CENTRE FOR GLOBAL BUSINESS DISCUSSION PAPER SERIES

Moral hazard and efficiency in a frictional market

Guillaume Roger, Monash University
Benoit Julien, UNSW

Discussion Paper Number 2021-04
January, 2021

Moral hazard and efficiency in a frictional market.

Guillaume Roger [*] **Benoit Julien**[†‡]

October 2, 2020

Abstract

Principals seek to trade with homogenous agents by posting incentive contracts, which direct their search. A contract solves the ex ante search problem, and moral hazard ex post; search and moral hazard interact in equilibrium. If using appropriate transfers the equilibrium allocation is always constrained welfare optimal, in contrast to the one-to-one principal-agent problem. Search frictions thus correct that inefficiency because search requires internalizing the utility of agents. Incentives are weaker than in bilateral contracting, and agents enjoy more efficient risk sharing. With a constraint on transfers the allocation may become inefficient; principal competition results in over-insurance of the agents, too little effort in equilibrium and excessive entry by principals.

Keywords: moral hazard, asymmetric information, contracts, directed search, search frictions, constrained efficiency. JEL Classification: D82, D83, D86.

1 Introduction

We study optimal contracts and the efficiency of equilibrium allocations in an economy with moral hazard and directed search frictions. Quoting Guerrieri, Shimer and Wright (2010) (now GSW), "it seems interesting to study models where both margins are operative to see how incentive problems manifests themselves in terms of distortions [...]". In this paper the

[*]Monash University, guillaume.roger72@gmail.com.

[†]UNSW Business School, Australia, benoit.julien@unsw.edu.au

[‡]Financial support from the Australian Research Council grant DP170104227 gratefully acknowledged.

two operative margins are search frictions and moral hazard, and their interaction induces distortions that have welfare implications.

The results of interest pertain to constrained efficiency and they are subtle: whether an equilibrium allocation is constrained efficient depends on both which contract is used and on the outside option of the agents.[1] Principals enter the search market at a cost and post incentive contracts non-cooperatively. Agents choose which principal to meet based on the observed terms of trade; hence search is directed by these offers. Because meeting is multilateral, not all agents meeting a principal can contract with him. Upon contracting a principal and an agent enter into a relationship under moral hazard. An important feature to bear in mind is this: attracting agents requires awarding them some rents; with risk averse agents this is most efficiently achieved by improving the insurance properties of the contract. But under moral hazard, better insurance implies lower a effort in response. This trade-off between search and moral hazard is central to the results. We explore it in some details through some comparative statics.

The main results are two-fold. First, for any weights assigned to principals and agents in the social welfare function, the equilibrium allocation is *always* constrained efficient under the optimal contract. That optimal contract includes compensatory transfers paid out to agents meeting a principal but unable to contract with him because they are crowded out. These transfers exactly price the congestion externality (on agents) that principals generate by competing to attract agents. Equivalently they insure against the search risk. In doing so they transfer rents to agents; therefore the rent to the one contracting agent need not be so large. This implies the distortions in effort are mitigated; these transfers assist in solving the trade-off between search and moral hazard. Second, without these compensatory transfers, the equilibrium allocation is not constrained efficient when the outside option of the agents is large. The reason is that competing principals are compelled to offer rents to agents by distorting the contract to excess; this weakens the incentives of the agents. Agents who manage to contract receive inefficiently large rents and produce inefficiently low effort. Throughout the welfare weights are irrelevant because they are completely internalized by the endogenous entry decision of principals.

Even though our model is cast in directed search, efficiency is far from immediate for two reasons. One, the equilibrium contract of the standard (bilateral) principal-agent problem

[1] "Constrained" means that a social planner is subject to the same informational constraints as the players.

(as in Holmström, 1979) is *not* welfare optimal. And two, introducing unfettered competition between principals often leads to a multiplicity of equilibria, or inefficiency, because contracts generate externalities on each other. However under directed search, and with the optimal contract, all these problems disappear. In fact, search frictions assist in delivering constrained efficiency; they induce just the right degree of competition. On the one hand, principal competition for agents forces them to internalise the utility of agents, just as a planner does. On the other, that competition is not unfettered: the frictions resolve the problem of multiplicity of equilibria and inefficient equilibria.

The works closest to ours are GSW, Moen and Rosen (2011), Auster and Gottardi (2018) and Lester et al. (2019). GSW study competitive search under adverse selection. They show the equilibrium may be Pareto inefficient. When there are few high types, a pooling allocation with transfers may Pareto dominate because the distortions required to separate types become socially too costly – as in Rothschild and Stiglitz (1976). Auster and Gottardi (2018) adapt GSW to multilateral meeting and echo their main results. Here, when the equilibrium allocation fails to mirror the allocation of the planner, we identify the source of this failure as the interaction of a mispriced search risk with a large outside option. Lester et al. (2019) adopt the model of GSW to study the impact of imperfect competition on the search equilibrium. Welfare may decrease as competition increases because more competition tightens incentive constraints.

Moen and Rosen (2011) adapt the Laffont-Tirole (1986) model to a dynamic search framework. Because type and effort are perfect substitutes, the contracting problem reduces to one of adverse selection only – "false moral hazard".[2] Instead we focus on moral hazard only; all distortions are attributable to it (and its interaction with search). In Moen and Rosen (2011) agents are risk neutral and effort increases in rents, which are necessary for screening. Here effort decreases in rents because higher rents correspond to better insurance, that is, to a lower marginal benefit of effort.

Like Jacquet and Tan (2012) we use compensatory transfers as part of the optimal contract. As in their paper, these transfers contribute to making a firm more attractive to agents. But they are even more important here than they are in Jacquet and Tan's work because they help in preserving the incentives for effort. Acemoglu and Shimer (1999), Golosov, Maziero and Menzio (2013) and Geromichalos (2015) study unemployment insurance in a

[2]This term was coined by Jean-Jacques Laffont.

search framework. These models all induce moral hazard in search; we focus on moral hazard in production.

Finally this work is related to the literature on contract design, which largely rests on one-to-one models. Exceptions include models of common agency under adverse selection (Martimort, 1996; Martimort and Stole, 2002, 2003, 2009a, 2009b; Stole, 1995), under moral hazard (Attar et al., 2006, 2007a, 2007b; Aubert, 2005; Bisin and Guaitoli, 2004), in which many principals seek to contract with a single agent. There is also a small number of models of "reduced-form" competition where the role of a market is subsumed in the participation decision only (Rochet and Stole, 2002; Roger, 2016).[3] So the extant literature offers a limited perspective on designing contracts when players operate in a broader market. We contribute to filling this gap.

2 Model: contracting and search

2.1 Basics

There is a large population of homogenous agents. The utility of an agent is $u(t) - c(a)$, with $u(\cdot)$ increasing, concave and at least three times differentiable; t is the transfer received and $a \in \mathcal{A} \subset \mathbb{R}$, the chosen action at cost $c(a)$ increasing and convex. The set $\mathcal{A}$ is compact. Action a is never observed by the principal and it governs the distribution $F(x|a)$ of outcomes $x \in \mathcal{X} := [\underline{x}, \overline{x}] \subset \mathbb{R}$, with density $f(x|a) > 0$ on $\mathcal{X}$. Agents have an outside option u_0. The likelihood ratio f_a/f is increasing, concave in x, hence $F(x|a) < F(x|a')$ for $a' < a$; it is also bounded. We also impose that $F_a(F^{-1}(x,a)|a)$ be convex in (x,a), which is sufficient for Concave Local Informativeness (CLI) (see Jewitt, Kadan and Swinkels, 2008), and that the functions $u(\cdot)$ and $u'(\cdot)$ do not diverge (their ratio is bounded; see Moroni and Zwinkels, 2014). The function $t(x) \in \mathcal{T}$ is a transfer that depends on outcome x. Throughout we suppose that the first-order approach to the agency set up is valid – see Jewitt (1988).

2.2 Competitive search with moral hazard

The economy is populated by an exogenous measure Λ^A of agents and a measure Λ^P of homogenous principals to be determined in equilibrium. The aggregate market tightness is $\Theta := \Lambda^A/\Lambda^P$ (agents per principals). Principals compete by offering contracts to attract

[3] Prescott and Townsend (1984) study Pareto allocations in perfectly competitive markets.

agents; they can commit to these contracts. In full generality, contracts may depend on the number n of agents present at a meeting.

Contracts. Principals use a set of contracts $\mathcal{C}^j = \{t_n^j(x), a_n^j, h_n^j\}_{n\geq 1}$ for principal j, where n is the number of agents present at a meeting, together with a rationing rule to be determined in equilibrium. Because agents are homogenous, the rationing rule is very simple: uniform random rationing is clearly optimal. The terms h_n^j are transfers paid to agents meeting with j but unable to contract with him.[4]

Market interaction. We contemplate an entry decision by principals only; entering the market costs k. Agents select over principals after observing the posted contacts. Each principal may meet more than one agent, but needs only one agent to contract with. This is without loss as long as the number of agents a principal may contract with is finite. Agents are not ubiquitous – each of them can only meet one principal; hence the extensive form rules out common agency and focusing on bilateral contracting is without loss.[5,6] We adopt the competitive search version of submarkets akin to the one in Shimer (1996), Moen (1997) and Mortensen and Wright (2002)). The timing is as follows:

1. principals enter by paying some cost $k > 0$;
2. principals posting identical sets of contracts form a submarket;
3. agents observe all contracts (all submarkets) and select a submarket to participate in, principals and agents meet according to some meeting technology;
4. if an agent is selected and accepts a contract, she chooses an action;
5. payoffs are realized. All agents meet at least one principal thanks to multilateral meeting; agents meeting but not contracting receive a transfer h_n^j yielding payoff $u(h_n^j+$

[4]Optimal contracts call for the use of these transfers. At times practical considerations may induce restrictions on transfers. While these considerations are valid we do not discuss alternative assumptions; what is assumed, or imposed, can only depend on the particular application under consideration. We do study the important case of $h_n^j \equiv 0$ in Section 4.3.

[5]Search is directed by the principals' offers but only matters for participation, not (directly) for incentive provision. We discuss this further in Section 5

[6]An alternative interpretation is that agents contact any subsets of principals and then choose one to approach; this principal may have a queue of agents interested in contracting with him. The mathematics are the same.

y); principals not matching receive 0. For convenience we recall $u(y) = u_0$.

Meeting technology. Principals offering the same set of contracts, say $\mathcal{C}^j$, form a submarket with measure $\Lambda^P(\mathcal{C}^j) \leq \Lambda^P$. Agents choose a submarket (equivalently, a contract) in which to search; this generates the measure $\Lambda^A(\mathcal{C}^j) \leq \Lambda^A$ of active agents in submarket j. Let the function $\theta(\mathcal{C}^j) := \Lambda^A(\mathcal{C}^j)/\Lambda^P(\mathcal{C}^j)$ represent the local market tightness in submarket j given the set of contracts $\mathcal{C}^j$. In general this differs from Θ depending on the menu $\mathcal{C}^j$; at times we may write θ^j as well. Let $p_0(\theta)$ be the probability to meet *no* agent given tightness θ, with $p_0'(\theta) < 0,\ p_0''(\theta) > 0$. Likewise for any $n \geq 1$ we can define the probability of meeting n agents: $p_1(\theta), p_2(\theta), ...$ and so on. The meeting rate for a principal is then $\sum_{n=1}^{\infty} p_n(\theta)$; it is equal to the contracting rate for a principal. Thanks to uniform rationing, the probability of contracting for an agent is $\sum_{n=1}^{\infty} \frac{p_n(\theta)}{n}$. Throughout we restrict attentions to probability functions $p_n(\theta)$ such that $\sum_{n=0}^{\infty} p_n(\theta) = \theta$.[7] It is handy to keep track of the quantity $1 - p_0(\theta)$ as the probability for a principal to meet at least one agent.

