Flexible Specification and Robust Estimation of Input Demand Systems

Jesse Tack[*], Rulon Pope, Jeffrey LaFrance, and Ricardo Cavazos

Abstract

Cost function and factor demand estimation and measurement are among the
most useful tools in economic analysis. This paper defines cost function flexibility in a
new way that extends results and concepts from consumer demand theory to models of
joint production. This applies to production models where input demand equations can be
represented in terms of input prices, quasi-fixed inputs, and cost. The model's flexibility is
determined by the rank of a matrix of price functions and the number and functional form
of a vector of cost functions. The new model is particularly useful when outputs are subject
to measurement errors, perhaps due to production risk. Input demands obtained with this
approach are robust to the form of the decision maker's risk preferences. An empirical application
to state-level annual time series data on U.S. agricultural input use in the 48 contiguous
states over the period 1960-1999 demonstrates the flexibility, generality, and value
of this new approach to production economics. The model is estimated by a new semiparametric
generalized method of moments method to address measurement errors in outputs,
endogenous regressors, technical change, heterogeneous production, heteroskedasticity,
and spatial-temporal error correlations. The estimated model is economically regular at
almost all (96 percent) observations, reflects a cross-sectional distribution of technologies
and economic responses, and the stochastic process for the residuals is stationary across
space and time. Full rank three is supported by the data. Common functional forms such as
the translog and quadratic are rejected.

Key Words: Aggregation, production, ex ante cost, spatial-temporal correlation
JEL Classification: C3, D2, D8

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Cost function measurement and/or conditional factor demand measurement are among the most useful tools for economic analysis. Analyses of factor substitution, returns to scale and scope, technical change, industry performance, and productivity all have relied on cost functions (Chambers, 1988). The usual econometric approach is to begin by specifying a flexible functional form of the cost function in prices, fixed inputs, and output and applying Shephard's Lemma with additive errors (McElroy, 1987) for input demands, expenditures, or cost shares.

For some questions, individual firm-level data are very useful. But most often, cost and input demand function estimates rely on aggregate time-series data. The recent availability of coherent state-level data sets invites consideration of less aggregate data, models, and methods. This is particularly true in agriculture, where pure competition is generally thought to apply at the firm level. These data have created opportunities for interesting analyses such as the level and convergence properties of productivity gains across states (Ball, et al., 2004) and the impacts of research and development (Huffman and Evenson, 2006).

One can analyze state-level production data in at least two ways: estimate a model for each state individually; or estimate a model with cross-equation restrictions on some parameters to incorporate common features across states, while allowing for heterogeneity across states in the remaining parameters. The first approach results in a very large number of parameters to be estimated, requiring a long time series to be feasible. Analyzing each state separately also cannot exploit any structure across states in the error covariances to

increase the efficiency of the estimates for the structural parameters. On the other hand, given a sufficiently large and rich data set, state-specific cost and input demand function estimates are unrestricted relative to one another, which can reduce the potential for estimation bias due to aggregating across different production technologies.

This paper presents a new model of conditional variable input demands and applies it to a balanced panel of annual state-level data on agricultural production for the period 1960-1999. The model and estimation methods address the following issues in this panel data set:

(a) measurement errors in outputs (e.g., due to production risk);

(b) flexibility in the number of terms and functional forms of the input demands and cost function;

(c) heteroskedasticity;

(d) exogenous technical change;

(e) exact aggregation (Gorman, 1981, Lewbel, 1990, Lewbel, 1991); and

(f) spatial-temporal correlation in the residuals.

The state-level panel data set available for our empirical application has 40 annual observations on 48 states. This makes it impractical to implement a non-parametric generalized method of moments (GMM) method, such as the Newey-West heteroskedasticity and autocorrelation consistent covariance (HACC) matrix estimator (Newey and West, 1987). Therefore, we develop and implement a new semi-parametric GMM method with cross-sectional heteroskedasticity and spatial-temporal correlations. The model estimates obtained using this method appear to reflect the structural and error covariance properties of

the data quite well.

The short time series also requires a judicious choice of structural parameters that are assumed to capture technological heterogeneity over cross-sectional observations. In this respect, the model chosen here is sufficiently flexible that a relatively small number of state-specific parameters can identify differences in the levels, slopes, and curvatures of input demand functions in prices, cost, quasi-fixed inputs, technical change, and any structural changes found in the sample period. The demand systems' errors have state-specific covariance matrices for cross-sectional heteroskedasticity. Time series correlations are modeled with an unrestricted AR(1) process, while spatial correlations are modeled with an exponential function of a 5th-order polynomial in the distance between states.

This spatial-temporal systems estimation method is called *nonlinear five stage least squares* (NL5SLS) because it involves the following five steps:

(1) Obtain consistent estimates of the structural parameters by nonlinear instrumental variables (commonly called nonlinear two-stage least squares or NL2SLS, e.g., Amemiya, 1985).

(2) Use the NL2SLS errors from (1) to estimate an unrestricted AR(1) time series process by linear seemingly unrelated regressions (SUR, Zellner, 1962);

(3) Use the errors from step (2) – which are asymptotically uncorrelated over time – to obtain consistent estimates of state-specific variance-covariance matrices for the variable input demand system (Malinvaud, 1990).

(4) Use the transformed errors from step (3) – which have zero means, unit variances, and are asymptotically uncorrelated across inputs and time – to estimate the spatial cor-

relation process as an exponential transformation of a polynomial in the geographic distance between pairs of states (Stohs and LaFrance, 2004).

(5) Given consistent estimates of the parameters of the stochastic process (variance-covariance matrices by cross-sectional unit and spatial-temporal correlations across time and cross-sectional units), use the standardized – i.e., mean zero, unit variance, and uncorrelated across inputs, time and space – residuals to estimate the structural parameters with a second nonlinear instrumental variables step.

Given the choice of instruments for identification, this procedure produces robust, consistent and asymptotically normal parameter estimates. If the maintained assumptions are true, then these estimators will be efficient in this class of semi-parametric GMM estimators. One can also check for any remaining heteroskedasticity of an unknown form by estimating robust standard errors (White, 1980, MacKinnon and White, 1985), which we do, and the results are reported in the empirical section. Steps (2)-(5) can be iterated until convergence, although there is no impact on the asymptotic properties of the resulting parameter estimates (Rothenberg and Leenders, 1964).

The model of the variable cost function applies the recent results of LaFrance and Pope (2010) and Ball, et al. (2010). These authors develop a theory for *ex ante cost functions* in joint production processes that accommodates output and output price risk for any expected utility function. This theory extends consumer choice theory and exact aggregation to joint production models, risk, and intertemporal production decisions. It characterizes flexible cost functions and input demand systems in terms of the rank of a matrix of price functions and functional forms of and relationships among elements of a vector of func-

tions of cost and quasi-fixed inputs. This is similar to the definition of flexibility for Engel curves (Gorman 1981, Lewbel, 1990, Lewbel, 1991).

The plan for the rest of the paper is as follows. The next section specifies a new form of flexible cost model studied in the empirical application. Section 3 develops the estimation procedure, Section 4 details the data employed in the application. Empirical results appear in section 5. The final section presents our conclusions.

The main findings of the paper can be summarized briefly as follows. We reject the full rank one (homothetic) and full rank two (price independent generalized logarithmic, PIGLOG, and price independent generalized linear, PIGL) models (Muellbauer, 1975, Muellbauer, 1976), which have been ubiquitous in empirical work since Berndt and Christensen (1971) and Binswanger (1974a, 1974b), in favor of a full rank three model. Second, the functional form of the full rank three model is explored by deriving a pair of Box-Cox transformations over cost and input prices to nest the generalized PIGL and PIGLOG functional forms. The empirical results reject the generalized PIGLOG and the generalized quadratic functional forms in favor of a PIGL alternative. Checks of economic regularity conditions find the empirical model to be consistent with cost minimizing behavior at nearly all data points in the panel. They also illustrate substantial differences across states in economic response levels and the direction and magnitude of structural changes due to the OPEC oil embargo and ensuing commodity price inflation that began in 1973 and the dramatic changes in agricultural policy that began with the 1986 omnibus farm bill.

2. The Cost Model

The model of variable costs in joint production follows LaFrance and Pope (2010) and ad-

dresses two common issues in econometric models of production. The available data have been aggregated across production units – across fields on a farm or farms in a county, state, country, or other region. In most, if not all, production systems, some or all variable inputs are committed to production long before outputs and output prices are realized. This leads to unobservable, or latent, variables in the cost function and input demand equations.

