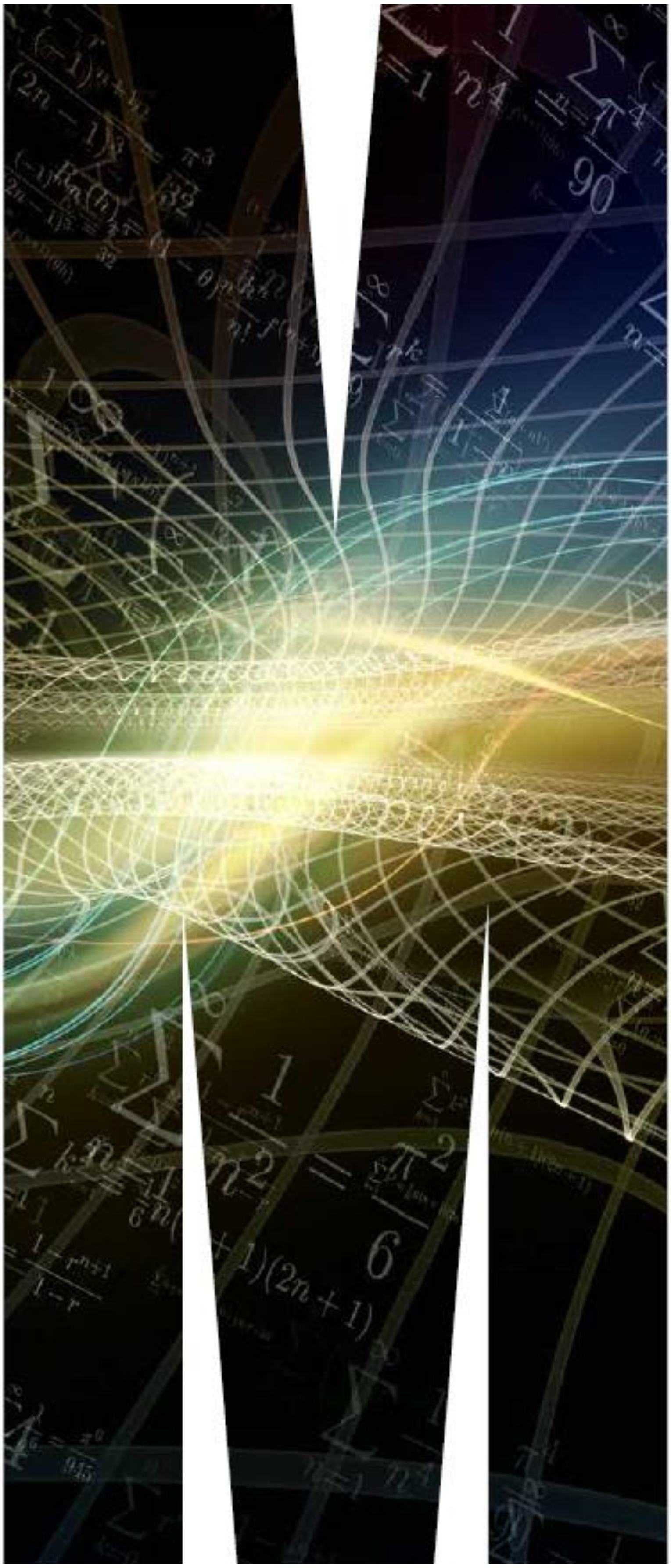
MONASH University

School of Mathematics

MTH3000 projects

2020/2021



MONASH
SCIENCE

Table of Contents

PURE MATHEMATICS

Rienmann surfaces

Supervisors: Julie Clutterbuck

Riemann surfaces are one-dimensional, complex manifolds: examples include the complex plane, the sphere and the torus. We will study their geometric and analytic properties using tools from complex analysis.

Topics in (Computational) Algebra

Supervisor: Heiko Dietrich

Project outline: Possible topics could involve (computations with) groups, Lie algebras, or other algebraic structures. If you are interested in the area of group theory or algebra, you are able to discuss an option for a project with Heiko Dietrich: heiko.dietrich@monash.edu .

The algebra of knots

Supervisor: Norm Do
Co-supervisor: Daniel Mathews

Background: A knot is made by taking a piece of string, tying it up in some fashion, and then gluing the ends together. For well over a century, mathematicians and scientists have been preoccupied with the question of how to distinguish two given knots. Over recent decades, inspirations from algebra and theoretical physics have led to a vast theory of knot polynomials, which help to answer this question. A particularly important example is the sequence of coloured Jones polynomials.

Project outline: There are many possible projects, depending on the preferences and strengths of the student. One is to examine the AJ conjecture, which relates the recursive structure of coloured Jones polynomials with the A-polynomial of a knot. Another is to consider modern approaches to the volume conjecture, which relates coloured Jones polynomials with the volume of a space related to the knot. Physicists have recently proposed that these conjectures can be investigated using a theory known as topological recursion, arising from the study of matrix models.

References:

* C. C. Adams, The knot book
* H. Murakami, An introduction to the volume conjecture

Matrix model techniques in enumerative geometry

Supervisor: Norm Do
Co-supervisor: Daniel Mathews

Background: How many ways are there to tile a given surface with polygons? The answer to this problem is elegantly solved by performing an integral over a space of matrices, otherwise known as a matrix model. The technique of using matrix models to solve such problems was discovered not by mathematicians, but by physicists about fifty years ago. There has been an explosion of activity in the field in recent years, particularly due to connections with algebraic geometry. In particular, the theory known as topological recursion is undergoing rapid development and its physical foundations need to be put on a rigorous mathematical footing.

Project outline: There are many possible projects, depending on the preferences and strengths of the student. For example, one could develop computational tools to calculate invariants arising from topological recursion. Another project would be to consider certain polynomials that arise from the surface tiling problem described above and explain the geometric meaning behind their coefficients. Yet another project could be to understand the relationship between topological recursion and the structure of geometric quantities known as Gromov-Witten invariants.

