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Real Savings, Entrepreneurship and Finance: A Monetary Model of Economic Growth

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1. Introduction

One of the most important questions in the field of finance is: does finance make a difference to economic growth, and if so, how? (Goldsmith, 1969). Since the 1990s, a large literature has sprung up to tackle this challenging question. Although a definitive answer is yet to be formed, many insights and some tentative conclusions have emerged. Among the tentative conclusions are: (1) finance appears to foster economic growth; and (2) the primary mechanism seems to be that finance promotes growth by improving resource allocation, facilitating technological change, and expanding economic opportunities (Levin, 2021).

But how exactly does this primary mechanism work? A useful method for investigation is to focus on the main functions of the financial system, namely, providing investment information and allocating capital; exerting corporate governance; diversifying and managing risks; mobilising savings; and facilitating exchange (Levine, 1997).

A number of influential studies have explored how specific functions of the financial system smooth market frictions to facilitate investment and economic growth. [1] For example, with regard to information provision and capital allocation, Greenwood & Jovanovic (1990) show that financial institutions allow for higher and safer returns as they facilitate trade by collecting and analysing information. King and Levine (1993a, 1993b) provide theoretical arguments and empirical evidence that financial institutions play an active role in evaluating, managing, and funding the entrepreneurial activities that lead to productivity growth. Buera, Kaboski and Shin (2011) find that financial frictions explain a substantial part of the misallocation of capital and entrepreneurial talent, implying that finance development would assist in directing funds to more promising investment projects. In relation to mobilising savings and facilitating exchange, Bencivenga & Smith (1991) show that financial intermediation reduces the need for self-financing, thereby promoting capital formation and growth. Acemoglu and Zilibotti (1997) demonstrate that financial intermediaries pool savings from many diverse individuals and invest in a diversified portfolio of investments. This enables the funding of profitable projects and reduces funding risks. Greenwood & Smith (1997) show that by lowering transaction costs, finance facilitates greater specialisation and larger trade volumes, which in turn creates productivity gains and new economic opportunities.

Building on the existing literature, this paper constructs a model to study how the quintessential bank – by issuing loans, taking deposits and providing payment services – plays a pivotal role in the sustainment and the growth of an economy. In this model, there are 4 occupations: Home Producer, Factory Worker, Entrepreneur and Banker. There are two possible equilibria. If the deposit interest rate is below the threshold which just compensates for time preference and expected inflation, then nobody saves and everyone has to be a Home Producer. If the deposit interest rate is

[1] Levin (2021) provides an excellent review of the literature that investigates each of these main functions of the financial system.

above the threshold, then everyone has real savings (i.e., real resources not consumed), and the equilibrium is one with factory production. To mobilise the real savings, the Banker lends to the Entrepreneur (at the lending interest rate) which enables the Entrepreneur to acquire the resources from everyone and transform them into capital. The capital formation in turn enables the adoption of a more advanced factory technology and the initiation of factory production. The purchase of real savings by the Entrepreneur is facilitated by the Bank's payment services. The Entrepreneur's expenditure becomes the savers' revenue, which the savers deposit with the Bank. At the end of a period, production is completed, savers receive deposit interests, workers receive wages and the Banker earns the difference between the lending and the deposit interests. All the income is then spent on the factory output and the Entrepreneur repays the Banker with the sales revenue.

All the transactions are monetary transactions facilitated by the Bank's payment services. We can therefore trace the flow of funds by examining the changes on the Bank's balance sheet.[2] The flow of funds from the loan issuance to its use and repayment mirrors the flow of real resources from the purchase of inputs and the hiring of workers to the sale of output.

The model shows that following the adoption of the more productive factory technology, there is a discontinuous jump in economic output. Numerical simulations suggest that the economy will continue to grow for some time, and the growth is accompanied by increasing amounts of lending. This result is consistent with Becker et al. (2023) who identify a positive association between liquidity creation by banks and economic growth. However, the economy in this model eventually reaches a steady state, and the steady state output depends on the rate of savings. Sustained economic growth over the long term requires continued development and adoption of ever better technologies – a well-known result of the Solow model (Solow, 1956).

This work contributes to the vast literature on the nexus of finance and growth in that it brings together the three important drivers of economic growth: real savings, entrepreneurship and finance. Savings (of real resources) are necessary for the formation of capital required by the adoption of new technologies; the entrepreneur initiates the formation of capital formation and the adoption of new technologies; the bank creates purchasing power to fund entrepreneurial activities, pays deposit interests which encourages real savings, and provides payment services to facilitate monetary transactions. In this context, the central role played by the bank in promoting economic development is mobilising real savings and facilitating the adoption of more productive technologies. As Schumpeter famously argues "[T]he banker, therefore, is not so much primarily a middleman … He authorizes people in the name of society … to [innovate]." (1912, p.74, quoted in King & Levin, 1993a).

[2] This is inspired by the stock-flow consistent approach developed by Wynne Godley (Godley & Lavoie, 2007).

Another contribution of this work is that it formalises two old insights. First, capital rather than technological advance per se is often the binding constraint on economic growth, as pointed out by Bagehot (1873, p10)

> "We have entirely lost the idea that any undertaking likely to pay, and seen to be likely, can perish for want of money; …A citizen of London in Queen Elizabeth's time … would have thought that it was no use inventing railways (if he could have understood what a railway meant), for you would have not been able to collect the capital with which to make them."

Hicks (1969) similarly points out that most of the products that were first manufactured during the early periods of the Industrial Revolution had been invented much earlier; and what caused the delay in the implementation of many of the existing inventions were the large capital requirements. In this model, the more advanced factory technology is available, but its implementation requires the mobilisation of savings.

Second, banks do not mobilise savings by collecting money and passing it on to the entrepreneur. Instead, they *create* purchasing power. Real savings are mobilised when the entrepreneur acquires them with the created purchasing power. As Schumpeter remarks:

> "The banker is therefore not so much an intermediary of the commodity 'purchasing power' as a *producer* [italics in original] of this commodity." (1934, p.62)

And

> "[C]redit is essentially the creation of purchasing power for the purpose of transferring it to the entrepreneur, but not simply the transfer of existing purchasing power. … By credit, entrepreneurs are given access to the social stream of goods before they have acquired the normal claim to it. And this function constitutes the keystone of the modern credit structure." (1954, p. 107)

This insight has been reiterated and elaborated in more recent work as well, including, for example, McLeay et al. (2014) and Werner (2014a, 2014b, 2016). In this model, the Banker's loan creates purchasing power which gives the Entrepreneur access to real savings (resources). The loan is provided before any deposits are taken. Indeed, it is only after the Entrepreneur has used the loan to purchase real resources, that the savers have any revenue to deposit with the Bank.

The rest of the paper is organised as follows. Section 2 presents the model, including the setup, the flow of funds analysis, the equilibrium solutions and numerical simulations. Section 3 discusses the performance of the model economy after the adoption of the factory technology. Section 4 concludes with a summary and an outline of possible future extensions of this work.

