Growth with backstop resources: The role of population and habitat

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Abstract

This paper analyses the joint effect of resources, population, and habitat constraints on long-run growth. It shows that the sustainability of growth obtained in the existing literature is not immune to extensions such as backstop resources with an upper bound and population growth. Specifically, under such a setting, both per capita income and population are bounded due to the Malthusian trap caused by resource scarcity. Only by accounting for the interaction between habitat and production, as well as habitat and fertility, can one show the feasibility of sustainable growth. The paper demonstrates that under these conditions, the population converges to a constant level in the long run and its growth becomes independent of income. On the other hand, due to habitat constraints, population growth may follow a non-monotonic path with a decline before reaching the stationary level. The only way to ameliorate such a decline is to promote 'green' technologies and policies that allow for growth with lesser pollution.

Key words: *long-run growth*, *resources*, *environment*, *habitat*
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1 Introduction

The possibility that growth may collapse with depletion of nonrenewable resources was pointed out by Hotelling (1931) and later by Meadows et al. (1972). This issue was seriously addressed by the strand of literature known as the Dasgupta-Heal-Solow-Stiglitz (DHSS) framework(Dasgupta and Heal, 1974; Solow, 1974a,b; Stiglitz, 1974a,b). The studies within the DHSS framework explicitly assume that resources are an essential input in production. The important finding of these studies is that the long-run sustainability of consumption is feasible as long as the rent obtained from natural capital (resources) are invested fully in produced capital (Hartwick, 1977). However, this finding is based on the assumption that the rate of substitution between natural and produced capital is unbounded. This does not appear to be in line with evidence.[1]

To overcome this problem, the DHSS framework proposes a concept of a backstop technology, which implies switching to an alternative resource that is too costly to use, while the traditional nonrenewable resources are still relatively abundant and cheap. However, the price of resources increases with their depletion rate, and at some point, the use of alternative backstop resources becomes viable. Discussions of this matter can be found in Ayres and Warr (2009), Endress et al. (2005), Dasgupta (1993), Dasgupta and Heal (1974), Heal (1976), Kamien and Schwartz (1978), and Prell (1996). In addition, Hart and Spiro (2011) and Spiro (2014) provide an explanation of long-run price dynamics of exhaustible resources.

As viable backstop technologies emerge, they can be used together with the traditional ones. The evolution of the use of resources along these lines has been considered by Chakravorty et al. (2012), Tahvonen and Salo (2001), Tsur and Zemel (2003, 2005), and van der Meijden and Smulders (2013). They introduce the possibility of substitution between the nonrenewable and backstop resources and consider

[1] With regard to the latter point, Daly (1997) argues, based on Georgescu-Rogen (1979), that natural capital and manufactured capital are complements rather than substitutes; hence, the sustainability of growth envisioned in the DHSS model is not feasible. In the same vein, Smulders (2003), Krautkraemer (1998), and Dasgupta (1993) argue that there is a limit to the level of substitution between physical capital and a nonrenewable resource. Hogan and Manne (1979) estimated the elasticity of substitution between energy and other production inputs to be around 0.25. This implies that the degree of substitution between energy and other inputs is quite low; hence, an increase in the relative price for energy will not lead to a significant change in relative input structure.

the stage of economic development with both types of resources used simultaneously. In general, a transition from traditional resources to backstop ones may not be smooth, as a discontinuity in production as a result of jumping from traditional resources to the backstop resources also can be possible (see D'Alessandro et al., 2010; Oren and Powell, 1985; Valente, 2011).[2] Nevertheless, all these studies indicate that the long-term solution towards sustainability of growth rests on the use of backstop resources.[3]

Following the rationale of the above-mentioned literature, in this paper, the existence of backstop resources is accepted as a solution for long-run sustainability. However, instead of exogenously given unlimited backstop resources, the backstop resources are assumed to have been obtained as a result of a deliberate effort to explore new ways of extraction. This paper postulates that the capacity to extract the backstop resources depends on the level of knowledge; hence, the level of human capital is the driving force of this capacity. It also assumes that there is a maximum level of backstop resources that potentially can be extracted. The rationale for the latter assumption stems from the law of energy conversion, which states that energy (matter in general) cannot be created or destroyed; it can only be transformed from one form to another. For example, the solar energy that can be harvested is limited by the radiation that the planet can receive from the sun. In addition, it has been highlighted that the marginal product of resources is bounded from above (e.g., Baumgärtner, 2004; Smulders, 2003). Therefore, contrary to what is proposed by the DHSS framework, the resource input cannot be reduced below some minimum fraction of the output. To accommodate for the low substitution between natural resources and other factor inputs, this paper follows the approach employed

[2]Le and Van (2014) analyse this dichotomy stemming from the use of nonrenewable and renewable resources within a single model and demonstrate its implications for the sustainability and long-run growth.

[3]A strand of literature has attempted to address the above-mentioned critique by considering other ways to resolve this problem, such as through an improvement of the resource efficiency, the development of backstop technologies, and an increase in the stock of usable resources through exploration and research. For example, Barbier (1999) shows that technological innovations can reduce resource use and help sustain economic growth. A somewhat similar conclusion is obtained by Bretschger and Smulders (2012), who find that sustainability should rely on structural changes that reduce resource use in the economy instead of high substitution elasticities between resources and produced capital or efforts on improving the productivity of resources. Nevertheless, it has been argued that production can be sustainable only by switching to use of alternative non-scarce resources (e.g., André and Cerdá, 2006).

by Bretschger and Smulders (2012) and Smulders and de Nooij (2003) and assumes a CES production function.

Next, alongside the existing literature pioneered by Malthus (1798),[4] which recognises that both demographic and economic dynamics are related and, thus, need to be considered jointly, this paper considers the link between population and long-run economic growth. Malthus's ideas about the sustainability of long-run growth have been addressed in unified growth theory (UGT), developed by Galor (2005, 2011) and Galor and Weil (2000). UGT argues that, after transitioning from agrarian to industrial production, land stops being a crucial factor that drives the growth of output and population. This implies that, in the long run, both income and population growth will not be constrained by a limited resource such as land. However, it should be noted that the feasibility of unconstrained population growth in a habitat of a limited size has been questioned (Brander and Taylor, 1998; Good and Reuveny, 2006; Peretto and Valente, 2015). Specifically, similar to UGT, assuming an inexhaustible but limited resource (land), Peretto and Valente (2015) consider a Schumpeterian growth model and show that, when individuals require some minimum resources for habitat, population will be static in the long run.[5]

To study the relationship between population and economic growth, this paper adopts an approach suggested by Peretto and Valente (2015), in which they take into account the effect of habitat on economic growth. To link population and economic growth, this paper extends a paternalistic family model employed in Doepke (2004), Jones and Schoonbroodt (2010), and Mookherjee et al. (2012). In particular, fertility is assumed to depend not only on income constraints but also on the limits imposed by the human habitat. It is also noted that not only the size of habitat but also the quality of the inhabitable environment may affect both production and

[4]See, for example, Barro and Becker (1989), Becker et al. (1990), Doepke (2004), and Kalemli-Ozcan (2003).

[5]The role of habitat on population growth has been studied in another strand of literature (bio-economic models) devoted to the analysis of the resource-growth nexus that considers exhaustible and renewable resources and their interaction with economic activity and population dynamics (Brander and Taylor, 1998; Dalton et al., 2005; Good and Reuveny, 2006; Pezzey and Anderies, 2003). These models explain how a Malthusian-type growth regime can develop and, in the case of over-harvesting, can lead to a collapse of the ecosystem and the economy with it. The main difference between the model presented in this paper and the bio-economic models is that this paper focuses on the evolution of the ecosystem that depends on the pollution stemming from production, but not on harvesting.

fertility in the long run (Mariani et al., 2010; Pautrel, 2009; Raffin and Seegmuller, 2014; Varvarigos, 2013). The current paper incorporates this mechanism by taking into account the quality of habitat, which depends on the environmental damage that stems from production. In light of these extensions, the model presented in this paper departs from Peretto and Valente (2015) in two important aspects: (i) the effective habitat depends on both the size and quality of the available environment and (ii) the effective habitat drives the cost of fertility.