References:
* A. Zvonkin, Matrix integrals and map enumeration: An accessible introduction
* B. Eynard, Counting surfaces: Combinatorics, matrix models and algebraic geometry

Knot polynomials and invariants

Supervisor: Daniel Mathews
Co-supervisor: Norm Do

Background: When you tie a string in a knot and join together the ends, you have a mathematical knot. How to tell whether two given knots are the same or different is a difficult question. Many mathematical techniques have been developed to tell apart knots - and many of these techniques use interesting ideas from algebra and topology.

Project outline: This project would start with some background reading on knots and knot invariants, and then cover some knot polynomials, such as the Alexander polynomial, A-polynomial, Jones polynomial, or coloured/quantum invariants. We could also examine knot complements and other ideas.

References:
* Colin Adams, The Knot Book
* Louis H Kauffman, On Knots
* Not knot (video) youtube.com/watch?v=AGLPbSMxSUM

The geometry of Hurwitz numbers

Supervisor: Norm Do
Background: Suppose that we have two surfaces and wish to wrap one around the other. Mathematically speaking, we would like to consider functions from the points of one surface to the points of the other. Of course, a sphere can wrap once around another sphere in a nice and simple way. However, if we use more complicated surfaces or try to wrap many times, then there are necessarily points where the function is no longer smooth. By controlling the location and behaviour of these so-called branch points, one can try to count the number of such coverings and the answer is known as a Hurwitz number. The study of Hurwitz numbers has fascinating connections to algebra, geometry, and mathematical physics.

Project outline: There are several possible projects, depending on the preferences and strengths of the student. For example, a great deal is known when the surface to be covered is a sphere, but what happens when it is a torus? There are also many results concerning simple Hurwitz numbers, though conjectures abound when it comes to the more general spin Hurwitz numbers. It is also possible to explore the generalisation to double Hurwitz numbers, in which case the connections to geometry remain conjectural and mysterious.

References: S. K. Lando, Combinatorial facets of Hurwitz numbers

Functional analysis and operator theory, applications to partial differential equations

Supervisor: Jérôme Droniou
Project outline: functional analysis deals with the study of infinite dimensional normed vector spaces, for example the space of continuous functions. A number of powerful techniques and results have been developed to understand the structure of such spaces. This understanding has practical consequence on the analysis of partial differential equations, in particular to ensure the existence and uniqueness of the solution to these equations.

Several projects can be conducted in this area, depending on the student's background and interests.

Discrete Models of Critical Phenomena

Supervisor: Tim Garoni
Co-supervisor: TBA

Background: Many probabilistic models defined on graphs possess special "critical" values of their parameters, at which long-range order develops and fractals emerge. Examples of such models include percolation, in which one studies the connectivity properties of random spanning subgraphs of some fixed graph, and the Ising/Potts models in which one studies random vertex colourings. The study of such "critical phenomena" has been a fertile area in mathematical physics for three quarters of a century. A wide variety of methods are used to study these models, ranging from rigorous combinatorial, algebraic and probabilistic methods, through to large-scale computer experiments and non-rigorous methods of theoretical physics. In the case of planar graphs, the conjectured conformal invariance of certain continuum limits of these discrete models has recently given rise to some very significant advances.

Project Outline: The project would start with a review of the required basic background in mathematical physics (statistical mechanics); the two references listed below being a good place to start. From there, a range of possible projects is available, and depending on your interests, the project could include combinatorial, computational, and/or probabilistic aspects to a greater or lesser degree. There would be scope for an investigation that may lead to new results, particularly if the project involves computer experiments.

Skills required: Programming would be helpful, but not essential, as would some background in physics.

References:
G. Slade, "Probabilistic Models of Critical Phenomena", in "The Princeton Companion to Mathematics" edited by T. Gowers, Princeton University Press (2008).
G. Grimmett, "Probability on Graphs", Cambridge University Press (2010), available online free (legally) at http://www.statslab.cam.ac.uk/~grg/books/pgs.html

Topics in graph decompositions and combinatorial design

Supervisor: Daniel Horsley
Project outline: Combinatorial design theory is an area of mathematics that studies arrangements of objects that are in some sense ``balanced''. It has applications to the design of experiments, to coding and cryptography, to traffic grooming in networks, and to numerous other areas. Many problems in combinatorial design theory can be considered as problems concerning decomposing graphs into edge-disjoint subgraphs, and this way of viewing them has led to many important results and insights.

This project will involve investigating one the many easily accessible problems in the area of graph decompositions and combinatorial designs. Possible problems could concern:
* embedding Steiner triple systems;
* colouring graph designs;
* resolutions of Steiner triple systems;
* decompositions of graphs into cycles; or
* infinite designs.

To discuss details of possible problems contact Daniel Horsley: **daniel.horsley@monash.edu**

Hyperbolic and non-Euclidean Geometry

Supervisor: Daniel Mathews

Background: The geometry we learn in school is Euclidean geometry. This geometry, which is close to our everyday intuition about space, is based upon axioms written down by Euclid in ancient times. However, Euclidean geometry is not the only type of geometry. By varying one axiom about parallel lines, we end up with hyperbolic geometry. This geometry has many fascinating and counterintuitive properties, has interesting relations to physics and algebra, and leads to beautiful pictures, such as those in the artwork of M.C. Escher.

Project outline: This project would work beginning with an introduction to non-Euclidean and hyperbolic geometry. Depending on interest, the project could then cover aspects relating to differential geometry, algebra, or topology.

References:
* James W. Anderson, Hyperbolic Geometry
* Jeffrey R. Weeks, The Shape of Space

Topics in Symplectic Geometry

Supervisor: Daniel Mathews
Co-supervisor: Norm Do

Background: Classical Newtonian mechanics was reformulated in the 19th century by William Hamilton. The mathematical language of Hamiltonian mechanics is symplectic geometry. This beautiful subject encompasses much of classical physics, but has also enjoyed several breakthroughs in recent years, connections with string theory, and the development of powerful invariants.

