Storage games

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Abstract

We study a long-horizon, oligopolistic market with random shocks to demand that can be arbitraged by two storage operators with finite capacity. This problem applies to any storable commodity – that is, most commodities. Because the arbitrage spread is so sensitive to market power, storage operators face strong incentives to restrain quantities by tacitly colluding. This cooperation takes new forms thanks to the multiplicity of actions they must take: selling, buying or both. We construct payoff-maximizing equilibria of this stochastic game, and uncover a new form of *Partial Cooperation* that trades off quantities and delay. While collusive, Partial Cooperation delivers higher consumer surplus thanks to the market power effect. Head-on competition is not always an equilibrium of the long-horizon game when market power becomes large enough. We draw implications for policy and suggest poorly competitive storage is a negative externality to the development of the underlying commodity – for example, renewable energy.

Keywords: *stochastic game, collusion, dynamic trading, storage*

JEL: *C73, D43, D47, Q41, Q42*

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1 Introduction

In the early afternoon of April 30th 2022, the state of California briefly met all its electricity demand with renewable energy; it even exported some surplus to neighboring states.[1] Likewise, the state of South Australia produced 120% of its needs in September 2023 – for a few hours.[2] These examples make it plain there is now a pressing need to store this energy. This observation applies in other large markets beyond California or Australia – for example, Spain or Texas. Upon inspection however, we know very little of the economics of merchant storage in general – that is, buying, holding and selling. This paper contributes, in part, to addressing this gap. In this nascent sector (of the electricity industry in particular), economists have a chance to understand the problem before its impact is fully manifested.

For any commodity, storage is the device by which to deliver it when needed rather than when available; it allows economic agents to better match the timing of production and consumption. Crops, whose production is highly correlated and driven by seasons, are more valuable thanks to storage; stores of minerals or fuel can be driven up or tapped into to address seasonal or random fluctuations in demand.

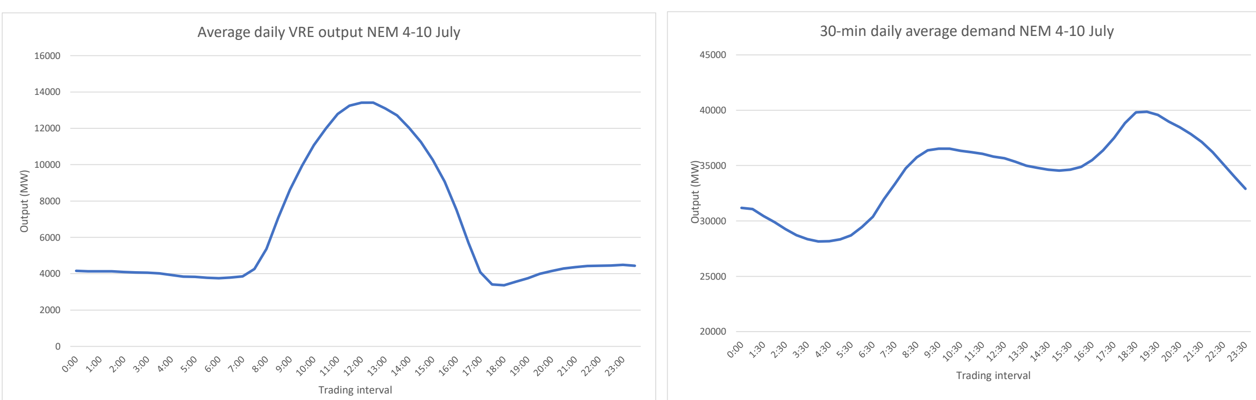

Figure 1: Mismatch between renewable (grid-scale) supply and (gross) energy demand. Daily 30-min averages, 4-10 July 2024, Australia. Source: AEMO data.

[1]Source: Electrek: https://electrek.co/2022/05/02/california-runs-on-100-clean-energy-for-the-first-time-with-solar-dominating/.

[2]Source: RenewEconomy, https://reneweconomy.com.au/solar-reaches-record-120-per-cent-of-electricity-demand-in-south-australia/

For electricity it is a revolution that is an essential complement to renewable energy: storage soaks up the sun and wind whenever available and prices are low, and delivers that energy when actually demanded and prices are high. This is shown in Figure 1 for the Australian market, and is quite typical. Storage can also take advantage of price differences induced by random shocks to either supply (the wind drops) or demand (the temperature changes). Because unexpected, these price swings can be very large as can be seen on Figure 2 for two Australian states. We focus on precisely this problem but point out it is not confined to electricity. Finally, storage revolutionizes the very exercise of trading, which can only be dynamic: all past trading decisions are summarized by the current inventory, which then conditions all future decisions.

Time	Queensland ($/MWh)	NSW ($/MWh)
6.05 pm	**15,500**	**12,465**
6.10 pm	**15,500**	**14,218**
6.15 pm	3,569	3,164
6.20 pm	**15,500**	**14,120**
6.25 pm	**15,500**	**14,507**
6.30 pm	399	358
6.35 pm	**15,500**	**13,519**
6.40 pm	355	308
6.45 pm	**9,999**	**9,026**
6.50 pm	**9,797**	**8,738**
6.55 pm	370	323
7 pm	304	265

Figure 2: Sample price sequence 16 March 2024 in the NEM. Source: RenewEconomy using AEMO data.

We study a long-horizon trading game based on storage in an oligopolistic market. There is a finite number n of (strategic) producers playing a static Cournot equilibrium in every period of the supergame. In each period, demand is subject to aggregate shocks that induce a sequence of low and high prices. This creates opportunities for two competing, strategic storage units to exploit price differences between periods. We refer to this as *intertemporal* arbitrage, and to the price differences as arbitrage spread.[3] These storage units have market power. In this environment, we are particularly interested in understanding the incentives to engage in cooperative behavior – tacit collusion. We find these incentives to be pervasive and

[3]The mere existence of arbitrage opportunities is typically regarded as an anomaly in economics. Here they exist because of the combination of *aggregate* shocks that cannot be diversified away and dynamics.

construct the payoff-maximizing equilibria of this stochastic game.

All our results follow from the sensitivity of the arbitrage surplus to market power – so, to capacity in this model. Indeed, the surplus is a concave function of the aggregate quantity traded and rapidly reaches zero as that quantity increases; the reason is that an arbitrageur must sell and *buy*, both of which are affected by market power. We determine the incentives to collude in terms of capacity levels. That is, which equilibria exist and which equilibrium is payoff-maximizing is determined by endogenous capacity thresholds that we compute. The extent of cooperation increases in the capacity of storage units. Collusion is implemented by taking turns to trade, which involves delays and so matters for incentives.

An important result is the emergence of a new class of equilibrium that we label "Partial Cooperation": players cooperate when buying but not when selling. This equilibrium arises in an intermediate range of capacity as a compromise between restraining quantities and costly delay. Partial Cooperation can exist thanks to the multiplicity of actions a storage unit faces, namely buying and selling, which enhances the scope of cooperative behavior and is not present when selling is the only relevant action. This equilibrium exists for all parameter values (discounting, shocks) in a sense we make precise. It is supported (in part) by a likewise new asymmetric, non-cooperative equilibrium that we dub "Follow-the-Leader". In this equilibrium, a player accepts to "follow" and trades only when the leader cannot (because of the state). In so doing, both players side-step direct competition. A fully collusive equilibrium can exist for larger capacities; operators coordinate on both buying and selling. However this equilibrium need not always exist, and even if it does, need not be payoff-maximizing because of the cost of delay required to implement it.

Head-on competition, in which storage units act simultaneously, is an equilibrium and can be payoff-maximizing, for low capacity levels; it ceases to be an equilibrium as capacity increases. The reason is that, absent any coordination, the joint quantities become so large that the arbitrage spread vanishes. Then any equilibrium requires some cooperation. Perhaps surprisingly, we show that consumers need not be harmed by this collusion. Specifically, the consumer surplus is larger under Partial Cooperation than under the equilibrium Competition.

At the payoff-maximizing capacity of each equilibrium, the total transfers from consumers are lower under Partial Cooperation because the traded quantities remain large enough. This potentially creates a new conundrum for antitrust authorities. Finally we highlight that a consequence of restraining quantities traded by storage is that the market for the underlying commodity is underdeveloped; simply less is traded – whether bought or sold. In the context of the energy transition this implies that the bottleneck persists.

This work can be made relevant to multiple commodities by adjusting the efficiency parameter and the discount factor, with different effects on equilibrium outcomes. A higher discount factor that corresponds to high-frequency trading such as electricity favors cooperation as an equilibrium. With a lower discount factor, as for crops, Partial Cooperation becomes more attractive. The model can also extend to market making in securities.[4] Market power is relevant in these markets. For example, there are five traders in the global commodities market; the NYSE only allows three Designated Market Makers to operate.[5]

The present model features two essential characteristics that are (jointly) absent in most other papers: a stochastic environment and market power; the latter is central. In electricity, which first piqued out interest, Andres-Cerezo and Fabra (2023b) study the question of market structure with storage, but leave aside how storage actually behaves. Andres-Cerezo and Fabra (2023a) present a model of cyclical storage and correlated renewable energy, in which all parties are price takers. This renders the dynamics moot: without price impact, storage buys to capacity and sells in full every cycle; therefore it is enough to analyze a single cycle. In our model, absent market power there are no incentives to restrain quantities and thus no need to collude. Butters et al. (Working Paper) use California data to estimate the equilibrium effect of large-scale storage. In that model however storage is assumed to behave competitively; this is almost orthogonal to our work. Karaduman (2020) is the first to study grid scale storage. He does allow for market power; however he does not compute the best reply but simulates it using Australian data. Geske and Green (2020) study arbitrage in a model of imperfect

[4]Indeed, this intermediation activity fits our model: assets are traded in response to aggregate shocks, a revenue is generated by arbitrage, holding inventory is necessary, price impact matters and capacity is bounded by capital constraints. Market making differs from idiosyncratic trading (e.g. Vayanos (1999)).

[5]The traders are Trafigura, Glencore, Vitol, Guvnor and Mercuria; the DMMs are Citadel, GTS and Virtu.

competition with demand uncertainty and diurnal, weekly and seasonal patterns. They confine themselves to numerical (approximate) solutions to the welfare maximization problem, and show quantity withholding. Williams and Green (2022) compute the welfare effects of storage on the current British market using simulations, and so without characterizing any equilibrium, with time-varying demand and no uncertainty. We construct equilibria. [6]

Further afield, Deaton and Laroque (1992) rationalize the volatility of 13 essential crops by introducing *perfectly competitive* storage in a model of speculative trading. Wright and Williams (1984) conduct a welfare analysis of the benefits of commodity storage, in which, again, all parties are price takers. In addition there are no actual dynamics in that model. Even Samuelson (1971) dabbles in the problem, however still under the assumption of perfect competition. Hence the storage problem in strategic environments demands more attention.[7]

While conceptually a model of arbitrage, our work departs from that rich literature (e.g. Shleifer and Vishny (1997), Oehmke (2009), Dávila et al. (2024) and many others). In these papers, arbitrage is *contemporaneous* (across segmented markets), fundamentally riskless and there is no aggregate risk. Here arbitrage can only be intertemporal (in a single market) and risky; there is aggregate risk in the economy.

2 Model

Consider a market with two storage units, n producers (for example, electricity generators) labeled $j = 1, 2, ...n$, and a pool of consumers. The storage units are identical and have finite capacity k. A storage operator can only either buy or sell in each period, so $b_t^i \cdot s_t^i = 0$ by assumption.[8] Retailers and consumers are confounded and retailing has no cost. Because the exact form of competition is not a central consideration, we let consumer behavior be

[6] Hydro-electric power differs from storage. The water inflow is free, exogenous and stochastic; a storage unit pays for the energy it buys and it makes that decision as part of its trading strategy. Most models of dam management take prices as *fixed* (not even moving in the aggregate).

[7] Storage differs from the typical inventory management problem (see Harrison and Taylor (1978) for example). That problem is strictly one of stochastic control – not a game – in which the per-unit payoffs (rewards or costs) are exogenous.

[8] In the case of electricity, it is a technical characteristic of the machines. However it is also clear that it cannot be optimal to buy and sell simultaneously.

described by the simple demand function

$$D(p_t, \varepsilon_t) = 1 - p_t + \varepsilon_t,$$

where ε_t is a shock that takes values in $\mathcal{A} := \{-a, a\}$ and follows the distribution:[9]

$$Pr\{\varepsilon = a\} = Pr\{\varepsilon = -a\} = 1/2, \qquad\qquad 0 < a < 1 \tag{1}$$

for any time period t.[10] At t, upon observing the shock realization, storage units simultaneously either buy $b_t^i \geqslant 0$ up to capacity, or sell any quantity $s_t^i \geqslant 0$. Then producers take the residual demand as given and simultaneously offer a quantity q_t^j each; we ignore capacity constraints on the producers.[11] For each unit $i = 1, 2$, this process gives rise to the equation of motion:

$$c_t^i = c_{t-1}^i + b_t^i - \frac{s_t^i}{\delta}, \qquad\qquad t \in \mathbb{N}, \qquad\qquad c_0^i = 0. \tag{2}$$

The quantity c_t^i is a current level of inventory ($0 \leqslant c_t^i \leqslant k$) and δ is a round-trip efficiency parameter ($0 < \delta \leqslant 1$) that applies to both units. All players face a common discount factor $\beta < 1$; they are exposed to a strictly positive interest rate. The parameters δ and β play a different role. Roughly speaking, δ captures the economic viability of storage: if it is too low, (2) implies a unit must buy a lot to operate, which is costly; δ can also be interpreted as a marginal cost. The term β represents the more standard patience (or an interest rate).[12]

[9]From now on we omit the subscript t in ε_t where it does not lead to confusion.

[10]One can also add renewable energy with stochastic supply and conceive of the demand function as residual demand without material consequences.

[11]The norm in some markets is to use the more elegant supply-function equilibrium (SFE); however the richness of the SFE is lost here since we rely throughout on binary shocks; see Klemperer and Meyer (1989). Further, the Cournot outcome is a possible equilibrium outcome of the SFE and constitutes an upper bound for the payoffs to suppliers (Klemperer and Meyer (1989)). Finally, quantity competition is used as a successful proxy in many papers (Acemoglu et al. (2017), Willems et al. (2009), Lundin and Tangerås (2020)); much of this work relies on the estimations of Borenstein and Bushnell (1999), Borenstein et al. (1999) or Bushnell et al. (2008).

[12]More precisely, these two parameters actually interact in the payoffs and jointly determine the incentives of the players; together they are the reason behind Proposition 6, in particular.

The market clears if, for any t,

$$D(p_t, \varepsilon_t) = \sum_{j=1}^{n} q_t^j + \sum_{i=1}^{2} [s_t^i - b_t^i].$$

Each storage operator maximizes the objective

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \; p_t \cdot (s_t^i - b_t^i)\right], \quad i = 1, 2, \tag{3}$$

by choice of (s_t^i, b_t^i) subject to the law of motion (2) and the capacity constraint

$$0 \leqslant c^i \leqslant k, \quad i = 1, 2. \tag{4}$$

Producers each seek to maximize

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \; p_t q_t^j\right], \quad j = 1, 2, ..., n, \tag{5}$$

by choice of the quantity q_t^j they produce each period. In both (3) and (5), the price p_t is a function of the aggregate quantity $Q_t = \sum_{j=1}^{n} q_t^j + \sum_{i=1}^{2} (s_t^i - b_t^i)$.

We seek subgame-perfect Nash equilibria of this game and focus on the problem of the storage units.[13] The state variables of this problem are a pair of states (of inventory) and demand shocks $(\mathbf{c}, \varepsilon) \in \mathcal{C} \times \mathcal{A}$, so actions are mappings $b_t^i, s_t^i : \mathcal{C} \times \mathcal{A} \times \mathcal{H}_t \mapsto [0, k]$ for each i, where $\mathcal{C} := [0, k]^2$ and $\mathcal{H}_t$ is the set of all histories up to time t with $\mathcal{H}_0 = \emptyset$. Hence the continuation game need not be a replica of the current stage game. Because actions $b^i(\mathbf{c}_t, \varepsilon; H_t)$ and $s^i(\mathbf{c}_t, \varepsilon; H_t)$ already encode the state $(\mathbf{c}, \varepsilon)$ of the system, histories $H_t \in \mathcal{H}_t$ are constructed in standard fashion. For each player i, a strategy is a sequence of actions from

[13] One could argue, given the stochastic environment, that the concept of Markov-perfection is more "natural" in the sense of less taxing on players; it is certainly less taxing on the analyst. Where the Markov-perfect equilibrium differs from the SPNE, it is not onerous to identify it using the same techniques we employ.

time 0 to ∞ : $\{(b_t^i, s_t^i)\}_{t=0}^{\infty}$. The corresponding value function to player i writes

$$\forall i, \quad V^i(\mathbf{c}) = \sup_{b^i, s^i} \mathbb{E}_\varepsilon \left[\sum_{t=0}^{\infty} \beta^t \, p_t \cdot (s_t^i - b_t^i) \right]. \tag{6}$$

A subgame-perfect Nash equilibrium of this game is a profile of strategies $(\hat{\mathbf{b}}_t, \hat{\mathbf{s}}_t, \hat{\mathbf{q}}_t)$ such that (i) $(\hat{b}_t^i, \hat{s}_t^i)$ maximizes (3) for each storage operator i, given j's equilibrium strategy $(\hat{b}_t^j, \hat{s}_t^j)$ and the equilibrium strategy $\hat{\mathbf{q}}_t$ of producers and (ii) $\hat{q}_t^k$ maximizes (5) for each producer k given $\hat{q}_t^{-k}$ and $(\hat{\mathbf{b}}_t, \hat{\mathbf{s}}_t)$. It is important to bear in mind that $\forall i,\ c_0^i = 0$; buying is the first action a storage unit can take. Transfers cannot be used to relax incentive constraints.

There are many such SNPEs, therefore we focus on equilibria that (a) maximize the payoffs to storage units and (b) (consequently) may feature some form of cooperation. A benefit of selecting payoff-maximizing equilibria is that they are supported by the most extreme off-equilibrium punishments, which are simple to describe. Characterizing equilibria for arbitrary policies $\{b_t^i, s_t^i\}_{t=0}^{\infty}$, $i = 1, 2$ remains an impossible task. Hence, most of our analysis focuses on the case, in which storage operators are restricted to buy and sell to capacity: $b^i, s^i \in \{0, k\}$. As a result, the scope of cooperation is limited to the *timing* of actions rather than their magnitude; they cooperate by taking turns in trading. Limiting attention to binary actions is common in much of the literature on dynamic games or repeated games. This restriction does induce some rigidity that renders cooperation more attractive than if we allowed for a more flexible play.[14] In Section B.1 of the Appendix, we do provide some results on more flexible actions and confirm this observation, but also our results. Flexibility in actions and cooperation are substitutes, but imperfect ones.

Even if restricting the strategy space so that storage units use binary actions, a general characterization remains elusive. In particular, we must detail the equilibrium behavior of the other n players (the producers). To make progress in this problem, we further reduce the space of admissible strategies and so impose that producers repeat the Cournot equilibrium of the stage game. This choice is further justified by the work of Bonatti et al. (2017), who

[14]That is, unable to soften competition by unilaterally reducing quantities, storage operators engage in cooperative play instead.

study a dynamic Cournot model under incomplete information with learning. The equilibrium converges to the repeated static Nash equilibrium. We start from this point. Because $s, b \in \{0, k\}$, storage units cannot distort their own quantities, but only the *aggregate* quantity; that is, there is no s, b strictly between 0 and k. Consequently, the best-reply choices of the n producers remain unchanged as well.

3 Constructing equilibria: competing, colluding and letting go

From now on we proceed systematically to construct our equilibria. First we must specify behaviors that can emerge as equilibrium. Then for each of them we must find the range of capacity over which a given behavior can be an equilibrium, which also requires a penalty regime to be constructed. Finally we collate these results to construct payoff-maximizing equilibria and evaluate their performance.

3.1 Behaviors and payoffs in the stochastic game.

Given that producers simultaneously offer quantities based on the residual demand, there are only n firms playing Cournot. The clearing prices can be easily computed.[15]

Lemma 1. *Let $l \in \{1, 2\}$ denote the number of active storage units. Clearing prices are*

$$p_l^b = \frac{1 - a + lk}{n + 1} \qquad \text{and} \qquad p_l^s = \frac{1 + a - lk}{n + 1}.$$

With this, to use throughout, we define purchasing costs under the negative shock as B_l if l storage units buy, and the revenue a storage unit earns when selling under the positive shock as A_l (again, depending on number of storage units l selling in that period):

$$B_l = \frac{1 - a + lk}{n + 1} \cdot k, \qquad A_l = \frac{1 + a - l\delta k}{n + 1} \cdot \delta k, \qquad l \in \{1, 2\}.$$

[15] For completeness, the quantities are $q_l^b = \frac{1-a+lk}{n+1}$ and $q_l^s = \frac{1+a-lk}{n+1}$; the proof is immediate.

In both expressions, the right multiplier (k or δk) is a quantity bought or sold. The left multiplier is the resulting Cournot price. It is easy to check that it is optimal to buy only when the shock ε is positive, and likewise to sell when the shock is negative.[16] Then we are left with n generators competing for the residual demand, which is $1-a+lk$ or $1+a-l\delta k$ for negative and positive shocks, respectively. Readily we see that the *arbitrage spread* $A_l - B_{l'}$ (where l need not be equal to l') is concave in k.[17] Indeed, the spread is the sum of concave parabolic functions of the capacity k, so its maximizer must be interior. That is why market power matters so much for storage.

To begin with, we provide a list of possible behaviors that may emerge as equilibria; not all these are equilibria – this analysis is forthcoming. This list need not be exhaustive in the class of behaviors that can be supported as an equilibrium by a grim-trigger strategy; rather they deliver upper bounds on the payoffs that can be achieved, and so are also stationary. We distinguish *behavior* – all in lower case – from *equilibrium*, which takes an Uppercase.

1. *competition.* There is no coordination at all. Empty storage units always buy when they face negative shocks. Full storage units always sell when they face positive shocks. Both stay idle otherwise.

2. *partial cooperation.* Storage units coordinate on buying but not on selling. That is, if both storage units face negative shocks while empty, they flip a coin to decide who is first to buy; the losing party stays idle. If both storage units face positive shocks while full, they sell simultaneously (i.e. without coordination). If the storage units are in different states (inventory), the empty one always buys under a negative shock, and the full one always sells under a positive shock. They stay idle otherwise.

3. *partial cooperation alt.* Storage units coordinate on selling but not on buying. If both storage units face negative shocks while empty, they buy simultaneously (no coordination). If both storage units face positive shocks while full, they flip a coin to decide who sells first; the losing unit stays idle. If the storage units are in different states, the empty

[16] Hence, in terms of notation, there is no loss in dispensing with the shock ε as a state variable.

[17] l need not be equal to l' because, for example, one unit may buy at t but both of them may sell at $t+1$.

one always buys under a negative shock, and the full one always sells under a positive shock. They stay idle otherwise.

4. *cooperation.* Storage operators coordinate on both buying and selling. If both storage units face negative shocks while empty, they flip a coin to decide who is first to buy; the losing party stays idle. If both storage units face positive shocks while full, they flip a coin again to decide who sells first; the losing unit stays idle. When they are in different states, the empty one always buys under a negative shock, and the full one always sells under a positive shock. They stay idle otherwise.

5. *follow the leader (ftl).* One of the storage units (the "leader") always buys (sells) first when both units are empty (full) under a negative (positive) shock. The second one (the "follower") stays idle. In different states, the empty one always buys under a negative shock, and the full one always sells under a positive shock. They stay idle otherwise. This is an asymmetric, non-cooperative play. Coordination is required, but not cooperation.

6. *ftl+competition.* The leader always buys first when both units are empty under a negative shock. The follower stays idle. If both storage units face positive shocks while full, they sell simultaneously (no coordination). In different states, the empty one always buys under a negative shock, and the full one always sells under a positive shock. They stay idle otherwise.

