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A Comment

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Abstract

In a directed search environment with adverse selection, Guerrieri, Shimer and Wright (2010) claim it is equivalent for principals to all post a full vector of contracts and to segment the market by offering each only one type of contract. This assertion is only true under a condition that is very restrictive, which renders it mostly impractical. This note also explains the source of this breakdown.

Keywords: adverse selection, asymmetric information, mechanisms, contracts, directed search. JEL Classification: D82, D83, D86.

1 Introduction

Guerrieri, Shimer and Wright (2010, now GSW) study a problem of search cum mechanisms in which homogenous principals post vectors of contracts (mechanisms) to attract and screen heterogenous agents. The search of agents is directed by the utility induced by these mechanisms. GSW derive a series of results, including an auxiliary result that asserts that all principals posting vectors of contracts and forming a single market is payoff equivalent to principals segmenting agents according to types by posting a single contract to attract one type of agent. Thus principals also separate in equilibrium, even though they are homogenous.

This assertion is often cited in the literature as a substantive result and used as a simplification device. However upon close inspection, this auxiliary result turns out to *not* hold

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in general. It only holds under a very restrictive condition, which effectively prevents it from being useful to study most problems of interest in mechanism design. Furthermore, in cases of interest from the literature (e.g. procurement, auctions, non-linear pricing), no payoff equivalence can hold. The equivalence claim of GSW leaves aside the requirement that principals be indifferent between the markets in which they may post contracts; such an indifference condition is not necessary in the game with mechanisms. Put another way, principals have to make an additional decision when posting contracts, which induces this indifference condition. Making this point is the object of this note. It likely does not invalidate the other results of GSW, provided they can be cast in the correct environment (using mechanisms and not simple contracts).

In related work, Auster and Gottardi (2019) study the lemons problem in a directed search model with multilateral meeting. Their equilibrium cannot be separating in the sense of GSW. Roger and Julien (2020) add a dimension of moral hazard (to adverse selection); the equilibrium cannot be separating either. In Jacquet and Tan (2007), markets are segmented in equilibrium however in a model of random matching and with heterogeneity on both sides.

2 Model

Take the model of GSW, which is exposed briefly for completeness. There is a measure 1 of heterogenous agents indexed by their type $i \in \mathbb{I} := \{1, 2, ..., I\}$ with each type in fraction $\pi_i > 0$. Type is an agent's private information. There is a large set of homogeneous principals, who must pay a cost $k > 0$ to enter the search market, where matching is bilateral.

A principal who matches with a type i agent receives a gross payoff $v_i(y)$ from contract y, and 0 if unmatched. An agent receives a payoff $u_i(y)$, or zero if unmatched. The functions v and u possess the usual conditions. To be clear, a mechanism $\mathbf{y} \in \mathbb{Y}$ is a menu of contracts $\{y_1, y_2, ..., y_I\}$, to which all parties can commit. All agents observe what all the principals post; this directs their search. Let $\theta(\mathbf{y})$ denote the local market tightness (here principal-agent ratio) associated with a mechanism $\mathbf{y}$

An agent matches with a principal with probability $\mu(\theta(\mathbf{y}))$, independent of type. The matching function $\mu : [0, \infty] \mapsto [0, 1]$ is nondecreasing. A principal offering mechanism $\mathbf{y}$ matches with a type i agent with probability $\eta(\theta(\mathbf{y}))\gamma_i(\mathbf{y})$, with $\eta : [0, \infty] \mapsto [0, 1]$ nonincreasing and where γ_i denotes the share of agents of type i applying to $\mathbf{y}$. The functions μ and η are connected by the consistency requirement $\mu(\theta) = \theta\eta(\theta)$. Throughout the same assumptions as in GSW are imposed. In particular, $v_1(y) \leq v_2(y) \leq ... \leq v_I(y)$ and the Spence-Mirrlees condition (single crossing) holds.

The one, small departure compared to GSW is that I maximize the utility of principals, who design contracts, subject to an entry condition, rather than that of agents. This is a more standard approach in contract theory, and a matter of convenience only; it is otherwise immaterial to the claim.