Project outline: This project would begin with a study of symplectic vector spaces, which are the building blocks of symplectic geometry, and symplectic manifolds. From there, several directions are possible, depending on the interests of the student, such as:

*Reformulating various physical theories (mechanics, optics, electromagnetism) in terms of the mathematics of symplectic geometry
*Dynamical systems in symplectic geometry
*Holomorphic curves and Gromov-Witten theory

Reference: Dusa McDuf and Dietmar Salamon, Introduction to Symplectic Topology

Topological quantum field theories

Supervisor: Daniel Mathews

Co-supervisor: Norm Do

Background: Quantum field theory describes our universe at a fundamental level, but it is plagued with difficulties. One approach mathematicians have taken is to define Topological Quantum Field Theories, or TQFTs, which are rigorously defined mathematical theories but possess some of the properties of quantum field theory. The mathematics that arises has fascinating connections to several branches of geometry and algebra.

Project outline: This project would start with a review of some necessary topology and algebra. Several projects are then possible, depending on the interests of the student, such as:

*Classification of low-dimensional TQFTs
*Khovanov homology, a powerful method for distinguishing knots based on TQFT ideas
*Relationship to mathematical physics and matrix integrals
*TQFT arising from contact and symplectic geometry

Reference: Joachim Kock, Frobenius Algebras and 2D Topological Quantum Field Theories

Planar Brownian motion and complex analysis

Supervisor: Greg Markowsky
Project outline: There is an intimate connection between planar Brownian motion and complex analysis, and the major theme of this project will be to tackle problems that lie in the intersection of these two topics. To be specific, a holomorphic map applied to a Brownian motion yields a new stochastic process, which can be realized as a Brownian motion run at a variable speed. This connection can be used to motivate and provide proofs of many of the major results in complex analysis; conversely, this conformal invariance, as it is often called, can also be used to prove many interesting results on planar Brownian motion via complex analytic techniques. We will study these techniques in more detail.

Complex analysis

Supervisor: Greg Markowsky
Co-supervisor: TBA

Project outline: The exact project needs is to be decided in consultation with Greg Markowsky taking into consideration your personal interests and mathematical background.

Topics in combinatorics

Supervisor: Burkard Polster
Possible project topics under this area include:

The mathematical modelling of juggling (e.g. juggling sequences, optimal control).
The mathematics of origami (e.g. constructability, map folding, fractal sequences, flat folding, Hilton-Pederson algorithm).
The mathematics of campanology (various graph-theoretical questions related to bell ringing sequences).
Mathematics in the movie “A Beautiful Mind” (in particular the Nash equilibrium theorem).
Finite group theory of Rubik’s-cube-like mechanical puzzles.

If you are interested in the area of combinatorics, you are able to discuss an option for a project with Burkard Polster: **burkard.polster@monash.edu**

A variety of undergraduate research projects are available in the following specialized areas in “Geometric Analysis”.

Geometric calculus of variations

Supervisors: Todd Oliynyk

Geometric evolution equations

Supervisors: Todd Oliynyk

Mathematical relativity

Supervisors: Todd Oliynyk

Project outline: The exact project will be decided in consultation with the relevant staff member and the research student, and taking into consideration the student’s personal interests and mathematical background. Descriptions of the areas can be found in pages 18-19 of the Honours Project and Essay handbook http://www.maths.monash.edu.au/honours/docs/hons-projects-essays.pdf

You are welcome to make an appointment with Todd Oliynyk Todd.Oliynyk@monash.edu to discuss your interests.

Here is a hodge-podge of some specific topics:
Approximation techniques in General Relativity – the Newtonian limit and post-Newtonian expansions; Classical mechanics – equations governing all motion; Construction of minimal surfaces (Weierstrass representation); Construction of minimal surfaces (geometric heat flow methods); Delaunay surfaces – axially symmetric constant mean curvature surfaces; Isoperimetric property of the sphere; Modelling the group of rigid body motions using Clifford Algebras; Remarkable properties of capillary surfaces; Renormalization Group Flow – geometric flows arising from scaling properties of Quantum Field Theories; Ricci flow - a powerful method for deforming geometric spaces; Stability of constant mean curvature surfaces – the Wente torus (soap bubbles no-one can blow); Symplectic integrators, theory and practice – dealing with differential equations arising in mathematical physics; Triangularizable matrix algebras; Spectral theory of compact self-adjoint operators; Measure theory and Lebesgue integration; Theory of elliptic equations with discontinuous coefficients.

Building 3-dimensional manifolds

Supervisor: Jessica Purcell

Background: A 3-dimensional manifold is a space in which every point has a neighbourhood that is a 3-dimensional ball. Such manifolds can always be built by gluing simple pieces, called compression bodies, along a shared surface boundary. We have learned much information about 3-dimensional manifolds by studying compression bodies and how they glue. There is still much to learn.

Project outlines: There are several possible projects depending on interest and background.

- Topological: Study gluings by investigating groups of boundary identifications, and the curve complex. Determine further information on the Cho-McCullough tree of compression bodies.
- Geometric: Compression bodies usually admit a hyperbolic structure. Investigate properties of geometric structures, such as how they change and how they degenerate.

- Computational: Develop an algorithm to visualize hyperbolic structures on compression bodies. Compute changes in volume based on location in the Cho-McCullough tree.

References:
One reasonable source for more information is Jesse Johnson's notes on Heegaard splittings, available from his website: http://users.math.yale.edu/~jj327/notes.pdf

Geometry of knots and links

Supervisor: Jessica Purcell

Background: A knot can be viewed as a copy of the circle embedded in 3-space. When you remove the knot from space, what is left over is a knot complement. We can often obtain interesting information about knots by studying knot complements. One way to study knot complements is to use geometry. Many knot complements are hyperbolic, meaning they admit a metric with constant negative curvature. In this project, we will study geometric properties of knots.

