are 'homogenising' the data. Purposeful adjustment, however, destroys the statistical properties of data and invalidates any hypothesis tests. Therefore, this study uses raw data. The effects of documented changes in weather stations are estimated, tested for significance and subtracted from the estimates of climate change and variability.

The stations at Cape Leeuwin in Western Australia, Robe in South Australia, Cape Otway in Victoria, Hobart (Ellerslie Road) in Tasmania, and Moruya Heads Pilot Station in New South Wales have remained in place with very little development around them. The documented changes include a standard sun-screen for thermometers around 1910 and a switch to automated weather stations in the 1990s (Bureau of Meteorology, 2017b). Out of curiosity, stations with little development are compared to Observatory Hill in the heart of Sydney. Figure 1 shows the location and year of first available data for the weather stations. All these stations are in southern Australia which is projected to be seriously affected by climate change (IPCC, 2013). By coincidence, these stations are along the coasts and are buffered by the oceans, which may be an advantage in estimating long-term climate change and variability.

Figure 1: Location of Weather Stations and Year of First Available Data.
(Source: Bureau of Meteorology, 2017b)

This study also uses a new statistical method for time-series analysis we have developed and reported elsewhere (Hertzler, 2017). Unlike other methods for estimating stochastic processes, our new method includes deterministic time trends. Unlike other methods for estimating deterministic trends, our new method does not assume stationarity. Assuming stationarity eliminates the underlying stochastic process from the estimation. A rapidly changing and highly variable process cannot be distinguished from a slowly changing but stable process. A stochastic process is never stationary, however, and interacts with deterministic time trends. It is impossible to consistently estimate one without the other. Our new method gives unbiased and consistent estimates of both the underlying stochastic process and deterministic trends for climate change and variability.

Generalized Ornstein-Uhlenbeck Process with Trends in Location and Scale

Arithmetic Brownian Motion (Bachelier, 1900) is non-ergodic and, hence, non-stationary. The Ornstein-Uhlenbeck Process (Ornstein and Uhlenbeck 1930) is ergodic and may become stationary, given an infinite amount of time to converge. Both processes are easily combined into a single stochastic process with constant location and scale (Kolmogorov, 1931). This process might be called a Generalized Ornstein-Uhlenbeck Process. Elsewhere, we have generalised this process even further to make the location and scale into functions of time (Hertzler, 2017). We have called this new process a Generalized Ornstein-Uhlenbeck Process with Trends in Location and Scale. We also provided proofs for the transition density, transition probability, transition likelihood, chi-square statistics and a pseudo-R^2 statistic. Here we present the transition likelihood, chi-square statistic and the pseudo-R^2 statistic for use in estimating a stochastic process with deterministic trends for climate change.

Transition Likelihood

The Generalized Ornstein-Uhlenbeck Process with Trends in Location and Scale has the transition likelihood

$$L(\delta,\rho,\mu,\sigma) = \frac{1}{(2\pi)^{n/2}} \prod_{i=1}^{n} \frac{\partial \varphi_i}{\partial y_i} e^{-1/2 \varphi_i^2(\delta,\rho,\mu,\sigma)};$$

where

$$\varphi_i(\delta,\rho,\mu,\sigma) = \frac{y_i - G_i(\delta,\rho,\mu)}{\left(H_i^2(\sigma)\right)^{1/2}};$$

$$\frac{\partial \varphi_i}{\partial y_i} = \frac{1}{\left(H_i^2(\sigma)\right)^{1/2}};$$

and

$$G_i(\delta,\rho,\mu) = x_i e^{-\rho(t_i - s_i)} + \rho e^{-\rho t_i} \int_{s_i}^{t_i} e^{\rho\tau} \mu(\tau) d\tau + \delta \frac{1 - e^{-\rho(t_i - s_i)}}{\rho};$$

$$H_i^2(\sigma) = e^{-2\rho t_i} \int_{s_i}^{t_i} e^{2\rho\tau} \sigma^2(\tau) d\tau.$$

