

Power, Energy, Dynamics - Renewable energy power from wind turbines

Lesson questions and answers

Q1: Kinetic energy of a mass of air

How can we determine the kinetic energy of a mass of air of density ρ that's moving at a speed v through a turbine of radius r ? (*Students to determine the general formula for calculating the kinetic energy of a mass of moving air*)

A1: Students will need to combine density ($\rho = \frac{m}{V}$) with the volume of air flow ($V = Av$), the area of a circle ($A = \pi r^2$) with the kinetic energy formula ($\frac{1}{2}mv^2$) to determine the kinetic energy of a mass of air as ($KE = \frac{1}{2}\pi r^2 \rho v^3$).

Q2: Blade kinetic energy

Consider a wind turbine with a span of 100m is situated at a site, subjected to a constant 8 ms^{-1} wind. If air density is 1.25 kgm^{-3} , how much kinetic energy passes through the plane of the blades every second?

A2: While we could directly substitute in the formula $KE = \frac{1}{2}\pi r^2 \rho v^3$, we are going to use a different strategy:

1. Determine area of plane
2. Determine volume of air passing through plane every second
3. Determine mass of air passing through plane every second
4. Calculate kinetic energy of the mass of air passing through the plane every second

$$A = \pi r^2$$

$$A = \pi \times 50^2$$

$$A = 7,854 \text{ m}^2$$

At 8 ms^{-1} , volume through plane

$$V = Av$$

$$V = 7854 \text{ m}^2 \times 8 \text{ ms}^{-1}$$

$$V = 62,832 \text{ m}^3 \text{ s}^{-1}$$

With density of $\rho = 1.25 \text{ kgm}^{-3}$

$$m = \rho V$$

$$m = 1.25 \text{ kg m}^{-3} \times 62,832 \text{ m}^3 \text{ s}^{-1}$$

$$m = 78,540 \text{ kg s}^{-1}$$

Kinetic energy every second

$$KE = \frac{1}{2}mv^2$$

$$KE_{/s} = \frac{1}{2}(78,540 \text{ kg s}^{-1}) \times 8^2 \text{ m}^2 \text{ s}^{-2}$$

$$KE_{/s} = 2.513 \times 10^6 \text{ J s}^{-1}$$

$$P = 2.513 \times 10^6 \text{ W} = 2513 \text{ kW}$$

Q3: Electricity in households

We want to sell this energy to households, and need a unit of measure that makes sense to consumers. This unit used is called a kilowatt-hour (kWh) and is defined as the energy delivered to a 1000 W appliance over 1 hour.

In pairs, determine how much 1 kWh is in joules.

A3: $P = \frac{E}{t}$

$$E = Pt$$

$$E = 1000 \text{ J s}^{-1} \times 3600 \text{ s}$$

$$E = 3.6 \times 10^6 \text{ J}$$

Q4: Independent practice

Consider a wind turbine with a span of 50m is situated at a site, subjected to a constant 12 ms^{-1} wind. If air density is 1.23 kg m^{-3} , how much kinetic energy passes through the plane of the blades every second? Round your answer to 3 s. f.

A4: Kinetic Energy every second

$$KE_{/s} = \frac{1}{2} \pi r^2 \rho v^3$$

$$KE_{/s} = \frac{1}{2} \pi (50^2) (1.23) (12^3) (\text{m}^2) \left(\frac{\text{kg}}{\text{m}^3}\right) \left(\frac{\text{m}}{\text{s}}\right)^3$$

$$KE_{/s} = 8,340,000 \text{ m}^2 \text{ kg s}^{-3}$$

$$KE_{/s} = 8.34 \times 10^6 \text{ J s}^{-1}$$

$$P = 8.34 \times 10^6 \text{ W} = 8,340 \text{ kW}$$