Under plausible conditions, joint production problems can be written in terms of planned, or expected, outputs. In such a case, if planned outputs are (weakly) separable from variable input prices in the cost function, then the variable input demands can be written as functions of input prices, quasi-fixed inputs, and variable cost in place of planned outputs. Let $X \in \mathbb{R}_+^N$ be a vector of variable inputs, let $\bar{Y} \in \mathbb{R}_+^M$ be a vector of planned outputs, let $Z \in \mathbb{R}_+^L$ be a vector of quasi-fixed inputs, let $W \in \mathbb{R}_+^N$ be a vector of input prices, and let $C(W, \bar{Y}, Z) \equiv \min\{W'X : (X, Y, Z) \in \mathcal{T}\}$ be the cost function, with $\mathcal{T} \subset \mathbb{R}_+^N \times \mathbb{R}_+^M \times \mathbb{R}_+^L$ a joint production possibilities set. Separability of $\bar{Y}$ from W in C implies that a scalar aggregator, $\tilde{\theta}(\bar{Y}, Z)$, exists such that

$$C(W, \bar{Y}, Z) = \tilde{C}(W, Z, \tilde{\theta}(\bar{Y}, Z)). \tag{1}$$

Apply Shephard's Lemma to (1) to obtain input demand functions of the form

$$X(W, \bar{Y}, Z) = \nabla_W \tilde{C}(W, Z, \tilde{\theta}(\bar{Y}, Z)). \tag{2}$$

At least one marginal cost must be strictly positive, implying that the cost function must be strictly increasing in outputs. Hence, $\tilde{C}$ must be strictly monotonic in $\tilde{\theta}$. Solve (1) for $\tilde{\theta} = \Gamma(W, Z, C)$, where Γ is the inverse of $\tilde{C}$ with respect to its last argument. Substitute this into the right-hand side of (2) to obtain

$$\nabla_W \tilde{C}(W, Z, \Gamma(W, Z, C)) \equiv \tilde{X}(W, Z, C). \tag{3}$$

Thus, input demands are functions of input prices, quasi-fixed inputs, and cost.[1]

If the joint production process also is subject to constant returns to scale – a well-accepted stylized fact in agriculture – then the variable cost function is homogeneous of degree one in $(\bar{Y}, Z)$. In such a case, divide through by the last element of the quasi-fixed input vector – acres of farmland in the application below – to obtain a per acre cost function

$$C(W, [Z'_{\sim L}/Z_L \ 1]', \tilde{\theta}(\bar{Y}/Z_L, [Z'_{\sim L}/Z_L \ 1]'))/Z_L \equiv \tilde{c}(W, z, \theta(\bar{y}, z)), \tag{4}$$

where $\tilde{c} = C/Z_L$, $\bar{y} = \bar{Y}/Z_L$, and $z = [Z_1/Z_L \ \cdots \ Z_{L-1}/Z_L]'$, are cost, planned outputs, and the first $L-1$ quasi-fixed inputs, respectively, all measured per unit of the Lth quasi-fixed input, and $\theta(\bar{y}, z) = \tilde{\theta}(\bar{y}, [z' \ 1]')$.

The cost function also is homogeneous of degree one in input prices. Hence, divide through by the Nth price – the farm wage rate in the empirical application – to obtain

$$\tilde{c}\left([W'_{\sim N}/W_N \ 1]', z, \theta(\bar{y}, z)\right)/W_N \equiv c(w, z, \theta(\bar{y}, z)), \tag{5}$$

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[1] One motivation is that what enters the decision maker's choice functions in (3) is what is observed and measured. Production risk is one reason this is a reasonable approach to modeling *ex ante* cost functions and input demands – output levels that are planned or expected at planting time are unobservable and unknown. One also can apply conventional errors-in-variables language, where $\bar{Y}$ is the true measure of outputs and Y is outputs measured with error. From a theoretical perspective, establishing the connection between models of consumer behavior and multiproduct production models leads to innovative and insightful new results. This allows a wealth of useful results from consumer theory to be extended to production theory, leading to a rich and broad range of potential new models of input demand and output supply. Regardless of the motivation or rationale that one prefers to accept, LaFrance and Pope (2010) show that, *inter alia*, (1) is necessary and sufficient for (3). They also find the complete class of full rank exactly aggregable input demand systems and cost functions implied by (3).

where $c = \tilde{c}/W_N$ and $\boldsymbol{w} = [W_1/W_N \quad \cdots \quad W_{N-1}/W_N]'$ are normalized cost per acre and normalized input prices, excluding the Nth, respectively.

Let capital, k, be a quasi-fixed input in addition to land and let t reflect exogenous technical change. Define the functions,

$$f(c) = (c^{\kappa} + \kappa - 1)/\kappa, \ \kappa \in \mathbb{R}_+, \ \text{with} \ c = [1 + \kappa(f-1)]^{1/\kappa},$$

$$g(x) = (x^{\lambda} + \lambda - 1)/\lambda, \ \lambda \in \mathbb{R}_+, \ \text{with} \ \boldsymbol{g}(\boldsymbol{w}) = [g(w_1) \cdots g(w_{N-1})]',$$

$$\alpha(\boldsymbol{w}, k, t) = \alpha_{N0} + \alpha_{N1}k + \alpha_{N2}t + (\boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 k + \boldsymbol{\alpha}_2 t)'\boldsymbol{g}(\boldsymbol{w}), \tag{6}$$

$$\beta(\boldsymbol{w}) = \boldsymbol{g}(\boldsymbol{w})'\boldsymbol{B}\boldsymbol{g}(\boldsymbol{w}) + 2\boldsymbol{\gamma}'\boldsymbol{g}(\boldsymbol{w}) + 1, \ \text{and}$$

$$\delta(\boldsymbol{w}) = \boldsymbol{\delta}'\boldsymbol{g}(\boldsymbol{w}) + \delta_N,$$

where $\alpha_{N0}, \alpha_{N1}, \alpha_{N2}, \kappa$, and λ are scalars, $\boldsymbol{\alpha}_0, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}$, and $\boldsymbol{\delta}$ are $(N-1)$-vectors, and $\boldsymbol{B}$ is an $(N-1) \times (N-1)$ matrix of constants. Then the normalized per acre cost function that is analyzed and estimated in this paper is defined by

$$c(\boldsymbol{w}, k, t, \theta(k, t, \overline{\boldsymbol{y}})) = \left\{ 1 + \kappa \left[\alpha(\boldsymbol{w}, k, t) + \frac{\beta(\boldsymbol{w})}{\delta(\boldsymbol{w}) + \theta(k, t, \overline{\boldsymbol{y}})\sqrt{\beta(\boldsymbol{w})}} - 1 \right] \right\}^{1/\kappa}. \tag{7}$$

Apply Hotelling's/Shephard's lemma to (7) and rearrange terms to obtain the first $N-1$ input demands as normalized expenditures per acre for state i in year t,[2]

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[2] One could also write the input demand system in terms of quantities or cost shares. This would change the exponents on cost and input prices in the leading terms on the right-hand side of (8), as well as the properties of the error terms, $\boldsymbol{u}_{it}$.

$$e_{it} = c_{it}^{1-\kappa} \Delta(w_{ijt}^{\lambda}) \left\{ \alpha_{0i} + \alpha_1 k_{it} + \alpha_2 t + \left[\frac{f(c_{it}) - \alpha_i(w_{it}, k_{it}, t)}{\beta(w_{it})} \right] [Bg(w_{it}) + \gamma] \right.$$

$$\left. + \left[\delta - \frac{\delta(w_{it})[Bg(w_{it}) + \gamma]}{\beta(w_{it})} \right] \frac{[f(c_{it}) - \alpha_i(w_{it}, k_{it}, t)]^2}{\beta(w_{it})} \right\} + u_{it}, \tag{8}$$

$$i = 1, \cdots, I, t = 1, \cdots, T,$$

where $e_{it} = [w_{i1t} x_{i1t} \quad \cdots \quad w_{iN-1t} x_{iN-1t}]'$ is an (N–1)-vector of normalized expenditures per acre for the first N–1 inputs, $\Delta(w_{ijt}^{\lambda})$ is an (N–1)×(N–1) diagonal matrix with w_{ijt}^{λ} as the jth main diagonal element, $j = 1, \cdots, N-1$, and u_{it} is an (N–1)-vector of random error terms.

The structural parameters include the $I+2$ N-vectors, $\alpha_{0i} = [\alpha_{10i} \cdots \alpha_{N0i}]', i = 1, \cdots, I$,

$\alpha_1 = [\alpha_{11} \cdots \alpha_{N1}]'$, and $\alpha_2 = [\alpha_{12} \cdots \alpha_{N2}]'$, the $N \times N$ matrix $\begin{bmatrix} B & \gamma \\ \gamma' & 1 \end{bmatrix}$, the N-vector $[\delta_1 \cdots \delta_N]'$,

and the scalars κ and λ. It is straightforward to show that the translated Box-Cox functions f and g in (7) are observationally equivalent to traditional Box-Cox transformations.