On the left-hand side of the first equation above, the transition likelihood, L, depends upon parameters δ, ρ, μ, and σ, where δ is the rate of divergence, ρ is the rate of convergence, μ is the location and σ is the scale. Location and scale may be functions of time. The special case with constant location and scale and with rate $\rho = 0$ is Arithmetic Brownian Motion; the special case with constant location and scale and with rate $\delta = 0$ is the Ornstein-Uhlenbeck Process. On the right-hand side of the first equation, the transition likelihood is a product which depends upon weighted residuals, φ_i, and derivatives of the residuals.

On the right-hand side of the second equation, a weighted residual is calculated as an unweighted residual divided by a covariance. The unweighted residual is $y_i - G_i$, where y_i is an observation of the forward variable and G_i is the mean as a function of δ, ρ, and μ. The covariance is H_i^2 as a function σ^2. On the right-hand side of the third equation, the derivative of a weighted residual is the square-root of the inverse of the covariance.

On the right-hand side of the fourth equation, the mean is an integral over time from backward time s_i to forward time t_i. Within the integral, the initial condition is backward variable x_i and location μ is a function of time. On the right-hand side of the fifth equation, the covariance is also an integral over time. Within the integral, the scale squared, σ^2, is a function of time.

The Generalized Ornstein-Uhlenbeck Process with Trends in Location and Scale is a linear transformation of a Levy Process (Lévy, 1937) which has independent increments over time. Hence, the integrals over time for mean G_i and covariance H_i^2 are independent and weighted residuals φ_i are independent. Therefore, the likelihood for a time-series sampled n times is the product of the likelihoods for all φ_i.

Data for estimating the parameters of the likelihood are observations for y_i and x_i, but also for t_i and s_i. Because times are part of the data, there are no problems with missing observations. Instead, the elapsed time between observations are used in calculating the likelihood.

Hypothesis Tests

Define the likelihood ratio as

$$\frac{L_0}{L_1} = \frac{L(\delta_0, \rho_0, \mu_0, \sigma_0)}{L(\delta_1, \rho_1, \mu_1, \sigma_1)};$$
$$0 \le L_0 \le L_1 \le 1;$$
$$k_0 < k_1.$$

Likelihood L_0 is for the null hypothesis and likelihood L_1 is for the alternate hypothesis. The number of parameters in each likelihood are k_0 and k_1. Likelihood L_0 must not exceed L_1 and have strictly fewer parameters. Define the likelihood ratio statistic

$$V = -2(\ln L_0 - \ln L_1);$$
$$0 \le V < +\infty.$$

If the null hypothesis is true, the transition density for V collapses to an invariant density

$$p(V) = \frac{V^{\frac{k_1 - k_0}{2} - 1}}{2^{\frac{k_1 - k_0}{2}} \Gamma\left(\frac{k_1 - k_0}{2}\right)} e^{-\frac{1}{2}V}$$

In the denominator, Γ is the gamma function. Thus V has an invariant $\chi^2(k_1 - k_0)$ distribution. In our other paper, we prove this. In the proof, there are no assumptions of large samples or maximization and the invariant χ^2 distribution applies to any two samples from a time-series which follows a Generalized Ornstein-Uhlenbeck Process with Trends in Location and Scale or its special cases.

Goodness-of-Fit

A pseudo-R^2 statistic can be formed from the likelihoods

$$R^2 = 1 - \frac{L_0^{\frac{2}{n - k_0}}}{L_1^{\frac{2}{n - k_1}}}.$$

If likelihood L_0 equals likelihood L_1, the pseudo-R^2 statistic is small, but greater than zero because $k_0 < k_1$. As L_0 decreases or L_1 increases, the pseudo-R^2 statistic increases toward one.

The usual R^2 statistic based on sums of squared residuals indicates the goodness-of-fit for location parameters only. Maximum likelihood estimation of the Generalized Ornstein-Uhlenbeck Process with Trends in Location and Scale is a compromise between fitting the location parameters and the scale parameters. The pseudo-R^2 based on likelihoods should be a better indicator of the goodness-of-fit.