The main findings of this study are as follows. The sustainability of growth obtained in the DHSS framework is not immune to extensions such as backstop resources with an upper bound and population growth. Only by accounting for the interaction between habitat and production, as well as habitat and fertility, can one show the feasibility of sustainable growth. More specifically, one implication of not taking into account the effect of habitat on fertility is that population growth can be non-monotonic due to falling income levels. This outcome is possible because an increasing population creates a drag on the use of resources. This ultimately limits both economic and demographic growth, because the rate of substitution between resource input and capital is limited, and the potentially extractable resources have an upper bound. Under these conditions, the economy reaches its steady state only when the population becomes static. This paper will show that income growth driven by technology is not sustainable, in this setup. If per capita income growth could be maintained even with falling resource input per capita, then population would expand continuously. On this growth path, given the limited substitution rate between resources and other factors, unrestricted population growth would reduce resource input per capita to levels that could not sustain income growth. As a result, per capita income and fertility rate would collapse. This implies that, under these conditions, the economy would face a Malthusian trap.

As aforementioned, the outcome of unconstrained population growth cannot be realistic, as it appears sensible to expect that the population size will be constrained by the carrying capacity of the ecosystem and, because of this, population growth will be limited. In light of this, the model in this paper is extended by integrating a minimum quality-quantity-level requirement for the habitat per capita to sustain population growth. This requirement can be challenging, as the quality of habitat is degraded by the pollution stemming from the use of resources. The extension of this model yields some new insights. In particular, it identifies four different growth paths, depending on the shape of the pollution function and its parameters. According to the first two growth scenarios, population grows monotonically until its further growth is limited by the habitat's carrying capacity. On the other two paths, a population overshoot is possible, and thus, the steady-state quality of the habitat can be achieved only by a decline in population to lower stationary levels. That is, the adjustment to the long-term path might be fraught with social and political upheavals.

Under such circumstances, continuous growth of output can be achieved without experiencing a decline in population only if technological progress can reduce the negative impact of production on habitat to the level where the damage can be stopped. This result highlights the crucial role of 'green' technologies and policies in overcoming the constraints imposed by the limits of the environment on long-run growth and sustainability. This finding complements the argument by Smulders et al. (2014), who emphasise that there is no assured way that monotonically leads to a 'green steady state' along an optimal growth path.

Overall, this paper contributes to the literature by (i) introducing a growth model with endogenous fertility that depends on both income and use of backstop resources with an upper bound; (ii) showing that sustained income growth is feasible only if habitat congestion and degradation is at or above certain minimum threshold levels; (iii) demonstrating that the limits imposed by habitat can lead to a population decline, which implies that the total population size can evolve along an inverted-U- shaped path; and (iv) showing that the sustained quality of the environment, along with continuous growth, is feasible. Moreover, if technological progress allows for both productivity increases and a cleaner ('greener') production process, a full abatement of pollution is feasible.

The above-stated findings are subject to a few caveats, as there are some important aspects that have not been taken into account in this study. First, the role of resources in technological change may have a 'sustainability bias' (Groth, 2007), as resources might actually put additional constraints on technological progress (Alcott, 2005; Tisdell, 1990). This channel, through which resource scarcity may affect long-run growth, requires further research. Second, this paper has not accounted for pollution and its effect on economic growth through its impact on longevity and fertility. Moreover, the endogenous mechanisms, that drive technological progress and raise productivity, as well as reduce the burden on environment, have not been considered. These shortcomings imply that there is a need to consider a fully endogenous growth model that examines resources and population in a setting that accounts for the quality of the environment, the limits it puts on population and economic growth, as well as the possible effects of resources on technological progress. Future research efforts along these lines will shed hopefully a new light on this important aspect of growth theory.

The remainder of the paper is structured as follows. Section 2 presents the setup of the model, lays out the optimisation problem, presents the solution, and discusses the dynamics of the economic system. Section 3 discusses the implications of accounting for the effect of habitat on population and economic growth in the long run. Section 4 concludes the paper.

2 The model

2.1 The basic setup

This section develops a two-period overlapping generations (OLG) model. To represent household preferences, the model follows the existing literature and assumes a paternalistic utility function with bequest, which specifies that the parents derive utility from the 'warm glow' of the bequest they leave and the quantity and quality of children they have, but not from their future welfare.[6] Therefore, the utility function of the representative agent is given by $u(c_t, d_t, h_{t+1}n_t)$, where $c_t$is consump-

[6]See Galor and Zeira (1993), Banerjee and Newman (1993), Kollmann (1997), and Aghion and Bolton (1997). See more on the "warm glow" utility function in Andreoni (1989, 1990).

tion, d_t is the bequest, h_{t+1} is the level of human capital obtained by the children, $n_t = b_t p$ is the number of surviving children, with b_t being the number of births and p $(0 < p < 1)$ being the probability of survival. Following Galor and Zeira (1993), a utility function of the following specification is adopted:

$$u_t = \ln(c_t) + \sigma \ln(p b_t h_{t+1}) + \delta \ln(d_t), \tag{1}$$

where σ and δ are the parameters that capture the degree of altruism of the parents. Human capital is assumed to evolve according to

$$h_{t+1} = e_t^{\tau} h_t^{1-\tau},$$

where e_t is the amount of income spent on the education of the children, and h_t is the parental human capital.

The cost of raising a child stems from the labour income forgone due to the time spent on a child (Becker et al., 1990; de la Croix and Doepke, 2003; Fioroni, 2010) and from the direct non-labour cost of child maintenance (Doepke, 2005; Jones et al., 2011; Kollmann, 1997). Given that the income share of labour is fixed over time, the income foregone due to the lost labour to look after a child is also a fixed fraction of the parental income. On the other hand, the non-labour cost of raising a child does not have to proportionally increase with the income level of the parents. If general living costs are falling behind the income growth, one would expect the non-labour cost of raising a child to fall as a share of income; however, if the living costs are growing faster than income, then the non-labour costs of raising a child would increase as a share of income. For example, if housing costs are increasing as a share of income due to urbanisation and congestion, it most likely would lead to an increase in the cost of raising a child, relative to the income level.

The intuition described above agrees with the evidence based on cross-country data that the relationship between fertility and income is of an inverted-U shape (see, for example, Fioroni, 2010). Based on this simple intuition, it can be postulated that the cost of raising a child, in general, is nonlinear in the level of disposable income, y_t, and the habitat related costs, v. Specifically, it can be assumed that the cost related to raising a child is given by

$$\psi \left(vy\right)^{\theta}, \tag{2}$$

where $\psi > 0$ and $0 < \theta$ are cost parameters. The above specification implies that, as mentioned above, urbanisation or congestion may lead to an increase in the cost of raising a child, relative to income. This type of structural change can be captured by allowing the child-raising cost to be a function of other non-income factors, such as the urbanisation rate or the congestion of the environment. This aspect will be revisited in Section 3, but meanwhile it is assumed that v is a fixed parameter.