Project outline: Projects depend on student interest and background.
Geometry of alternating links and Dehn fillings. This reading project would involve working through notes on hyperbolic geometry and knot theory, available from my website. It would include working carefully with many examples, and learning the basic theory.
Geometric properties of classes of links and their parent manifolds. There are a few known classes of links, which have very cool geometry, including 2-bridge links. There are other classes of knots that seem to have interesting geometric properties experimentally, but there are fewer known results. These include (1,1)-knots and links, and some generalizations. This project would involve selecting a class of knots and links, and learning everything we could about their geometry.

References:
Jessica Purcell, Hyperbolic Knot Theory notes, available at:
http://users.monash.edu.au/~jpurcell/papers/hyp-knot-theory.pdf

Orthogonal Latin Squares

Supervisor: Ian Wanless
Background: Latin squares are 2-dimensional permutations. You find them everywhere from sudoku puzzles to Cayley tables of groups. They are used for designing codes for communication and schedules for sporting tournaments.

Here's a challenge for you. Take the court cards (aces, kings, queens and jacks) from a standard pack of cards and arrange them in a 4 x 4 array so that every row and column contains exactly one card of each suit, and exactly one card of each rank (A/K/Q/J). What you have just built is a pair of orthogonal Latin squares. One tells you the suits, and the other tells you the ranks.

In the 18th Century, Euler famously conjectured that there are no k x k orthogonal Latin squares if k = 2 and mod 4. Euler was spectacularly wrong, but we did not know that for more than 150 years, and when the news broke it made the front page of the New York Times!

Project outline: You could learn a little of the history and applications, then study recently discovered combinatorial and algebraic techniques for tackling such questions. The goal would be to use these techniques to find a new proof that Euler was correct when k = 6 (even that took more than a century to establish the first time round, but we have much better techniques now!).

Reference: * Laywine and Mullen, Discrete Mathematics Using Latin Squares.

Permutation Polynomials

Supervisor: Ian Wanless

Background: Every function from a finite field to itself can be represented by some polynomial. If the function is bijection then the corresponding polynomial is a permutation polynomial. For example, $x^4 + 3x$ is a permutation polynomial in the field of order 7, because its values are 0, 4, 1, 6, 2, 3, 5.

Project outline: You'd learn some of the theory for testing whether a polynomial produces a permutation. If you want to do some original work, there would be scope, but you would need to be able to do some computations in finite fields (e.g. with the free software GAP).

Reference: * Lidl and Niederreiter, Finite Fields.

How fast do rumours spread?

Supervisors: Nick Wormald

Background: In the beginning, somebody starts a rumour by telling a friend. Each day, everybody who knows the rumour calls one of their friends at random and tells them the rumour. How long do we expect it to take before everybody knows the rumour?

Project outline: The answer to the question above depends on the structure of the 'network' of friends. We can ask which network structures of n friends are likely to take longest for the rumour to spread. We can ask if the rules of spreading are changed, how does it affect the spread time. In considering these questions you will learn about simple probability processes, some graph theory, and possible something about random graphs.

Skills required: some familiarity with probability is highly desirable.

Random graphs

Supervisors: Nick Wormald

A graph, or network, is a set of points called 'vertices' and links between the nodes called 'edges'. Random graphs involve some random choices in determining the edges. Random graph theory was first used last century to show the existence of graphs with special hard-to-construct properties. Since then, it has been used to answer many questions arising in computer science, as well as generating questions of intrinsic interest. These questions usually ask for the properties of a large random graph. Sometimes the problem is to make a random graph in some weird way so that it is likely to have some desirable property.

Besides learning about techniques used to study random graphs, there are many different unsolved problems that can be focussed on. Here is one. First, some notation. The 'degree' of a vertex is the number of other vertices linked to it. A Hamilton path is a path going along edges, through all vertices in the graph, without repeating any vertices. The question is, if you take a random graph G with all vertices of degree 3 and 4, how likely is G to have a Hamilton path?

Randomly evolving networks

Supervisors: Nick Wormald

Background: Random networks theory was invented to show that certain special networks exist, but lately has been used to answer many problems from computer science. Some of these problems ask for the properties of a network that grows randomly.

Project outline: A network is a set of ‘nodes’ and ‘links’. Dynamic networks change with time. Random networks involve some random choices. Even with simple-sounding rules for their evolution, randomly evolving networks can have some surprising properties. One of the well-known examples of these regards the degree of the nodes, i.e. the numbers of links they have to other nodes. Another property was popularised under ‘six degrees of separation’: there is usually a ‘short’ path between any two nodes. The aim of the project is to look at some new kinds of networks and learn how to investigate their properties.

Skills required: some familiarity with probability is highly desirable.

APPLIED AND COMPUTATIONAL MATHEMATICS

Monotone schemes for flows in porous media

Supervisor: Jérôme Droniou

Background: Flows in porous media describe phenomena that occur, in particular, during underground oil extraction or leakage in nuclear waste disposals. The set of (partial differential) equations describing these phenomena are too complex to be exactly solved. Numerical simulation is thus the only method to predict the behaviours of fluids in this context, and industrial applications require to use robust methods which results can be trusted. Robustness of a method is assessed through several characteristics, one of them being its monotony, i.e. its ability to provide approximate solutions that respects physical bounds. If the unknown is a concentration, for example, then it should remain between 0 and 1. However, some numerical methods happen to give approximate solutions that do not respect these bounds. This is then quite confusing since it's not obvious what to do with a negative concentration...

Project Outline: In this project, we will study some numerical methods to approximate diffusion equations involved in flows in porous media. We will in particular concentrate on methods which are monotone, trying to understand why they are monotone and to assess their efficiency. Depending on the preference of the student, the project might concentrate on the theoretical analysis of the methods and/or on practical implementations.