Interacting Trends and System Dynamics

The transition likelihood above has no restrictions on functional forms for the time trends. Estimation, however, requires specific functional forms. The same functional forms can be used for location and scale squared if the integrals for location and scale squared are expressed as

$$\rho e^{-\rho t}\int_s^t e^{\rho\tau}\mu(\tau)\,d\tau = \alpha e^{-\alpha t}\int_s^t e^{\alpha\tau}\beta(\tau)\,d\tau;$$
$$e^{-2\rho t}\int_s^t e^{2\rho\tau}\sigma^2(\tau)\,d\tau = e^{-\alpha t}\int_s^t e^{\alpha\tau}\beta(\tau)\,d\tau.$$

The first equation is for location with $\alpha = \rho$ and the second equation is for scale squared with $\alpha = 2\rho$. The deterministic time trend in both equations is $\beta(\tau)$. The integrals equal zero when $t - s = 0$ and solutions lose their connection to s as $t - s \to \infty$. If rate $\rho = 0$, the solutions for location are all zero but the solutions for scale squared collapse to the integral of the trends over time. If rate $\rho \to \infty$, the trends must also go to infinity in a way that $\beta(\tau)/\alpha$ remains finite. These conditions will become clear for exponential and cyclical trends in location and scale squared.

Exponential Trends

The function $\beta(\tau)$ can be exponential growth or decay

$$\beta(\tau) = \beta_0 e^{\beta_1(\tau - \beta_2)}.$$

Parameter β_0 is the intercept, parameter β_1 is the rate of change and parameter β_2 is the time shift for the start of the trend. A constant is a special case if the rate of change is zero. The integral has the solution

$$e^{-\alpha t}\int_s^t e^{\alpha\tau}\beta(\tau)\,d\tau = \frac{\beta_0}{\alpha+\beta_1}\left(e^{\beta_1(t-\beta_2)} - e^{-\alpha(t-s)+\beta_1(s-\beta_2)}\right).$$

Cyclical Trends

The function $\beta(\tau)$ can model cycles, using cosines for example

$$\beta(\tau) = \beta_0 \cos\left(\tfrac{2\pi}{\beta_1}(\tau-\beta_2)\right).$$

Parameter β_0 is the amplitude of the cycle, parameter β_1 is the length of the cycle and parameter β_2 is a time shift for the peak of the cycle. If the length of the cycle becomes infinite, the cosine goes to one and a constant is a special case. The solution for the integral is

$$\begin{aligned} e^{-\alpha t}\int_s^t e^{\alpha\tau}\beta(\tau)d\tau = \frac{\beta_0}{\alpha^2+(2\pi/\beta_1)^2}\Big[\alpha\cos\left(\tfrac{2\pi}{\beta_1}(t-\beta_2)\right)+\tfrac{2\pi}{\beta_1}\sin\left(\tfrac{2\pi}{\beta_1}(t-\beta_2)\right) \\ -e^{-\alpha(t-s)}\left(\alpha\cos\left(\tfrac{2\pi}{\beta_1}(s-\beta_2)\right)+\tfrac{2\pi}{\beta_1}\sin\left(\tfrac{2\pi}{\beta_1}(s-\beta_2)\right)\right)\Big]. \end{aligned}$$

For comparison, stationarity can be modelled by assuming $t-s\to\infty$. The terms containing s are eliminated from the time trends. Parameters α and β become confounded and an assumption about α is required to estimate β. Usually α is chosen for the special case of a Normal Process in which rate $\rho = 1/2$.

Estimation

General Circulation Models project that Australia will be seriously affected by climate change (IPCC, 2013). Here, we estimate climate change and variability as time trends in the Generalized Ornstein-Uhlenbeck Process with Trends in Location and Scale. Then we formally test the hypotheses that temperatures are rising and becoming more variable.