7. *competition+ftl.* If both storage units are facing negative shocks while empty, they buy simultaneously (no coordination). When both units are full, the leader always sells first under a positive shock. The follower stays idle. If the storage units are in different states, the empty one always buys under a negative shock, and the full one always sells under a positive shock. They stay idle otherwise.

The first four behaviors deliver symmetric (expected) payoffs, while the last three do not. Because of efficiency losses ($\delta \leqslant 1$), behaviors 3 and 7 are always (payoff-) dominated by 2 and 6, respectively. They are discarded from now on. Next we compute the payoff function that is induced by each of these behaviors to understand their desirability, as well as their capacity to either become an equilibrium, or to support an equilibrium. That is, we can write recursive equations of the form

$$\mathbf{V}(\mathbf{c^1}, \mathbf{c^2}) = \mathbf{P} + \beta \mathbf{Q} \mathbf{V}(\mathbf{c^1}, \mathbf{c^2}), \tag{7}$$

that correspond to each of these behaviors, and compute them in terms of B_l, A_l and the discount factor β.[18] In Equation (7), $\mathbf{c^i}$ are infinite sequences, $\mathbf{V}$ is a vector of continuation values, $\mathbf{P}$ a vector of flow payoffs and $\mathbf{Q}$ a square matrix. Our first Proposition greatly simplifies the forthcoming analysis; these value functions reduce to simple polynomials. With this important step, equilibria can then be constructed with relative ease.

Proposition 2. *The payoffs for the equilibrium-candidate behaviors are:*

- *competition:*

$$U_{com} = -\frac{B_2}{2} + \frac{\beta}{4(1-\beta)} \left(A_2 - B_2\right).$$

- *partial cooperation:*

$$U_{pc} = -\frac{2+\beta}{8} B_1 + \frac{\beta}{16(1-\beta)} \Big((2-\beta)\left(A_1 - B_1\right) + 2\beta \left(A_2 - B_1\right) \Big).$$

- *cooperation:*

$$U_{col} = \frac{1}{2(2-\beta)} \left(-B_1 + \frac{\beta}{(1-\beta)(2+\beta)} \left(A_1 - B_1\right) \right).$$

- *ftl:*

  The leader's payoff:

$$\bar{U}_{ftl} = -\frac{B_1}{2} + \frac{\beta}{4(1-\beta)} \left(A_1 - B_1\right).$$

  The follower's payoff:

$$U_{ftl} = \frac{\beta}{2(2-\beta)} \left(-B_1 + \frac{\beta^2}{2(1-\beta)(2+\beta)} \left(A_1 - B_1\right) \right).$$

[18] That is, as functions of a, k and β.

- *ftl+competition:*

  The leader's payoff:

$$\bar{U}_{fc} = -\frac{B_1}{2} + \frac{\beta}{8(1-\beta)}\Big((2-\beta)\,(A_1 - B_1) + \beta\,(A_2 - B_1)\Big).$$

  The follower's payoff:

$$U_{fc} = \frac{\beta}{4}\left(-B_1 + \frac{\beta}{2(1-\beta)}\,(A_2 - B_1)\right).$$

Each of these payoff functions are also drawn in Figure 3. They are linear in the terms A_l and B_l, and therefore are quadratic functions of capacity k. The arbitrage spread $A - B$ may depend on the number $l \in \{1, 2\}$ of firms buying or selling at any time t. This spread is then discounted by the factor β net of the initial inventory. For example, under competition, everything is symmetric and $l = 2$ always. The first term is the cost of the first purchase, which occurs at time zero with probability $1/2$. The spread is then discounted starting from time $t = 1$, and weighed by $1/4$, which is the frequency of a full cycle $(-a, a)$. Under partial cooperation, the spread is made of a linear combination of A_2, A_1 and B_1, which are discounted a different rate since they do not occur with the same frequency, and the initial purchase cost is B_1 rather than B_2 since they buy sequentially.

Cooperation (green) is an attractive behavior for large enough a capacity in that its payoff is the largest, but competition is more attractive for low capacities. The dotted lines are payoffs arising from *asymmetric* behaviors – a leader and a follower – and so deliver asymmetric payoffs; the leader always receives larger payoffs.

For behaviors relying on cooperation to exist as an equilibrium, one must find an off-equilibrium punishment to support them. This exercise is not obvious. For example, for very large capacities, competition is *not* an equilibrium – it is trivially dominated by $b = s = 0$ and so cannot support cooperation as an equilibrium.

Figure 3 suggests that whether a behavior can be sustained as an equilibrium depends on the storage capacity k. Hence we introduce three of many thresholds we need in our

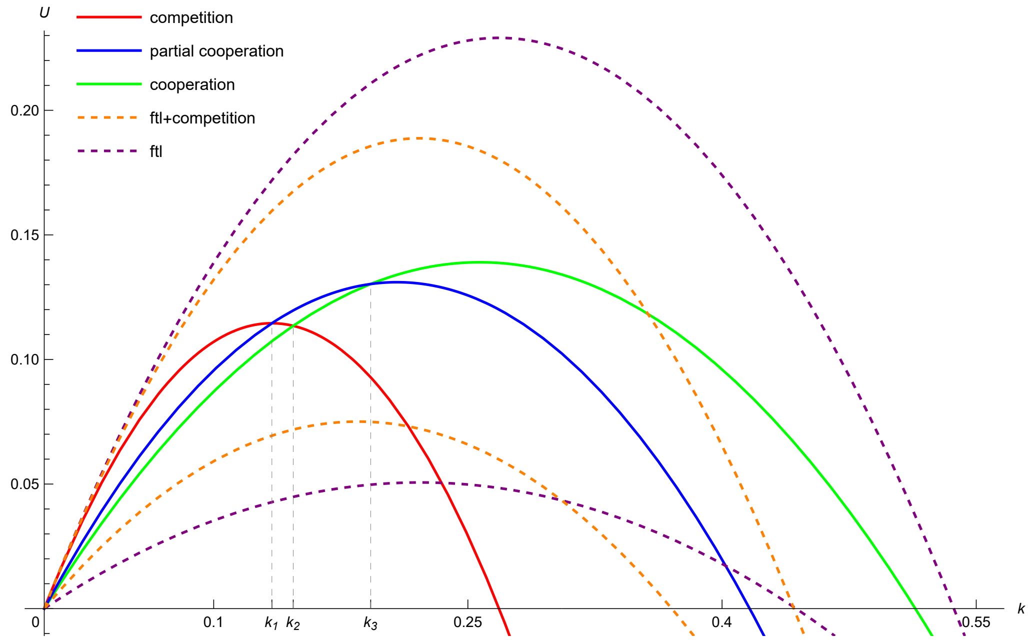

Figure 3: Payoff functions for different strategies as a function of capacity for $n = 2$, $a = 0.6$, $\beta = \delta = 0.95$.

analysis. They are labeled k_1 to k_3 and denote the capacity level at which a storage operator is indifferent *in terms of payoffs* between any two behaviors. k_1 is that capacity level such that the payoffs to competition and partial cooperation are the same; k_2 identifies indifference between competition and cooperation, and k_3 between partial cooperation and cooperation. Throughout we require a condition that connects the magnitude of the shocks to the technical parameters β and δ for storage to have a role to play:

Lemma 3. *If condition*

$$\frac{1-a}{1+a} < \frac{\beta\delta}{2-\beta} \tag{8}$$

holds, then there exist thresholds $0 < k_1 < k_2 < k_3$.

The right-hand side of (8) is the discounted surplus of a full cycle of operation; the left-hand side is a measure of volatility of the demand. Condition (8) then requires the shock a to be large enough so that there is room for storage to pay itself from the variations in demand. Before studying which of the listed behaviors are, or not, an equilibrium, the following observation complements Proposition 2.

Remark 4. *A single firm owning both storage units receives payoffs that are the upper envelope of the payoff functions listed in Proposition 2. It adopts the corresponding behavior depending on whether* $k < k_1,\ k_1 < k < k_3$ *or* $k > k_3$*. It may adopt a symmetric or asymmetric behavior, which results in the same aggregate payoffs.*

The single entity need not be concerned with equilibrium; it can commit to itself, internalize all conflicts and implement "frictionless cooperation". Instead, competing firms must satisfy incentive constraints when they are relevant, which depends on the state of the game.

Not all equilibria are supported by the same punishment threat. The description of these equilibria is subtle at times; we pare down the details and notation as much as possible and relegate these to the Proofs. To ease exposition we break down this exercise in three parts, in which we study a specific equilibrium. In a last step we combine these equilibria to determine payoff-maximizing equilibria. For emphasis, any cooperation requires taking turns; the order of play matters a great deal for the exact equilibrium construction.

3.2 Competing and letting go

This first case is the most natural one, yet not immediately intuitive. Let $C \equiv -(2-\beta)(1-a) + \beta(1+a)\delta$, then Condition (8) rewrites as $C > 0$ and

Proposition 5. *There exists a capacity threshold* $\overline{\kappa}_r \equiv \frac{C}{4-\beta+2\beta\delta^2}$ *such that, if Condition (8) holds, then Competition is a subgame-perfect Nash equilibrium for* $0 < k < \overline{\kappa}_r$*.*

When capacity k is small enough, the symmetric non-cooperative play is an equilibrium; it ceases to be an equilibrium as soon as capacity exceeds $\overline{\kappa}_r$.[19] In this case, one of the players is better off *letting go* and following the other one – the leader – under the behavior ftl+competition. The reason is that capacity becomes large enough to erode the arbitrage spread; this erosion is so acute from $\overline{\kappa}_r$ on that even the follower is better off under this new regime. The threshold $\overline{\kappa}_r$ is the point of indifference for the *follower*; it is the relevant incentive since only the player that becomes the follower changes behavior at that point. At the

[19] In the proof we show $\overline{\kappa}_r$ to be the smallest of many thresholds that correspond to many possible deviations that may combine both buying and selling.

threshold k_4 the payoff-maximizing non-cooperative behavior switches from ftl+competition to ftl, and at k_p from ftl to zero; that is, the follower stops being active. These equilibrium payoffs and the relevant thresholds are depicted in Figure 4. It is perhaps surprising that competition

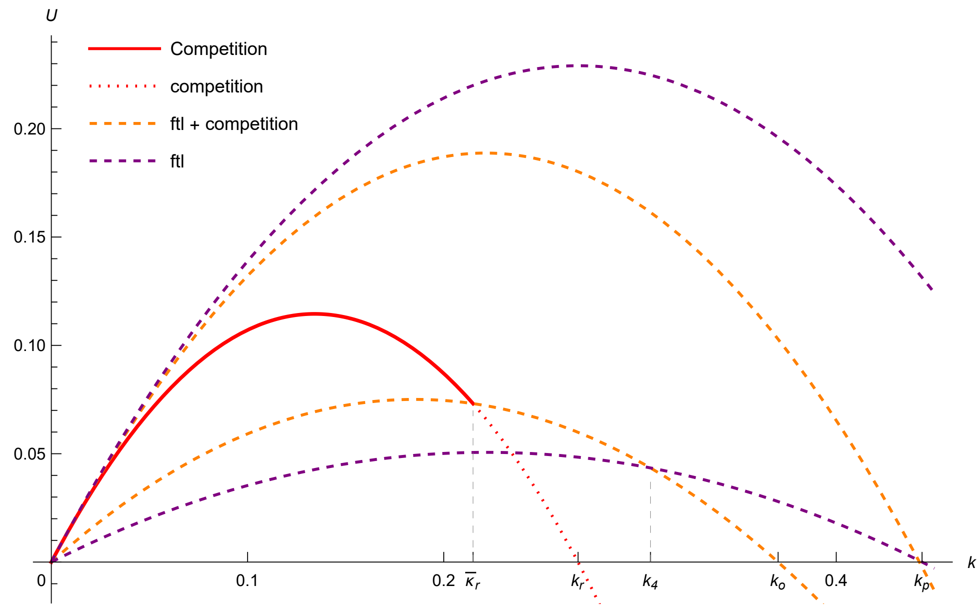

Figure 4: Non-cooperative payoff functions and Competition payoffs as a function of capacity for $n = 2$, $a = 0.6$, $\beta = \delta = 0.95$.

is not always an equilibrium; after all, it is an equilibrium in the repetition of the Cournot game. Part, but only part, of the explanation lies with the rigidity of the action space $\{0, k\}$. The other part of the explanation probably best highlights the profound difference between "the storage game" and conventional production-for-sale. When only selling, players only need to care about $A(l, k)$. But a storage unit also exerts market power when *buying*; the function $B(l, k)$ is increasing in both arguments and convex in k. Selling "too much" by imperfectly internalizing one's infra-marginal impact also requires buying "too much", which very rapidly destroys any arbitrage revenue. Then letting go dominates.

To be clear, follow-the-leader requires no cooperation at all; it is an asymmetric, non-cooperative, coordination behavior of this game that can be played repeatedly without any supporting penalty regime. It can produce strictly positive payoffs for the follower because of the combination of uncertainty and capacity constraint: once the leader has moved and the next shock is in the same direction (e.g. negative), then the follower can move (e.g. buy), and wait for the opportunity to sell later. This characterization is important for another reason:

this behavior can be used to support cooperative play.[20]

3.3 Competition, Partial Cooperation – and letting go

Partial cooperation is one such cooperative play. As we know since the work of Abreu (1988), a cooperative equilibrium can be supported with simple penal codes; here, the simple penal code is reversion to any non-cooperative equilibrium – for example, Competition, but not exclusively. For emphasis, this is a stochastic game (and not a repeated game), so the exact time at which each player can take turns depends on the combination of the realizations of the state (c_t^1, c_t^2) and the play. This feeds into the incentive constraints an equilibrium must satisfy.

Proposition 6. *Assume that (8) holds. There exist capacity thresholds* $0 < \underline{\kappa}_b$ *and* $\underline{\kappa}_b < \overline{\kappa}_b$ *such that, for* $\underline{\kappa}_b < k < \overline{\kappa}_b$*, the following is a subgame-perfect Nash equilibrium that we label Partial Cooperation:*

- *The storage units play the behavior partial cooperation.*
- *If one of the units deviates by purchasing under a negative shock when both units are empty and it is not its turn, the other unit switches to competitive behavior forever as a punishment. That is, it starts buying (selling) every time it is empty (full) under a favorable shock. A subgame-perfect Nash Equilibrium of this subgame off the equilibrium path is as follows. There exist capacity thresholds* k_4, k_o *and* k_p *such that:*
  - *both units play Competition if* $\underline{\kappa}_b < k < \overline{\kappa}_r$*;*
  - *the deviating unit plays FTL+Competition if* $\overline{\kappa}_r < k < \min\{k_4, k_o\}$*;*
  - *the deviating unit plays FTL if* $k_4 < k_o$ *and* $k_4 < k < \min\{k_p, \overline{\kappa}_b\}$*;*
  - *the deviating unit sells (either competitively or using ftl strategy) as soon as possible and exits the market forever afterwards for any remaining* $k \leqslant \overline{\kappa}_b$*.*

[20] At this point we do not claim that ftl is an equilibrium; it is an appealing suggestion but we do not check whether it is immune from deviation. Instead we focus on symmetric equilibria. Later we do check that ftl and its variations are an equilibrium for some k when constructing cooperative equilibria.

Here the threshold $\underline{\kappa}_b$ is the capacity level such that the incentive to play the Partial Cooperation equilibrium dominates the deviation gain (competing when one should not) and enduring the punishment forever after. As in Proposition 5, above the threshold $\overline{\kappa}_b$ Partial Cooperation can no longer be an equilibrium because its payoff is dominated by either follow-the-leader, or by playing zero if being the second mover. As before, one of the players accepts to be a follower; this is "letting go". Depending on capacity k, partial cooperation can be supported as an equilibrium thanks to the existence of one of the four non-cooperative plays to which players can revert in case of deviations that are identified in Section 3.2. We show the equilibrium payoff and the range on which the equilibrium exists in Figure 5, where some subtleties demand explaining. A SPNE must be subgame perfect at every node; here it

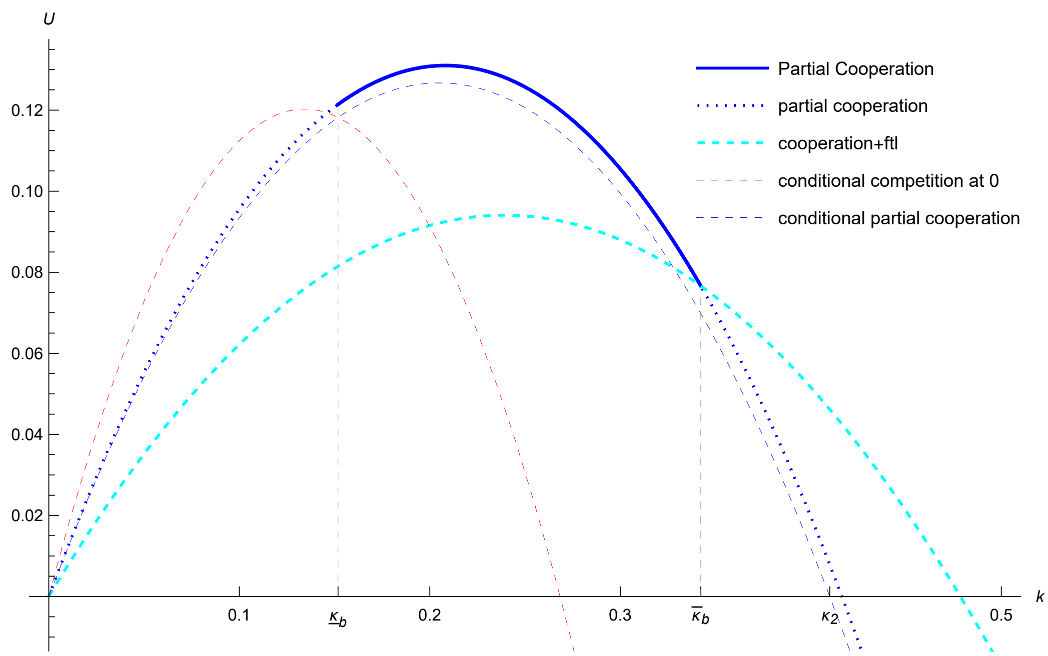

Figure 5: Partial Cooperation payoffs with the payoffs of possible deviations for $n = 2$, $a = 0.6$, $\beta = \delta = 0.95$.

implies it must continue to be an equilibrium for every state $(\mathbf{c}_t, \varepsilon_t)$. However the incentives to cooperate do differ depending on that state and on the order of play: with discounting, the payoff to the first-moving unit is always higher than that to the second-moving unit – on and off the equilibrium path. The solid blue line represents the *ex ante* expected payoff from Partial Cooperation, that is, before the order of play is revealed. The dashed blue line passing through the point κ_2 is the *interim* expected payoff from playing the same equilibrium for the party who moves *second*; the order of play is known. The *interim* expected payoff that accrues to the second mover, *when s/he knows s/he is the second mover*, is the relevant payoff to determine whether the behavior can be sustained as an equilibrium. The reason is that it is also the worse payoff from partial cooperation. That payoff is contrasted to the payoff from competition when in the same state $\mathbf{c}_t = 0$ (simultaneous move); the indifference point is the threshold $\underline{\kappa}_b$. That threshold is then applied to the *ex ante* expected payoff (the solid line) to determine the equilibrium. Conversely, the upper bound $\overline{\kappa}_b$ is determined by the incentives of the players *ex ante*. At that point, both storage units are in the same state and are supposed to sell simultaneously. At $\overline{\kappa}_b$, even the player who is becoming the follower is better off giving up on partial cooperation. Rather than competing symmetrically when selling, it becomes more profitable to follow the leader when selling too; this is labeled "cooperation+ftl". The intersection with the solid blue line (partial cooperation) identifies $\overline{\kappa}_b$.

Partial Cooperation, which involves playing partial cooperation all the time, is an equilibrium for capacity levels between $\underline{\kappa}_b$ and $\overline{\kappa}_b$. On this range, there may be other equilibria that involve playing partial cooperation some of the time. In particular, since the interval $[\underline{\kappa}_b, \overline{\kappa}_r]$ is never empty (see the Proof of Proposition 6), Partial Cooperation and Competition are both equilibria for some k. One can therefore conceive of more intricate equilibrium play that involve some combination of both.

3.4 Competition, Cooperation – and letting go.

Cooperation is a behavior that becomes increasingly attractive as capacity k becomes large. It is also supported as an equilibrium by the same non-cooperative equilibria as Partial Cooperation, however with some details that are important to the actual construction of the equilibrium. For this we require

$$\frac{1-a}{1+a} < \frac{G_1(\beta) + G_2(\beta)\delta^2}{G_3(\beta) + G_4(\beta)\delta^2} \cdot \delta, \tag{9}$$

where the coefficients $G_i(\beta)$ are polynomial functions of the discount factor β (laid out in the Proof). Condition (9) implies (8) as $\beta < 1$.[21] In other words, cooperation is a little more onerous than any other play so far. The reason, as is apparent from the definition of the behavior in Section 3.1, is the delay that is involved. An interpretation one can make is that a larger volatility is required to compensate for the delay incurred by the second-moving unit.

Proposition 7. *Assume that Condition (9) holds. There exist thresholds $0 < \underline{\kappa}_g$ and $\underline{\kappa}_g < \overline{\kappa}_g < 1$, and a discount factor $\bar{\beta}$, such that for $\bar{\beta} < \beta$ and for $\underline{\kappa}_g < k < \overline{\kappa}_g$, the following is a subgame-perfect Nash equilibrium labeled Cooperation:*

- *The storage units play cooperation.*
- *If one of the units deviates to competition when both units are either empty (state $(c_t^1 = 0, c_t^2 = 0)$) or full (state (k, k)), the other unit switches to competitive behavior forever as a punishment. That is, it starts buying (selling) every time when it is empty (full) under a favorable shock. A subgame-perfect Nash Equilibrium of this subgame off the equilibrium path is either Competition, or FTL+Competition, or FTL, or the deviating unit quits the market (after selling if full).*

Here the threshold $\underline{\kappa}_g$ plays the same role as $\underline{\kappa}_b$ in Proposition 6: at that point, not only is cooperation more attractive than competition, it can be sustained as an equilibrium. Which non-cooperative equilibrium supports the cooperative one depends on the capacity level. We show the payoff of the Cooperation equilibrium, and the range of capacity over which it can exist, in Figure 6, which also requires some comments. Characterizing $\underline{\kappa}_g$ is a little more demanding than first appears.

As before, the subgame perfect criterion requires of us to consider deviations for all states $(\mathbf{c}_t, \varepsilon_t)$. To do so, one must compute the payoffs from cooperation for $\mathbf{c}_t = (0, 0)$ and $\mathbf{c}_t = (k, k)$, as well as the deviation payoff (to competition) for $c_t^i = k$. In Figure 6 these are labeled "conditional cooperation", with a clear ranking depending on c_t^i. That is, playing cooperation

[21] It also implies the weaker condition $\frac{1-a}{1+a} < \frac{\delta\beta(1+\beta)}{4-\beta-\beta^2}$, which naturally implies (8) and guarantees a positive payoff to the second mover when cooperation is played. But it is not sufficient for cooperation to be an equilibrium.

