Australian Government
Australian Research Council

FLEET
ARC CENTRE OF EXCELLENCE IN FUTURE LOW-ENERGY ELECTRONICS TECHNOLOGIES

MONASH University

Universal Efimov Scaling in the Rabi-Coupled Few-Body Spectrum

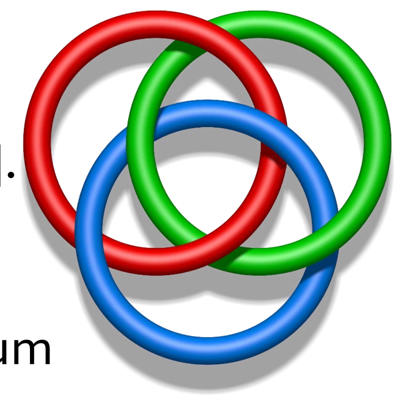
Anthony N. Zulli, Brendan C. Mulkerin, Meera M. Parish, Jesper Levinsen

School of Physics and Astronomy, Monash University, Victoria 3800, Australia and ARC Centre of Excellence in Future Low-Energy Electronics Technologies, Monash University, Victoria 3800, Australia

We investigate the behavior of the Efimov effect—a universal quantum few-body phenomenon—in the presence of an external driving field. Specifically, we consider up to three bosonic atoms, such as 133Cs, interacting with a light atom, such as 6Li, where the latter has two internal spin states {↑,↓} that are Rabi coupled. Assuming that only the spin-↑ light atom interacts with the bosons, we find that the Rabi drive transposes the entire Efimov spectrum such that the Efimov trimers and tetramers are centered around the Rabi-shifted two-body scattering resonance. Crucially, we show that the Rabi drive preserves the trimers' discrete scaling symmetry, while universally shifting the Efimov three-body parameter, leading to a log-periodic modulation in the spectrum as the Rabi drive is varied. Our results suggest that Efimov physics can be conveniently explored using an applied driving field, opening up the prospect of an externally tunable three-body parameter.

Efimov Physics?

Efimov physics is a quantum mechanical phenomena where even in the absence of a two-body bound state, there exists an infinite number of these three-body bound states (trimer) centred around the two-body unitarity, forming a geometric tower-like structure [1]. These trimers can be discretely mapped from one to another following a universal scaling relation $E^{(n)} = \lambda^{-2n}E_0$ and $a_-^{(n)} = \lambda^n a_-$, where a_- is the point at which the trimer enters the continuum and λ is the scaling factor, set by details of the system [2].

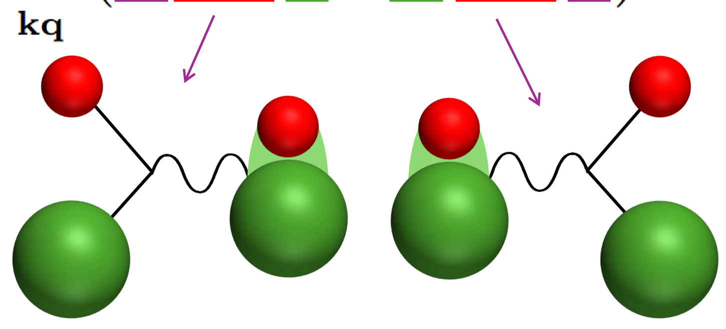


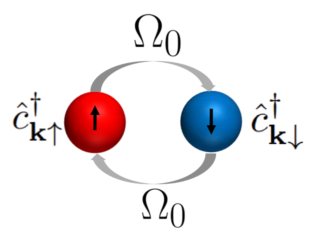
Model Hamiltonian

Boson energy — Impurity energy — Dimer energy — Boson-Impurity interactions through a closed channel molecule