As a first example, Figure 2 shows 105 years of maximum daily temperatures at the Cape Leeuwin lighthouse on the southwestern tip of Australia. The data are raw with no 'homogenisation'.

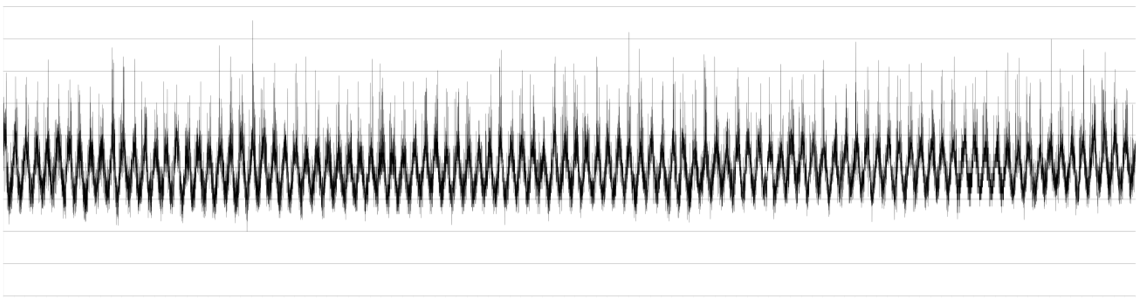

Figure 2: Maximum Daily Temperatures at Cape Leeuwin lighthouse in Western Australia, C°, 1/1/1910-12/31/2014.
(Source: Bureau of Meteorology, 2017b)

To the naked eye, maximum daily temperatures are seasonal, variability also appears to be seasonal and there may be long term trends due to climate change. Any effects of changes in the weather station are indiscernible. Official records began on 1 January 1910, the mercury thermometer was replaced by an automated weather station on 3 April 1993 and the station was moved a few metres further from buildings on 14 April 1999 (Bureau of Meteorology, 2017b). These changes are included with the raw data as categorical (dummy) variables.

Table 1 presents estimates for parameters of the Generalized Ornstein-Uhlenbeck Process with exponential and cyclical trends in both location and scale. In the upper right of the table, the pseudo-R^2 statistic compares the full model to a stationary process with constant location and scale. The full model explains 68.5% of the variation in a sample of 38,156 observations by estimating 18 parameters.

Near the upper left of the table, the rate of divergence, δ, is estimated to be zero and the rate of convergence, ρ, is estimated to be 256.225. This is in units of days/year, the equivalent daily rate being 0.701509. For comparison, a Normal Process would have a daily rate of 0.5. In the column to the right of each parameter, the p-values from a $\chi^2(1)$ distribution test the simple null hypotheses that each parameter is zero. The probability is one that $\delta = 0$ and the probability is zero that $\rho = 0$. Therefore, the stochastic process is ergodic and converges toward deterministic trends in location and scale squared.

Table 1: Estimates for Maximum Daily Temperatures at Cape Leeuwin.

Rates				pseudo-R^2	0.685
δ	0	1		n	38,156
ρ	256.225	0		k	18
Location			**Scale Squared**		
Exponential Trend			*Exponential Trend*		
μ_{93}	-0.164202	0.020	σ_{93}	-241.062	6.9e-4
μ_{99}	-0.0220979	0.81	σ_{99}	-191.857	0.033
μ_0	19.4647		σ_0	3139.15	
μ_1	0.000400554	0	σ_1	-0.00247284	0
μ_2	1910		σ_2	1910	
Cyclical Trend			*Cyclical Trend*		
μ_{93}	0.189988	0.012	σ_{93}	0	1
μ_{99}	0.185034	0.15	σ_{99}	0	0.99
μ_0	3.40374	0	σ_0	1103.81	0
μ_1	1		σ_1	1	
μ_2	0.0971118		σ_2	0.0697729	

Lower in the table, location on the left and scale squared on the right have exponential and cyclical trends. Not all of the parameters are estimated; the exponential trends start in 1910 and the lengths of the cycles are 1 year. Not all of the parameters are tested against zero; location and scale squared must at least be constant and a cycle may start at any time rather than the start of a calendar year. The p-values for these parameters are blank.