3 The claim

GSW assert that for principals, posting a mechanism – that is, a vector $\mathbf{y}$ of contracts, one for each type of agent – is payoff equivalent to posting a single contract y_i for agents $i \in \mathbb{I}$. That is, an equilibrium in which principals segment by posting a single contract each (thus forming I submarkets), is payoff equivalent to one in which all principals form a single submarket and post each a menu of I contracts.

3.1 A condition for payoff equivalence

The crucial step is the following. For principals to choose to post a single contract rather than a menu, they must not only be indifferent between the equilibrium payoffs in each of the market structures, the following equality must also hold in the segmenting equilibrium:

$$\eta(\theta_i)v_i(y_i) = \eta(\theta_j)v_j(y_j), \quad \forall\, i, j, \tag{3.1}$$

for any principal facing agents i or j, where $\gamma_i(y_i) = \gamma_j(y_j) = 1$. That is, principals must be indifferent to posting contracts that attract either i or j. If this condition fails, one of these (groups of) principals should instead offer the contract that yields higher payoffs. Since $v_i \leq v_j$ for $i < j$, and η is non-increasing, Condition (3.1) is equivalent to $\theta_i \leq \theta_j$.

When agents face mechanisms, the incentive constraint, as specified in GSW, reads

$$u_j(y_j) \geq u_j(y_i), \quad \forall\, i, j, \tag{3.2}$$

which clearly requires that at the time of reporting her type, agent j also has choice y_i available to her. The participation condition of the agents is given by

$$\mu(\theta)u_j(y_j) \geq \overline{U}_j, \tag{3.3}$$

with $\overline{U}_j := \max\left\{0, \max_{\mathbf{y}} \mu(\theta')u_j(y'_j)\right\}$, reflecting the fact there is a single market. Incentive compatibility and participation can be summarised by the condition that agents search optimally, as used by GSW (Definition 1).

When principals post *contracts* rather than mechanisms, the same optimal-search condition $\overline{U}_j := \max\{0, \max_{\mathbf{y}'} \mu(\theta')u_j(y')\}$ can be used to summarise participation and constraint and incentive compatibility. This condition subsumes both the participation constraint

$$\mu(\theta_j)u_j(y_j) \geq \overline{U}_j, \tag{3.4}$$

with $\overline{U}_j := \max\left\{0, \max_{y'_j} \mu(\theta'_j)u_j(y'_j)\right\}$ since only contracts y_j, y'_j, y''_j... are available to agent j (and likewise y_i, y'_i... to agent i), and the accompanying incentive constraint

$$\mu(\theta_j)u_j(y_j) \geq \mu(\theta_i)u_j(y_i), \quad \forall\ i, j, \tag{3.5}$$

which now reflects the fact that selecting contract y_i also implies facing tightness θ_i, not θ_j (nor the aggregate θ). In a game in which markets are segmented, once agent j elects to enter market j, she cannot select contract y_i; that choice is simply not available. The optimal search condition allows for any other contract than y_j – and so can stand for both (3.4) and (3.5).

To make progress one may first consider an intermediate question: "when are (3.2) and (3.5) equivalent conditions?" To answer this first question it is useful to define $\varphi_j(y_i) := u_j(y_i) - u_i(y_i)$ as the standard incentive rent of agent j mimicking i and $\psi(\theta_i, \theta_j) := \frac{\mu(\theta_i)}{\mu(\theta_j)}$. Then rewrite (3.2) as

$$u_j(y_j) \geq u_i(y_i) + \varphi_j(y_i) \tag{3.6}$$

and Condition (3.5) as

$$u_j(y_j) \geq \psi(\theta_i, \theta_j)\left[u_i(y_i) + \varphi_j(y_i)\right] \tag{3.7}$$

Proposition 1 *Conditions (3.2) and (3.5) are equivalent only if* $\theta_i \equiv \theta_j,\ \forall\ i, j$.

