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## **Optimal Monetary Policy with Asymmetric Targets<sup>\*</sup>**

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### **Abstract:**

We investigate the derivation of optimal interest rate rules in a simple stochastic framework. The monetary authority chooses to minimise an asymmetric loss function made up of the sum of squared components, where the monetary authority places positive weight on squared negative (positive) deviations of output (inflation) and zero weight on squared positive (negative) deviations. Recent approaches to monetary policy under asymmetric preferences have emphasised the adoption of a linear exponential (linex) preference structure. This paper presents a new and different analytic methodology that is based on the explicit calculation of semi-variances. This approach can be used to derive precise coefficients of the optimal interest rate rules. We derive optimal interest rate rules based on two different informational assumptions. In the first case, which we call a fixed interest rate rule, the monetary authority knows only the structure of the economy and the variance of sectoral shocks so that interest rates must take a constant value. In the second case, which we call a flexible interest rate rule, the monetary authority also has access to additional information in that it can observe the contemporaneous inflation rate. In this second case, we restrict our analysis to the class of linear interest rate rules. The more standard approach in the literature derives optimal monetary policy rules using symmetric loss functions, where monetary policy is designed to minimise the sum of squared components. We also compare optimal interest rate rules under both symmetric and asymmetric loss functions.

JEL classification: C61; E43; E58.

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## 1. INTRODUCTION

Following from the seminal paper by Poole (1970), an extensive literature has developed which examines the properties of optimal money supply rules in economies that are faced with stochastic disturbances to different sectors. Poole considered optimal money supply rules in a stochastic IS/LM framework. Optimal money supply rules have also been derived in models with full market clearing when there is an asymmetry of information between the public and private sector; and when money supply responses are fully anticipated. Studies for open economies tend to emphasise the relationship between monetary policy and exchange rate policy. More recent work has emphasised the distinction between anticipated, unanticipated, permanent and temporary shocks and the relationship between wage indexation, lagged feedback rules, and monetary policy.

While this approach has become standard in the literature, even to the point of being adopted as one approach to monetary policy in the standard textbooks on monetary policy, standard monetary operating procedures adopted by central banks now tend to focus on a short-term interest rate as the primary instrument of monetary policy rather than the manipulation of monetary aggregates. As a consequence, recent literature on monetary policy rules has tended to focus on interest rate rules rather than money supply rules (see Walsh, 1998, and Woodford, 2003).

All these approaches to monetary policy tend to assume that monetary policy rules are chosen so as to minimise a quadratic loss function, made up of a weighted sum of squared terms, typically including major macroeconomic indicators, such as output and inflation. Such loss functions are characterised by symmetric properties, in that they give the same weight to positive and negative deviations about some chosen path.

In this paper we will focus on optimal interest rate rules that are chosen so as to minimise a loss function with asymmetric properties, that is, a loss function that gives different weights to positive and negative deviations from a chosen path. Intuitively this makes sense because, for example, agents in the economy are more concerned about negative deviations in output (high unemployment) than they are about positive deviations (overfull employment) and they are more concerned about positive deviations of inflation from some desired path than they are about negative deviations. Recent studies (Nobay and Peel, 2003; Surico, 2007) make a compelling case that, in practice, policymakers are likely to adopt a loss function with asymmetric properties.

Aizenman and Frenkel (1985, Eq. A8, p. 420) show how the microfoundations for a loss function that has asymmetric properties can be developed. Their results demonstrate that by expanding the production function in a Taylor series around the general equilibrium up to second-order terms the loss function will be symmetric; but by also including third-order terms of the Taylor expansion, an asymmetric loss function results.

Previous studies have used computer simulation techniques to examine the implications of asymmetric loss functions for the properties of optimal rules (Friedman, 1975; Kunstman, 1984). Waud (1976) used analytic techniques to examine the properties of optimal responses under asymmetric criteria. Cukierman and Meltzer (1986) considered optimal monetary policy under an asymmetric criterion by considering the case when the policymaker's loss function comprised both quadratic and linear components. Stemp (1993) calculated the semi-variance of a normally distributed variable and showed how these theoretical results could be used to calculate optimal money supply rules when preferences are asymmetric.

Recent studies have focused on the specification of asymmetric preferences using a linear-exponential (linex) specification. This approach to determining optimal monetary rules was first introduced in a theoretical framework by Nobay and Peel (2003). Analysis of asymmetric preferences using the linex specification, or some generalisation of the linex specification, has also been employed in recent empirical studies of monetary policy (Ruge-Murcia, 2003; Surico, 2007; Boinet and Martin, 2008).

The linex specification has the advantage that its theoretical distribution can be determined in a straightforward manner and that the quadratic distribution is nested within the linex specification as a special case. This makes this specification particularly useful for the empirical testing for asymmetric preferences versus symmetric (quadratic) preferences. It has the disadvantage that it only provides an approximation to any particular form of asymmetric preferences and cannot be used to precisely construct a loss structure that gives positive weight to squared positive (negative) deviations, but zero weight to squared negative (positive) deviations of a particular variable.

This paper draws on ideas first presented in Stemp (1993) and subsequently extended further in Stemp (2009). It is possible to use this approach to paste together different components comprising the semi-variances of particular variables (such as output and inflation). The sum of zero weighted and different, but positive, weighted components to form a range of asymmetric loss functions more precisely reflects the objectives of a monetary authority that has truly one-sided targets. As far as this author is aware, the approach to the construction of asymmetric preferences adopted in this paper has not been employed anywhere else, apart from the Stemp (1993, 2009) papers.

The rest of this paper proceeds as follows: Section 2 provides a formula for calculating the semi-variance components of an asymmetric loss function and shows how each of these components can be minimised by appropriate choice of mean and variance. A diagrammatic approach to implement this calculation is introduced in Section 3. A simple monetary model that will be used as the basis for analyses in the rest of the paper is introduced in Section 4. This model is used in Section 5 to calculate optimal fixed interest rate rules under asymmetric preferences. In Section 6, these results are compared with results under a symmetric loss function. The model is used again in Sections 7, 8 and 9 to calculate optimal interest rate rules when the interest rate rule also depends on contemporaneous shocks. Comparisons are also made in these sections with the interest rate rules under the standard symmetric (quadratic) preference structure. Concluding comments are provided in Section 10.

## 2. COMPONENTS OF AN ASYMMETRIC LOSS FUNCTION

Assume that  $X$  is a normally distributed random variable with mean,  $\mu$ , and variance,  $\sigma^2$ . We will write this as:  $X : N(\mu, \sigma^2)$ . We then define two random variables,  $X^+$  and  $X^-$ , as follows:

$$X^+ = \begin{cases} X, & \text{if } X > 0, \\ 0, & \text{otherwise,} \end{cases} \quad (1a)$$

And

$$X^- = \begin{cases} X, & \text{if } X < 0, \\ 0, & \text{otherwise.} \end{cases} \quad (1b)$$

Define  $F(z)$  as the cumulative distribution function for a variable which is normally distributed with mean of zero and variance equal to 1, so that:

$$\Pr(Z < z) = F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z \exp\left(-y^2/2\right) dy. \quad (2)$$

It then follows from Stemp (1993, Appendix), as further developed in Stemp (2009), that:

$$E(X^+)^2 = (\sigma^2 + \mu^2)F\left(\frac{\mu}{\sigma}\right) + \frac{\sigma\mu}{\sqrt{2\pi}} \exp\left(-\frac{\mu^2}{2\sigma^2}\right) \quad (3a)$$

And

$$E(X^-)^2 = (\sigma^2 + \mu^2)F\left(-\frac{\mu}{\sigma}\right) - \frac{\sigma\mu}{\sqrt{2\pi}} \exp\left(-\frac{\mu^2}{2\sigma^2}\right) \quad (3b)$$

If we define the function,  $H(z)$ , as follows:

$$H(z) = (1 + z^2)F(z) + \left(\frac{z}{\sqrt{2\pi}}\right) \exp\left(-z^2/2\right) \quad (4)$$

Then equations (3a, 3b) can be rewritten as:

$$E(X^+)^2 = \left\{H\left(\frac{\mu}{\sigma}\right)\right\} \sigma^2 \quad (5a)$$

$$E(X^-)^2 = \left\{H\left(-\frac{\mu}{\sigma}\right)\right\} \sigma^2 \quad (5b)$$

Equations (5a, 5b), when written in this form, have the useful property that they are separable in the two arguments,  $\frac{\mu}{\sigma}$  and  $\sigma^2$ .

