Long-Term Capital Market Assumptions
Public Equities

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Abstract

This document presents our Capital Market Assumptions (CMA) for public equities, which outline the expected returns, volatility, and correlation estimates of different equity markets in the next 10 years. In order to forecast real equity returns, we consider two widely used approaches including cyclically adjusted price-to-earnings ratio (CAPE) and building blocks, and rely on backtesting to select the best model for each market. We also take into account effects of inflation and foreign exchange rate changes to reflect the real returns to investors and the increasing trend of cross-border investments.

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1 Introduction

To assist investors in developing their long-term strategic asset allocation, we present in this paper our capital market assumptions (CMA) for public equity, that is the forward estimates of returns, volatilities, and correlations across different equity markets based on 10-year time horizon. Such forward-looking estimates are useful to develop long-term strategic asset allocations and build diversified portfolios as they are key inputs for the modern portfolio theory (Markowitz, 1952). Within the context of this paper, we particularly focus on equity markets in 23 developed and 11 emerging countries/regions, and three multi-country markets as follows:

- Developed markets: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Israel, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, United Kingdom, United States.
- Emerging markets: Brazil, China, India, Indonesia, Malaysia, Mexico, Poland, South Africa, South Korea, Taiwan, Turkey.
- Multi-country markets: Emerging Markets, Europe, World.

We first present details of theoretical background and methodologies used to forecast real equity returns in Section 2. When it come to forecasting, there are a number of alternative methods available. Our modelling choices reflect what we believe are the best to adequately capture fundamentals while avoiding unnecessary complexity. Thus, we adopt two most commonly used approaches namely CAPE and building blocks. While the CAPE approach solely makes use of the Campbell and Shiller's 1998 cyclically adjusted price-to-earnings ratio, the building block approach forecasts each component of equity returns including income returns, earnings growth and valuation change. Using backtesting, we compare the performance of each model if it had been used in the past and select the model that outperforms. Our assumptions on inflation, real foreign exchange rates, and volatility and correlation are respectively presented in Sections 3, 4, and 5. We also discuss the implications of arithmetic and geometric average returns in Section 6.

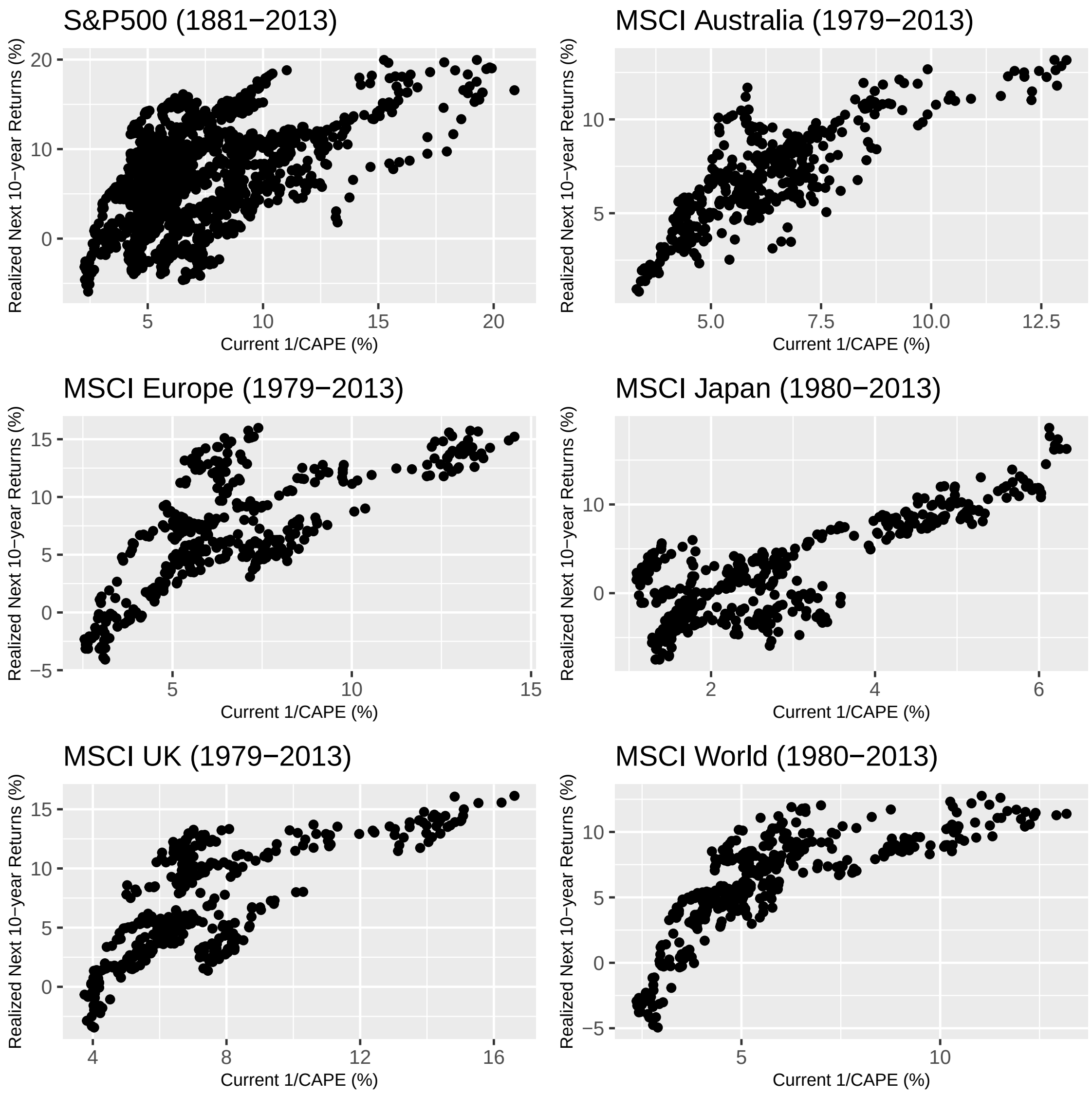
We note that while we provide point estimates, they come with significant uncertainty. Thus, our estimates are best understood as forecasts of what believe are the central tendencies of future returns and risks. Our backtesting results presented in the main text and also in the Appendices on the one hand assess the accuracy of our models, on the other hand provide some ideas about the uncertainty associated with the current point forecasts. Throughout the document, we also describe in details the data sources so that readers, if desired, could not only replicate our models but also modify them to reflect their own views.