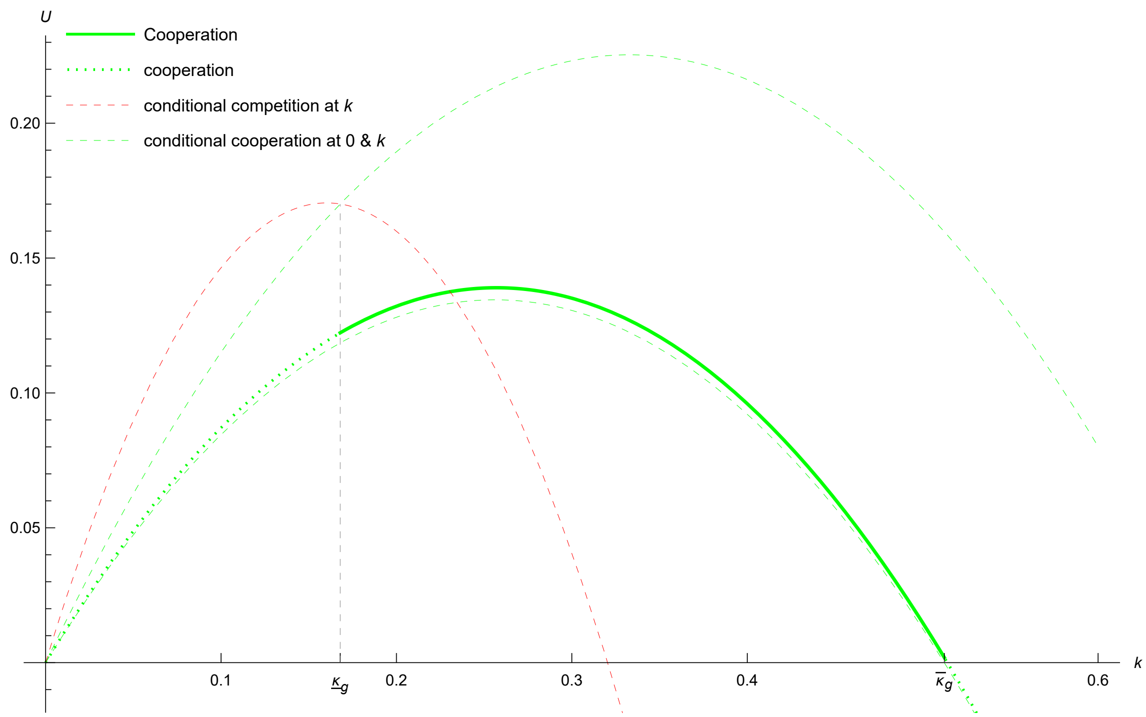

Figure 6: Collusion payoffs with the payoffs of possible deviations for $n = 2$, $a = 0.6$, $\beta = \delta = 0.95$.

when $c_t^i = k$ is more profitable, as in a position to sell, than if $c_t^i = 0$ because i must first buy. We also label "conditional competition", which is the payoff from symmetric cooperation when $c_t^i = k$; this is the relevant deviation payoff in that state. The threshold $\underline{\kappa}_g$ is the level of capacity at which an operator with state $c_t = k$ is indifferent between cooperation and reverting to competition. Again, these are *interim* payoffs, and they capture the relevant incentive here: what strategy should a full unit pursue at this stage of the game? A full unit that plays Cooperation must wait, whereas it can sell immediately if deviating to competition. The threshold $\underline{\kappa}_g$ is then applied to the *ex ante* expected payoff from playing cooperation to pin the equilibrium. Hence that equilibrium is determined by the player with the strongest incentives to deviate on the equilibrium path (the full unit with $c_t^i = k$). Likewise, the threshold $\overline{\kappa}_g$ is the level of capacity at which the *second-mover* with state $c_t^j = 0$ receives 0 from cooperation and so is better off doing something else. At that point, the empty unit that elects to cooperate must wait the longest: it must have a turn, buy, wait and then sell. The green solid line shows the ex ante expected payoff to cooperation starting in state $\mathbf{c}_t = 0$, and is an equilibrium over the range $[\underline{\kappa}_g, \overline{\kappa}_g]$. While not obvious from Figure 6, the ex ante equilibrium payoff is higher than the interim payoff with $\mathbf{c}_t = 0$ even at $\overline{\kappa}_g$.

For capacities in excess of $\overline{\kappa}_g$, Cooperation can no longer be an equilibrium. Then players must revert to the asymmetric equilibria we identify in Section 3.2. Proposition 7 can be complemented with

Corollary 8. *There exists β^* such that for any $\beta > \beta^*$ the equilibrium Cooperation always exists for some values of a and δ; it does not exist otherwise.*

Corollary 8 identifies a necessary condition for the equilibrium Cooperation to exist. It can also be interpreted as a reaffirmation that Cooperation requires patience as players take turns to trade.

3.5 Payoff-maximizing equilibria

Now we are in a position to collect our results and to characterize all payoff-maximizing equilibria supported by a grim-trigger strategy. Recall the thresholds k_1 to k_3. As we know from Propositions 5 to 7, equilibria do not switch at these thresholds, however they do matter for incentives, and therefore ultimately to determine payoff-maximizing equilibria. We know from Lemma 3 that these thresholds are ordered; therefore competition, partial cooperation, and cooperation are always payoff-maximizing behaviors (if positive) in this order.

Proposition 9. *Take $s(c), b(c) \in \{0, k\}$. If condition (8) holds, payoff-maximizing symmetric equilibria are characterized as follows:*[22]

1. *if Condition (9) holds and $k_3 \leqslant \overline{\kappa}_g$,*
   - *for $k \in (0, \underline{\kappa}_b]$, players engage in Competition;*
   - *for $k \in [\underline{\kappa}_b, k_3]$, players engage in Partial Cooperation;*
   - *for $k \in [k_3, \overline{\kappa}_g]$, full Cooperation prevails;*
2. *if Condition (9) fails, or $k_3 > \overline{\kappa}_g$,*

[22] The relation between k_3 and $\overline{\kappa}_g$ is established for ease of exposition. Both are functions of the underlying parameters β, δ and a. So these conditions are equivalent to conditions on these underlying parameters, which we show in the Appendix but are too cumbersome to be helpful here.

- *for* $k \in (0, \underline{\kappa}_b]$*, players engage in Competition;*
- *for* $k \in [\underline{\kappa}_b, \overline{\kappa}_b]$*, players engage in Partial Cooperation.*

Item 1 is shown on Figure 7 while Figure 8 shows item 2 of the Proposition. In both Figures, the solid lines depict equilibrium maximum payoffs; the dashed lines show payoffs arising from the same equilibria, but are payoff-dominated by another equilibrium.

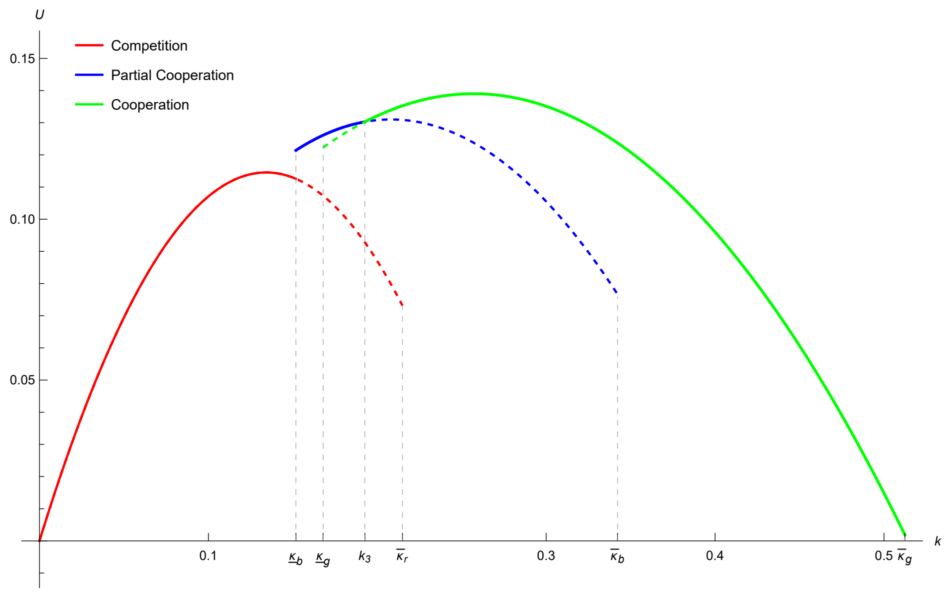

Figure 7: Profit-maximizing equilibria for $k_3 \leqslant \overline{\kappa}_g$, $n = 2$, $a = 0.6$, $\beta = \delta = 0.95$.

In Figure 7, the discount factor is such that the behavior cooperation is not only an equilibrium, it is also profit-maximizing for the storage operators. In Figure 8 Cooperation is also an equilibrium for some capacities, but it is payoff-dominated by Partial Cooperation. The reason is that the threshold k_3 falls to the right of $\overline{\kappa}_g$. At the point k_3, the payoffs to either behavior is negative (and neither is an equilibrium).

There are important details that justify the richness of Proposition 9 and the plethora of figures. The cooperative equilibria are both determined in relation to a version of a non-cooperative equilibrium (Competition, FTL+Competition or FTL). That is, for incentive purposes, these are the only equilibria that matter. There is no connection between Partial Cooperation and Cooperation as equilibria; in particular, one does not support the other. Rather, for a set of parameters (a, β, δ), they may just dominate one another in terms of payoffs only.

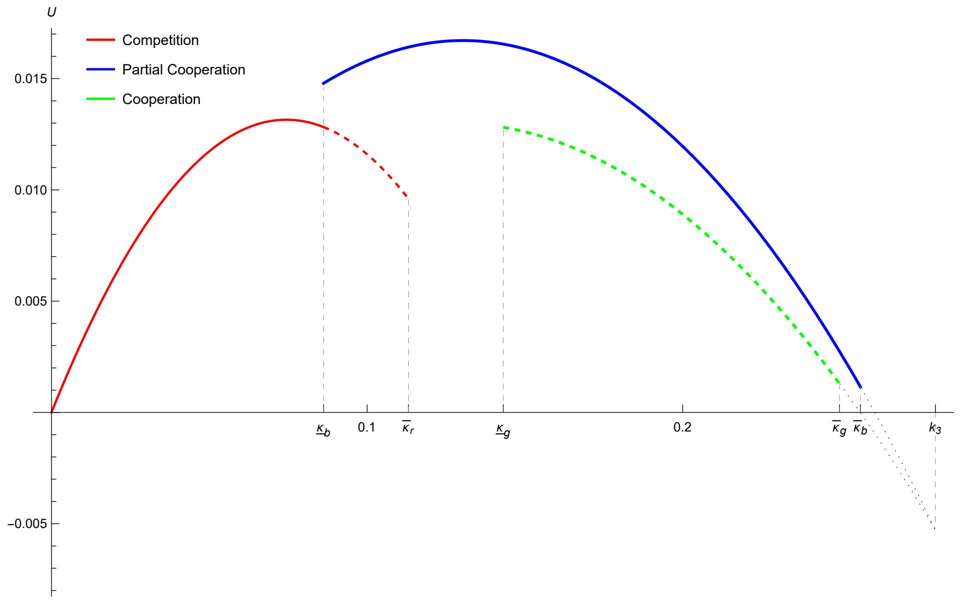

Figure 8: Equilibrium payoffs for different strategies for $n = 2$, $a = 0.6$, $\beta = 0.8$, $\delta = 0.85$.

To illustrate, in both figures, at $\underline{\kappa}_b$, Partial Cooperation must deliver a discretely larger payoff than Competition otherwise the deviation is too tempting (equivalently, the benefit of Cooperation too small). This reflects the fact that Partial Cooperation must be robust to deviations at the interim stage – when one of the players is revealed to be the second mover and contemplates her options then (see Section 3.3). Likewise, at $\underline{\kappa}_g$ (in Figure 7), Cooperation must also deliver a discretely larger payoff than Competition: it must be robust to deviation at another interim stage (see Section 3.4). (In contrast, k_3 is an indifference point between the behaviors partial cooperation and cooperation.)

Remark 10. *If in addition, Condition* (9) *fails, Cooperation does not even exist as an equilibrium because its payoff can never be positive.*

The contrast between Figure 3 and Remark 4 on the one hand, and Figures 7–8 and Proposition 9 is evident. Even under symmetric information, there exist frictions to cooperation because the incentives to the players differ in each state $(\mathbf{c}, \varepsilon)$.

Remark 11. *Instead of flipping a coin, one may consider allocating a token. Players flip a coin only once to decide who owns the token first. Whoever holds the token moves (buys or sells) first and passes the token to the other player. It's easy to show that the expected payoffs for cooperation and partial cooperation are the same, but the pivotal values of k change.*

3.6 Consumer welfare

The welfare implications of these results are not immediate. One the one hand, a large capacity erodes the market power of storage. On the other, when storage plays the equilibrium Competition, it is not available half the time – from state $(0, 0)$ when the shock is positive, and from (k, k) when the shock in negative. If playing Partial Cooperation, at least one storage unit can buy in states $(0, k)$ and $(k, 0)$.

To speak to consumer welfare we compute the cumulative (expected) payment functions that consumers face; these are the transfers to the collection of producers and storage operators and are written for each behavior as a function of k. Because storage operators must buy from producers, these payment functions actually represent the whole producer surplus in this model. We also compute the total (expected) consumer surplus functions. These results are collated in Lemma 16 in the Appendix (Section C). We denote them

$$P_{com},\ P_{pc},\ P_{col} \text{ and } S_{com},\ S_{pc},\ S_{col},$$

respectively, and then use them to establish that, for the same capacity, the behavior competition is more favorable to consumers, which largely accords with intuition. (The ranking between partial cooperation and cooperation is ambiguous.)

Proposition 12. *If condition (8) holds, for any capacity k we have*

$$P_{com} < P_{pc} \quad \textit{and} \quad P_{com} < P_{col}, \ \textit{and}$$

$$S_{com} > S_{pc} \quad \textit{and} \quad S_{com} > S_{col}.$$

This result provides an incomplete picture of the efficiency of the equilibria we construct because different equilibria arise for different capacity levels. For some capacity level $\underline{k}$, storage operators play Competition while for some other capacity level $\overline{k} > \underline{k}$ they may play Cooperation. In addition, one can conceive of an entry stage, in which storage units choose their capacity. To address this point, first we find the maximizers of the equilibrium payoff functions, and denote them k_{com}, k_{pc} and k_{col} (Lemma 17 in Section C; see also Figure 9).

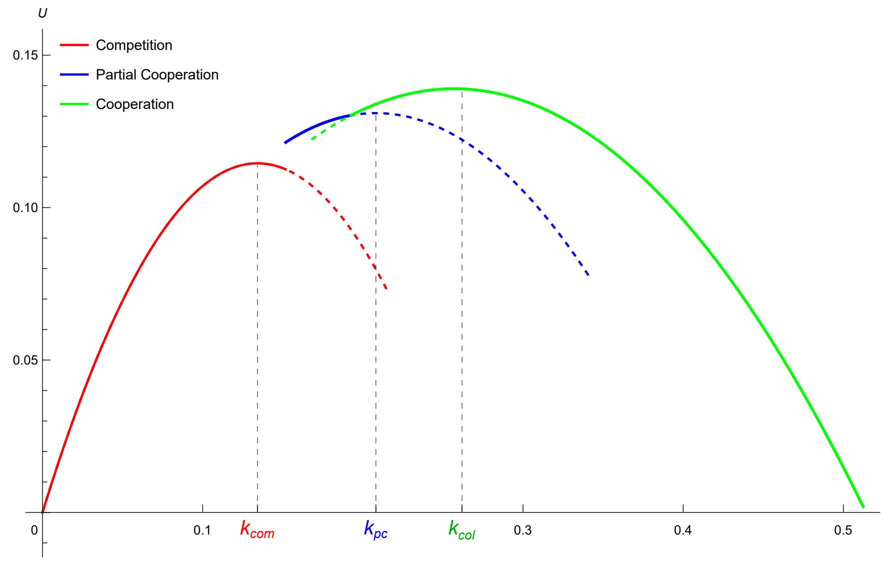

Figure 9: Maximizers of the payoff functions for $n = 2$, $a = 0.6$, $\beta = \delta = 0.95$.

With this we can evaluate the same transfer and surplus functions at the relevant capacity levels; that is, we can compute the equilibrium values of these functions and compare these values across equilibria.

Proposition 13. *At the maximizers k_{com}, k_{pc} and k_{col} and if Condition (8) holds,*

$$P_{pc}(k_{pc}) < P_{com}(k_{com}) \text{ and } S_{com}(k_{com}) < S_{pc}(k_{pc}).$$

There are two drivers behind this result: capacity utilization, and capacity level. The average capacity utilization under the behavior partial cooperation is lower than under competition – that is the point of it. Indeed, for competition, the utilization is $2k$ when buying or selling under the favorable shock; this happens with probability $1/2$. Thus, the average capacity utilization in each period is k. For partial cooperation, the probabilities of each state are $1/2$ for the state $(0, 0)$ (both units are empty), $1/4$ for the state $(0, k)$ (one unit is empty, the other one is full), and $1/4$ for the state (k, k) (both units are full).[23] Then the expected

[23]This stable state may be calculated using the formula $\lim_{n\to\infty}(1, 0, 0) \cdot \mathbf{Q}_{pc}^n = (1, 0, 0) \cdot \mathbf{Q}_{pc}^2 = (1/2, 1/4, 1/4)$. Here, the initial vector is equal to $(1, 0, 0)$, because units always start from being empty.

capacity utilization is equal to

$$\frac{1}{2} \cdot \frac{1}{2} \cdot k + \frac{1}{4} \cdot k + \frac{1}{4} \cdot \frac{1}{2} \cdot 2k = \frac{3k}{4}.$$

However, this lower utilization, in line with the idea that operators seek to restrain quantities, is dominated by the quantity effect: not only $k_{pc} > k_{com}$ but also

$$\frac{3}{4}k_{pc} - k_{com} = \frac{C \cdot (4 - \beta^2 + \beta(2 - 3\beta)\delta^2)}{8\,(2 - \beta + \beta\delta^2)\,(4 - \beta^2 + \beta(2 + 3\beta)\delta^2)} > 0.$$

That is, even under Partial Cooperation, storage operators trade larger quantities on average – they can only partially restrain quantities. With market power this implies lower prices too, all of which makes consumers better off.[24]

4 Discussion

In this section we briefly discuss the effects of heterogeneity in discount factor and capacity, and then draw some implications of our work.

4.1 Heterogeneous players

Departing from homogeneity of the players is not completely straightforward. If the differences are small, restricting attention to symmetric equilibria can remain valid. For large differences in the characteristics of the players, it may be more appropriate to consider asymmetric play as well. We confine ourselves to the former to contrast to our results so far.

Heterogeneous discount factors. When players differ only in their discount rate, so do their payoff functions, but they remain independent. More precisely, the system (7) consists of independent equations for each player. For a symmetric equilibrium, we can identify the

[24] For most storage facilities, returns to scale are small at best: storing more of a commodity requires more storage space in a constant fashion – more battery packs, more storage volume and so on. An implication of Proposition 13 is that where there are returns to scale, as for pumped hydroelectricity, the quantity effect may be sufficient to counter the curtailment in capacity utilization these operators may seek. Absent returns to scale (batteries), one ought to be wary of large storage facilities as there is no social benefit to their large size.

relevant thresholds using the same techniques as to this point. Figure 10 displays these payoff functions for $\beta_1 = 0.95, \beta_2 = 0.9$, and we observe two effects.

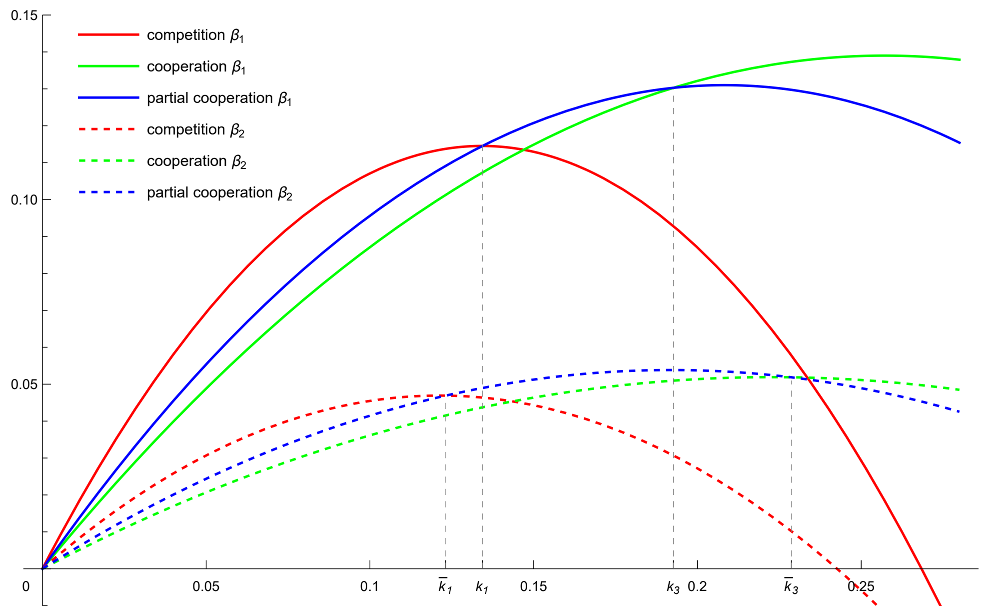

Figure 10: Payoffs for different strategies for $n = 2$, $a = 0.6$, and asymmetric discount factors $\beta_1 = 0.95$, $\beta_2 = 0.9$, $\delta = 0.95$.

The first one conforms with expectations: the less patient firm is less willing to engage in cooperation *for a large capacity.* For example, it has incentives to switch from partial cooperation to cooperation for a higher capacity than the more patient firm *for a large capacity* ($\overline{k}_3 > k_3$ in Figure 10). In this case, the behavior of the less patient player determines the equilibrium. As a result, Partial Cooperation remains an equilibrium for a larger capacity than under symmetry ($\beta_1 = \beta_2 = 0.95$).

The second effect is novel and intimately tied to the fact that storage units must *buy* and sell. For *smaller capacities*, the less patient firm seeks to engage in partial cooperation for *lower* capacities than the more patient player. As can be seen in Figure 10, it has incentives to switch from competition to partial cooperation for smaller ks than the other firm ($\overline{k}_1 < k_1$). This is counterintuitive, but explained as follows. As traded quantities increase, so does the price impact and the spread $A_2 - B_2$ generated under competition shrinks; this is true for both operators. However the continuation value of the impatient agent is lower because of her lower discount factor. She would like to increase this continuation value by changing play (to partial cooperation). In doing so she would decrease the effective discount factor because trading less frequently, which is costly. But the benefit of restraining quantities through partial cooperation would dominate. This phenomenon arises because the behavior *partial cooperation* exists; it cannot in a model of sale only. In the class of symmetric equilibria, these new incentive effects do not change the equilibrium though. Here, the more patient player faces different incentives and so does not conform to the behavior partial cooperation for all the same value of k as the impatient player. This pins the equilibrium.

Heterogeneous capacities. When players differ (only) in their capacities, k_1 and k_2, the payoff functions become functions of two variables: $U^i(k_1, k_2)$ for each player $i \in \{1, 2\}$. Thus, to analyze possible equilibria and how they depend on capacity, we need to operate not on an interval but on a plane (k_1, k_2). Payoff sets are surfaces, which significantly complicates the calculations. Similar patterns hold though with a couple of caveats. The equilibrium Competition exists. Cooperation (if it exists) is still a payoff-dominant equilibrium if both capacities and their sum $k_1 + k_2$ are large enough. A strip of Partial Cooperation emerges in between, as with symmetric players. But asymmetry in capacities does matter and players tend to play Competition if at least one of k_i is small enough, and the other one not too large. Indeed, the unit with higher capacity (j) faces weaker incentives to cooperate because the price impact of i's small capacity added to k_j is lower than the cost of delay. That is, a small opponent weakens any incentive for cooperation. Finally, for sufficiently asymmetric players and $k_1 + k_2$ large enough, a new coordination equilibrium of the stage game emerges, in which either 1 or 2 trades and the other one selects 0.[25]

4.2 Implications

The results we derive lead to reasonably direct implications for competition policy. It is quite immediate that only small-capacity storage operators have any incentives to behave competitively. But for intermediate capacity levels, the quantity effect dominates the restraining of capacity utilization, and so is beneficial to consumers (Proposition 13). This last claim must be appended with two caveats. First, as shown in Section B.1, a more flexible model

[25] There is also a mixed-strategy equilibrium that we do not investigate.

dilutes the incentives to cooperate precisely because flexibility tames the quantity effect, which drives the result of Proposition 13.[26] Second, capacity is free in this model; any capacity cost reduces the optimal capacity level and renders idle capacity (possibly excessively) expensive. Nonetheless, one must note that whether to strictly enforce antitrust provisions or to limit capacity size is not a foregone conclusion but very much a matter of empirical analysis. Again, the genesis of this implication is the impact of market power on the arbitrage spread of storage units; that is also the sense in which the quantity effect is really important for storage.