2. A monetary model of economic development

2.1. Setup: preferences, endowments, technologies and finance

Consider an economy with N individuals who derive utility from consuming a single good X. The individuals live for an indefinite number of periods. However, they only consider 2 periods (the present and the future) at a time when making decisions. The individuals can choose one of 4 occupations: Home Producer, Factory Worker, Entrepreneur, and Banker. Their utility function is:

$$U = c_1 + \theta c_2 \tag{1}$$

where c_1 is the quantity of good X consumed in period 1 ("the present"), and c_2 is the quantity of good X available for consumption in period 2 ("the future"). θ $(\theta < 1)$ is the individual's time preference parameter; a larger θ indicates a smaller discount rate applied to future utility.

Each individual is endowed with 1 unit of labour in each period and labour cannot be transferred inter-temporally. Good X can be produced at home using only labour (l):

$$X_h = al \tag{2}$$

Or it can be produced by a firm using a more productive factory technology which requires hired workers (N_{FW}), capital good (K), and one entrepreneurial input (E).

$$X_f = \begin{cases} A[\min(N_{FW}, K)], & E = 1, \\ 0, & E < 1 \end{cases} \tag{3}$$

where A $(A > a)$ is the productivity parameter of the factory technology. This specification assumes that labour and capital are perfect complements – one unit of labour is required to work with one unit of capital good K. The capital good fully depreciates in one period.

The capital good is transformed from Good X with an entrepreneurial input.[3]

$$K = \begin{cases} b\, X_k, & E = 1,\ X_k > a - c_0 \\ 0, & E < 1 \end{cases} \tag{4}$$

The entrepreneurial input (E) captures the Entrepreneur's time and energy devoted to making the capital good available, hiring workers and organising factory production. The Entrepreneur must devote all her labour endowment to starting and running the Firm. That is, firm production can take place only if $E = 1$. $(a - c_0)$ is the maximum amount an individual can save in period 1, where a is the output an individual can produce at home in one period, and c_0 is the minimum consumption required to survive a period. $X_k > a - c_0$ is a minimum input requirement, which implies that an individual cannot create capital using their savings alone.

Since firm production requires capital good K which needs to be transformed from good X, it can only begin from period 2 – after good X has been produced. It is assumed that capital good K can

[3] We can add factory labour to the production function of the capital good, but it complicates calculation without adding more insight. The main feature of capital good production is that it requires someone (the entrepreneur) to initiate and complete a project; it costs real resources (X and E) and the output is used for production instead of consumption.

be made instantaneously so that firm production can commence with hired workers at the beginning of period 2. Factory Workers are paid at the end of period 2. It takes one period to produce good X regardless of the technology used. Individuals consume at the end of each period.

The Entrepreneur needs finance from the Bank. In particular, she needs a one-period loan (L_E) at the beginning of period 2 to purchase good X to transform into capital good K. She also needs a ("short-term") loan (L_{Es}) to pay wages at the end of period 2. She retains some factory output for her own consumption and sells the rest at the end of period 2. She then uses the sales revenue to repay the Bank.

The Bank is started and run by the Banker who, like the Entrepreneur, devotes all his labour endowment to his business. The Bank provides the required loan amounts to the Entrepreneur at the lending interest rate (i_l), pays depositors at the deposit interest rate (i_d) and earns the interest spread.

2.2. Flow of funds

As discussed, to start factory production, the Entrepreneur borrows from the Bank and buys good X to turn into the capital good. She then hires labour to produce good X, borrows again to pay wages, sells the factory output, and repays her loans. All transactions are monetary transactions mediated by the Bank's payment services. We can therefore trace the flow of funds step by step through changes in the Bank's balance sheet.

(1) The Bank makes a one-period loan to the Entrepreneur (L_E) to buy good X to make the capital good

Assets	Liabilities and Net Worth
Loan to Entrepreneur: L_E	Entrepreneur's deposit: L_E
L_E	L_E

In modern banking practice, a bank creates money "out of nothing" when it makes a loan (Werner, 2014, 2016). In fact, "[m]ost of the money in the economy is created … by banks when they provide loans".[4]

[4] See Bank of England, Explainers, How is money created? https://www.bankofengland.co.uk/explainers/how-is-money created#:~:text=Banks%20create%20around%2080%25%20of,central%20bank%20money%2C%20or%20reserves.

(2) The Entrepreneur buys good X from Home-Producers, Factory Workers and the Banker

Assets	Liabilities and Net Worth
Loan to Entrepreneur: L_E	Entrepreneur's deposit: 0 Home Producers' deposit: D_{HP} Factory Workers' deposit: D_{FW} Banker's net worth: NW_B
L_E	L_E

When the Entrepreneur pays for good X, funds are transferred from her deposit account to the accounts of the sellers (i.e., the Home Producers, the Factory Workers and the Banker). Thus, the Entrepreneur's deposit falls to 0, while the increases in the Home Producers' deposits (D_{HP}), the Home producers' deposits (D_{FW}), and the Banker's net worth (NW_B) add up to the total expenditure of the Entrepreneur, which is the loan amount (L_E).

(3) The Banker earns loan interests and pays deposit interests

Assets	Liabilities and Net Worth
Loan to Entrepreneur: L_E Interest receivable from Entrepreneur: I_E	Entrepreneur's deposit: 0 Home Producers' deposit: $D_{HP} + I_{HP}$ Factory Workers' deposit: $D_{FW} + I_{FW}$ Banker's net worth: $NW_B - I_{HP} - I_{FW} + I_E$
$L_E + I_E$	$L_E + I_E$

When the loan interest is earned, the available funds in the economy increase by the amount of the loan interests ($I_E = L_E i_l$). The increased funds are divided between the depositors and the Banker. The Home Producers' funds increase by their deposit interests $I_{HP} = D_{HP} i_d$. The Factory Workers' funds increase by their deposit interests $I_{FW} = D_{FW} i_d$. The Banker's net worth increases by the difference between the loan interests and the deposit interests ($I_E - I_{HP} - I_{FW}$).

(4) The Entrepreneur gets a short-term loan (L_{Es}) from the Banker and pays workers

Assets	Liabilities and Net Worth
Loan to Entrepreneur: $L_E + L_{Es}$ Interest receivable from Entrepreneur: I_E	Entrepreneur's deposit: 0 Home Producers' deposit: $D_{HP} + I_{HP}$ Factory Workers' deposit: $D_{FW} + I_{FW} + L_{Es}$ Banker's net worth: $NW_B - I_{HP} - I_{FW} + I_E$
$L_E + I_E + L_{Es}$	$L_E + I_E + L_{Es}$

The short-term loan becomes the wage income of Factory Workers. It increases the available funds in the economy by the amount of the short loan (L_{Es}).

(5) The Banker earns an interest on the short-term loan (I_{Es}).

Assets	Liabilities and Net Worth
Loan to Entrepreneur: L_E+ L_{Es} Interest receivable from Entrepreneur: I_E + I_{Es}	Entrepreneur's deposit: 0 Home Producers' deposit: $D_{HP} + I_{HP}$ Factory Workers' deposit: $D_{FW} + I_{FW} + L_{Es}$ Banker's net worth: $NW_B - I_{HP} - I_{FW} + I_E + I_{Es}$
$L_E + L_{Es} + I_E + I_{Es}$	$L_E + L_{Es} + I_E + I_{Es}$

When the loan interest is earned, the available funds in the economy increase by the amount of the loan interest (I_{Es}).[5]

(6) Home Producers, Factory Workers and the Banker buy from the Entrepreneur

Assets	Liabilities and Net Worth
Loan to Entrepreneur: L_E+ L_{Es} Interest receivable from Entrepreneur: I_E + I_{Es}	Entrepreneur's deposit: $L_E + L_{Es} + I_E + I_{Es}$ Home-producers' deposit: 0 Factory-workers' deposit: 0 Banker's net worth: 0
$L_E + L_{Es} + I_E + I_{Es}$	$L_E + L_{Es} + I_E + I_{Es}$

When the buyers spend all their funds to buy good X from the Entrepreneur, the Entrepreneur receives the total funds available in the economy, which is equal to the Entrepreneur's loan amounts plus interest.