This model is nonlinear in variable cost, creating aggregation properties similar to those in the theory of consumer choice (see LaFrance, 2008 and the references therein). In the present case, three functions of cost appear on the right-hand side of (8),

$$e_{it} = A_1\left(w_{it}, k_{it}, t\right) c_{it}^{1-\kappa} + A_2\left(w_{it}, k_{it}, t\right) c_{it} + A_3\left(w_{it}, k_{it}, t\right) c_{it}^{1+\kappa} + u_{it} \tag{9}$$

where A_1, A_2, and A_3 are (N–1)-vector-valued functions that do not depend on cost. The rank of the input demand system is the rank of $A = [A_1 \quad A_2 \quad A_3]$. It is straightforward to see from (8) that $[\delta_1 \cdots \delta_N]' = 0$ implies no more than rank 2 since then A_3 vanishes, while $B = 0$, $\gamma = 0$, and $[\delta_1 \cdots \delta_N]' = 0$ jointly imply rank 1, since then A_2 and A_3 vanish. Thus, this framework nests the rank of the input demand system.

On the other hand, if $\kappa = 1$, then $f(x) = x,$ while if $\kappa = 0$, then $f(x) = 1 + \ln x.$ The same results apply to $g(x)$ for $\lambda = 1$ or 0, respectively. For any other values of $(\kappa, \lambda) \in \mathbb{R}_+ \times \mathbb{R}_+,$ we obtain a PIGL functional form in normalized input prices and cost. Therefore, this choice of functional forms for f and g nests the generalized translog and the generalized quadratic multiple output production models within a class of PIGL functional forms.

3. Robust Estimation of a Spatial-Temporal Panel System

This section develops a semi-parametric GMM estimation procedure to consistently estimate the structural parameters of a simultaneous econometric system for panel data. The econometric methodology is not specific to the cost model in the previous section, and we use more general notation throughout this section to illustrate this.

Let $i = 1, \cdots, I$ index states in the panel, let $j = 1, \cdots, N-1$ index the dependent variables in the system, and let $t = 1, \cdots, T$ index time. The system of estimating equations can be written in general form as

$$e_{ijt} = f_{ij}(\boldsymbol{w}_{it}, k_{it}, c_{it}, t; \boldsymbol{\theta}) + u_{ijt}, \; i = 1, \cdots, I, \; j = 1, \cdots, N-1, \; t = 1, \cdots, T, \tag{10}$$

where $\boldsymbol{\theta}$ is a $K \times 1$ vector of parameters to be estimated and u_{ijt} is a mean zero random error term. Let $\boldsymbol{Z}_i$ denote the matrix of instrumental variables for state i and let $\mathbb{N}_i = \boldsymbol{Z}_i(\boldsymbol{Z}_i'\boldsymbol{Z}_i)^{-1}\boldsymbol{Z}_i'$ the associated projection matrix.[3] Let $\boldsymbol{\tau} = [1\, 2 \cdots T]',$ and stack equation (10) by j and then t. We use nonlinear two-stage least squares (NL2SLS) to estimate $\boldsymbol{\theta}$

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[3] In the empirical application, we use the same instruments for all states, so that $\mathbb{N}_i \equiv \mathbb{N} \; \forall \; i = 1, \cdots, I.$

consistently,

$$\hat{\theta}_{2SLS} = \underset{\theta}{\text{argmin}} \sum_{i=1}^{I} [e_{i\cdot\cdot} - f_{i\cdot}(w_{i\cdot\cdot}, k_{i\cdot}, c_{i\cdot}, \tau; \theta)]' (\mathbb{N}_i \otimes I_T)[e_{i\cdot\cdot} - f_{i\cdot}(w_{i\cdot\cdot}, k_{i\cdot}, c_{i\cdot}, \tau; \theta)]. \quad (11)$$

This estimator is then used to generate consistent estimates of the errors,

$$\hat{u}_{ijt} = e_{ijt} - f_{ij}(w_{it}, k_{it}, c_{it}, t; \hat{\theta}_{2SLS}), \ i=1,\cdots,I, \ j=1,\cdots,N-1, \ t=1,\cdots,T. \quad (12)$$

For $t = 2,\cdots,T$, estimate an unrestricted $(N–1)\times(N–1)$ intertemporal correlation matrix, Φ, by linear seemingly unrelated regressions (SUR) methods,

$$\hat{\Phi} = \arg\min_{\Phi} \left\{ \sum_{i=1}^{I} \sum_{t=2}^{T} (\hat{u}_{i\cdot t} - \Phi\hat{u}_{i\cdot t-1})^\top \hat{\Sigma}_i^{-1} (\hat{u}_{i\cdot t} - \Phi\hat{u}_{i\cdot t-1}) \right\}. \quad (13)$$

Since the weight matrix does not affect consistency, one can complete this step using

$\hat{\Sigma}_i = I_N \ \forall \ i$ or $\hat{\Sigma}_i = \sum_{t=1}^{T} u_{i\cdot t} u'_{i\cdot t}/T$ calculated from the NL2SLS estimates, $\hat{\theta}_{2SLS}$. The

first method is robust to departures from the assumed covariance structure. The second

method may be more efficient if the model is correct. In favor of robustness, we apply the

first method in the empirical application.

It is useful to explain one additional way in which the Nth input, labor, is treated asym-

metrically in the stochastic error component of the econometric model. The errors for the

N–1 estimation equations are assumed to follow an unrestricted AR(1) process,

$$u_{i\cdot t} = \Phi u_{i\cdot t-1} + \varepsilon_{i\cdot t}, \ \varepsilon_{i\cdot t} \ i.i.d. \ (0, \Sigma_i), \ t=1,\cdots,T. \quad (14)$$

Let the N-vector of random variables, $[u_{i1t} \quad \cdots \quad u_{iN-1t} \quad u_{iNt}]'$, satisfy $u_{iNt} = -\iota'u_{i\cdot t}$, where

$u_{i\cdot t} \in \mathbb{R}^{N-1}$ satisfies (14), so that

$$u_{i \cdot t} = \boldsymbol{\Phi} u_{i \cdot t-1} + \boldsymbol{\varepsilon}_{i \cdot t},$$

$$E(\boldsymbol{\varepsilon}_{i \cdot t} \boldsymbol{\varepsilon}'_{i \cdot t}) = \boldsymbol{\Omega}_i \ \forall \ t,$$

$$E(\boldsymbol{u}_{i \cdot t} \boldsymbol{\varepsilon}'_{i \cdot t \pm s}) = [\mathbf{0}] \ \forall s, t,$$

$$E(\boldsymbol{u}_{i \cdot t} \boldsymbol{u}'_{i \cdot t}) = \boldsymbol{\Sigma}_i = \boldsymbol{\Phi} \boldsymbol{\Sigma}_i \boldsymbol{\Phi}' + \boldsymbol{\Omega}_i \ \forall \ t.$$

(15)

Then,

$$u_{iNt} = -\boldsymbol{\iota}' \boldsymbol{\Phi} u_{i \cdot t} - 1 - \boldsymbol{\iota}' \boldsymbol{\varepsilon}_{i \cdot t},$$

$$E(u_{iNt}^2) = \boldsymbol{\iota}' \boldsymbol{\Sigma}_i \boldsymbol{\iota} = \boldsymbol{\iota}'(\boldsymbol{\Phi} \boldsymbol{\Sigma}_i \boldsymbol{\Phi}' + \boldsymbol{\Omega}_i) \boldsymbol{\iota},$$

$$E(u_{iNt} \boldsymbol{u}_{i \cdot t}) = -\boldsymbol{\Sigma}_i \boldsymbol{\iota} = -(\boldsymbol{\Phi} \boldsymbol{\Sigma}_i \boldsymbol{\Phi}' + \boldsymbol{\Omega}_i) \boldsymbol{\iota},$$