Now specifically turning to electricity markets, it is already not rare to see the same owner operate multiple units; for example, in Australia the French operator Neoen owns and operates three – and soon more – large units in the same wholesale electricity market. With batteries (in particular) it is trivial to coordinate the action of units that operate in the same portfolio. We suggest that this form of portfolio concentration presents real risks of anti-competitive behavior. First, they can easily coordinate on the units they own; second, as coordinated entity, they face exactly the incentives we study here. In the same vein, some new business models emerge in the form of "Virtual Power Plants" (VPP), which propose to take control of multiple small storage units to trade in the wholesale market. Here too, a competition authority may need to guide (and possibly cap) the extent of the consolidation these businesses imply. And it may need to also want to prevent an entity from owning and operating multiple VPPs.

Exercising market power is a lot easier on a network. So is colluding, for the same reasons. Our results suggest that market operators need to upgrade their surveillance capacity to also include the tacit collusion that we describe. In California, the operator CAISO already uses automated "market power mitigation mechanisms" to combat the exercise of unilateral market power at nodes where it is likely to arise. It needs to also account for storage collusion on nodes where that is becoming relevant. This may be particularly relevant in California, where already at least three large operations (Crimson, Oberon and Desert Sunlight) are located in close proximity just East of Joshua Tree, presumably on the same transmission line.

Market design can be used in conjunction with competition policy instruments to foster

[26]The impact of flexible actions is more acute for cooperation than for partial cooperation.

competition. The market we model is a spot-only market; there is no forward market. Since the work of Allaz and Vila (1993), we know that forward markets are pro-competitive. In electricity, one such form of a forward market is a day-ahead market (DAM), in which suppliers make binding financial commitments 24 hours ahead of delivery. While stochastic shocks cannot be anticipated, the underlying demand could be serviced using a DAM, with only the unanticipated variations being settled on the spot market. Jurisdictions that do not use DAMs may find it increasingly attractive.

The restraining of quantities that we describe here – and which takes the form of cooperation between nominally competing storage units – has further consequences. Because purchase prices are kept low, they undervalue the traded commodity, which in turn leads to inefficiently low investment levels in the production of that commodity. Put another way, storage units do not add enough to the low-state demand; not enough quantities are traded. In the context of electricity, this means insufficient investment in (renewable) generation capacity. Since we argue that storage is the bottleneck to the energy transition, this may call for some kind of intervention to alter the equilibrium behavior of storage operators.

As argued before, our work applies beyond electricity to any storable commodity, such as those studied by Deaton and Laroque (1992) but also fuels and metals.[27] It may in fact also explain the emergence of bubbles in the trading of these commodities: speculators may hoard large stockpiles of commodities, which they then cannot get rid of without the price collapsing. In fact, this is what happened to Sumitomo's trader Yasuo Hamanaka in the mid-1990s on the London Metal Exchange.[28] While there may not be enough aggregate storage capacity in any electricity market yet to be immediately concerned, that aggregate capacity is bound to become enormous as the energy transition progresses. Market operators need to become aware of this possibility.

[27] Arguably, there is little arbitrage in the market for fresh tomatoes, for example; for such crop, simply set a low value to δ to render arbitrage uneconomical.

[28] Source: Reuters https://www.reuters.com/article/us-lme-warehousing-insight/insight-fixing-the-worlds-metals-warehousing-why-so-long-idUSBRE9AE06U20131115

5 Conclusion

Storage operators that have market power have very strong incentives to eschew competition because the arbitrage spread they need to earn squeezes rapidly to oblivion from their selling and their *buying*. These incentives continue to exist even when storage units can unilaterally decrease their traded quantities. We construct selected cooperative equilibria of this game. Because of the dynamic nature of the game and of these multiple actions (selling and buying), the construction of these equilibria can be intricate at times. The reason is that the exact incentives depend on both the state of the units and of their current action.

Here cooperation can be implemented in multiple ways because the decision whether to collude has to be made for the actions of buying, selling or both. We construct payoff-maximizing equilibria and in so doing we uncover a new class of equilibrium that we call Partial Cooperation that reflects this feature. Partial Cooperation is the embodiment of a compromise between restraining quantities (when buying) and facing the cost of delay from discounting, which leads to selling simultaneously. Perhaps surprisingly, Partial Cooperation can deliver welfare improvements to consumers. The reason is that this equilibrium emerges when capacity is large enough that the traded quantities still depress prices; then the aggregate transfers paid by consumers to their suppliers are lower than under Competition.

We also find that head-on, repeated competition is not always an equilibrium. Again, the reason is that with large enough capacities, head-on competition induces the arbitrage spread to vanish. Instead, asymmetric equilibria emerge, in which one of the players accepts to be a follower and to play only when the circumstances allow it, rather than seeking to compete.

Finally we draw some implications of these results, both in terms of policy and of the development of an industry, in which storage plays an important role. When storage acts non-competitively, it depresses the purchase price of the underlying commodity and therefore its value to potential investors.

Appendix

This Appendix contains three parts. The first one complements Proposition 9; the second one is an extensive robustness analysis that allows for more flexible actions and changes in the discount factor. The last one contains all proofs, including the details of the critical thresholds (k and κ) that characterize our equilibria.

A Complement to Proposition 9

In Proposition 9 we restrict attention to large enough a discount factor β, as is standard in the literature on repeated games and stochastic games; it also assists in the exposition. Other (payoff-maximizing) equilibria exist for smaller β, such that $0.631 < \beta < 0.81$, and for some values of a and δ, even if $k_3 < \underline{\kappa}_g$. Here we present these complementary cases in pictures.

Two options emerge: $\overline{\kappa}_b \geqslant \underline{\kappa}_g$ and $\overline{\kappa}_b < \underline{\kappa}_g$, described by Figures 11 and 12, respectively. In the first case, we observe a new jump from the blue line to the green line at point $\underline{\kappa}_g$. In the second case, where β is smaller, there is a gap $(\overline{\kappa}_b, \underline{\kappa}_g)$, where none of those two equilibria exist. These two additional cases exhaust all possibilities. We present them here for completeness and regard them more as curiosities rather than central to our analysis.

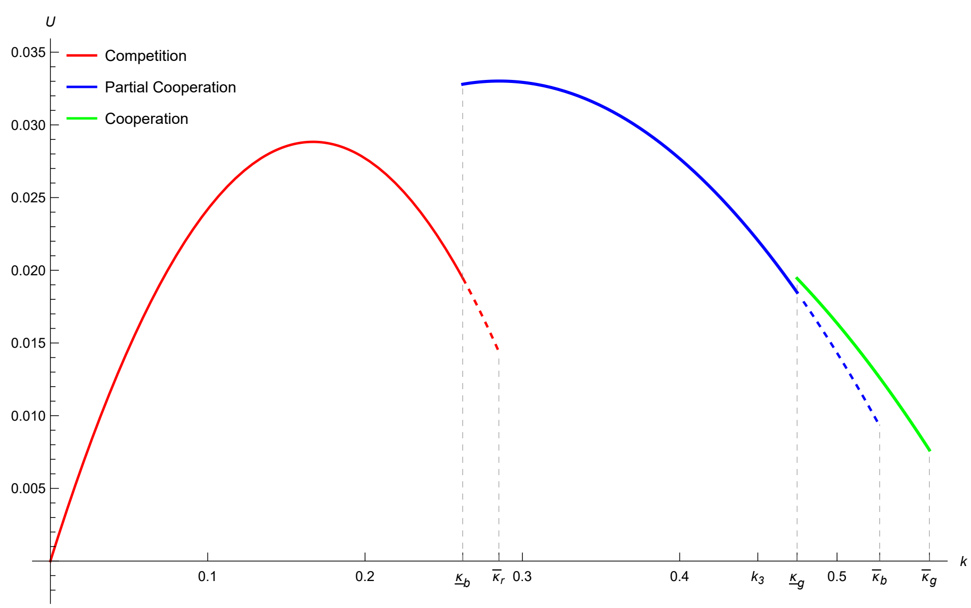

Figure 11: Equilibrium payoffs for different strategies for $n = 2$, $a = 0.99$, $\beta = 0.68$, $\delta = 0.99$.

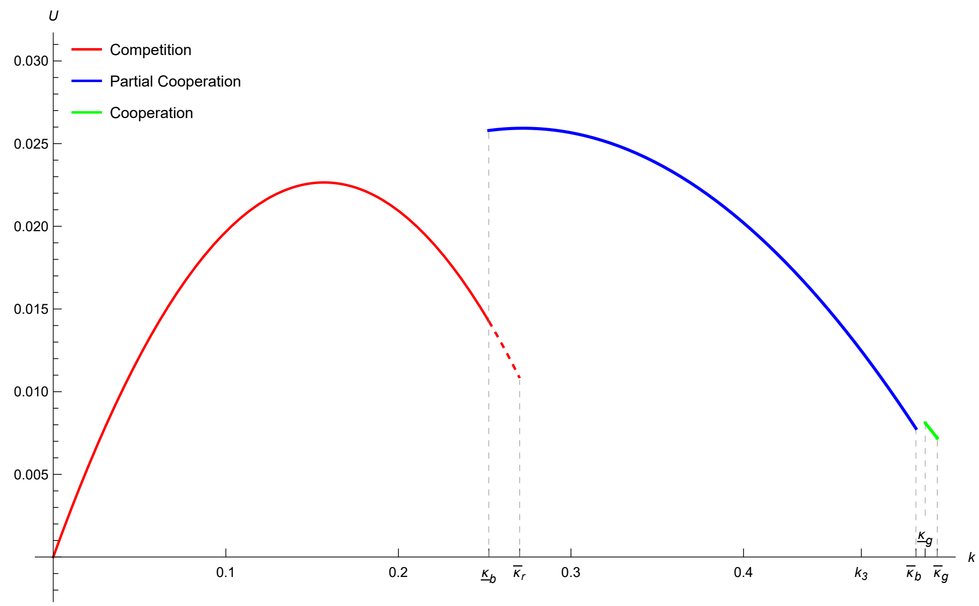

Figure 12: Equilibrium payoffs for different strategies for $n = 2$, $a = 0.99$, $\beta = 0.64$, $\delta = 0.99$.

B Robustness: flexible actions and discounting

The results derived in the main text rely on a rather inflexible set of actions: $b^i, s^i \in \{0, k\}$. It is natural then to wonder whether these results are not contrived and excessively specific to this rigid model. That is, do the incentives to collude remain so strong if players can soften competition unilaterally by altering the quantities they trade? And even if so, do the same equilibria still arise? Finally, we know behavior is sensitive to delay, which is why Partial Cooperation emerges as an equilibrium. So, how robust are these equilibria to changes in the discount factor?

B.1 More flexible actions

We are confident that our results are robust to a more flexible action space. In what follows, we carry out a similar exercise as in Section 3.1; that is, we compute the payoffs to the same behaviors when players can buy and sell in two or three steps, rather than one. Employing two or three steps need not be optimal; indeed, the optimal strategy ought to be a function of the state $(\mathbf{c}, \epsilon)$ and history of play rather than be pre-specified. However one must accept that such a task lies beyond what is feasible today. To make progress we employ a similar approach as in our companion paper (Balakin and Roger (2023)) and rely on a *heuristic* to approximate an optimal strategy. This heuristic is to use constant quantities up to capacity.[29]

Except for a small subset of parameters, payoffs to these behaviors are ranked in the same order as for the single-step case. That is, for most parameter values, there exist similar indifference points to k_1, k_2, k_3. In addition, partial cooperation remains an attractive behavior for most parameter values. We do stop short from constructing equilibria for two and three steps, which is extremely tedious. However the same approach can be employed to (i) verify that cooperative equilibria exist, (ii) construct them and (iii) find the payoff-maximising equilibria.

Suppose now that the storage units can operate either in halves or thirds of their total capacity. As before let l denote the number of active units and m the number of steps (2 or 3) they use to buy or sell their capacity k. Then we define purchasing costs under the negative shock as $B_l(k/m)$ if l units buy k/m units of energy, and likewise the revenue a storage unit earns when selling k/m units under the positive shock as $A_l(k/m)$:

$$B_l\left(\frac{k}{m}\right) = \frac{1-a+lk/m}{n+1} \cdot \frac{k}{m}, \qquad A_l = \frac{1+a-l\delta k/m}{n+1} \cdot \frac{\delta k}{m}, \qquad l \in \{1,2\}, \quad m \in \{2,3\}.$$

As before, in both expressions, the right multiplier (k/m or $\delta k/m$) is just a quantity bought or sold. The left multiplier is the resulting Cournot price. The residual demand is now $1-a+lk/m$ or $1+a-l\delta k/m$ for negative and positive shocks, respectively. We can extend to $m = 1$, in which case we are back in Section 3.1.

In light of the added flexibility provided by this heuristic, we need to amend the descriptions of the behaviors. By the same dominance argument, we dispense with two of the seven behaviors we start with.

1. *competition.* Storage units that are not full always buy together when they face negative shocks. They always sell together when they face positive shocks as long as they are not empty. Both storage units stay idle otherwise.

[29] Please see details of our companion paper to gain confidence in the robustness of this approach.

2. *partial cooperation.* If both storage units face negative shocks while not full, they flip a coin to decide who buys; the losing one remains idle. If both storage units face positive shocks while not empty, they sell simultaneously. If only one unit is full, the other one always buys under a negative shock. If only one unit is empty, the other one always sells under a positive shock. They stay idle otherwise.

3. *cooperation.* If both storage units face negative shocks while not full, they flip a coin to decide who buys; the losing one remains idle. If both storage units face positive shocks when not empty, they flip a coin again to decide who sells; the losing party remains idle. If only one unit is full, the other one always buys under a negative shock. If only one unit is empty, the other one always sells under a positive shock. They stay idle otherwise.

4. *follow the leader (ftl).* One of the units (the"leader") always buys (sells) first when it is not full (nonempty) under a negative (positive) shock. The second one (the "follower") remains idle. Only if the leader is full, the follower who is not full can buy under a negative shock. Only if the leader is empty, the follower who is not empty can sell under a positive shock. They stay idle otherwise.

5. *ftl+competition.* The leader always buys first when both units are not full under a negative shock. The follower stays idle and can buy (under a negative shock) only when the leader is full. If both units face positive shocks while nonempty, they sell simultaneously. If the follower is empty and the leader is not, the latter sells alone under a positive shock. They stay idle otherwise.

The next two Propositions echo Proposition 2 and list the payoffs from the three behaviors of interest as functions of the capacity k.

Proposition 14. *Let $m = 2$. The payoffs from competition, partial cooperation and cooperation read:*

- *competition:*

$$U'_{com} = \frac{1}{2-\beta}\left(-B_2\left(\frac{k}{2}\right) + \frac{\beta}{(1-\beta)(2+\beta)}\left(A_2\left(\frac{k}{2}\right) - B_2\left(\frac{k}{2}\right)\right)\right).$$

- *partial cooperation:*

$$\begin{aligned} U'_{pc} &= \frac{1}{64-8\beta^2-12\beta^3+\beta^5}\Bigg[-4\left(4+2\beta+\beta^2\right)B_1\left(\frac{k}{2}\right) + \frac{\beta}{4(1-\beta)} \\ &\times \left(\left(4+\beta\right)\left(8-4\beta-\beta^3\right)\left(A_1\left(\frac{k}{2}\right) - B_1\left(\frac{k}{2}\right)\right) + 2\beta\left(8+4\beta+\beta^3\right)\left(A_2\left(\frac{k}{2}\right) - B_1\left(\frac{k}{2}\right)\right)\right)\Bigg] \end{aligned}$$

- *cooperation:*

$$U'_{col} = \frac{1}{4-2\beta-\beta^2}\left(-B_1\left(\frac{k}{2}\right) + \frac{\beta(2-\beta^2)}{(1-\beta)(4+2\beta-\beta^2)}\left(A_1\left(\frac{k}{2}\right) - B_1\left(\frac{k}{2}\right)\right)\right).$$

Proposition 15. *Let $m = 3$. The payoffs from competition, partial cooperation and cooperation read:*

- *competition:*

$$U''_{com} = \frac{1}{2(2-\beta^2)}\left(-(2+\beta)B_2\left(\frac{k}{3}\right) + \frac{\beta(4-\beta^2)}{4(1-\beta)}\left(A_2\left(\frac{k}{3}\right) - B_2\left(\frac{k}{3}\right)\right)\right).$$

- *partial cooperation:*

$$\begin{aligned} U''_{pc} &= \frac{1}{2H_1(\beta)}\Bigg[-H_2(\beta)B_1\left(\frac{k}{3}\right) \\ &+ \frac{\beta}{1-\beta}\left(2H_3(\beta)\left(A_1\left(\frac{k}{3}\right) - B_1\left(\frac{k}{3}\right)\right) + \beta H_4(\beta)\left(A_2\left(\frac{k}{3}\right) - B_1\left(\frac{k}{3}\right)\right)\right)\Bigg], \end{aligned}$$

where

$$H_1(\beta) = 2048 - 512\beta^2 - 896\beta^3 + 160\beta^5 + 40\beta^6 + 4\beta^8 - \beta^9,$$

$$H_2(\beta) = 1024 + 512\beta + 128\beta^2 - 288\beta^3 - 96\beta^4 + 24\beta^5 + 8\beta^6 + 2\beta^7 + \beta^8,$$

$$H_3(\beta) = 256 - 64\beta - 48\beta^2 - 96\beta^3 + 20\beta^4 + 10\beta^5 + \beta^6 - 2\beta^7,$$

$$H_4(\beta) = 256 + 64\beta - 80\beta^3 + 4\beta^5 + 8\beta^6 + \beta^7.$$

- *cooperation:*

$$U''_{col} = \frac{4-\beta^2}{2\left(8-4\beta-4\beta^2+\beta^3\right)}\left(-B_1\left(\frac{k}{3}\right) + \frac{\beta(4-3\beta^2)}{(1-\beta)\left(8+4\beta-4\beta^2-\beta^3\right)}\left(A_1\left(\frac{k}{3}\right) - B_1\left(\frac{k}{3}\right)\right)\right).$$

Next we depict these payoffs, sequentially in Figures 13 to 15, for $m = 1, 2, 3$ and for the same parameter values we use throughout Section 3. It is quite apparent that these payoffs are ordered in the same way throughout – for these parameters. We also point out that introducing flexibility in operations ($m = 2, 3$) has its own benefits, which we explain in detail in our companion paper.[30]

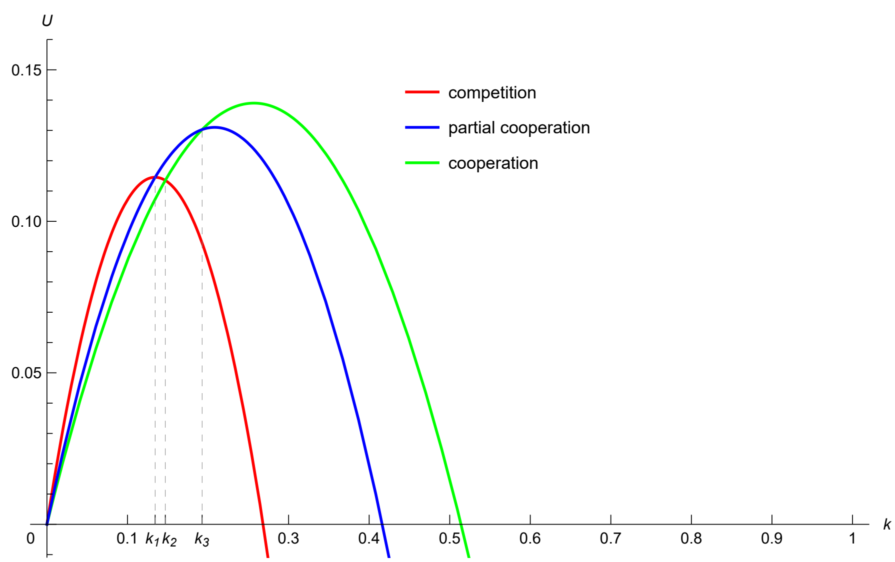

Figure 13: Payoffs for competition, pc, and cooperation for $n = 2$, $a = 0.6$, $\beta = \delta = 0.95$ and $m = 1$.

[30] Balakin and Roger (2023).

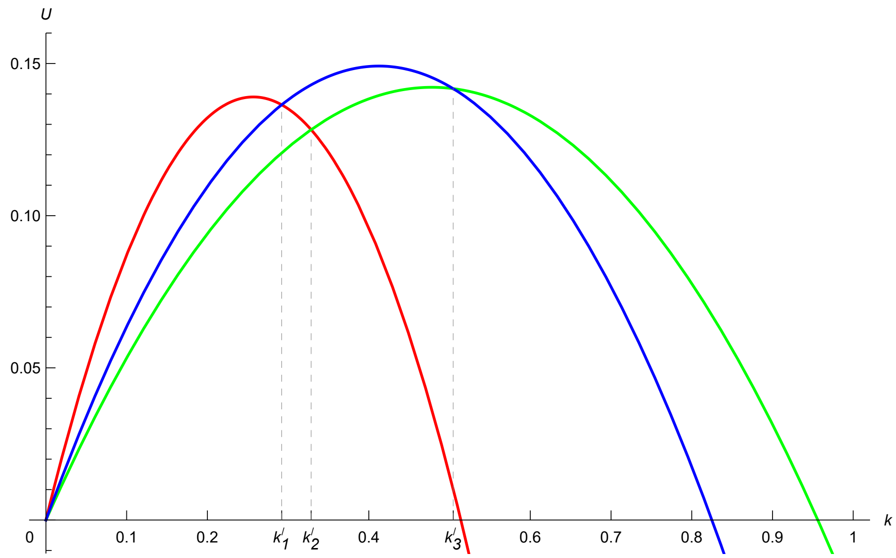

Figure 14: Payoffs for competition, pc, and cooperation for $n = 2$, $a = 0.6$, $\beta = \delta = 0.95$ and $m = 2$.

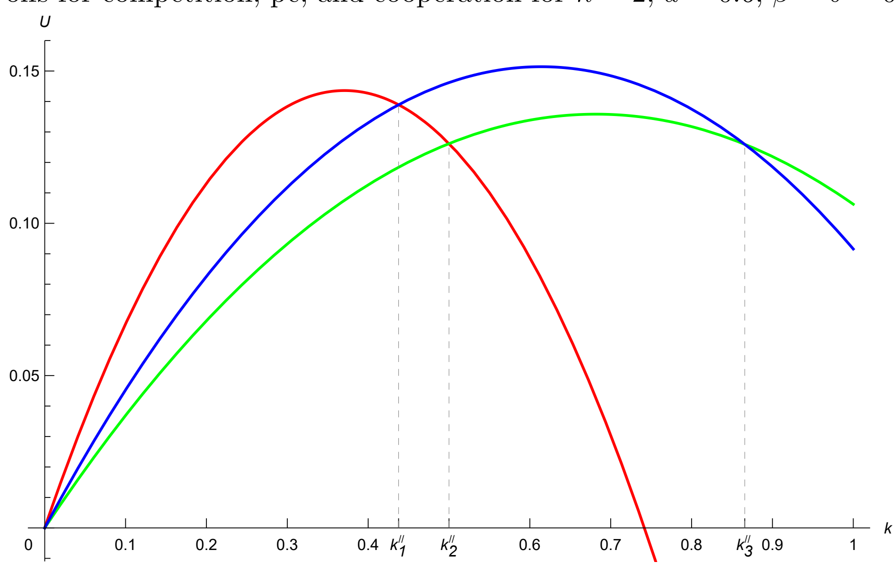

Figure 15: Payoffs for competition, pc, and cooperation for $n = 2$, $a = 0.6$, $\beta = \delta = 0.95$ and $m = 3$.

This series of pictures accords with the intuition that cooperation becomes less attractive with more flexible actions. We see in particular that partial cooperation delivers the highest payoffs when $m = 2$ and $m = 3$, however less so in the latter. As m increases, competing become relatively more attractive and cooperating can only payoff for a large capacity. Let us try to make sense of this intuitive point, which actually has a quite subtle explanation.