Proof: Consider the game with mechanisms being posted; take a submarket k in which principals post some mechanism, and in which a subset $\mathbb{A} \subseteq \mathbb{I}$ with cardinality A of agents search over this vector of contracts. Let θ^k denote the aggregate local tightness in k, and for each type i, θ_i^k the local tightness for that type. Then for this submarket there are $A \leq I$ relevant incentive constraints, which read

$$u_j(y_j^k) \geq u_j(y_i^k), \quad \forall\ i \neq j \in \mathbb{A}, \tag{3.8}$$

for each k. Participation in submarket k requires

$$\mu(\theta^k)u_j(y_j^k) \geq \overline{U}_j, \tag{3.9}$$

where $\overline{U}_j$ is defined as before. In equilibrium this condition binds for each k (see GSW), so that Condition (3.9) turns into (3.3). Therefore Condition (3.8) also rewrites as (3.2). The rest is immediate given the properties of $\mu(\cdot)$ and that $\theta_i \leq \theta_j$. ■

Per se Proposition 1 is not spectacular and almost obvious. The incentive constraints (3.2) and (3.5) are equivalent if they characterize sets that are payoff equivalent to the principals. However, the implication of Proposition 1 is then $\forall\ i, j,\ v_j(y_j) \equiv v_i(y_i)$ is a necessary condition at equilibrium to preserve the indifference condition (3.1) of the principals. This is problematic because it precludes, for example:

- the canonical model of regulation of Baron and Myerson (1982) and the didactic model of procurement of Laffont and Martimort (2002) – adapting their notation to this model:

$$v(q,t) := S(q) - t \quad \text{and} \quad u(t,q;\omega) = t - \omega q,\ \omega \in \{\underline{\omega}, \overline{\omega}\}$$

  in which, at equilibrium $\overline{q} < \underline{q}$ and $v(\overline{q}, \overline{t}) < v(\underline{q}, \underline{t})$;

- the richer but closely related procurement model of Laffont and Tirole (1986):

$$v(q,t) := S(q) - (1+\lambda)\mathbb{E}[t + C] \quad \text{and} \quad u(t,q,e;\omega) := \mathbb{E}[t] - \psi(e),\ C := (\omega - e)q + \epsilon,$$

  with $\omega \in \Omega \subset \mathbb{R},\ e \in E \subset \mathbb{R}_+$, which delivers similar inequalities;

- standard auctions, like that of Myerson (1981), which may be written, for $\omega \in \Omega$:

$$v(q,t) = \mathbb{E}_\Omega\left[w_0\left(1 - \sum_{k\in N} q_k\right) + \sum_{k\in N} t_k(\omega)\right] \quad \text{and} \quad u(t,q;\omega) = \mathbb{E}_{\Omega\setminus j}\left[w_j(\omega_j)q_j(\omega_j) - t_j(\omega_j)\right],$$

  in which it is immediate that for $\omega_i < \omega_j,\ t_i < t_j$ in equilibrium so that $v(q_i,t_i) < v(q_j,t_j)$; or

- the work of Maskin and Riley (1984)

$$v(q,t) = t_s - cq_s \quad \text{and} \quad u(t,q;\omega) = \int_0^q w(x,\omega_j)dx - t_j,\ \omega \in \Omega,$$

  and most of the literature on non-linear pricing, among many examples.

In other words, holding on this equivalence between mechanisms and contracts requires a condition that precludes using the GSW model for many environments where mechanism design is useful and tractable.

3.2 No payoff equivalence

If the primitives of the GSW model are such that for $i < j$, $v_i(y_i) < v_j(y_j)$ at equilibrium, then the indifference condition of the principals requires that $\theta_i < \theta_j$, which violates Proposition 1. In this case, under contract posting, at an equilibrium allocation $\mathbf{y}^*$ of the game in mechanisms,

$$u_j(y_j^*) > \psi(\theta_i, \theta_j)\left[u_i(y_i^*) + \varphi_j(y_i^*)\right],$$

since $\psi(\theta_i, \theta_j) < 1$. That is, the rent that is optimal if posting mechanisms is excessive under contract posting. To see why, observe that at an equilibrium of the mechanism posting game the allocation $\mathbf{y}^*$ satisfies, in particular, the complementary slackness conditions