(7) The Entrepreneur repays both loans and interest and the Banker's balance sheet shrinks to zero.

Assets	Liabilities and Net Worth
Loan to Entrepreneur: 0 Interest receivable: 0	Entrepreneur's deposit: 0
0	0

After the Entrepreneur repays her loans and interest, money is destroyed. To start a new round of factory production, new loans must be issued to enable the Entrepreneur to buy back some good X to turn into capital goods, and to hire workers.

2.3. Equilibrium

The equilibrium of the model economy may be defined as a situation where four conditions are met: (1) the value of money is stable; (2) every individual's utility is maximised; (3) markets

[5] We will assume zero interest for the short-term loan in later analysis. This simplifies calculations without changing the main results of the analysis.

clear; and (4) individuals are content with their occupations so that they do not switch between occupations. We discuss each of these 4 conditions in the following.

(1) Stable money

By stable money, we mean zero inflation, that is, the money price of good X remains unchanged from period 1 to period 2. If we choose the money price of good X in period 1 as the *numerie* so that $p_1 \equiv 1$, stable money requires that in equilibrium (we use the asterisk symbol to denote equilibrium values), $p_2^* = p_1^* = 1$

The stable money condition can be relaxed to mean that inflation ($\frac{p_2^*}{p_1^*}$) is fully anticipated, so that the value of $\frac{p_2^*}{p_1^*}$ is known, but does not need to be 1.

(2) Utility maximisation

Individuals can choose one of 4 occupations in the economy: Home Producer, Factory Worker, Banker or Entrepreneur. We consider the utility maximisation problem for an individual in each occupation below.

(i) Home Producer

Home producers use the home production technology to produce good X in each period. At the end of period 1, they consume some of their products and sell the rest to the Entrepreneur for money which they deposit with the Bank. At the end of period 2, they use their deposits plus interests to buy good X from the Entrepreneur to supplement their consumption. A Home Producer's decision problem is:

$$max\, U_{HP} = c_1 + \theta c_2 \tag{5}$$

subject to:

$$x_1 = al_1$$
$$x_2 = al_2$$
$$x_1 = c_1 + s_1$$
$$p_1 s_1(1 + i_d)/p_2 + x_2 = c_2$$
$$l_1 = l_2 = 1$$
$$c_0 \leq c_1 \leq x_1$$

where c_0 is the minimum consumption required to survive a period; s_1 is real savings in period 1; i_d is deposit interest rate; p_1 and p_2 are the money prices of good X in period 1 and period 2, respectively.

Substituting the constraints into the utility function and simplifying, we have:

$U_{HP} = (1 + \theta)a + [\theta p_1(1 + i_d)/p_2 - 1]s_1$

If $\theta(1 + i_d)(p_1/p_2) - 1 > 0$, or equivalently, $i_d > (p_2/p_1 - \theta)/\theta$, U_{HP} is maximised when s_1 is the largest. Thus, we have:

$c^*_{1HP} = c_0,\ s^*_{1HP} = a - c_0$

$c^*_{2HP} = (a - c_0)(1 + i_d)(p_1/p_2) + a$

$U^*_{HP} = c_0 + \theta[(a - c_0)(1 + i_d)(p_1/p_2) + a]$

If $i_d \leq (p_2/p_1 - \theta)/\theta$, U_{HP} is maximised when $s_1 = 0$. Thus, we have:

$c^*_{1HP} = a$

$c^*_{2HP} = a$

$U^*_{HP} = a(1+\theta)$

The solutions indicate that if the deposit interest rate (i_d) is below the threshold required to compensate for time preference (θ) and expected inflation (p_2/p_1), the Home Producer will consume all that they produce and will not save at all. If the deposit interest rate exceeds the threshold, the Home Producer will only consume the minimum amount necessary in period 1, and save the rest.

(ii) Factory Worker

Factory Workers produce at home in period 1. Like the Home Producers, they save some of their output, sell them to the Entrepreneur and deposit the revenue with the Bank. They work for the factory in period 2 and receive a wage (at the end of the period) which they use to buy good X from the Entrepreneur. A Factory Worker's decision problem is:

$$\max U_{FW} = c_1 + \theta c_2 \tag{6}$$

subject to:

$$x_1 = al_1$$

$$x_1 = c_1 + s_1$$

$$[p_1 s_1(1 + i_d) + wl_2]/p_2 = c_2$$

$$l_1 = l_2 = 1$$

$$c_0 \leq c_1 \leq x_1$$

where *w* is the wage rate; other variables are the same as those in the Home Producer's decision problem.

Substituting the constraints into the utility function and simplifying, we obtain the following solutions:

If $i_d > (p_2/p_1 - \theta)/\theta$

$c^*_{1FW} = c_0,\ s^*_{1FW} = a - c_0$

$c^*_{2FW} = (a - c_0)(1 + i_d)\left(\frac{p_1}{p_2}\right) + \frac{w}{p_2}$

$U^*_{FW} = c_0 + \theta[(a - c_0)(1 + i_d)\left(\frac{p_1}{p_2}\right) + \frac{w}{p_2}]$

If $i_d \leq (p_2/p_1 - \theta)/\theta$

$c^*_{1FW} = a,\ \ s^*_{1FW} = 0$

$c^*_{2FW} = a$

$U^*_{FW} = a(1+\theta)$

The solutions indicate that the Factory Worker has the same threshold deposit interest rate as the Home Producer. If the deposit interest rate is below the threshold, the Factory Worker will not save, and there will be no factory work available in period 2. If the deposit interest rate exceeds the threshold, the Factory Worker will save the maximum amount in period 1, and work for the Entrepreneur in period 2. Their period 2 consumption will be funded by both their wage income and their period 1 savings plus interests.

(iii) The Banker

The Banker produces at home in period 1. He devotes his entire labour endowment to running the banking business in period 2. At the beginning of period 2, the Banker provides a loan to the Entrepreneur at the lending interest rate (i_l).[6] The Entrepreneur uses the loan to buy good X from everyone who saved in period 1. The Entrepreneur's expenditure is the sellers' revenue. The Home Producers and the Factory Workers deposit their revenue with the Bank at the deposit interest rate (i_d). The Banker's own revenue becomes the bank capital (i.e., the Banker's net worth at the beginning of period 1). The Banker earns the difference between the interests earned from the Entrepreneur and the interests paid to depositors. He uses his net worth at the end of period 2 (i.e., initial bank capital plus earnings) to buy good X from the Entrepreneur. The Banker's decision problem is:

$$\max U_B = c_1 + \theta c_2 \tag{7}$$

subject to:

$$x_1 = al_1$$

$$x_1 = c_1 + s_1$$

$$l_1 = l_2 = 1$$

$$L_E i_l - (L_E - s_1 p_1) i_d + s_1 p_1 = p_2 c_2 \;\; (\text{if } \; B = l_2 = 1)$$

$$c_0 \le c_1 \le x_1$$

where B is the Banking input which equals the Banker's period 2 labour endowment of 1. L_E is the one-period loan to the Entrepreneur, i_l is the lending interest rate, i_d is the deposit interest rate. Other variables are the same as those in the Home Producer's decision problem.