(16)

and the full $N \times N$ error covariance structure is

$$E \begin{bmatrix} \boldsymbol{u}_{i \cdot t} \boldsymbol{u}'_{i \cdot t} & u_{iNt} \boldsymbol{u}_{i \cdot t} \\ \boldsymbol{u}'_{i \cdot t} u_{iNt} & u_{iNt}^2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Sigma}_i & -\boldsymbol{\Sigma}_i \boldsymbol{\iota} \\ -\boldsymbol{\iota}' \boldsymbol{\Sigma}_i & \boldsymbol{\iota}' \boldsymbol{\Sigma}_i \boldsymbol{\iota} \end{bmatrix}$$

$$= \begin{bmatrix} \boldsymbol{\Phi} \boldsymbol{\Sigma}_i \boldsymbol{\Phi}' + \boldsymbol{\Omega}_i & -(\boldsymbol{\Phi} \boldsymbol{\Sigma}_i \boldsymbol{\Phi}' + \boldsymbol{\Omega}_i) \boldsymbol{\iota} \\ -\boldsymbol{\iota}'(\boldsymbol{\Phi} \boldsymbol{\Sigma}_i \boldsymbol{\Phi}' + \boldsymbol{\Omega}) & \boldsymbol{\iota}'(\boldsymbol{\Phi} \boldsymbol{\Sigma}_i \boldsymbol{\Phi}' + \boldsymbol{\Omega}_i) \boldsymbol{\iota} \end{bmatrix}.$$

(17)

The full $N \times N$ AR(1) Markov process therefore satisfies

$$\begin{bmatrix} \boldsymbol{u}_{i \cdot t} \\ u_{iNt} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Phi}_i & \mathbf{0}_{N-1} \\ -\boldsymbol{\iota}' \boldsymbol{\Phi}_i & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_{i \cdot t-1} \\ u_{iNt-1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon}_{i \cdot t} \\ \varepsilon_{iNt} \end{bmatrix}.$$

(18)

The eigenvalues of this singular stochastic process therefore are the solutions to the characteristic equation,

$$\left| \begin{bmatrix} \boldsymbol{\Phi} & \mathbf{0}_{N-1} \\ -\boldsymbol{\iota}' \boldsymbol{\Phi} & 0 \end{bmatrix} - \lambda \boldsymbol{I}_N \right| = \begin{vmatrix} \boldsymbol{\Phi} - \lambda \boldsymbol{I}_{N-1} & \mathbf{0}_{N-1} \\ -\boldsymbol{\iota}' \boldsymbol{\Phi} & -\lambda \end{vmatrix} = -\lambda \left| \boldsymbol{\Phi} - \lambda \boldsymbol{I}_{N-1} \right| = 0.$$

(19)

Hence, one eigenvalue is 0 and the remaining eigenvalues are those for $\boldsymbol{\Phi}$. Since labor is the Nth input, we model the time series properties of this input through those of u_{iNt} and

the adding up condition. This allows the AR(1) matrix of intertemporal serial correlation parameters, $\boldsymbol{\Phi}$, to be fully flexible – i.e., there is no requirement that it must be symmetric or otherwise restricted, except for the conditions for a stationary stochastic process.

The next step constructs consistent estimates of the spatially correlated error terms,

$$\hat{\varepsilon}_{ijt} = \sum_{j'=1}^{N-1} \hat{\ell}_{ijj'} \hat{v}_{ij't} \tag{20}$$

where $\hat{v}_{ijt} = \hat{u}_{ijt} - \sum_{j'=1}^{N-1} \hat{\phi}_{jj'} \hat{u}_{ij't-1}$ and $\hat{\boldsymbol{L}}_i = [\hat{\ell}_{ijj'}]_{j,j'=1,\cdots N-1}$ satisfies $\tilde{\boldsymbol{\Sigma}}_i^{-1} = \hat{\boldsymbol{L}}_i \hat{\boldsymbol{L}}_i'$. Consistent empirical (i.e., sample) estimates for the spatial correlations between states are,

$$\hat{\rho}_{ii'} = \sum_{j=1}^{N} \sum_{t=2}^{T} \hat{\varepsilon}_{ijt} \hat{\varepsilon}_{i'jt} \big/ N(T-1), \ \ i,i' = 1,\cdots, I. \tag{21}$$

These $\frac{1}{2}I(I-1)$ empirical spatial correlations are then used to estimate the relationship between the spatial correlations and the geographic distance between pairs of distinct states using robust nonlinear least squares to obtain $\hat{\boldsymbol{R}} = [\hat{\rho}(d_{ii'})]$. In this study, we use an unrestricted fifth-order exponential polynomial for the spatial correlation function,

$$\rho(d_{ii'}) = \exp\left\{\eta_0 + \sum_{k=1}^{5} \eta_k d_{ii'}^k\right\}. \tag{22}$$

Let $\boldsymbol{R}^{-1} = \boldsymbol{Q}\boldsymbol{Q}'$, where $\boldsymbol{Q}$ is a lower triangular Cholesky factorization of the inverse spatial correlation matrix, and write

$$\omega_{ijt} = \sum_{i'=1}^{I} q_{ii'} \varepsilon_{i'jt}, i = 1,\cdots, I, \ j = 1,\cdots, N-1, t = 2,\cdots, T. \tag{23}$$

The random variables ω_{ijt} are mean zero, unit variance, and uncorrelated across inputs, states, and time. Replacing the unknown parameters and error terms with the consistent

estimates developed with the previous estimation steps and substituting backwards recursively gives

$$
\begin{aligned}
\hat{\omega}_{ijt} &= \sum_{i'=1}^{I} \hat{q}_{ii'} \hat{\varepsilon}_{i'jt} \\
&= \sum_{i'=1}^{I} \hat{q}_{ii'} \sum_{j'=1}^{N} \hat{\ell}_{i'jj'} \hat{v}_{i'j't} \\
&= \sum_{i'=1}^{I} \hat{q}_{ii'} \sum_{j'=1}^{N} \hat{\ell}_{i'jj'} \left(\hat{u}_{i'j't} - \sum_{j''=1}^{N} \hat{\phi}_{j'j''} \hat{u}_{i'j''t-1} \right) \\
&\xrightarrow{P} \omega_{ijt},
\end{aligned}
\tag{24}
$$

with $E(\omega_{ijt}) = 0$, $E(\omega_{ijt}^2) = 1$, $E(\omega_{ijt}\omega_{i'j't'}) = 0$, $\forall (i,j,t) \neq (i',j',t')$.

Stack the standardized residuals across inputs and time, fix the estimates of each state's covariance matrix, the AR(1) matrix of intertemporal covariance terms, and the spatial correlation parameters fixed at the above values, and complete a nonlinear instrumental variables estimation step of the form,

$$
\hat{\theta}_{3SLS} = \arg \min_{\boldsymbol{\theta}} \left\{ \sum_{i=1}^{I} \hat{\boldsymbol{\omega}}_{i\cdot\cdot}(\boldsymbol{\theta})' \left(\mathbb{N}_i \otimes \boldsymbol{I}_N \right) \hat{\boldsymbol{\omega}}_{i\cdot\cdot}(\boldsymbol{\theta}) \right\}.
\tag{25}
$$

This generates consistent and asymptotically normal estimates of $\boldsymbol{\theta}$. If the model's assumptions are correct, then given the choice of instruments, these estimators will be efficient among GMM estimators in this class. Alternatively, as T increases the orders of the AR process and spatial correlation polynomial can increase accordingly (e.g., at rates proportional to $T^{1/4}$) to increase the robustness of the covariance matrix estimator. White's heteroskedasticity consistent covariance matrix estimator (White, 1980, MacKinnon and White, 1985) can be applied to the ω_{ijt} at the fifth and final stage of the estimation proce-

dure for standard errors that are robust to heteroskedasticity beyond the state-specific co-variance matrices.

4. Data

The main data set analyzed in the empirical application is a state-level annual time series panel on U.S. agricultural production. This data set is compiled and maintained by the Economic Research Service (ERS) of the United States Department of Agriculture (USDA), is described in detail in Ball, et al. (2004), and is commonly known as the Ball data. This data contains measures of variable input quantities, prices, and expenditures, farm capital, land in farms, along with variables relating to realized (i.e., *ex post*) farm production and revenues.

To match the data used in the empirical application as closely as possible to the above theory, we modify the Ball data in three important ways. The measure of own labor costs in the Ball data is calculated from off-farm employment surveys and nonfarm wage rates. We use the farm wage rate so that management skill is assigned to the owner/operator's net return to farming.[4] The Ball data also contains an imputed value of capital that relies on a host of assumptions and secondary value estimates. We use direct estimates of the value of farm capital obtained from annual surveys reported by the ERS. Finally, we relate total farmland in each state to the results contained in the Census of Agriculture reports, which were published in 1954, 1959, 1964, 1969, 1974, 1978, 1982, 1987, 1992, 1997, 2002, and 2007. In a sample year that coincides with a Census of report, we use the Census data for

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[4] A recent ERS survey found that 98 percent of U.S. farms are family farms (Hoppe and Banker, 2006).

land in farms. In other years, we use ERS estimates of harvested acres for each state's major crops to adjust the Ball data. First, the difference between the farmland measure in the Ball data and the ERS measure of harvested acres is calculated in each non-census year in each state. For each three- to four-year period between adjacent census years, the average of this difference is added to the annual harvested acres measure in those years. This relates changes in farmland directly to changes in crop acres and uses the best available data on farmland whenever and wherever it is available.

