Applications are open for the 2021 / 2022 AMSI and Monash summer vacation scholarships.

Please contact the supervisor for more details, prerequisites and to obtain the required formal letter of support, before applying.

How to apply ........................................ ........................................ ........................................ ........................................ ........................................ ........................................ ........................................ ........................................ 2

Projects........................................................................................................................................................................... 3

1. Taking the fast train to Makassar................................................................................................................................... 3
2. Optimal classification trees in machine learning ........................................................................................................ 3
3. Computational (optimal) transport for domain adaptation .......................................................................................... 3
4. Hadamard matrices and perfect sequences .................................................................................................................. 4
5. The percolation on cellular automata .............................................................................................................................. 4
6. Nash Equilibria in random games ................................................................................................................................... 4
7. Topics in geometric partial differential equations ......................................................................................................... 5
8. Topics in discrete probability and statistical mechanics ................................................................................................ 5
9. Properties of Brownian motion ........................................................................................................................................... 5
10. Planar Brownian motion and complex analysis ............................................................................................................. 5
11. Advanced topics in complex analysis ............................................................................................................................ 6
12. Knots and combinatorics .................................................................................................................................................. 6
13. Study of buckling and bifurcations in nonlinear elasticity ................................................................................................ 6
14. Coupling of reaction-diffusion systems and rotational flows ........................................................................................... 7
15. Simulating the consolidation of porous interfaces .......................................................................................................... 7
16. Modelling the coupling of subsurface flow and forest dynamics ...................................................................................... 7
17. Random walks (generic) and their applications .............................................................................................................. 8
18. Identifying Machine Learning Algorithms for satellites’ orbits ..................................................................................... 8
How to apply

What does this AMSI/Monash scholarships offer me?

• the chance to work on a real research project for six weeks
• travel and accommodation to attend the national AMSICONNECT student conference
• a six-week award of $350/week (total $2,100)

Eligibility

These scholarships are open to students who:

• are currently enrolled at an AMSI Member university
• are in their third year doing a major in the mathematical sciences (outstanding second year students with the support of their department may apply)
• have a strong academic record
• intend to go on to honours and/or postgraduate study in the mathematical sciences, this includes students doing joint degrees that include mathematics and statistics.

To apply visit:   http:vrs.amsi.org.au/

Applications close: Friday 27 August (Monash) and late September 2021 (AMSI)

IMPORTANT application information for students applying through AMSI website

There are two steps to this application process, namely that you are required to submit two applications for a scholarship:

1. First, apply to Monash University at https://www.monash.edu/students/scholarships/current/research-projects online from Monday 9 August 2021 until Friday 27 August (5pm).

2. Second, apply to AMSI at http:vrs.camsi.org.au/.

AMSI and Monash summer vacation scholarship offers

Successful Monash scholarship applicants will receive an offer first from Monash and then a subsequent offer directly from AMSI, for the same project (not a second scholarship). You must accept both offers.

Note: If you did not apply for, and get, a Monash scholarship, you will not be offered an AMSI scholarship

Scholarships offered, by both AMSI and Monash, are administered by Monash University, including your weekly payments.
Projects

1. **Taking the fast train to Makassar**

**Project title:** Single track scheduling for the Makassar railway line.  
**Supervisor/s:** Prof Andreas Ernst & Dr Simon Bowly

**Prerequisites:** Some programming skills required. At least one of MTH3330 or MTH3170.

This project will look at the railway line between Makassar & Pare-Pare in the province of South Sulawesi, Indonesia. This railway line is currently being built to carry both slow freight trains and fast (150km/hr) passenger trains. However, many parts of the railway are just single track with passing loops at occasional stations. This means both the overtaking of trains in the same direction and the passing of trains in opposite directions has to be scheduled very carefully. This project will develop optimisation models and scheduling algorithms to try to maximise the number of trains that can be scheduled on this new railway. This project combines discrete mathematics (directed graphs), computational mathematics (optimisation) and a practical application focus.