$E(X^+)^2$  and  $E(X^-)^2$  are both always positive valued or zero and so have a minimum value of zero. We explore below, under what circumstances that this minimum value occurs.

Note, from equation (4), that  $H(z) \rightarrow 0$ , as  $z \rightarrow -\infty$ . As a consequence, it is straightforward to show that, for  $\sigma^2 > 0$ ,

$$E(X^+)^2 \rightarrow 0, \text{ as } \frac{\mu}{\sigma} \rightarrow -\infty \quad (6a)$$

$$E(X^-)^2 \rightarrow 0, \text{ as } \mu/\sigma \rightarrow +\infty \quad (6b)$$

Essentially, equations (6a, 6b) tell us that, if we wish to minimise the positive part of the variable,  $X$ , this can be achieved by driving the mean of  $X$  as far away from any positive value as possible. Similarly, if we wish to minimise the negative part of the variable,  $X$ , this can be achieved by driving the mean of  $X$  as far away from any negative value as possible.

It also follows that, as  $\sigma^2 \rightarrow 0$ ,  $H\left(\frac{\mu}{\sigma}\right) < \infty$ , whenever  $\mu \leq 0$ . As a consequence, it is straightforward to show that, as  $\sigma^2 \rightarrow 0$ ,

$$E(X^+)^2 \rightarrow 0, \text{ whenever } \mu \leq 0 \quad (7a)$$

$$E(X^-)^2 \rightarrow 0, \text{ whenever } \mu \geq 0 \quad (7b)$$

In particular, whenever  $\mu = 0$  and  $\sigma^2 \rightarrow 0$ , the expectations terms,  $E(X^+)^2$  and  $E(X^-)^2$ , simultaneously approach zero.

As  $\sigma^2 \rightarrow 0$ , the whole of the value taken by the variable,  $X$ , is concentrated at its mean,  $\mu$ , so that

$$\lim_{\sigma^2 \rightarrow 0} E(X^+)^2 = \begin{cases} \mu^2, & \text{if } \mu > 0 \\ 0, & \text{if } \mu \leq 0 \end{cases} \quad (8a)$$

$$\lim_{\sigma^2 \rightarrow 0} E(X^-)^2 = \begin{cases} 0, & \text{if } \mu \geq 0 \\ \mu^2, & \text{if } \mu < 0 \end{cases} \quad (8b)$$

Essentially, equations (7a, 7b and 8a, 8b) tell us that, if we wish to minimise the positive part of the variable,  $X$ , as  $\sigma^2 \rightarrow 0$ , this can be achieved by  $\mu$  taking a value that is in the non-positive part of  $X$ . Similarly, if we wish to minimise the negative part of the variable,  $X$ , as  $\sigma^2 \rightarrow 0$ , this can be achieved by  $\mu$  taking a value that is in the non-negative part of  $X$ .

### 3. USING DIAGRAMS TO MINIMISE COMPONENTS

In this section, we present a methodology for minimising  $H(z)$  using a diagrammatic approach. Our approach will then be employed later in the paper. We begin by first establishing properties of the function,  $H(z)$ .

$$F(z) = \left( \frac{1}{\sqrt{2\pi}} \right) \int_{-\infty}^z \exp\left(-y^2/2\right) dy \quad (9a)$$

Hence,

$$F_z(z) = \left( \frac{1}{\sqrt{2\pi}} \right) \exp\left(-z^2/2\right) \quad (9b)$$

So that

$$F_{zz}(z) = -\left( \frac{z}{\sqrt{2\pi}} \right) \exp\left(-z^2/2\right) = -zF_z(z) \quad (9c)$$

Since

$$\begin{aligned} H(z) &= (1+z^2)F(z) + \left( \frac{z}{\sqrt{2\pi}} \right) \exp\left(-z^2/2\right) \\ &= (1+z^2)F(z) + zF_z(z) \end{aligned} \quad (10a)$$

It follows that

$$\begin{aligned} H_z(z) &= (1+z^2)F_z(z) + 2zF(z) \\ &\quad + \left( \frac{1}{\sqrt{2\pi}} \right) \exp\left(-z^2/2\right) - \left( \frac{z^2}{\sqrt{2\pi}} \right) \exp\left(-z^2/2\right) \end{aligned} \quad (10b)$$

$$= (1+z^2)F_z(z) + 2zF(z) + F_z(z) - z^2F_z(z) \quad (10c)$$

$$= 2F_z(z) + 2zF(z). \quad (10d)$$

Also,



$$H_{zz}(z) = 2[F_{zz}(z) + F(z) + zF_z(z)] \quad (10e)$$

$$= 2[-zF_z(z) + F(z) + zF_z(z)] \quad (10f)$$

$$= 2F(z) \quad (10g)$$

From Equation (10g), since  $F(z) > 0$ , for all finite values of  $z$ , it follows that the function  $H(z)$  satisfies the second-order conditions for a minimum. From Equation (10d), the first-order conditions for a minimum are satisfied when

$$H_z(z) = 2F_z(z) + 2zF(z) = 0. \quad (11)$$

(Figure 1A about here)

Figure 1 plots  $H_z(z)$ . On the basis of the plot we can determine that the function  $H(z)$  achieves a minimum value when equation (11) is satisfied, that is when  $z \rightarrow -\infty$ . Since  $E(X^+)^2 = \{H(z)\}\sigma^2$ , where  $z = \mu/\sigma$  we can assert that, for fixed variance,  $E(X^+)^2$  takes its minimum value when  $\mu \rightarrow -\infty$ .

A similar methodology can be used, for fixed variance, to choose  $\mu$  so as to minimise  $E(X^-)^2$ . Note that  $E(X^-)^2 = \{H(-z)\}\sigma^2$ , where  $z = \mu/\sigma$ . Thus, in order to determine the minimum value of  $E(X^-)^2$ , for fixed variance, we need to find the value of  $z$  that minimises  $H(-z)$ . For fixed variance,  $H(-z)$  is minimised when

$$-H_z(-z) = 0 \quad (12a)$$

Or equivalently, when

$$H_z(-z) = 0 \quad (12b)$$

Note that the plot of  $H(-z)$  is the mirror image of  $H(z)$  when rotated through the " $z = 0$ " axis. Similarly, the plot of  $H_z(-z)$  is the mirror image of  $H_z(z)$  when rotated through the " $z = 0$ " axis.

(Figure 1B about here)

Figure 1 plots  $H_z(-z)$ . As above and on the basis of the plot we can determine that the function  $H(-z)$  achieves a minimum value when  $z \rightarrow +\infty$ . As a consequence, we can assert that, for fixed variance,  $E(X^-)^2$  takes its minimum value when  $\mu \rightarrow +\infty$ .

#### 4. A SIMPLE MONETARY MODEL

Our purpose in presenting the results of Sections 2 and 3 was to use those results as components of an asymmetric loss functions. To this end, we consider a simple model of the following form:

$$y = \alpha p + u \tag{13a}$$

$$y = -\beta r + v \tag{13b}$$

$$r = \gamma_0 + \gamma_1 p \tag{13c}$$

where

$y$  = real output (expressed in deviations about some target level);

$p$  = inflation (expressed in deviations about some target level);

$r$  = nominal interest rate;

$u, v$  are independent normally distributed variables, both with mean zero and with respective variances given by  $\sigma_u^2, \sigma_v^2$ .

$\alpha, \beta$  are exogenously fixed constants;

$\gamma_0, \gamma_1$  are parameters chosen to minimise a suitable loss function.

Equation (13a) defines the supply-side of the economy through a Phillips curve relationship. Equation (13b) defines the demand-side of the economy through a simple cut-down specification of an IS curve. Equation (13c) defines a simple interest rate rule.