If selling in multiple steps, for a fixed capacity each trade is small(er) and so does not erode the arbitrage spread as badly. Moreover, there are more opportunities to follow the behavior competition; there are m such relevant states. For example, for $m = 2$ *each* unit can buy in states $(0,0)$ and $(k/2,k/2)$ and sell in states $(k/2,k/2)$ and (k,k).[31] The number of such opportunities, combined with the lesser price impact, render competition more attractive. On the other hand, any form of cooperation requires taking turns, precisely in these states where unilateral actions are available to *both*, which does include states $(0,k/2),(k/2,0),(k,k/2)$ and $(k/2,k)$ as well. This is costly because of discounting. It is most costly under (full) cooperation, where units must forego trading opportunities buying and selling.[32] Hence for $m>1$, units simultaneously face more opportunities to engage in competition (with a lesser price impact), and must forego unilateral action in more states if cooperating (for a lesser benefit).

None of this implies cooperation has no value; for large enough a capacity, it is still the more attractive behavior. But it is less compelling simply because flexibility in trade mutes the price impact. That is, fixing the discount factor, cooperation only becomes viable for capacities so large that even if selling in two or three steps, market power dissipates the arbitrage spread. The indifference points increase with flexibility – see Figures 14 and 15. For large enough a capacity, the payoff from cooperation remains the most attractive, even as m increases.

Repeatedly we write "for almost all parameter values"; let's explain this now. The analogue of Lemma 3 fails to hold in general; that is, for some values of β,δ and a, partial cooperation is payoff-dominated by either competition or cooperation. On Figures 14 and 15 we see the indifference thresholds shift rightward. For some parameter values, for example $\beta=0.998$, $\delta=0.99$, $a=0.8$, the thresholds k_2', k_2'' move right of k_3', k_3'', respectively; that is, partial cooperation is always payoff-dominated. The relevant parameter constellation requires very large values in all dimensions; indeed on Figures 14 and 15, where parameters take more intermediate values, the thresholds follow the same ordering as in Lemma 3. Hence, for some trading environments characterized by (β,δ,a), partial cooperation remains attractive. For others, in particular when β is very large and so corresponds to rapid trading, it does not. We

[31]The other states $(0,k/2),(k/2,0)$ and $(k,k/2),(k/2,k)$ are never reached since players act symmetrically starting from $(0,0)$.

[32]The only instance where cooperation is costless is when a unit must stay idle anyway, that is, in states $(k,0)$ or $(0,k)$. But now of course the frequency of these events is lower.

illustrate that point below.

The most sensitive parameter is the discount factor β. We compute and graph payoffs for the case where partial cooperation still has a role to play ($\beta = 0.996$, $\delta = 0.99$, $a = 0.9$ for $m = 2$ and $\beta = 0.99$, $\delta = 0.99$, $a = 0.9$ for $m = 3$) and where it is dominated ($\beta = 0.998$, $\delta = 0.99$, $a = 0.9$ for $m = 2$ and $\beta = 0.992$, $\delta = 0.99$, $a = 0.9$ for $m = 3$). We "zoom in" to show how minor an event it is. The axes and colors are the same as for all other figures. In Figures 16 and 17 the range of relevant capacities is $[0.5, 0.53]$. In Figure 16, even for a large value of β, the ranking of the indifference thresholds k'_1, k'_2, k'_3 is the same as in Lemma 3.

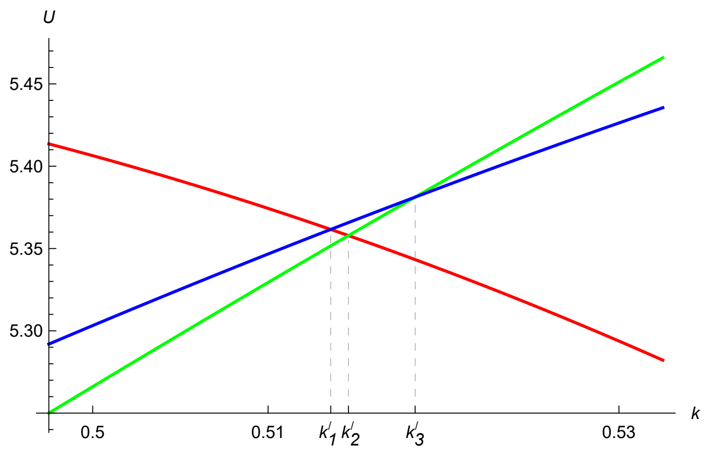

Figure 16: Payoffs for competition, partial cooperation, and cooperation for $n = 2$, $a = 0.9$, $\beta = 0.996$, $\delta = 0.99$ and $m = 2$.

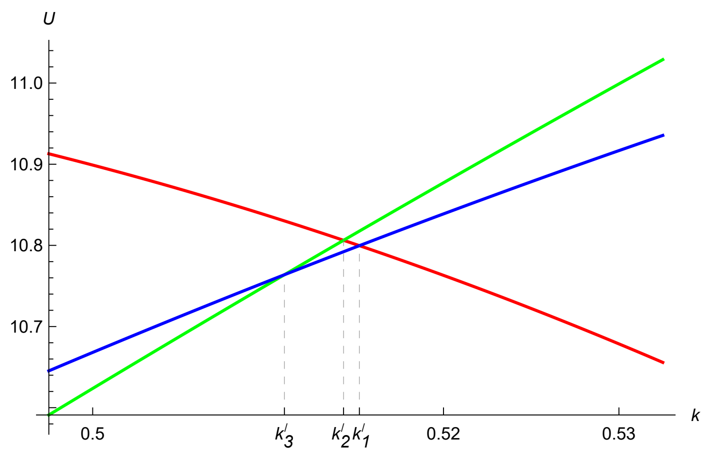

Figure 17: Payoffs for competition, partial cooperation, and cooperation for $n = 2$, $a = 0.9$, $\beta = 0.998$, $\delta = 0.99$ and $m = 2$.

In Figure 17, for a slightly larger value of β, the ranking of these indifference thresholds is reversed. In Figures 18 and 19 the range of relevant capacities is higher: $[0.795, 0.82]$.

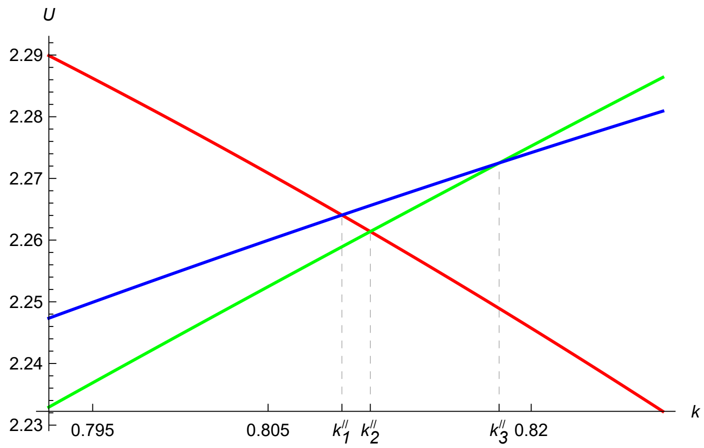

Figure 18: Payoffs for competition, partial cooperation, and cooperation for $n = 2$, $a = 0.9$, $\beta = 0.99$, $\delta = 0.99$ and $m = 3$.

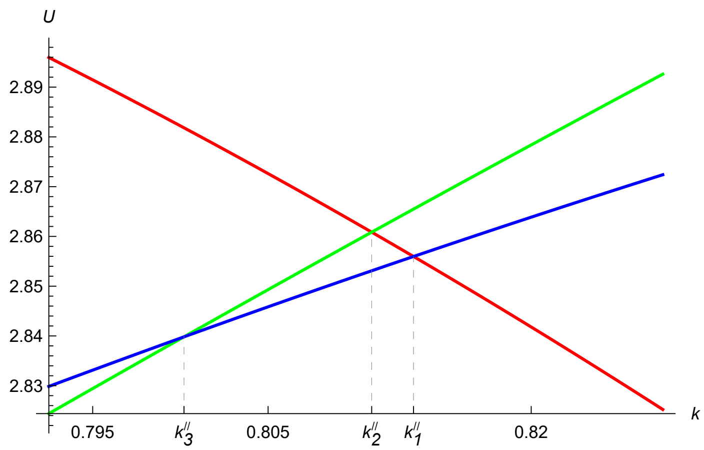

Figure 19: Payoffs for competition, partial cooperation, and cooperation for $n = 2$, $a = 0.9$, $\beta = 0.992$, $\delta = 0.99$ and $m = 3$.

We observe the same ranking and its reversal for lower values of the discount factor β (however still large). This also confirms the idea that partial cooperation is a behavior that is a compromise between quantity restriction and time delay that emerges most naturally when actions are inflexible.

B.2 Discounting

We know from the extensive literature on dynamic games that the existence of a cooperative equilibrium is sensitive to the discount factor. This is the dimension we want to explore here; to do so we fix $m = 2$. Again we stop short from exactly constructing equilibria; rather we compute the payoff functions from the three behaviors of interest as the discount factor varies from 0.6 to 0.999. The high values of the discount rate are meant to reflect the high frequency at which some commodities are traded – for example, electricity is traded every five minutes in Australia and every fifteen minutes in California. Commodities like grain may be traded less frequently. We display the graphs of these payoff functions in Figure 20.

First we observe that the ordering of the payoff functions does not change as the discount factor varies. Second, cooperation becomes increasingly attractive as the discount factor increases, which is quite intuitive. More precisely, as capacity k increases, first partial cooperation and then cooperation deliver the highest payoffs. So, while flexibility in trade ($m = 2, 3, ...$) mitigates the incentives to collude, these incentives still remain and, as one may expect, become stronger the more patient the players are – or the higher the frequency of trade.[33] We conjecture with confidence that equilibria can be constructed as we proceed in Sections 3.2 to 3.4. This gives us comfort in the robustness of the analysis we carry out and the results we lay out in Section 3.

Figure 20 shows that Partial Cooperation is not just a "funny" equilibrium. For low-to-intermediate values of the discount factor, the behavior partial cooperation generates the highest payoffs and payoff-dominates cooperation for all relevant values of the capacity k (see the first three panels). This further illustrates the trade-off between restraining quantities and the cost of delay; for low discount factors, or equivalently, infrequent trading, delay is simply too costly.

In line with our earlier discussion, the last panel of Figure 20 ($\beta = 0.999$) suggests partial cooperation is almost payoff-dominated (it is not completely here). Bearing in mind that $m = 2$ (a small number), when the discount factor becomes (very) large, the cost of delay becomes negligible. This illustrates that the compromise that partial cooperation presents, becomes irrelevant.

[33] In Sannikov and Skrzypacz (2007), cooperation breaks down as the frequency of play increases because of the inference problem; their game is one of incomplete information. There is no such inference to be made here.

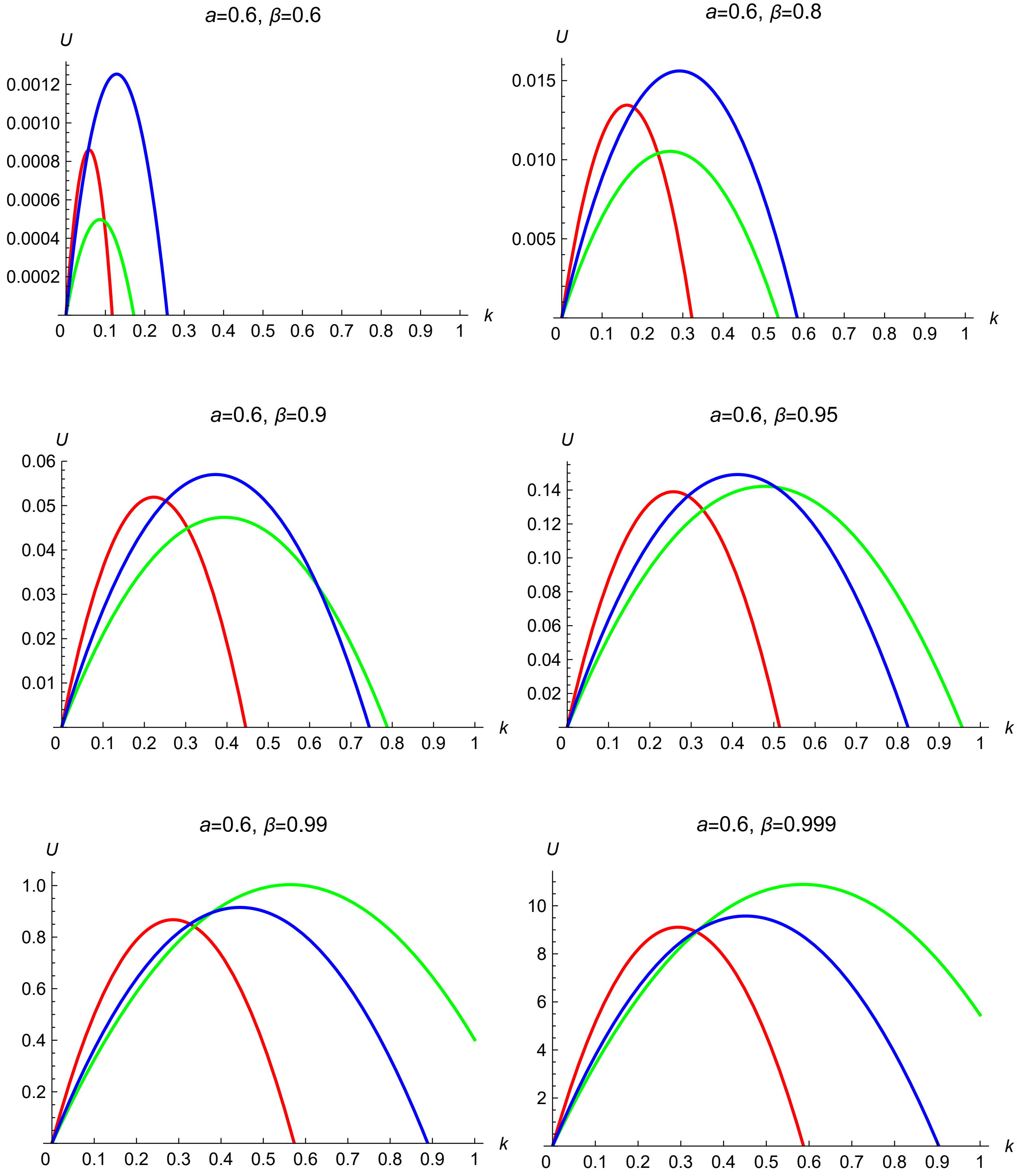

Figure 20: Payoffs for competition, partial cooperation, and cooperation for $n = 2,\ a = 0.6,\ \delta = 0.95$ and $m = 2$ as function of different values of β.

C Proofs

Proof of Proposition 2. We start in the order of the proof with the payoff formula for competition first. Since storage units do everything symmetrically, there are only two states of charge here, $(0,0)$ and (k,k). The system of equations (7) takes the following form:

$$\begin{cases} V(0,0) = \frac{\beta}{2}V(0,0) + \frac{1}{2}\left(-B_2 + \beta V(k,k)\right), \\ V(k,k) = \frac{\beta}{2}V(k,k) + \frac{1}{2}\left(A_2 + \beta V(0,0)\right). \end{cases}$$

Indeed, if both units are empty, they either remain empty in the case of a positive shock (with probability 1/2) or compete while purchasing energy (and spending B_2) under a negative shock. In the latter case, the new state is (k,k). Likewise, if both units are full, they either remain full in the case of a negative shock (with probability 1/2) or compete while selling energy (and gaining A_2) under a positive shock. Then they return to state $(0,0)$.

Using the notation from equation (7),

$$\mathbf{P} = \begin{pmatrix} -B_2 \\ A_2 \end{pmatrix}, \qquad \mathbf{Q} = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}.$$

Note that $\mathbf{Q}^2 = \mathbf{Q}$. For $\beta < 1$, we can find that

$$\mathbf{V} = \mathbf{P} + \sum_{i=1}^{t} \beta^i \mathbf{Q}^i \cdot \mathbf{P} + \beta^{t+1}\mathbf{Q}^{t+1} \cdot \mathbf{V} =$$

$$= \mathbf{P} + \frac{\beta(1-\beta^t)}{1-\beta} \cdot \mathbf{Q} \cdot \mathbf{P} + \beta^{t+1} \cdot \mathbf{Q} \cdot \mathbf{V} \xrightarrow[t\to\infty]{} \mathbf{P} + \frac{\beta}{1-\beta} \cdot \mathbf{Q} \cdot \mathbf{P} = \frac{1}{4(1-\beta)} \begin{pmatrix} \beta A_2 - (2-\beta)B_2 \\ (2-\beta)A_2 - \beta B_2 \end{pmatrix}.$$

The upper term $V(0,0)$ is exactly U_{com}.

Note now that the leader's payoff for ftl is the same except for substituting A_2 and B_2 for A_1 and B_1, respectively. It is implied by the fact that the leader acts exactly the same way as the units do under competition (always buying when empty under a negative shock and always selling when full under a positive shock), but it doesn't face any competition from the follower.

Next we turn to partial cooperation. The system of equations (7) takes the following form:

$$\begin{cases} V(0,0) = \frac{\beta}{2}V(0,0) + \frac{1}{4}\left(-B_1 + \beta V(k,0)\right) + \frac{1}{4}\beta V(0,k), \\ V(0,k) = \frac{\beta}{2}V(0,0) + \frac{1}{2}\left(-B_1 + \beta V(k,k)\right), \\ V(k,0) = \frac{\beta}{2}V(k,k) + \frac{1}{2}\left(A_1 + \beta V(0,0)\right), \\ V(k,k) = \frac{\beta}{2}V(k,k) + \frac{1}{2}\left(A_2 + \beta V(0,0)\right). \end{cases}$$

Indeed, if both units are empty, with probability $1/2$ they experience a negative shock and remain empty (with discounting). However, if the shock is positive (probability $1/2$ again), they flip the coin. With resulting probability $1/4$ the first unit buys energy alone, pays B_1, and we end up with state $(k,0)$ where this unit is full and the other one is still empty. With the same resulting probability $1/4$ the other unit purchases up to its full capacity, while the first unit stays idle. In this case, we move to state $(0,k)$.

If the first unit is empty and the second one is full (state $(0,k)$), the first unit either stays idle in the case of a positive shock (and we turn to $(0,0)$ afterwards because the second unit sells) or buys energy alone paying B_1 in the case of a negative shock (and the new state will be (k,k)). Likewise, in the case of state $(k,0)$ the first unit stays either idle in the case of negative shock (and the other unit buys shifting the state to (k,k)) or sells energy alone gaining A_1 in the case of positive shock (the new state is $(0,0)$).

Finally, when both units are full, they stay idle with probability $1/2$ in the case of negative shock and compete otherwise. In this case both units get A_2 for selling their energy and become empty (state $(0,0)$).

Again using the notation from equation (7),

$$\mathbf{P} = \begin{pmatrix} -B_1/4 \\ -B_1/2 \\ A_1/2 \\ A_2/2 \end{pmatrix}, \qquad \mathbf{Q} = \begin{pmatrix} 1/2 & 1/4 & 1/4 & 0 \\ 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \end{pmatrix}.$$

Note that for any $n > 2$

$$\mathbf{Q}^n = \mathbf{Q}^2 = \begin{pmatrix} 1/2 & 1/8 & 1/8 & 1/4 \\ 1/2 & 1/8 & 1/8 & 1/4 \\ 1/2 & 1/8 & 1/8 & 1/4 \\ 1/2 & 1/8 & 1/8 & 1/4 \end{pmatrix}.$$

Then we can find $\mathbf{V}(\mathbf{c^1}, \mathbf{c^2}) = \mathbf{V}$ for any $\beta < 1$.

$$\mathbf{V} = \mathbf{P} + \beta \cdot \mathbf{Q} \cdot \mathbf{P} + \sum_{i=2}^{t} \beta^i \cdot \mathbf{Q}^i \cdot \mathbf{P} + \beta^{t+1} \cdot \mathbf{Q}^{t+1} \cdot \mathbf{V} =$$

$$= \mathbf{P} + \beta \cdot \mathbf{Q} \cdot \mathbf{P} + \frac{\beta^2(1-\beta^{t-1})}{1-\beta} \cdot \mathbf{Q}^2 \cdot \mathbf{P} + \beta^{t+1} \cdot \mathbf{Q}^2 \cdot \mathbf{V} \xrightarrow[t\to\infty]{} \mathbf{P} + \beta \cdot \mathbf{Q} \cdot \mathbf{P} + \frac{\beta^2}{1-\beta} \cdot \mathbf{Q}^2 \cdot \mathbf{P} =$$

$$= \frac{1}{16(1-\beta)} \begin{pmatrix} 2\beta^2 A_2 + \beta(2-\beta)A_1 - (4-\beta^2)B_1 \\ 2\beta(2-\beta)A_2 + \beta^2 A_1 - (2-\beta)(4-\beta)B_1 \\ 2\beta(2-\beta)A_2 + (8-8\beta+\beta^2)A_1 - \beta(2+\beta)B_1 \\ 2(4-2\beta-\beta^2)A_2 + \beta^2 A_1 - \beta(2+\beta)B_1 \end{pmatrix}.$$

The uppermost term is exactly $V(0,0) = U_{pc}$.

The leader's and the follower's payoffs $\bar{U}_{fc}$ and U_{fc} for ftl+competition can be obtained the same way as U_{pc}. Indeed, for the leader we have:

$$\mathbf{P} = \begin{pmatrix} -B_1/2 \\ -B_1/2 \\ A_1/2 \\ A_2/2 \end{pmatrix}, \qquad \mathbf{Q} = \begin{pmatrix} 1/2 & 0 & 1/2 & 0 \\ 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \end{pmatrix},$$

$$\mathbf{Q}^n = \mathbf{Q}^2 = \begin{pmatrix} 1/2 & 0 & 1/4 & 1/4 \\ 1/2 & 0 & 1/4 & 1/4 \\ 1/2 & 0 & 1/4 & 1/4 \\ 1/2 & 0 & 1/4 & 1/4 \end{pmatrix}, \qquad n > 2.$$

Then

$$V(0,0) = \frac{1}{8(1-\beta)} \left(\beta^2 A_2 + (2-\beta)(-2B_1 + \beta A_1)\right),$$

which is exactly $\bar{U}_{fc}$.

For the follower in ftl+competition we have:

$$\mathbf{P} = \begin{pmatrix} 0 \\ -B_1/2 \\ A_1/2 \\ A_2/2 \end{pmatrix}, \qquad \mathbf{Q} = \begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \end{pmatrix},$$

$$\mathbf{Q}^n = \mathbf{Q}^2 = \begin{pmatrix} 1/2 & 1/4 & 0 & 1/4 \\ 1/2 & 1/4 & 0 & 1/4 \\ 1/2 & 1/4 & 0 & 1/4 \\ 1/2 & 1/4 & 0 & 1/4 \end{pmatrix}, \qquad n > 2.$$

Then

$$V(0,0) = \frac{\beta}{8(1-\beta)} \left(\beta A_2 - (2-\beta)B_1\right),$$

which is exactly U_{fc}.

Now let's obtain the payoff formula for cooperation. The vector $\mathbf{P}$ and matrix $\mathbf{Q}$ from (7) take the following form:

$$\mathbf{P} = \begin{pmatrix} -B_1/4 \\ -B_1/2 \\ A_1/2 \\ A_1/4 \end{pmatrix}, \qquad \mathbf{Q} = \begin{pmatrix} 1/2 & 1/4 & 1/4 & 0 \\ 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \\ 0 & 1/4 & 1/4 & 1/2 \end{pmatrix}.$$

To calculate power t of matrix $\mathbf{Q}$, we find the Jordan decomposition $\mathbf{Q} = \mathbf{T} \cdot \mathbf{J} \cdot \mathbf{T}^{-1}$ of $\mathbf{Q}$. Here,

$$\mathbf{J} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad \mathbf{T} = \begin{pmatrix} 1 & 1 & -1 & 0 \\ 1 & -2 & 0 & -1 \\ 1 & -2 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix},$$

so $\mathbf{Q}^t = \mathbf{T} \cdot \mathbf{J}^t \cdot \mathbf{T}^{-1}$. For $\beta < 1$,

$$\mathbf{V} = \mathbf{P} + \sum_{i=1}^{\infty} \beta^i \mathbf{Q}^i \cdot \mathbf{P} = \mathbf{P} + \mathbf{T} \cdot \begin{pmatrix} \frac{\beta}{1-\beta} & 0 & 0 & 0 \\ 0 & -\frac{\beta}{2+\beta} & 0 & 0 \\ 0 & 0 & \frac{\beta}{2-\beta} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \mathbf{T}^{-1} \cdot \mathbf{P} =$$

$$= \frac{1}{4(1-\beta)(2-\beta)(2+\beta)} \begin{pmatrix} 2\left(\beta A_1 - (2-\beta^2)B_1\right) \\ (2-\beta)\left(\beta(1+\beta)A_1 - (4-\beta-\beta^2)B_1\right) \\ (2-\beta)\left((4-\beta-\beta^2)A_1 - \beta(1+\beta)B_1\right) \\ 2\left((2-\beta^2)A_1 - \beta B_1\right) \end{pmatrix}.$$

The uppermost term is exactly $V(0,0) = U_{col}$.