2 Real Equity Returns

2.1 CAPE Approach

Since its introduction by Campbell and Shiller (1998), CAPE, or cyclically adjusted price-to-earnings ratio (P/E), has become one of the most popular metrics used to forecast future equity returns. This stems from the significant empirical reverse relation between the current level of the ratio and the real total return of the market over the subsequent decade. A high CAPE ratio has been associated with below-average (above-average) 10-year-ahead stock returns. The relation holds not only in the U.S. but also in various non-US markets as shown in Figure 1.

Figure 1: 1/CAPE vs. Future Real 10-Year Return (Annualized)



Philips (1999) discusses the theoretical background on why valuation ratios are predictive of long-run equity returns. While Campbell and Shiller's CAPE is arguably the most popular valuation ratio, prior studies also consider alternatives such as price-to-book, price-to-cash flow, or price-to-sales (Philips and Ural, 2016), or alternative measures of earnings in CAPE (Siegel, 2016). Since the conclusions from these studies are mixed and we also consider in our CMA non-US markets where data availability is limited, we stick to the conventional CAPE measure, which is the ratio of an equity index's real price level to the 10-year average of its real earnings:

$$CAPE = \frac{P_t^r}{\frac{1}{120}\sum_{i=1}^{120} EPS_{t-i}^r} \tag{1}$$

where P_t^r is the real price of the equity index at the end of month t, and EPS_{t-i}^r is the real earnings per share at the end of month $t-i$.

The forecasting model of the next 10-year returns using 1/CAPE can be specified as:

$$R_{t\to t+120}^r = \alpha + \beta \times \frac{1}{CAPE_t} + \epsilon_t \tag{2}$$

The coefficients α and β are conventionally estimated using historical data (e.g., Davis, Aliaga-Díaz, Ahluwalia, and Tolani, 2018). However, in our particular context, such estimations might be significantly biased due to short history of data in non-US markets (especially when backtesting is considered in Section 2.3). Thus, we adopt the most simple form of the model, that is, assuming $\alpha = 0$, and $\beta = 1$. While this model choice seems arbitrary, it avoids unnecessary complexity and is also supported by the U.S. data where we find an insignificant α and a β of 0.91 using a long-history of the S&P500 index from 1871 to 2023.[1]

2.2 Building Block Approach

From investors' perspective, equity returns can be broken into income return (Inc) and capital gain (Cg). Income return is distributed to investors through dividends and reinvestment of dividends, whereas capital gain is distributed through price appreciation. Capital gain is the sum of real capital gain (Cg^r) and inflation (π). The decomposition can be formulated as:

$$R_t = Inc_t + [(1+\pi_t) \times (1 + Cg_t^r) - 1] \tag{3}$$

The real capital gain component (Cg^r) can be further broken into real EPS growth

[1]The data are obtained from Professor Robert Shiller's website, at `http://www.econ.yale.edu/ /~shiller/data.htm`

(g_{EPS^r}) and growth in P/E ratio ($g_{P/E}$):

$$\begin{aligned} Cg_t^r &= \frac{P_t^r}{P_{t-1}^r} - 1 \\ &= \frac{P_t^r/EPS_t^r}{P_{t-1}^r/EPS_{t-1}^r} \times \frac{EPS_t^r}{EPS_{t-1}^r} - 1 \\ &= (1 + g_{P/E,t}) \times (1 + g_{EPS^r,t}) - 1 \end{aligned} \tag{4}$$

Taken together,

$$R_t = Inc_t + [(1 + \pi_t) \times (1 + g_{P/E,t}) \times (1 + g_{EPS^r,t}) - 1] \tag{5}$$

Since the geometric interactions between terms in the model are relatively small, real equity returns can eventually be approximated as sum of income returns, growth in real EPS, growth in P/E ratio which is commonly referred as valuation change:

$$R_t^r \approx Inc_t + g_{P/E,t} + g_{EPS^r,t} \tag{6}$$

Based on the described framework, we respectively forecast each component (i.e., building block) to arrive at the final projected real equity return in the next 10 years.

Income Returns Forecast

Exploiting the high autocorrelation nature of dividends (e.g., Balduzzi, Bertola, and Foresi, 1995), we model the income return component (Inc) as the exponentially weighted average of 10-year monthly dividend yields with a half-life of five years. Monthly dividend yields are computed as the ratio between trailing 12-month dividend per share and the current stock price. For simplicity, we omit the reinvestment return component in our income return projection due to its insignificant magnitude.[2]

It is also common in the literature to use the most recent trailing 12-month dividend per share as the forecast of dividend yield in the next 10 years for the U.S. equity market (e.g., Grinold, Kroner, and Siegel, 2011). However, as we also consider a large number of non-US markets, we opt for the 10-year average to smooth out the anomalies which are frequent especially in emerging markets.

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[2]For example, Ibbotson and Chen (2003) show that the reinvestment returns in the U.S. over the 1926-2000 period is only about 0.2%.