Payoffs and market utility. Upon contracting the number n is fixed. Principal j receives $\pi(t_n^j, a_n^j) := \int_{\mathcal{X}} [z - t_n^j(z)]\, dF(z|a_n^j)$; likewise an agent receives $U(t_n^j, a_n^j) := \int_{\mathcal{X}} u(t_n^j(z)) dF(z|a_n^j) - c(a_n^j)$.

Finally we need to contemplate the participation decision of agents into a submarket – before they observe how many other agents meet the same principal as they. The *expected* number of agents that are present at a given meeting is the tightness that is induced by the set of contract $\mathcal{C}^j$ at the meeting with principal j – that is, $\theta(\mathcal{C}^j)$. Thus the expected utility of an agent participating in submarket j is

$$V(\theta(\mathcal{C}^j)) := \sum_{n=1}^{\infty} \left[\frac{p_n(\theta^j)}{n} U(t_n^j, a_n^j) + \left(1 - \frac{p_n(\theta^j)}{n}\right) u(h_n^j + y) \right]. \tag{2.1}$$

In this definition we let $V(\infty) = u_0$: in the limit agents all receive their exogenous outside option, and $V(0) \equiv u_0$ by convention. An agent is exposed to two sources of risk: a search risk, in that she may meet but not contract with a principal, and a stochastic payoff $U(t^j, a^j)$ if contracting. The latter is necessary to generate incentives for effort and the former is the result of search frictions. Throughout there is no insurance market where agents may insure against either risk.

[7] Examples include the binomial distribution or the Poisson distribution.

In a large economy, when principals deviate in a submarket the deviation does not affect the maximum expected utility agents receive by participating in any contracts offered by non-deviating principals. This is the *market utility property* (MUP), as used by McAfee (1993), Shimer (1996) and Moen (1997). Let $\mathcal{C}^{-j}$ be the symmetric contract posted in all other submarkets other than j and yielding expected utility $\widetilde{V}$ defined as

$$\widetilde{V}(\theta) := \max \left\{ \max_{\mathcal{C}} V(\mathcal{C}^{-j}), u_0 \right\}.$$

Then, given $\widetilde{V}(\theta)$ participation in submarket j requires

$$V(\theta(\mathcal{C}^j)) \geq \widetilde{V}(\theta), \quad 0 < \theta(\mathcal{C}^j) < \infty. \tag{2.2}$$

Finally we denote by $\Pi(\mathcal{C}^j)$ the expected payoff of principal j

$$\Pi(t^j, a^j, h^j; \theta^j) := \sum_{n=1}^{\infty} p_n(\theta^j) \left[\pi(t_n^j, a_n^j) - (n-1)h_n^j \right]. \tag{2.3}$$

The term $(n-1)h_n$ is the total payment made to the $n-1$ agents meeting, but not contracting with, principal j.

Definition 1 *An equilibrium is an allocation defined by a tuple* $\left(\widehat{\mathcal{C}}, \theta(\widehat{\mathcal{C}}), \Pi(\widehat{\mathcal{C}}), V(\widehat{\mathcal{C}})\right)$ *of contracts, market tightness, and expected payoffs to principals and agents, such that*

- *for each principal in submarket* j
  - $\widehat{\mathcal{C}}^j \in \arg\max_{\mathcal{C}^j} \Pi(\mathcal{C}^j)$ *with associated tightness* $\theta(\widehat{\mathcal{C}}^j) = \Theta \; \forall j$ *in equilibrium only; and*
  - *under free entry,* $\Pi(\widehat{\mathcal{C}}^j) = k$*;*
- *agents*
  - *optimally select submarkets such that :* $V(\mathcal{C}^j) \geq \widetilde{V}$*; and*
  - *for each agent contracting with a principal,* $a \in \arg\max_{a'} U(t^j, a')$ *given any contract* $\mathcal{C}^j$*.*

Without loss we look for symmetric, subgame-perfect equilibria of this game – as is implicitly assumed in the definition of $\Pi(\mathcal{C}^j)$.

3 Characterization: contracts and entry.

We first lay out the incentive constraints that arise from the moral hazard problem. Then we solve for the optimal contract for an arbitrary market tightness Θ, and finally we determine the equilibrium tightness via free entry.

3.1 Incentive compatibility

Consider an agent who has met a principal; n is known. Facing the contract $(t_n^j(x), a_n^j, h_n^j)$, she chooses action $\widehat{a}_n^j$ (if selected) satisfying

$$\int_{\mathcal{X}} u(t_n^j(z))dF_a(z|\widehat{a}_n^j) = c'(\widehat{a}_n^j), \quad n \geq 1. \tag{3.1}$$

Under the assumptions of the first-order approach the maximizer $\widehat{a}_n^j$ is unique for each n. These actions induce utility $U(t_n^j(x), \widehat{a}_n^j)$ for each n.

3.2 Characterization.

With the MUP and the restriction to symmetric equilibria, we can formulate the problem as one of constrained optimization and drop the superscript j. For some arbitrary tightness Θ, the program is[8]

Problem 1

$$\max_{\{t_n(x),h_n,a_n\}_{n=0}^{\infty},\theta} \Pi(t_n, a_n, h_n; \theta) \quad s.t.$$

the MUP (2.2), the moral hazard constraint (3.1) and

$$\forall n \geq 0, \quad \int_{\mathcal{X}} u(t_n(z))dF(z|a) - c(a_n) \geq u(h_n + y), \tag{3.2}$$

$$\forall n \geq 0, \quad h_n \geq 0, \tag{3.3}$$

The first constraint is the (ex post) participation constraint of the agent, who is always free to reject a contract after meeting a principal. The second one is convenient to establish our results. Attach multipliers, respectively, $\{\nu_n, \mu_n, \gamma_n, \epsilon_n\}_{n\geq 0}$ to each of these constraints.

Proposition 1 *There exists a competitive search equilibrium. The optimal contract is a* uniform *contract* $\left(t^S(x), a^S, h^S\right)$ *that is independent of* n. *It is unique and characterized by*

[8]Maximizing with respect to θ is common practice in the literature on competitive search. It amounts to directly exploiting an envelop condition because $\theta := \theta(\mathcal{C})$.

the conditions:

$$\frac{1-p_0(\theta)}{u'(t(x))} = \gamma + \nu\frac{1-p_0(\theta)}{\theta} + \mu\frac{f_a(x|a)}{f(x|a)}, \tag{3.4}$$

$$h = -p_0'(\theta)\pi(t,a) \tag{3.5}$$

for the transfers, and the effort prescription

$$[1-p_0(\theta)]\int_{\mathcal{X}}[z-t(z)]dF_a(z|a) + \mu\left[\int_{\mathcal{X}} u(t(z))dF_{aa}(z|a) - c''(a)\right] = 0 \tag{3.6}$$

with $\gamma > 0,\ \mu > 0,\ \nu > 0$ *and* $\theta = \Theta$.

All proofs are confined to Appendix A. It is helpful to explain this characterization in some details to better grasp our forthcoming results.

Uniform contract. The result of Proposition 1 is both substantive and simplifying: the optimal contract is independent of the number n of agents meeting a principal. Selcuk (2012) shows that using transfers contingent on the number n of agents present at a meeting amounts to exposing risk-averse agents to a lottery over payoffs, which is costly. Here the principals prefer a single contract because it minimizes the total cost of implementing their preferred action. That cost is convex and increasing in the action, so principals are better off avoiding a lottery over actions a_n. This result complements Selcuk's.

Explaining the conditions. Condition (3.4) shows that the slope of the transfer is related to the likelihood ratio f_a/f, as we know from Holmström (1979); this is what generates the incentives for effort. However this condition departs from Holmström's. Because of the search problem, (3.4) includes the term ν/θ: the MUP (2.2) binds at a solution, and clearly the search problem interacts with the incentive problem. This captures a new trade off, which lies at the heart of this model; we return to it throughout the paper.

Condition (3.5) is central to the main results of this paper. It shows that the compensatory transfer h amounts to the marginal benefit of increasing the queue length θ at any principal. Marginally increasing the queue length increases the probability of receiving the profit $\pi(t,a)$, but also imposes an externality on the other agents already present: they risk not contracting. The optimal transfer makes the principals indifferent and insures the agents against the failure to contract; it compensates them for the congestion externality at the meeting stage. We further discuss the role of transfers h in Section 4.3. For now, because constraints (3.2) and (2.2) bind,

$$V(\theta) = U(t,a) = u(h+y) = \widetilde{V}(\theta)$$

In words, there is perfect risk sharing in the search problem since the expected value of participating in the search market is exactly the value of contracting, which also equates the value of meeting but failing to contract.[9]

Finally Expression (3.6) is standard in a moral hazard problem, and takes the same form as in the bilateral contract of Holmström (1979), for example. It results from subgame perfection: for any transfer $t(x)$, the agent chooses the action that is optimal for her. Thus search does not distort this equation; it only alters the incentives through $t(x)$.

Some intuition. Principals compete to attract agents, which affords the latter some bargaining power; in equilibrium agents receive rents: $U(t, a) > u_0$. Attracting risk-averse agents is most efficiently achieved through insurance of two sources: first, by offering the transfer h (to insure against the search risk) and second by reducing the variability (the slope) of the transfer $t(x)$ to deliver rents. As a result, contracting agents face weaker incentives to exert effort.[10] The lump-sum transfer h mitigates, but does not eliminate, the distortion of the transfer function $t(x)$ (compared to the bilateral problem of Holmström). The reason is that principals still face a trade-off between incentives and participation probability, hence between the transfers h and $t(x)$. This is crystallized by the fact that both the moral hazard constraint (3.1) and the MUP (2.2) simultaneously bind, and that the multiplier ν enters the FOC (3.4).

3.3 Entry.

To complete the characterization we let the tightness Θ be determined by a free entry condition, which writes

$$\Pi(t, a, h; \Theta) = [1 - p_0(\Theta)]\pi(t^S, a^S) - \Theta h^S(\Theta) = k.$$

Expected profits to principals are exhausted by the entry cost. Using (3.5) to substitute in this condition, one has

$$[1 - p_0(\Theta)]\pi(t^S, a^S) - \Theta h^S(\Theta) = [1 - p_0(\Theta) + \Theta p_0'(\Theta)]\pi(t^S, a^S) = k. \tag{3.7}$$

Condition (3.7) shows, under a different lens, that the transfer h induces the principals to exactly internalize the externality $p_0'(\theta)\pi(t, a) < 0$ generated by their competition when

[9] However, imperfect insurance of the contracting agent remains essential to generate any incentives for effort.

[10] This is formally established in Lemma 3 in the Appendix.

making their entry decision. This is critical to the forthcoming main results. The entry condition can be related to known results in the search literature. Denote by

$$\eta(\theta) := \frac{\partial(1-p_0(\theta))}{\partial\theta}\frac{\theta}{1-p_0(\theta)} = -p_0'(\theta)\frac{\theta}{1-p_0(\theta)} \tag{3.8}$$

the elasticity of the matching rate for principals for any tightness θ. After some simple manipulations, Condition (3.7) rewrites as the well-known Hosios sharing rule:

$$[1-p_0(\Theta)]\left[[1-\eta(\Theta)]\pi(t^S,a^S)\right] = k, \tag{3.9}$$

We can readily see that only a fraction $1-\eta(\Theta)$ of the expected payoff of any one principal can be in fact appropriated by the principal; the balance is paid out to unsuccessful agents through the transfer h. This device is important to the results of Section 4.

With this (and other results in the proof) we can present some comparative statics results. We especially attract attention to the fact that the equilibrium action varies with the entry cost k – equivalently, with the market tightness. Naturally, all payoffs vary with the action a^S, and therefore with k. For example, in Condition (3.9), it implies not only that Θ varies with k on the LHS, but also a^S, t^S and therefore $\pi(\cdot,\cdot)$ vary with k.