The instrumental variables that are used to obtain consistent estimates in the presence of simultaneously determined regressors in the model include national averages of the normalized input prices and costs per acre, and the real value of farm capital per acre, all lagged two periods, a time trend, and variables on the general economy that include real per capita disposable income, the unemployment rate, the real rate of return on AAA corporate 30-year bonds, the real manufacturing wage rate, the real producer's price index (PPI) for materials and components, and the real PPI for fuels, energy and power. Per capita income is deflated by the consumer price index (CPI) for all items, the manufacturing wage rate and wholesale price indices are deflated by the implicit price deflator (IPD) for gross domestic product (GDP), and the real rate of return on bonds is the nominal annual rate minus the annual inflation rate (the percentage change in the CPI).

The aggregate farm sector instrumental variables are lagged two years in order that these will be predetermined even with AR(1) errors. U.S. agriculture is a small sector of the U.S. economy, currently with approximately 2% of employment and less than 1½% of GDP. Each individual state is substantially smaller than U.S. agriculture as a whole, and

the largest state's agricultural economy – California – accounts for less than ¼% of national GDP. As a result, the causal effects of changes in a state's farm prices, production costs, or capital stocks will be much smaller on the general economy than the corresponding impacts of changes in the general economy on each state's farm economy. The general economy variables are intended to be instruments for specific right-hand-side variables: the manufacturing wage is an instrument for the farm wage; income is an instrument for the cost of agricultural production; the real rate of return on corporate AAA 30-year bonds is a measure of the opportunity cost of investing in agriculture; the PPI for fuels, energy and power is an instrument for the farm costs of fuels and energy and agricultural chemicals (largely hydrocarbon based); and the PPI for materials and components is an instrument for the farm cost of materials.

To satisfy the adding up condition and to incorporate the singularity of the residual covariance matrix for all N demands, farm labor is excluded from the system of estimated equations. All input prices and variable cost are normalized by the farm wage rate to satisfy zero degree homogeneity.[5] The estimated input demand equations include energy, agricultural chemicals, and materials. Lagging the aggregate farm-level instruments two periods shortens the sample two years. Thus, 1,824 observations on the 48 contiguous states in the 38 years 1962-1999 are used in the final model estimation.

A recent paper, Gutierrez, et al. (2007), investigates the role that structural breaks play

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[5] As discussed above, we also treat labor asymmetrically in the stochastic part of the econometric model, permitting estimation of an unrestricted 3×3 AR(1) process for the serial correlation part of the empirical model, thereby substantially increasing the flexibility of this component of the model.

in the agricultural economy with respect to finding a stable relationship between farmland prices and land rents. These authors use a panel data set of 31 U.S. states over the time period 1960–2000. They find that all states have at some point been subject to structural breaks.

In this paper, tests for structural breaks also are conducted. There are many diagnostic procedures for testing the adequacy of a model's specification and parameter stability (e.g. Brown, et al., 1975, Ploberger and Krämer, 1992). The test procedure in LaFrance (2008) works well in situations like the present one, where the econometric model includes a relatively large nonlinear simultaneous equation system with a short time series sample.

The results of Gutierrez, et al. (2007) show considerable evidence of structural breaks in 1973 and 1986. The former is the beginning of the period of rapid commodity price inflation that followed the OPEC oil embargo. The latter is the start of major reforms in agricultural policy during the Reagan administration, including a movement to decouple farm subsidy payments from agricultural production, and lower price and income support levels overall. During the mid to late 1970s, the U.S. agriculture experienced oil price shocks, a high rate of growth in farm income, growth in net agricultural exports with a falling exchange rate, and poor weather conditions in competing production regions. On the other hand, increased uncertainty in the expected returns to agricultural investment, high real interest rates, and lower commodity prices and support levels have characterized the farm economy since the second half of the 1980s.

To allow for these impacts, two dummy variables and a set of associated structural break parameters are added to the above model. Specifically, α_{0i} is specified as

$$\boldsymbol{\alpha}_{0it} = [\alpha_{10i} + \tau_{1i}^{73}D_t^{73} + \tau_{1i}^{86}D_t^{86} \quad \cdots \quad \alpha_{N0i} + \tau_{Ni}^{73}D_t^{73} + \tau_{Ni}^{86}D_t^{86}]', \tag{26}$$

and α_i as

$$\alpha_{it}(\boldsymbol{w}_{it},k_{it},t) = \alpha_{N0i} + \tau_{Ni}^{73}D_t^{73} + \tau_{Ni}^{86}D_t^{86} + \alpha_{N1}k_{it} + \alpha_{N2}t + (\boldsymbol{\alpha}_{0i} + \boldsymbol{\alpha}_1 k_{it} + \boldsymbol{\alpha}_2 t)'\boldsymbol{g}(\boldsymbol{w}_{it}), \tag{27}$$

where the τs are parameters to be estimated and the dummy variables D_t^{73} and D_t^{86} are

defined as

$$D_t^{73} = \begin{cases} 1, & \text{if } t \geq 1973, \\ 0, & \text{otherwise, and} \end{cases}$$

$$D_t^{86} = \begin{cases} 1, & \text{if } t \geq 1986, \\ 0, & \text{otherwise.} \end{cases} \tag{28}$$

The τs are included in a flexible way, unrestricted across states and input demands.

The top panel of Figure 1 reports box plots of the 48 state-specific test statistics for each demand equation and for the system overall without structural breaks. The null hypothesis of parameter stability is rejected at the 5% significance level 26 times for individual equation tests (18% of 144 tests, i.e., 48 states times 3 equations) and 7 times (15% of 48 tests) for the state-level systems tests. The bottom panel of Figure 1 presents the results of the same parameter stability tests with the modified specification that includes the dummy variables for post-1973 and 1986. In this set of Box plots, the null hypothesis of parameter stability fails to be rejected for any equation or the system as a whole at the 5% significance level. In addition, equation-specific Wald tests of the null hypotheses

$$H_0 : \tau_{j1}^{73} = \cdots = \tau_{j48}^{73} = \tau_{j1}^{86} = \cdots = \tau_{j48}^{86} = 0 \tag{29}$$

strongly support inclusion of the structural change parameters (all p-values are less than

0.001). The results that follow are therefore reported for the model specification that includes these two structural breaks.

5. Results

The estimated 3×3 intertemporal autocorrelation matrix $\hat{\boldsymbol{\Phi}}$, with White/Huber robust asymptotic standard errors in parentheses, is reported in Table1. The parameter estimates imply positive semidefiniteness and stable dynamics, as the Eigen values for the 4×4 difference equation are all non-negative and less than 1 (the largest Eigen value is 0.48). The F-test that all parameter estimates are jointly zero is rejected at the 1% significance level, and the Durbin-Watson statistics do not suggest higher order serial correlation (the average Durbin-Watson statistics across states for each equation are 1.911, 1.757, and 1.775, respectively).[6]

The estimated spatial correlation function, with White/Huber robust standard errors in parentheses, is:

$$
\hat{\rho}(d_{ii'}) = \exp\left\{ -\underset{(.0266)}{1.33} - \underset{(.0502)}{.370}\ d_{ii'} - \underset{(.0569)}{.0134}\ d_{ii'}^2 \right.
$$

$$
\left. + \underset{(.0383)}{.102}\ d_{ii'}^3 + \underset{(.0212)}{.0313}\ d_{ii'}^4 - \underset{(.00925)}{.0187}\ d_{ii'}^5 \right\},
$$

(30)

where $i \neq i', i, i' = 1, \cdots, 48,$ indexes pairs of states. A Wald test for the joint hypothesis that all parameters are equal to zero yields a p-value of 0.000, suggesting spatial correlation is important in this panel data set. The empirical spatial correlations, estimated correlation

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[6] Detailed empirical results and kernel density estimates of the distribution of Durbin-Watson statistics across states are available from the authors upon request.

function, and 95% confidence band are presented in Figure 2. An interesting property is that the spatial correlation is quite flat from a distance of approximately 800 miles out to approximately 2,500 miles.