Skills required: For subject revolving around theoretical analysis, good basis of multivariable calculus and elliptic partial differential equations.

For subject involving practical implementations, good knowledge of a scientific programming language is necessary. FORTRAN is preferred (an existing code will then be provided but some modification will be necessary), but Matlab can also be an alternative.

To discuss details of possible problems contact Jerome Droniou:jerome.droniou@monash.edu

Numerical methods for flows in porous media

Supervisor: Jérôme Droniou

Background: Flows in porous media describe phenomena that occur, in particular, during underground oil extraction or leakage in nuclear waste disposals. The set of (partial differential) equations describing these phenomena are too complex to be exactly solved. Numerical simulation is thus the only method to predict the behaviours of fluids in this context, and industrial applications require to use robust methods which results can be trusted. Robustness of a method is assessed through several characteristics, for example its capacity to provide discrete energy estimates and/or to respect the expected physical bounds of the solution (e.g. that a concentration should remain between 0 and 1).

Practical applications also require the method to be implementable on a very generic grid, with possibly distorted cells.

Project outline: In this project, we will study some numerical methods to approximate diffusion equations involved in flows in porous media. We will concentrate on some particular aspects, trying to understand which method preserves which property. Depending on the preference of the student, the project might be centred around the theoretical analysis of the numerical schemes and/or their practical implementations.

Skills required: For subject revolving around theoretical analysis, good basis of multivariable calculus and elliptic partial differential equations.

For subject involving practical implementations, good knowledge of a scientific programming language is necessary. FORTRAN is preferred (an existing code will then be provided but some modification will be necessary), but MATLAB can also be an alternative.

To discuss details of possible problems contact Jerome Droniou:jerome.droniou@monash.edu

Modelling traffic on road networks using stochastic cellular automata

Supervisor: Tim Garoni
Co-supervisor: TBA

Background: In addition to being of significant real-world importance, the study of vehicular traffic has played an increasingly significant role in the development of non-equilibrium statistical mechanics over recent years. The use of cellular automata has been particularly popular within the statistical mechanics community, ever since the introduction of the Nagel-Schreckenberg (NaSch) model. A cellular automaton is a model which is discrete in time, space, and state variables, whose dynamical rules are local. The NaSch model is generally considered to be the minimal model for traffic on freeways. A huge literature dealing with various extensions of the NaSch model has evolved since its first appearance, and our understanding of freeway traffic has benefited greatly as a result. The behaviour of traffic networks, by contrast, is far less well understood. The NetNaSch model, recently developed by our group, provides a simple way to extend the NaSch model to arbitrary urban road networks.

Project Outline: The basic code for the NetNaSch model is already written (in C++), and this project would involve modifying certain subroutines of the code, using the code to run simulations, and then performing statistical analyses of the simulated data. This project would therefore provide training and hands-on experience in scientific computing, mathematical modelling, simulation, and statistical analysis, as well as in relevant aspects of statistical mechanics. Some previous experience in programming would be very useful, but prior C++ experience is not essential. Some specific questions that could be studied are:

- Compare the efficiency of different traffic signal systems
- Study the effect of increased tram priorities at traffic signals
- Study fluctuations of key traffic observables, such as travel time, speed, and queue length, under different signal-control strategies
- Study parameter sensitivity, and how it relates to fluctuations in key traffic observables

Skills required: Some experience with programming would be helpful, but not essential.

References:
J. de Gier, T. Garoni and O. Rojas, “Traffic flow on realistic road networks with adaptive traffic lights”, J. Stat. Mech. P04008 (2011).
A. Schadschneider, D. Chowdhury and K. Nishinari, “Stochastic transport in complex systems: From molecules to vehicles”, Elsevier (2010).

Accurate reaction simulations in the presence of diffusion and advection

Supervisor: Mark Flegg
Co-supervisor: TBA

Description: Models for simulating chemical systems of diffusing and reacting molecules have been well established. These models break down if the molecules in the system are under the influence of external forces (such as electrostatic forces). In this project we will be looking at accurate state-of-the-art algorithms that are used to simulate reaction-diffusion systems and improving them to work in the presence of weak advection.

Skills required: Experience with numerical solutions of PDEs. Good programming skills (e.g. MATLAB).

Topics in Geological Fluid Dynamics

Supervisor: Anja Slim

Project outline: My currently active projects keep evolving; please email me (anja.slim@monash.edu) with a copy of your transcript and a brief description of your interests and what you hope to get from a research project and I will send you a summary of possible projects. If there is a particular topic that fascinates you, let me know and I would be happy to try to come up with a relevant project.

Prerequisites: MTH3360 and competence in Matlab

Plasma Physics

Supervisor: Paul Cally

Project Outline: Plasma Physics is the science of ionized gases, in which large scale magnetic or electric fields play an important role in determining how they move. This is relevant to most of the baryonic matter in the universe. This project will explore some of the basic concepts and approximations of plasma physics, starting with Magnetohydrodynamics at one extreme, and particle orbit theory at the other. We shall also explore such concepts as Debye shielding and the plasma frequency.

Skills required: A good understanding of vector field theory; some knowledge of complex variable and Fourier theory.

Numerical analysis of optimal control problems

Supervisor: Janosch Rieger

Outline: Optimal control problems arise in an abundance of applications. An aircraft controller needs to stabilise the vessel with minimal effort. An airline needs to know the optimal flight path from one location to another in terms of profit. An investor needs to determine an optimal investment strategy over time. Almost every problem in which optimal behaviour over a finite or infinite time interval is sought can be recast as an optimal control problem.

The solution of optimal control problems is, in general, difficult. Only for very simple systems, it is possible to write down a closed form solution. For that reason, numerical methods for optimal control problems are needed. These can be divided into *first-optimise-then-discretise* approaches, which exploit optimality criteria such as the Pontryagin maximum principle, and *first-discretise-then-optimise* approaches, which discretise the problem first and then look for an optimal solution of the resulting finite-dimensional problem.