The model can be solved to give solutions for  $y, p$  as follows:

$$y = -\left(\frac{\alpha\beta}{\alpha + \beta\gamma_1}\right)\gamma_0 + \left(\frac{\beta\gamma_1}{\alpha + \beta\gamma_1}\right)u + \left(\frac{\alpha}{\alpha + \beta\gamma_1}\right)v \quad (14a)$$

$$p = -\left(\frac{\beta}{\alpha + \beta\gamma_1}\right)\gamma_0 - \left(\frac{1}{\alpha + \beta\gamma_1}\right)u + \left(\frac{1}{\alpha + \beta\gamma_1}\right)v \quad (14b)$$

Next, we define:

$$y^- = \begin{cases} y, & \text{if } y < 0 \\ 0, & \text{otherwise} \end{cases} \quad (15a)$$

And

$$p^+ = \begin{cases} p, & \text{if } p > 0 \\ 0, & \text{otherwise} \end{cases} \quad (15b)$$

We will choose an interest rate rule (given by some variant of Equation 13c) to minimise an asymmetric loss function of the following form:

$$L_A = \delta E(y^-)^2 + (1 - \delta)E(p^+)^2, \text{ where } 0 < \delta < 1. \quad (16)$$

We consider two types of interest rate rules:

- ***A fixed interest rule*** of the form:

$$r = \gamma_0 \quad (17a)$$

This rule assumes that the setter of interest rates (the central bank) has access to the structure of the economy, given by Equations (13a-13c, 15a-15b and 16)

and the mean and variance of contemporaneous shocks, but does not have access to any additional information. The optimal rule is derived by fixing  $\gamma_1 = 0$ , keeping  $\gamma_0$  as a choice parameter in Equations (13a, 13b), and choosing  $\gamma_0$  so as to minimise Equation (16).

- ***A flexible interest rate rule*** of the form:

$$r = \gamma_0 + \gamma_1 p \tag{17b}$$

This rule assumes that, in addition to the information available for the fixed interest rate rule, the central bank also has access to the contemporaneous value of inflation,  $p$ . An interest rate rule that is non-linear would do better than the linear rule considered here. However, as we wish to compare our results with those derived under the standard Poole (1970) methodology based on symmetric loss functions and also in order to keep the analysis tractable, we will restrict our analysis to the linear rule given by Equation (17b). The optimal rule is derived by keeping both  $\gamma_0$  and  $\gamma_1$  as choice parameters in Equations (14a, 14b), and choosing  $\gamma_0, \gamma_1$  so as to minimise Equation (16).

We consider the implications of each of these rule types in turn.

## **5. OPTIMAL POLICY UNDER A FIXED INTEREST RATE RULE**

Under the type of fixed interest rate rule considered here, the monetary authority is unable to influence the impact of stochastic shocks in the economy and can only influence the means of  $y, p$ .

Then  $y : N(\mu_y, \sigma_y^2)$  and  $p : N(\mu_p, \sigma_p^2)$  where

$$\mu_y = -\beta\gamma_0 \quad (18a)$$

$$\sigma_y^2 = \sigma_v^2 \quad (18b)$$

$$\mu_p = -\left(\frac{\beta}{\alpha}\right)\gamma_0 \quad (18c)$$

$$\sigma_p^2 = \left(\frac{1}{\alpha}\right)^2 [\sigma_u^2 + \sigma_v^2] \quad (18d)$$

So that the loss function, given by Equation (16), can be written in the form:

$$L_A = \delta H\left(\frac{-\mu_y}{\sigma_y}\right) \sigma_y^2 + (1-\delta) H\left(\frac{\mu_p}{\sigma_p}\right) \sigma_p^2 \quad (19a)$$

**Case of positive demand-side shocks ( $\sigma_u^2 \geq 0, \sigma_v^2 > 0$ )**

We first focus on the case when there are positive demand shocks ( $\sigma_v^2 > 0$ ).

Equation (19a) can be reduced to:

$$L_A = \delta \left\{ H\left(\frac{\beta\gamma_0}{\sigma_v}\right) \right\} \sigma_v^2 + (1-\delta) \left\{ H\left(\frac{-\beta\gamma_0}{\sqrt{\sigma_u^2 + \sigma_v^2}}\right) \right\} \left[ \frac{(\sigma_u^2 + \sigma_v^2)}{\alpha^2} \right] \quad (19b)$$

Choosing  $\gamma_0$  to minimise Equation (19b) is equivalent to choosing  $z$  to minimise:

$$J(z) = \alpha^2 \theta^2 \delta H(z) + (1-\delta) H(-\theta z), \quad (19c)$$

$$\text{where } z = \frac{(\beta\gamma_0)}{\sigma_v} \text{ and } \theta = \frac{\sigma_v}{\sqrt{\sigma_u^2 + \sigma_v^2}} > 0$$

Taking first derivatives of  $J(z)$  with respect to  $z$  yields the first-order condition:

$$\alpha^2 \theta \delta H_1(z) = (1-\delta) H_1(-\theta z) \quad (20)$$

This also satisfies appropriate second-order conditions for a minimum.

We define:

$$g(z; \delta, \theta) = \alpha^2 \theta \delta H_1(z) \quad (21a)$$

$$h(z; \delta, \theta) = (1 - \delta) H_1(-\theta z) \quad (21b)$$

Figure 1 shows plots of  $g(z; \delta, \theta)$  and  $h(z; \delta, \theta)$  and illustrate how these curves adjust as the parameters,  $\delta$  and  $\theta$ , are allowed to vary.

(Figure 2 about here)

It will be observed that  $g(z; \delta, \theta)$  moves upward as  $\delta$  and/or  $\theta$  increase. Similarly,  $h(z; \delta, \theta)$  moves downwards as  $\delta$  and/or  $\theta$  increase. From Equations (20, 21a, 21b), observe that the minimum occurs at the intersection of these two curves. Thus, as  $\delta$  and/or  $\theta$  increase, the value of  $z$  (and hence of  $\gamma_0$ ) becomes smaller. Since the two curves can be plotted numerically, the graphical methodology described here can be used to derive precise numerical values for  $\gamma_0$ , limited only by the constraints imposed by machine precision. The following special cases arise:

$$\gamma_0 = +\infty \ (z = +\infty), \text{ when } \delta = 0 \quad (22a)$$

$$\gamma_0 = 0 \ (z = 0), \text{ when } \delta = \frac{1}{1 + \alpha^2 \theta} \quad (22b)$$

$$\gamma_0 = -\infty \ (z = -\infty), \text{ when } \delta = 1 \quad (22c)$$

### ***Intuitive justification of results***

The polar cases for  $\delta$  coincide with the cases where the policy maker is focused solely on minimising  $E(p^+)^2$ , when  $\delta = 0$ ; and focused solely on minimising  $E(y^-)^2$ , when  $\delta = 1$ . Thus, a highly contractionary monetary policy is appropriate when  $\delta = 0$ ; and a highly expansionary monetary policy is appropriate

when  $\delta = 1$ . This is consistent with the results given from Figure 1 and by Equations (22a, 22c).

The polar cases for  $\theta$  are demonstrated in Figures 3A and 3B. Figure 3A illustrates the case when there is primarily a supply-side shock, so that  $\theta \rightarrow 0$ . Figure 2B illustrates the case when there is primarily a demand-side shock, so that  $\theta \rightarrow 1$ . For simplicity, assume that, in each diagram,  $E(p^+)^2$ , which can be loosely aligned with the aggregate shock above  $L_1L_1'$ , is equal to 1. Then, from Figures 3A and 3B, for any given value of  $\gamma_0$ , it will be observed that  $E(y^-)^2$ , which can be loosely aligned with the aggregate shock to the left of  $L_2L_2'$ , is less for the supply shock than it is for the demand shock.

(Figures 3A and 3B about here)

Thus, for a given value of  $\delta$ , if we let  $\gamma_0^*$  be the optimal value of  $\gamma_0$  under a supply-side shock, then, under a demand-side shock,  $\gamma_0^*$  will be associated with too high a value for  $E(y^-)^2$ . We can correct this by reducing the value of  $E(y^-)^2$  and increasing the value of  $E(p^+)^2$ ; that is, by increasing the expected value of  $y$ . This can be achieved by adopting a more expansionary monetary policy stance, that is by reducing  $\gamma_0$ . Thus, as  $\theta$  increases, the optimal choice of  $\gamma_0$  will be reduced. This is consistent with the results derived from Figure 1.