Finally, the follower's payoff U_{ftl} for ftl may be obtained the same way as U_{col}. Vector $\mathbf{P}$

and matrix $\mathbf{Q}$ from (7) take the following form:

$$\mathbf{P} = \begin{pmatrix} 0 \\ -B_1/2 \\ A_1/2 \\ 0 \end{pmatrix}, \qquad \mathbf{Q} = \begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \\ 0 & 0 & 1/2 & 1/2 \end{pmatrix}.$$

Then matrices $\mathbf{J}$ and $\mathbf{T}$ are the following:

$$\mathbf{J} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad \mathbf{T} = \begin{pmatrix} 1 & 1 & -1 & -1 \\ 1 & -2 & 0 & 1 \\ 1 & -2 & 0 & -1 \\ 1 & 1 & 1 & 1 \end{pmatrix}.$$

For $\beta < 1$, we have

$$V(0,0) = \frac{\beta}{4(1-\beta)(2-\beta)(2+\beta)} \left(\beta^2 A_1 - (4 - 2\beta - \beta^2)B_1\right),$$

which is exactly U_{ftl}. □

Proof of Lemma 3. The real numbers k_1, k_2, k_3 are nonzero roots of equations $U_{com}(k) = U_{pc}(k)$, $U_{com}(k) = U_{col}(k)$, and $U_{col}(k) = U_{pc}(k)$, respectively. Solving them, we obtain:

$$k_1 = \frac{-(2-\beta)(1-a) + \beta(1+a)\delta}{6 - \beta + 3\beta\delta^2} > 0, \qquad k_2 = \frac{-(4 - 4\beta + \beta^3)(1-a) + \beta(2-\beta^2)(1+a)\delta}{2\left(6 - 4\beta - \beta^2 + \beta^3 + \beta(3-\beta^2)\delta^2\right)},$$
$$k_3 = \frac{-\beta^2(1-a) + (4 - 2\beta - \beta^2)(1+a)\delta}{\beta^2 + (12 - 2\beta - 3\beta^2)\delta^2}.$$

Then, using (8), we get:

$$k_2 - k_1 = \frac{\beta^2\Big((-\beta^2 + (6-4\beta-\beta^2)\delta^2)(1-a) + (6-4\beta-\beta^2-\beta^2\delta^2)(1+a)\delta\Big)}{2\big(6-4\beta-\beta^2+\beta^3+\beta(3-\beta^2)\delta^2\big)\big(6-\beta+3\beta\delta^2\big)} >$$

$$> \frac{\beta^2\left((-\beta^2 + (6-4\beta-\beta^2)\delta^2)(1-a) + (6-4\beta-\beta^2-\beta^2\delta^2)(1+a)\frac{2-\beta}{\beta}\frac{1-a}{1+a}\right)}{2\big(6-4\beta-\beta^2+\beta^3+\beta(3-\beta^2)\delta^2\big)\big(6-\beta+3\beta\delta^2\big)} =$$

$$= \frac{\beta(1-\beta)(1-a)}{6-4\beta-\beta^2+\beta^3+\beta(3-\beta^2)\delta^2} > 0,$$

$$k_3 - k_2 = \frac{(2+\beta)(2-\beta)^2\Big((-\beta^2 + (6-4\beta-\beta^2)\delta^2)(1-a) + (6-4\beta-\beta^2-\beta^2\delta^2)(1+a)\delta\Big)}{2\big(\beta^2+(12-2\beta-3\beta^2)\delta^2\big)\big(6-4\beta-\beta^2+\beta^3+\beta(3-\beta^2)\delta^2\big)} >$$

$$> \frac{(2+\beta)(2-\beta)^2\left((-\beta^2 + (6-4\beta-\beta^2)\delta^2)(1-a) + (6-4\beta-\beta^2-\beta^2\delta^2)(1+a)\frac{2-\beta}{\beta}\frac{1-a}{1+a}\right)}{2\big(\beta^2+(12-2\beta-3\beta^2)\delta^2\big)\big(6-4\beta-\beta^2+\beta^3+\beta(3-\beta^2)\delta^2\big)} =$$

$$= \frac{(2+\beta)(1-\beta)(2-\beta)^2\big(6-\beta+3\beta\delta^2\big)(1-a)}{\beta\big(\beta^2+(12-2\beta-3\beta^2)\delta^2\big)\big(6-4\beta-\beta^2+\beta^3+\beta(3-\beta^2)\delta^2\big)} > 0,$$

□

Proof of Proposition 5. Before fully engaging into the proof we note that the critical value of capacity is

$$\overline{\kappa}_r = \frac{-(2-\beta)(1-a) + \beta(1+a)\delta}{4-\beta+2\beta\delta^2} > 0.$$

Now, Condition (8) guarantees that the competitive payoff is positive for some $k > 0$. Indeed, the second nonzero root k_r of the equation $U_{com} = 0$ is equal to

$$k_r = \frac{-(2-\beta)(1-a) + \beta(1+a)\delta}{2\left(2-\beta+\beta\delta^2\right)},$$

which is positive if and only if (8) holds.

Since payoffs are expressed as a function of the capacity k, we construct equilibria for ranges of the value of that capacity. So whether a behavior is an equilibrium is determined in terms of that capacity. Deviations from competition may be only in the form of not competing but allowing the opponent to buy or sell first alone. Consider different deviations:

- Both players are empty (so $c_t^1 = c_t^2 = 0$) and ready to buy under the negative shock, but one player deviates by **not buying** competitively just for **one period**. They continue competing after that. This deviation is profitable when

$$k > x_b = \frac{-(2-\beta)(1-a) + \beta(1+a)\delta}{4 - \beta + 2\beta\delta^2}.$$

Indeed, the payoff U_0 of each player if no deviation is observed (conditional on a negative shock and on both storage units being empty) is equal to

$$U_0 = -B_2 + \beta V(k,k) = -B_2 + \frac{\beta}{2}A_2 + \frac{\beta^2}{4(1-\beta)}(A_2 - B_2),$$

The payoff U_d of a deviating player is

$$U_d = \beta\left(\frac{1}{2}\left(-B_1 + \beta V(k,k)\right) + \frac{1}{2}\beta V(0,0)\right) = \beta\left(-\frac{B_1}{2} + \frac{\beta}{4(1-\beta)}\left(A_2 - B_2\right)\right).$$

Here, $V(k,k)$ and $V(0,0) = U_{com}$ are continuation values for competition when both players are full (state $(c_t^1 = k, c_t^2 = k)$) and empty (state $(c_t^1 = 0, c_t^2 = 0)$), respectively (see Proof of Prop. 2). The deviation is profitable if $U_0 < U_d$, which is equivalent to $k > x_b$.

The threshold x_b stays constant in the case of deviation for not buying competitively for two, three, four, etc. periods of time up to infinity. The latter means that the deviating player completely switches to the behavior ftl+competition. Thus, x_b is the point where the red competition curve intersects with the orange ftl+competition line (see Fig. 4).

- Both players are full ($c_t^1 = k, c_t^2 = k$) and ready to sell under the positive shock, but one player deviates by **not selling** competitively just for **one period**. They continue competing after that. This deviation is profitable when

$$k > x_s = \frac{-\beta(1-a) + (2-\beta)(1+a)\delta}{2\beta + (4-\beta)\delta^2}.$$

Indeed, the payoff U_k of each player if no deviation is observed (conditional on a positive shock and on both storage units being full) is equal to

$$U_k = A_2 + \beta V(0,0) = A_2 - \frac{\beta}{2}B_2 + \frac{\beta^2}{4(1-\beta)}(A_2 - B_2),$$

The payoff $U_{d'}$ of a deviating player is

$$U_{d'} = \beta\left(\frac{1}{2}\left(A_1 + \beta V(0,0)\right) + \frac{1}{2}\beta V(k,k)\right) = \beta\left(\frac{A_1}{2} + \frac{\beta}{4(1-\beta)}\left(A_2 - B_2\right)\right).$$

The deviation is profitable if $U_k < U_{d'}$, which is equivalent to $k > x_s$.

Again, the threshold x_s remains constant for the *same* deviation for two, three, four, etc. periods of time up to infinity. The latter means that the deviating player completely switches to competition+ftl. Now we prove that $x_b < x_s$. Indeed, using (8), we get

$$\begin{aligned} x_s - x_b &= \frac{\left(-\beta^2 + (8-6\beta-\beta^2)\delta^2\right)(1-a) + \left(8-6\beta-\beta^2-\beta^2\delta^2\right)(1+a)\delta}{\left(2\beta+(4-\beta)\delta^2\right)\left(4-\beta+2\beta\delta^2\right)} \\ &> \frac{\left(-\beta^2 + (8-6\beta-\beta^2)\delta^2\right)(1-a) + \left(8-6\beta-\beta^2-\beta^2\delta^2\right)(1+a)\frac{2-\beta}{\beta}\frac{1-a}{1+a}}{\left(2\beta+(4-\beta)\delta^2\right)\left(4-\beta+2\beta\delta^2\right)} \\ &= \frac{4(1-\beta)(1-a)}{\beta\left(2\beta+(4-\beta)\delta^2\right)} > 0. \end{aligned}$$

There are potentially many more deviations, but now that x_b does not change by adding "buying" deviations, the only way the pivotal value of k can change is by combining instances of buying and selling. Likewise with x_s and selling.

For k in the interval $[x_b, x_s]$, all deviations ("letting go") when buying are profitable, but not when selling. Starting from x_b, adding "selling" deviations moves the pivotal point to the right towards x_s – that is, the pivotal $k > x_b$ so a deviation is more demanding in that it requires a larger capacity to be profitable. Conversely, starting from x_s, adding "buying" deviations moves the pivotal point to the left towards x_b – that is, the pivotal $k < x_s$. For example, consider pivotal values for the following deviations:

– x_{bs} – deviation to one instance of buying and maximum one instance of selling starting from $(c_t^1 = 0, c_t^2 = 0)$ and facing the negative shock;

– x_{sb} – the converse, however starting from $(c_t^1 = k, c_t^2 = k)$ and facing the positive shock;

– $x_{b\infty s\infty}$ – deviation forever starting from $(c_t^1 = 0, c_t^2 = 0)$ and facing the negative shock (switch to ftl strategy);

– $x_{s\infty b\infty}$ – deviation forever starting from $(c_t^1 = k, c_t^2 = k)$ and facing the positive shock;

– x_{b2s} – deviation to instances of buying and at most one of selling starting from $(c_t^1 = 0, c_t^2 = 0)$ and facing the negative shock;

– x_{bs2} – deviation to one instance of buying and at most two of selling starting from $(c_t^1 = 0, c_t^2 = 0)$ and facing the negative shock;

– x_{b2s2} – deviation to at most two instances of buying and at most two of selling starting from $(c_t^1 = 0, c_t^2 = 0)$ and facing the negative shock.

Then we can rank the respective indifference thresholds in the following order:

$$x_b \leqslant x_{b2s} \leqslant x_{bs} \leqslant x_{b2s2} \leqslant x_{bs2} \leqslant x_{b\infty s\infty} \leqslant x_{s\infty b\infty} \leqslant x_{sb} \leqslant x_s.$$

We remark that

- $x_{b\infty s\infty} = x_{bs\infty} = x_{b2s\infty}$ with the same idea as for $x_b = x_{b2} = x_{b3} = \ldots = x_{b\infty}$. That is, fixing one behavior renders the thresholds constant.

- *All* pivotal values for all the deviations that start from $(c_t^1 = 0, c_t^2 = 0)$ are lower than the pivotal values for all the deviations that start from $(c_t^1 = k, c_t^2 = k)$. Moreover, these intervals do not intersect except for the case $\beta = 1$. Then $x_{b\infty s\infty} = x_{s\infty b\infty}$, otherwise $x_{b\infty s\infty} < x_{s\infty b\infty}$.

- The effect of additional instances of buying and selling cancel each other if and only if $\beta = 1$ and $\delta = 1$. In this case, not only $x_{bs} = x_{b2s2}$ but also $x_b = x_s$; the one pivotal value is the same for all candidate deviations.

Finally, we prove that $x_b \leqslant k_r$ to make sure that at least (some of) our candidate deviations are not trivial.

$$k_r - x_b = \frac{\beta\left(-(2-\beta)(1-a)+\beta(1+a)\delta\right)}{2\left(2-\beta+\beta\delta^2\right)\left(4-\beta+2\beta\delta^2\right)} > 0.$$

Putting $\bar{\kappa}_r = x_b$ concludes the proof. The entire picture can be seen in Fig. 4. □

Proof of Proposition 6. Here the threshold value for k are

$$\begin{aligned}
\underline{\kappa}_b &= \frac{(8-8\beta+\beta^2)(-(2-\beta)(1-a)+\beta(1+a)\delta)}{32-40\beta+14\beta^2-\beta^3+(16\beta-16\beta^2+3\beta^3)\delta^2},\\
\overline{\kappa}_b &= \min\left\{\frac{-\beta(2+\beta)(1-a)+(8-4\beta-\beta^2)(1+a)\delta}{\beta(2+\beta)+(16-4\beta-3\beta^2)\delta^2}, \frac{(4-\beta)(-(2-\beta)(1-a)+\beta(1+a)\delta)}{8-6\beta+\beta^2+\beta(8-3\beta)\delta^2}\right\},
\end{aligned}$$

rewritten as

$$\begin{aligned}
\underline{\kappa}_b &= \frac{C}{4-\beta+2\beta\delta^2+\beta_1\left(2+\beta\delta^2\right)},\\
\overline{\kappa}_b = \overline{\kappa}_b &= \min\left\{\frac{-\beta_2(1-a)+(1+a)\delta}{\beta_2+(1+2(2-\beta)\beta_2/\beta)\delta^2}, \frac{C}{2-\beta+2\beta\delta^2-\frac{\beta^2}{4-\beta}\delta^2}\right\},
\end{aligned}$$

on the equilibrium path, and

$$\begin{aligned}
k_4 &= \frac{-\beta^2(1-a)+(4-2\beta-\beta^2)(1+a)\delta}{\beta^2+2\left(4-\beta-\beta^2\right)\delta^2}\\
k_o &= \frac{-(2-\beta)(1-a)+\beta(1+a)\delta}{2-\beta+2\beta\delta^2}\\
k_p &= \frac{-(4-2\beta-\beta^2)(1-a)+\beta^2(1+a)\delta}{4-2\beta-\beta^2+\beta^2\delta^2}
\end{aligned}$$

off the equilibrium path.

Again Condition (8) guarantees that the partially cooperative payoff is positive for some $k > 0$. Indeed, equation $U_{pc} = 0$ has two roots $k = 0$ and

$$k_b = \frac{(2-\beta)\left(-(2-\beta)(1-a)+\beta(1+a)\delta\right)}{4-\beta^2+\beta(2+3\beta)\delta^2},$$

which is positive if and only if (8) holds.

Consider first the deviation from the equilibrium Partial Cooperation (PC) to competition. Whoever has to remain idle (and empty) after the coin flip deviates by competitively purchasing; that is, simultaneously with the first-mover in the equilibrium. The harshest punishment consists in competing forever, and this punishment starts immediately. In this case, the deviating player faces competition payoff U_0 that starts from zero level and is conditional on the negative shock having already occurred:

$$U_0 = -B_2 + \beta V_{com}(k,k) = -B_2 + \frac{\beta}{4(1-\beta)}\big((2-\beta)A_2 - \beta B_2\big)$$

(where $V_{com}(k,k)$ is a payoff for competition when the profile is (k,k) – both players are full.) In Fig. 5, U_0 is drawn as the dashed red line. Instead, the equilibrium play delivers

$$U_- = \beta V_{pc}(0,k) = \frac{\beta}{16(1-\beta)}\left(2\beta(2-\beta)A_2 + \beta^2 A_1 - (2-\beta)(4-\beta)B_1\right),$$

where $V_{pc}(0,k)$ is a payoff for PC when the state profile is $(0,k)$ – the unit is empty unit and its opponent is full (see Proof of Prop. 2). In Fig. 5, U_- is drawn as the thin dashed blue line. This payoff differs from the PC line U_{pc}, because it is conditional on the already occurred negative shock and unfortunate outcome of the coin. The deviation is profitable if $U_0 > U_-$. Solving this inequality with respect to k, we obtain the condition

$$k < \underline{\kappa}_b = \frac{(8-8\beta+\beta^2)(-(2-\beta)(1-a)+\beta(1+a)\delta)}{32-40\beta+14\beta^2-\beta^3+\beta(16-16\beta+3\beta^2)\delta^2},$$

with $\underline{\kappa}_b > 0$ as long as (8) holds. When $k < \underline{\kappa}_b$, PC is not an equilibrium: the player who has to wait her turn prefers to deviate to competition. When $k \geqslant \underline{\kappa}_b$ this particular deviation is not profitable. We still need to calculate all the Nash equilibria in that subgame off the equilibrium path, which we turn to later.

The payoff U_- plays an important role in finding the Partial Cooperation equilibrium. Even if the *ex ante* payoff U_{pc} is positive, a storage unit may find unprofitable to participate (under this equilibrium) after an adverse coin toss; that is, $U_- < 0$. Since U_- is a quadratic function of k with two roots $k = 0$ and some other nonzero root

$$k = \kappa_2 = \frac{(4-\beta)(-(2-\beta)(1-a) + \beta(1+a)\delta)}{8 - 6\beta + \beta^2 + \beta(8-3\beta)\delta^2},$$

inequality $U_- > 0$ is equivalent to $k < \kappa_2$. Note that $\kappa_2 < k_b$. For larger capacities, a deviation to competition may be either dominated or even no viable (i.e. negative payoff). Then another deviation consists in letting the opponent sell first. The deviating unit continues to play cooperatively when buying but reverts to ftl when selling. To compute a payoff from this deviation, we use the same tools as we did in Prop. 2. Namely, for cooperation+ftl, we have:

$$\mathbf{P} = \begin{pmatrix} -B_1/4 \\ -B_1/2 \\ A_1/2 \\ 0 \end{pmatrix}, \qquad \mathbf{Q} = \begin{pmatrix} 1/2 & 1/4 & 1/4 & 0 \\ 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \\ 0 & 0 & 1/2 & 1/2 \end{pmatrix},$$

$$\mathbf{J} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad \mathbf{T} = \begin{pmatrix} 1 & 1 & -1 & -1 \\ 1 & -2 & 0 & 3 \\ 1 & -2 & 0 & -1 \\ 1 & 1 & 1 & 1 \end{pmatrix}.$$

Then, for $\beta < 1$ the payoff U_{colf} for cooperation+ftl is the following:

$$U_{colf} = V_{col+ftl}(0,0) = \frac{\beta\left(4 - 2\beta + \beta^2\right) A_1 - \left(8 - 4\beta^2 - \beta^3\right) B_1}{8(1-\beta)(2-\beta)(2+\beta)},$$

and this deviation is profitable when $U_{colf} > U_{pc}$. Note that we can calculate payoffs starting from state $(c_t^1 = 0, c_t^2 = 0)$ rather than from state $(c_{t'}^1 = k, c_{t'}^2 = k)$ because these strategies are the same between those two states. We get

$$U_{colf} > U_{pc} \quad \Leftrightarrow \quad k > \kappa_1 = \frac{-\beta(2+\beta)(1-a) + (8-4\beta-\beta^2)(1+a)\delta}{\beta(2+\beta) + (16-4\beta-3\beta^2)\delta^2}.$$

The minimum of κ_1 and κ_2 gives us the value of $\overline{\kappa}_b$, above which ($k > \overline{\kappa}_b$) the PC equilibrium is not sustainable anymore. Hence, PC is an equilibrium for $k \in [\underline{\kappa}_b, \overline{\kappa}_b]$. Our findings are represented in Figure 5. There, κ_1 is the intersection of cooperation+ftl and PC payoff lines (dashed cyan line and solid blue line, respectively), and κ_2 is the point where the dashed blue line U_- turns from positive to negative. On the graph, $\kappa_1 < \kappa_2$, and so $\overline{\kappa}_b = \kappa_1$.

Since $\kappa_2 < k_b$, we conclude that $\overline{\kappa}_b < k_b$. Next we prove that $\underline{\kappa}_b < \overline{\kappa}_b$. We show $\underline{\kappa}_b < \overline{\kappa}_r$ first and then that $\overline{\kappa}_r < \overline{\kappa}_b$. When (8) holds,

$$\begin{aligned}
\overline{\kappa}_r - \underline{\kappa}_b &= \frac{\beta^2\left(2+\beta\delta^2\right)\left(-(2-\beta)(1-a)+\beta(1+a)\delta\right)}{\left(4-\beta+2\beta\delta^2\right)\left(32-40\beta+14\beta^2-\beta^3+\beta(16-16\beta+3\beta^2)\delta^2\right)} > 0,\\
\kappa_1 - \overline{\kappa}_r &= \frac{\left(-2\beta(2+\beta)+\left(32-24\beta-6\beta^2+\beta^3\right)\delta^2\right)(1-a)+\left(2\left(16-12\beta-\beta^2\right)-\beta^2(4-\beta)\delta^2\right)(1+a)\delta}{\left(\beta(2+\beta)+\left(16-4\beta-3\beta^2\right)\delta^2\right)\left(4-\beta+2\beta\delta^2\right)}\\
&> \frac{\left(-2\beta(2+\beta)+\left(32-24\beta-6\beta^2+\beta^3\right)\delta^2\right)(1-a)+\left(2\left(16-12\beta-\beta^2\right)-\beta^2(4-\beta)\delta^2\right)(1+a)\frac{2-\beta}{\beta}\frac{1-a}{1+a}}{\left(\beta(2+\beta)+\left(16-4\beta-3\beta^2\right)\delta^2\right)\left(4-\beta+2\beta\delta^2\right)}\\
&= \frac{16(1-\beta)(1-a)}{\beta\left(\beta(2+\beta)+\left(16-4\beta-3\beta^2\right)\delta^2\right)} > 0,\\
\kappa_2 - \overline{\kappa}_r &= \frac{\left(8-2\beta+\beta^2\delta^2\right)\left(-(2-\beta)(1-a)+\beta(1+a)\delta\right)}{\left(8-6\beta+\beta^2+\beta(8-3\beta)\delta^2\right)\left(4-\beta+2\beta\delta^2\right)} > 0.
\end{aligned}$$

Hence, as long as (8) holds, $0 < \underline{\kappa}_b < \overline{\kappa}_b < k_b$; so the PC equilibrium *always* exists.

Of course, off the equilibrium path competition is not always an equilibrium for all $k \in (\underline{\kappa}_b, \overline{\kappa}_b)$ – as shown in Proposition 5. That is, upon observing a deviation (off the equilibrium path), players enter the punishment equilibrium; what is an equilibrium depends on the value of k. It is still Competition for $\underline{\kappa}_b < k \leqslant \overline{\kappa}_r$, but for $k \geqslant \overline{\kappa}_r$ the best response varies from ftl+competition to ftl to leaving the market immediately after selling energy. The next paragraphs conclude the proof with finding the conditions on k for SPNE out of equilibrium path.