Parameters μ_0 and σ_0 are the reference parameters for the automated weather station in the recent period. Under location, the reference temperature, adding and subtracting the amplitude of its annual cycle, is 19.4647 ± 3.40374 degrees. Under scale squared, the reference variability, adding and subtracting the amplitude of its annual cycle, is 3139.15 ± 1103.81 degrees squared. Parameters μ_{93} and σ_{93} are for a mercury thermometer in the period before 1993. Under location, temperatures from a mercury thermometer are significantly lower with a significantly greater annual cycle. Under scale squared, variability from a mercury thermometer is significantly less but with the same amplitude of its annual cycle. Parameters μ_{99} and σ_{99} are for the automated weather station in the period before it was moved in 1999. Under location, moving the weather station caused no significant differences in temperatures or its annual cycle. Under scale squared, however, moving the weather station significantly increased variability but with the same annual cycle. Parameters μ_2 and σ_2 are for the start of the trends. Curiously, the cycle in scale squared starts about 0.027 years or 10 days before the cycle in location. Although not shown in the table, the hypothesis of equal starting times $(\mu_2 = \sigma_2)$ has a p-value of zero and can be rejected.

More importantly, in the exponential trends for location and scale squared, rates μ_1 and σ_1 estimate climate change and variability. Maximum temperatures are estimated to grow at an annual rate of 0.000400554 but the scale squared is estimated to decline at an annual rate of 0.00247284. Both rates have p-values from $\chi^2(1)$ distributions equal to zero. Jointly they have a p-value from a $\chi^2(2)$ distribution equal to zero. The null hypothesis that there have been no changes in climate can be rejected.

Suppose temperatures over the 105 years of the sample had been measured by the current weather station. The exponential trend in location grew from 19.4647 to 20.3009 degrees, a growth of 0.836111 degrees. Also over the 105 years, the exponential trend in scale squared declined from 3139.15 to 2421.30 degrees squared, a decline of 717.851 degrees squared. For comparison with the location, the square root decreased from 56.0281 to 49.2067 degrees, a decline of 6.82142 degrees. Overall, the temperature has increased but, contrary to predictions, the variability has decreased. There are fewer extremely hot or cold days at the Cape Leeuwin lighthouse.

Table 2 tests special cases. Judging by the pseudo-R^2 statistics, the special cases appear to fit the data very well. Because of the large data set, however, the likelihood ratio statistics, *V*, are very large and the p-values from χ^2 distributions are zero. All special cases can be rejected, even the Normal Process, which would apparently fit the data almost as well as the Generalized Ornstein-Uhlenbeck Process with Trends in Location and Scale.

Table 2: Special Cases at Cape Leeuwin.

Special case#	pseudo-R^2	V	$k_1 - k_0$	p
GOUPTLS	0.685			
Normal, trends	0.680	614.785	2	0
GOUP, no trends	0.606	8,624.73	10	0
Stationary, trends	0.583	10,714.4	2	0
Stationary, constant		44,124.1	12	0

#Normal, trends $H_0 : \delta = 0, \rho = 182.624$; GOUP, no trends $H_0 : \mu_1$, $\sigma_1 = 0$ in exponential trends, $\mu_{93}, \mu_{99}, \mu_0, \mu_2, \sigma_{93}, \sigma_{99}, \sigma_0, \sigma_2 = 0$ in cyclical trends; Stationary, trends $H_0 : \delta = 0, \rho \to \infty, \mu_0 \to \infty, \mu_0 / \rho$ finite, $\sigma_0 \to \infty, \sigma_0 / 2\rho$ finite in exponential trends, $\mu_0 \to \infty, \mu_0 / \rho^2$ finite, $\sigma_0 \to \infty, \sigma_0 / 4\rho^2$ finite in cyclical trends; Stationary, constant $H_0 : \delta = 0, \rho \to \infty, \mu_0 \to \infty, \mu_0 / \rho$ finite, $\sigma_0 \to \infty, \sigma_0 / \rho$ finite, μ_1, $\sigma_1 = 0$ in exponential trends, $\mu_{93}, \mu_{99}, \mu_0, \mu_2, \sigma_{93}, \sigma_{99}, \sigma_0, \sigma_2 = 0$ in cyclical trends.