In the given setup, the budget constraint faced by the adult agent is expressed as

$$c_t + d_t + (\psi \left(vy\right)^{\theta} + pe_t)b_t \leq w_t + i_t k_{t-1} = y_t, \tag{3}$$

where w_t is the market wage rate and i_t is the rate of return to capital. The law of motion of the adult population is given as

$$L_{t+1} = L_t[1 + (pb_t - \mu)], \tag{4}$$

where μ is the death rate given exogenously. This implies that the population growth rate is given as $s_t \equiv b_t p - \mu$. The amount of bequest is saved in the form of physical capital which is used by the firms. Therefore, the capital accumulation process in per-worker terms is governed by

$$k_{t+1} = \frac{d_t}{pb_t}. \tag{5}$$

2.2 The representative agent's problem

The representative agent maximises intertemporal utility given by

$$\max_{c,d,b,e} U = \ln(c_t) + \sigma \ln(pb_t h_{t+1}) + \delta \ln(d_t), \tag{6}$$

subject to the following constraint:

$$c_t = y_t - d_t - \left[\psi\left(y_t v\right)^\theta + pe_t\right] b_t. \tag{7}$$

Substituting for c_t from (7), the agent's problem in an unconstrained form can be stated as

$$\max_{d,b,e} U = \ln\left\{y_t - d_t - \left[\psi\left(vy\right)^\theta + pe_t)\right] b_t\right\} + \sigma \ln(pb_t e_t^\tau h_t^{1-\tau}) + \delta \ln(d_t). \tag{8}$$

Solving the first-order conditions of this optimisation problem for the equilibrium values of e_t, d_t and b_t yields the following:

$$e_t = \frac{\tau\psi(vy_t)^\theta}{p\sigma(1-\tau)}, \tag{9}$$

$$b_t = \frac{By_t^{1-\theta}}{v^\theta}, \tag{10}$$

$$d_t = \frac{\delta y_t}{1+\delta+\sigma}, \tag{11}$$

where $B \equiv \frac{\sigma^2(1-\tau)}{\psi[(\sigma(1-\tau)+\tau)(1+\delta+\sigma)]}$.

The results obtained above lead to the following lemma.

Lemma 2.1 *The birth rate b_t, the spending on education e_t, and the amount of bequest d_t, are increasing in the per capita income level. On the other hand, the birth rate is decreasing in the habitat-related cost, v.*

Proof The result is immediate from (10) and (11). ■

The above results are in line with the findings in the literature. With increasing income, agents tend to spend more on education for the children and leave large bequests. The only result that may invoke a concern is a positive relationship between fertility and income, given as $\theta < 1$. However, as mentioned earlier, cost parameter v can be viewed as endogenous and driven by the development characteristics of the economy. For example, if the habitat-related costs increase faster than income levels, one can verify by using (9) and (10) that the well-known 'quality–quantity trade-off' can occur. That is, an increase in income levels would lead to a fall in fertility, accompanied with greater spending on education of children. This line of reasoning indicates that to capture the inter-relationship between fertility and income fully, the model needs to be modified to account for a negative feedback from development to fertility. This point will be revisited later when the role of habitat in economic growth is discussed.

2.3 Production, extraction, and exploration sectors

Households own firms and supply capital to their firms. Firms produce a single final good. It is assumed that the production function for the final good is given as

$$Y_t = A\left(\gamma {R_t}^{\rho} + (1-\gamma)\left(K_t^{\alpha}(H_t)^{1-\alpha}\right)^{\rho}\right)^{\frac{1}{\rho}}, \tag{12}$$

where A stands for a Harrod-neutral technology coefficient, R is the amount of resources input, and K and H are total physical and human capital, respectively, engaged in the final consumption goods sector. Output of final goods is divided among consumption, investment into physical and human capital, and the cost of providing the resource as input to the production process. Following Endress et al. (2005), it is assumed that the unit cost of extracting the natural resource and providing it as input to production is given by ϕ. However, in this case, the cost does not depend on the stock of resources, as it is assumed that only inexhaustible backstop resources are used. Instead, it is postulated that this cost is a decreasing function of the overall productivity measured by the technology coefficient, A_t. Due to the cost related to resource extraction, the disposable income of per capita then is given as $y_t = \frac{Y_t}{L_t} - \phi r_t$.

Another distinctive feature of this model is that the resources are assumed to be of a backstop nature only, which, in this context, implies that they are fully renewable. All nonrenewable resources are assumed away, as the DHSS framework demonstrated that the long-run sustainability hinges upon the availability of backstop resources (see D'Alessandro et al., 2010; Oren and Powell, 1985; Valente, 2011). Therefore, straight away, it is assumed that the economy exhausted all nonrenewable resources and that production is based only on renewable backstop resources. However, this differs from the existing models with backstop resources by assuming that there is an upper bound for such resources. The simple rationale is that even the solar energy that can be harvested, as well as other resources that can be reused, is limited at some point. That is, the law of conversion of energy holds; therefore, resources cannot be made out of nothing. They can only be transformed.

In light of the discussion above, the resource extraction is modelled as the intermediate goods sector with an output price, ϕ, in terms of final goods. This resource extraction output is an increasing function of the maximum level of extractable backstop resource X. In light of this, this function is given as

$$R_t = \pi X, \tag{13}$$

where $0 < \pi \leq$ is the extractive capacity. It is assumed that the extractive capacity depends on the level of human capital and is given as

$$\pi_t = \frac{\underline{\pi} + \eta H_t}{1 + H_t}. \tag{14}$$

Since $0 < \pi \leq$, then $0 < \underline{\pi} < 1$ and $0 < \eta$. The extractive capacity converges to η when $H_t \to \infty$.

Firms maximise their profits and take the interest rate, wage rate, and cost of resources as given. This optimisation yields the following equilibrium values: the rate of return to physical capital i, and the rate of return to labour w, and the unit cost of resource extraction ϕ. The values are found as the cost of the marginal product of the respective factor:

$$i = \frac{\partial F}{\partial K}, \; w = \frac{\partial F}{\partial L}, \; \phi = \frac{\partial F}{\partial R}. \tag{15}$$

Now let $r = \frac{R}{L}$, $h = \frac{H}{L}$, $k = \frac{K}{L}$ and, recalling that y denotes disposable income after paying the cost of resource extraction, given as $y = \frac{Y}{L} - \phi r$, output of final goods per capita is rewritten as follows:

$$y = A\left(\gamma r^{\rho} + (1-\gamma)(k^{\alpha} h^{1-\alpha})^{\rho}\right)^{\frac{1}{\rho}} - \phi r. \tag{16}$$

2.4 Dynamics of the model

The solution of the model allows the dynamics of the model to be described analytically. For that purpose, recall from (5) that $k_{t+1} = \frac{d_t}{pb_t}$ and from (11) that $d_t = \frac{\delta y_t}{1+\sigma+\delta}$ and $b_t = \frac{By_t^{1-\theta}}{v^\theta}$. Then the capital evolution is given by

$$k_{t+1} = \frac{\delta(vy_t)^\theta}{pB(1+\sigma+\delta)} = \frac{\delta v^\theta \left[A\left(\gamma r_t^\rho + (1-\gamma)(k_t^\alpha h_t^{1-\alpha})^\rho\right) - \phi r_t\right]^{\frac{\theta}{\rho}}}{pB(1+\sigma+\delta)}.$$

To obtain the balanced-growth path (BGP) values; the effective capital per worker needs to be determined. That is, the following modified variables are employed: $\hat{k}_{t+1} \equiv \frac{k_{t+1}}{h_{t+1}}$, $\hat{r}_t \equiv \frac{r_t}{h_t}$, $\hat{k}_t \equiv \frac{k_t}{h_t}$, and the following equation is derived:

$$\hat{k}_{t+1} = \frac{\delta v^\theta \left[A\left(\gamma \hat{r}_t^\rho + (1-\gamma)\hat{k}_t^{\alpha\rho}\right) - \phi \hat{r}_t\right]^{\frac{\theta}{\rho}}}{pB(1+\sigma+\delta)}. \tag{17}$$

From the equality of the marginal product of human and physical capital on the BGP, it can be postulated that, in the steady state, $\hat{k}_t = \hat{k}_{t+1} = \bar{\hat{k}}$ is constant. Therefore, both human and physical capital grow at the same rate on the BGP.