The aim of this project is to review some literature such as

- Dontchev, Hager; The Euler approximation in state constrained optimal control; Math. Comp. 70 (2001), no. 233, 173-203
- Dontchev, Hager, Veliov; Second-order Runge-Kutta approximations in control constrained optimal control; SIAM J. Numer. Anal. 38 (2000), no. 1, 202-226

on the latter type of method, work out the theoretical details, implement the corresponding solvers in Matlab and test the code on model problems such as the nonlinear inverted pendulum.

Prerequisites: For this project, a solid background in ordinary differential equations and programming skills are required. Basic knowledge on numerical methods for ordinary differential equations is recommended.

Viability techniques in computational control theory

Supervisor: Janosch Rieger

Outline: A typical task from control theory is to steer a number of vehicles from given initial positions to their respective targets, preventing collisions with each other and obstacles in the environment. One technique for treating collision avoidance is the concept of a viability kernel, which is the set of all states of the ensemble of vehicles from which future collisions can definitely be avoided. Knowing this set reduces the original problem, which is computationally expensive, to the very manageable problem of staying within the viability kernel while approaching the targets.

Computing the viability kernel is a nontrivial task, and several different approaches to the problem have been proposed. Many ideas originate from the engineering community, because the algorithms discussed in the mathematical community do not suit the needs of particular applications.

The first aim of this project is to review some of this literature, in particular recent work by John Lygeros and Ian Mitchell.
The second aim is to implement the most promising algorithm for viability kernel computation from this literature and apply it to the model of a racing car on a complex track.

Prerequisites: For this project, a solid background in ordinary differential equations and programming skills are required. Basic knowledge on numerical methods for ordinary differential equations is recommended.

Cyclic flow shop scheduling problem

Supervisor: Andreas Ernst
Co-supervisor: Atabak Elmi

Cyclic scheduling has been an effective scheduling method for the repetitive discrete manufacturing environment. Cyclic or periodic scheduling repetitively produces the smallest set of part types called the minimal part set (MPS) which is proportional to the production requirement. The cyclic flow shop scheduling problem is an extended version of the flow shop scheduling problem, where the jobs are executed repeatedly. The aim of the problem is to minimise the cycle time while sequencing a number of parts (same or multiple types) to be processed repetitively. The parts enter the system via an input device. They are then processed on a number of machines, in sequence, and exit the system via an output device. There is no buffer for intermediate storage between machines, and hence, each machine can hold only one part at a time.

Such scheduling problems sit at the intersection of pure (discrete) mathematics and applied mathematics (operations research). The project will give the student an understanding of how these

problems are modelled using graph structures and integer linear programming methods. Based on this a heuristic algorithm will be developed for the problem.
Pre-requisites: Knowledge in programming, linear algebra is desirable.

The exact project can be decided in consultation with the supervisors based on your experience, skills and interest.

References:

1. Bożejko, Wojciech, Mariusz Uchroński, and Mieczysław Wodecki. "Parallel metaheuristics for the cyclic flow shop scheduling problem." Computers & Industrial Engineering 95 (2016): 156-163.
2. Elmi, Atabak, and Seyda Topaloglu. "Multi-degree cyclic flow shop robotic cell scheduling problem with multiple robots." International Journal of Computer Integrated Manufacturing30.8 (2017): 805-821.

Train plan scheduling

Supervisor: Andreas Ernst
Co-supervisor: Atabak Elmi

Background: We are working with Pacific National, Australia’s largest rail company, on advanced optimisation methods for their logistics. Planning and scheduling the operations for train and container movements is complex. This project will look at one aspect of this real world application of operations research.

Project: We consider to model to find optimal origin-destination (O-D) itineraries for all weekly train services across Australia. An O-D itinerary is a chain of paths leading from an origin to a destination of a service matching the departure/arrival time requirements. Each path between two nodes is a time slot for rail link between two stations. This project consider to determine the appropriate itineraries and relevant paths for each service. The optimal solutions of this model will be the best way of scheduling paths for all of the services. In addition the schedule must assign wagons and locomotives appropriately to create train trips for each path that is used.

In this project the student will introduce a mathematical model to represent the characteristics of the problem employing ideas from graph theory and optimisation. The exact project can be decided in consultation with the supervisors based on your experience, skills and interest. With either a stronger focus on mathematical modelling of the real world problem, or a focus on optimisation methods for an abstracted version of the problem.

Pre-requisites: Knowledge in programming, linear algebra and graph theory are desirable.
References:

1. Ghoseiri, Keivan, Ferenc Szidarovszky, and Mohammad Jawad Asgharpour. "A multi-objective train scheduling model and solution." Transportation research part B: Methodological38.10 (2004): 927-952.
2. Singh, Gaurav, Andreas T. Ernst, Matthew Baxter, and David Sier. "Rail schedule optimisation in the hunter valley coal chain." RAIRO-Operations Research 49, no. 2 (2015): 413-434.
3. Petering, Matthew EH, Mojtaba Heydar, and Dietrich R. Bergmann. "Mixed-integer programming for railway capacity analysis and cyclic, combined train timetabling and platforming." Transportation Science 50.3 (2015): 892-909.

Approximating PDEs on evolving geometries

Supervisor: Santiago Badia

Background: The method of lines, which segregates spatial and time discretisation, is ubiquitous in the approximation of transient partial differential equations because of its simplicity. However, this approach is not suitable for domains that evolve in time. Besides, discretisation techniques for partial differential equations, e.g., finite elements, require a geometry discretisation. It involves unstructured mesh generation, a non-automatic process that requires human intervention and amounts to 80% of the simulation time. The idea to have 4D (spacetime) body-fitted mesh generators for complex evolving domains is unrealistic.
As a result of the standard time integration techniques and the complications related to geometrical discretisations, the accurate approximation of PDEs on evolving geometries (e.g., problems with interfaces or free boundaries) is a challenging and open problem. On the other hand, many applications of interest, e.g., cell migration in biology, additive manufacturing of metals, or fluid-structure interaction in aerospace vehicles, involve such simulations. Novel mathematical algorithms for these problems are required.