Figures 3, 4 and 5 compare the stationary process with trends, the Generalized Ornstein-Uhlenbeck Process without trends and the Generalized Ornstein-Uhlenbeck Process with Trends in Location and Scale. For illustration, only the last decade of the 105 years are shown, followed by daily forecasts for one year beyond the data. Near the top of each figure, observations y_i are compared with means G_i. Any differences are unweighted residuals. Near the bottom of each figure, covariances H_i^2, after taking the square root, are weights for the weighted residuals φ_i. In Figures 4 and 5, spikes in the covariances are due to missing observations.

In Figure 3, the stationary process is assumed to converge immediately to trends, leaving large residuals. After weighting by the covariances, however, the weighted residuals are well-behaved. Beyond the data, the forecast means and covariances of the forecasts appear reasonable. In Figure 4, the Generalized Ornstein-Uhlenbeck Process without trends converges slowly, leaving small residuals. The weighted residuals are cyclical, however. The forecast means converge to a constant location but covariances of the forecasts more than double to a constant scale. In Figure 5, the Generalized Ornstein-Uhlenbeck Process with Trends in Location and Scale is a compromise. It converges at a medium rate toward trends, leaving mediumsized residuals. The covariances are small and the weighted residuals are well-behaved. For several days, the forecast means and covariances of the forecasts are conditional upon the last observation at the end of year 2014. Subsequently, these converge to deterministic trends almost indistinguishable from those of the stationary process in Figure 3.

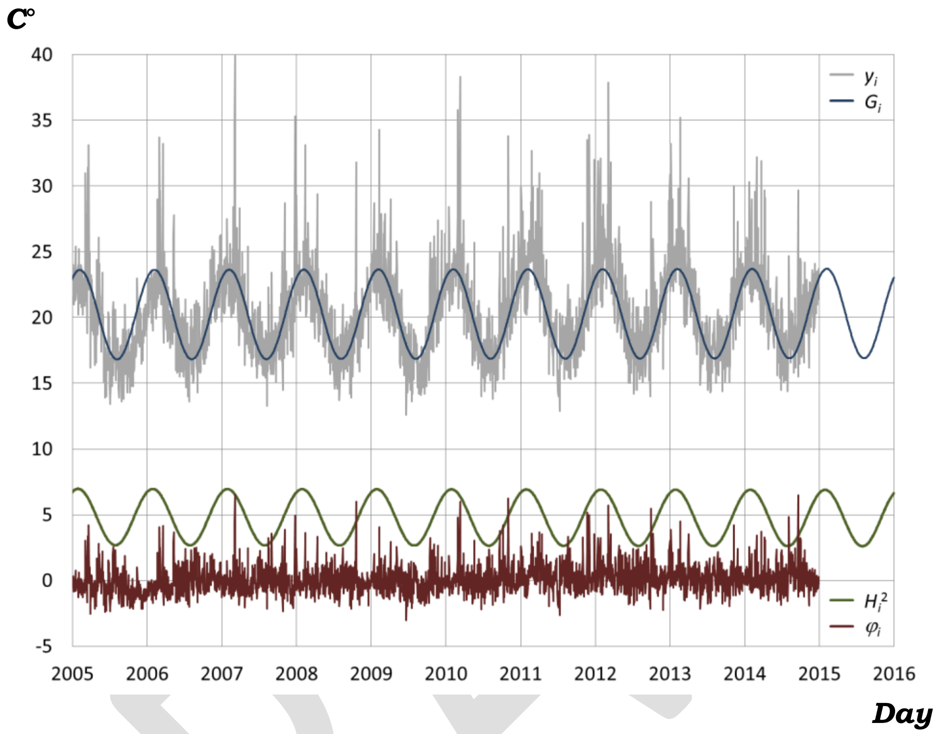

Figure 3. Stationary Process with Trends.