Proposition 2 *Consider the competitive search equilibrium characterized in Proposition 1.*

1. *Market tightness Θ increases in the entry cost k;*
2. *Ex post profits $\pi(t^S,a^S)$ increase in the entry cost k; and*
3. *The equilibrium action a^S increase in the entry cost k.*

These statements are intuitive but not trivial. First, facing a higher entry cost, fewer principals choose to enter the search market. It is obvious when the surplus from entry is fixed, but no so here, where it varies with the equilibrium action. That action is parametrized by market tightness (Lemma 4 in the Appendix), hence by the exogenous cost k. This is the essence of the second statement: ex post profits $\pi(t^S,a^S)$ increase with k. This too is evident when the ex post profits are exogenous: the entry cost saturates the ex post profits. However here (again), these larger ex post profits are generated by a higher action – the third statement. Implementing a higher action is only possible when there less competition among principals. This stands at the heart of the next results.

4 Efficiency properties

Now we come to our main results. Efficiency is assessed against a weighted social welfare function with arbitrary weights. To fix ideas, the exercise we conduct is akin to the Second Welfare Theorem: can the market equilibrium deliver the planner's solution? And how does this compare to standard bilateral contracting? These are our benchmarks. As a preview, under the optimal contract, the decentralized equilibrium is always constrained welfare efficient. Absent the optimal contract, the equilibrium may not be efficient. Whether it is depends on the tradeoff between participation and incentives, that is, ultimately, on the value of the outside option u_0. Perhaps surprisingly all these results are *independent* of the arbitrary weights $\alpha, 1-\alpha$ of the welfare function. We explain why.

4.1 Benchmarks

Both benchmarks feature a single agent and a single principal. The first one is the model of Holmström (1979); the second one is the solution of the social planner.

The agency problem. The payoff to the principal and the agent are, respectively:

$$\pi(t,a) := \int_{\mathcal{X}} [z - t(z)]\, dF(z|a) \quad \text{and} \quad U(t,a) := \int_{\mathcal{X}} u(t(z))dF(z|a) - c(a).$$

The principal maximizes $\pi(t,a)$ by choice of the pair $(t(x), a)$, subject to the moral hazard constraint $U_a = 0$ and to $U(t,a) \geq u_0$ for some known outside option u_0. The solution is denoted with the superscript B and is characterized by

$$\frac{1}{u'(t)} = \lambda^B + \mu^B \frac{f_a}{f} \quad \text{and} \quad \pi_a + \mu^B U_{aa} = 0 \tag{4.1}$$

where λ^B, μ^B are Lagrange multipliers. Some of our result are contrasted to this benchmark.

The planner's solution. For arbitrary weights $\alpha, 1-\alpha,\ \alpha \in (0,1)$ placed on the principal and the agent surplus, a planner maximizes the sum of utilities

$$W(t,a;\alpha) := \alpha\pi(t,a) + (1-\alpha)U(t,a)$$

subject to the same constraints. For a utilitarian planner, for example, $\alpha = 1/2$.[11] At a solution $\left(t^W(x), a^W\right)$ it holds that

$$\frac{\alpha}{u'(t)} = (1-\alpha) + \lambda^W + \mu^W \frac{f_a}{f} +, \quad \forall x \in \mathcal{X} \tag{4.2}$$

The planner always engages in some redistribution when the agent is risk-averse (this is the term $1-\alpha$ in (4.2)). Also,

$$\alpha \pi_a + \mu^W U_{aa} = 0 \tag{4.3}$$

Even when $\alpha = 1/2$, there is no reason to expect the solution to the standard agency problem to also be the solution to the planner's problem. It would require, for example, that $(1-\alpha+\lambda^W)/\alpha = \lambda^B$ and $\mu^W/\alpha = \mu^B$. To be clear, Holmström's solution is Pareto optimal but not welfare efficient. Yet, for any utility function, any distribution $F(x|a)$, any weights $\alpha, 1-\alpha$ and any given outside option, the frictional market delivers constrained welfare optimality *under the optimal contract*, even where the initial (bilateral) trading environment does not.[12]

4.2 Main result 1: efficient equilibrium.

With the optimal contract characterised in Proposition 1, the frictional market exactly "corrects" the contracting problem highlighted in the benchmarks. The equilibrium allocation is identical to that chosen by the planner. Moreover, this is true for any weights $\alpha \in (0,1)$. In contrast, in the benchmark problems (Section 4.1), the planner offers a different transfer function and prescribes a different action than the principal, so that the expected output differs across the two benchmarks. To establish this result, we construct the social welfare function from the principals' and agents' payoffs

$$W(t,a,h;\alpha) := \Lambda^P \alpha \Pi(t,a,h) + \Lambda^A (1-\alpha) V(t,a) - \alpha \Lambda^P k,$$

[11] We are agnostic as to the relative importance of principal and agent; rather we are interested in the total net surplus generated by a relationship in a bilateral problem. However this formulation allows for a more general treatment, and to show that the results are invariant in α.

[12] In many other models (e.g. Coles and Eeckhout, 2003; Jacquet and Tan, 2012), the bilateral trade is already constrained efficient – so directed search only preserves efficiency. Here it restores it.

net of the entry cost k paid by principals.[13] Dividing by Λ^A, welfare per agent reads

$$w(t,a,h) := -\alpha\frac{k}{\theta}+\frac{[1-p_0(\theta)]}{\theta}\left[\alpha\pi(t,a)+(1-\alpha)[U(t,a)-u(h+y)]+(1-\alpha)u(h+y)-\alpha h\right] \tag{4.4}$$

The last two terms in the brackets sum to the "meeting surplus" – the net utility of paying h to the unsuccessful agents. Next we supply a definition of constrained efficiency, which is the yardstick by which we measure the efficiency of the equilibrium allocation.

Definition 2 *A constrained welfare efficient allocation denoted* $(t^P(x), a^P, h^P)$ *maximizes* $w(t,a,h)$ *subject to moral hazard and the agents' participation (Constraints (3.1) and (3.2)).*

Proposition 3 *The competitive search equilibrium characterized by Proposition 1 and the entry condition (3.7) is constrained welfare optimal under the optimal contract. That is, consider a planner maximizing* $w(t,a,h)$ *for any weights* $\alpha \in (0,1)$*; the competitive search equilibrium always implements the planner's allocation:*

$$t^S(x) = t^P(x),\ a^S = a^P,\ h^S = h^P \text{ and } \Theta^S = \Theta^P.$$

In the proof we show that the entry conditions for the planner and the equilibrium are identical, that the endogenous variables $\left(t^P(x), a^P, h^P\right)$ are equal to the equilibrium allocation $\left(t^S(x), a^S, h^S\right)$. Therefore the equilibrium tightness Θ^S and Θ^P must also be identical. The mathematics are simple: in the search model, the constrained problem of the principals is isomorphic to that of the planner. Hence they have the same solution.

The economic intuition is remarkably robust; it is easiest to explain if taking $\alpha = 1/2$. Then a welfare optimal allocation maximizes the (unweighted) sum of the principals' surplus and the agents' surplus – subject to constraints. But this is exactly what the principals do when they are subject to the Market Utility Property (2.2). Principal competition forces them to internalize the agents' utility, as does a planner. Thus, the principals and the planner present the agents with the same incentives.[14]

This logic persists when $\alpha \neq 1/2$. In terms of economics, altering the weights on the payoffs of principals and agents is reflected in the endogenous tightness Θ. That is, a planner who decreases the relative importance of principals, for example, equivalently wishes to

[13]When $\alpha = 1/2$, this welfare function can be found in Moen (1997), Mortensen and Pissarides (1994) or in Rogerson, Shimer and Wright (2005), albeit with linear utility.

[14]The first-best remains out of reach because imperfect insurance remains necessary to induce effort on the part of agents.

increase the rents to agents. However, the same planner also weighs the entry cost k by α; therefore α is neutral on principal entry. The entry condition of the planner is the same as that of principals in the decentralized equilibrium. With the same entry decision, the tightness is identical and the contracts offered by principals are also identical (see Proposition 2) across the problems. The Lagrange multipliers simply adjust to be equal (modulo a linear constant).

Thus we see the *search market* implements perfect risk sharing of the search risk because that minimizes the sum of transfers for a given action. In contrast the *planner* implements perfect risk sharing out of his objective function. The motives are different but the outcome is the same. In closing, efficiency cannot be attributed solely to competition; without frictions competition typically delivers inefficient solutions and multiple equilibria (see for example Aubert, 2005; Attar and Chassangnon, 2009; Attar, Piaser and Porteiro, 2007; Attar, Campioni and Piaser, 2007). Directed search brings about just enough competition.[15] However even this relies on the right contract to be used, as we show next.

4.3 Main result 2: not always efficiency.

Our first result relies on the principals using the optimal contract. Here we further explore the role of the compensatory transfers h; we show they are sometimes essential to welfare efficiency. Constraint (2.2) gives us a telling indication: suppose $h \equiv 0$ for exogenous reasons, then (2.2) becomes

$$\frac{1 - p_0(\theta)}{\theta} U(t, a) + \left[1 - \frac{1 - p_0(\theta)}{\theta}\right] u_0 \geq \widetilde{V}(\theta).$$

An agent who fails to contract receives only her outside option, while $U(t, a)$ need not be equal to u_0. Even in equilibrium, the LHS of this constraint remains a lottery, which introduces costly search risk. This stands in contrast from the first result. The agent facing this participation decision requires some kind of risk premium to compensate for the gamble; that is, more distortions are required. We begin with the counterpart to Proposition 1 when $h \equiv 0$. To be clear, this is not the optimal contract.

Lemma 1 *Let $h \equiv 0$ in Problem 1. The optimal contract $(t^N(x), a^N)$ is characterized by the necessary and sufficient first-order conditions*

$$\frac{1 - p_0(\theta)}{u'(t)} = [1 - p_0(\theta)]\frac{\nu}{\theta} + \mu \frac{f_a}{f} \tag{4.5}$$

[15] Agents are not infinitely elastic with respect to their payoffs, as they are under Bertrand competition.

$$-p_0'(\theta)\pi(t,a) = \frac{\nu}{\theta}\frac{1-p_0(\theta)+\theta p_0'}{\theta}\left[U(t,a)-u_0\right] \tag{4.6}$$

$$[1-p_0(\theta)]\pi_a(t,a) + \mu U_{aa}(t,a) = 0 \tag{4.7}$$

with $U(t,a) > u_0$ *(so* $\lambda^N = 0$*),* $\mu^N, \nu^N > 0$ *and* $\theta = \Theta^N$*. Market tightness* Θ^N *is determined by the entry condition*

$$\left[1-p_0(\Theta^N)\right]\pi(t^N, a^N) = k. \tag{4.8}$$

The key difference to Proposition 1 is Condition (4.6), which balances the benefit (to principals) of a longer queue with its cost. This cost is a marginal increase in the net rent to agents – one can think of this as their "risk premium". That cost is now measured as a departure from the outside option u_0 rather than by the transfer h in Proposition 1. The trade-off between participation and incentives is more acute than if using transfer h because now it is handled by a single instrument: the transfer function $t^N(x)$.

The material consequence of losing the flexibility provided by the transfer h is that the equilibrium allocation fails to be efficient when the exogenous outside option u_0 is large enough.[16] Denote by the superscript *np* a solution to the planner's problem.