2. **Optimal classification trees in machine learning**

**Project title:** Optimal classification trees in machine learning  
**Supervisor/s:** Prof Andreas Ernst

Most forms of machine learning involves solving an optimisation problem to find the best fit between a mathematical model and a collection of training data. Typically, the optimisation problem is solved heuristically (that is in a somewhat ad-hoc manner). A recent paper showed that for a class of models called classification trees, the fitting of the model to the data can be optimally using integer programming approaches. The advantage is with a better fitting, models of the same complexity tend to produce a more accurate classification. However, the integer programs that arise in this application are challenging to solve, particularly for big data sets. This project will look at improving the performance through better integer programming formulations and smarter algorithms.

**Prerequisites:** Solid programming skills required. At least one of MTH3330, MTH3310 or MTH3170.

3. **Computational (optimal) transport for domain adaptation**

**Project title:** Computational (optimal) transport for domain adaptation  
**Supervisor:** Dr Tiangang Cui

Because of the limited availability of labelled data and computing resources, it is often challenging to deploy machine learning algorithms to real-world scientific applications. Domain adaptation, which aims to transfer a well-trained model for a specific machine learning task to similar tasks within the same class, offers a viable route to solve scientific machine learning problems of this type. For example, how can we adapt the decision strategy for the COVID management plan of City M to City S? We surely want to find the optimal plan without repeating the expensive experiments. By casting scientific machine learning tasks into a probabilistic
framework, this project will investigate various avenues in applying (optimal) transport methods to address problems in domain adaptation.

This project has two aspects. Firstly, we want to identify sufficient conditions on the underlying problem and the target learning task (e.g., classification, regression, and decision-making), which allows us to apply the transport-based domain adaptation framework in a certified manner.

Then, we aim to implement high-performance solvers that can address the domain adaptation problems at scale. The student(s) will have the opportunity to apply the resulting tools to one of the application areas, including (but not limited to) remote sensing, environmental science, medical imaging, or structural mechanics.

**Prerequisites:** Competence with MATLAB or Python, completed at least one of MTH3320 and MTH3330 or equivalent.

4. **Hadamard matrices and perfect sequences**

**Project title:** Hadamard matrices and perfect sequences  
**Supervisor:** Dr Santiago Barrera Acevedo

Hadamard matrices and perfect sequences are combinatorial objects with applications in cryptography and information security. Recently an interesting connection between Hadamard matrices and perfect sequences over quaternions was established. In this project we aim to understand and use this connection to construct Hadamard matrices from families of perfect sequences over quaternions and vice versa.

**Prerequisites:** The project is open to students who have completed at least two years of their undergraduate degree and are interested in pursuing honours in Pure Mathematics. Algebra and Number Theory I is essential. Some experience with computer programming would be desirable, but not essential.

5. **The percolation on cellular automata**

**Project title:** The percolation on cellular automata  
**Supervisor/s:** A/Prof Andrea Collevecchio & Prof Kais Hamza

Abstract. Consider the following self-organized system on $\mathbb{Z}^+ \times \mathbb{Z}^+$. The vertices are labelled using the familiar euclidean coordinates. To each vertex we assign a value in \{-1,1\} according to the following rule. The value at vertex $(i,j)$ is the product of the values at $(i, j - 1)$ and $(i - 1, j)$. The system is well defined once we provide the boundary conditions, which are the following. The vertices with coordinates $(1, j)$, with $j \geq 1$ have all the same values. The values of the vertices of the form $(i, 1)$, with $i \geq 1$ are generated at random according to a certain rule. This project investigates the main features of this system.

**Prerequisites:** A strong background in probability and stochastic processes is necessary for this project. You must have completed at least the unit MTH3241 (or equivalent).

**Duration and period:** 4 to 6 weeks in January and/or February.

6. **Nash Equilibria in random games**

**Project title:** Nash Equilibria in random games  
**Supervisor/s:** A/Prof Andrea Collevecchio & Prof Kais Hamza
Abstract: we study the geometry of Nash Equilibria in games where payoffs are generated at random. We use methods from Probability, Game Theory, Statistical Mechanics.

**Prerequisites:** A strong background in probability and stochastic processes is necessary for this project. You must have completed at least the unit MTH3241 (or equivalent).

7. **Topics in geometric partial differential equations**

**Project title:** Topics in geometric partial differential equations  
**Supervisor:** Dr Julie Clutterbuck

Geometric shapes that arise in nature (such as liquid droplets, the shape of a hanging thread, or biomembranes) can be understood as minimisers of an energy, and as such often are solutions to partial differential equations. In this project we will examine some physical energies and the geometries they give rise to.