***Case when there are supply-side shocks, but no demand-side shocks ( $\sigma_u^2 > 0, \sigma_v^2 = 0$ )***

In the case where  $\sigma_v^2 = 0$ , so that  $\theta = 0$ , the above analysis needs to be modified as follows.

Let  $z = (\beta\gamma_0)/\sigma_u$ . Then, firstly,  $g(z; \delta, \theta)$  should be replaced with:

$$g^*(z; \delta) = \begin{cases} 0, & \text{if } z \leq 0 \\ 2\alpha^2\delta z, & \text{if } z > 0 \end{cases} \quad (23a)$$

Secondly,  $h(z; \delta, \theta)$  should be replaced with:

$$h^*(z; \delta) = (1 - \delta)H_1(-z) \quad (23b)$$

(Figure 4 about here)

The optimal value of  $\gamma_0$  can then be derived using Figure 4. Once again, the optimal value of  $z$ , and hence of  $\gamma_0$ , is determined by the intersection of the two curves. The following special cases arise:

$$\gamma_0 = +\infty \quad (z = +\infty), \text{ when } \delta = 0 \quad (24a)$$

$$-\infty < \gamma_0 \leq 0 \quad (-\infty < z < 0), \text{ when } \delta = 1 \quad (24b)$$

The results when  $\theta = 0$  can be interpreted as limiting cases of the results when  $\theta > 0$  and, as such, are consistent with the intuitive justification of results provided above.

## 6. COMPARISON WITH OPTIMAL RULE UNDER A SYMMETRIC LOSS FUNCTION

It is informative to compare the results derived above under an asymmetric function with those derived under the more commonly used symmetric quadratic loss function. In this section we examine optimal interest rules derived under the following symmetric loss function:

$$L_S = \delta E(y)^2 + (1 - \delta)E(p)^2 \quad (25a)$$

When  $y : N(\mu_y, \sigma_y^2)$  and  $p : N(\mu_p, \sigma_p^2)$ , this loss function can be rewritten as:

$$L_S = \delta(\mu_y^2 + \sigma_y^2) + (1 - \delta)(\mu_p^2 + \sigma_p^2) \quad (25b)$$

This can be rewritten as:



$$L_S = [\delta\mu_y^2 + (1-\delta)\mu_p^2] + [\delta\sigma_y^2 + (1-\delta)\sigma_p^2] \quad (25c)$$

The results derived under the symmetric loss function are standard in the literature and, in this case, are given by the following:

Under the ***fixed interest rule***, the optimal interest rate rule under a symmetric loss function is given by:

- $r = 0$ , irrespective of the relative magnitude of the shocks (26)

The optimal fixed interest rate rule is one where  $\gamma_0$  is chosen so as to drive  $\mu_y$  and  $\mu_p$  to zero and  $\gamma_1$  is not a parameter of choice but rather fixed at zero.

Under the ***flexible interest rate rule***, the optimal interest rate rule under a symmetric loss function is given by:

- $r = \gamma_1 p$  where  $\gamma_0 = 0$  and  $\gamma_1$  is a function of  $\sigma_u^2, \sigma_v^2, \delta$  as well as other parameters in the economy. (27)

The optimal flexible interest rate rule is one where  $\gamma_0$  is chosen so as to drive  $\mu_y$  and  $\mu_p$  to zero and  $\gamma_1$  is chosen so as to minimise  $\delta\sigma_y^2 + (1-\delta)\sigma_p^2$ .

It will be observed that, for a symmetric loss function, under both types of interest rate rule it is optimal to drive  $\mu_y$  and  $\mu_p$  to zero. For our analyses, this is equivalent to requiring that the optimal value of  $z$  is zero. This is not the case for the fixed interest rate rule, unless  $\delta = \frac{1}{(1+\alpha^2\theta)}$ . Thus, in general for a fixed interest rate rule, the optimal  $\gamma_0$  will not drive  $\mu_y$  and  $\mu_p$  to zero and the optimal interest rate rule under an asymmetric loss function is generally different to that under a symmetric

loss function. Comparison of results for the fixed interest rate rule are summarised in Table 1.

(Table 1 about here)

Similarly, we will show below that, under a flexible interest rate rule, in general, the optimal value of  $z$  is non-zero. But there are significant exceptions.

## 7. APPROACH UNDER A FLEXIBLE INTEREST RATE RULE

Under the flexible interest rate rule considered here, the monetary authority is able to influence the impact of stochastic shocks in the economy and so can influence both the means and variances of  $y, p$ . To aid in tractability, we restrict our analysis to the class of linear interest rate rules defined by Equation (17b).

Then  $y : N(\mu_y, \sigma_y^2)$  and  $p : N(\mu_p, \sigma_p^2)$  where

$$\mu_y = -\left(\frac{\alpha\beta}{\alpha + \beta\gamma_1}\right)\gamma_0 \quad (28a)$$

$$\sigma_y^2 = \frac{[(\beta\gamma_1)^2 \sigma_u^2 + \alpha^2 \sigma_v^2]}{(\alpha + \beta\gamma_1)^2} \quad (28b)$$

$$\mu_p = -\left(\frac{\beta}{\alpha + \beta\gamma_1}\right)\gamma_0 \quad (28c)$$

$$\sigma_p^2 = \frac{[\sigma_u^2 + \sigma_v^2]}{(\alpha + \beta\gamma_1)^2} \quad (28d)$$

It follows that the asymmetric loss function, given by Equation (16), can be written in the form:

$$L_A = \delta H\left(\frac{-\mu_y}{\sigma_y}\right)\sigma_y^2 + (1 - \delta)H\left(\frac{\mu_p}{\sigma_p}\right)\sigma_p^2 \quad (29a)$$

Choosing  $\gamma_0$  and  $\gamma_1$  to minimise Equation (29a) is equivalent to choosing  $z$  (thus determining  $\gamma_0$ ) and making consistent choices of  $\theta$  and  $\gamma_1$  (thus determining  $\gamma_1$ ) so as to minimise:

$$J(z, \theta, \gamma_1) = \frac{\left[ \theta^2 \delta H\left(\frac{\alpha z}{\theta}\right) + (1 - \delta)H(-z) \right]}{(\alpha + \beta \gamma_1)^2} \quad (29b)$$

$$\text{Where } z = \frac{\beta \gamma_0}{\sqrt{\sigma_u^2 + \sigma_v^2}} \text{ and } \theta = \sqrt{\frac{[(\beta \gamma_1)^2 \sigma_u^2 + \alpha^2 \sigma_v^2]}{(\sigma_u^2 + \sigma_v^2)}} \quad (29c)$$

For the flexible interest rate rule, we will first consider the two special cases when the shocks come from only one sector at a time.

## 8. FLEXIBLE INTEREST RATE RULE: SHOCKS TO ONE SECTOR AT A TIME

### *Demand-side shock only ( $\sigma_u^2=0, \sigma_v^2>0$ )*

When there are only demand-side shocks, it follows that  $\theta = \alpha$  and  $z = \frac{\beta \gamma_0}{\sigma_v}$ . Then, Equation (29b) reduces to:

$$J(z, \theta, \gamma_1) = J(z, \gamma_1) = \frac{\left[ \alpha^2 \delta H(z) + (1 - \delta)H(-z) \right]}{(\alpha + \beta \gamma_1)^2} \quad (30)$$

This can be minimised by choosing  $\gamma_0 = 0$  ( $z = 0$ ) and letting  $\gamma_1 \rightarrow \infty$ . This is equivalent to choosing an interest rate rule to ensure that the aggregate demand curve given by Equation (13b) is horizontal in  $p$ - $y$  space where  $p$  is on the vertical axis and  $y$  is on the horizontal axis. Since any demand-side shock will just move the aggregate demand curve to the right or left, thus the aggregate demand curve remains unchanged by a demand-side shock and so both output and inflation will be unchanged by such a shock.