Proposition 4 *Suppose* $h \equiv 0$ *for exogenous reasons. The competitive search equilibrium is constrained welfare efficient only if* u_0 *is low enough, and otherwise not. For* u_0 *large enough there is excessive entry of principals in equilibrium:* $\Theta^N < \Theta^{np}$*.*

Why this result? Not using h means the "risk premium" $U(t,a) - u_0$ must be large enough to attract agents. That is, the transfer function $t^N(x)$ must be more distorted than the function $t^S(x)$ to generate the rents $U(t^N, a^N)$ necessary to attract agents; it must provide better insurance than $t^S(x)$. We know that in response the equilibrium action a^N is also lower. As u_0 increases, the incentives for effort provided by the function $t^N(x)$ become increasingly weak and the action a^N of the agent increasingly low. This alone does not imply inefficiency. What does is the fact that the planner prefers curtailing these rents for large enough values of the outside option u_0 by setting $U(t^{np}, a^{np}) = u_0$, lest the action become too low. However each principal *must* offer rents to the agents to have a chance to match; each distorts $t^N(x)$ to excess compared to the welfare-maximizing transfer $t^{np}(x)$.[17] In other words, they do not have the instrument necessary to internalise the effect of their

[16] This is true for u_0 beyond a threshold, not for a carefully chosen value of u_0.

[17] To be obvious, a planner does not care about which principals match, only that the optimal measure of principals enter and match.

competition on each other. In Proposition 3 instead, the competition externality is priced by h^S for any outside option u_0, which leaves t^S less distorted than t^N. A notable aspect of this second result is this. Sharing of the search risk is now imperfect since $U(t^N, a^N) > u_0$ in equilibrium; the parties inability to efficiently share the search risk has consequences for the incentives provided through the contract.

The tradeoff between efficiency and participation is the source of the breakdown in efficiency. This new margin does not exist in GSW, for example; it is also irrelevant in Moen and Rosen (2011) because effort remains "conditionally first best".[18] This new margin is mitigated when using the compensatory transfer h because it captures exactly the externality that principals exert on agents through their competition, and so mutes the distortions of $t(x)$.

When the equilibrium is not constrained efficient the planner can nonetheless correct the allocation using lump-sum taxes and subsidies. Tax all entering principals at a flat rate $\tau^P > 0$ so the entry cost is $k + \tau^P$. For some subsidy τ^A to agents failing to contract, budget balance requires

$$\Lambda^P \tau^P = \Lambda^A \tau^A \text{ or } \Theta = \frac{\tau^P}{\tau^A}.$$

Hence by setting transfers τ^A, τ^P such that

$$\frac{\tau^P}{\tau^A} \equiv \Theta^{np},$$

the market implements the planner's allocation. Re-arranging terms, notice $\tau^P = \Theta^{np}\tau^A$, and letting $\tau^A = h^S$, indeed the transfer h has exactly the same effect as the fiscal scheme. When the equilibrium is inefficient it is because too many principals enter and inefficiently compete for agents. The fiscal scheme curbs entry, as does the transfer h. We conclude therefore that a) directed search alone does not guarantee constrained welfare optimality, and b) using the optimal contract is important, and at times, essential to efficiency.

4.4 Applications

We turn to two short applications to illustrate our results and the relevance of this model. For these examples we suppose that $h \equiv 0$, which may be in keeping with most real-life insurance contracts and financial contracts.

[18] No distortion conditional on type; see Laffont and Tirole (1986). It is rooted in the perfect substitutability of effort and type.

4.4.1 Insurance

Let principals be insurers and agents be customers (seeking insurance). A customer faces a damage $-d \in [-D, 0]$ and $-d \sim F(-d|a)$, where $F(\cdot|a)$ is FOSD with respect to the action $a \in \mathcal{A}$. A contract is a pair of *net transfer* and action recommendation $(t(-d), a)$.[19] Damages are mapped into monetary equivalent though $\phi : [-D, 0] \mapsto \mathbb{R}$. The wealth W of any agent is large enough in the sense that $W + \phi(-D) \geq 0$ and $W + t(-d) \geq 0$.[20] Therefore, conditional on contracting, the return to insurance underwriters is

$$\pi(t, a) := \int_{-D}^{0} \phi(-d) - t(-d) dF(-d|a), \tag{4.9}$$

and that to their customers is

$$U(t, a) := \int_{-D}^{0} u(W + t(-d)) dF(-d|a) - c(a). \tag{4.10}$$

For any contract offer, the moral hazard constraint follows that in the main text, and pins the maximiser $\widehat{a}$:

$$U_a(t, a) = \int_{-D}^{0} u(W + t(-d)) dF_a(-d|a) - c'(a) = 0.$$

Applying Proposition 4 we know that for u_0 large enough, efficiency breaks down. Customers are overinsured and exert too little diligence. For example, there may be too small a deductable. In terms of interpretation it may be necessary to clarify what u_0 may be in this model. In most countries outside the United States, some health insurance is provided by Government and so defines a clear outside option for customers of private health insurance. The model suggests that as long as this outside option is not too generous, the frictional market delivers a constrained efficient allocation, but not if the outside option becomes "too good".

4.4.2 Financial contracting

Let principals be financiers and agents be entrepreneurs. A project requires an investment $I > 0$ and an entrepreneur has some wealth A, $0 \leq A < I$; therefore she needs a financier. In line with Holmström and Tirole (1997), we deliberately do not label financiers as lenders or equity investors; they just contribute capital $I - A$ to the venture. A project delivers

[19] A net transfer can be decomposed into a premium $-p$ and a claim payment $P(-d)$, for example.

[20] So there is no limited liability; see below.

a return $R \sim F(R|a)$ on $\mathcal{R} \subset \mathbb{R}$, and the principal shares this return R with the agent: $R_e(R) + R_f(R) \equiv R$. Therefore, conditional on contracting, the return to financiers is

$$\pi(A, a) := \int_{\mathcal{R}} R_f(R) dF(R|a) - (I - A), \tag{4.11}$$

and that to entrepreneurs is

$$U(A, a) := \int_{-A}^{\overline{R}} u(A + R_e(R)) dF(R|a) - c(a). \tag{4.12}$$

Explicit in this formulation is the fact that entrepreneurs are protected by limited liability below zero; that is, $0 \leq R_e(R) + A$ at all times.[21] Throughout we suppose that $\pi + U \geq I$ at the equilibrium of the bilateral contracting problem (the benchmark): investing is socially valuable. The moral hazard constraint reads

$$\int_{-A}^{\overline{R}} u(A + R_e(R)) dF_a(R|a) = c'(a), \tag{4.13}$$

and given the conditions on $u(\cdot)$ and $F(\cdot|a)$, the wealthier an agent is, the more effort she exerts (see also Jewitt, Kadan and Swinkels, 2008; Roger, 2013). Again Proposition 4 tells us that for u_0 large enough, efficiency breaks down. A small effort may be necessary to make an attractive interpretation. We may think of principals as banks subject to regulation, while the outside option u_0 is provided by "shadow banks" – unregulated providers. In addition we note that the credit market may even be rationed. Indeed for a set of production (equivalently, distribution) functions $F(R|a)$ and initial wealth A, we may have

$$0 < \pi(A, t^N, a^N) < I - A,$$

in which case a principal should refuse to trade altogether. The reason is that the effort produced by the agent is too low to generate sufficient (expected) returns to the financier. If we contemplate a distribution $G(I - A)$ of (net) investment costs, some projects are not funded in equilibrium, even though entry is ex ante attractive for principals.

5 Discussion

In light of our results we raise a few items for discussion and comment on institutional design.

[21] There are known implications of this lower bound on transfers to the agent: the agent exerts less effort because the downside is not harmful and so the incentives are weaker (see Jewitt, Kadan and Swinkels, 2008). We take this in our stride and abstract from this issue.

5.1 Departures from GSW

This work differs from GSW in two important ways. First, and most obviously, it treats moral hazard while GSW focus on adverse selection. In doing so we uncover a new source of inefficiency, which stems from the interaction between moral hazard and participation. Under moral hazard the Pareto frontier depends on the action of the agent, and so it is endogenous to the problem. This is exactly why the equilibrium may sometimes be inefficient under the circumstances we describe in the paper.[22] This cannot occur in GSW, where there is no effort to induce – but sorting instead. Second, we allow for multilateral meetings, whereas the model in GSW features bilateral meetings. With multilateral meetings there is a queue at each principal, so this queue must be (endogenously) rationed. It is also why the transfer h arise as part of the optimal contract. These have no object with bilateral meetings for there can be no congestion.

5.2 Bilateral meeting

It is reasonable to wonder how results are affected by the meeting technology; a natural alternative is to consider bilateral meetings – as in GSW. It turns out that not much changes. The intuitive reason is that under bilateral meeting there is no role for the transfer h, so that the problem can essentially be recast as in Section 4.3.

The meeting technology is a generic constant returns to scale meeting function $m(\theta)$, increasing and concave. The meeting rates for principals and agents become, respectively, $m(\theta)$ and $m(\theta)/\theta$. Conditional on meeting a partner, the contracting probability is 1 – modulo satisfying the participation constraint; there is no longer any compensatory transfer h. An agent meeting no principal receives u_0 and a principal meeting no agent receives 0. The expected utility of an agent participating in submarket j is

$$V(\theta(\mathcal{C}^j)) := \left[\frac{m(\theta^j)}{\theta^j}U(t^j, a^j) + \left(1 - \frac{m(\theta^j)}{\theta^j}\right)u_0\right]. \tag{5.1}$$

The expected payoff of principal j, gross of entry cost k, reads

$$\Pi(t^j, a^j; \theta^j) := m(\theta^j)\pi(t^j, a^j). \tag{5.2}$$

Setting $m(\theta) = 1 - p_0(\theta)$, it is immediate that these payoffs are simple transformations of the payoffs to agents and principals in Section 4.3. It is no stretch to assert the same result

[22] To be clear, the equilibrium is always Pareto efficient: all constraints are satisfied. This differs from welfare optimality.

follows.[23] That is, under bilateral contracting, the equilibrium allocation remains efficient as long as the outside option u_0 is not too large; beyond a threshold, inefficiency obtains. Under bilateral meeting, the meeting technology already encodes the uniform rationing rule: the meeting probability for agents is $m(\theta)/\theta$. Under multilateral meeting it is $m(\theta)$ but uniform rationing $1/\theta$ implies the same contracting probability. The more essential difference is that bilateral meeting does not permit the use of compensatory transfers h; it is more restrictive. However, the same fiscal scheme as in Section 4.3 can be used.

5.3 Allowing for adverse selection

Our model can be extended, at a cost, to allow for adverse selection.[24] We do so in a working paper (cited, and available upon request). The moral hazard problem persists; it does not reduce to a case of "false moral hazard" (as labelled by Laffont and Tirole (1986, 1993)). In that environment our efficiency results continue to hold, for exactly the same reason as we describe here. So the breakdown in efficiency we describe is clearly driven by moral hazard, not adverse selection. In that paper we also discuss other aspects of efficiency that are connected to adverse selection.

5.4 Common agency and contracting externalities.

The extensive form of the game that is induced by the search stage rules out common agency and thus neutralizes contracting externalities beyond the participation stage. We do not mean to advocate the merit of an extensive form over another, however three points deserve mention. First, searching for trading partners is a natural process that needs little defense. Second, it is precisely these contracting externalities that lay at the source of inefficiencies in many models of common agency under moral hazard (e.g. Bisin and Guaitoli, 2004; Aubert, 2005; Attar and Chassangnon, 2009; Attar, Piaser and Porteiro, 2007; Attar, Campioni and Piaser, 2007). Whether these externalities affect incentives does depend on the details of the game, so the inefficiency results of these papers need not hold universally.[25] Third, the extensive form results from the search stage; it is not protracted nor chosen for convenience.

[23]It is no stretch because we have duly verified this claim.

[24]In addition to moral hazard one must write incentive constraints to sort agents, as well as an endogenous rationing rule that depends on the information revealed by agents as part of the mechanism.

[25]Here the absence of common agency also implies that no further constraint need be placed on the planner; see Bisin and Guaitoli (2004).

5.5 Different environments.

Constraints on transfers. The analysis of the optimal contract takes transfers h to be unconstrained. While the optimality condition (3.5) characterizing h make it plain they are always bounded, the total value $\Theta \cdot h$ may be large. A cap on transfers h may arise from a binding cash constraint or a liquidity constraint. Even when capped, compensatory transfers go a long way in providing the necessary insurance against search risk, and so assist in inducing the efficient allocation.[26]

Random matching. Since Diamond (1971) we know that random matching is not welfare efficient: each seller behaves like a monopolist. So too with this model if casting it in a random matching framework. Left to their own devices, principals offer the standard second-best contracts and too many of them enter compared to the welfare-maximizing planner's solution. We provide details in Appendix B.