Real EPS Growth Forecast

To estimate the growth in real EPS, we first calculate the annualized 10-year EPS growth from historical data as:

$$g_{EPS^r,t} = \left(\frac{EPS^r_t}{EPS^r_{t-120}}\right)^{\frac{1}{10}} - 1 \tag{7}$$

where EPS^r_t is the real earnings per share at the end of month t, and EPS^r_{t-120} is the real earnings per share 10 years before. We consider both parametric and non-parametric approaches in estimating the real EPS growth. Specifically, for the non-parametric approach, we build on the work of Chan, Karceski, and Lakonishok (2003) who show that there is no persistence in long-term earnings growth beyond chance. As a result, it is reasonable to assume that real EPS growth in the next 10 years would equal its historical median. For the parametric approach, we exploit the stationary nature of g_{EPS^r} to model the time series using an Autoregressive Moving Average ($ARMA(p, q)$) model:

$$g_{EPS^r,t} = \mu + \sum_{i=1}^{p} \phi_i g_{EPS^r,t-i} + \sum_{i=1}^{q} \theta_i \epsilon_{t-i} + \epsilon_t \tag{8}$$

The ARMA orders (p, q) are chosen so that the model residuals are not distinguishable from a white noise series.[3] The final set of these parameters for each country is presented in Appendix A. To satisfy the requirement of an ARMA model, we only apply the model to countries/regions for which $g_{EPS^r,t}$ are stationary.

The empirical results show that the two methods yield similar results, as the absolute differences between them exhibit cross-sectional mean and median of 0.7%, and a maximum of 1.8%.[4] Ultimately, to reconcile the differences, we take the average of the two values as the final EPS growth forecast in the next 10-year. For equity indexes whose $g_{EPS^r,t}$ do not satisfy the stationarity requirement, we only use the historical median instead.

Valuation Change Forecast

To forecast the valuation change component ($g_{P/E}$), instead of using the standard price-to-earnings (P/E) ratio, we employ CAPE that uses the ten year average of inflation adjusted earnings as oppose to single year earnings (Campbell and Shiller, 1998). Using a long term average helps to smooth out the short-term noises, rendering CAPE better suited for long-term forecast. Our first step in estimating the change in equity valuation is to calculate a "fair" value of the CAPE ratio, i.e., the value that CAPE would eventually revert to

[3]The stationarity is accessed based on the Augmented Dickey-Fuller model with constant. The number of lags in the model is chosen based on the Akaike Information Criterion where the maximum number of lags to be considered is 12.

[4]Estimations are as of September 30, 2023 based on 34 MSCI country indexes.

in the long run. The fair CAPE (denoted as $CAPE_t^*$) is computed as the exponentially weighted average of monthly CAPE over the last 20 years with a half-life of 10 years. In the next step, we assume that the current $CAPE_t$ fully reverts to its fair value in 20 years. Therefore, the average annual change in valuation is calculated as:

$$Valuation\,Change_t\,(g_{P/E,t}) = \left(\frac{CAPE_t^*}{CAPE_t}\right)^{\frac{1}{20}} - 1 \tag{9}$$

We then take the sum of income return, real EPS growth, and valuation change forecasts to obtain the real equity return forecast.

Due to tax benefits and financial flexibility, share repurchases (also known as buybacks) are increasingly used by firms to distribute cash to shareholders apart from dividend. According to Zeng and Luk (2020), the total amount of buybacks has exceed the cash dividends paid by U.S. firms since 1997. A decomposition of equity returns that explicitly incorporate buybacks can be expressed as:

$$R_t = \frac{D_t}{P_t} - \Delta Shares_t + g_{E^r,t} + g_{P/E,t} + \pi_t \tag{10}$$

where $\frac{D_t}{P_t}$ is the dividend yield, $-\Delta Shares$ is the percentage change in the number of shares outstanding which reflects the net effect of outstanding shares repurchased and new shares issued, and $g_{E^r,t}$ is the real earnings (not earnings per share) growth over the period. Thus, an alternative method is forecasting $\Delta Shares$ and g_{E^r} in place of g_{EPS^r} as in model (6).[5] However, we do not opt for this option due to two reasons. First, advocates of this approach usually rely on real GDP growth to forecast real earnings growth under the assumption that the two values should be equal. This is because if corporate profits grow faster than GDP, they eventually take over the economy, leaving nothing for labor, government, natural resource owners, or other claimants. Meanwhile, if profits grow more slowly than GDP, they eventually disappear and business will have no profit motive to continue operating (Grinold et al., 2011). While this could be true in the very long run, we believe that a 10-year period is not long enough for a country to reach this equilibrium, especially emerging markets. Second, while buybacks have gained popularity in many countries outside the U.S., empirical data for those markets are very limited. Thus, using the readily available long history data of EPS is certainly a better choice.

[5]It is easy to show that $-\Delta Shares + g_{E^r}$ and g_{EPS^r} are mathematically equivalent. The term $-\Delta Shares$ has a negative sign because a decrease of the number of shares outstanding adds to returns and vice versa.

2.3 Backtesting: 1/CAPE vs. Building Blocks

Backtesting is essential to evaluate the performance of a forecasting model. In this section, we assess how each approach presented above would have performed if it had been used in the past. Specifically, at the end of each month we estimate the predicted 10-year real equity returns using historical data up to that month and compare with the realized values. We utilize two metrics that are widely used when evaluating the time series forecasting models, including RMSE (Root Mean Squared Error) and MAE (Mean Absolute Error) which are computed as follows:

$$\begin{aligned} RMSE &= \sqrt{\frac{1}{N}\sum_{t=1}^{N}(Actual_t - Predicted_t)^2} \\ MAE &= \frac{1}{N}\sum_{t=1}^{N}|Actual_t - Predicted_t| \end{aligned} \tag{11}$$

We source monthly equity data, including price and earnings per share (EPS) from MSCI. We calculate real price (P^r) and real EPS (EPS^r) at any month t using the following formulas:

$$\begin{aligned} P^r_{t-k} &= P_{t-k} \times \frac{CPI_t}{CPI_{t-k}} \\ EPS^r_{t-k} &= EPS_{t-k} \times \frac{CPI_t}{CPI_{t-k}} \end{aligned} \tag{12}$$

where $k \in \{0, 1, ..., t-1\}$, and CPI is the consumer price index of the country/region where the asset is based.