Through consideration of the evolution of resource use on the path where growth rates of physical and human capital are balanced, the following lemma is stated.

Lemma 2.2 *A balanced growth path of human and physical capital implies declining resource use in per capita terms. With production given as a CES function, this also implies falling per capita income.*

Proof Clearly, $L_t \to \infty$ implies that $H_t \to \infty$, since $\lim_{H_t\to\infty} \pi_t \to \eta$ and $\lim_{H_t\to\infty} x_t \equiv \frac{X}{H_t} \to 0$. Then given that $\hat{r}_t \equiv \frac{\pi_t X}{H} = \pi_t x_t$, it follows that $\lim_{H_t\to\infty} \hat{r}_t \to \eta x_t \to 0$. Production following a CES function implies that $\lim_{L_t\to\infty} y_t \equiv \lim_{\bar{r}_t\to 0} y_t \to 0$. ■

Using the above lemma, a further conclusion can be made about the steady state.

Proposition 2.3 *If fertility is increasing along with income, and resources are bounded from above, the economy achieves a stationary point, where $y = const$ and $pb_t - \mu = 0$ hold.*

Proof Recall that the population evolves according to $L_{t+1} = L_t [1 + (pb_t - \mu)]$. Given that $b_t = \frac{By_t^{1-\theta}}{v^\theta}$, the birth rate is positive as soon as output is positive. If the birth rate is high enough that the population growth rate satisfies the condition $pb_t - \mu > 0$, then population growth is positive. According to Lemma (2.2), this implies that $\lim_{L_t \to \infty} \hat{r} \to 0$; thus, $\lim_{L_t \to \infty} y_t \to 0$, so that, due to falling income per capita, the birth rate declines, and at some low-enough levels of per capita income, $pb_t - \mu < 0$ becomes possible. Therefore, at some low-enough output per capita, population stops growing. That is $\lim_{L_t \to \bar{L}} \hat{r} \to \underline{r}$; thus, $\lim_{L_t \to \bar{L}} y_t \to \underline{y}$, so that $pb_t - \mu = 0 | y = \underline{y}$. In the absence of technological improvements, at this point, the static population implies constant output $y = \underline{y}$, as any change in output would also change the size of the population. ■

The above result indicates that a stationary point is possible only if there is no income growth. However, this steady state is reached only at very low levels of income and, most likely, with a serious overcrowding.

Now consider the case when technology is growing. Recall the function of per disposable capita output:

$$y = A_t \left(\gamma r^\rho + (1 - \gamma)(k^\alpha h^{1-\alpha})^\rho \right)^{\frac{1}{\rho}} - \phi r.$$

In the above analysis, it was assumed that the technological coefficient, A_t, was a fixed parameter. Now, this assumption is relaxed and it is assumed that the technology improves according to the exogenously given process as follows:[7]

$$\Delta A_t = a.$$

Under such a condition, per capita output, y_t, keeps growing at the same rate as technology, even if all the inputs to production are fixed. However, growing output implies a growing population, L_t, given that the fertility rate, $b_t = \frac{By_t^{1-\theta}}{v^\theta}$, where

[7] In the endogenous growth literature, this process usually is given by a vertical innovation process, such as $\Delta A_t = aH_t L_{At}$, where L_{At} is labour employed in the research and development (R&D) sector. In this setting, it implies that $\Delta A_t > 0$ is possible as $H_t > 0$, and it can be assume that some fraction of labour can be dedicated to R&D. This implies that, with rising human capital H_t, the productivity growth is accelerating. To avoid this situation, it is simply assumed that $\Delta A_t = const$.

$\theta < 1$ and $B > 0$. This consideration leads to the following corollary.

Corollary 2.4 *If fertility is increasing along with income, and resources are bounded from above, growth cannot be sustained by technological improvements only.*

Proof In this case, if income grows due to technology while h and k are fixed, this results in population growth. Hence, $H = hL_t$ keeps growing, which according to Lemma (2.2), entails $\lim_{H_t \to \infty} \hat{r} \to 0$ and hence, implies that $\lim_{\hat{r}_t \to 0} y_t \to 0$. Therefore, given a CES technology in a steady state, technological improvements cannot solve the problem of resource scarcity. ■

The main conclusion that can be drawn from the analysis of the modified DHSS model is as follows. When (i) the resources are entirely based on the backstop technology and there is an upper bound on its potentially extractable amount, (ii) the substitution between resources and other factors are limited, and (iii) the growth of population are endogenous, the overall characteristics of the dynamics of this system are different than the original DHSS model. Most importantly, contrary to the Solow model, in this modified model, technological progress cannot lead to sustained growth, but only can guarantee collapse of income per capita due to scarcity of resources. A steady state is possible under a Malthusian-type trap, where per capita income levels are low enough to sustain a constant population. In general, the results stated in Proposition (2.3) and Corollary (2.4) seem quite pessimistic, as if back in the Mathusian world. The next subsection will consider the effects of habitat on population expansion along with income growth and ascertain whether the new assumptions will yield a different result.

3 The effect of the habitat on population growth

Incorporating the effect of habitat on fertility choice can be considered as a way to avoid a situation where population growth limits long-run growth through its effect on resources. If the effect of boundedness of habitat can offset the positive effect of income on fertility, it may be possible to have long-run income growth without any drag from population growth. This rationale draws upon the argument suggested by Peretto and Valente (2015) that the limited size of the planet will impose constraints on population growth. In addition, the quality of the inhabitable environment also is likely to affect both production and fertility in the long run (Mariani et al., 2010; Pautrel, 2009; Raffin and Seegmuller, 2014; Varvarigos, 2013). In light of these arguments, it is postulated that not only the size of the habitat but also its quality are the factors affecting production and fertility.

The simplest way will be to assume that the marginal cost of having and raising a child increases with the decline of the quality and quantity of the habitat. This implies that the amount of effective habitat evolves according to $\Omega_t = \bar{\Omega} q_t$, where q_t is the quality and $\bar{\Omega}$ is the fixed size of the habitat, respectively. In this setting, to sustain the growth of population, the agents require a minimum amount of habitat use, $\underline{\epsilon}$. Since both congestion and degradation are important factors that affect habitat, it is reasonable to expect that the threshold level of the habitat use index is reached as soon as either $\frac{\bar{\Omega}}{L_t} \equiv \bar{\omega}_t = \underline{\omega}$ or $q_t = \underline{q}$ holds, where $\underline{\omega}$ and $\underline{q}$ stand for the threshold values of congestion and environment quality, respectively. Thus, the following function can be employed as the index of effective habitat use:

$$\epsilon_t = f(\bar{\omega}_t, q_t) \text{ such that } \epsilon_t = \underline{\epsilon}, \text{ if } \bar{\omega}_t = \underline{\omega} \text{ or } q_t = \underline{q}, \quad \frac{\partial \epsilon}{\partial \bar{\omega}} > 0 \text{ and } \frac{\partial \epsilon}{\partial q} > 0. \tag{18}$$

Now, it is assumed that the child-raising cost stemming from the habitat constraints, v, is a convex function and depends on the habitat use index. Specifically, it is assumed that $\frac{\partial v}{\partial \epsilon} \gtreqless 0$ and $\frac{\partial^2 v}{\partial \epsilon^2} > 0$. Applied to the existing setting, this implies that the birth rate given by (10) is adjusted as follows:

$$b_t = \frac{By^{1-\theta}}{v(\epsilon)}. \tag{19}$$

Thus, if at some relatively large values of ϵ (plenty of high quality habitat), $\frac{\partial v}{\partial \epsilon} < 0$ is observed, then rising income affects population growth positively. On the other hand, if at some relatively small values of ϵ, such that $\epsilon < \underline{\epsilon}$ (habitat degradation and congestion), $\frac{\partial v}{\partial \epsilon} > 0$ is observed, rising income is accompanied with increasing child-raising cost stemming from habitat degradation and congestion; this condition will reduce population growth.[8] If the pressure of overcrowding on population growth exactly offsets the effect on population growth stemming from increasing incomes, then a stagnant population is feasible at some point of economic development. In other words, there is some threshold value for the habitat use, $\underline{\epsilon}$, such that when $\epsilon < \underline{\epsilon}$, fertility falls with income, and hence, $\frac{\partial b}{\partial y} + \frac{\partial y}{\partial v}\frac{\partial v}{\partial \epsilon} < 0$. This also implies that, when $\epsilon = \underline{\epsilon}$, fertility does not depend on income level; thus, $\frac{\partial b}{\partial y} + \frac{\partial y}{\partial v}\frac{\partial v}{\partial \epsilon} = 0$.