$$\hat{H} = \sum_{\mathbf{k}} \varepsilon^b_{\mathbf{k}} \hat{b}^\dagger_{\mathbf{k}} \hat{b}_{\mathbf{k}} + \sum_{\mathbf{k}\sigma} \varepsilon^c_{\mathbf{k}} \hat{c}^\dagger_{\mathbf{k}\sigma} \hat{c}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}} \varepsilon^d_{\mathbf{k}} \hat{d}^\dagger_{\mathbf{k}} \hat{d}_{\mathbf{k}} + g \sum_{\mathbf{kq}} \left(\hat{d}^\dagger_{\mathbf{q}} \hat{c}_{\mathbf{q-k}\uparrow} \hat{b}_{\mathbf{k}} + \hat{b}^\dagger_{\mathbf{k}} \hat{c}^\dagger_{\mathbf{q-k}\uparrow} \hat{d}_{\mathbf{q}} \right)$$
$$+ \frac{\Omega_0}{2} \sum_{\mathbf{k}} (\hat{c}^\dagger_{\mathbf{k}\uparrow} \hat{c}_{\mathbf{k}\downarrow} + \hat{c}^\dagger_{\mathbf{k}\downarrow} \hat{c}_{\mathbf{k}\uparrow}) + \Delta_0 \sum_{\mathbf{k}} \hat{c}^\dagger_{\mathbf{k}\downarrow} \hat{c}_{\mathbf{k}\downarrow}.$$

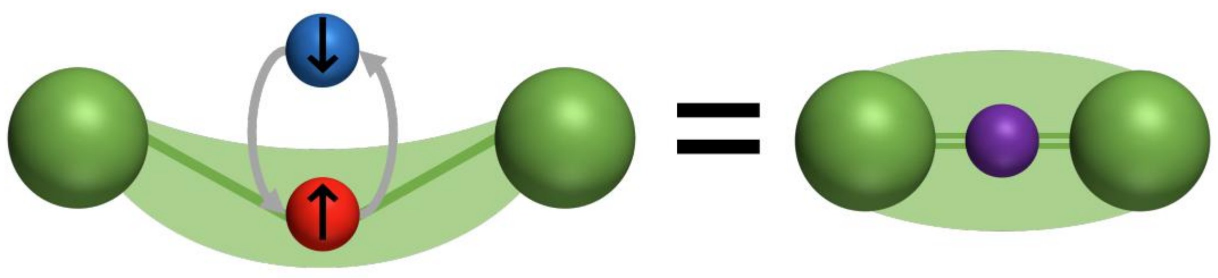
Rabi-Coupling — Detuning





Detuning is the energy difference between the ↑−↓ transition

Schematic

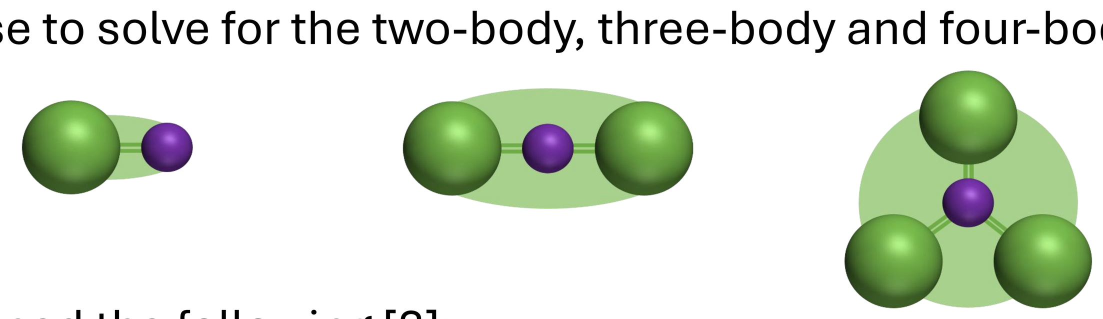


Few-Body Equations

We can solve for the coupled T matrix equation with a finite range parameter (R^*) governing the two-body interactions

$$\mathcal{T}^{-1}_{\uparrow}(E,\mathbf{k}) = \frac{m_r^2 R^*}{\pi}(E - \epsilon^d_{\mathbf{k}}) + \frac{m_r}{2\pi a} - \frac{m_r^{3/2}}{\sqrt{2}\pi}\left(u^2\sqrt{\epsilon_- + \epsilon^d_{\mathbf{k}} - E} + v^2\sqrt{\epsilon_+ + \epsilon^d_{\mathbf{k}} - E}\right)$$

Which we can use to solve for the two-body, three-body and four-body bound states



and we have defined the following [3]:

$$u^2 = \frac{1}{2}\left(1 + \frac{\Delta_0}{\sqrt{\Omega_0^2 + \Delta_0^2}}\right) \qquad \epsilon_\pm = (\Delta_0 \pm \sqrt{\Omega_0^2 + \Delta_0^2})/2$$
$$v^2 = \frac{1}{2}\left(1 - \frac{\Delta_0}{\sqrt{\Omega_0^2 + \Delta_0^2}}\right)$$

Rabi-split energies!