For consumer price index, since we are unable to find any single data source that covers all countries in our CMA, we compile data from different sources including Bloomberg, Federal Reserve Economic Data (Fred), and OECD Data. When a country presents in multiple data sources, we select one with the longer history. It is common that by the time we update our CMA, CPI data for some countries are not yet available due the lag in reporting. In that case, we extrapolate the data using the most recent CPI.[6] Finally, we adjust the CPI series for seasonality using the X12 method of the US Census Bureau.

We require at least 60 observations of realized 10-year returns to calculate RMSE and MAE. Combined with 30 years of data required to compute long-term CAPE, an equity index must have at least 45 years of historical data to be included in the backtest. Out of 37 indexes under consideration, only 17 satisfy this condition.

Table 1 show that in spite of its simplicity, the CAPE model surprisingly outperforms in 10 out of 17 indexes, while the more complicated building block model outperforms in

[6] The lag is usually one month, thus does not significantly affect our estimations.

5. Denmark and Japan exhibit mixed results as the two criteria disagree with one another.

Table 1: Backtesting comparisons between CAPE and building blocks

A model is defined as outperforming if its RMSE and MAE are smaller than that of the other, otherwise the result is marked as "Mixed". Numbers are presented in percentage. Results are based on the estimations as of September 30, 2023.

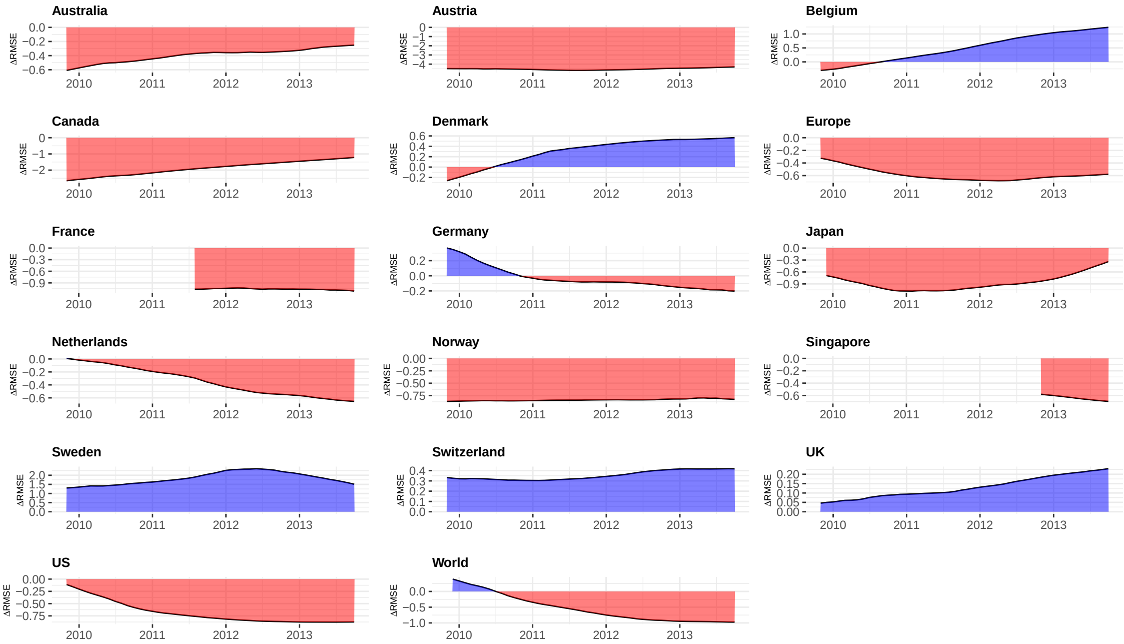
	RMSE (%)		MAE (%)		Outperforming
Equity Market	1/CAPE	Building Block	1/CAPE	Building Block	Model
Australia	1.1	1.7	1.0	1.4	1/CAPE
Austria	7.2	11.4	6.4	10.8	1/CAPE
Belgium	6.1	5.6	5.8	5.2	Building Block
Canada	1.7	3.9	1.3	3.0	1/CAPE
Denmark	7.3	7.2	6.7	6.9	Mixed
Europe	2.7	3.0	2.3	2.8	1/CAPE
France	1.5	2.5	1.2	2.0	1/CAPE
Germany	2.3	2.1	1.8	1.7	Building Block
Japan	3.5	3.5	3.2	2.8	Mixed
Netherlands	3.3	3.5	2.5	2.9	1/CAPE
Norway	2.4	3.3	2.1	3.0	1/CAPE
Singapore	3.4	4.1	2.8	3.3	1/CAPE
Sweden	3.9	2.8	3.6	2.4	Building Block
Switzerland	2.5	2.1	2.2	1.8	Building Block
UK	3.3	3.1	3.0	2.8	Building Block
US	4.2	4.7	3.5	4.0	1/CAPE
World	2.6	2.9	2.1	2.4	1/CAPE

We next compute the difference in RMSE and MAE of results from the CAPE and building block approaches for 10-year rolling windows and present the results in Panels A and B of Figure 2 respectively. The results show that except for Belgium, Denmark, Sweden, Switzerland and the UK, the CAPE model consistently outperforms the building block model in almost all 10-year backtesting windows.