Solving equation $U_{fc} = U_{ftl}$ with respect to k, we obtain the pivotal value

$$k_4 = \frac{-\beta^2(1-a) + (4-2\beta-\beta^2)(1+a)\delta}{\beta^2 + 2\,(4-\beta-\beta^2)\,\delta^2},$$

above which the ftl behavior becomes more profitable than ftl+competition. Note that this value is valid not only for state $c_t^1 = c_t^2 = 0$ but also for state $c_t^1 = c_t^2 = k$.

Next, solving for $U_{fc} = 0$ with respect to k, we obtain the pivotal value

$$k_o = \frac{-(2-\beta)(1-a) + \beta(1+a)\delta}{2-\beta+2\beta\delta^2},$$

above which trading zero quantities becomes more profitable than the ftl+competition behavior for state $c_t^1 = c_t^2 = 0$. In the case of state $c_t^1 = c_t^2 = k$, it means that selling k and quitting the market right afterwards is better than playing ftl+competition. For example, Figure 4 reflects the case $k_4 < k_o$.

It can be proven that $\min\{k_o, k_4\} \leqslant \overline{\kappa}_b$, whence we conclude that the behavior ftl+competition is always an equilibrium strategy for a deviating player if and only if $\overline{\kappa}_r \leqslant k \leqslant \min\{k_o, k_4\}$. The case $k_o < k_4$ does not allow any room for the ftl behavior because it becomes dominated by either ftl+competition or trading zero. As for $k_o \geqslant k_4$, ftl is the best response of a deviating unit for any k such that $k_4 < k < \min\{k_p, \overline{\kappa}_b\}$, where k_p is the nonzero root of equation $U_{ftl} = 0$:

$$k_p = \frac{-(4-2\beta-\beta^2)(1-a) + \beta^2(1+a)\delta}{4-2\beta-\beta^2+\beta^2\delta^2}.$$

k_p is a pivotal value starting from which never trading positive quantities (so that $c_t^1 = c_t^2 = 0$) or quitting (trading zero) right after selling all the energy ($c_t^1 = c_t^2 = k$) becomes more profitable than ftl. Then the deviating unit quits the market right after selling energy (either competitively or by "letting go") for any $k_o < k < \overline{\kappa}_b$ in the case $k_o < k_4$ and for any $k_p < k < \overline{\kappa}_b$ in the case $k_o \geqslant k_4$ and $k_p < \overline{\kappa}_b$. □

Proof of Proposition 7. The coefficients G_1 to G_4 of Condition (9) read

$$G_1(\beta) = -16 + 28\beta - 8\beta^2 - \beta^3 - \beta^4, \qquad G_2(\beta) = \beta(1+\beta)(2-\beta)^2,$$
$$G_3(\beta) = \beta(2-\beta)(4-\beta-\beta^2), \qquad G_4(\beta) = 32 - 48\beta + 14\beta^2 + 5\beta^3 - \beta^4.$$

and the capacity threshold values used in the Proposition are

$$\underline{\kappa}_g = \frac{-\beta(4-\beta-2\beta^2)(1-a) + (8-8\beta-\beta^2+2\beta^3)(1+a)\delta}{\beta(8-\beta-3\beta^2) + (16-12\beta-3\beta^2+3\beta^3)\delta^2},$$
$$\overline{\kappa}_g = \frac{-(4-\beta-\beta^2)(1-a) + \beta(1+\beta)(1+a)\delta}{4-\beta-\beta^2+\beta(1+\beta)\delta^2},$$

We begin by ensuring that the cooperative payoff U_{col} is positive under condition (9). Equation $U_{col} = 0$ has two roots, $k = 0$ and

$$k_g = \frac{-(1-a)(2-\beta^2) + \beta(1+a)\delta}{2-\beta^2+\beta\delta^2}.$$

Inequality $U_{col} > 0$ is satisfied if and only if $k_g > 0$, which is equivalent to

$$\frac{1-a}{1+a} < \frac{\beta\delta}{2-\beta^2}. \tag{10}$$

It can be proven that condition (9) is stronger than (10) for any values of a, β, and δ.

Consider the deviations from Cooperation to competition, of which there can be multiple variations. That is, whoever has to remain idle after the coin flip, may deviate by either (competitively) buying or selling. The harshest punishment in this case is to compete forever, and it starts immediately. Consider first the case ($c_t^1 = 0, c_t^2 = 0$) – both units are empty. Then the deviating player faces competition payoff U_0 that starts from $(0,0)$, and is conditional on the negative shock having already occurred:

$$U_0 = -B_2 + \beta V_{com}(k,k) = -B_2 + \frac{\beta}{4(1-\beta)}\big((2-\beta)A_2 - \beta B_2\big),$$

where $V_{com}(k,k)$ is the continuation payoff for competition in state (k,k) – both players are full. The alternative is not to deviate from the equilibrium prescription and wait to buy, which delivers

$$U_{-}^{c}=\beta V_{col}(0,k)=\frac{\beta}{4\left(2-\beta-\beta^{2}\right)}\left(\beta(1+\beta)A_{1}-\left(4-\beta-\beta^{2}\right)B_{1}\right),$$

where $V_{col}(0,k)$ is the continuation payoff from cooperation for an empty unit when its opponent is full – state $(0,k)$ (see Proof of Prop. 2). The deviation is unprofitable if $U_0 < U_{-}^{c}$. Solving this inequality with respect to k, we obtain

$$k<\kappa_{0}=\frac{-\left(8-8\beta-\beta^{2}+2\beta^{3}\right)(1-a)+\beta\left(4-\beta-2\beta^{2}\right)(1+a)\delta}{16-12\beta-3\beta^{2}+3\beta^{3}+\beta(8-\beta-2\beta^{2})\delta^{2}}.$$

Suppose the profile of charge is $(c_t^1 = k, c_t^2 = k)$. Deviating yields the competition payoff U_k conditional on the positive shock:

$$U_{k}=A_{2}+\beta V_{com}(0,0)=A_{2}+\frac{\beta}{4(1-\beta)}\big(\beta A_{2}-(2-\beta)B_{2}\big),$$

where $V_{com}(0,0)$ is a payoff for competition when both players are empty (state $(c_t^1 = 0, c_t^2 = 0)$). If playing equilibrium instead, one receives

$$U_{+}^{c}=\beta V_{col}(k,0)=\frac{\beta}{4\left(2-\beta-\beta^{2}\right)}\left(\left(4-\beta-\beta^{2}\right)A_{1}-\beta(1+\beta)B_{1}\right),$$

where $V_{col}(k,0)$ is a payoff for cooperation when the state is $(c_t^1 = k, c_t^2 = 0)$ (see Proof of Prop. 2). The deviation is unprofitable if $U_k < U_{+}^{c}$. Solving this inequality with respect to k, we obtain

$$k<\kappa_{k}=\frac{-\beta\left(4-\beta-2\beta^{2}\right)(1-a)+\left(8-8\beta-\beta^{2}+2\beta^{3}\right)(1+a)\delta}{\beta(8-\beta-2\beta^{2})+\left(16-12\beta-3\beta^{2}+3\beta^{3}\right)\delta^{2}}.$$

Next, compute

$$\kappa_{k}-\kappa_{0}=\frac{2(1-\beta)(2+\beta)K(a,\beta,\delta)}{\left(\beta(8-\beta-2\beta^{2})+\left(16-12\beta-3\beta^{2}+3\beta^{3}\right)\delta^{2}\right)\left(16-12\beta-3\beta^{2}+3\beta^{3}+\beta(8-\beta-2\beta^{2})\delta^{2}\right)},$$

where

$$
\begin{aligned}
K(a,\beta,\delta) &= \left(-(2-\beta)\beta^2 + \left(32-40\beta+2\beta^2+7\beta^3\right)\delta^2\right)(1-a) \\
&+ \left(32-40\beta+2\beta^2+7\beta^3-(2-\beta)\beta^2\delta^2\right)(1+a)\delta.
\end{aligned}
$$

Using (10), we obtain

$$
\begin{aligned}
K(a,\beta,\delta) &> \left(-(2-\beta)\beta^2 + \left(32-40\beta+2\beta^2+7\beta^3\right)\delta^2\right)(1-a) \\
&+ \left(32-40\beta+2\beta^2+7\beta^3-(2-\beta)\beta^2\delta^2\right)(1+a)\frac{2-\beta^2}{\beta}\frac{1-a}{1+a} \\
&= \frac{(1-\beta)(1-a)}{\beta}\left(64-16\beta-44\beta^2+8\beta^3+7\beta^4+\beta\left(32-12\beta-8\beta^2+\beta^3\right)\delta^2\right) \\
&> 0.
\end{aligned}
$$

Thus $\kappa_0 < \kappa_k$ if (10) holds, and any deviation to competition is unprofitable only if $k > \kappa_k$. Putting $\underline{\kappa}_g = \kappa_k$ finishes the first part of the proof.

In Fig. 6, U_k is drawn as the thin dashed red line, and U_+^c is the highest of the two thin dashed green arches. The intersection of these lines is exactly $\underline{\kappa}_g$.

Even if (10) holds, it may become unprofitable to stick to cooperation if moving second, that is, when $U_-^c < 0$. Since U_-^c is a quadratic function of k with two roots $k = 0$ and some other nonzero root

$$
\overline{\kappa}_g = \frac{-(4-\beta-\beta^2)(1-a)+\beta(1+\beta)(1+a)\delta}{4-\beta-\beta^2+\beta(1+\beta)\delta^2},
$$

inequality $U_-^c > 0$ is equivalent to $k < \overline{\kappa}_g$. Note that $\overline{\kappa}_g < k_g$.

In Fig. 6, U_-^c is drawn as the lowest of the two thin dashed green arches, and intersects the horizontal axe at $\overline{\kappa}_g$. In the small interval $(\overline{\kappa}_g, k_g)$, cooperation still delivers a positive payoff *on average*, but the second mover stops immediately; that is, cooperation is not an equilibrium. Hence, the Cooperation equilibrium exists in the interval $(\underline{\kappa}_g, \overline{\kappa}_g)$, if this interval exists. Indeed,

$$\overline{\kappa}_g - \underline{\kappa}_g = \frac{(2+\beta)\left(-\left(G_3(\beta)+G_4(\beta)\delta^2\right)(1-a) + \left(G_1(\beta)+G_2(\beta)\delta^2\right)(1+a)\delta\right)}{\left(4-\beta-\beta^2+\beta(1+\beta)\delta^2\right)\left(\beta(8-\beta-3\beta^2)+(16-12\beta-3\beta^2+3\beta^3)\delta^2\right)},$$

and $\underline{\kappa}_g < \overline{\kappa}_g$ is equivalent to condition (9).

Of course, competition is not always an equilibrium for all $k \in (\underline{\kappa}_g, \overline{\kappa}_g)$ along the off-equilibrium path after deviation, and the best response to the punishment in this subgame may be different, similar to what we observe in the proof of Proposition 6. The best response of the deviating unit depends on the order of parameters $\underline{\kappa}_g$, $\overline{\kappa}_g$, $\overline{\kappa}_r$, k_4, k_o, and k_p and may result in playing competition, or ftl+competition, or ftl, or even quitting the market after selling all the energy. □

Proof of Corollary 8. The existence of the equilibrium Cooperation is guaranteed by condition (9), so we seek conditions for which it is known to hold. The LHS of (9) is always positive, so we need to guarantee the RHS is also positive; in that case, there is a value of a such that (9) holds. Since the denominator of the RHS is always positive, we just need to find the conditions that provides $G_1(\beta) + G_2(\beta)\delta^2 > 0$. Note that $G_2 > 0$ for any β, so we can put $\delta = 1$. Then we have

$$G_1(\beta) + G_2(\beta) = 4(-4 + 8\beta - 2\beta^2 - \beta^3),$$

which is positive if $\beta > \beta^* = 0.6309$. □

Proof of Proposition 9. The second part is obvious. If (9) fails to hold, cooperation can never be an equilibrium behavior for any k. Even if (9) holds but $k_3 > \overline{\kappa}_g$, $U_{pc} > U_{col}$ for any $k \in [\underline{\kappa}_g, \overline{\kappa}_g]$, so while playing cooperation is an equilibrium, it cannot be payoff-maximizing. Competition is the only option for $k \in (0, \underline{\kappa}_b]$, and starting from $\underline{\kappa}_b$, $U_{pc} > U_{com}$ until $\overline{\kappa}_b$, when partial cooperation stops being an equilibrium.

To prove the first case, it is enough to show that $\underline{\kappa}_g < k_3$ under (9) and high enough β. We have

$$k_3 - \underline{\kappa}_g = \frac{(2+\beta)(1+a)\bar{K}(\frac{1-a}{1+a}, \beta, \delta)}{\left(\beta^2+(12-2\beta-3\beta^2)\delta^2\right)\left(\beta(8-\beta-3\beta^2)+(16-12\beta-3\beta^2+3\beta^3)\delta^2\right)},$$

where

$$\bar{K}\left(\frac{1-a}{1+a},\beta,\delta\right) = -\beta\Big((2-\beta)\beta^2-(24-30\beta+4\beta^2+3\beta^3)\delta^2\Big)\frac{1-a}{1+a} + \Big(\beta(16-22\beta+6\beta^2+\beta^3)-(2-\beta)(8-8\beta-2\beta^2+3\beta^3)\delta^2\Big)\delta.$$

According to (9), we have

$$0<\frac{1-a}{1+a}<\frac{G_1(\beta)+G_2(\beta)\delta^2}{G_3(\beta)+G_4(\beta)\delta^2}\cdot\delta.$$

Since the function $\bar{K}$ is linear in $(1-a)/(1+a)$, it is enough to show that $\bar{K}$ is positive at the ends of the interval. We have

$$\begin{aligned}\bar{K}(0,\beta,\delta) &= \big(\beta(16-22\beta+6\beta^2+\beta^3)-(2-\beta)(8-8\beta-2\beta^2+3\beta^3)\delta^2\big)\,\delta\\ &\geqslant 2(1-\beta)(-8+12\beta-\beta^2-2\beta^3),\end{aligned}$$

which is positive if $\beta\geqslant 0.81$. Also,

$$\bar{K}\left(\frac{G_1(\beta)+G_2(\beta)\delta^2}{G_3(\beta)+G_4(\beta)\delta^2}\cdot\delta,\beta,\delta\right)= \frac{2(1-\beta)(2-\beta)\delta\Big(\beta(4-\beta)-(8-6\beta-3\beta^2)\delta^2\Big)\Big(\beta(8-\beta-3\beta^2)+(16-12\beta-3\beta^2+3\beta^3)\delta^2\Big)}{(2-\beta)\beta(4-\beta-\beta^2)+(32-48\beta+14\beta^2+5\beta^3-\beta^4)\delta^2}.$$

Since

$$\beta(4-\beta)-(8-6\beta-3\beta^2)\delta^2\geqslant 2(-4+5\beta+\beta^2)$$

and the last polynomial is positive if $\beta\geqslant 0.71$, we may conclude that $k_3-\underline{\kappa}_g\geqslant 0$ at least for all $\beta\geqslant 0.81$. This is laid out in Figure 7 (for $k_3\leqslant\overline{\kappa}_g$) and Figure 8 (for $k_3>\overline{\kappa}_g$). The solid lines depict equilibrium maximum payoffs that arise from the equilibria listed in Proposition 9. The dashed lines show payoffs arising from the same equilibria, but are payoff-dominated by another equilibrium. For example, in Figure 7, between $\underline{\kappa}_g$ and k_3, Cooperation is an equilibrium but it is dominated by Partial Cooperation, which delivers higher payoffs.

This is reversed for k larger than k_3. In both Figures, at $\underline{\kappa}_b$, Partial Cooperation must deliver a discretely larger payoff than competition otherwise the deviation is too tempting (equivalently, the benefit of Cooperation too small). This reflects the fact that Partial Cooperation must be robust to deviations at the interim stage – when one of the players is revealed to be the second mover and contemplates her options then (see Section 3.3). However, in Figure 7, storage operators are indifferent between either cooperative equilibrium at k_3. At that point, the ex ante incentives are relevant. □

Lemma 16. *For each of the behaviors competition, partial cooperation and cooperation, cumulative payments from consumers take, respectively, the form*

$$P_{com} = \frac{2\big(1+a^2\big)n - \big(a(2-\beta+\beta\delta) - 2 + \beta + \beta\delta\big)k(n-1) - 2\big(2-\beta+\beta\delta^2\big)k^2}{2(1-\beta)(n+1)^2},$$

$$P_{pc} = \frac{8\big(1+a^2\big)n - (2+\beta)\big(a(2-\beta+\beta\delta) - 2 + \beta + \beta\delta\big)k(n-1) - \big(4-\beta^2+\beta(2+3\beta)\delta^2\big)k^2}{8(1-\beta)(n+1)^2},$$

$$P_{col} = \frac{(4-\beta^2)(1+a^2)n - \big(a(2-\beta^2+\beta\delta) - 2 + \beta^2 + \beta\delta\big)k(n-1) - \big(2-\beta^2+\beta\delta^2\big)k^2}{(1-\beta)(4-\beta^2)(n+1)^2}.$$

For the same behaviors, consumer surplus is computed as

$$S_{com} = \frac{\big(1+a^2\big)n^2 + \big(a(2-\beta+\beta\delta) - 2 + \beta + \beta\delta\big)kn + \big(2-\beta+\beta\delta^2\big)k^2}{2(1-\beta)(n+1)^2},$$

$$S_{pc} = \frac{8\big(1+a^2\big)n^2 + 2(2+\beta)\big(a(2-\beta+\beta\delta) - 2 + \beta + \beta\delta\big)kn + \big(4-\beta^2+\beta(2+3\beta)\delta^2\big)k^2}{16(1-\beta)(n+1)^2},$$

$$S_{col} = \frac{(4-\beta^2)(1+a^2)n^2 + 2\big(a(2-\beta^2+\beta\delta) - 2 + \beta^2 + \beta\delta\big)kn + \big(2-\beta^2+\beta\delta^2\big)k^2}{2(1-\beta)(4-\beta^2)(n+1)^2}.$$

Proof of Lemma 16. Let $V_t(i,j) = V(i,j)$, $i,j \in \{0,k\}$ be the value function which is the cumulative expected payoff of consumers from moment t if the current states of the storage units are i and j. We have a system of recursive equations for competition, partial cooperation, and cooperation with respective sub-indices:

$$\begin{cases} V_{com}(0,0) = \frac{1}{2} \cdot (C_0 + \beta V_{com}(0,0)) + \frac{1}{2} \cdot (C_5 + \beta \cdot V_{com}(k,k)), \\ V_{com}(k,k) = \frac{1}{2} \cdot (C_4 + \beta \cdot V_{com}(0,0)) + \frac{1}{2} \cdot (C_3 + \beta \cdot V_{com}(k,k)), \end{cases}$$

$$\begin{cases} V_{pc}(0,0) = \frac{1}{2} \cdot (C_0 + \beta V_{pc}(0,0)) + \frac{1}{2} \cdot (C_1 + \beta \cdot V_{pc}(0,k)), \\ V_{pc}(0,k) = \frac{1}{2} \cdot (C_2 + \beta V_{pc}(0,0)) + \frac{1}{2} \cdot (C_1 + \beta \cdot V_{pc}(k,k)), \\ V_{pc}(k,k) = \frac{1}{2} \cdot (C_4 + \beta \cdot V_{pc}(0,0)) + \frac{1}{2} \cdot (C_3 + \beta \cdot V_{pc}(k,k)), \end{cases}$$

$$\begin{cases} V_{col}(0,0) = \frac{1}{2} \cdot (C_0 + \beta V_{col}(0,0)) + \frac{1}{2} \cdot (C_1 + \beta \cdot V_{col}(0,k)), \\ V_{col}(0,k) = \frac{1}{2} \cdot (C_2 + \beta V_{col}(0,0)) + \frac{1}{2} \cdot (C_1 + \beta \cdot V_{col}(k,k)), \\ V_{col}(k,k) = \frac{1}{2} \cdot (C_2 + \beta \cdot V_{col}(0,k)) + \frac{1}{2} \cdot (C_3 + \beta \cdot V_{col}(k,k)), \end{cases}$$

where

$$C_3 = \frac{1-a}{n+1} \cdot \frac{1-a}{n+1} n, \qquad C_2 = \frac{1+a-\delta k}{n+1} \cdot \left(\frac{1+a-\delta k}{n+1} n + \delta k \right),$$

$$C_0 = \frac{1+a}{n+1} \cdot \frac{1+a}{n+1} n, \qquad C_1 = \frac{1-a+k}{n+1} \cdot \left(\frac{1-a+k}{n+1} n - k \right),$$

$$C_4 = \frac{1+a-2\delta k}{n+1} \cdot \left(\frac{1+a-2\delta k}{n+1} n + 2\delta k \right), \qquad C_5 = \frac{1-a+2k}{n+1} \cdot \left(\frac{1-a+2k}{n+1} n - 2k \right).$$

It can be rewritten in a matrix form

$$\mathbf{V} = \mathbf{C} + \beta \cdot \mathbf{Q} \cdot \mathbf{V},$$

where the corresponding $\mathbf{V}$, $\mathbf{C}$, and $\mathbf{Q}$ for competition are the following:

$$\mathbf{V}_{com} = \begin{pmatrix} V_{com}(0,0) \\ V_{com}(k,k) \end{pmatrix}, \qquad \mathbf{C}_{com} = \frac{1}{2} \begin{pmatrix} C_0 + C_5 \\ C_3 + C_4 \end{pmatrix}, \qquad \mathbf{Q}_{com} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix};$$

for partial cooperation:

$$\mathbf{V}_{pc} = \begin{pmatrix} V_{pc}(0,0) \\ V_{pc}(0,k) \\ V_{pc}(k,k) \end{pmatrix}, \qquad \mathbf{C}_{pc} = \frac{1}{2}\begin{pmatrix} C_0 + C_1 \\ C_1 + C_2 \\ C_3 + C_4 \end{pmatrix}, \qquad \mathbf{Q}_{pc} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix};$$

finally, for cooperation:

$$\mathbf{V}_{col} = \begin{pmatrix} V_{col}(0,0) \\ V_{col}(0,k) \\ V_{col}(k,k) \end{pmatrix}, \qquad \mathbf{C}_{col} = \frac{1}{2}\begin{pmatrix} C_0 + C_1 \\ C_1 + C_2 \\ C_3 + C_2 \end{pmatrix}, \qquad \mathbf{Q}_{col} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix};$$

.

Note that $\mathbf{Q}_{com}^2 = \mathbf{Q}_{com}$. For $\beta < 1$, we obtain

$$\mathbf{V}_{com} = \mathbf{C}_{com} + \sum_{i=1}^{t} \beta^i \mathbf{Q}_{com}^i \cdot \mathbf{C}_{com} + \beta^{t+1}\mathbf{Q}_{com}^{t+1} \cdot \mathbf{V}_{com} \xrightarrow[t\to\infty]{} \mathbf{C}_{com} + \frac{\beta}{1-\beta} \cdot \mathbf{Q}_{com} \cdot \mathbf{C}_{com} =$$
$$= \frac{1}{4(1-\beta)} \begin{pmatrix} (2-\beta)(C_0 + C_5) + \beta(C_3 + C_4) \\ \beta(C_0 + C_5) + (2-\beta)(C_3 + C_4) \end{pmatrix}.$$

The upper term after substituting C_i is exactly P_{com}.

For partial cooperation, note that $\mathbf{Q}_{pc}^2 = \mathbf{Q}_{pc}^n$ for any $n \geqslant 2$. Then we have

$$\mathbf{V}_{pc} = \mathbf{C}_{pc} + \beta\mathbf{Q}_{pc} \cdot \mathbf{C}_{pc} + \sum_{i=2}^{t} \beta^i \mathbf{Q}_{pc}^i \cdot \mathbf{C}_{pc} + \beta^{t+1}\mathbf{Q}_{pc}^{t+1} \cdot \mathbf{V}_{pc}$$
$$\xrightarrow[t\to\infty]{} \mathbf{C}_{pc} + \beta\mathbf{Q}_{pc} \cdot \mathbf{C}_{pc} + \frac{\beta^2}{1-\beta} \cdot \mathbf{Q}_{pc}^2 \cdot \mathbf{C}_{pc} =$$
$$= \frac{1}{8(1-\beta)} \begin{pmatrix} (2-\beta)\big(2C_0 + (2+\beta)C_1 + \beta C_2\big) + \beta^2(C_3 + C_4) \\ 2\beta C_0 + \big(4 - 2\beta + \beta^2\big)C_1 + (2-\beta)\big((2-\beta)C_2 + \beta(C_3 + C_4)\big) \\ \beta\big(2C_0 + (2+\beta)C_1 + \beta C_2\big) + \big(4 - 2\beta - \beta^2\big)(C_3 + C_4) \end{pmatrix}.$$

The uppermost term after substituting C_i is exactly P_{pc}.