Project Outline: There are different projects that can be considered within this topic, that will depend on the background and interests of the student. One line of research is the development of spacetime variational formulations on non-trivial domains, combined with unfitted geometrical discretisations. Such approach would be well-suited to these problems because they can deal with evolving geometries and reduce the constraints of the geometrical discretisation (the mesh is not enforced to respect the spacetime geometry). Students can focus on numerical analysis, e.g., stability and convergence analysis of the methods, the implementation of the formulations in the Gridap project (a Julia library for the discretisation of PDEs using advanced numerical methods), and/or the application of these techniques to applications in biology, manufacturing, etc.

Skills required: Numerical background (numerical differentiation, integration, linear solvers, etc), and ideally some basic knowledge on numerical PDEs. Some programming skills (Julia, Python, or MATLAB).

Neural networks and PDE approximations

Supervisor: Santiago Badia

Background: ReLU deep neural networks have transformed artificial intelligence. New approximation results and their relation with adaptive finite elements are starting to explain their excellent approximability properties. They generate piece-wise linear functions on meshes defined as the intersection of hyper-planes that depend on weights, which can be optimised via stochastic gradient descent. These discretisations have the potential to become a breakthrough in numerical PDEs, as an alternative to adaptive finite elements. However, it is unclear how to deal with complex geometries in such settings; the deep networks produce approximations on n-cubes whereas PDE applications usually involve complex domains. On the other hand, many of the steps in the simulation pipeline (e.g., the solver step) for these discretisations have not even been considered yet.

Project outline: There are different projects that can be considered within this topic, that will depend on the background and interests of the student. One line of research is the development of unfitted tools to generate PDE solvers on general geometries with deep neural network functional discretisation. Students can focus on numerical analysis,
e.g., stability and convergence analysis of the methods and/or the implementation of the formulations in the Gridap project (a Julia library for the discretisation of PDEs using advanced numerical methods) using Flux (machine learning Julia suite).

Skills required: Numerical background (numerical differentiation, integration, linear solvers, etc), and ideally some basic knowledge onn numerical PDEs, neural networks, and data science, and machine learning. Some programming skills (Julia, Python, or MATLAB).

PROBABILITY AND STATISTICS

Markov-chain Monte Carlo methods in Statistical Mechanics

Supervisor: Tim Garoni
Co-supervisor: TBA

Background: Statistical mechanics began life as a branch of mathematical physics, but is now a central paradigm for studying all manner of complex systems, across fields as diverse as physics, chemistry, biology, economics and sociology. An important branch of statistical mechanics concerns discrete models, in which one studies random structures defined on graphs. These studies have significant overlap not only with probability theory, but also combinatorics and computer science. Since models in statistical mechanics are often mathematically intractable, Markov-chain Monte Carlo (MCMC) methods have become an indispensable computational tool in this field.

However, not all Markov chains are created equal, and while two Markov chains may have the same stationary distribution (and therefore approximate the same statistical mechanical model), their rates of convergence to stationarity (and therefore their practical efficiency) can be very different. While the classical theory of Markov chains considers the late-time asymptotics of fixed chains, the relevant asymptotics in statistical mechanics concerns the growth of “mixing times” as the size of the state space becomes large. Quantifying the size of such mixing times, and designing new Markov chains with reduced mixing times, are the central tasks in this field.

Project Outline: The project would start with a review of the required basic background in discrete statistical mechanics, and Markov-chain Monte Carlo; the references listed below being a good place to start. From there, a range of possible projects is available, studying specific classes of Monte Carlo methods for specific classes of statistical-mechanical models. Depending on your interests, the project will involve a combination of both theoretical studies and computer experiments; the focus could range from being largely computational, to entirely theoretical.

Skills required: Programming would be helpful, but not essential.

References:
A. Levin, Y. Peres and E. Wilmer, “Markov Chains and Mixing Times”, available online free (legally) at http://pages.uoregon.edu/dlevin/MARKOV/markovmixing.pdf
Recorded lectures at http://www.msri.org/web/msri/scientific/show/-/event/Wm406 and http://www.msri.org/web/msri/scientific/show/-/event/Wm318
G. Grimmett, “Probability on Graphs”, Cambridge University Press (2010), available online free (legally) at http://www.statslab.cam.ac.uk/~grg/books/pgs.html.

Probability theory

Supervisor: Kais Hamza
Co-supervisor: TBA

The exact project needs are to be decided in consultation with Kais Hamza taking into consideration your personal interests and mathematical background.

Markov chains

Supervisor: Kais Hamza

Co-supervisor: TBA

The exact project needs are to be decided in consultation with Kais Hamza taking into consideration your personal interests and mathematical background.

Stylometry – resolving questions of authorship

Supervisor: Jonathan Keith
Co-supervisor: TBA

Project outline: The science of stylometry is concerned with quantifying the characteristic components of an author's writing style. In this project, Bayesian sequence segmentation techniques will be used to segment a stream of text into parts that may have been written by different authors. The techniques will be used both to resolve questions of who composed certain historical documents, and to detect parts of historical documents that may have been inserted or modified by a different author subsequent to the original composition.

Financial mathematics

Supervisor: Fima Klebaner
Co-supervisor: Kais Hamza

The exact project needs are to be decided in consultation with Fima Klebaner, taking into consideration your personal interests and mathematical background.

Stochastic differential equations

Supervisor: Fima Klebaner
Co-supervisor: Kais Hamza

The exact project needs are to be decided in consultation with Fima Klebaner, taking into consideration your personal interests and mathematical background.