The solutions are:

If $i_d > (p_2/p_1 - \theta)/\theta$

$c^*_{1B} = c_0,\; s^*_1 = a - c_0$

$c^*_{2B} = L_E i_l - (L_E - s_1 p_1) i_d + s_1 p_1] / p_2 = [L_E(i_l - i_d) + (a - c_0)\, p_1 (1 + i_d)] / p_2$

$U^*_B = c_0 + (\theta/p_2)[L_E(i_l - i_d) + (a - c_0) p_1 (1 + i_d)]$

[6] The Banker also provides a short-term loan to the Entrepreneur to pay wages at the end of period 2 as noted in the Flow of Funds section. Since this short-term loan is assumed to be interest free, it does not enter the Banker's decision problem.

If $i_d \leq (p_2/p_1 - \theta)/\theta$

$c^*_{1B} = a$

$c^*_{2B} = a$

$U^*_B = a(1+\theta)$

The solutions indicate that the Banker has the same threshold deposit interest rate as the Home Producer and the Factory Worker. If the deposit interest rate is above the threshold, the Banker will save at the end of period 1 and start a Bank. For the Banker, his own savings earn an implicit deposit interest rate. If the deposit interest rate is less than the threshold, he will not save and there will be no Bank in period 2.

(iv) The Entrepreneur

The Entrepreneur produces at home in period 1. At the end of period 1, she uses her real savings (in the form of good X) to start a firm (a factory). However, her own savings are not sufficient to meet the minimum capital requirement to initiate factory production. She takes a one-period loan from the Bank at the lending interest rate of i_l and buys *all the good X available* for sale.[7] She transforms all the good X at her disposal into capital good K, and hires workers to produce good X in her factory. At the end of period 2, she gets a short-term loan from the Bank (which we assume to be interest free to simplify calculations) to pay workers. She then sells good X, repays her loans plus interest, and retains the residual product for herself. The Entrepreneur's decision problem is:

$$max\ U_E = c_1 + \theta c_2 \tag{8}$$

subject to:

$$x_1 = al_1$$

$$x_1 = c_1 + s_1$$

$$x_2 = \begin{cases} A[\min(N_{FW}, K)], & E = 1 \\ 0, & E < 1 \end{cases}$$

$$K = \begin{cases} b\,X_k, & E = 1,\ X_k > a - c_0 \\ 0, & E < 1 \end{cases}$$

$$l_1 = l_2 = 1$$

$$L_E = p_1(X_k - s_1)$$

$$p_2(x_2 - c_2) = L_E(1 + i_l) + L_{Es}$$

$$c_0 \leq c_1 \leq x_1$$

where N_{FW} is the number of Factory Workers; K is the quantity of the capital good K; X_k is the quantity of good X purchased to turn into the capital good; L_E is loan for buying good X, i_l is the loan interest rate for L_E. L_{Es} is the loan for paying workers. The other variables are the same as those in the Home Producer's decision problem.

[7] This assumption implies that the optimal quantity of real investment is all the real savings (good X not consumed at the end of period 1). In other words, the available real resources are the binding constraint of capital investment.

Solving the problem, we obtain that

if $i_l > (p_2/p_1 - \theta)/\theta$

$c^*_{1E} = c_0, \; s^*_1 = a - c_0$

$c^*_{2E} = [p_2 \mathrm{A}\, b\, X_k - \mathrm{L_{Es}} - (X_k - a + c_0)(1 + i_l)p_1]/p_2$

$U^*_E = c_0 + (\theta/p_2)[p_2 \mathrm{A}\, b\, X_k - \mathrm{L_{Es}} - (X_k - a + c_0)(1 + i_l)p_1]$

If $i_l \leq (p_2/p_1 - \theta)/\theta$

$c^*_{1E} = a$

$c^*_{2E} = a$

$U^*_E = a(1 + \theta)$

The solutions indicate that if the lending interest rate (i_l) is above than the threshold that compensates for time preference and expected inflation, the Entrepreneur will save at the end of period 1 and start a Firm. For the Entrepreneur, her own savings earn an implicit lending interest rate because she has to borrow less from the Bank. If the lending interest rate is below the threshold, she would not save and there would be no Firm in period 2.

Since the lending rate is greater than the deposit rate, the Entrepreneur's threshold for saving is lower than other individuals. However, unless other individuals save, the Entrepreneur would not be able to meet the minimum input requirement to form capital. Thus, the relevant threshold for the firm production is the higher threshold that applies to other individuals, namely, $i_d > (p_2/p_1 - \theta)/\theta$.

(3) Market clearance

Markets exist only when there is factory production in period 2; and markets exist only in period 2.

(i) Market for the input to capital production

The Entrepreneur's demand for good X to transform into the capital good K (X_K^d) equals the total quantity of good X saved by individuals *other than the Entrepreneur* at the end of period 1.

$$X_K^d = (N - 1)(a - c_0) \tag{9}$$

The Entrepreneur's purchase of good X is financed by a loan (L_E) from the Bank:

$$L_E = (N - 1)(a - c_0)p_1 \tag{10}$$

The amount of good X transformed into the capital good is total quantity of good X (X_k) saved by all individuals *including the Entrepreneur* at the end of period 1.

$$X_k = N(a - c_0) \tag{11}$$

(ii) Market for labour

The demand for labour (N_{FW}) is determined by the factory production technology, which requires one unit of labour (one worker) for each unit of capital good. We assume that the population size (*N*) is larger than the quantity demanded for labour.

$$N_{FW} = K = \ b\,X_k = bN(a - c_0) \quad (12)$$

The Entrepreneur finances the wage payment with a short-term loan (L_{Es}).

$$L_{Es} = wN_{FW} = w\,bN(a - c_0) \quad (13)$$

(iii) Market for final goods

The total quantity supplied of the final good (X^s) is the total factory output minus the quantity the Entrepreneur retains for consumption in period 2 (c^*_{2E}):

$$X^s = AN_{FW} - c^*_E = A\,bN(a - c_0) - c^*_{2E} \quad (14)$$

The total quantity demanded (X^d) is the quantity that can be bought with the total funds available to the buyers. The total funds available equals the total assets on the Bank's balance sheet before the sale of the factory-produced good X.

$$p_2 X^d = L_E(1 + i_l) + L_{Es} = (a - c_0)[(N - 1)(1 + i_l) + wbN] \quad (15)$$

Market clearance implies:

$$AN_{FW} - c^*_{2E} = L_E(1 + i_l) + L_{Es} \quad (16)$$

$$\text{where } c^*_{2E} = \text{A}\,b\,X_k - L_{Es} - (X_k - a + c_0)(1 + i_l)$$

(4) No switch between occupations

Individuals will not switch to a different occupation if there is no net gain from switching. This is the case when the utility levels associated with different occupations are equalised. Since everyone is a Home Producer and has the same consumption level in period 1, we only need to compare period 2 consumption levels across occupations.