3.1 The effect of production on the quality of the habitat

Now, to ascertain the mechanisms through which output growth affects population growth, we need to model the evolution of the quality of the habitat and its scarcity. The quality of the habitat is assumed to depend on the pollution stemming from resource use. The quantity of the inhabitable environment is fixed and equal to Ω, whereas the quality, q_t, depends on the flow of pollution stemming from production and the improvements generated by the abatement activities carried out by the government. Following Varvarigos (2013), it assumed that the quality of the habitat is given by

$$q_t = Q - \chi_t, \tag{20}$$

where Q denotes the maximum value of the habitat quality measure and χ_t is the stock of pollution. This formulation implies that the quality of the habitat has a certain fixed maximum value. The stock of pollution evolves according to

$$\chi_{t+1} = m\chi_t + D_t, \tag{21}$$

where $m \in (0, 1)$ is the natural rate of absorption of the pollution, whereas D_t is the net flow of pollution. Substituting (21) in (20), the quality of the habitat evolves according to

$$q_{t+1} = Q(1 - m) + mq_t - D_t.$$

[8] Given the historical evidence, such as the Easter Island ecological catastrophe, this type of population dynamics is not far-fetched.

Since the environmental damage is determined by the net flow of pollution, its functional form is important in defining how the environmental degradation is related to the output produced in the economy. In the literature, there are two approaches to model the relationship between production and environmental degradation. In one case (see, for example, Economides and Philippopoulos, 2008; Raffin and Seegmuller, 2014), an increase in output can lead to improvements in the quality of the environment, as the abatement can more than compensate for the pollution that is caused by the production. The other view is that an increase in production always leads to the degradation of the environment, despite the abatement activities (see, for example, Palivos and Varvarigos, 2010; Pautrel, 2009; Varvarigos, 2013). To accommodate both views in one framework, it is assumed that the amount of pollution is related to the resources used in production in a nonlinear way and given as $a_1 R_t^{\zeta}$, where a_1 and ζ are parameters relating to resources used to pollution. The abatement activities are financed by taxing the total income of the economy and are given as $a_2 \tau Y_t$, where τ is the tax rate. Therefore, the net pollution flow is given by

$$D_t = a_1 R_t^{\zeta} - a_2 \tau Y_t. \tag{22}$$

3.1.1 Evolution of the quality of the habitat

If $\frac{\partial D}{\partial R} < 0$, then the evolution of the quality of the habitat will be optimistic, as it keeps improving with economic growth, which is driven by extensive use of resources until it reaches the maximum possible quality. Nevertheless, this might be seen as another type of a 'green bias' in modelling the effect of economic growth on the environment. The common knowledge so far is that, with economic growth, a deteriorating quality of the environment is observed. Hence, if it is assumed, based on the latter rationale, that net pollution rises with the growth of resources used, then $\frac{\partial D}{\partial R} > 0$. However, to include a possible evolution path where marginal net pollution decreases with the level of resources used, $\frac{\partial D}{\partial R} < 0$ is allowed.

Whether habitat degradation has a maximum level depends on the condition that $\frac{\partial^2 D}{\partial R^2} < 0$ holds. If this condition does not hold, then net pollution grows until the upper bound of the resource input is reached. If $\frac{\partial^2 D}{\partial R^2} < 0$ holds, then there is a maximum level of resource use, beyond which an increase in resources input leads to a marginal decrease in pollution. By analysing net pollution function (22), the following lemma is stated.

Lemma 3.1 *If the pollution function is given by* (22) *and the production function is given by* (12)*, the amount of net pollution is either a concave or convex function in the level of resources used, depending on the parameter values.*

Proof It can be verified that $\frac{\partial^2 D}{\partial R^2} \gtrless 0$ holds for the given net pollution and production function. The sign depends on the values of parameters.■

This implies that there are two possible paths along which pollution may evolve.

1. Suppose that $\frac{\partial D}{\partial R} > 0$ and $\frac{\partial^2 D}{\partial R^2} > 0$; hence, the net pollution is given as a convex function of resource use with a range in R^+.

2. Suppose that $\frac{\partial D}{\partial R} \gtrless 0$ and $\frac{\partial^2 D}{\partial R^2} < 0$; hence, pollution is given as a concave function of resource use. In this case, net pollution can also be negative.

Based on the assumption of the shape of net pollution, the following corollary is formulated.

Corollary 3.2 *If the pollution function is of a convex form, then the amount of resources that maximises net pollution is limited only by the upper bound of the resources, that is* $R_{max} = \arg\max(D_t) = \bar{R}$*. However, if the pollution function is of a concave form, then* $R_{max} = \arg\max(D_t) \gtrless \bar{R}$*.*

Proof If $D(R)$ is a convex function, pollution keeps growing until it reaches the maximum stemming from the upper bound of resources. If $D(R)$ is a concave function, then the maximum point can be within the feasible range determined by the upper bound of the resources; hence, $R_{max} = \arg\max(D_t) < \bar{R}$. However, it is also admissible that the pollution function attains its maximum beyond the feasible range; hence, $R_{max} = \arg\max(D_t) > \bar{R}$ is also possible.■

Now, the equation describing the evolution of the quality of habitat can be rewriten as

$$q_{t+1} = Q(1-m) + mq_t - a_1 R_t^{\zeta} + a_2 \tau Y_t. \tag{23}$$

By analysing Equation (23), the following corollary can be stated about the quality of the environment.

Corollary 3.3 *For the environment degradation to stabilise or reverse, it is required that the net pollution is non-positive.*

Proof This can be verified that, for the stock of pollution to decline, net pollution should satisfy $D_t \leq 0$. This condition is satisfied only if $\frac{R_t^{\zeta}}{Y_t} \leq \left(\frac{a_2\tau}{a_1}\right)$ holds.■

This implies that total output has no upper bound if the growth of output is driven by technology only. That is, even if $R_t < R_{max}$, output can grow without damaging the environment further, as soon as $\frac{R_t^{\zeta}}{Y_t} \leq \left(\frac{a_2\tau}{a_1}\right)$ holds. On the other hand, Corollary (3.3) implies that if $\frac{R_t^{\zeta}}{Y_t} > \left(\frac{a_2\tau}{a_1}\right)$, a growth path driven by extensive resources use leads to habitat degradation. Since profit maximisation and economic efficiency might lead to a heavier use of resources, this outcome is feasible.

The above results indicate that for environment-friendly growth, there is a need for more a proactive approach to keep pollution under control. In particular, having balanced growth, where increase in output is not coming mainly from the resource-intensive, high-pollution production, makes it possible to preserve the quality of the habitat. The other important policy issue is that linking the cost of abatement of pollution to the output produced may not be enough; the policies should make sure that the pollution is fully abated by resources collected through taxing output. Overall, the above results also indicate that continuous income growth driven by technology (as in the endogenous growth models) is possible in principle. The habitat degradation might be an obstacle on the continuous income growth path; however, if there is an upper bound on the pollution stemming from resources use, then this obstacle might be overcome. The next section considers possible paths of how the habitat and growth can evolve in the long run.