The reason for this outcome is simple: under random matching, the matching process is orthogonal to the utility profile, so the principals have no tool by which to compete for agents. Contracts are independent to the matching process and no rent need be generated to attract agents. Mathematically the MUP (2.2) does not enter the principals' problem, so they do not internalize the utility of the agents. Instead the principals implement the standard second-best solution (Condition (4.1)). This suggests that understanding the details of the nature of the matching process is critical to assess whether it leads to efficient outcomes, and if not, what correction may be warranted.

Output maximization. Output maximization is the criterion used by Acemoglu and Shimer (1999). In contrast to a welfare-maximizer, an output-maximizing planner does not care about the utility of agents in a match; it is as if $\alpha = 1$ in the social welfare function. So the planner solves exactly the same problem as a single principal dealing with a single agent, up to a multiplying constant. It is as if principals and agents matched randomly. The social value of a match is understated as the utility of agents is ignored.

[26] We do not provide details of this intuitive claim, which we have duly verified – details available upon request.

6 Conclusion

This paper presents a contracting model under moral hazard cast in a competitive search environment. We characterize the optimal contract for quasi-linear preferences and show that, with the optimal contract, the equilibrium allocation is always constrained welfare efficient. That optimal contract specifies compensatory transfers paid to an agent meeting, but not contracting with, a principal. Absent these transfers the allocation need not be efficient. This inefficiency result has roots in the interaction of the directed search problem and the moral hazard problem (of risk averse agents).

Our main contribution is to allow for moral hazard; the social surplus depends on the action of the agent(s). Because there is a search stage, and because search is directed, principals compete to attract agents by increasing their rents. Thus search frictions affect the social surplus to be shared – through the level of action governing output – and not just the sharing rule. Markets and frictions matter a great deal in that they alter the agents' outside option: search frictions restore some bargaining power on the side of the agents, however incompletely so. In equilibrium this process of competition generates the (constrained) efficient allocation, precisely because search entails frictions that render the agent imperfectly elastic. Thus we also affirm that the nature of the matching process is relevant to efficiency; random matching does not induce efficiency.

We believe that combining agency with directed search is a natural model of problems such as employment or procurement, in addition to the applications we suggest. The transfers h may be construed as optimal unemployment insurance that is voluntarily offered by employers when there is asymmetric information in production rather than search. Search models of monetary policy may also benefit from this innovation. It is already known that paying with debt is not the same as paying with cash, not because of record-keeping problems but because of ex post moral hazard (see DeMarzo, Kremer and Skrzypacz (2005) in the context of auctions). That is, there may be reasons that traders have to hold money balances beyond the standard explanation of unavailability of credit.

Appendix

This Appendix has two parts. The first one contains the proofs. The second part presents some additional material on random matching and output maximization to support our Discussion.

A Proofs

Proof of Proposition 1: We begin by characterizing the solution to a variant of Problem 1, in which the number of agents at each principal is restricted to be finite: $n < \infty$, so that the number of constraints remains finite. Then we demonstrate that solution is dominated by a uniform contract. Therefore any solution to Problem 1 is also dominated by a uniform contract. For clarity we recall the following. Denote a n *meeting contingent* contract by $\mathcal{C} = (\mathbf{t}(x), \mathbf{a}, \mathbf{h})$ posted in any submarket with $\mathbf{t}(x) = (t_1(x), ..., t_N(x)), \mathbf{a} = (a_1, ..., a_N), \mathbf{h} = (h_1, ..., h_N)$ with $N \in \mathbb{N}$. The contract $\mathcal{C}$ offers transfers $t_n(x), h_n$ and prescribes actions a_n that may depend on the number n of agents meeting one principal in the submarket.

Let the profit to a principal contracting with an agent when there are n agents be $\pi_n = \pi(t_n, a_n)$. The ex ante expected net profit to a principal being in a n meeting in the submarket from the text writes

$$\Pi(\theta) = \sum_{n=1}^{N} p_n(\theta)[\pi_n - (n-1)h].$$

We adopt a similar notation for the agents. The ex ante expected payoffs to an agent being in a n-agent meeting in the submarket are

$$V(\theta) = \sum_{n=1}^{N} \frac{p_n(\theta)}{n} U_n + \left[1 - \sum_{n=1}^{N} \frac{p_n(\theta)}{n}\right] u(h_n + y).$$

Principals creating a submarket implies that they can compute the anticipated tightness $\theta(\mathcal{C})$ – contingent on the posted contract $\mathcal{C}$. This anticipation is also shared by agents observing the contract $\mathcal{C}$. Off equilibrium path $\theta(\mathcal{C}) \neq \Theta$ can be anything.

Lemma 2 *Let $N < \infty$ in Problem 1; a unique solution exists. It is characterized by the*

Conditions

$$\begin{aligned}
\frac{p_n(\theta)}{u'(t_n)} - \left[\gamma_n + \mu_n \frac{f_a(x|a_n)}{f(x|a_n)} + \nu_n \frac{p_n(\theta)}{n}\right] &= 0 \\
p_n(\theta)(n-1) + \left[\gamma_n - \nu_n \left[1 - \frac{p_n(\theta)}{n}\right]\right] u'(h_n + y) &= 0 \\
\sum_{n=1}^{N} p_n'(\theta)[\pi_n - (n-1)h_n] + \nu_n \sum_{n=1}^{N} \frac{p_n'(\theta)}{n}[U_n - u(h_n + y)] &= 0
\end{aligned}$$

for $n \geq 0$. *For each* n *all the constraints bind except the non-negativity constraints on payments – i.e.* $h_n > 0$.

Proof: For all $n \leq N$, the first-order conditions of Problem 1 are as in the Lemma (after simple manipulations). The complementary slackness conditions are

$$\forall n \geq 1, \quad \gamma_n[U_n - u(h + y)] = 0, \;\; \nu_n[V(\theta) - \widetilde{V}] = 0, \;\; \epsilon h_n = 0,$$

Since $\gamma_n \geq 0$ the second condition immediately implies $\nu_n > 0$ by simple inspection. Suppose that $\gamma_n > 0$; then we have $V(\theta) = U(t_n, a_n) = u(h_n + y)$. From the third condition

$$\sum_{n=1}^{N} p_n'(\theta)\left[\pi_n - (n-1)h_n\right] = 0,$$

which then implies $h_n > 0$; thus $\epsilon_n = 0$ for all n. Next we argue that $\gamma_n > 0$ in three steps. One, it is cheaper to the risk-neutral principal. Fix the action a_n, whenever $U_n = u(h_n + y)$, the quantity $h_n + c(a_n)$ is the certainty equivalent of the lottery $F(x|a_n)$ over $[t_n(\underline{x}), t_n(\overline{x})]$. That is, $\mathbb{E}_{\mathcal{X}}\left[t_n(z)|a_n\right] = h_n + c(a_n)$, and if $U_n > u(h_n + y)$, $\mathbb{E}_{\mathcal{X}}\left[t_n(z)|a_n\right] > h_n + c(a)$: that is, delivering $\widetilde{V}(\theta)$ becomes more expensive. Two, it induces at least weakly more effort.

Indeed, the MUP (2.2) must continue to bind at $\widetilde{V}(\theta)$ for each n, that is,

$$\begin{aligned}
\int_{\mathcal{X}} u'(t_n)\Delta_h t_n(z) dF(z|a_n) + \overbrace{\left[\int_{\mathcal{X}} u(t_n(z)) dF_a - c'(a_n)\right]}^{=0} \frac{da}{dt}\Delta_h t_n \;\; &+ \;\; \left(1 - \frac{p_n(\theta)}{n}\right) u'(h_n + y) = 0 \\
\int_{\mathcal{X}} u'(t_n)\Delta_h t_n(z) dF(z|a_n) \;\; &= \;\; -\left(1 - \frac{p_n(\theta)}{n}\right) u'(h_n + y) \\
&< \;\; 0
\end{aligned}$$

and therefore $\int_{\mathcal{X}} \Delta_h t_n(z) dF(z|a_n) < 0$. The notation $\Delta_h t_n(z)$ denotes a variation in the function $t_n(x; h)$ that is parametrized by h_n. By monotonicity of $t_n(x; h)$ in x, there exists some $\hat{x} \in (\underline{x}, \overline{x}]$ such that either

$$\int_{\underline{x}}^{\hat{x}} \Delta_h t_n(z) dF(z|a_n) < 0, \int_{\hat{x}}^{\overline{x}} \Delta_h t_n(z) dF(z|a_n) \geq 0 \text{ and } \int_{\mathcal{X}} \Delta_h t_n(z) dF(z|a_n) < 0,$$

in which case the new function t_n (after adjusting for a marginal change in h_n) is steeper, or the converse:

$$\int_{\underline{x}}^{\hat{x}} \Delta_h t_n(z) dF(z|a_n) \geq 0, \int_{\hat{x}}^{\overline{x}} \Delta_h t_n(z) dF(z|a_n) < 0 \text{ and } \int_{\mathcal{X}} \Delta_h t_n(z) dF(z|a_n) < 0,$$

in which case t_n is shallower. However monotonicity of $t_n(x;h)$ in x and monotonicity of $u(\cdot)$ imply the latter case is impossible. Now to show this must result in more effort, recall that because $\int f_a(z|a)dz = 0$, for some $\tilde{x}$,

$$f_a \begin{cases} < 0, & x < \tilde{x}; \\ = 0, & x = \tilde{x}; \\ > 0, & x > \tilde{x}. \end{cases}$$

Take any $h_n^1 \downarrow h_n^2$, the corresponding transfers $t_n(x;h_n^2), t(x;h_n^1)$ parametrized by h_n must take the form characterized by (3.4). By continuity in t and a, if $t_n(x;h_n^1)$ passes through the point $\tilde{x}$, $t_n(x;h_n^2)$ passes arbitrarily close to it. Then,

$$t(x;h_n^2) \begin{cases} < t(x;h_n^1), & x < \tilde{x} \text{ and} \\ > t(x;h_n^1), & x > \tilde{x}. \end{cases}$$

so that

$$\Delta_h t_n \begin{cases} < 0, & x < \tilde{x} \text{ and} \\ > 0, & x > \tilde{x}. \end{cases}$$

Let $\hat{x} = \tilde{x}$, then $\int u' \Delta_h t_n dF_a > 0$ necessarily. Now contract offers must satisfy the constraint $U_a = 0$. The identity $\frac{d}{dh} U_a \equiv 0$ stemming from this constraint rewrites

$$\int u' \Delta_h t_n dF_a + U_{aa}\Big|_{a_n} \frac{da_n}{dh_n} \equiv 0$$

and since $U_{aa} < 0$, the implicit function $a(h_n)$ is increasing in h_n. Thus we must have $\gamma_n > 0$ as well.