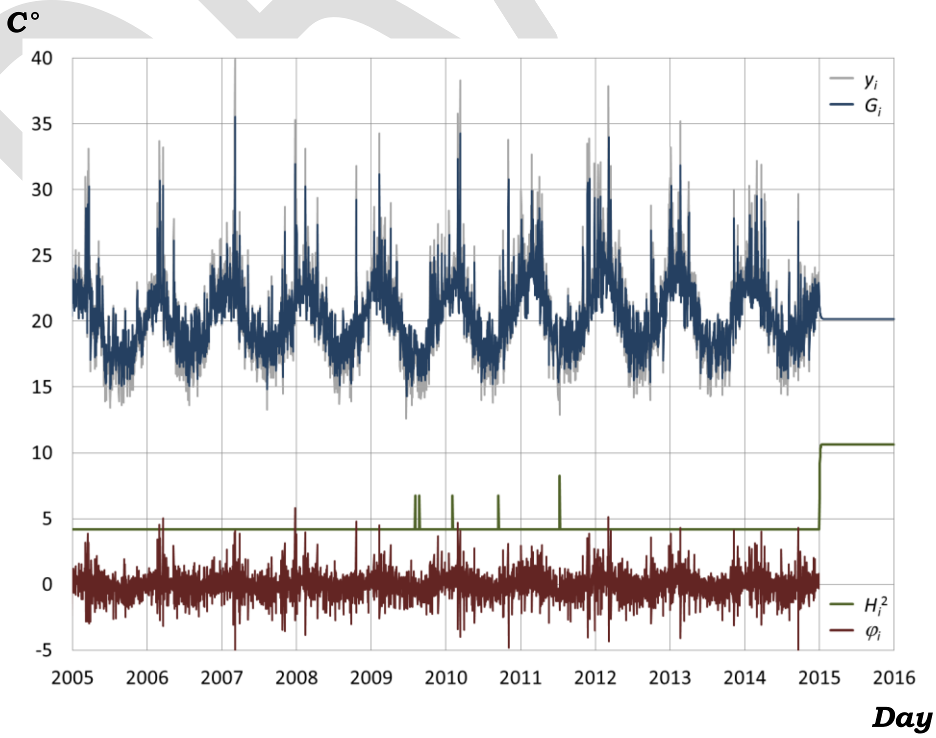

Figure 4. Generalized Ornstein-Uhlenbeck Process, without Trends.