(i) Equalisation of period 2 consumption between Home producers and Factory workers

From the solutions to the Home Producer's and the Factory Worker's utility maximisation problems, we have period 2 consumption of the Home Producer's (c^*_{2HP}) and that of the Factory Worker (c^*_{2FW}):

$$c^*_{2HP} = (a - c_0)(1 + i_d)\left(\frac{p_1}{p_2}\right) + a = (a - c_0)(1 + i_d) + a \quad (17)$$

$$c^*_{2FW} = (a - c_0)(1 + i_d)\left(\frac{p_1}{p_2}\right) + \frac{w}{p_2} = (a - c_0)(1 + i_d) + w \quad (18)$$

Equalisation of period 2 consumption between Home Producers and Factory Workers determines the equilibrium wage rate:

$$w^* = a \quad (19)$$

(ii) Equalisation of period 2 consumption between the Banker and the Entrepreneur

The solutions to the Banker's and the Entrepreneur's utility maximisation problems give us their period 2 consumption levels, respectively:

$$c^*_{2B} = L_E(i_l - i_d) + (a - c_0)(1 + i_d) \tag{20}$$

$$c^*_{2E} = \mathrm{A}\, b\, X_k - L_{Es} - (X_k - a + c_0)(1 + i_l) \tag{21}$$

Equalisation of period 2 consumption between the Banker and the Entrepreneur requires:

$$L_E(i_l - i_d) + (a - c_0)(1 + i_d) = \mathrm{A}\, b\, X_k - L_{Es} - (X_k - a + c_0)(1 + i_l) \tag{22}$$

which, together with equations (10), (11) and (13) define a relationship between the lending interest rate and the deposit interest rate:

$$i_l = \frac{AbN - w\,bN - N + N\,i_d - 2i_d}{(2N - 2)} \tag{23}$$

(iii) "Semi-equalisation" of period 2 consumption between the Banker and the Home producer

If all individuals are *ex ante* identical so that everyone could potentially become the Banker (or the Entrepreneur), then period 2 consumption between the Banker and the Home Producer should be equalised. However, if some talent and/or higher tolerance for risk are required to be a Banker (or the Entrepreneur), then the Banker (and the Entrepreneur) will be rewarded with a higher level of period 2 consumption than the Home Producer (or the Factory Worker). We use λ ($\lambda \geq 1$) to represent the "talent/risk premium".

The Banker's and the Home Producer's period 2 consumption are, respectively:

$$c^*_{2B} = L_E(i_l - i_d) + (a - c_0)(1 + i_d) \tag{24}$$

$$c^*_{2HP} = (a - c_0)(1 + i_d) + a \tag{25}$$

"Semi-equalisation" of period 2 consumption between the Banker and the Home Producer requires:

$$L_E(i_l - i_d) + (a - c_0)(1 + i_d) = \lambda[(a - c_0)(1 + i_d) + a] \tag{26}$$

Jointly solving equations (23) and (26), and using equation (10) we obtain:

$$i^*_d = \frac{(AbN - abN - N + 2 - 2\lambda)(a - c_0) - 2\lambda a}{(a - c_0)(N - 2 + 2\lambda)} \tag{27}$$

$$i^*_l = \frac{AbN - abN - N}{(2N - 2)} + \left(\frac{N - 2}{2N - 2}\right)\frac{(AbN - abN - N + 2 - 2\lambda)(a - c_0) - 2\lambda a}{(a - c_0)(N - 2 + 2\lambda)} \tag{28}$$

(5) Summary, discussion and numerical simulations

In summary, the economy may have 2 different equilibria depending on the relationship between deposit interest rate (i^*_d) and time preference (θ).

If $i_d^* \leq (1-\theta)/\theta$, where $i_d^* = \frac{(AbN - abN - N + 2 - 2\lambda)(a - c_0) - 2\lambda a}{(a - c_0)(N - 2 + 2\lambda)}$, the equilibrium is one in which all individuals are Home Producers in both periods. They consume what they produce in each period which are: $c_1^* = c_2^* = a$.

If $i_d^* > (1-\theta)/\theta$, the equilibrium is one in which real savings, entrepreneurship and finance together enable capital investment and factory production. Specifically, in period 1, everyone saves; in period 2, they take one of the four occupations, Home Producer, Factory worker, Banker or Entrepreneur. The equilibrium prices and quantities are summarised in Table 1.

As shown in Table 1, the total output in period 2 is: $X_2^* = (A - a)bN((a - c_0) + aN$, which is obviously higher than that in period 1 ($X_1^* = aN$). This (discontinuous) growth in output is brought about by the adoption of the factory technology, which is in turn enabled by real savings, entrepreneurship and finance. It is easy to see that $\frac{\partial X_2^*}{\partial A} > 0$, $\frac{\partial X_2^*}{\partial b} > 0$, $\frac{\partial X_2^*}{\partial c_0} < 0$. In other words, period 2 output is positively related to the productivity of the factory technology, and negatively related to period 1 consumption (or put another way, positively related to period 1 real savings).

For the Home Producer and the Factory Worker, their period 2 goods available for consumption is: $c_{2HP}^* = c_{2FW}^* = [bN(A - a)(a - c_0) + aN]/(N - 2 + 2\lambda)$. This quantity is higher than that available in period 1, provided the talent/risk premium (λ) received by the Banker and the Entrepreneur is not too high. Specifically, $c_{2HP}^* = c_{2FW}^* > a$ if $\lambda < [(A - a)bN(a - c_0) + 2a]/2a$. It is easy to show that, $\frac{\partial c_{2HP}^*}{\partial A} > 0$, $\frac{\partial c_{2HP}^*}{\partial b} > 0$, $\frac{\partial c_{2HP}^*}{\partial c_0} < 0$, $\frac{\partial c_{2HP}^*}{\partial \lambda} < 0$. It is also clear that $\frac{\partial c_{2HP}^*}{\partial N} = 0$ if $\lambda = 1$, but $\frac{\partial c_{2HP}^*}{\partial N} > 0$ if $\lambda > 1$. This is because for the Home Producer and the Factory Worker, the talent/risk premium paid to the Banker and the Entrepreneur is like a fixed cost; the average fixed cost gets smaller as the number of people paying it gets larger.

To get a better feel for the relative magnitudes of the equilibrium values and how they change in response to variations in parameter values, we perform a set of numerical simulations. The results are presented in Table 2.

In the base case, we set population $N = 100$, productivity of home production $a = 3$, minimum consumption $c_0 = 1$, productivity of factory production $A = 6$, efficiency of capital goods transformation $b = 0.4$, talent/risk premium $\lambda = 2$. We also assume time preference parameter $\theta = 0.9$, which implies that the threshold deposit interest rate for saving is 0.111. The equilibrium deposit interest rate ($i_d^* = 0.118$) is higher than saving threshold; therefore, individuals will save in period 1. The Entrepreneur borrows $L_E^* = 198$ from the Banker to start a factory at the lending interest rate $i_l^* = 0.159$ and hires 80 workers ($N_{FW}^* = 80$). At the end of period 2, Home Producers and Factory Workers have per capita good X available for consumption $c_{2HP}^* = c_{2FW}^* = 5.235$. The Banker and the Entrepreneur have twice as much good X available for consumption: $c_{2B}^* = c_{2E}^* = 10.471$. The utility level of the Home Producer and the Factory Work is: $U_{HP}^* = U_{FW}^* = 6.712$; and the utility level of the Banker and the Entrepreneur is: $U_B^* = U_E^* = 11.424$. Both are higher than 5.7, which is the utility of Autarky where there is no factory production and everyone is a Home Producer.