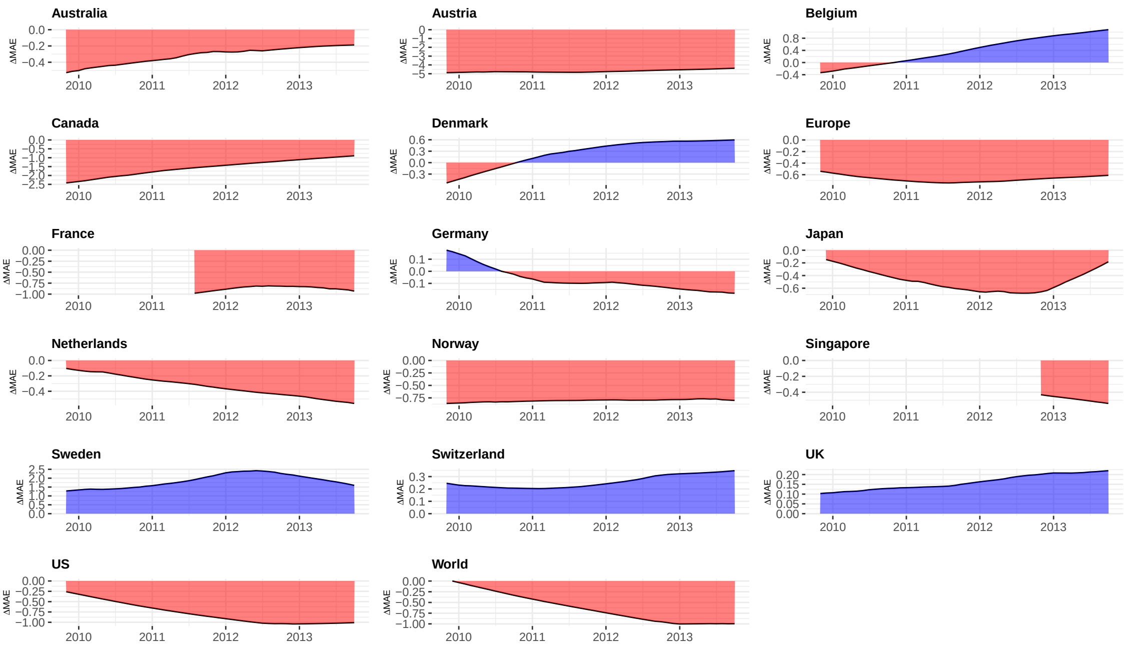
Figure 2: Differences in RMSE and MAE of CAPE and building block models, 10-year rolling windows

This figure presents the differences in RMSE (Panel A) and MAE (Panel B) of the CAPE and building block models in 10-year rolling windows. Negative numbers (represented by red areas) indicate the outperformance of the CAPE model compared to the building block model. Positive numbers (represented by blue areas) indicate the outperformance of the building block model. Numbers are presented in percentage. Estimations are as of September 30, 2023.

Panel A. Difference in RMSE



Panel B. Difference in MAE



2.4 The Next 10-Year Outlook

The backtesting results show that neither the CAPE nor the building block model is a "silver bullet" one-size-fits-all approach for all equity markets. Thus, for markets with backtesting results, we use the model that historically outperforms to forecast the equity returns in the next 10 years, specified as belows:

- CAPE model: Australia, Austria, Canada, Europe, France, Germany, Japan,[7] Netherlands, Norway, Singapore, US Large, World.
- Building block model: Belgium, Denmark, Sweden, Switzerland, UK.

Without backtesting results, it is difficult to choose an appropriate model for the remaining equity markets. Nevertheless, due to the historical outperformance of the CAPE approach in a majority of countries where backtesting is possible, we opt for this model in our main results for those markets. For comparison, we present the alternative projections using the building block model in Appendix B.

In a final step, we compare the projected real returns with the historical returns sourced from the Dimson-Marsh-Staunton database (Dimson, Marsh, and Staunton, 2002) which compiles the yearly equity returns of 35 countries from 1899 to 2022. If our projected returns are either smaller than the 10^{th} or larger than the 90^{th} percentiles, we trim the estimates at these values. For countries that are not included in the DMS database, we use data from MSCI which cover much shorter period albeit are available at higher (monthly) frequency.

Table 2 presents the forecasted real equity returns in the next 10 years for all markets under consideration.

[7] For the case of Japan, while the backtesting results presented in Table 1 are mixed where RMSE supports the CAPE model but MAE is in favor of the building block model, we decide to use the former for two reasons. First, unlike MAE, RMSE is capable of punishing large errors in predictions. Second, the results using 10-year rolling windows are consistently in favor of the CAPE model regardless of whether RMSE or MAE is used.

Table 2: Expected returns vs. historical return distribution

This table presents our forecasted 10-year real equity returns as of September 30, 2023, along with the corresponding historical 10-year returns sourced from the Dimson-Marsh-Staunton (DMS) and MSCI databases. * indicates countries with backtesting results.