**Supply-side shock only ( $\sigma_u^2 > 0, \sigma_v^2 = 0$ )**

In this case, note that  $\theta = \beta\gamma_1$  and  $z = \beta\gamma_0 / \sigma_u$ . Equation (26b) then reduces to

$$J(z, \theta, \gamma_1) = J(z, \gamma_1) = \frac{\left[ (\beta\gamma_1)^2 \delta H\left(\frac{\alpha z}{\beta\gamma_1}\right) + (1-\delta)H(-z) \right]}{(\alpha + \beta\gamma_1)^2} \quad (31)$$

We next apply the first-order conditions for a minimum. Firstly, setting  $\frac{\partial J}{\partial z} = 0$  ensures that:

$$\alpha\beta\gamma_1 \delta H_1\left(\frac{\alpha z}{\beta\gamma_1}\right) = (1-\delta)H_1(-z) \quad (32)$$

Secondly, setting  $\left(\frac{\partial J}{\partial \gamma_1}\right) = 0$  ensures that:

$$\left(\frac{-2\beta}{(\alpha + \beta\gamma_1)}\right) J(z, \gamma_1) + \frac{2\beta(\beta\gamma_1) \delta H\left(\frac{\alpha z}{\beta\gamma_1}\right)}{(\alpha + \beta\gamma_1)^2} - \frac{z\alpha\beta \delta H_1\left(\frac{\alpha z}{\beta\gamma_1}\right)}{(\alpha + \beta\gamma_1)^2} = 0 \quad (33a)$$

The first two terms of Equation (50a) can be reduced to:

$$2\beta \left[ \alpha\beta\gamma_1 \delta H\left(\frac{\alpha z}{\beta\gamma_1}\right) - (1-\delta)H(-z) \right] \frac{1}{(\alpha + \beta\gamma_1)^3} \quad (33b)$$

Note that  $H(0) \neq 0$  and  $H_1(0) \neq 0$ . Hence, using Equations (32, 33b) it is straightforward to show by substitution that the first-order necessary conditions are satisfied when the following two equations are satisfied simultaneously:

$$z = 0 \quad (34a)$$

$$\alpha\beta\gamma_1\delta = (1-\delta) \quad (34b)$$

Thus the parameters of the optimal rule are given by:

$$\gamma_0 = 0 \quad (35a)$$

$$\gamma_1 = \frac{(1-\delta)}{\alpha\beta\delta} \quad (35b)$$

And the optimal flexible interest rate rule when there is only a supply-side shock can be written as

$$r = \gamma_1 p \quad \text{where} \quad \gamma_1 = \frac{(1-\delta)}{\alpha\beta\delta} \quad (36)$$

### ***Comparison with results under symmetric loss function***

In the case when there are demand-side shocks only ( $\sigma_u^2 = 0, \sigma_v^2 > 0$ ), the symmetric loss function reduces to:

$$L_s = \frac{(\delta\alpha^2 + 1 - \delta)\sigma_v^2}{(\alpha + \beta\gamma_1)^2} \quad (37)$$

Thus, the optimal rule under a symmetric loss function is given by letting  $\gamma_1 \rightarrow \infty$ , or equivalently by allowing the interest rate to vary so as to ensure:

$$p = 0 \quad (38)$$

This is also the optimal rule under the asymmetric loss function.

The case when there are supply-side shocks only ( $\sigma_u^2 > 0, \sigma_v^2 = 0$ ) shows that these are not the only situations when rules are identical under symmetric and asymmetric loss functions. As given by Equations (35a, 35b) the parameters of the optimal flexible interest rate rule under an asymmetric loss function are given by:

$$\gamma_0 = 0 \quad (39a)$$

$$\gamma_1 = \frac{(1-\delta)}{\alpha\beta\delta} \quad (39b)$$

When  $\gamma_0 = 0$ ,  $\mu_y = \mu_p = 0$ . Also the asymmetric loss function can be reduced to the form:

$$L_A = \frac{(\delta\sigma_y^2 + (1-\delta)\sigma_p^2)}{2} \quad (40)$$

Hence, the optimal rule, given by Equations (39a, 39b), chooses  $\gamma_0$  so as to drive  $\mu_y$  and  $\mu_p$  to zero and chooses  $\gamma_1$  so as to minimise  $\delta\sigma_y^2 + (1-\delta)\sigma_p^2$ . Thus, this rule also minimises the symmetric loss function given by:

$$L_S = \delta(\mu_y^2 + \sigma_y^2) + (1-\delta)(\mu_p^2 + \sigma_p^2) \quad (41)$$

The optimal flexible interest rate rule, when there are supply-side shocks only, is therefore identical under both symmetric and asymmetric loss functions.

(Table 2 about here)

Comparison of results for the flexible interest rate rule when shocks come from only one sector are summarised in Table 2.

## 9. FLEXIBLE INTEREST RATE RULE: SIMULTANEOUS SHOCKS TO BOTH SECTORS

### *Simultaneous output and inflation targeting* ( $0 < \delta < 1$ )

We will now consider the general case when there are simultaneous demand-side and supply-side shocks and  $0 < \delta < 1$ . We now apply the first-order conditions to Equations (29b, 29c) to establish necessary conditions for an interior minimum.

Firstly, setting  $\frac{\partial J}{\partial z} = 0$  ensures that:

$$\alpha\theta\delta H_1(\alpha z/\theta) = (1-\delta)H_1(-z) \quad (42)$$

Secondly, noting that  $\theta$  is a function of  $\gamma_1$  and setting:

$$\left(\frac{\partial J}{\partial \gamma_1}\right) + \left(\frac{\partial J}{\partial \theta}\right)\left(\frac{d\theta}{d\gamma_1}\right) = 0 \quad (43a)$$

ensures that:

$$\left(\frac{-2\beta}{(\alpha + \beta\gamma_1)}\right) J(z, \theta, \gamma_1) + \frac{2\theta\delta\left(\frac{d\theta}{d\gamma_1}\right)H(\alpha z/\theta)}{(\alpha + \beta\gamma_1)^2} - z\alpha\delta\left(\frac{d\theta}{d\gamma_1}\right)H_1(\alpha z/\theta) \Big/ (\alpha + \beta\gamma_1)^2 = 0 \quad (43b)$$

When  $z$  equals zero, the third term of Equation (43b) is eliminated. Hence, for what follows, we can focus on the first two terms of Equation (43b). The first two terms of Equation (42b) reduce to the following:

$$\left(\frac{2\beta}{(\alpha + \beta\gamma_1)^3}\right) \left\{ \delta \left[ \frac{(\alpha\beta\gamma_1\sigma_u^2 - \alpha^2\sigma_v^2)}{(\sigma_u^2 + \sigma_v^2)} \right] H(\alpha z/\theta) - (1-\delta)H(-z) \right\} \quad (43c)$$

Equation (43c) equals zero if

$$\delta \left[ \frac{(\alpha\beta\gamma_1\sigma_u^2 - \alpha^2\sigma_v^2)}{(\sigma_u^2 + \sigma_v^2)} \right] H(\alpha z/\theta) = (1-\delta)H(-z) \quad (43d)$$

Using Equations (42, 43d) it is straightforward to show by substitution that the first-order necessary conditions are not simultaneously satisfied when  $\gamma_0 = 0$  ( $z = 0$ ) and hence that the results under the asymmetric loss function are not the same as the results under the symmetric loss function. We prove this by contradiction.