From the base case, we vary each of the parameters, N, a, c_0, A, b, λ, one at a time. The results show that when N is reduced to 50, factory production cannot take place because the equilibrium deposit interest rate (i_d^*) is 0.038, which is below the saving threshold of 0.111. When N is increased, the saving threshold is exceeded. As N increases, the per capita quantities of good X available for consumption also increase (due to the fixed cost nature of the Banker's and Entrepreneur's talent/risk premium noted earlier).

When productivity of home production a is raised to 3.5 or higher, the equilibrium deposit interest rates turn negative, so there can be no factory production in period 2. Factory production can take place when a is reduced. However, since a lower a means a smaller savings in period 1, the quantity of good X produced in period 2 is lower. Also, the deposit and leading interest rates are higher, reflecting the relatively higher productivity of factory production.

When minimum consumption c_0 is raised to 1.5 or higher, there can be no factory production as the equilibrium deposit interest rates fall below the saving threshold. When c_0 is reduced, factory production takes place. A smaller c_0 means a larger period 1 saving and consequently a larger quantity of good X produced in period 2. Also, the deposit and leading interest rates are higher.

When the productivity of factory production (A) is lowered to 5, or the efficiency of capital goods transformation (b) is lowered to 0.35, the equilibrium deposit rates turn negative, and there is no factory production in period 2. If A or b increases, more good X are produced in period 2, and the higher quantities are distributed to the population via higher deposit and lending interest rates.

A change in the talent/risk premium (or wealth distribution indicator) λ does not change the factory output; it only changes the distribution of the output. A higher λ increases the output shares of the Banker and the Entrepreneur, and decreases those of the Home Producers and the Factory Workers. However, if λ is increased to 2.2 or higher, the equilibrium deposit interest rate falls below the saving threshold, and no factory production can take place in period 2.

From the numerical simulations, we see the familiar positive effects of savings and technological progress on economic performance. Provided that factory production takes place, a higher savings level (a larger a, a smaller c_0), and a more productive technology (a larger A, a larger b) result in a larger per capita period 2 output, and a higher individual utility level over the 2 periods. We also see that a higher period 2 output is associated with higher deposit and lending interest rates. This is because interest rates play the role of distributing the additional wealth to the capital contributors who have made the factory production possible. The lending interest rate reflects the average gross return to capital; and the deposit rate reflects the average net return to capital (net of the cost of Banking).

3. Economic performance after the adoption of factory technology

As described earlier, individuals in this model live indefinitely, but their decision horizon is just 2 periods. We have so far looked at the individuals' decisions at the end of period 1 (or the beginning of period 2). [8] The decisions include how much to consume and save, and what occupation to choose in period 2. These decisions jointly determine period 2 output of good X and the prices, namely, the wage rate, and the deposit and lending interest rates. The prices allocate resources (labour and capital goods) to factory production, and distribute the factory output to individuals according to their factor contributions.

To study the economic performance beyond period 2 after the adoption of the factory technology, we can repeat individual decision-making in each subsequent period, taking as given the resources available at the time. Thus, if factory production takes place in period 2, then the output of period 2 is allocated to individuals through the market and becomes the individual's resources available for consumption at the beginning of period 3. Individuals then decide again how much to consume and save. These decisions will determine the output of period 3, which become their resources available at the beginning of period 4, and so on.[9]

We start from the base case with $N = 100, a = 3, A = 6, b = 0.4, \lambda = 1, \theta = 0.9$.[10] Assuming no change in the population or technology (i.e., N, a, A, b remain constant), we study how the economy performs over time. The simulation results are presented in Table 3.

Given the base case parameter values, at the end of period 2, each individual has a quantity of 5.340 good X produced ($x_t^* = 5.340$, at t = 2). This becomes the individual's resources *available for consumption* at the beginning of period 3 ($c_t = 5.340$, at t = 3). If the individuals consume 33% of the total sources as they do in period 2 ($c_0 = 0.33c_t$, savings rate = 67%), the model breaks down at t = 3 because the demand for Factory Workers will be 142.4, which is greater than the total population of 100. If, in the face of the significant increase in wealth, individuals increase their consumption and savings rate falls to 45% ($c_0 = 0.55c_t$), then the economy continues to grow in period 3. The individuals will have $c_t = 5.824$ at t = 4. However, the model breaks down in period 4 as the high savings rate drive the demand for Factory Workers to a level higher than the total population.

If we lower the savings rate to 40% ($c_0 = 0.60c_t$), the economy will continue to grow from per capita output $x_t^* = 5.340$, at t = 2, to $x_t^* = 5.503$, at t = 3, to $x_t^* = 5.582$, at t = 4, and so on. Per capita output reaches a peak of $x_t^* = 5.654$, at t = 12, and stays at that level beyond period 12.[11] The economy reaches a steady state at t = 20 where all variables remain unchanged over time.

[8] Individuals have no choice at the beginning of period 1; everyone has to use the home technology to produce. At the end of period 1, individuals have a quantity of output, and they decide how much to consume and how much to save.
[9] If factory production does not take place in period 2, then there is no saving in period 1, and consequently period 2 (and all subsequent periods) will be the same as period 1.
[10] The assumption of $\lambda = 1$ removes the impact of uneven distribution on economic performance.
[11] The reported figures are accurate to 3 decimal places.

If we lower the savings rate further to 30% ($c_0 = 0.70c_t$), the economy will shrink from $x_t^* = 5.340$, at t = 2, to $x_t^* = 4.862$ at t = 3, to $x_t^* = 4.690$, at t = 4, and so on. Per capita output falls to a trough of $x_t^* = 4.594$, at t = 9, and stays there. The steady state is reached at t = 15.

These simulations suggest a familiar conclusion: given the constant-return factory technology, there cannot be continuing economic growth over the long term. Eventually the economy reaches a steady state with zero per capita output growth. The steady state level of per capita output is positively related to savings rate (provided that the savings rate is not too high to break the system). Long-term growth can be achieved through continued development and implementation of better technologies (i.e., increases in *A* and/or *b*) as shown by the simulation results in Table 2.

4. Concluding remarks

In this paper, we have developed a monetary model of economic growth that incorporates three main drivers of growth: savings, entrepreneurship and finance. In this model, real savings are the precondition for capital formation. Capital formation in turn is the precondition for adopting a more productive factory technology. The Entrepreneur initiates capital formation and organises factory production using the more productive technology.

The Entrepreneur's activities need to be financed. The Banker provides finance by issuing loans and by taking deposits with interests. Loans create purchasing power which mobilises the real resources from the savers to the Entrepreneur. Interest payment to depositors provides an incentive to save in the first place. The deposit interest rate reflects the net average return to capital (net of the banking cost).

In this model (as in the real world), loans are created *before* deposits are taken – without the loans, the Entrepreneur cannot buy real resources from the savers, and the savers will have nothing to deposit with the Bank. Thus, the Bank does not collect deposits to make loans. Rather, it makes loans to create deposits. From changes in the Bank's balance sheet, we can trace the flow of funds between market participants. We see that provided factory production takes place, economic activities begin with lending, and end with loan repayment. A growing economy involves an increasing amount of lending.