We turn next to a subset of the parameter estimates for the structural model. Table 2 presents the estimates of the parameters in $\beta(w)$ and $\delta(w)$, and the parameters (κ, λ). Two interesting hypotheses with regard to functional form are the transformations of input prices and variable costs. The industry standards are the logarithmic and linear transformations, which are tested with the following null hypotheses:

(1) linear-linear, H_0: $\kappa = \lambda = 1$, $\chi^2(2) = 686.2$, p-value=0.000;

(2) log-log, H_0: $\kappa = \lambda = 0$, $\chi^2(2) = 59.35$, p-value=0.000;

(3) log-linear, H_0: $\kappa = 0, \lambda = 1$, $\chi^2(2) = 709.2$, p-value=0.000; and

(4) linear-log, H_0: $\kappa = 1, \lambda = 0$, $\chi^2(2) = 1736.3$, p-value=0.000.

Thus, all four hypotheses are rejected at the 1% significance level. From Table 2, the null hypothesis H_0: $\kappa = 0$ has p-value=0.231. However, the log-log and log-linear transformations are not supported by this data set. It is clear that the additional flexibility of the power functions in prices and variable cost due to the Box-Cox transformation is quite useful in explaining these data. This is consistent with results found in models of consumer choice behavior (e.g., LaFrance, et al., 2006, LaFrance, 2008).

A Wald test for the null hypothesis that the nine parameters of the quadratic form $\beta(w)$ are jointly zero is rejected at the 1% significance level. The conclusion from a similar test for the four parameters in $\delta(w)$ is less clear, with a p-value of 0.0886. In general,

we conclude that there is strong evidence in favor of including higher order price effects and some evidence that the extension to rank 3 is warranted for this data.

Testing whether per acre variable input demand functions depend on capital per acre is equivalent to testing the hypothesis $H_0 : \alpha_{11} = \alpha_{21} = \alpha_{31} = \alpha_{41} = 0$. This test produces a p-value of 0.005, supporting the inclusion of capital per acre in the demand model. We also find evidence that including the trend parameters is warranted. A Wald test for the null hypothesis $H_0 : \alpha_{12} = \alpha_{22} = \alpha_{32} = \alpha_{42} = 0$ produces a p-value of 0.059.

The remaining parameter estimates are the state-specific terms $\alpha_{j0i}, \quad j = 1, \cdots, 4,$ and $i = 1, \cdots, 48$. There are too many estimates to report in tables here. As an alternative, Figure 3 presents kernel density functions across states of the parameter estimates for each input along with similar plots of estimates of the structural break parameters. These plots strongly suggest that there is substantial heterogeneity across states both in technologies and the impacts of structural breaks post-1973 and post-1986. Wald tests for the null hypotheses $H_0 : \alpha_{10i} = \alpha_{20i} = \alpha_{30i} = \alpha_{40i} = 0$ reject the null for all but two states at a 1% significance level and for all states at a 5% significance level.

As noted above, the Durbin-Watson statistics averaged across states do not suggest that there is any remaining serial correlation. Tests for mean zero residuals for each state and input are conducted as in LaFrance (2008), where

$$z_{ij} = \frac{\sqrt{T}\,\bar{\hat{\omega}}_{ij}}{\hat{\sigma}_{ij}} \xrightarrow[T \to \infty]{D} N(0,1). \tag{31}$$

Figure 4 reports these test statistics for all inputs, which fail to reject zero means at the 10% significance level (the largest test statistic in absolute value is 1.19).

Economic regularity of the variable cost function requires monotonicity in the input prices and a negative semi-definite Hessian matrix. Monotonicity is confirmed as the fitted demands are positive for at all data points. For our variable cost specification, the full 4×4 Hessian is given by

$$H = \begin{bmatrix} H_{33} & -(1/w_N)H_{33}^{\top}w \\ -(1/w_N)w^{\top}H_{33} & (1/w_N^2)w^{\top}H_{33}w \end{bmatrix}, \tag{32}$$

where the upper left 3×3 block is given by

$$H_{33} = \frac{1}{w_N}\left[(\lambda - 1)\Delta\left(w_i^{-1}\right)\Delta\left(\partial c / \partial w_i\right) + c^{1-\kappa}\Delta\left(w_i^{\lambda-1}\right)\frac{\partial f(c)}{\partial w \partial w^{\top}}\Delta\left(w_i^{\lambda-1}\right) \right.$$
$$\left. + (1-\kappa)c^{-1}\frac{\partial c}{\partial w}\frac{\partial c}{\partial w^{\top}} \right]. \tag{33}$$

Note that H_{33} negative semi-definite is necessary and sufficient for H negative semi-definite, since the input prices are positive. We calculate $\widehat{H}_{33}$ for each state-year observation in the data and find that 96% of the associated eigenvalues are negative.

Finally, we evaluate the model's ability to capture heterogeneous technology across states. Because the demand equations are quadratic forms in the $\alpha_{it}(w_{it}, k_{it}, t)$ terms, the state-specific parameters impact levels, slopes, and curvatures of the input demands with respect to all other variables in the model. We calculate demand elasticities of the form

$$E = \Delta\left(w_i\right)H\Delta\left(q_i^{-1}\right) \tag{34}$$

where w_i and q_i are non-normalized input prices and quantities. These elasticities are calculated for all state-year pairs in the data set. Figure 5 reports histograms and kernel density plots of the average own-price elasticities across states, clearly demonstrating the mod-

el's ability to capture a wide range of price responsiveness. As is apparent from the densities, own-price elasticities are negative and distributed over reasonable supports. Chemicals and energy are among the most elastic factor demands. Note also that all of the elasticity distributions are left (negatively) skewed with the exception of energy. Considering two inputs often of policy interest, energy (chemical) demands have a relatively large number of the elasticities that are less (more) elastic than the mean or modal response.

6. Conclusions

Adopting two conceptual innovations, accounting for output risk and the notion of flexibility under aggregability, our focus has been to provide a robust and holistic econometric approach for estimating aggregate state-level cost functions by estimating conditional factor demands. Because cost minimizing behavior has been widely used to infer factor substitution and scale elasticities, obtaining consistent flexible estimates across agents and factor demands is important.

Using a systems approach to state-level agricultural data, we have estimated cost minimizing factor demands accounting for fixed effects, auto and spatial correlation, technical change and output uncertainty. The form used is consistent with exact aggregation across economic agents. We reject rank one and two (PIGLOG and PIGL based) models which are ubiquitous in agricultural research (beginning with Berndt and Christensen (1971) and Binswanger (1974a, 1974b) using the translog) in favor of rank three models. Simple forms yielding flexibility via a power function parameter and rank three are useful for this nested test. These tests reject the generalized PIGLOG and quadratic forms in favor of a more generalized alternative.

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Table 1. First-Order Autocorrelation Parameter Estimates.

|	Other Materials	Fuels and Energy	Agricultural Chemicals
Other Materials	0.202** (0.081)	0.075 (0.290)	-0.323*** (0.115)
Fuels and Energy	0.015* (0.009)	0.535*** (0.064)	0.051 (0.049)
Agricultural Chemicals	0.013 (0.015)	-0.155** (0.076)	0.410*** (0.095)

White's heteroskedasticity consistent asymptotic standard error is in parentheses below each point estimate. *, **, and *** indicate significantly different from zero at the 10%, 5%, and 1% level, respectively.

Table 2. Estimates of B, δ, and Box-Cox Parameters.

Coefficient	Estimate	Standard Error	T-Ratio	P-Value
B_{11}	.0783	.0256	3.06	.002
B_{12}	$.982 \times 10^{-2}$	$.416 \times 10^{-2}$	2.36	.018
B_{13}	$-.657 \times 10^{-2}$	$.302 \times 10^{-2}$	-2.17	.030
γ_1	$-.239 \times 10^{-2}$	.0349	-.069	.995
B_{22}	$-.465 \times 10^{-2}$	$.229 \times 10^{-2}$	-2.03	.042
B_{23}	$-.322 \times 10^{-2}$	$.142 \times 10^{-2}$	-2.26	.023
γ_2	-.0190	$.432 \times 10^{-2}$	-4.29	.000
B_{33}	$.873 \times 10^{-2}$	$.252 \times 10^{-2}$	3.46	.001
γ_3	-.0129	$.739 \times 10^{-2}$	-1.75	.081
δ_1	$.122 \times 10^{-2}$	$.111 \times 10^{-2}$	1.10	.270
δ_2	$-.831 \times 10^{-3}$	$.445 \times 10^{-3}$	-1.87	.062
δ_3	$-.429 \times 10^{-3}$	$.339 \times 10^{-3}$	-1.27	.206
δ_4	.0306	.0131	2.33	.020
κ	.0438	.0366	1.20	.231
λ	.254	.0395	6.44	.000

Standard errors are White's heteroskedasticity consistent asymptotic standard errors.