Once again, note that  $H(0) \neq 0$  and  $H_1(0) \neq 0$ . Assume that the first-order necessary conditions satisfy  $\gamma_0 = 0$  ( $z = 0$ ). Then, from Equations (42, 43c) the following two equations are satisfied simultaneously:

$$\alpha\theta\delta = (1 - \delta) \quad (44a)$$

$$\delta(\alpha\beta\gamma_1\sigma_u^2 - \alpha^2\sigma_v^2) = (1 - \delta)(\sigma_u^2 + \sigma_v^2) \quad (44b)$$

Combining Equations (44a, 44b) and noting that  $0 < \delta < 1$  and  $\alpha > 0$

$$\beta\gamma_1\sigma_u^2 - \alpha\sigma_v^2 = \theta(\sigma_u^2 + \sigma_v^2) \quad (45a)$$

where

$$\theta = \sqrt{\frac{[(\beta\gamma_1)^2\sigma_u^2 + \alpha^2\sigma_v^2]}{[\sigma_u^2 + \sigma_v^2]}} \quad (45b)$$

Hence, squaring both sides of Equation (45a),

$$(\beta\gamma_1)^2\sigma_u^4 + \alpha^2\sigma_v^4 - 2\alpha(\beta\gamma_1)\sigma_u^2\sigma_v^2 = \theta^2(\sigma_u^2 + \sigma_v^2)^2 \quad (45c)$$

$$= [(\beta\gamma_1)^2\sigma_u^2 + \alpha^2\sigma_v^2](\sigma_u^2 + \sigma_v^2) \quad (45d)$$

Equation (44d) can be reduced to:

$$(\alpha + \beta\gamma_1)^2\sigma_u^2\sigma_v^2 = 0 \quad (45e)$$

But then, since  $\sigma_u^2 > 0$  and  $\sigma_v^2 > 0$ , it follows that:

$$\alpha + \beta\gamma_1 = 0 \quad (45f)$$

Then, by substitution in Equation (44b),

$$\theta = \alpha \quad (45g)$$

And by substitution in Equation (43a)

$$\alpha^2\delta = (1 - \delta) \quad (45h)$$

Then, given Equation (45h) and assuming  $z = 0$ , the asymmetric loss function summarised by Equations (29a - 29c) reduces to:

$$L_A = \frac{[\alpha^2\delta(\sigma_u^2 + \sigma_v^2)]}{(\alpha + \beta\gamma_1)^2} \quad (46)$$



And, using Equation (45f), this diverges to  $+\infty$  whenever  $0 < \delta < 1$ . Clearly, if  $\gamma_1 \rightarrow \infty$ ,  $L_A$  will take a finite value, so Equation (45h) is not consistent with a minimum. This is a contradiction. Hence, our initial assumption that  $z = 0$  is not valid. It follows that, when  $\sigma_u^2 > 0$ ,  $\sigma_v^2 > 0$  and  $0 < \delta < 1$ , the optimal interest rate rules under the symmetric and asymmetric loss functions cannot be the same.

While we are not able to say more about the general case, we can cast further light on the two polar cases excluded from the above analysis: inflation targeting (when  $\delta = 0$ ) and output targeting (when  $\delta = 1$ ).

### ***Inflation targeting ( $\delta=0$ )***

In this case, Equations (29b, 29c) reduce to:

$$J(z, \theta, \gamma_1) = J(z, \gamma_1) = \frac{H(-z)}{(\alpha + \beta\gamma_1)^2}, \text{ where } z = \frac{\beta\gamma_0}{\sqrt{\sigma_u^2 + \sigma_v^2}} \quad (47)$$

This can be minimised by choosing  $\gamma_0 = 0$  ( $z = 0$ ) and letting  $\gamma_1 \rightarrow \infty$ . This is equivalent to choosing an interest rate rule to ensure that inflation is constantly maintained at  $p = 0$ . Such a rule will ensure that  $E(p^+)^2 = 0$ . This is also the optimal rule under a symmetric loss function.

### ***Output targeting ( $\delta=1$ )***

In this case, Equations (29b, 29c) reduces to:

$$J(z, \theta, \gamma_1) = \frac{[(\theta^2)H(\alpha z/\theta)]}{(\alpha + \beta\gamma_1)^2} \quad (48a)$$

$$\text{Where } z = \frac{\beta\gamma_0}{\sqrt{\sigma_u^2 + \sigma_v^2}} \text{ and } \theta = \sqrt{\frac{[(\beta\gamma_1)^2 \sigma_u^2 + \alpha^2 \sigma_v^2]}{[(\sigma_u^2 + \sigma_v^2)]}} \quad (48b)$$

It is not possible to choose  $\gamma_1$  so that  $\sigma_y^2$  is driven to zero. The optimal rule requires choosing an interest rate rule where  $\gamma_0 \rightarrow -\infty$  ( $z \rightarrow -\infty$ ). This is equivalent to choosing an interest rate rule that is highly expansionary. Such a rule will ensure that  $E(y^-)^2 \rightarrow 0$ . This is not the optimal rule under a symmetric loss function.

### ***Comparison with results under symmetric loss function***

In general, under a flexible interest rate rule and asymmetric loss function, the optimal value of  $z$  is non-zero. Thus the optimal interest rate rule is different under symmetric and asymmetric loss functions. This applies in those cases where  $\sigma_u^2 > 0, \sigma_v^2 > 0$  and  $0 < \delta \leq 1$ . As shown above, exceptions arise in other cases: when  $\delta = 0$  and when shocks emanate in only one sector at a time.

As can be demonstrated using Equations (8a, 8b), one exception arises when it is possible to choose  $\gamma_1$  so that  $\delta\sigma_y^2$  and  $(1-\delta)\sigma_p^2$  are simultaneously driven to zero. Thus the optimal flexible interest rate rule under inflation targeting ( $\delta = 0$ ) is given by  $p = 0$  (consistent with  $\gamma_1 \rightarrow \infty$ ). This rule drives  $\sigma_p^2$  to zero, resulting in identical rules for this case under both symmetric and asymmetric loss functions.

## **10. CONCLUDING COMMENTS**

This paper has investigated the derivation of optimal interest rate rules in a simple stochastic framework where the monetary authority chooses to minimise an asymmetric loss function. We have focused on deriving optimal interest rate rules based on two different informational assumptions. In the first case, which we call a

fixed interest rate rule, the monetary authority knows only the structure of the economy and the variance of sectoral shocks so that interest rates must take a constant value. In the second case, which we call a flexible interest rate rule, the monetary also has access to additional information in that it can observe the contemporaneous inflation rate. In this second case, we restrict our analysis to the class of linear interest rate rules. The paper has presented an analytic methodology that could be used, in conjunction with computing techniques, to derive precise coefficients of the optimal interest rate rules in each of these cases.

While this paper has focused on deriving optimal interest rate rules under an asymmetric preference structure, the more standard approach, first developed by Poole (1970), derives optimal monetary policy rules using symmetric loss functions. Our analysis also compares optimal interest rate rules under both symmetric and asymmetric loss functions.

We have shown that, in general, the optimal rules derived under asymmetric loss functions will be different than the optimal rules derived under symmetric loss functions. Under the fixed interest rate rule, the optimal rules differ. However, under the flexible interest rate rule, it is possible to construct special cases where the optimal rules are the same.

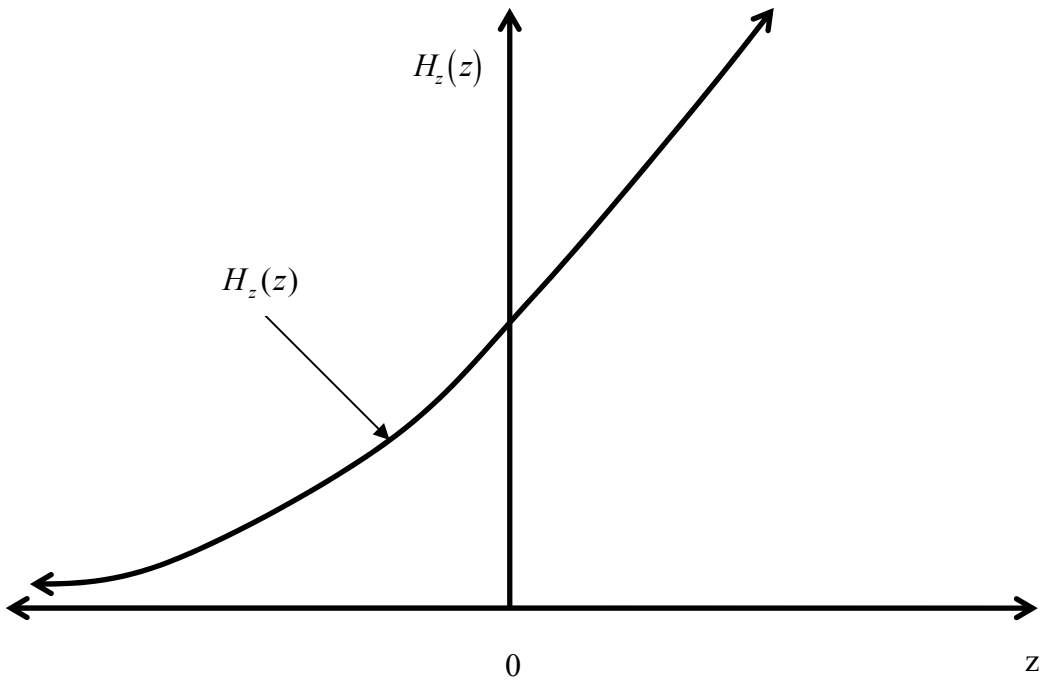
The results of this paper could easily be extended in a variety of ways along the lines of extensions to the Poole (1970) literature. For example, our simple monetary model could be extended to incorporate different informational assumptions and different expectations assumptions. It would also be possible to consider asymmetric loss functions in conjunction with a dynamic monetary modelling framework.