One more first-order condition is required

$$p_n(\theta)\pi_n|_{a_n} + \mu_n \frac{\partial^2 U_n}{\partial a_n^2} = 0$$

The U_a-terms are zero (by the moral hazard constraints (3.1). The multiplier μ_n is positive for each n by direct application of Jewitt's proof (Jewitt, 1988). To complete the argument, note that with these contracts $V(\mathcal{C}) = \widetilde{V}$ for each type, hence given some market utility $\widetilde{V}$, the contract characterized in Lemma 2 does constitute a best response. In other words, $\widetilde{V}$ is a fixed point of $V(\mathcal{C})$. ■

This characterizes the solution to the original problem for any finite n. To show a uniform contract dominates, consider the characterization in Lemma 2 where $U(t_n, a_n) = u(h_n + y)$. A principal can offer the contract (t, a, h) such that, first, by Jensen's inequality:

$$u(h + y) \geq \mathbb{E}_n \left[u(h_n + y)\right] \text{ and } h \leq \mathbb{E}_n \left[h_n\right].$$

There also exists a transfer function $t(x)$ that induces action $a = \mathbb{E}[a_n]$ at a lower cost than the array of transfers $(t_1, t_2, ...t_N)$. To establish this, let $L(\cdot)$ denote the Lagrangian of the cost-minimization problem. The total cost of implementing an action a_n that is induced by the transfer function t_n is defined as

$$\begin{aligned} C(a_n) &= \max_{\nu_n, \mu_n} \min_{u(t_n)} L(u(t_n)) \\ &= \max_{\nu_n, \mu_n} \nu_n \left[V(\mathcal{C})\right] + \mu_n c'(a_n) - \int \rho \left(\nu_n + \gamma_n + \mu_n \frac{f_a(z|a_n)}{f(z|a_n)}\right) dF(z|a_n^j) \end{aligned}$$

for some fixed n and where $V((\theta(\mathcal{C})) = u(h_n + y)$. The cost function $C(a_n)$ is convex in a_n under CLI (see JKS). The first line is an application of the Lagrange duality theorem, and the second one uses the definition of the optimal transfer t_n. In the second line the function $\rho \left(\nu_n + \gamma_n + \mu_n \frac{f_a(z|a_n^j)}{f(z|a_n^j)}\right) := \max_{u(t_n)} \left(\nu_n + \gamma_n + \mu_n \frac{f_a(z|a_n^j)}{f(z|a_n^j)}\right) u(t_n) - u^{-1}(u(t_n))$ is a convex function for any argument $\left(\nu_n + \gamma_n + \mu_n \frac{f_a(z|a_n^j)}{f(z|a_n^j)}\right)$ and $c''(a_n) > 0$ by assumption. Take any $a_1 \leq a_2 \leq ... \leq a_N$ (w.l.o.g.) induced by $t_1 \neq t_2 \neq ... \neq t_N$ and define the expected cost over realizations of n

$$\mathbb{E}_n[C(a_n)] := \sum_{n=1}^{N} \Pr(a_n) C(a_n)$$

for any probability distribution. This is a convex function for it is necessarily bounded (below and above). Furthermore, there also exists a convex function

$$C\left(\mathbb{E}_n[a_n]\right) := C\left(\sum_{n=1}^{N} \Pr(a_n) a_n\right).$$

For each n, let a_n^* denote the optimal action; then $\mathbb{E}_n[a_n^*] \equiv \sum_{n=1}^{N} \Pr(a_n) a_n^*$ and

$$C\left(\mathbb{E}_n[a_n^*]\right) \leq \mathbb{E}_n[C(a_n^*)],$$

which contradicts the premise that the transfers $t_1 \neq t_2 \neq ... \neq t_N$ are the least-cost transfers to implement $\mathbb{E}_n[a_n^*]$. Finally we rewrite the payoffs of all parties under uniform contracts. $\sum_{n=1}^{\infty} p_n(\theta) = 1 - p_0(\theta)$, and evaluating $\sum_{n=1}^{\infty} p_n(\theta)(n-1)h = \theta h$. Hence

$$\Pi = [1 - p_0(\theta)]\pi - \theta h,$$

and the expected payment is θh. So for the agent so under uniform contract

$$
\begin{aligned}
V(\theta) &= \sum_{n=1}^{\infty}\left[\frac{p_n(\theta)}{n}U + [1-\frac{p_n(\theta)}{n}]u(h+y)\right] \\
&= \sum_{n=1}^{\infty}\frac{p_n(\theta)}{n}\left[U - u(h+y)\right] + u(h+y) \\
&= \frac{1-p_0(\theta)}{(\theta)}\left[U - u(h+y)\right] + u(h+y).
\end{aligned}
$$

To complete the proof it suffices to take the first-order conditions of that new program and repeat the steps of the proof of Lemma 2. ■

Proof of Proposition 2: To prove the Proposition we need some intermediate results.

Lemma 3 *Under search with friction, the optimal action $\widehat{a}$ solving (3.6) under (3.1) is lower than the standard optimal action a^B solving (4.2) under (3.1) for each type.*

Proof: To show this we call on the representation of the cost of effort that was first suggested by Jewitt (1997) and used in JKS.

$$
C(a) := \max_{\gamma,\mu}\left[\gamma(u(h+y)+c(a)) + \mu c'(a) - \int \rho\left(\gamma + \frac{\nu}{\theta} + \mu\frac{f_a}{f}\right) dF(z|a)\right]
$$

and follow the proof of Roger (2016), who shows that $C(a)$ is an increasing function of the rent (here $u(h+y)$. As a sketch, take any two rent levels $u_1 = u(h_1+y) < u_2 = u(h_2+y)$, for any given action a, the corresponding cost functions follow the same ranking: $C(a,u_1) < C(a,u_2)$. In addition, $C(a,u)$ is increasing convex in a, and it is a super-modular function. Hence for higher rents the principals ask for a lower action. ■

Furthermore,

Lemma 4 *The equilibrium profits of the principals increase with market tightness Θ : $d\Pi^S/d\Theta > 0$.*

Proof: We prove the Lemma in two Claims.

Claim 1 *For any type the equilibrium expected profit increases in the action a^S : $d\Pi/da > 0$.*

Proof: Write equilibrium profit

$$
\Pi = [1 - p_0(\Theta) + \Theta p_0'(\Theta)]\pi(t,a) - k
$$

where all constraints are satisfied. From Conlon (2008) we know that the transfer

$$
T(a) := \int_{\mathcal{X}} t(z)dF(z|a)
$$

is an increasing, concave function in a, for $t(x)$ is concave in x as it track the likelihood ratio f_a/f. Differentiate with respect to a:

$$\frac{d\Pi(a)}{da} = [1 - p_0(\Theta) + \Theta p_0'(\Theta)] \left[\int_{\mathcal{X}} \pi(t,a) dF_a(z|a) - T'(a) \right] > 0.$$

This term is necessarily positive because the principal is constrained in his choice of action: $\mu > 0$. ■

Claim 2 *The equilibrium action increases in the market tightness: $da/d\Theta > 0$.*

Proof: In Problem 1 principals take market tightness as exogenous and their contract offers must satisfy the constraint $U_a = 0$. The identity $\frac{d}{d\Theta} U_a \equiv 0$ stemming from this constraint rewrites

$$\int u' \Delta t dF_a + U_{aa} \frac{da}{d\Theta} \equiv 0$$

where Δt denotes a variation in t with respect to Θ : $\Delta t = \lim_{\Theta_2 \to \Theta_1} \frac{t[\Theta_2](q) - t[\Theta_1](q)}{\Theta_2 - \Theta_1}$ and $U_{aa} < 0$. (Continuity and smoothness of t in Θ follows from the first-order conditions (3.4) and (3.5)). So the sign of $da/d\Theta$ follows that of the first term. In that first term the action a remains constant. Because $\int f_a(z|a)dz = 0$, for some $\tilde{x}$,

$$f_a \begin{cases} < 0, & x < \tilde{x}; \\ = 0, & x = \tilde{x}; \\ > 0, & x > \tilde{x}. \end{cases}$$

Take any $\Theta_2 \downarrow \Theta_1$, the corresponding transfers $t[\Theta_2], t[\Theta_1]$ parametrized by market tightness must take the form characterized in Proposition 1, that is, $(1 - p_0(\Theta))(1/u') = \gamma + \nu/\Theta + \mu f_a/f$. By continuity in t and a, if $t[\Theta_1]$ passes through the point $\tilde{x}$, $t[\Theta_2]$ passes arbitrarily close to it. Then, if the contract $t[\Theta_2](x)$ is steeper,

$$t[\Theta_2](x) \begin{cases} < t[\Theta_1](x), & x < \tilde{x} \text{ and} \\ > t[\Theta_1](x), & x > \tilde{x}. \end{cases}$$

so that

$$\Delta t \begin{cases} < 0, & x < \tilde{x} \text{ and} \\ > 0, & x > \tilde{x}. \end{cases}$$

and $\int u' \Delta t dF_a > 0$ necessarily. The converse holds when $t[\Theta_2](x)$ is shallower. To complete the argument we show that indeed a steeper transfer $t[\Theta_2](x)$ strictly satisfies the moral hazard constraint that is binding under $t[\Theta_1](x)$. Take $t[\Theta_2](x)$ steeper than $t[\Theta_1](x)$ (so $t[\Theta_2](x)$ single-crosses $t[\Theta_1](x)$ from below arbitrarily close to $\tilde{x}$). Then

$$\begin{aligned}
& \int_{\underline{x}}^{\tilde{x}} u(t[\Theta_2](x))dF_a(x|a) + \int_{\tilde{x}}^{\overline{x}} u(t[\Theta_2](x))dF_a(x|a) \\
= & \int_{\underline{x}}^{\tilde{x}} u(t[\Theta_1](x))dF_a(x|a) - \int_{\underline{x}}^{\tilde{x}} \left[u(t[\Theta_1](x)) - u(t[\Theta_2](x))\right] dF_a(x|a) \\
+ & \int_{\tilde{x}}^{\overline{x}} u(t[\Theta_1](x))dF_a(x|a) - \int_{\tilde{x}}^{\overline{x}} \left[u(t[\Theta_1](x)) - u(t[\Theta_2](x))\right] dF_a(x|a) \\
= & \int_{\underline{x}}^{\tilde{x}} u(t[\Theta_1](x))dF_a(x|a) - \int_{\underline{x}}^{\tilde{x}} \left[u(t[\Theta_1](x)) - u(t[\Theta_2](x))\right] \frac{f_a}{f} dF(x|a) \\
+ & \int_{\tilde{x}}^{\overline{x}} u(t[\Theta_1](x))dF_a(x|a) - \int_{\tilde{x}}^{\overline{x}} \left[u(t[\Theta_1](x)) - u(t[\Theta_2](x))\right] \frac{f_a}{f} dF(x|a) \\
= & c'(a) - \left(\int_{\underline{x}}^{\tilde{x}} \left[u(t[\Theta_1](x)) - u(t[\Theta_2](x))\right] \frac{f_a}{f} dF(x|a) + \int_{\tilde{x}}^{\overline{x}} \left[u(t[\Theta_1](x)) - u(t[\Theta_2](x))\right] \frac{f_a}{f} dF(x|a) \right) \\
> & c'(a)
\end{aligned}$$

where the penultimate line comes from the moral hazard constraint under Θ_1. The last line uses the fact that $u(t[\Theta_1](x)) - u(t[\Theta_2](x)) > 0$ and $\frac{f_a}{f} < 0$ to the left of $\tilde{x}$, and conversely to its right. So under $t[\Theta_2](x)$ the agent would rather pick a higher action. Hence $\int u' \Delta t dF_a > 0$ so that $da/d\Theta > 0$. ■

Combining the two Claims it follows that $d\Pi(a)/d\Theta > 0$. ■

A corollary of Claim 1 is that $d\pi(t,a)/da > 0$, directly from its proof – for each type. Combine with Claim 2 to conclude that $\pi(t,a)$ an increasing function of Θ; therefore the implicit function $\pi(t,a;\Theta)$ is almost everywhere differentiable in Θ. Next differentiate the entry condition $\Pi(t,a,h) - k = 0$ at $\theta = \Theta$ with respect to k; the first term is necessarily differentiable, so:

$$\frac{d\theta}{dk} \left[\theta p_0''(\theta)\pi(t,a) + [1 - p_0(\theta) + \theta p_0'(\theta)] \frac{d\pi(t,a)}{d\theta} \right] = 1,$$

and one concludes $\frac{d\Theta}{dk} > 0$ as claimed. To prove the second item combine $\frac{d\pi(t,a)}{d\theta} > 0$ and $\frac{d\Theta}{dk} > 0$ to obtain the result. Last, Claim 2 and $\frac{d\Theta}{dk} > 0$ together yield the last line. ■

Proof of Proposition 3: Take the per capita social welfare function $w(t,a,h)$ and divide by α for convenience; a planner solves

Problem 2

$$\max_{t(x),h,a,\theta} \frac{1 - p_0(\theta)}{\theta} \left(\pi(t,a) + \frac{1-\alpha}{\alpha} U(t,a) \right) + \left[1 - \frac{1 - p_0(\theta)}{\theta} \right] \frac{1-\alpha}{\alpha} u(h+y) - h - \frac{k}{\theta}$$

s.t. (3.2) and (3.1).