We can transform from the spin basis to the upper/lower dressed basis and perform a scattering expansion:

$$\mathcal{T}_{--}(E) = u^2 T_{\uparrow\uparrow}(E) \qquad k\cot\delta + ik = \frac{2\pi}{m_r}\mathcal{T}^{-1}_{--}(E + \epsilon_-) \simeq a_{\rm eff}^{-1} + R^*_{\rm eff}k^2 + ik$$

Coupled Two-Body Scattering

Due to the Rabi-coupling, the two-body scattering phase shift is altered, resulting in a new scattering length, parameterised by the details of the coupling.

$$a_{\rm eff} = u^2\left(\frac{1}{a} - \frac{1}{a_c}\right)^{-1} \qquad \frac{1}{a_c} = v^2\sqrt{2m_r(\epsilon_+ - \epsilon_-)} - 2R^*\epsilon_- m_r$$

Unitarity has now shifted to $1/a_c$

The scattering continuum has now shifted to E_0^-

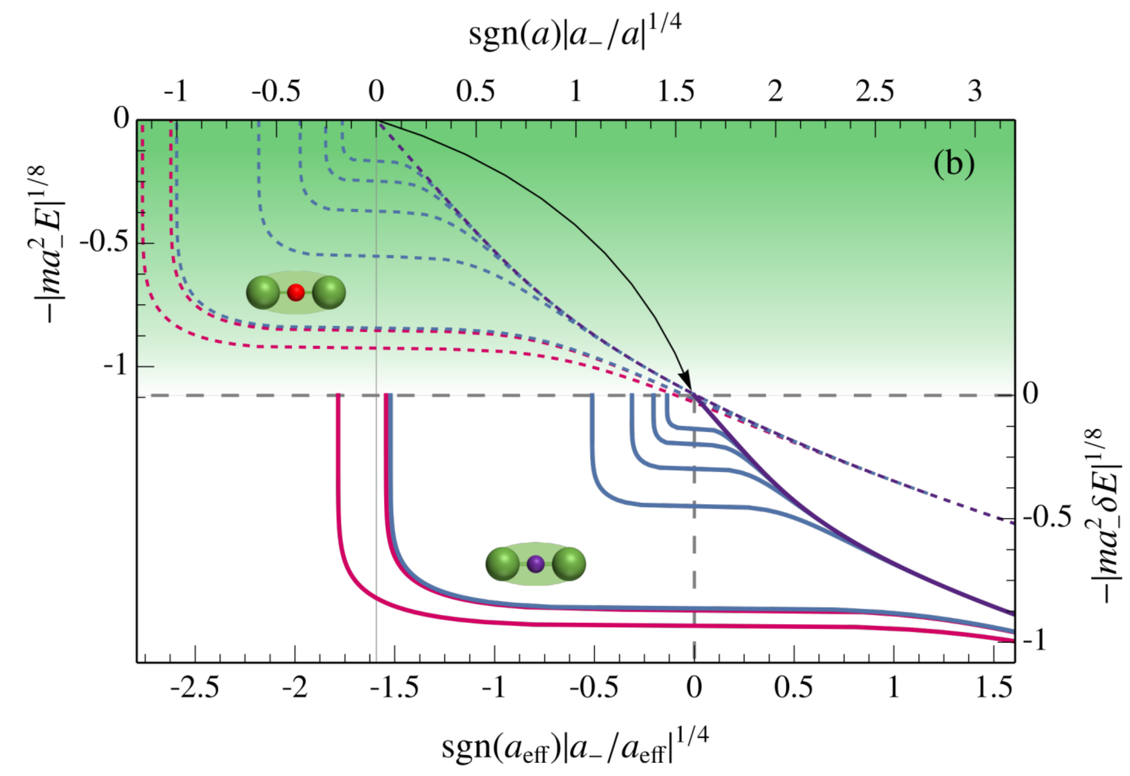