Branching processes

Supervisor: Fima Klebaner
Co-supervisor: Kais Hamza

The exact project needs are to be decided in consultation with Fima Klebaner, taking into consideration your personal interests and mathematical background.

Random walks and electrical resistance on graphs .

Supervisor: Greg Markowsky

Project outline: There is a beautiful and well-known connection between the properties of electricity in a circuit and the properties of a walker moving randomly amongst the vertices of a graph. We will go deeply into this topic. An excellent source concerning this is Doyle and Snell's book, given in the references.

Reading:
• Doyle, P. and Snell, J. 1992 Random Walks and Electric Networks, MAA.

• Biggs, N. 1993 Potential theory on distance-regular graphs. Combinatorics, Probability and Computing 2, p. 243-255.
• Biggs, N. 1997 Algebraic Potential Theory on Graphs, Bulletin of the London Mathematical Society, Volume 29, Number 6, p. 641-682.

Probability Theory

Supervisor: Greg Markowsky
Co-supervisor: TBA

Project outline: The exact project needs are to be decided in consultation with Greg Markowsky taking into consideration your personal interests and mathematical background.

Random walks

Supervisor: Kais Hamza
Co-supervisor: Andrea Collevecchio

The exact project needs are to be decided in consultation with Aidan Sudbury, taking into consideration your personal interests and mathematical background.

Reinforced random walks and preferential attachment

Supervisor: Andrea Collevecchio

We study stochastic processes which heavily depend on their past. These processes can be used to build random networks (e.g. preferential attachment schemes), or simply describe a possible "conservative" way of exploring a fixed graph (reinforced random walks). In order to study these objects we first study generalisations of Polya's Urn.
References Pemantle: A survey of random processes with reinforcement.

MATHEMATICAL BIOLOGY AND BIOINFORMATICS

The influence of DNA sequence alignments on gene discovery

Supervisor: Jonathan Keith
Co-supervisor: TBA

Project outline: Sequence alignments involve placing two or more DNA sequences into a matrix in such a way that columns of the matrix contain bases that evolved from a common ancestor. Sequence alignments play an important role in the discovery of new genes. However, errors in sequence alignments can potentially hinder such methods. This project will explore the effect of errors in sequence alignments on a Bayesian method for discovering non-protein-coding genes.

Non-protein-coding genes responsible for disease

Supervisor: Jonathan Keith
Co-supervisor: TBA

Project Outline: Several recent studies have identified disease susceptibility loci in parts of the human genome that do not code for proteins. Evidence is mounting that non-protein coding genes play an important role in complex genetic diseases. This project will use Bayesian segmentation algorithms to combine data on seven common human genetic diseases with data from multiple species comparisons to identify non-protein-coding genes responsible for disease.

Bayesian algorithms for whole-genome association studies

Supervisor: Jonathan Keith
Co-supervisor: TBA

Project outline: Association studies aim to identify genetic loci that are associated with a particular disease or phenotype. Increasingly, such studies are being performed on a whole-genome scale, and are providing exciting new insights into the causes of complex genetic diseases. This project will develop advanced Bayesian statistical methods for analysing data from such studies.

Effective numerical methods for simulating chemical reaction systems

Supervisor: Tianhai Tian
In recent years, the stochastic simulation algorithm (SSA) has been successfully applied for simulating genetic reactions in which the molecular population of a critical reactant species is relatively small. The bottleneck in the application of the SSA arises from the huge amount of computing time, which is an obstacle to develop stochastic models for complex biological systems. This challenge has stimulated the development of effective methods for simulating large-scale stochastic systems during the last 10 years. This project will design efficient simulation methods for simulating chemical reaction systems.

Skill required: Computer programming skills in MATALAB or another major language.
Please contact: Tianhai Tian, tianhai.tian@monash.edu.au

Mathematical modelling of cell signalling pathways

Supervisor: Tianhai Tian

Cell signalling pathway communicates signal from the growth factor receptors on the cell surface to effector molecules located inside the cell. In recent years a number of mathematical models have been designed to study the cell signalling pathway. One of the major challenges in the modelling of signalling pathways is the lack of experimental data for determining the rate constants in mathematical models. The advances in high-throughout technologies such as microarray and proteomics have produced an unprecedented amount of genome-scale data. This project will develop mathematical models for the cell signalling pathways based on omics datasets.

Skill required: basic knowledge in biology. Computer programming skills in MATALAB or another major language.
Please email Tianhai Tian: tianhai.tian@monash.edu.au

Stochastic modelling of genetic regulatory networks

Supervisor: Tianhai Tian

Recent advances in experimental genetics have shown that gene expression is governed by stochastic process. Randomness in transcription and translation leads to cell-to-cell variations at mRNA and protein levels. The experimental discoveries have stimulated an increasing number of mathematical modelling studies to discover the origin and consequences of stochastics in gene expression. This project will develop stochastic models for a particular genetic regulatory network. The candidate networks include the lactose operon in E coli, the p53 gene network, the NF-kB gene network, and the network controlling the differentiation of stem cells.

Skill required: basic knowledge in biology. Computer programming skills in MATALAB or another major language.

Please email Tianhai Tian: tianhai.tian@monash.edu.au

Mathematical modelling of telomere length regulation in ageing research

Supervisor: Tianhai Tian
A long-standing proposition in ageing study is the "telomere length hypothesis". Telomeres with shortened length are related to various age- and inactivity-related diseases including cancer. Extensive experimental studies have been carried out since the discovery of telomere in the 1970s, leading to the Nobel Prize in Physiology/Medicine for 2009. This project will develop new mathematical models to explore the molecular mechanisms governing telomere length regulation and elucidate the function of telomere in determining cell fate.

Skill required: basic knowledge in biology. Computer programming skills in MATALAB or another major language.

Please email Tianhai Tian: tianhai.tian@monash.edu.au

-