$$\forall i, j \in \mathbb{A}, \quad \lambda_j[u_j(y_j^*) - (u_i(y_i^*) + \varphi(y_i^*))] = 0, \quad \lambda_j \geq 0, \tag{3.10}$$

$$\forall j \in \mathbb{A}, \quad \nu_j[\mu(\theta)u_j(y_j^*) - \overline{U}_j] = 0, \quad \nu_j \geq 0, \quad \mu(\theta)u_1(y_1^*) = \overline{U}_1 \tag{3.11}$$

where λ_j, ν_j are Lagrange multipliers. At an optimum any one, or both, of the relevant conditions may be slack ($\lambda_j, \nu_j = 0$). If $\lambda_j \equiv 0$ for all types, screening has no object, or equivalently, is free; no distortions are required. Therefore I focus on the substantive case, in which (3.6) binds for an least some types at the optimum. Then, thanks to the Spence-Mirrlees condition ($\partial^2\varphi(y_k)/\partial y_k \partial k \geq 0$), at an optimum, (3.10) is such that $\lambda_j > 0$ for at least some j. This pins the rent $\varphi(y_j^*)$ under mechanism posting. Then, given payoff equivalence, Condition (3.7) becomes

$$u_j(y_j') \geq \psi(\theta_i, \theta_j)\left[u_i(y_i') + u_j(y_j^*) - u_i(y_i^*)\right],$$

for some equilibrium allocation y_j' under contract posting. Payoff equivalence of $\mathbf{y}^*$ and (the vector) $\mathbf{y}'$, and optimality of the allocation $\mathbf{y}'$ under contract posting then imply

$$u_j(y_j') \geq \psi(\theta_i, \theta_j)u_j(y_j^*),$$

which can only hold as a strict inequality when $\theta_i < \theta_j$ given payoff equivalence. But then clearly the allocation $\mathbf{y}'$ cannot be optimal. If it is optimal to have $u_j(y_j^*) = u_j(y_i^*) = u_i(y_i^*) + \varphi(y_i^*)$ and $\mathbf{y}^*$ and $\mathbf{y}'$ are payoff equivalent, it cannot be optimal to have $u_j(y_j') > u_j(y_i') = u_i(y_i') + \varphi(y_i')$. That is, if paying attention to the cases in which mechanism design has shown to be useful in the literature, payoff equivalence cannot hold at an optimum.

This almost general failure of payoff equivalence owes to the interaction of the equivalence of the incentive constraints (3.2) and (3.5), and the indifference condition of principals (3.1).

This indifference condition is not required when principals post mechanisms, because they all operate in the same market; more precisely, their expected payoffs are necessarily identical in equilibrium in that unique market: $\eta(\theta)\sum_i \pi_i v_i(y_i)$.

3.3 An example

Let us call on the model of a Baron and Myerson (1982), reproduced in Laffont and Martimort (2002) as a didactic example with $\alpha = 0$ without loss. The payoffs are

$$v(q) := S(q) - t \quad \text{and} \quad u(\omega, q) := t - \omega q, \qquad \omega \in \{\underline{\omega}, \overline{\omega}\},$$

to the principal and agent, respectively, where $S(\cdot)$ is a concave function and q denotes quantity. Let also $\rho = Prob(\omega = \underline{\omega})$. Since there are two types, only two contracts $\big((\overline{t}, \overline{q}); (\underline{t}, \underline{q})\big)$ are necessary to separate agents. First I study the problem of principals forming a single market and offering two contracts each, skipping a few steps for brevity. Incentive compatibility (Condition (3.2)) amounts to

$$\overline{u} = \overline{t} - \overline{\omega q} \quad \geq \quad \underline{t} - \overline{\omega}\underline{q} \tag{3.12}$$

$$\underline{u} = \underline{t} - \underline{\omega q} \quad \geq \quad \overline{t} - \underline{\omega}\overline{q} = \overline{u} + (\overline{\omega} - \underline{\omega})\overline{q}; \tag{3.13}$$