3.1.2 Multiple growth paths

In general, the dynamics of the quality of the environment, given by (23) and Lemma (3.1), imply that there can be multiple paths along which the quality of the environment can evolve.[9] Under the given setup, it is possible that the quality of the habitat falls to the level that, based on (18), it implies $\epsilon_{min} < \underline{\epsilon}$. In this case, an increase in output, before reaching the highest degradation of habitat, leads to a population decline. However, if $\epsilon_{min} > \underline{\epsilon}$, then population keeps growing until $\underline{\epsilon}$ is reached. Therefore, in this setting, the habitat in per capita terms can change, either due to (i) population increase, (ii) degradation of the environment, (iii) or both factors become binding. The common thing in both cases is that the habitat congestion will converge to its upper limit, $\underline{\epsilon}$. Consider all of these cases.

(a) Monotonic population growth

Convex pollution function

Let the pollution function, D, be convex in R, and hence, the quality of the environment is declining in R. The quality of the habitat reaches the threshold, $\underline{q}$, at the resource use level equal to or greater than its upper bound. That is, $R = arg[\underline{q}(R)] \geq \bar{R}$. This implies that the lower bound of habitat quality is not reached; hence, $q_t > \underline{q}$ and $\epsilon_t > \underline{\epsilon}$, thereby implying a growing population with rising income. Then, the index of habitat use will depend only on the size of the population, as $\bar{\omega}_t \to \underline{\omega}$ implies $\epsilon_t \to \underline{\epsilon}$. The population grows until the maximum level of habitat congestion is reached, $\bar{\omega}_t = \underline{\omega}$, and hence, $\epsilon = \underline{\epsilon}$ holds. After that point, income growth is decoupled from population growth. Figure 1 illustrates this growth path. As one can see, the population keeps growing until it reaches the congestion threshold at point B. In this case, the quality of the habitat is higher than the minimum threshold.

[9] Notably, these possible growth paths appear to be in line with the possible dynamics of growth in a finite world, as described in Meadows et al. (1992, p.123).

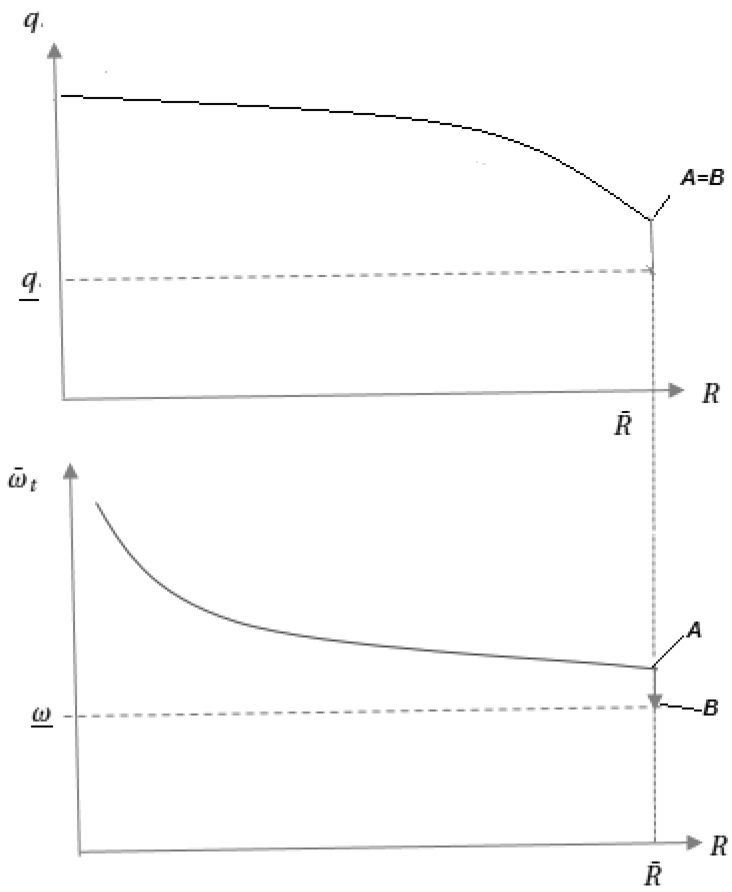

Figure 1: Convex pollution function and monotonic population growth.

Concave pollution function

Let the pollution function, D, be concave in R, and hence, the quality of the habitat is declining in R up to the $R = R_{max}$. After that point, the quality starts improving. In this case, it is assumed that $q(R_{max}) \geq \underline{q}$; hence, the quality of the habitat stays above the threshold. This implies that the lower bound of habitat quality is not reached; hence, $q_t > \underline{q}$ and $\epsilon_t > \underline{\epsilon}$, thereby implying a growing population with rising income. Then, the index of habitat use will depend only on the size of the population, as $\bar{\omega}_t \to \underline{\omega}$ implies $\epsilon_t \to \underline{\epsilon}$. The population grows until the maximum level of the habitat congestion is reached, $\bar{\omega}_t = \underline{\omega}$, and hence, $\epsilon = \underline{\epsilon}$ holds. After that point, income growth is de-coupled from population growth. However, the quality of habitat at point $A = B$ can be better than in the case with a convex pollution function shown in Figure 1. An illustration of this path is given in Figure 2.

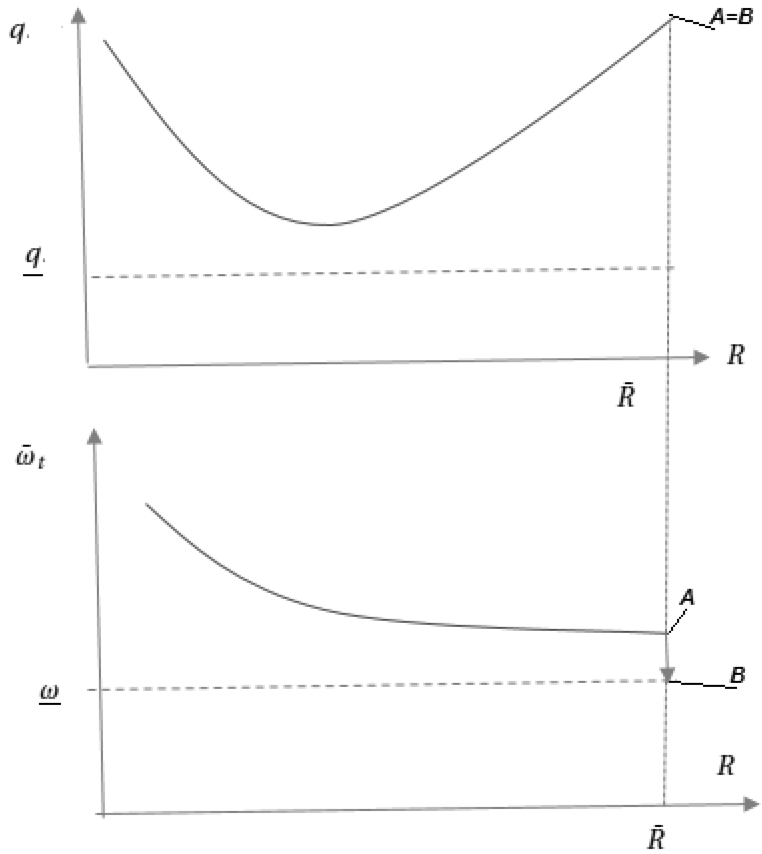

Figure 2: Concave pollution function and monotonic population growth.