Figure 1. Box Plots for First-Order Parameter Stability Tests.
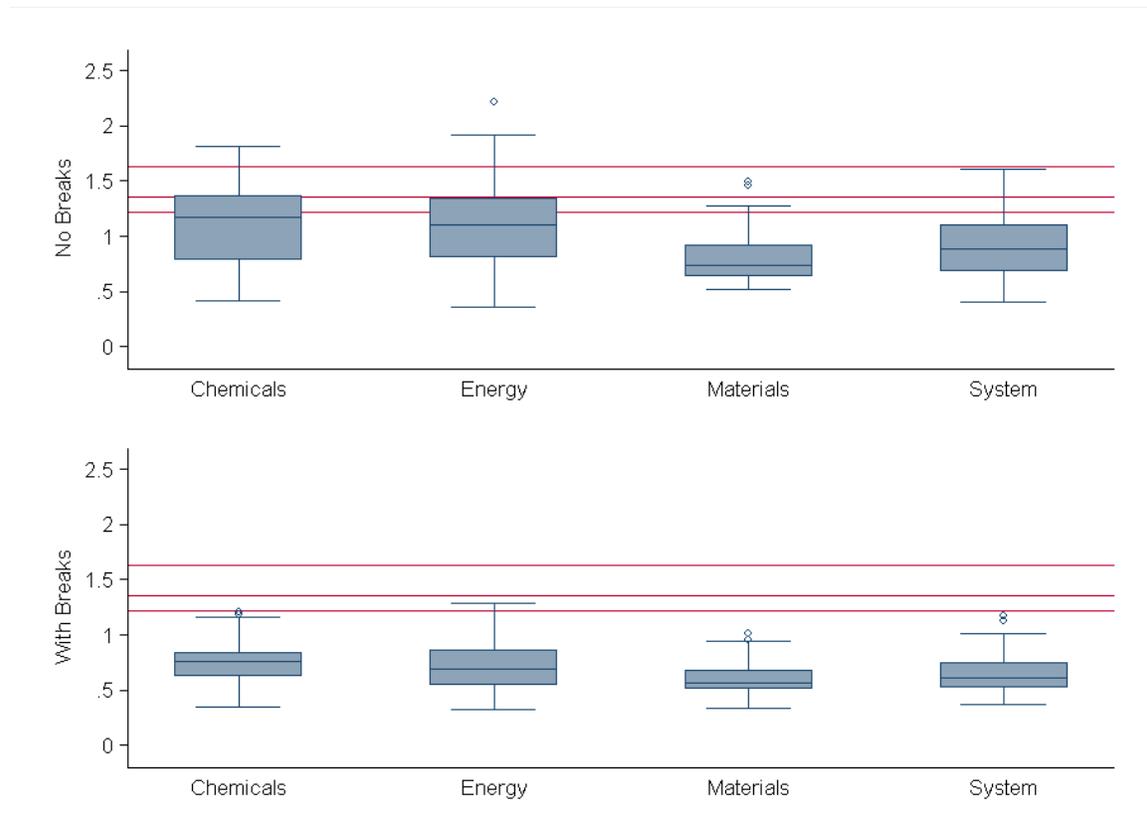
For each equation (Chemicals, Energy, and Materials) and for the overall system, box-plots are reported for the 48 state-specific first order parameter stability test statistics. The top panel omits structural break parameters from the demand system, while the lower panel includes breaks at 1973 and 1986. The three solid lines correspond to critical values at the 10% (1.22), 5% (1.36), and 1% (1.63)

Figure 2. Spatial Correlations Between the 48 Contiguous States in the Rank 3 Cost Model.
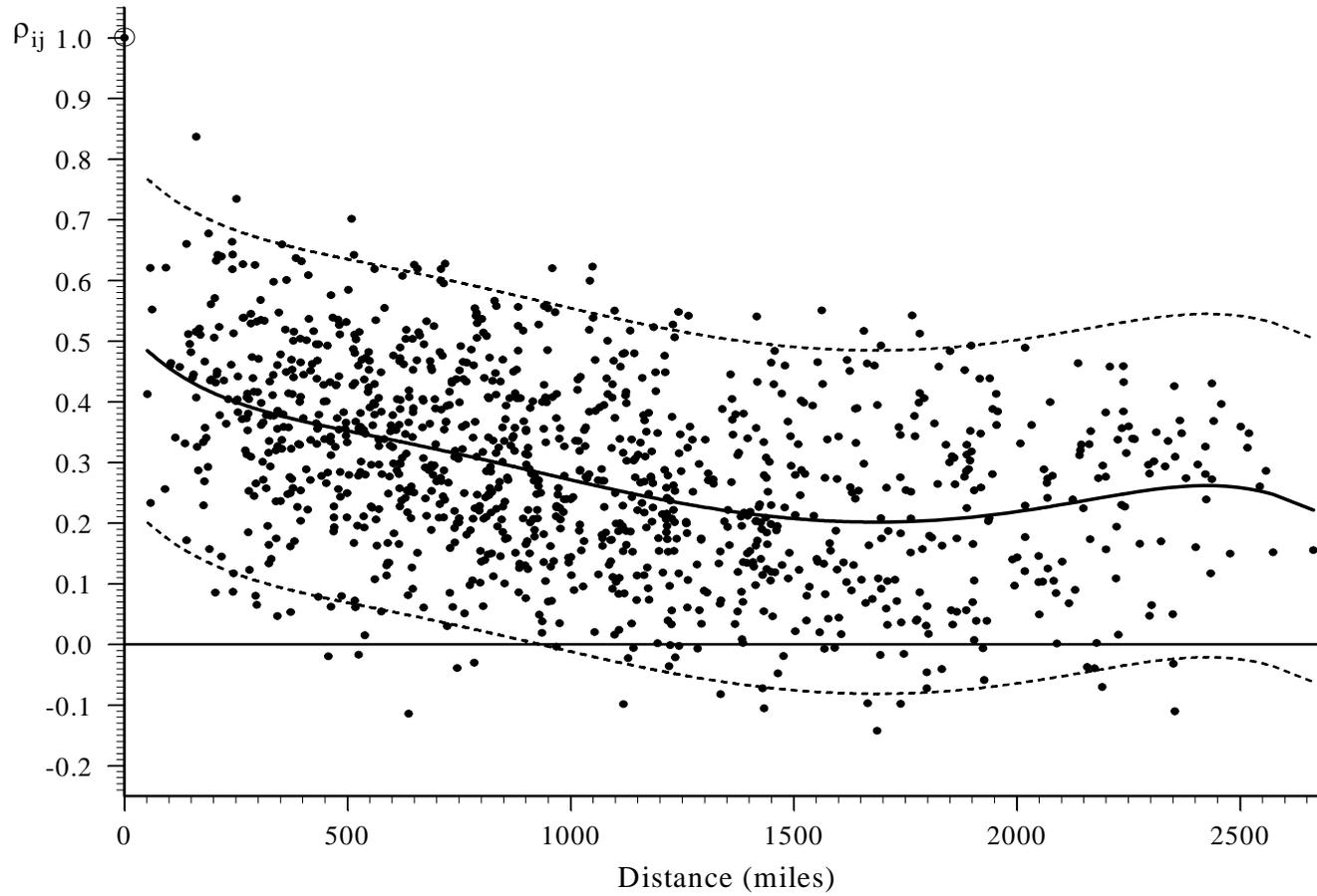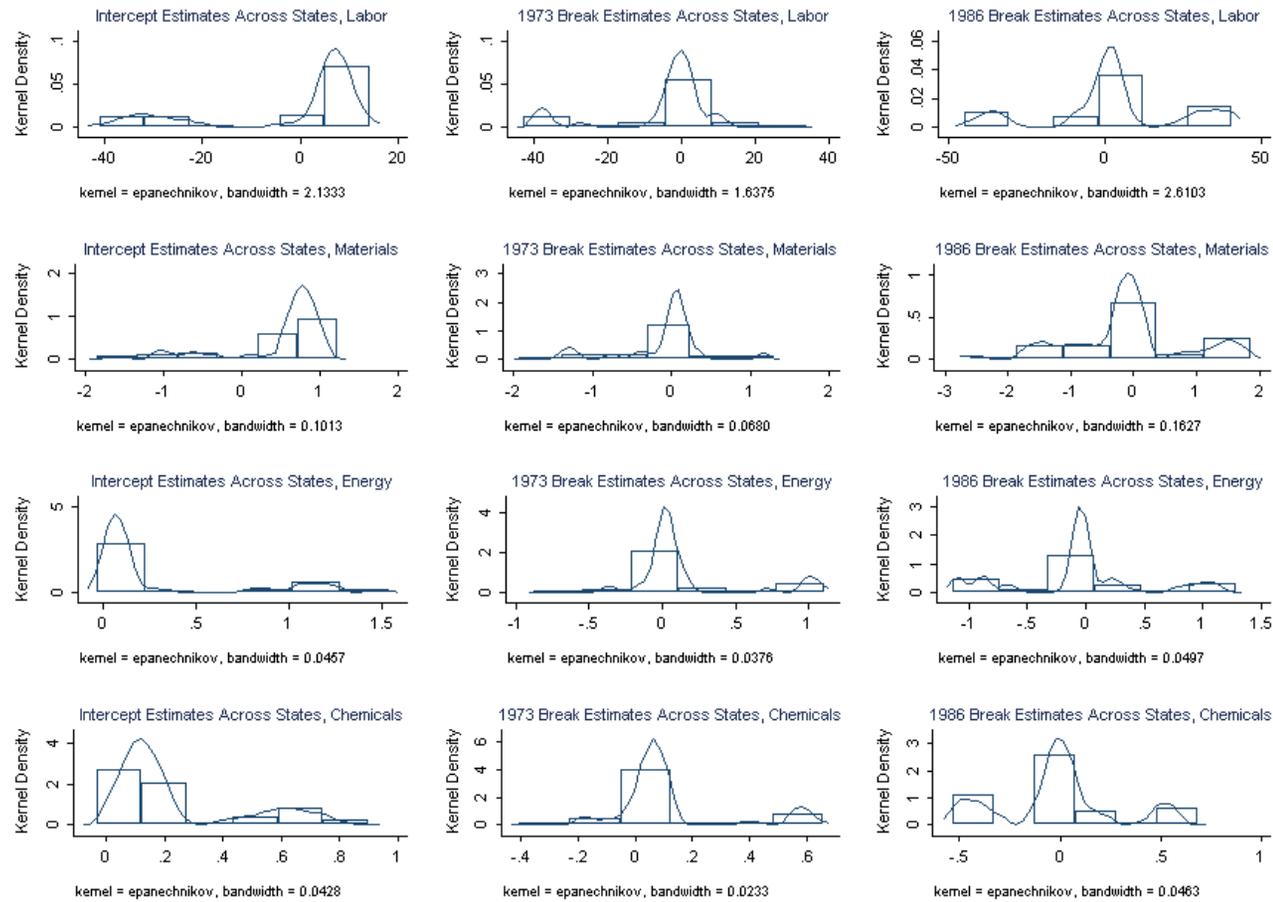
Figure 3. Kernel Density Plots for State-Specific Parameter Estimates.