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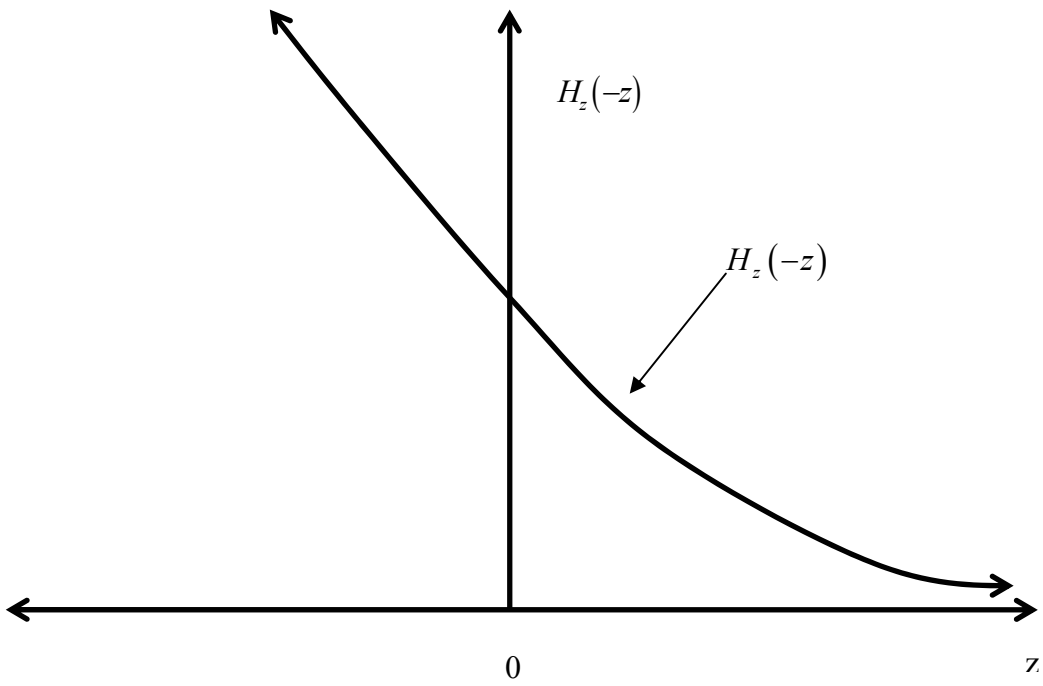
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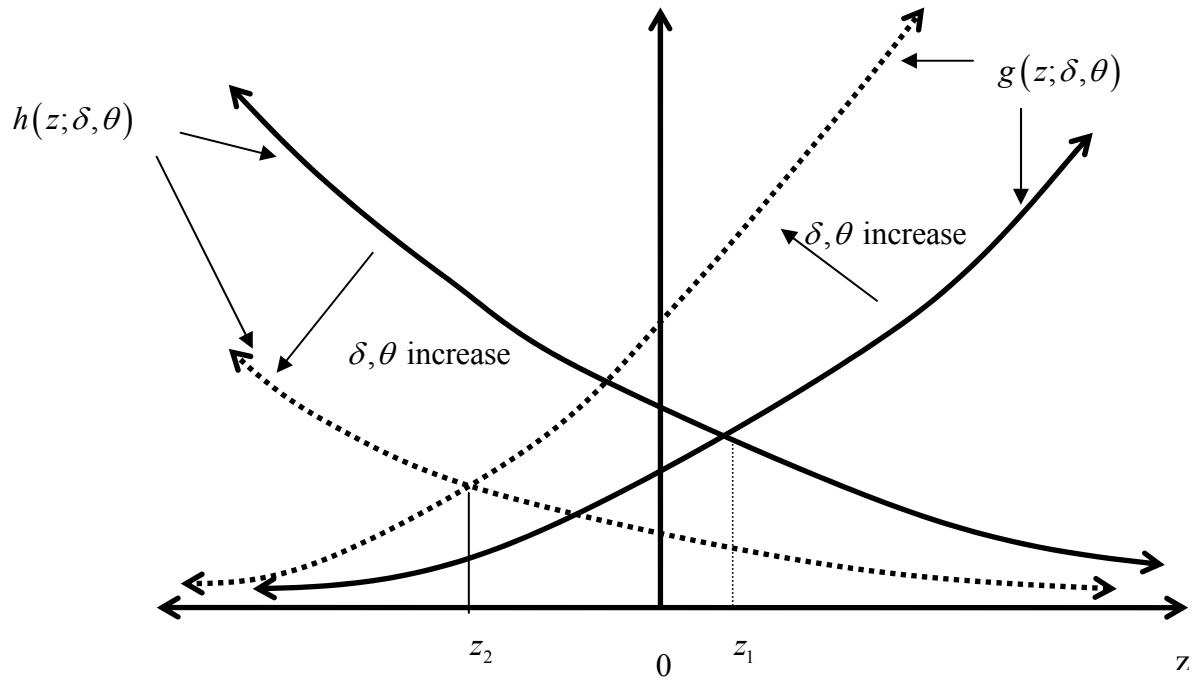
**Figure 1A**  
**Minimising  $E(X^+)^2$**



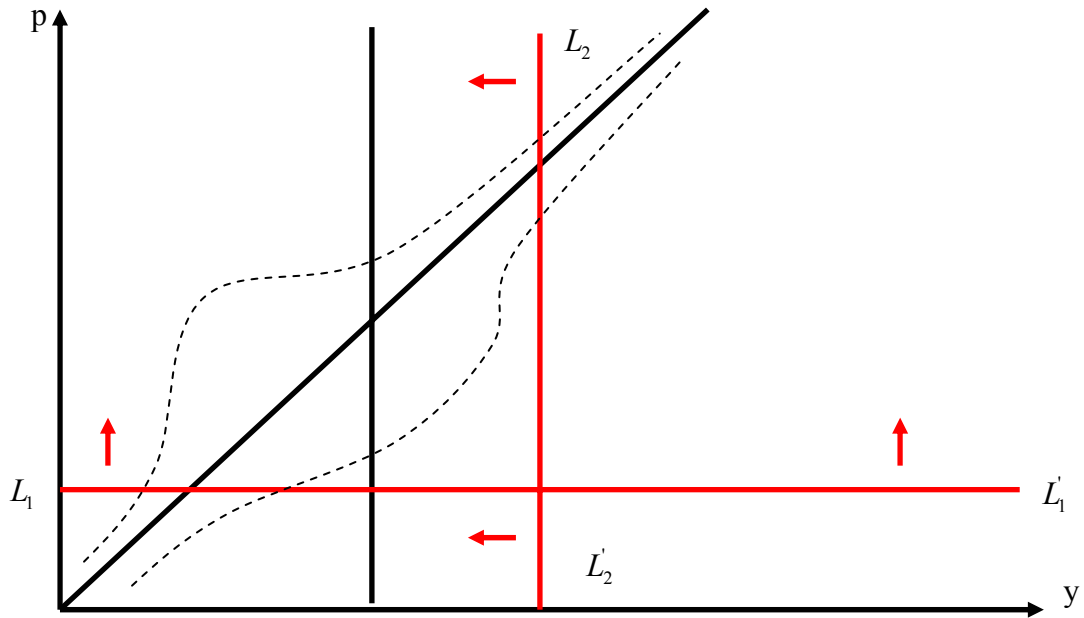
**Figure 1B**  
**Minimising  $E(X^-)^2$**