(b) Non-monotonic population growth

The above described growth paths look quite optimistic– steady-state growth will be achieved eventually without degrading the human habitat to the conditions when the very quality of life will be threatened. However, there are other possible growth paths that might be less optimistic. The main problem identified on these paths is that the quality of the habitat, rather than habitat congestion, might prove to be a significant factor that determines the long-term welfare of people and might cause a population decline if the worst case scenarios of habitat degradation are not avoided.

Convex pollution function

Let the pollution function, D, be convex in R, and hence, the quality of the environment is declining in R. The quality of the environment reaches the threshold,

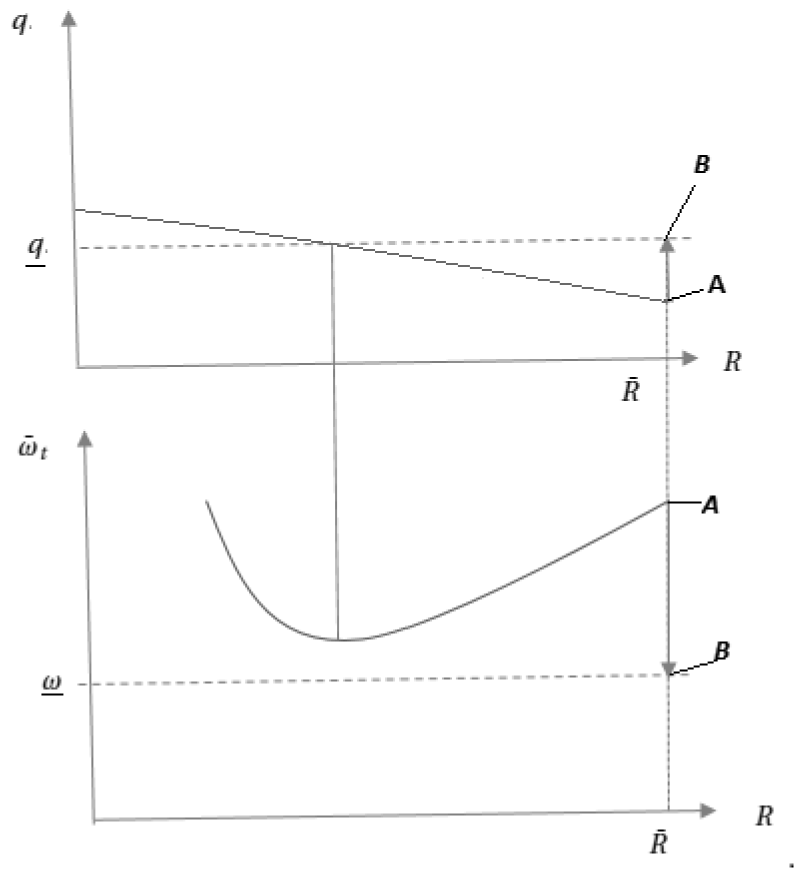

Figure 3: Convex pollution function and non-monotonic population growth

$\underline{q}$, at the resource use level that is below the upper bound of resources. That is, $R = arg[\underline{q}(R)] < \bar{R}$. In this case, a further increase in resource use and a deterioration of the quality of habitat implies that $q_t < \underline{q}$, and hence, $\epsilon_t < \underline{\epsilon}$. The congestion level can be given either by $\bar{\omega} > \underline{\omega}$ or $\bar{\omega}_t = \underline{\omega}$. In either case, this condition entails a shrinking population, as $\frac{\partial b}{\partial y} + \frac{\partial b}{\partial \epsilon}\frac{\partial \epsilon}{\partial y} < 0$. That is, this growth in output can be accommodated only by a falling population. A fall in the population size will increase $\bar{\omega}_t$. However, this change will not be enough to have $\epsilon_t \to \underline{\epsilon}$ because the quality of the habitat is still $q_t < \underline{q}$. The process of habitat quality degradation stops only when $R_t = \bar{R}$. At that point, $\bar{\omega}_t \geq \underline{\omega}$ and $q_t < \underline{q}$; hence, $\epsilon_t < \underline{\epsilon}$. Therefore, population keeps shrinking. Only further output growth driven by technology, enabling greater abatement, would lead to improvements in the quality of the habitat. When the quality of the habitat reaches the threshold, $q_t \geq \underline{q}$, the condition $\epsilon_t \geq \underline{\epsilon}$ is restored. This will lead to population growth up to the point when $\bar{\omega}_t = \underline{\omega}$ is achieved and population growth stops. If abatement dominates pollution in the long run, then the quality of the habitat will be restored to its highest level.

In Figure 3, one can see a version of such a path, where the degradation of the habitat is the main force leading to a population decline. In particular, when the level of resource use reaches its upper bound (point A), the quality of the habitat can become lower than the minimum that can sustain a constant population. Only if economic growth is possible due to technological improvements and pollution abatement increases is it possible to restore the quality of the habitat to $q_t > \underline{q}$. This will allow an increase in population; thus, the congestion rate will move from point A to point B, where $\bar{\omega}_t = \underline{\omega}$.

Concave pollution function

Let the pollution function, D, be concave in R, and hence, the quality of the habitat is declining in R up to the $R = R_{max}$. After that point, the quality starts improving. That is, for some $R^* > R_{max}$, $q(R^*) > \underline{q}$. In this case, the quality of the habitat reaches the threshold $\underline{q}$, at the resource use level before the pollution function reaches its maximum. That is, $R = arg[\underline{q}(R)] < R_{max}$. This condition implies that the habitat degradation index $\epsilon < \underline{\epsilon}$. This condition leads to a population decline. It is also possible that population density is either $\bar{\omega}_t = \underline{\omega}$ or $\bar{\omega}_t > \underline{\omega}$. In any case, population shrinks; hence, $\bar{\omega}_t$ rises. However, when further increase in output and, hence, resources use leads to $R^* > R_{max}$, this implies $q(R^*) > \underline{q}$. In this range of resource use, if $\bar{\omega}_t < \underline{\omega}$, then population starts growing, and if $\bar{\omega}_t = \underline{\omega}$, then population stays static, as $\epsilon = \underline{\epsilon}$ will hold. Therefore, the population would decline, first, due to habitat constraints, and grow later after reaching the habitat improving stage of development. The population stops growing when the condition is reached due to a larger size of the population. An illustration of this path is given in Figure 4.

The above discussion leads to the following lemma.

Lemma 3.4 *If the upper bound threshold of habitat degradation is reached before the pollution maximising level of resource use, population growth follows a non-monotonic path caused by habitat degradation.*

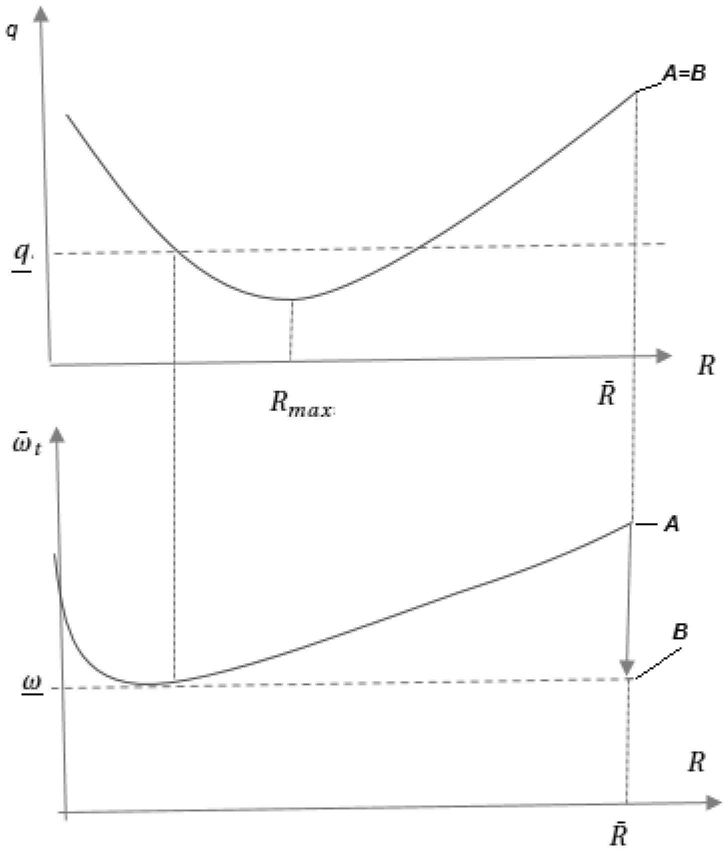

Figure 4: Concave pollution function and non-monotonic population growth.