One might ask: why does the economy described in the model *need* a bank? Why can't the Entrepreneur finance directly from each saver and get rid of the middleman? The answer is, of course, transaction costs.[12] It would be very costly for the Entrepreneur to obtain direct finance from the savers. The Entrepreneur has no means of payment, the savers would in effect become shareholders of the Firm and need to monitor its performance. Moreover, the economy would be akin to a planned economy as there would be no monetary transactions. In contrast, the Banker creates purchasing

[12] Apparently, Professor Douglas North believes that the answer to almost all economic questions is transaction costs. (https://taitc.buzzsprout.com/2186249/12817992-taitc-episode-1)

power for the Entrepreneur and provides payment services. These turn the transactions between the Entrepreneur and the savers from investment contracts to current sales transactions, greatly reducing transaction costs. The Banker monitors the Entrepreneur instead of every saver having to do it, again generating large cost savings. In real economies, direct financing does occur, but it is hard to imagine direct financing in modern economies without a banking system that creates money and provides payment services.

This model may be extended in several dimensions. For example, instead of assuming exogenous technologies, future research can introduce endogenous technological progress by making the productivity of factory technology (A) and/or the efficiency of capital goods formation (b) a function of labour input. Endogenous technological progress may overcome the issue of labour shortage (demand for factory workers exceeding the population size) we have encountered in the numerical simulations.

Also, we have assumed in this model that all income (wages, interests, and the Banker's earnings) is spent at the end of each period. This is necessary for the Entrepreneur to receive sufficient revenue to repay her loans. We can relax this assumption to study "the paradox of thrift", that is, we can investigate what happens if *financial* savings result in a deficiency in demand, and what can be done about it.

Finally, in this model, loans are provided only to the Entrepreneur for productive activities. It would be interesting to study how the economy may be affected if loans are issued to finance consumption or financial speculation.

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Table 1. Equilibrium with Savings, Entrepreneurship and Finance

Prices	**Quantities**
Price of good X at the end of period 1: $p_1^* = 1$	Total quantity of good X produced in period 1: $X_1^* = Na$ Total quantity of good X saved for transforming into capital goods at the end of period 1: $X_k^* = N(a - c_0)$
Wage rate in period 2: $w^* = a$	Number of Factory Workers: $N_{FW}^* = bN(a - c_0)$
Deposit rate in period 2: $i_d^* = \frac{(AbN - abN - N + 2 - 2\lambda)(a - c_0) - 2\lambda a}{(a - c_0)(N - 2 + 2\lambda)}$	Total deposits at the beginning of period 2: $D^* = (N - 2)(a - c_0)$
Lending rate in period 2: $i_l^* = \frac{AbN - abN - N}{(2N - 2)} + (\frac{N - 2}{2N - 2}) \frac{(AbN - abN - N + 2 - 2\lambda)(a - c_0) - 2\lambda a}{(a - c_0)(N - 2 + 2\lambda)}$	Total loans at the beginning of period 2: $L_E^* = (N - 1)(a - c_0)$
Price of good X in period 2: $p_2^* = 1$	Total quantity of good X produced in the factory in period 2: $X_{2f}^* = A\, bN(a - c_0)$ Total quantity of good X produced at home in period 2: $X_{2HP}^* = aN[1 - b(a - c_0)]$ Total quantity of good X produced in period 2: $X_2^* = bN(A - a)(a - c_0) + aN$ Period 2 good X available for consumption for Home Producers and Factory Workers: $c_{2HP}^* = c_{2FW}^*$ $= [bN(A - a)(a - c_0) + aN]/(N - 2 + 2\lambda)$ Period 2 goods available for consumption for the Banker and the Entrepreneur: $c_{2B}^* = c_{2E}^*$ $= \lambda[bN(A - a)(a - c_0) + aN]/(N - 2 + 2\lambda)$

Table 2. Numerical Simulations: Equilibrium Values

Base case														
Parameter values ($\theta = 0.9$. saving threshold $(1-\theta)/\theta = 0.111$)						**Equilibrium prices and quantities**						**Utility levels**		
N	a	c_0	A	b	λ	N_{FW}^*	L_E^*	i_d^*	i_l^*	$c_{2HP}^* = c_{2FW}^*$	$c_{2B}^* = c_{2E}^*$	$U_{HP}^* = U_{FW}^*$	$U_B^* = U_E^*$	**Autarky Utility**
100	3	1	6	0.4	2	80	198	0.118	0.159	5.235	10.471	6.712	11.424	5.7
Change N														
50	3	1	6	0.4	2	40	98	**0.038**	0.121	5.077	10.154	6.569	11.138	5.7
200	3	1	6	0.4	2	160	398	0.158	0.179	5.317	10.634	6.785	11.570	5.7
300	3	1	6	0.4	2	240	598	0.172	0.186	5.344	10.689	6.810	11.620	5.7
400	3	1	6	0.4	2	320	798	0.179	0.190	5.358	10.716	6.822	11.645	5.7
500	3	1	6	0.4	2	400	998	0.183	0.192	5.367	10.733	6.830	11.660	5.7
Change a														
100	*1.5*	1	6	0.4	2	20	49.5	0.647	0.724	2.324	4.647	2.591	4.682	2.85
100	*2*	1	6	0.4	2	40	99	0.490	0.546	3.490	6.980	4.141	7.282	3.8
100	*2.5*	1	6	0.4	2	60	148.5	0.307	0.354	4.461	8.922	5.515	9.529	4.75
100	***3.5***	1	6	0.4	2	**100**	247.5	**-0.075**	-0.037	5.814	11.627	7.732	12.965	6.65
100	***4***	1	6	0.4	2	**120**	297	**-0.268**	-0.234	6.196	12.392	8.576	14.153	7.6
Change c_0														
100	3	*0.8*	6	0.4	2	88	217.8	0.123	0.162	5.471	10.941	7.124	12.047	5.7
100	3	*0.9*	6	0.4	2	84	207.9	0.120	0.161	5.353	10.706	6.918	11.735	5.7
100	3	***1.5***	6	0.4	2	60	148.5	**0.098**	0.150	4.647	9.294	5.682	9.865	5.7
100	3	***2***	6	0.4	2	40	99	**0.059**	0.130	4.059	8.118	4.653	8.306	5.7
100	3	***2.5***	6	0.4	2	20	49.5	**-0.059**	0.072	3.471	6.941	3.624	6.747	5.7

Table 2. Numerical Simulations: Equilibrium Values (continued)