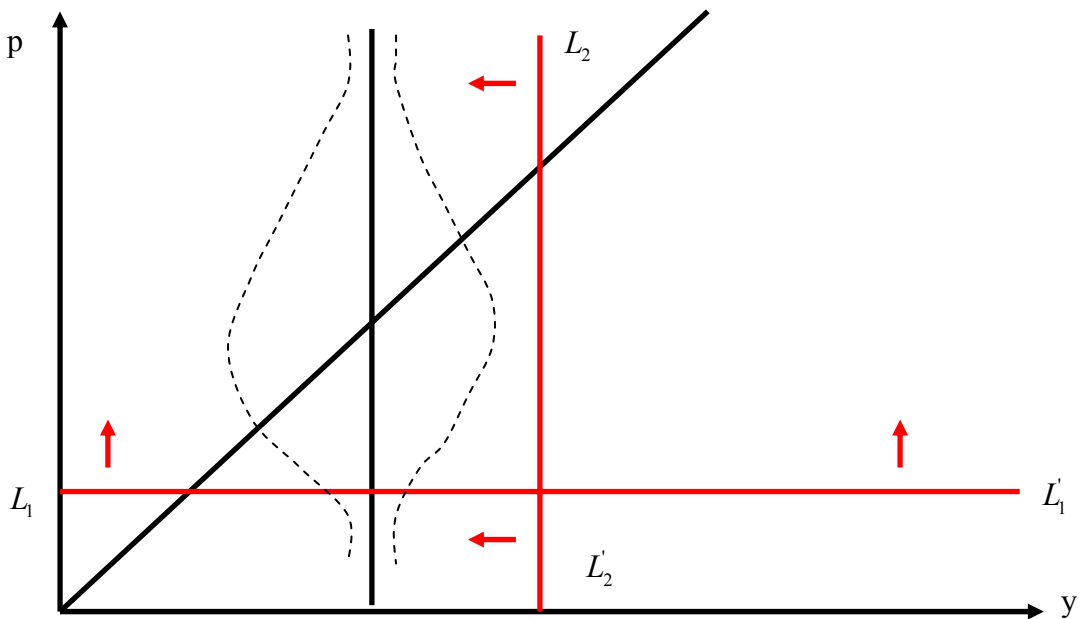
**Figure 2**  
**Minimising  $L_A$  under Fixed Interest Rate Rule when  $\theta > 0$ ;**  
**Following increase in  $\delta$  or  $\theta$ ,  $z$  moves from  $z_1$  to  $z_2$ .**



**Figure 3A**  
**Fixed Interest Rate Rule:**  
**Comparing  $E(p^+)^2$  and  $E(y^-)^2$  under a Supply-Side Shock.**

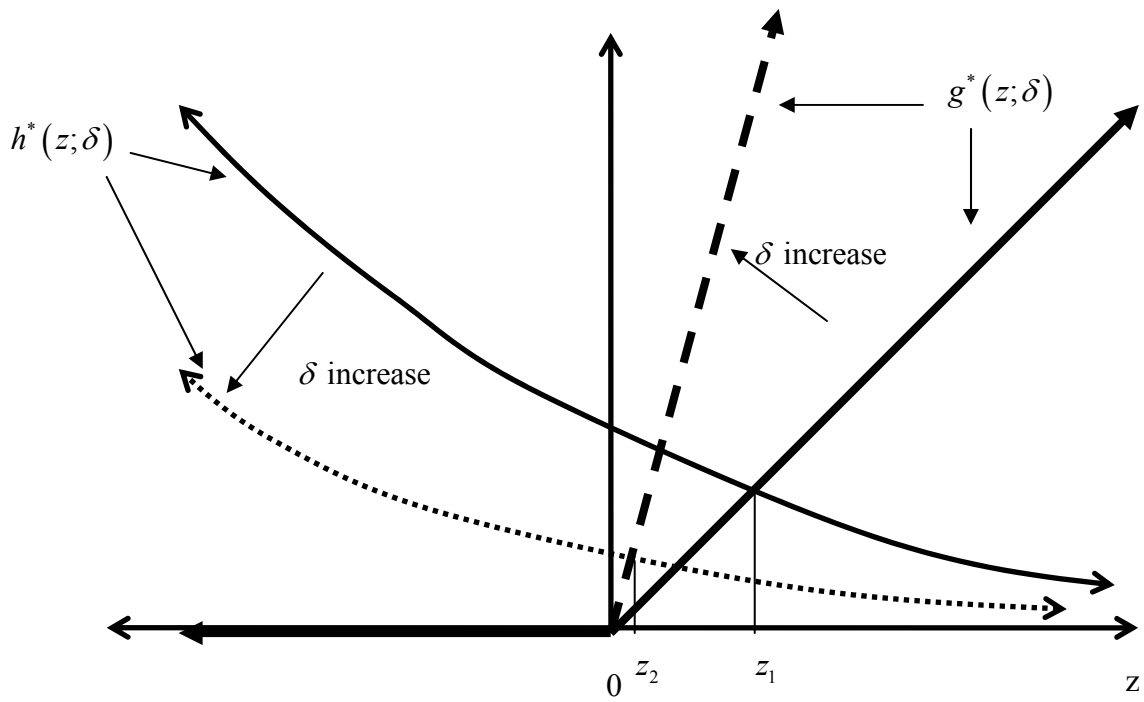


**Figure 3B**  
**Fixed Interest Rate Rule:**  
**Comparing  $E(p^+)^2$  and  $E(y^-)^2$  under a Demand-Side Shock.**





**Figure 4**  
**Minimising  $L_A$  under Fixed Interest Rate Rule when  $\theta = 0$ ;**  
**Following increase in  $\delta$ ,  $z$  moves from  $z_1$  to  $z_2$ .**



**Table 1**  
**Comparison of Optimal Fixed Interest Rate Rules under Symmetric and Asymmetric Loss Functions**

	Source of Shocks	Asymmetric Loss Function $L_A$	Symmetric Loss Function $L_S$
<b>Fixed Interest Rate Rule</b> $(r = \gamma_0)$	<b>Demand-Side Shock</b> $(\sigma_u^2 \geq 0, \sigma_v^2 > 0)$	$r = f(\delta)$ , where $f_\delta(\delta) < 0$ $f(0) = +\infty$ $f(1) = -\infty$	$r = 0$
	<b>Supply-Side Shock but no Demand-Side Shock</b> $(\sigma_u^2 > 0, \sigma_v^2 = 0)$	$r = g(\delta)$ , where $g_\delta(\delta) < 0$ $g(0) = +\infty$ $-\infty < g(1) \leq 0$	$r = 0$

**Table 2**  
**Shocks to One Sector: Comparison of Optimal Flexible Interest Rate Rules**  
**under Symmetric and Asymmetric Loss Functions**

	Source of Shocks	Asymmetric Loss Function $L_A$	Symmetric Loss Function $L_S$
<b>Flexible Interest Rate Rule</b> $(r = \gamma_0 + \gamma_1 p)$	<b>Demand-Side Shock Only</b> $(\sigma_u^2 = 0, \sigma_v^2 > 0)$	$p = 0$ where $\gamma_0 = 0$ $\gamma_1 \rightarrow \infty$	<b>Same as Asymmetric Loss Function</b>
	<b>Supply-Side Shock Only</b> $(\sigma_u^2 > 0, \sigma_v^2 = 0)$	$r = \gamma_1 p$ where $\gamma_0 = 0$ $\gamma_1 = \frac{(1-\delta)}{\alpha\beta\delta}$	<b>Same as Asymmetric Loss Function</b>