The MUP (2.2) does not enter the planner's problem because the he can dictate the terms of trade to the principals; more precisely, the planner can impose a contract. Attach multipliers γ and μ to these constraints, and denote a solution with the superscript P. The optimality conditions are

$$\frac{1}{u'(t)} = \frac{1-\alpha}{\alpha} + \gamma\frac{\theta}{1-p_0(\theta)} + \mu\frac{\theta}{1-p_0(\theta)}\frac{f_a}{f}, \quad \gamma \geq 0 \tag{A.1}$$

$$1-\epsilon = \left[\left(1 - \frac{1-p_0(\theta)}{\theta}\right)\frac{1-\alpha}{\alpha} - \gamma\right] u'(h+y) \tag{A.2}$$

$$0 = \pi_a + \frac{\theta}{1-p_0(\theta)}\mu U_{aa} \tag{A.3}$$

$$\frac{k}{\theta^2} = \frac{1-p_0(\theta)+\theta p_0'(\theta)}{\theta^2}\left[\pi(t,a) + \frac{1-\alpha}{\alpha}\left(U(t,a) - u(h+y)\right)\right] \tag{A.4}$$

Using the same elasticity as in the Proof of Proposition 1, the condition with respect to θ writes

$$[1-p_0(\theta)][1-\eta(\theta)]\left[\pi + \frac{1-\alpha}{\alpha}\left(U - u(h+y)\right)\right] = k,$$

and at an optimum, the solution to (A.4) is Θ^P. To establish our claim, first divide the Lagrangian of the principals' equilibrium Problem 1 by θ to revert to the Lagrangian of the planner. Second, observe then that at an optimum the Lagrange multipliers satisfy

$$\gamma^S = \Theta^P\gamma^P \text{ and } \mu^S = \Theta^P\mu^P \text{ and } \frac{\nu}{\theta} = \frac{1-\alpha}{\alpha}$$

which is always possible as Lagrange multipliers are real numbers. Note that $\gamma^P > 0$, so that $U(t,a) = u(h+y)$; therefore the entry condition in the problem of the planner mirrors that in the equilibrium problem, and the equilibrium transfers offered to the agents are identical (across problem). Therefore the equilibrium actions are the same across environments. We must conclude that $\Theta^P = \Theta^S$.

Since $\lambda^P > 0$, $U(t,a) - u(h+y) = 0$ and the planner's entry condition above is the same as the equiliibrium (3.19). Principals competition replicates the planner's problem in that it internalizes the agents' utility through the MUP. ■

Proof of Lemma 1: Suppose that $h \equiv 0$ exogenously so $u(h+y) = u_0$; constraint (3.2) disappears and the program becomes.

Problem 3

$$\max_{t,a}[1-p_0(\theta)]\int_{\mathcal{X}}[z - t(z)]dF(z|a)$$

s.t. (2.2), (3.1) and the participation constraint

$$\int_{\mathcal{X}} u(t(z))dF(z|a) - c(a) \geq u_0 \tag{A.5}$$

where (2.2) is made to reflect $u(h+y) = u_0$. Attach multiplier $\lambda \geq 0$ to (A.5). The FOC of interest are

$$
\begin{aligned}
-[1-p_0(\theta)]f(z|a) + \lambda u' f(z|a) + \mu u' f_a(z|a) + \nu \frac{1-p_0(\theta)}{\theta} u' f(z|a) &= 0 && \text{(A.6)} \\
-p_0'(\theta) \int [z - t(z)] dF(z|a) & && \text{(A.7)} \\
+[1-p_0(\theta)] \int [z - t(z)] dF(z|a) - \left[\frac{1-p_0+\theta p_0'}{\theta}\right] \frac{\nu}{\theta} [U(t,a) - u_0] &= 0
\end{aligned}
$$

with complementary slackness conditions $\nu[V - \widetilde{V}] = 0$ and $\lambda[U - u_0] = 0$. A standard argument implies $\lambda = 0$ – else a deviating principal can attract more agents at almost no cost. So now

$$(1-p_0(\theta))\frac{1}{u'(t)} = (1-p_0(\theta))\frac{\nu}{\theta} + \mu \frac{f_a(x|a)}{f(x|a)}.$$

Integrate over $\mathcal{X}$,

$$\mathbb{E}\left[\frac{1}{u'(t)}\middle| a\right] = \frac{\nu}{\theta} > 0,$$

whence $\nu^N > 0$ as well. Finally Θ^N is determined by the now familiar entry condition (4.8), as the text. The objective function is monotone increasing, hence (4.8) identifies a unique solution. ■

Proof of Proposition 4: The analysis of the planner's problem has one wrinkle compared to Proposition 3. Here it is not clear that the planner lets the participation constraint of agent remain slack with respect to the outside option u_0. We denote a solution with a superscript np rather than P. As before, divide the social welfare function by α for convenience.

Problem 4

$$\max_{t(x),a,\theta} \frac{1-p_0(\theta)}{\theta}\left(\pi(t,a) + \frac{1-\alpha}{\alpha}U(t,a)\right) + \frac{1-\alpha}{\alpha}\left[1 - \frac{1-p_0(\theta)}{\theta}\right]u_0 - \frac{k}{\theta}$$

s.t. (3.1) and (3.2).

Attach multipliers λ, μ to these constraints,

$$
\begin{aligned}
\frac{1-p_0(\theta)}{u'(t)} &= [1-p_0(\theta)]\frac{1-\alpha}{\alpha} + \lambda\theta + \mu\theta\frac{f_a}{f}, \quad \lambda, \mu \geq 0 && \text{(A.8)} \\
0 &= \frac{1-p_0(\theta)}{\theta}\pi_a + \mu U_{aa} && \text{(A.9)} \\
\frac{k}{\theta^2} &= \frac{1-p_0(\theta)+\theta p_0'(\theta)}{\theta^2}\left[\pi(t,a) + \frac{1-\alpha}{\alpha}\left(U(t,a) - u_0\right)\right] && \text{(A.10)}
\end{aligned}
$$

and again the entry condition writes

$$[1-p_0(\theta)+\theta p_0'(\theta)]\left[\pi(t^{np},a^{np})+\frac{1-\alpha}{\alpha}\left(U(t^{np},a^{np})-u_0\right)\right]=k \tag{A.11}$$

where however $U(t^{np},a^{np})-u_0$ needs not be strictly positive (while it is in the equilibrium). We need to distinguish two cases, depending on whether $U(t^{np},a^{np})>u_0$ in the planner's problem – whether $\lambda^{np}=0$.

Case 1: $U(t^{np},a^{np})>u_0$**.** As in the equilibrium, the participation constraint is slack at the planner's solution and the number of principals entering is determined by

$$[1-p_0(\theta)+\theta p_0'(\theta)]\left[\pi(t^{np},a^{np})+\frac{1-\alpha}{\alpha}\left(U(t^{np},a^{np})-u_0\right)\right]=k$$

which we must compare to the principals' entry condition

$$[1-p_0(\theta)]\pi(t^N,a^N)=k.$$

First divide the Lagrangian of the principals' equilibrium Problem 3 by θ; then set the Lagrange multipliers to satisfy

$$\lambda^N=\theta^{np}\lambda^{np},\ \mu^N=\theta^{np}\mu^{np}\text{ and }\frac{\nu^N}{\theta^N}=1$$

at an optimum. Next, from (A.8) in the principals' (equilibrium) problem,

$$\frac{\nu^N}{\theta^N}=-\frac{\theta^N p_0'(\theta^N)}{1-p_0(\theta^N)+\theta^N p_0'(\theta^N)}\frac{\pi(t^N,a^N)}{\frac{1-\alpha}{\alpha}\left(U(t^N,a^N)-u_0\right)}=1$$

so that $-\theta^N p_0'(\theta^N)\pi(t^N,a^N)=[1-p_0(\theta^N)+\theta^N p_0'(\theta^N)]\left[\frac{1-\alpha}{\alpha}\left(U(t^N,a^N)-u_0\right)\right]$. The the marginal benefit $\theta p_0'(\theta)\pi(t^N,a^N)$ to the principals of a queue length θ^N is equal to the marginal rent to the agents. Adding this on both sides of the principals' entry condition, we can rewrite it:

$$[1-p_0(\theta)+\theta p_0'(\theta)]\left[\pi(t^N,a^N)+\frac{1-\alpha}{\alpha}\left(U(t^N,a^N)-u_0\right)\right]=k,$$

which is an identical condition to that of the planner (A.11). As before, principal competition replicates the planner's problem in that it internalizes the agents' utility through the MUP. By the same arguments as in the proof of Proposition 3, the entry levels and the allocations must be identical at an optimum: $\Theta^N=\Theta^{np},\ t^N(x)=t^{np}(x),\ a^N=a^{np}$.

Case 2: $U(t^{np}, a^{np}) = u_0$. Unlike the equilibrium, the participation constraint binds at the planner's solution (so $\lambda^{np} > \lambda^N = 0$) and the number of principals entering is determined by

$$[1 - p_0(\theta) + \theta p_0'(\theta)]\, \pi(t^{np}, a^{np}) = k$$

which we compare to the same entry condition for principals:

$$[1 - p_0(\theta)]\pi(t^N, a^N) = k.$$

The result is immediate nothing that $\theta^N p_0'(\theta^N) < 0$. Too many principals enter this market.

■

B Random matching, welfare and output maximization.

We follow the same notation as in the main next for ease of comparison, without confusion. Let's introduce the matching function $m(\Theta)$, which depends only on the aggregate market tightness Θ – there is no submarket j, so no θ^j. This function satisfies $\forall\Theta,\ m(\Theta) = 1 - p_0(\Theta)$; in particular its first and second derivatives also agree with those of $1 - p_0(\Theta)$. With this matching technology there is no role for the transfers h because principals do not post contracts for the agents to see. The principals' problem is

$$\max m(\Theta) \int [z - t(z)]dF(z|a) \quad s.t\ (A.5)\ and\ (3.1).$$

There is no MUP to worry about since search is not directed. Then it is immediate that the solution follows the standard benchmark; up to the exact value of the multipliers λ, μ, the solution is the second-best solution characterized by (4.1) in the benchmark. Each principal entry conditions is then

$$m(\Theta)\pi(t^B, a^B) = k,$$

with solution Θ^B. A planner seeking *welfare* maximization under random matching solves

$$\max_{\mathbf{t},\mathbf{a},\Theta} \frac{m(\Theta)}{\Theta} \int [z - t(z)]dF(z|a) + \left(\int u(t(z))dF(z|a) - c(a) - u_0 \right) + u_0 - \frac{k}{\Theta}$$

subject to the same constraints. Optimizing w.r.t. t yields the conditions (4.2) to (4.3) of the second benchmark, simply because it is a replication of that problem. Letting RP denote a planner's allocation under random matching, entry is determined by

$$[m(\Theta^{RP}) - \Theta^{RP} m'(\Theta^{RP})]\pi(t^W, a^W) + U(t^W, a^W) - u_0 = k,$$

where, as before, $-\Theta^{RP} m'(\Theta^{RP}) \left[\pi(t^W, a^W) + U(t^W, a^W) - u_0\right]$ is the social marginal benefit of entry when internalizing the agents' utility. Using (3.8) again, the condition rewrites

$$m(\Theta^{RP})[1 - \eta(\Theta^{RP})]\mathbb{E}_\alpha \left[\pi(t^W, a^W) + U(t^W, a^W) - u_0\right] = k. \tag{B.1}$$

To proceed further, consider the following thought experiment: suppose that instead of selecting an agent after she revealed her type, the principal picked an agent randomly and then played the revelation mechanism. Then the entry condition of the planner would exactly be (B.1). Likewise, the equilibrium entry condition of such a game could be written

$$m(\Theta)\pi(t^{\cdot}a) - h = k, \;\; h > 0 \text{ and } h \text{ unique} \tag{B.2}$$

The idea is to establish a comparison between these two conditions. Readily we know $\mathbb{E}_\alpha \left[\pi(t^{\cdot}a) - h\right] < \mathbb{E}_\alpha \left[\pi(t^B, a^B)\right]$ from the prequel. Therefore the solution $\Theta^B < \Theta^{RP}$. That is, Θ^B cannot be constrained efficient: too many principals enter.