Figure 1: The standard Efimov spectra (dashed) for the CsLi mixture using the left/top axis and the coupled spectra using the right/bottom axis. The coupled spectra has shifted to centre around a critical scattering length and the scattering continuum has been shifted down. Here we have plotted the two-body binding energies (purple), the three-body trimers (blue) and the four body tetramers (pink).

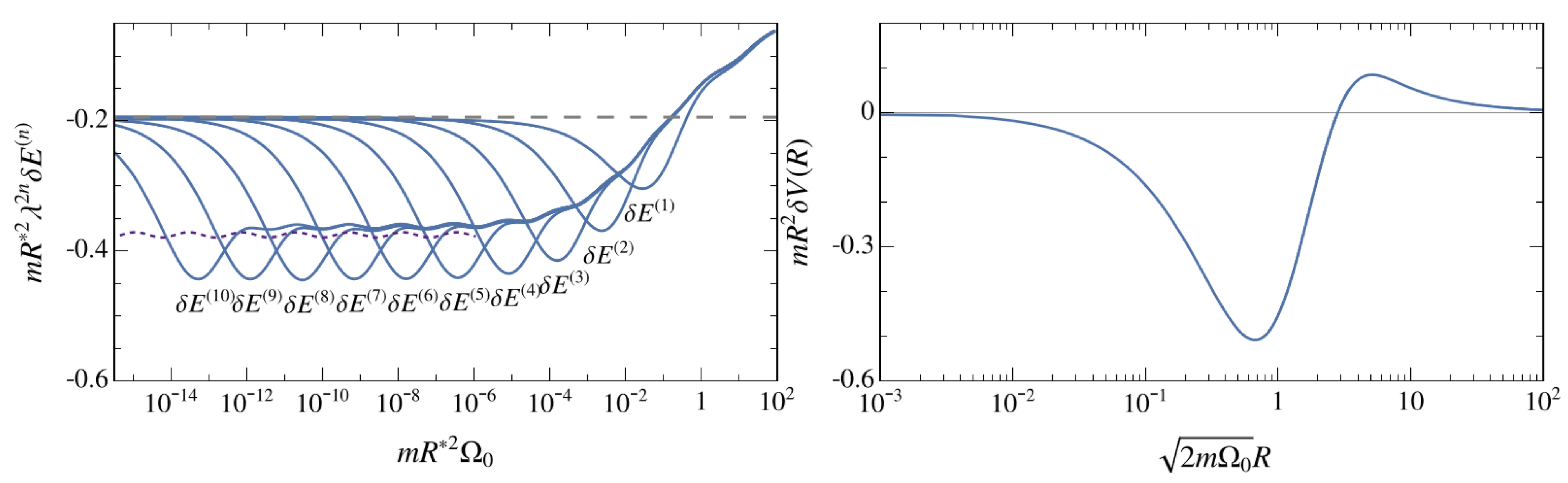


Figure 2: a) The binding energies of the first 10 excited Efimov trimers as a function of Coupling strength. Here we have performed the inverse Efimov mapping ($E_0 = \lambda^{2n}E^{(n)}$) to collapse all states on top of each other, highlighting the emergence of a new three-body parameter. b) The difference in the Born-Oppenheimer [4] effective potentials as a function of the separation between the two heavy atoms. This repulsive deviation is what sets the three-body parameter.