**Prerequisites:** Real analysis is essential.

8. **Topics in discrete probability and statistical mechanics**

**Project title:** Topics in discrete probability and statistical mechanics  
**Supervisor/s:** A/Prof Tim Garoni

**Additional details:** Please contact Tim Garoni for project details.

9. **Properties of Brownian motion**

**Project title:** Properties of Brownian motion  
**Supervisor/s:** Dr Greg Markowsky

Brownian motion is the most famous and important continuous-time stochastic process. In this project, we will examine some of the more advanced properties of the process.

Here are some things we can look at: Hausdorff dimension, which is a method of determining the dimension of complicated sets. There are many fascinating results on the Hausdorff dimension of various sets related to Brownian motion, such as the zero set of the process, the graph of the process in 1 dimension, and the range of the process in higher dimensions. Local time, which is a measure placed on the zero sets of Brownian motion. Fourier analysis plays a major role in its study. Potential theory, which is the study of harmonic functions, and which has many connections with Brownian motion. Self-intersections of the paths of Brownian motion in higher dimensions. There is a rich theory of this topic, and it is closely related to local time.

**Prerequisites:** MTH3020 or equivalent, familiarity with Brownian motion.

10. **Planar Brownian motion and complex analysis**

**Project title:** Planar Brownian motion and complex analysis  
**Supervisor/s:** Dr Greg Markowsky
The image of a planar Brownian motion under a non-constant analytic function is again a planar Brownian motion. This fact has a large number of interesting consequences. The purpose of this project is to study this connection in depth.

**Prerequisites:** MTH3020 or equivalent, familiarity with Brownian motion.

11. **Advanced topics in complex analysis**

**Project title:** Advanced topics in complex analysis  
**Supervisor/s:** Dr Greg Markowsky

Complex analysis is one of the most beautiful and important subjects in classical mathematics, and remains an active area of research today. In this project, we will explore some of the more advanced topics in the subject, in accordance with the student’s interests. Possible topics include conformal mapping, including Mobius transformations and the Schwarz-Christoffel transformation; boundary behaviour of conformal maps; the maximum modulus principle, uniqueness principle, Schwarz’s Lemma, the argument principle, the Phragmen-Lindelof principle, Picard’s theorems, and Rouche’s Theorem; Schwarz’s reflection principle and consequences; Infinite products, including Weierstrauss’ Theorem; convergence of sequences of analytic functions and normal families; Hardy spaces; special functions; the hyperbolic metric; harmonic measure.

12. **Knots and combinatorics**

**Project title:** Knots and combinatorics  
**Supervisor/s:** Dr Dan Mathews & Dr Jian He

Details: Knot invariants have given rise to a large family of conjectural combinatorial identities, which are surprisingly difficult to prove on their own. In this project we will investigate some of these, including numerical verifications (who knows, we may even find a counter-example!).

**Prerequisites:** A second or third year pure maths unit. Familiarity with a mathematical programming software such as Mathematica/Matlab/Maple is preferred but not required.

13. **Study of buckling and bifurcations in nonlinear elasticity**

**Project title:** Study of buckling and bifurcations in nonlinear elasticity  
**Supervisor/s:** A/Prof Ricardo Ruiz Baier

A large class of nonlinear problems depend on parameters operating on the unknown variable solution. If such solution exists, we would like to know how many solutions are to the problem, and how that number of solutions varies when we modify the parameters. We want to focus on bifurcations (where a solution to the problem splits into two or more solutions) that are encountered in the deformation of so-called hyperelastic solids (such as rubber). A way of finding and identifying solutions consists of continuation methods (and their variants), which follow a particular solution along a solution branch. This can be done with the help of iterative schemes and, for example, finite element methods. The tasks of the project involve understanding the essential aspects of bifurcation in nonlinear PDEs, and to investigate the properties of numerical solutions obtained for the buckling of beams.