Clearly, the above-described growth path scenarios indicate that long-run steady-state growth is feasible. However, output growth affects the quality of the habitat through pollution and population growth. Both of these factors are stabilised when the capacity of the habitat reaches its upper bounds, which would limit population growth.

Proposition 3.5 *In the long run, sustained growth in per capita income with stable habitat is possible only with a static population.*

Proof The above discussion of the different paths of environment and population evolutions demonstrates that the steady state is achieved only when the resources use is at its maximum and the quality of the environment is stabilised. This also implies that the population becomes static. However, under these conditions, from Corollary (3.3), it follows that continuous output (income) growth driven by technological progress is feasible.■

Notably, the above proposition extends the result obtained in the neoclassical growth framework to the modified DHSS framework. Namely, in the long run, population growth will not have a significant impact on sustainable growth, as its growth will be constrained by the size and quality of the habitat. However, if technological progress improves productivity but does not make production processes less polluting, then there may be a large decline in the human population with unforeseeable implications. To address this issue adequately, government has to tackle the 'tragedy of commons' effectively, as myopic private producers might be interested in lifting productivity but may not have a strong natural urge to decrease the footprint of production. As can be seen, theoretically, if the degradation of the world's habitat is disregarded, its population can be driven into decline, as Lemma (3.4) attests. Overall, the above analysis clarifies that, in the given setup, the boundedness of backstop resources will not hinder economic growth once the productivity driven by technology is sustainable and the fertility rate is consistent with the per capita capacity of the habitat.

3.2 The role of 'green' technologies and policies

The main point of Proposition (3.5) is that accounting for the quality and size of the habitat imposes binding constraints on the size of the total population. This condition will make sustained economic growth quite challenging, as requires a trade-off between income growth and population growth. On the other hand, if parameter a_1 (which relates the amount of resources to the level of pollution) was endogenous and declining with the level of technology ('green technologies'), such that $\frac{\partial a_1}{\partial A} < 0$, then an optimistic outcome can be expected. As the description of possible growth paths indicate, the best outcome case appears to be when the pollution function is concave and population growth is monotonic. Clearly, to get onto this path, net pollution should be adjusted by decreasing the negative impact of production and relying more on output growth driven by technological improvements rather than resource use. Therefore, the effect of technologies are twofold in this context: reduce pollution directly, through $\frac{\partial a_1}{\partial A} < 0$ effect, and indirectly, through decreasing resource use. This consideration leads us to the following proposition.

Proposition 3.6 *If technological progress can reduce the negative impact of production on the habitat, output growth without habitat degradation is possible.*

Proof Recall that the net pollution function is given by $D_t = a_1 R_t^{\zeta} - a_2 \tau Y_t$. Given that $\frac{\partial a_1}{\partial A} < 0$ and if $\frac{\partial D}{\partial a_1}\frac{\partial a_1}{\partial A} = \frac{\partial D}{\partial Y}\frac{\partial Y}{\partial A} - \frac{\partial D}{\partial Y}\frac{\partial Y}{\partial R}$ holds, then the increase in output under this condition would not change the flow of net pollution. ■

The above result indicates that if the effect of 'green' technologies on pollution is high enough, it is possible to raise output, not only based on technological improvements, but also by increased resource use, while keeping the quality of the habitat stable.

It should also be emphasised that not only is less pollution important for the sustainability of the habitat, but also the efforts to abate the pollution emitted in the production process. As Corollary (3.3) points out, a higher environmental tax rate can be conducive for long-term habitat sustainability. That is, output needs to be taxed high enough that the collected resources are enough to combat the negative effects on the environment, on the one hand, and on the other hand, discourage overproduction. In this sense, these results are supportive of the ideas suggested by D'Alessandro et al. (2010) that lower growth rates are one of the conditions to achieve feasible paths of economic and environmental sustainability.

The above results need to be interpreted with caution. There are at least two important factors that have not been taken into account in this model. First, the role resources play in technological change may have a 'sustainability bias' (Groth, 2007); in fact, technological improvements may lead to a greater demand for natural resources (Alcott, 2005), or technological progress may not be enough to compensate for the depletion of resources (Tisdell, 1990). Therefore, this channel through which resource scarcity may affect long-run growth requires further research. Second, the quality of the habitat and the role of abatement policies in long-run growth requires a deeper analysis than the stylised approach employed in this study. For example, this analysis does not consider the effect of habitat quality on fertility and longevity, as well as on productivity; accounting for this channel may lead to different outcomes. Another aspect that was not considered is how the damaging effect of production can be reduced through endogenous technological progress. Unless the feasibility of such an endogenous process is demonstrated analytically, the claim of Proposition(3.6) could prove to be wishful thinking.

4 Conclusions

This paper extends the DHSS framework in two directions. First, unlike the DHSS framework, which assumes that when depleted, the fixed and a priori known stock of nonrenewable resources is replaced by the unlimited exogenous backstop technology, it is assumed that there is an upper bound on the backstop resources. The level of extraction of this type of resource depends on human capital. Second, it is assumed that population growth is endogenous and that it ultimately depends on the dynamics of the quantity-quality trade-off faced by parents.

By analysing this extended DHSS model, a few conclusions have been drawn. The availability of resources is an important factor that drives not only the long-run level of per capita consumption but also population growth. In such a setting, with an upper-bound constraint imposed by the resource sector, the economy cannot grow driven by technological progress only. This is because unconstrained income growth results also in unlimited population growth. Falling resource input per capita, due to limited substitutability between resources and other factors of production, results in a decline of output in per capita terms. Falling income levels lead to a static population size, as low-income levels would not be enough to sustain population growth. In other words, a Malthusian trap is inevitable under such a setting.

To address the above issue, the effect of habitat on fertility choices of the households has been incorporated into the model. The quality of habitat depends on the pollution stemming from production, whereas quantity of habitat per person is determined by the size of total population. The results of the analysis indicate that it is possible to have sustained growth when a decline in the availability of habitat and its quality affects fertility negatively by increasing the cost of child-rearing. When production strictly leads to the degradation of the environment, and if the abatement activities cannot overcome all the damage from the pollution, growth in population at some point will be constrained by the sustainability of the habitat. This type of outcome can lead to a decline in population (quasi-Easter Island scenario), although per capita income can keep growing. With such pollution dynamics, the sustained per capita output growth, without causing a population decline, is possible only if technological progress can lessen the negative impact of production on the environment to a level where it can be fully abated. Recapping the above results, technology-driven income growth can be seen as possible only with static population; however, the level of resources used may not be a constraint on growth as long as 'green' technologies can keep net pollution levels in a non-increasing trend.

There are a few caveats in this study that should be mentioned with regard to the above interpretations of the results of this analysis. In this study, pollution and its effect on the quality of the environment are considered in a very simplified way. Accounting for the effects of these factors on economic growth through their impact on longevity and fertility could lead to new insights. Endogenous mechanisms that drive productivity and reduce pollution simultaneously are not considered to assure the feasibility of positive growth rates in the long run. This implies that there is a need to consider a fully endogenous growth model with resources and population that accounts for the quality of the environment and the limits it puts on population and economic growth. These questions deserve a thorough analysis in future research.

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