For cooperation, note that $\mathbf{Q}_{col} = \mathbf{T} \cdot \mathbf{J} \cdot \mathbf{T}^{-1}$, where

$$\mathbf{J} = \begin{pmatrix} -1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \mathbf{T} = \begin{pmatrix} 1 & -1 & 1 \\ -2 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

Thus, $\mathbf{Q}_{col}^t = \mathbf{T} \cdot \mathbf{J}^t \cdot \mathbf{T}^{-1}$. For $\beta < 1$, we obtain:

$$\mathbf{V}_{cool} = \mathbf{C}_{col} + \sum_{i=1}^{\infty} \beta^i \mathbf{Q}_{col}^i \cdot \mathbf{C}_{col} = \mathbf{C}_{col} + \mathbf{T} \cdot \begin{pmatrix} -\frac{\beta}{2+\beta} & 0 & 0 \\ 0 & \frac{\beta}{2-\beta} & 0 \\ 0 & 0 & \frac{\beta}{1-\beta} \end{pmatrix} \cdot \mathbf{T}^{-1} \cdot \mathbf{C}_{col} =$$

$$= \frac{1}{2(1-\beta)(4-\beta^2)} \begin{pmatrix} \big(4 - 2\beta - \beta^2\big) C_0 + 2\big(2 - \beta^2\big) C_1 + \beta\big(2C_2 + \beta C_3\big) \\ (2-\beta)\big(\beta(C_0 + C_3) + 2(C_1 + C_2)\big) \\ \beta\big(\beta C_0 + 2C_1\big) + \big(4 - 2\beta^2\big) C_2 + \big(4 - 2\beta - \beta^2\big) C_3 \end{pmatrix}.$$

The uppermost term after substituting C_i is exactly P_{col}. For consumer surplus, the proof repeats the same steps, with the only difference in the constant term for the recursive equation. Indeed, let $V_t(i,j) = V(i,j)$, $i, j \in \{0, k\}$ be the value function which is now the cumulative consumer surplus of consumers from moment t if the current states of the storage units are i and j. We have a system of recursive equations for competition, partial cooperation, and cooperation with respective sub-indices:

$$\begin{cases} V_{com}(0,0) = \frac{1}{2} \cdot \left(\bar{C}_0 + \beta V_{com}(0,0)\right) + \frac{1}{2} \cdot \left(\bar{C}_5 + \beta \cdot V_{com}(k,k)\right), \\ V_{com}(k,k) = \frac{1}{2} \cdot \left(\bar{C}_4 + \beta \cdot V_{com}(0,0)\right) + \frac{1}{2} \cdot \left(\bar{C}_3 + \beta \cdot V_{com}(k,k)\right), \end{cases}$$

$$\begin{cases} V_{pc}(0,0) = \frac{1}{2} \cdot \left(\bar{C}_0 + \beta V_{pc}(0,0)\right) + \frac{1}{2} \cdot \left(\bar{C}_1 + \beta \cdot V_{pc}(0,k)\right), \\ V_{pc}(0,k) = \frac{1}{2} \cdot \left(\bar{C}_2 + \beta V_{pc}(0,0)\right) + \frac{1}{2} \cdot \left(\bar{C}_1 + \beta \cdot V_{pc}(k,k)\right), \\ V_{pc}(k,k) = \frac{1}{2} \cdot \left(\bar{C}_4 + \beta \cdot V_{pc}(0,0)\right) + \frac{1}{2} \cdot \left(\bar{C}_3 + \beta \cdot V_{pc}(k,k)\right), \end{cases}$$

$$\begin{cases} V_{col}(0,0) = \frac{1}{2} \cdot \left(\bar{C}_0 + \beta V_{col}(0,0)\right) + \frac{1}{2} \cdot \left(\bar{C}_1 + \beta \cdot V_{col}(0,k)\right), \\ V_{col}(0,k) = \frac{1}{2} \cdot \left(\bar{C}_2 + \beta V_{col}(0,0)\right) + \frac{1}{2} \cdot \left(\bar{C}_1 + \beta \cdot V_{col}(k,k)\right), \\ V_{col}(k,k) = \frac{1}{2} \cdot \left(\bar{C}_2 + \beta \cdot V_{col}(0,k)\right) + \frac{1}{2} \cdot \left(\bar{C}_3 + \beta \cdot V_{col}(k,k)\right), \end{cases}$$

where

$$\bar{C}_3 = \frac{1}{2}\left(\frac{1-a}{n+1}n\right)^2, \qquad \bar{C}_2 = \frac{1}{2}\left(\frac{1+a-\delta k}{n+1}n + \delta k\right)^2,$$

$$\bar{C}_0 = \frac{1}{2}\left(\frac{1+a}{n+1}n\right)^2, \qquad \bar{C}_1 = \frac{1}{2}\left(\frac{1-a+k}{n+1}n - k\right)^2,$$

$$\bar{C}_4 = \frac{1}{2}\left(\frac{1+a-2\delta k}{n+1}n + 2\delta k\right)^2, \qquad \bar{C}_5 = \frac{1}{2}\left(\frac{1-a+2k}{n+1}n - 2k\right)^2.$$

The difference between $\bar{C}_i$ and C_i is dictated by the fact that C_i denotes the consumers' payments in each period: $C_i = p^* \cdot q^*$, while $\bar{C}_i$ denotes consumer surplus in each period: $\bar{C}_i = \frac{1}{2} \cdot (1 + \varepsilon - p^*) \cdot q^*$. It can be rewritten in a matrix form

$$\mathbf{V} = \bar{\mathbf{C}} + \beta \cdot \mathbf{Q} \cdot \mathbf{V},$$

where $\mathbf{V}$ and $\mathbf{Q}$ are the same as above and $\bar{\mathbf{C}}$ for competition, partial cooperation and cooperation, respectively, are the following vectors:

$$\mathbf{C}_{com} = \frac{1}{2}\begin{pmatrix} C_0 + C_5 \\ C_3 + C_4 \end{pmatrix}, \qquad \mathbf{C}_{pc} = \frac{1}{2}\begin{pmatrix} C_0 + C_1 \\ C_1 + C_2 \\ C_3 + C_4 \end{pmatrix}, \qquad \mathbf{C}_{col} = \frac{1}{2}\begin{pmatrix} C_0 + C_1 \\ C_1 + C_2 \\ C_3 + C_2 \end{pmatrix}.$$

.

Using the same algorithm as earlier, we obtain expressions for $\mathbf{V}_{com}$, $\mathbf{V}_{pc}$ and $\mathbf{V}_{col}$. Their first coordinates are the desired functions S_{com}, S_{pc} and S_{col}. □

Proof of Proposition 12. For the first part, we can derive

$$P_{pc} - P_{com} = \frac{(2-\beta)k\Big(\big(a(2-\beta+\beta\delta)-2+\beta+\beta\delta\big)(n-1)+\big(6-\beta+3\beta\delta^2\big)k\Big)}{8(1-\beta)(n+1)^2}.$$

Since (8) is equivalent to $a > \frac{2-\beta-\beta\delta}{2-\beta+\beta\delta}$, we conclude that $P_{pc} > P_{com}$. Also,

$$P_{col} - P_{com} = \frac{k\big(L_1(n-1)+2L_2k\big)}{2(1-\beta)(4-\beta^2)(n+1)^2},$$

where

$$L_2 = 6 - 4\beta - \beta^2 + \beta^3 + \beta(3-\beta^2)\delta^2 > 0$$

for any β and δ and

$$L_1 = a(4-4\beta+\beta^3+\beta(2-\beta^2)\delta) - 4 + 4\beta - \beta^3 + \beta(2-\beta^2)\delta,$$

and using condition (8) we have

$$L_1 > \frac{2-\beta-\beta\delta}{2-\beta+\beta\delta}(4-4\beta+\beta^3+\beta(2-\beta^2)\delta) - 4 + 4\beta - \beta^3 + \beta(2-\beta^2)\delta = \frac{4(1-\beta)\beta^2\delta}{2-\beta+\beta\delta} > 0.$$

Hence, $P_{col} > P_{com}$. Likewise, we can derive

$$S_{com} - S_{pc} = \frac{(2-\beta)k\Big(\big(a(2-\beta+\beta\delta) - 2 + \beta + \beta\delta\big)n + \big(6-\beta+3\beta\delta^2\big)k\Big)}{16(1-\beta)(n+1)^2}.$$

Since (8) is equivalent to $a > \frac{2-\beta-\beta\delta}{2-\beta+\beta\delta}$, we conclude that $S_{com} > S_{pc}$. Further,

$$S_{com} - S_{col} = \frac{k\big(L_1 n + L_2 k\big)}{2(1-\beta)(4-\beta^2)(n+1)^2},$$

where

$$L_2 = 6 - 4\beta - \beta^2 + \beta^3 + \beta(3-\beta^2)\delta^2 > 0$$

for any β and δ and

$$L_1 = a(4 - 4\beta + \beta^3 + \beta(2-\beta^2)\delta) - 4 + 4\beta - \beta^3 + \beta(2-\beta^2)\delta,$$

and using condition (8) we have

$$L_1 > \frac{2-\beta-\beta\delta}{2-\beta+\beta\delta}(4 - 4\beta + \beta^3 + \beta(2-\beta^2)\delta) - 4 + 4\beta - \beta^3 + \beta(2-\beta^2)\delta = \frac{4(1-\beta)\beta^2\delta}{2-\beta+\beta\delta} > 0.$$

Hence, $S_{com} > S_{col}$, as claimed. □

Lemma 17. *The levels of capacity, under which competitive, partially cooperative, or cooperative storage units maximize their profits, are equal to k_{com}, k_{pc}, k_{col}, respectively:*

$$k_{com} = \frac{C}{4\left(2-\beta+\beta\delta^2\right)}, \qquad k_{pc} = \frac{C}{2\left(2-\beta+\beta\frac{2+3\beta}{2+\beta}\delta^2\right)}, \qquad k_{col} = \frac{C-\beta(1-\beta)(1-a)}{2\left(2-\beta^2+\beta\delta^2\right)}.$$

The proof is trivial and therefore omitted; the maximizers obtain from the functions computed in Proposition 2.

Proof of Proposition 13. Using Lemmata 16 and 17, we obtain:

$$P_{com}(k_{com}) = \frac{16(2-\beta+\beta\delta^2)(1+a^2)n - (a(2-\beta+\beta\delta)-2+\beta+\beta\delta)^2(2n-1)}{16(1-\beta)(2-\beta+\beta\delta^2)(n+1)^2},$$
$$P_{pc}(k_{pc}) = \frac{32(4-\beta^2+\beta(2+3\beta)\delta^2)(1+a^2)n - (2+\beta)^2(a(2-\beta+\beta\delta)-2+\beta+\beta\delta)^2(2n-1)}{32(1-\beta)(4-\beta^2+\beta(2+3\beta)\delta^2)(n+1)^2},$$
$$S_{com}(k_{com}) = \frac{16(2-\beta+\beta\delta^2)(1+a^2)n^2 + (a(2-\beta+\beta\delta)-2+\beta+\beta\delta)^2(4n+1)}{32(1-\beta)(2-\beta+\beta\delta^2)(n+1)^2},$$
$$S_{pc}(k_{pc}) = \frac{32(4-\beta^2+\beta(2+3\beta)\delta^2)(1+a^2)n^2 + (2+\beta)^2(a(2-\beta+\beta\delta)-2+\beta+\beta\delta)^2(4n+1)}{64(1-\beta)(4-\beta^2+\beta(2+3\beta)\delta^2)(n+1)^2}.$$

Then

$$P_{com}(k_{com}) - P_{pc}(k_{pc}) = \frac{\beta(2-\beta)(2+\beta-\beta\delta^2)(a(2-\beta+\beta\delta)-2+\beta+\beta\delta)^2(2n-1)}{32(1-\beta)(2-\beta+\beta\delta^2)(4-\beta^2+\beta(2+3\beta)\delta^2)(n+1)^2} > 0,$$
$$S_{com}(k_{com}) - S_{pc}(k_{pc}) = -\frac{\beta(2-\beta)(2+\beta-\beta\delta^2)(a(2-\beta+\beta\delta)-2+\beta+\beta\delta)^2(4n+1)}{64(1-\beta)(2-\beta+\beta\delta^2)(4-\beta^2+\beta(2+3\beta)\delta^2)(n+1)^2} < 0$$

for any $0 < a < 1$, $0 < \beta < 1$, $0 < \delta \leqslant 1$, and $n \in \mathbb{N}$. □

Proof of Proposition 14. We use the same algebra as in Proposition 2. For competition, the system of equations (7) takes the following form:

$$\begin{cases} V(0,0) = \frac{1}{2}\left(\beta V(0,0) - B_2\left(\frac{k}{2}\right) + \beta V\left(\frac{k}{2},\frac{k}{2}\right)\right), \\ V\left(\frac{k}{2},\frac{k}{2}\right) = \frac{1}{2}\left(A_2\left(\frac{k}{2}\right) + \beta V(0,0) - B_2\left(\frac{k}{2}\right) + \beta V(k,k)\right), \\ V(k,k) = \frac{1}{2}\left(A_2\left(\frac{k}{2}\right) + \beta V\left(\frac{k}{2},\frac{k}{2}\right) + \beta V(k,k)\right), \end{cases}$$

so (now omitting the argument $k/2$ in all A_i and B_i throughout the proof)

$$\mathbf{P} = \frac{1}{2}\begin{pmatrix} -B_2 \\ A_2 - B_2 \\ A_2 \end{pmatrix}, \qquad \mathbf{Q} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}.$$

To calculate the power t of matrix $\mathbf{Q}$, we find the Jordan decomposition $\mathbf{Q} = \mathbf{T}\cdot\mathbf{J}\cdot\mathbf{T}^{-1}$

of $\mathbf{Q}$. Here,

$$\mathbf{J} = \begin{pmatrix} -1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \mathbf{T} = \begin{pmatrix} 1 & -1 & 1 \\ -2 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix},$$

so $\mathbf{Q}^t = \mathbf{T} \cdot \mathbf{J}^t \cdot \mathbf{T}^{-1}$.

For $\beta < 1$, we obtain:

$$\begin{aligned} \mathbf{V} &= \mathbf{P} + \sum_{i=1}^{\infty} \beta^i \mathbf{Q}^i \cdot \mathbf{P} = \mathbf{P} + \mathbf{T} \cdot \begin{pmatrix} -\frac{\beta}{2+\beta} & 0 & 0 \\ 0 & \frac{\beta}{2-\beta} & 0 \\ 0 & 0 & \frac{\beta}{1-\beta} \end{pmatrix} \cdot \mathbf{T}^{-1} \cdot \mathbf{P} \\ &= \frac{1}{(1-\beta)(2-\beta)(2+\beta)} \begin{pmatrix} \beta A_2 - (2-\beta^2) B_2 \\ (2-\beta)(A_2 - B_2) \\ (2-\beta^2) A_2 - \beta B_2 \end{pmatrix}. \end{aligned}$$

The uppermost term is exactly $V(0,0) = U_{com}$.

For cooperation and partial cooperation, there are not three states anymore but nine. The elements of (7) are

$$\mathbf{V} = \begin{pmatrix} V(0,0) \\ V(0,k/2) \\ V(0,k) \\ V(k/2,0) \\ V(k/2,k/2) \\ V(k/2,k) \\ V(k,0) \\ V(k,k/2) \\ V(k,k) \end{pmatrix}, \quad \mathbf{P}_{col} = \frac{1}{4} \begin{pmatrix} -B_1 \\ -B_1 \\ -2B_1 \\ 2A_1 - B_1 \\ A_1 - B_1 \\ A_1 - 2B_1 \\ 2A_1 \\ A_1 \\ A_1 \end{pmatrix}, \quad \mathbf{Q}_{col} = \frac{1}{4} \begin{pmatrix} 2 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 2 \end{pmatrix},$$

$$\mathbf{P}_{pc} = \frac{1}{4}\begin{pmatrix} -B_1 \\ -B_1 \\ -2B_1 \\ 2A_1 - B_1 \\ 2A_2 - B_1 \\ 2A_2 - 2B_1 \\ 2A_1 \\ 2A_2 \\ 2A_2 \end{pmatrix}, \qquad \mathbf{Q}_{pc} = \frac{1}{4}\begin{pmatrix} 2 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 \end{pmatrix}.$$

Again we find the payoffs using Jordan's decomposition. For example, for cooperation the corresponding Jordan form and transition matrix are

$$\mathbf{J}_{col} = \frac{1}{4}\begin{pmatrix} -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1-\sqrt{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1-\sqrt{5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{5}-1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1+\sqrt{5} \end{pmatrix},$$

$$\mathbf{T}_{col} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 & -1 & 1 & -1 \\ -1 & 1 & 0 & -1 & 1 & \frac{-3-\sqrt{5}}{2} & \frac{1+\sqrt{5}}{2} & \frac{-3+\sqrt{5}}{2} & \frac{1-\sqrt{5}}{2} \\ 2 & 0 & 1 & -2 & 1 & 1+\sqrt{5} & 0 & 1-\sqrt{5} & 0 \\ 1 & -1 & 0 & 1 & 1 & \frac{-3-\sqrt{5}}{2} & \frac{1+\sqrt{5}}{2} & \frac{-3+\sqrt{5}}{2} & \frac{1-\sqrt{5}}{2} \\ 0 & 0 & -1 & 0 & 1 & 1+\sqrt{5} & 0 & 1-\sqrt{5} & 0 \\ -1 & -1 & 0 & -1 & 1 & \frac{-3-\sqrt{5}}{2} & \frac{-1-\sqrt{5}}{2} & \frac{-3+\sqrt{5}}{2} & \frac{-1+\sqrt{5}}{2} \\ -2 & 0 & 1 & 2 & 1 & 1+\sqrt{5} & 0 & 1-\sqrt{5} & 0 \\ 1 & 1 & 0 & 1 & 1 & \frac{-3-\sqrt{5}}{2} & \frac{-1-\sqrt{5}}{2} & \frac{-3+\sqrt{5}}{2} & \frac{-1+\sqrt{5}}{2} \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Using the same steps as we did for competition, we can obtain U_{col}. We repeat the algorithm for the payoff function U_{pc} but omit this last iteration here to conserve space. □

Proof of Proposition 15. Here too we use the same algorithm as for proving Proposition 14. Now we have four states in competition (that we list below), and sixteen states for cooperation and partial cooperation. For competition, the system of equations (7) takes the following form:

$$\begin{cases} V(0,0) = \frac{1}{2}\left(\beta V(0,0) - B_2\left(\frac{k}{3}\right) + \beta V\left(\frac{k}{3},\frac{k}{3}\right)\right), \\ V\left(\frac{k}{3},\frac{k}{3}\right) = \frac{1}{2}\left(A_2\left(\frac{k}{3}\right) + \beta V(0,0) - B_2\left(\frac{k}{3}\right) + \beta V\left(\frac{2k}{3},\frac{2k}{3}\right)\right), \\ V\left(\frac{2k}{3},\frac{2k}{3}\right) = \frac{1}{2}\left(A_2\left(\frac{k}{3}\right) + \beta V(\frac{k}{3},\frac{k}{3}) - B_2\left(\frac{k}{3}\right) + \beta V\left(k,k\right)\right), \\ V(k,k) = \frac{1}{2}\left(A_2\left(\frac{k}{3}\right) + \beta V\left(\frac{2k}{3},\frac{2k}{3}\right) + \beta V(k,k)\right), \end{cases}$$

so (now omitting the argument $k/3$ in all A_i and B_i throughout the proof)

$$\mathbf{P} = \frac{1}{2}\begin{pmatrix} -B_2 \\ A_2 - B_2 \\ A_2 - B_2 \\ A_2 \end{pmatrix}, \qquad \mathbf{Q} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}.$$

Here the Jordan decomposition of $\mathbf{Q}$ is $\mathbf{Q} = \mathbf{T} \cdot \mathbf{J} \cdot \mathbf{T}^{-1}$:

$$\mathbf{J} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1/\sqrt{2} \end{pmatrix}, \qquad \mathbf{T} = \begin{pmatrix} 1 & 1 & -1 & -1 \\ -1 & 1 & 1+\sqrt{2} & 1-\sqrt{2} \\ -1 & 1 & -1-\sqrt{2} & -1+\sqrt{2} \\ 1 & 1 & 1 & 1 \end{pmatrix},$$

and its power t is given by $\mathbf{Q}^t = \mathbf{T} \cdot \mathbf{J}^t \cdot \mathbf{T}^{-1}$. For $\beta < 1$, we obtain:

$$\mathbf{V} = \mathbf{P} + \sum_{i=1}^{\infty} \beta^i \mathbf{Q}^i \cdot \mathbf{P} = \mathbf{P} + \mathbf{T} \cdot \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{\beta}{1-\beta} & 0 & 0 \\ 0 & 0 & -\frac{\beta}{\sqrt{2}+\beta} & 0 \\ 0 & 0 & 0 & \frac{\beta}{\sqrt{2}-\beta} \end{pmatrix} \cdot \mathbf{T}^{-1} \cdot \mathbf{P} =$$

$$= \frac{1}{8(1-\beta)(2-\beta^2)} \begin{pmatrix} \beta(4-\beta^2)A_2 - (8-4\beta^2-\beta^3)B_2 \\ (2+\beta)(2-\beta)^2 A_2 - (8-6\beta^2+\beta^3)B_2 \\ (8-6\beta^2+\beta^3)A_2 - (2+\beta)(2-\beta)^2 B_2 \\ (8-4\beta^2-\beta^3)A_2 - \beta(4-\beta^2)B_2 \end{pmatrix}.$$

The uppermost term is exactly $V(0,0) = U_{com}$.

For cooperation and partial cooperation, there are 16 states possible. The elements of (7) are

$$\mathbf{V} = \begin{pmatrix} V(0,0) \\ V(0,k/3) \\ V(0,2k/3) \\ V(0,k) \\ V(k/3,0) \\ V(k/3,k/3) \\ V(k/3,2k/3) \\ V(k/3,k) \\ V(2k/3,0) \\ V(2k/3,k/3) \\ V(2k/3,2k/3) \\ V(2k/3,k) \\ V(k,0) \\ V(k,k/3) \\ V(k,2k/3) \\ V(k,k) \end{pmatrix}, \qquad \mathbf{P}_{col} = \frac{1}{4} \begin{pmatrix} -B_1 \\ -B_1 \\ -B_1 \\ -2B_1 \\ 2A_1 - B_1 \\ A_1 - B_1 \\ A_1 - B_1 \\ A_1 - 2B_1 \\ 2A_1 - B_1 \\ A_1 - B_1 \\ A_1 - B_1 \\ A_1 - 2B_1 \\ 2A_1 \\ A_1 \\ A_1 \\ A_1 \end{pmatrix}, \qquad \mathbf{P}_{pc} = \frac{1}{4} \begin{pmatrix} -B_1 \\ -B_1 \\ -B_1 \\ -2B_1 \\ 2A_1 - B_1 \\ 2A_2 - B_1 \\ 2A_2 - B_1 \\ 2A_2 - 2B_1 \\ 2A_1 - B_1 \\ 2A_2 - B_1 \\ 2A_2 - B_1 \\ 2A_2 - 2B_1 \\ 2A_1 \\ 2A_2 \\ 2A_2 \\ 2A_2 \end{pmatrix},$$

$$
\mathbf{Q}_{col} = \frac{1}{4}\begin{pmatrix}
2 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 2
\end{pmatrix},
$$

$$
\mathbf{Q}_{pc} = \frac{1}{4}\begin{pmatrix}
2 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2
\end{pmatrix}.
$$

Here too we omit the repetition of the algorithm for the functions U_{col} and U_{pc}. $\square$

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