**Prerequisites:** Having completed a course on solid mechanics (MTE2546, MEC3455, MTH3310 or equivalent), and having some experience with the implementation of numerical methods.
14. Coupling of reaction-diffusion systems and rotational flows

**Project title:** Coupling of reaction-diffusion systems and rotational flows  
**Supervisor/s:** A/Prof Ricardo Ruiz Baier

Reaction-diffusion systems can explain many phenomena taking place in diverse disciplines such as industrial and environmental processes, biomedical applications, population dynamics, etc. These models allow to reproduce chaos, spatio-temporal patterns, rhythmic and oscillatory scenarios, and so on. Nevertheless, in most of the applications mentioned above, the reactions do not occur in complete isolation. The species are rather immersed in a fluid, or they move within (and interact with) a fluid-solid continuum. We want to perform linear stability analysis for a simplified coupled system to determine how the interaction between non-autonomous patterns arise in response to changes in the advecting fluid velocity and other flow conditions typical in rotation-based formulations. We also want to compute numerical solutions using advanced mixed finite element methods.

**Prerequisites:** Having done a course on PDEs and a course on mathematical modelling (MTH3310 or equivalent) and/or a course on fluids (MTH3360, CHE2161, MEC2404 or equivalent).

15. Simulating the consolidation of porous interfaces

**Project title:** Simulating the consolidation of porous interfaces  
**Supervisor/s:** A/Prof Ricardo Ruiz Baier

The project involves setting appropriate boundary or transmission conditions for the interaction between free flow and a deformable porous medium. We want to investigate how a material such as a sponge, deteriorates in time and its filtration ability is progressively lost. This involves solving equations for fluid on a subdomain, and equations of poroelasticity on the other subdomain, with the particularity that the interface evolves in time. The developed model will be used to investigate the early stages of glaucoma, which encompasses a series of mechanisms leading to a decreased retinal function of the eye, impairing visual fields and eventually leading to blindness.

The project is suitable for graduates with background in quantitative disciplines such as mathematics, physics or engineering. Some experience and/or interest in numerical computation and fluid dynamics will be needed, while existing background in biology is not required.

**Prerequisites:** Having completed a course on fluid mechanics (MTH3360, CHE2161, MEC2404 or equivalent) and on mathematical modelling (MTH3310 or equivalent). Having programming skills in python.

16. Modelling the coupling of subsurface flow and forest dynamics

**Project title:** Modelling the coupling of subsurface flow and forest dynamics  
**Supervisor/s:** A/Prof Ricardo Ruiz Baier

Vegetation can influence the development of soil properties. The main biochemical interaction between plants and soil is via litter deposition. We are interested in refining a mathematical model capturing the qualitative behaviour of the soil-tree ecological system, in which one of the species favours the high calcium soil concentration as a competitive strategy for extension growth. The aims of the project involve implementation of finite element methods for a 2D/3D mixed-dimensional model (forest growth in a 2D surface and nutrient intake on a 3D subsurface domain), testing computational efficiency, and validation of results.

**Prerequisites:** Having completed a course on numerical methods for PDEs (MTH3011, MEC2456 or equivalent), and having programming skills in python.
17. **Random walks (generic) and their applications**

**Project title:** Random walks (generic) and their applications  
**Supervisor/s:** A/Prof Andrea Collevecchio & Prof Kais Hamza

Study of random walks in one and more dimensions. Its connections with combinatorics, analysis. In particular, we focus on questions like: How fast does the range grow? How many times each vertex is visited by the process given that is visited once.

18. **Identifying Machine Learning Algorithms for satellites’ orbits**

**Project title:** Identifying Machine Learning Algorithms for satellites’ orbits  
**Supervisor/s:** Dr Alina Donea

The aim of this program is to:
- become familiar with the mathematics and exact solutions of Newtonian orbits and perturbation solutions for orbits and then
- together with the group leader to identify the machine learning algorithms that are useful to improve satellite orbit prediction.

Research papers such as below are useful for a start:  

We will search the literature to understand the problem, to model orbits given initial conditions, to identify data, to identify the ML algorithms for tracking orbits and make our own codes. This is a learning curve for the whole team and we seek an enthusiastic person who knows python and is not afraid of calculating some fancy integrals.

**Prerequisites:**  
Competence in equivalent, Linear Algebra, Discrete Mathematics and some experience in programming in Python I (intermediate), Support Vector Machine

You should be able to write a simple source code in python (with comments about the mathematics behind satellite’s 2D simplest orbits) to calculate, using a simple runge-kutta method, a basic orbit of a body mass around Earth (your choice) - with sample code sent to Alina - This is not about machine learning. This is a first step in identifying the best candidate for the project.

**Additional details:** Only available after the end of the exam period (2021)