Base case														
Parameter values ($\theta = 0.9$, saving threshold $(1-\theta)/\theta = 0.111$)						**Equilibrium prices and quantities**						**Utility levels**		
N	a	c_0	A	b	λ	N^*_{FW}	L^*_E	i^*_d	i^*_l	$c^*_{2HP} = c^*_{2FW}$	$c^*_{2B} = c^*_{2E}$	$U^*_{HP} = U^*_{FW}$	$U^*_B = U^*_E$	**Autarky Utility**
100	3	1	6	0.4	2	80	198	0.118	0.159	5.235	10.471	6.712	11.424	5.7
Change *A*														
100	3	1	***5***	0.4	2	80	198	**-0.275**	**-0.237**	4.451	8.902	6.006	10.012	5.7
100	3	1	*7*	0.4	2	80	198	0.510	0.555	6.020	12.039	7.418	12.835	5.7
100	3	1	*8*	0.4	2	80	198	0.902	0.951	6.804	13.608	8.124	14.247	5.7
100	3	1	*9*	0.4	2	80	198	1.294	1.348	7.588	15.176	8.829	15.659	5.7
100	3	1	*10*	0.4	2	80	198	1.686	1.744	8.373	16.745	9.535	17.071	5.7
Change *b*														
100	3	1	6	***0.35***	2	70	198	**-0.029**	0.011	4.941	9.882	6.447	10.894	5.7
100	3	1	6	*0.41*	2	82	198	0.147	0.189	5.294	10.588	6.765	11.529	5.7
100	3	1	6	*0.42*	2	84	198	0.176	0.219	5.353	10.706	6.818	11.635	5.7
100	3	1	6	*0.43*	2	86	198	0.206	0.248	5.412	10.824	6.871	11.741	5.7
100	3	1	6	*0.44*	2	88	198	0.235	0.278	5.471	10.941	6.924	11.847	5.7
Change λ														
100	3	1	6	0.4	*1*	80	198	0.170	0.185	5.340	5.340	6.806	6.806	5.7
100	3	1	6	0.4	*1.5*	80	198	0.144	0.172	5.287	7.931	6.758	9.138	5.7
100	3	1	6	0.4	*2.1*	80	198	0.113	0.157	5.225	10.973	6.703	11.875	5.7
100	3	1	6	0.4	***2.2***	80	198	**0.107**	0.154	5.215	11.473	6.693	12.325	5.7
100	3	1	6	0.4	***2.3***	80	198	**0.102**	0.152	5.205	11.971	6.684	12.774	5.7

Table 3. Numerical Simulations: Growth Beyond Period 2

Base case: $c_0 = 0.33c_t$, **savings rate = 67%**											
Parameter values ($\theta = 0.9$, saving threshold $(1-\theta)/\theta = 0.111$)							**Equilibrium values of endogenous variables**				
t	N	a	c_t	c_0	A	b	N_{FW}^*	L_E^*	i_d^*	i_l^*	x_t^*
t = 2	100	3	3	1	6	0.4	80	198	0.170	0.185	5.340
$c_0 = 0.33c_t$, **savings rate = 67%**											
t = 3	100	3	5.340	1.78	6	0.4	**142.4**	352.44	0.183	0.192	7.212
$c_0 = 0.55c_t$, **savings rate = 45%**											
t = 3	100	3	5.340	2.937	6	0.4	96.120	237.897	0.175	0.188	5.824
t = 4	100	3	5.824	3.203	6	0.4	**104.825**	259.441	0.177	0.189	6.085
$c_0 = 0.60c_t$, **savings rate = 40%**											
t = 3	100	3	5.340	3.204	6	0.4	85.440	211.464	0.172	0.186	5.503
t = 4	100	3	5.503	3.302	6	0.4	88.051	217.927	0.173	0.187	5.582
t = 5	100	3	5.582	3.349	6	0.4	89.305	221.029	0.173	0.187	5.619
t = 6	100	3	5.619	3.371	6	0.4	89.906	222.518	0.173	0.187	5.637
t = 7	100	3	5.637	3.382	6	0.4	90.195	223.233	0.173	0.187	5.646
t = 8	100	3	5.646	3.388	6	0.4	90.334	223.576	0.173	0.187	5.650
t = 9	100	3	5.650	3.390	6	0.4	90.400	223.740	0.173	0.187	5.652
t = 10	100	3	5.652	3.391	6	0.4	90.432	223.819	0.173	0.187	5.653
t = 11	100	3	5.653	3.392	6	0.4	90.447	223.857	0.173	0.187	5.653
t = 12	100	3	5.653	3.392	6	0.4	90.455	223.875	0.173	0.187	5.654
t = 13	100	3	5.654	3.392	6	0.4	90.458	223.884	0.173	0.187	5.654
t = 14	100	3	5.654	3.392	6	0.4	90.460	223.888	0.173	0.187	5.654
t = 15	100	3	5.654	3.392	6	0.4	90.461	223.890	0.173	0.187	5.654
t = 16	100	3	5.654	3.392	6	0.4	90.461	223.891	0.173	0.187	5.654
t = 17	100	3	5.654	3.392	6	0.4	90.461	223.892	0.173	0.187	5.654
t = 18	100	3	5.654	3.392	6	0.4	90.461	223.892	0.173	0.187	5.654
t = 19	100	3	5.654	3.392	6	0.4	90.461	223.892	0.173	0.187	5.654
t = 20	100	3	5.654	3.392	6	0.4	90.462	223.892	0.173	0.187	5.654
t = 21	100	3	5.654	3.392	6	0.4	90.462	223.892	0.173	0.187	5.654

Table 2. Numerical Simulations: Growth beyond period 2 (continued)

Base case: $\boldsymbol{\lambda = 1, c_0 = 0.33c_t}$**, savings rate = 67%**											
Parameter values ($\theta = 0.9$, saving threshold $(1-\theta)/\theta = 0.111$)							**Equilibrium values of endogenous variables**				
$\boldsymbol{t}$	$\boldsymbol{N}$	$\boldsymbol{a}$	$\boldsymbol{c_t}$	$\boldsymbol{c_0}$	$\boldsymbol{A}$	$\boldsymbol{b}$	$\boldsymbol{N^*_{FW}}$	$\boldsymbol{L^*_E}$	$\boldsymbol{i^*_d}$	$\boldsymbol{i^*_l}$	$\boldsymbol{x^*_t}$
t = 2	100	3	3	1	6	0.4	80	198	0.170	0.185	5.340
$\boldsymbol{c_0 = 0.70c_t}$**, savings rate = 30%**											
t = 3	100	3	5.340	3.738	6	0.4	64.080	158.598	0.163	0.181	4.862
t = 4	100	3	4.862	3.404	6	0.4	58.349	144.413	0.159	0.180	4.690
t = 5	100	3	4.690	3.283	6	0.4	56.286	139.307	0.157	0.179	4.629
t = 6	100	3	4.629	3.240	6	0.4	55.543	137.468	0.157	0.179	4.606
t = 7	100	3	4.606	3.224	6	0.4	55.275	136.807	0.157	0.179	4.598
t = 8	100	3	4.598	3.219	6	0.4	55.179	136.568	0.157	0.178	4.595
t = 9	100	3	4.595	3.217	6	0.4	55.144	136.483	0.156	0.178	4.594
t = 10	100	3	4.594	3.216	6	0.4	55.132	136.452	0.156	0.178	4.594
t = 11	100	3	4.594	3.216	6	0.4	55.128	136.441	0.156	0.178	4.594
t = 12	100	3	4.594	3.216	6	0.4	55.126	136.437	0.156	0.178	4.594
t = 13	100	3	4.594	3.216	6	0.4	55.125	136.435	0.156	0.178	4.594
t = 14	100	3	4.594	3.216	6	0.4	55.125	136.435	0.156	0.178	4.594
t = 15	100	3	4.594	3.216	6	0.4	55.125	136.434	0.156	0.178	4.594
t = 16	100	3	4.594	3.216	6	0.4	55.125	136.434	0.156	0.178	4.594