For each equation (going down: Labor, Materials, Energy, and Chemicals), the model includes state-specific intercept and structural break parameters (going across for each equation). Each graph is a kernel density plot (overlying corresponding histogram) of the forty eight parameter estimates across states.

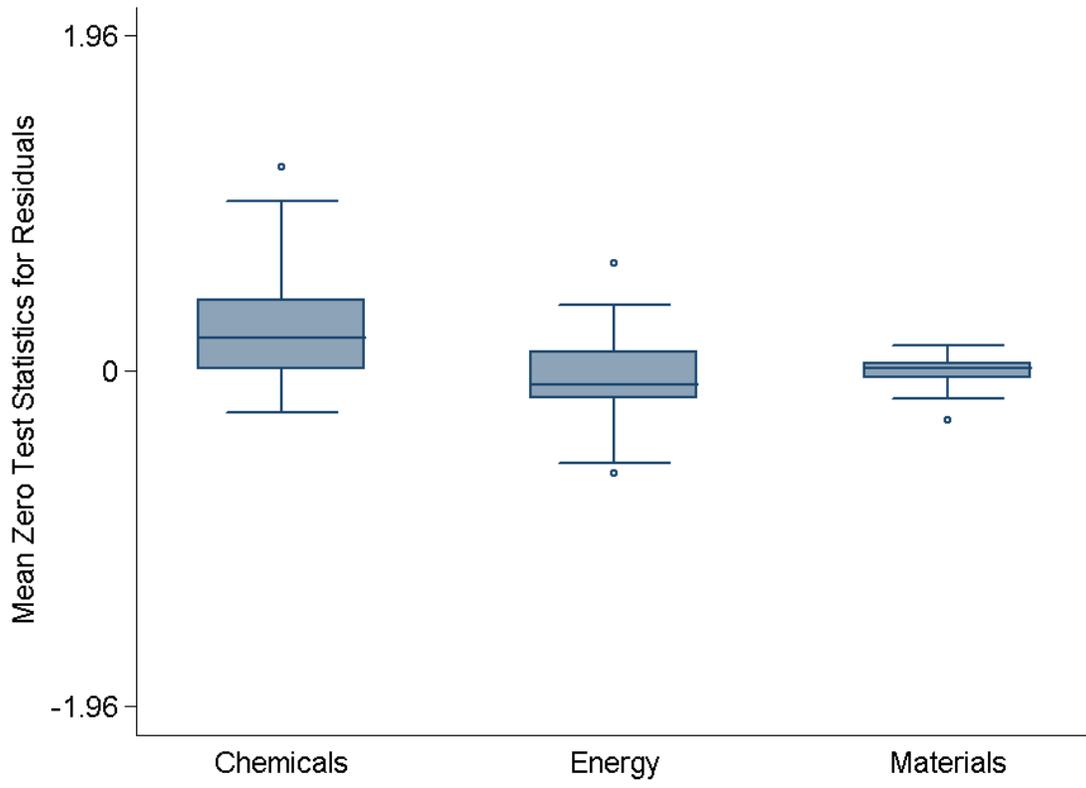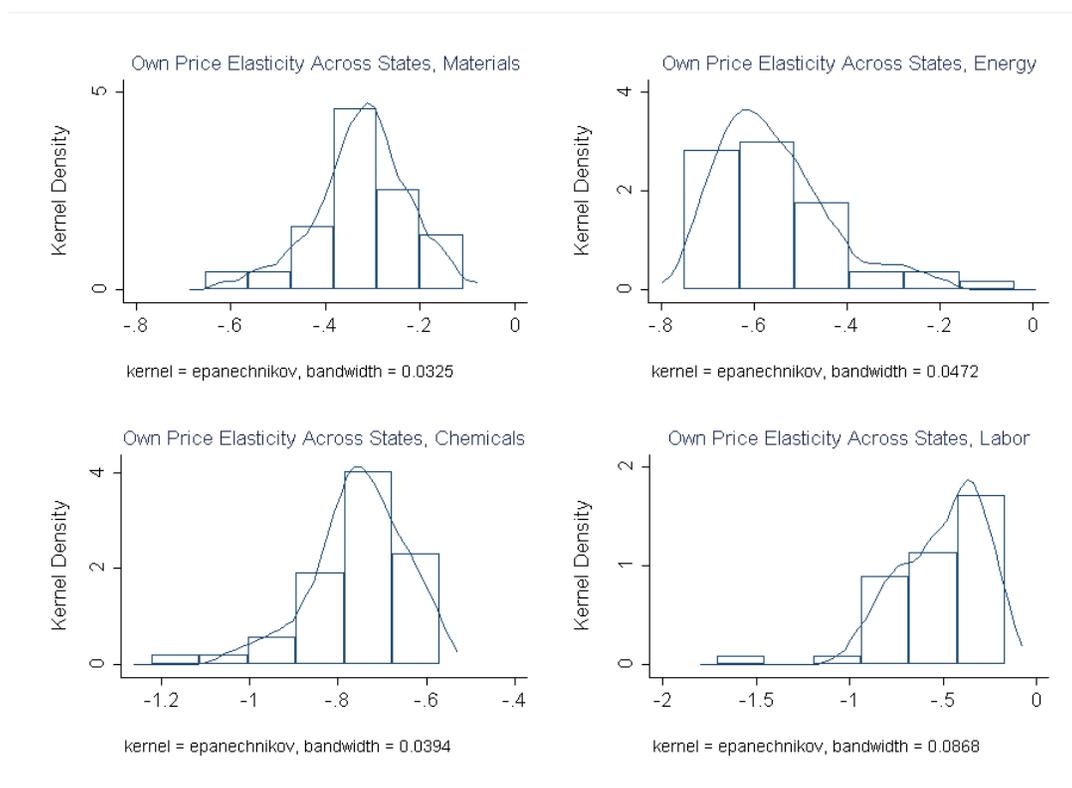**Figure 4. Mean Zero Test Statistics for Residuals.**

Figure 5. Kernel Density Plots for Average State-Level Own-Price Elasticities.

For each equation (Materials, Energy, Chemicals, and Labor), time-average own-price demand eladticities are calculated for each state. Each graph is a kernel density plot (overlying corresponding histogram) of the forty eight time-averaged elasticities.