Equity Market	Expected Real Returns	Historical Return Distribution						
		Mean	5pct	10pct	Median	90pct	95pct	Start Year
Australia*	6.0	6.7	-0.4	0.8	6.9	12.1	13.0	1900
Austria*	9.8	1.3	-16.5	-11.9	2.7	13.1	15.0	1900
Belgium*	5.0	3.1	-7.4	-4.7	2.8	11.5	14.1	1900
Brazil	9.6	8.0	-5.8	-3.9	8.7	19.1	20.0	1951
Canada*	5.4	5.9	-0.8	1.3	5.6	11.2	12.6	1900
China	9.2	0.5	-16.6	-11.1	1.5	9.2	10.7	1992
Denmark*	5.6	5.7	-0.4	0.3	5.3	12.0	13.4	1900
Emerging Markets	9.1	4.0	-15.5	-7.6	6.5	11.8	12.9	1900
Europe*	5.9	4.5	-5.6	-4.0	4.4	12.0	14.3	1900
Finland	6.0	5.7	-10.9	-6.7	6.7	15.2	18.6	1900
France*	4.6	3.6	-7.2	-4.2	3.2	12.2	15.4	1900
Germany*	7.1	3.7	-12.5	-8.7	3.8	11.4	16.2	1900
Hong Kong	8.9	9.1	1.3	1.9	8.4	15.5	16.7	1963
India	3.3	6.2	-0.7	0.6	5.0	12.8	16.7	1953
Indonesia	5.9	1.4	-12.9	-11.2	1.5	14.4	16.6	1987
Ireland	4.0	4.7	-5.7	-4.1	4.0	13.5	14.9	1900
Israel	7.0	1.1	-4.1	-3.6	0.3	7.0	7.7	1992
Italy	5.7	2.2	-10.9	-9.4	3.2	13.8	14.7	1900
Japan*	4.7	4.4	-18.9	-5.1	6.7	14.6	19.1	1900
Malaysia	8.0	6.1	-2.4	-0.3	6.1	13.3	15.3	1970
Mexico	6.2	11.0	-1.5	-0.0	11.0	21.0	31.9	1969
Netherlands*	4.3	5.3	-4.2	-2.3	4.2	14.5	16.0	1900
New Zealand	4.6	6.2	-0.1	1.3	5.9	11.5	12.3	1900
Norway*	6.5	4.4	-6.3	-3.2	6.0	10.4	11.3	1900
Poland	3.9	-1.1	-7.2	-6.3	-0.7	3.9	4.7	1992
Portugal	4.8	3.9	-10.2	-5.5	4.0	12.4	16.2	1900
Singapore*	9.0	4.9	-0.3	0.5	5.1	9.0	9.8	1966
South Africa	7.4	7.3	-0.5	0.8	7.2	14.1	16.6	1900
South Korea	8.6	10.3	-5.0	-3.0	8.2	26.7	31.0	1963
Spain	7.1	3.6	-9.8	-4.4	3.3	12.3	14.2	1900
Sweden*	10.4	6.2	-5.8	-0.9	6.4	12.4	15.8	1900
Switzerland*	6.0	4.7	-4.6	-2.5	5.1	11.7	13.9	1900
Taiwan	5.1	9.2	-1.0	0.1	8.0	22.3	23.2	1967
Turkey	7.6	1.3	-5.1	-4.2	1.2	7.6	9.8	1987
UK*	5.3	5.8	-2.7	-0.3	5.4	13.2	13.6	1900
US*	3.6	6.4	-2.6	-1.1	6.3	13.4	14.2	1900
World*	4.3	5.3	-1.9	-0.9	5.4	11.8	12.8	1900

3 Inflation

Inflation reflects the change in the price level of goods and services within a country/region over a period of time. Our inflation forecast relies on the historical year-over-year inflation (π_t), computed as:

$$\pi_{i,t} = \frac{CPI_{i,t}}{CPI_{i,t-12}} - 1 \tag{13}$$

where CPI_t is the seasonally adjusted consumer price index of country/region i at the end of month t.

We postulate that inflation would ultimately revert to its fair value in the long run. We believe that this is a valid assumption as economists have widely agreed that while high inflation is harmful, a small amount of inflation can help drive economic growth. As a result, although inflation can be caused by various market factors such as demand-pull, cost-push, or households' inflation expectations, central banks usually intervene to maintain inflation within certain ranges through their monetary policy. For example, the US's Federal Reserve's official target for inflation is 2%, and the Reserve Bank of Australia set the target between 2 and 3% on average in the long run.

We compute the fair value of inflation for each country (denoted as $\pi^*_{i,t}$) by employing the exponentially weighted average of monthly year-over-year inflation over the last 10 years with a half-life of 5 years.[8] Since high inflation is undesirable, we do not expect the gap between the current inflation and the fair inflation to be closed on a linear basis. Instead, we employ the following model that allows it to be closed at exponential rate:

$$\pi_{i,t+1} - \pi_{i,t} = \rho \times (\pi_{i,t} - \pi^*_{i,t}) \tag{14}$$

We assume that half of the gap between current inflation ($\pi_{i,t}$) and long-term inflation ($\pi^*_{i,t}$) will be closed on a two year time horizon, resulting in the parameter ρ of -0.029 (more precisely, $\rho = -\frac{log(2)}{24}$). In other words, we expect that about 3% of the remaining gap will be closed every month. Due to the dynamic nature of the model, it allows recursively forecasting year-over-year inflation each month in the next 10 years.

Finally, we calculate the forecasted annual geometric average inflation over a 10 year horizon as:

$$\hat{\pi}_{i,t \to t+120} = \left(\prod_{k=1}^{10} (\hat{\pi}_{i,12 \times k} + 1) \right)^{\frac{1}{10}} - 1 \tag{15}$$

Inflation volatility forecasts are estimated as follows. First, we compute rolling stan-

[8] We do not use central banks' inflation targets due to the fact that many countries have consistently failed to reach the targets. Two typical examples are Japan and Turkey.

dard deviation of monthly year-over-year inflation using a 10-year window (i.e., 120 observations). Second, we take the median of the historical 10-year standard deviation series as the forecasted volatility in the next 10-year. We use 50 years of data for developed countries and 20 years of data for emerging countries. The shorter data window for the latter eliminates the hyperinflation period, for example, in Brazil, Mexico and Poland, when inflation reached more than 100%. We believe that such high inflation is very unlikely given the current stage of development of these countries.