Effective scattering length around new unitarity:

$$a_{\rm eff}(\Delta_0) \propto \frac{1}{\Delta_0 - \Delta_{2B}}$$

The scattering length from a Feshbach resonance:

$$a = a_{bg}\left(1 - \frac{\Delta}{B - B_0}\right)$$

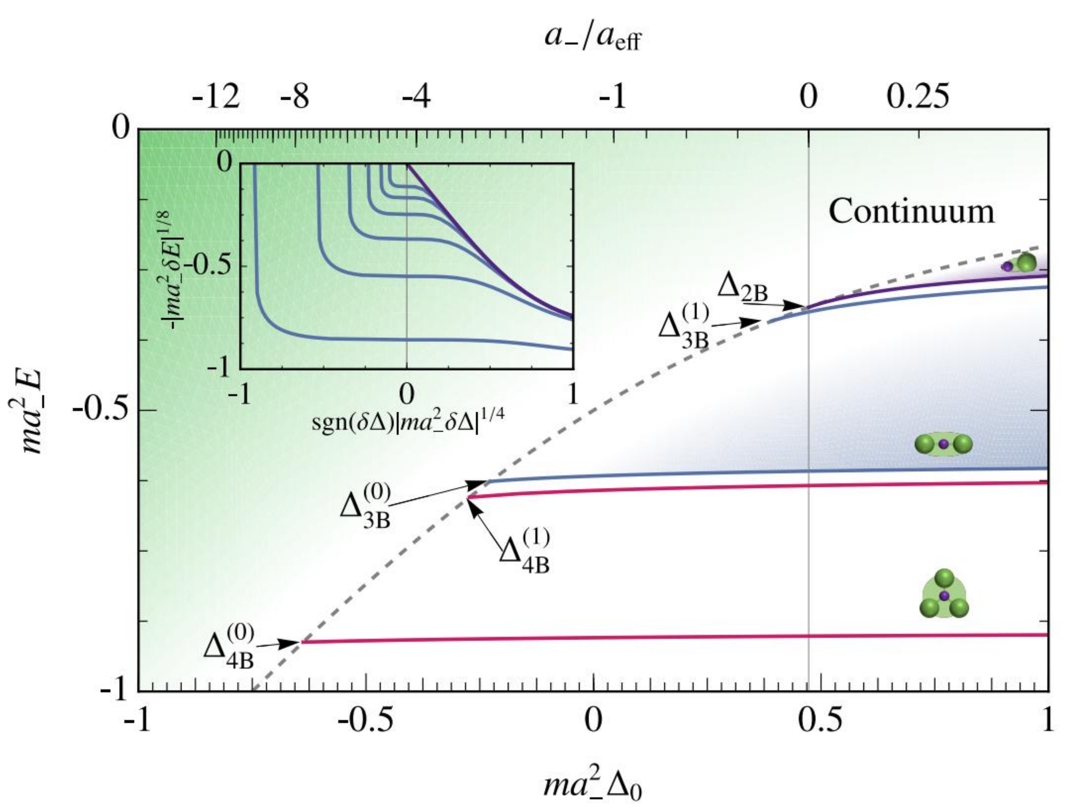


Figure 3: The Efimov spectrum as a function of detuning for a fixed coupling strength and scattering length. We find all the same features as the original spectrum. **Inset:** The Efimov spectrum as a function of renormalised detuning $\delta\Delta = \Delta_0 - \Delta_{2B}$ and renormalised energy $\delta E = E - E_0^-$. We find an essentially identical Efimov spectrum as Fig. 1, with the original tower-like structure and discrete scale invariance.

FOR FURTHER INFORMATION

Anthony Nicholas Zulli

e Anthony.Zulli1@monash.edu

w www.fleet.org.au/innovate/

REFERENCES

[1] V. Efimov, Nuclear Physics A 210, 157 (1973).

[2] P. Naidon And S. Endo, Reports on Progress in Physics 80, 056001 (2017).

[3] B. C. Mulkerin, J. Levinsen, And M. M. Parish, Physical Review A 109, 023302(2024).

[4] A. C. Fonseca, E. F. Redish, and P. E. Shanley, Nuclear Physics A 320, 273 (1979).

[5] A. N. Zulli, B. C. Mulkerin, M. M. Parish, and J. Levinsen, Physical Review Letters 135, 053401 (2025)

ACKNOWLEDGEMENTS

This research is supported by an Australian Government Research Training Program (RTP) Scholarship and by the ARC Center of Excellence in Future Low-energy Electronics Technologies (FLEET).