that is, $\varphi(q) = (+/-)(\overline{\omega} - \underline{\omega})q$ here.[1] Between this and the participation constraints for each type, the problem of any one principal amounts to solving

$$\max_{(\underline{t},\underline{q};\overline{t},\overline{q})} \eta(\theta)\left[\rho[S(\underline{q}) - \underline{t}] + (1-\rho)[S(\overline{q}) - \overline{t}]\right]$$

subject to (3.12), (3.13) and

$$\mu(\theta)\left[\overline{t} - \overline{\omega q}\right] \quad \geq \quad \overline{U} \tag{3.14}$$

$$\mu(\theta)\left[\underline{t} - \underline{\omega q}\right] \quad \geq \quad \underline{U}. \tag{3.15}$$

Because principals do not separate here there is a single tightness parameter θ. Attach multipliers $\overline{\lambda}, \underline{\lambda}, \overline{\nu}, \underline{\nu}$ to each of these constraints. In these problems Condition (3.12) is slack in equilibrium while (3.13) binds, which is not formally verified in this note; both $\overline{\nu}, \underline{\nu} > 0$. Re-arranging the optimality conditions, one has

$$S'(\underline{q}) = \underline{\omega} \quad \text{and} \quad S'(\overline{q}) = \overline{\omega} + (\overline{\omega} - \underline{\omega})\frac{\rho}{1-\rho},$$

[1] And $(\underline{\omega} - \overline{\omega})\underline{q} < 0 < (\overline{\omega} - \underline{\omega})\overline{q}$.

and $\overline{q} < \underline{q}$ by concavity of S.

Now turning to the problem of separating principals, the same allocation $(\overline{q}, \underline{q})$ must satisfy incentive compatibility

$$\mu(\overline{\theta})\overline{u} \geq \mu(\underline{\theta}) \left[\underline{t} - \overline{\omega}\underline{q}\right] \tag{3.16}$$

$$\mu(\underline{\theta})\underline{u} \geq \mu(\overline{\theta}) \left[\overline{t} - \underline{\omega}\overline{q}\right]. \tag{3.17}$$

and payoff equivalence implies that this allocation also satisfies conditions (3.12) and (3.13). Principals also need the all important

$$\eta(\underline{\theta})\underline{v} = \eta(\overline{\theta})\overline{v} \Leftrightarrow \underline{v} = \overline{v}, \tag{3.18}$$

since (3.12), (3.13), (3.16) and (3.17) all holding implies $\underline{\theta} = \overline{\theta}$. The indifference condition (3.18) rewrites equivalently[2]

$$\begin{aligned}
S(\underline{q}) - \underline{t} &= S(\overline{q}) - \overline{t} \\
S'(\underline{q})\underline{q} - \underline{t} &= S'(\overline{q})\overline{q} - \overline{t} \\
\underline{\omega}\underline{q} - [\overline{t} + \underline{\omega}(\underline{q} - \overline{q})] &= \left[\overline{\omega} + (\overline{\omega} - \underline{\omega})\frac{\rho}{1-\rho}\right]\overline{q} - \overline{t} \\
\underline{\omega} &= \overline{\omega} + \underbrace{(\overline{\omega} - \underline{\omega})\frac{\rho}{1-\rho}}_{>0},
\end{aligned}$$

which clearly cannot hold. Hence a payoff-equivalent allocation cannot satisfy the indifference condition of the principals, which is necessary for them to remain ambivalent between contract posting and mechanism posting.

4 Conclusion

The model of GSW makes an important contribution to the literature by casting contract design in a market environment. However the claim of equivalence of mechanism and contract posting is too far reaching and must be re-examined. Technically it is a device that is in fact too restrictive to be useful. Substantively this note establishes that separating agents by screening them via mechanisms is not equivalent to separating them into submarkets, except in a special case that has little applicability in light of the extant literature.

[2]The first line is (3.18), the second one its linear approximation, the third one uses the FOC of the prior problem (by payoff equivalence), as well as the IC (3.13) and the last one simplifies.

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