We present our inflation projections for various countries/regions as of September 30, 2023 in Panel A of Table 3. By combining forecast real equity returns and inflation, we arrive at the forecasts for nominal equity returns which appears in Panel B of Table 3.

Table 3: Expected Inflation and Expected Nominal Equity Returns

Panel A: Expected Inflation			*Panel B: Expected nominal equity returns*		
Country	Expected Inflation	Expected Inflation Volatility	Equity Market	Expected Real Returns	Expected Nominal Returns
Australia	3.3	2.1	Australia	6.0	9.3
Austria	4.2	1.1	Austria	9.8	14.1
Belgium	3.1	1.5	Belgium	5.0	8.1
Brazil	5.7	2.7	Brazil	9.6	15.3
Canada	3.1	1.6	Canada	5.4	8.5
China	1.3	2.3	China	9.2	10.5
Denmark	2.1	1.1	Denmark	5.6	7.6
Finland	3.3	1.8	Finland	6.0	9.3
France	2.7	1.0	France	4.6	7.3
Germany	3.3	1.4	Germany	7.1	10.4
Hong Kong	1.9	2.6	Hong Kong	8.9	10.9
India	5.7	3.1	India	3.3	9.0
Indonesia	3.3	3.6	Indonesia	5.9	9.2
Ireland	3.2	2.8	Ireland	4.0	7.2
Israel	2.1	4.2	Israel	7.0	9.1
Italy	3.0	1.5	Italy	5.7	8.7
Japan	1.5	1.2	Japan	4.7	6.3
Malaysia	2.0	1.7	Malaysia	8.0	10.0
Mexico	4.8	1.1	Mexico	6.2	11.0
Netherlands	3.1	1.2	Netherlands	4.3	7.4
New Zealand	3.4	2.1	New Zealand	4.6	8.0
Norway	3.6	1.5	Norway	6.5	10.1
Poland	6.2	2.0	Poland	3.9	10.1
Portugal	2.4	3.6	Portugal	4.8	7.2
Singapore	2.4	2.2	Singapore	9.0	11.4
South Africa	5.0	2.6	South Africa	7.4	12.4
South Korea	2.5	1.2	South Korea	8.6	11.2
Spain	2.6	1.7	Spain	7.1	9.7
Sweden	4.2	2.3	Sweden	10.4	14.5
Switzerland	1.0	1.8	Switzerland	6.0	7.0
Taiwan	1.7	1.5	Taiwan	5.1	6.9
Turkey	29.6	9.1	Turkey	7.6	37.2
UK	3.9	2.0	UK	5.3	9.2
US	3.3	1.4	US	3.6	6.9
			Emerging Markets	9.1	12.5
			Europe	5.9	9.2
			World	4.3	7.7

Note: Numbers are presented in percentage. Estimations are as of September 30, 2023.

4 Foreign Exchange Rates

Let S_t denotes bilateral exchange rate or spot rate at time t, which is measured as the amount of the investor's home currency can be purchased with one unit of the foreign currency (i.e., the asset's local currency), and R_{t+1} denotes the local currency denominated return earned from the asset over the period from t to $t+1$. Assuming the investment is unhedged, the one-period return from the asset in home currency (R^U_{t+1}) can be expressed as:

$$\begin{aligned} R^U_{t+1} &= (1 + R_{t+1}) \times \frac{S_{t+1}}{S_t} - 1 \\ &= (1 + R_{t+1}) \times (1 + \Delta S_{t+1}) - 1 \end{aligned} \tag{16}$$

where ΔS_{t+1} denotes the percentage change in the bilateral exchange rate over the period. By ignoring the small geometric interaction, returns in the investor's home currency can be approximated as:

$$R^U_{t+1} \approx R_{t+1} + \Delta S_{t+1} \tag{17}$$

The expression indicates that unhedged expected returns approximately equal the sum of asset's expected returns in local currency and the expected foreign exchange (FX) returns.

Based on the Purchasing Power Parity (PPP) theory, we compute real exchange rate S^r_t from nominal exchange rate and price levels as:

$$S^r_t = \frac{CPI^f_t}{CPI^h_t} \times S_t \tag{18}$$

where S^r_t is real exchange rate, and CPI^h_t and CPI^f_t are consumer price indexes, which are used as proxies for price levels, of countries where the investor and the asset are based, respectively. For the equation to hold, both CPI^h_t and CPI^f_t must have the same base year. Thus, when our raw CPI data are not based on the same year, we rebase them to meet this condition.

It can also be derived from Eq. (18) a relation between real exchange rate change and nominal exchange rate change over the period from t to $t+1$ as:

$$\Delta S_{t+1} = \Delta S^r_{t+1} + \pi^h_{t+1} - \pi^f_{t+1} \tag{19}$$

where π_{t+1} is inflation (i.e., percentage change in CPI) during the period. Combined with Eq. (17), unhedged returns can be expressed as sum of return in asset's local currency, real FX returns, and inflation differential:

$$\begin{aligned} R^U_{t+1} &\approx R_{t+1} + \Delta S^r_{t+1} + \pi^h_{t+1} - \pi^f_{t+1} \\ &\approx R^r_{t+1} + \Delta S^r_{t+1} + \pi^h_{t+1} \end{aligned} \tag{20}$$

This expression can be generalised to a 10-year horizon by simply changing the subscripts in the equation.