Now a planner seeking *output* maximization under random matching maximizes

$$\max_{t,a,\Theta} \Lambda^P m(\Theta) \int [z - t(z)] dF(z|a) - k$$

subject to the same constraints. Divide by Λ^A and rewrite the objective as

$$\max_{t,a,\Theta} \frac{m(\Theta)}{\Theta} \int [z - t(z)] dF(z|a) \frac{k}{\Theta}.$$

Again the first-order conditions with respect to t, a mirror those of the benchmark: (4.1). Optimizing by choice of Θ and simplifying yields

$$m(\Theta^O)[1 - \eta(\Theta^O)]\pi(t^B, a^B) = k.$$

Because the planner can always avail the market solution, $\pi(t^W, a^W) + U(t^W, a^W) - u_0 \geq \pi(t^B, a^B)$ for a given Θ. Therefore under random matching, $\Theta^{RP} < \Theta^O$: when the principal cares about the utility of agents he curbs principal entry (compared to the output-maximizing entry). The reason is that the utility $U(t^W, a^W)$ of the agents is higher under welfare maximization – higher than $U(t^B, a^B) = u_0$.

Likewise we can say that $\Theta^B < \Theta^O$: left to their own devices, too many principals enter compared to the planner's solution. (Maximizing output or principal payoff is effectively the same, up to the entry decision.) Under random matching, contracts have no influence on meeting prospects. In equilibrium they generate larger payoff to contracting principals; more principals enter. Only the tightness Θ influences meeting probabilities; agents' elasticity cannot be manipulated through contracts.

References

[1] Acemoglu, D. and Robert Shimer (1999) "Efficient unemployment insurance." *Journal of Political Economy*, Vol 107, pp. 893-928.

[2] Aubert, C. (2005) "A note on common agency with moral harzard." *mimeo*, Universite de Toulouse.

[3] Attar, A., Eloisa Campioni, and Gwenaël Piaser (2006), "Multiple Lending and Constrained Efficiency in the Credit Market", *The B. E. Journal of Theoretical Economics (Contributions)*, vol. 6, n. 1.

[4] Attar, A. and Arnold Chassangnon (2009) "On moral hazard and nonexclusive contracts." Volume 45, Issues 9–10, pp 511-525 *Journal of Mathematical Economics*

[5] Andrea Attar, Gwenaël Piaser, and Nicolas Porteiro, "A Note on Common Agency Models of Moral Hazard", *Economics Letters*, vol. 95, n. 2, pp. 278–284.

[6] Bisin, A. and Danilo Guaitoli (2004) "Moral Hazard and Non-exclusive Contracts." *Rand Journal of Economics*, 35(2), pp. 306-28

[7] Bland, J. and Simon Loertscher (2012) "Monotonicity, Non-Participation, and Directed Search Equilibria. "*mimeo* The University of Melbourne.

[8] Burdett, K., Shouyong Shi and Randall Wright (2001) "Pricing and Matching with Frictions." *Journal of Political Economy* 2001, No 109, pp 1060-1085.

[9] Chade, H., Jan Eeckhout and Lones Smith "Sorting through Search and Matching Models in Economics." *Journal of Economic Literature, forthcoming.*

[10] Coles, M. and J. Eeckhout (2003) "Indeterminacy and directed search," *Journal of Economic Theory*, Elsevier, vol. 111(2), pages 265-276, August.

[11] Conlon, John R. (2008) "Supplement to Two New Conditions Supporting the First-Order Approach to Multisignal Principal-Agent Problems." *mimeo*, University of Mississippi.

[12] Conlon, John R. (2009) "Two New Conditions Supporting the First-Order Approach to Multisignal Principal-Agent Problems." *Econometrica*, Vol. 77(1), pp. 249-278

[13] Dam K. and D. Perez-Castrillo (2006) "The Principal-Agent Matching Market," *The B.E. Journal of Theoretical Economics*, De Gruyter, vol. 2(1), pages 1-34, August.

[14] DeMarzo, P., Ilan Kremer and Andrzej Skrzypacz (2005) "Bidding with Securities: Auctions and Security Design. "*American Economic Review*, Vol 95, No 4, pp 936-959.

[15] Diamond, P.A. (1971) "A model of price adjustment. "*Journal of Economic Theory*, Vol. 3 (2), pp 156–168

[16] Eeckhout, J. and P. Kircher (2010) "Sorting versus screening: Search frictions and competing mechanisms." *Journal of Economic Theory*, 145 1354–1385.

[17] Engelhardt, B and Peter Rupert (2012) "Competitive Search: A Test for Direction, Efficiency, and submarkets." *working paper, UC Santa Barbara*

[18] Galenianos, M. and Philipp Kircher (2012) "On the Game-theoretic Foundations of Competitive Search Equilibrium."*International Economic Review*, Vol 53 (1), 1-21.

[19] Geromichalos, A. (2015) "Unemployment Insurance and Optimal Taxation in a Search Model of the Labor Market," *Review of Economic Dynamics* 18 (2), 365-380.

[20] Golosov, M. Pricila Maziero and Guido Menzio. (2013) "Taxation and Redistribution of Residual Income Inequality,," *Journal of Political Economy* 121 (6), 1160-1204.

[21] Guerrieri, V., Robert Shimer and Randall Wright (2010) "Adverse Selection in Competitive Search Equilibrium Econometrica", *Econometrica* No 78 (6), pp 1823-1862.

[22] Hall, R.E. and A.B. Krueger (2008) "Wage Formation between Newly Hired Workers and Employers: Survey Evidence," NBER Working Papers 14329, National Bureau of Economic Research, Inc.

[23] Holmström, B. (1979) "Moral hazard and observability." *The Bell Journal of Economics*, Vol. 10, pp. 74-91.

[24] Holmström, B. and Paul Milgrom (1987) "Aggregation and Linearity in the Provision of Intertemporal incentives."*Econometrica*, Vol. 55, pp. 303-328.

[25] Holzer, H.J., L.F. Katz and A.B. Krueger (1991) "Job Queues and Wages," *The Quarterly Journal of Economics*, Oxford University Press, vol. 106(3), pages 739-768.

[26] Hosios, A. (1990) "On the Efficiency of Matching and Related Models of Search and Unemployment. " *The Review of Economic Studies*, Vol. 57, No. 2, pp. 279-298.

[27] Jacquet, N.L. and S. Tan (2012) "Wage-vacancy contracts and coordination frictions," *Journal of Economic Theory*, Elsevier, vol. 147(3), pages 1064-1104.

[28] Jewitt, I. (1988) "Justifying the first-order approach to principal-agent problems." *Econometrica*, Vol. 56, pp. 1177-1190

[29] Jewitt, I. Ohad Kadan and Jeroen Swinkels (2008) "Moral hazard with bounded payments.", *Journal of Economic Theory* 143(1) pp. 59-82.

[30] Julien, B. Ian King and John Kennes (2000) "Bidding for labor." *Review of Economic Dynamics* Vol. 3 (4), 619-649.

[31] Julien, B. and Guillaume Roger (2020) "Incentive contracts and efficiency in a frictional market." working paper *mimeo* Monash University and UNSW.

[32] Laffont, Jean-Jacques and Jean Tirole (1986) "Using Cost Observation to Regulate Firms." *Journal of Political Economy*, Vol. 94, No. 3, pp. 614-641

[33] Li, K., M. Peters, M. and P. Xu (2014) "Understanding Transitions: Insights from Directed-Search", mimeo.

[34] Martimort, D. (1996) "Exclusive Dealing, Common Agency and Multiprincipals Incentive Theory," *The RAND Journal of Economics*, Vol 27(1), pp. 1-31.

[35] Martimort, D. and Lars Stole (2002) "The Revelation and Delegation Principles in Common Agency Games," *Econometrica*, 70(4), pp. 1659-1673.

[36] Martimort, D. and Lars Stole (2003) "Contractual Externalities and Common Agency Equilibria," *Advances in Theoretical Economics*, 3(1) http://www.bepress.com.

[37] Martimort, D. and Lars Stole (2009a) "Selecting Equilibria in Common Agency Games," *Journal of Economic Theory*, Vol 144 pp. 604-644.

[38] Martimort, D. and Lars Stole (2009b) "Market Participation under Delegated and Intrinsic Common Agency Games," *The RAND Journal of Economics*, Vol 40 pp. 78-102.

[39] McAfee, R. P. (1993) "Mechanism Design by Competing Sellers.", *Econometrica* No 61, Vol 6, pp. 1281-1312.

[40] Moen, E. (1997) "Competitive Search Equilibrium." *Journal of Political Economy,* 105, 385-411.

[41] Moen, E. and Åsa Rosen (2011) "Incentives in competitive search equilibrium." *Review of Economic Studies,* 78, 733-761.

[42] Mortensen D. and Christopher Pissarides (1994) "Job Creation and Job Destruction in the Theory of Unemployment." *The Review of Economic Studies*, Vol. 61 (3), pp 397-415.

[43] Mortensen D. and Randall Wright (2002) "Competitive pricing and efficiency in competitive search equilibrium." *International Economic Review*, Vol. 43 (1), pp 1-20.

[44] Peters, M. (1984) "Bertrand Equilibrium with Capacity Constraints and Restricted Mobility", *Econometrica* Vol 52.

[45] Peters, M. (1991) "Ex Ante Price Offers in Matching Games Non-Steady States." *Econometrica*, Vol. 59, No. 5, pp. 1425-1454.

[46] Peters, M. (2000) "Limits of Exact Equilibria for Capacity Constrained Sellers with Costly Search." *Journal of Economic Theory* 95(2) pp. 139-168.

[47] Peters, M, and S. Severinov, Competition Among Sellers Who Offer Auctions Instead of Prices, *Journal of Economic. Theory* 75 (1997), 141-179.

[48] Prescott, E.C. and R. Townsend (1984) "Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard,"Econometrica, 52, 21-45.

[49] Roger, G. (2016) "Participation in Moral Hazard Problem." *Games and Economic Behavior*. Vol 95, pp 10-24.

[50] Rothschild, M. and J.E. Stiglitz (1976) "Equilibrium in competitive insurance markets: an essay on the economics of imperfect information," *The Quarterly Journal of Economics* (Oxford Journals) 90 (4): 629–649.

[51] Salop, S. (1979) "Monopolistic competition with outside goods. " *Bell Journal of Economics*, Vol. 10, No 1.

[52] Selcuk, C. (2012) "Trading Mechanism Selection with Directed Search when Buyers are Risk Averse. "*Economics Letters*, Vol. 115, Issue 2.

[53] Shimer, R. (1996) "Contracts in Frictional Labor Markets."*mimeo* MIT.

[54] Stole, L. (1995) "Nonlinear Pricing and Oligopoly,"*Journal of Economics & Management Strategy*, Vol 4(4) pp. 529-562.

[55] Tirole, J. (2006) "The Theory of Corporate Finance."*Princeton University Press.*