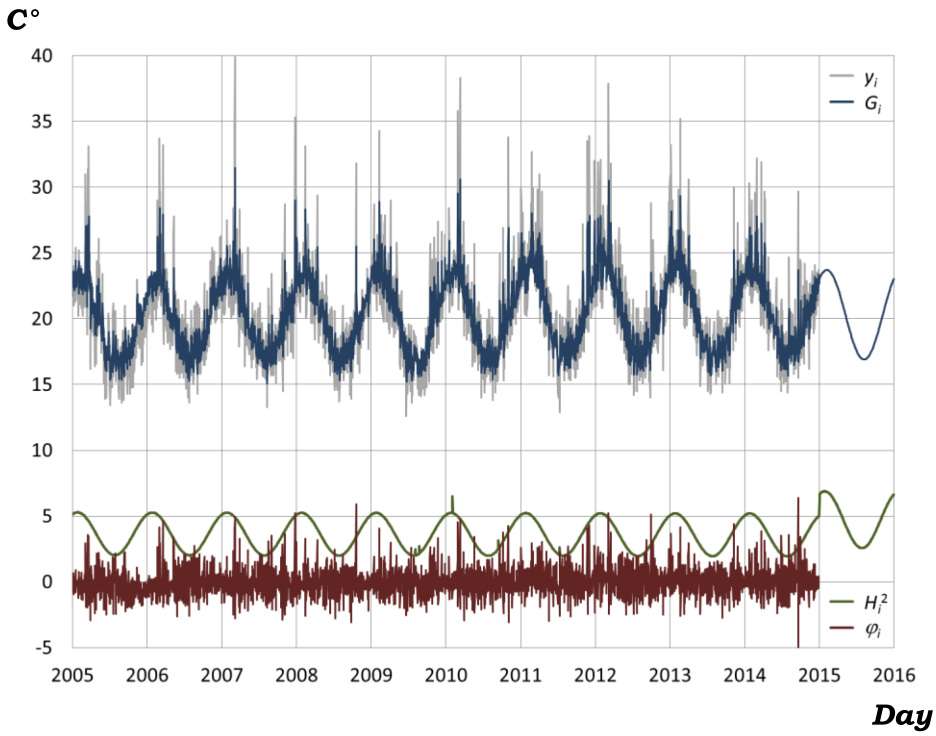

Figure 5: Generalized Ornstein-Uhlenbeck Process with Trends.

Conclusions and Further Research

We emphatically reject the null hypothesis of no climate change at Cape Leeuwin lighthouse in southwestern Australia. Over the 105 years of observations, after accounting for changes in the weather station and annual cycles, the temperature grew from 19.4647 to 20.3009 degrees, a growth of 0.836111 degrees. But there is a surprise. Over the 105 years, variability declined from 3139.15 to 2421.30 degrees squared, a decline of 717.851 degrees squared. The square root decreased from 56.0281 to 49.2067 degrees, a decline of 6.82142 degrees. Maximum daily temperatures at Cape Leeuwin lighthouse have become slightly hotter but much less variable.

Our results are preliminary and our investigations continue. The finding of declining variability requires scrutiny. In addition to raw weather data, the Australian Bureau of Meteorology also publishes 'homogenised' data (Bureau of Meteorology, 2017c). A comparison of homogenised and raw data reveals that stark changes are being made at undocumented break points. Perhaps these break points could be included in the statistical analysis to test whether undocumented changes in the weather station might account for the decreasing variability. Or perhaps, Cape Leeuwin is an isolated case. Our further work with data from other long-running weather stations in Australia may confirm or contradict these results.

Our future research will also investigate different types of time trends. Exponential trends go to infinity or zero in the very long term. Certainly temperatures will go to neither extreme. Instead, asymptotic trends in location and scale might reveal how high and how variable maximum daily temperatures will become in the long term. Future research should also investigate changes in minimum temperatures and rainfall. Because daily rainfall is bounded below by zero a new stochastic process, extending the process discovered by Feller (1951) and called the Cox-Ingersoll-Ross process in finance, will be required.

Most studies of adaptation to climate change use spatial analogues without estimating local climate change or variability. The Generalized Ornstein-Uhlenbeck Process with Trends in Location and Scale is easy to estimate using standard statistical software without special algorithms. Once estimated, the stochastic process, can be used in dynamic decision models under uncertainty. Future studies of adaptation can use our method to understand how best to adapt to climate change.

The rise in temperatures estimated in this study accords with the projections of General Circulation Models. The declining variability does not. A General Circulation Model, however, is deterministic. It has no formal statistical method for calculating errors and estimating variability. Instead, a calculation is done using the spread of projections from an ensemble of models. An ensemble of models has no statistical meaning. Meaningful statistics require model errors to be calculated and variability to be estimated using real data. Our new method for combining stochastic processes with deterministic time trends is surely a better way to estimate variability. Because our results are local, however, many more estimations are required to reach a consensus about the variability of climate change in Australia.

Or perhaps we should ask the old-timers about weather in the olden days. Weather may just seem more variable to us because of 24/7 news.

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