The forecasting process for the inflation component is described in Section 3. To forecast the real FX return component, we build on prior literature that real exchange rates in logarithmic form tend to exhibit mean reversal (e.g., Cheung and Lai, 1994; Gil-Alana, 2000). Using monthly real exchange rates calculated from Eq. (18), we estimate the speed of reversion to the long-term mean of the log real exchange rate as:

$$ln(S^r_{i,t}) - ln(S^r_{i,t-1}) = \rho \times [ln(S^r_{i,t-1}) - ln(S^r_{i,t})^*] \tag{21}$$

where $ln(S^r_{i,t})^*$ is the long-term mean of log real exchange rates of the currency pair i at the end of month t, calculated as the exponential weighted moving average with a 20-year window (i.e., from month $t-239$ to month t) and a 10-year half life. To estimate the model, we run a pool regression across all currency pairs. While our CMA covers 25 currencies corresponding to 600 pairs, to avoid overly complicated calculations, we only include in the regression 24 pairs in which USD is quoted as home currency, as returns for all other pairs can be easily approximated from that of those 24 pairs.[9]

The estimated regression coefficient $\hat{\rho}$,[10] combined with the current value of $S^r_{i,t}$ and $S^{r*}_{i,t}$, are then used to forecast the real exchange rates in the next 10 years ($\hat{S}^r_{i,t+120}$). The average annual change of real exchange rates is calculated as:

$$\Delta S^r_{i,t\to t+120} = \left(\frac{\hat{S}^r_{i,t+120}}{S_{i,t}}\right)^{\frac{1}{10}} - 1 \tag{22}$$

Table 4 presents our forecasts of real exchange rate returns in the next 10 years for various currency pairs. Numbers in the matrix are reported in percentage. A positive (negative) number indicates that the currency in the corresponding row is expected to depreciate (appreciate) against the currency in the corresponding column. For example, the Japanese Yen (JPY) is expected to appreciate 3.6% against the US Dollar (USD) in the next 10 years.

[9] For example, if returns on AUD/USD is 3% and returns on GBP/USD is 2%, returns on AUD/GBP is approximately 1%.

[10] As of September 30, 2023, $\hat{\rho} = -0.012$ and statistically significant at 1%, confirming the mean-reverting property of real exchange rates.

Table 4: 10-year forecasted real exchange rate changes

	AUD	USD	BRL	CAD	CHF	CNY	DKK	EUR	GBP	HKD	IDR	ILS	INR	JPY	KRW	MXN	MYR	NOK	NZD	PLN	SEK	SGD	TRY	TWD	ZAR
AUD	0.0	1.6	-0.3	0.5	0.8	0.4	0.1	0.7	0.4	1.1	0.3	0.5	1.2	-2.1	-0.1	2.0	-0.5	-1.0	0.4	1.0	-0.6	1.2	-3.7	0.3	-0.9
USD	-1.6	0.0	-1.9	-1.1	-0.8	-1.2	-1.5	-0.9	-1.2	-0.5	-1.3	-1.1	-0.4	-3.7	-1.7	0.4	-2.1	-2.6	-1.2	-0.6	-2.2	-0.4	-5.3	-1.3	-2.5
BRL	0.3	1.9	0.0	0.8	1.1	0.7	0.4	1.0	0.7	1.4	0.6	0.8	1.5	-1.8	0.2	2.3	-0.2	-0.7	0.7	1.3	-0.3	1.5	-3.4	0.6	-0.6
CAD	-0.5	1.1	-0.8	0.0	0.3	-0.1	-0.4	0.2	-0.1	0.6	-0.2	-0.0	0.7	-2.6	-0.6	1.5	-1.0	-1.5	-0.1	0.5	-1.1	0.7	-4.2	-0.2	-1.4
CHF	-0.8	0.8	-1.1	-0.3	0.0	-0.4	-0.7	-0.1	-0.4	0.3	-0.5	-0.3	0.4	-2.9	-0.9	1.2	-1.3	-1.8	-0.4	0.2	-1.4	0.4	-4.5	-0.5	-1.7
CNY	-0.4	1.2	-0.7	0.1	0.4	0.0	-0.3	0.3	-0.0	0.7	-0.1	0.1	0.8	-2.5	-0.5	1.6	-0.9	-1.4	-0.0	0.6	-1.0	0.8	-4.1	-0.1	-1.3
DKK	-0.1	1.5	-0.4	0.4	0.7	0.3	0.0	0.6	0.3	1.0	0.2	0.4	1.1	-2.2	-0.2	1.9	-0.6	-1.1	0.3	0.9	-0.7	1.1	-3.8	0.2	-1.0
EUR	-0.7	0.9	-1.0	-0.2	0.1	-0.3	-0.6	0.0	-0.3	0.4	-0.4	-0.2	0.5	-2.8	-0.8	1.3	-1.2	-1.7	-0.3	0.3	-1.3	0.5	-4.4	-0.4	-1.6
GBP	-0.4	1.2	-0.7	0.1	0.4	-0.0	-0.3	0.3	0.0	0.7	-0.1	0.1	0.8	-2.5	-0.5	1.6	-0.9	-1.4	-0.0	0.6	-1.0	0.8	-4.1	-0.1	-1.3
HKD	-1.1	0.5	-1.4	-0.6	-0.3	-0.7	-1.0	-0.4	-0.7	0.0	-0.8	-0.6	0.1	-3.2	-1.2	0.9	-1.6	-2.1	-0.7	-0.1	-1.7	0.1	-4.8	-0.8	-2.0
IDR	-0.3	1.3	-0.6	0.2	0.5	0.1	-0.2	0.4	0.1	0.8	0.0	0.2	0.9	-2.4	-0.4	1.7	-0.8	-1.3	0.1	0.7	-0.9	0.9	-4.0	-0.0	-1.2
ILS	-0.5	1.1	-0.8	-0.0	0.3	-0.1	-0.4	0.2	-0.1	0.6	-0.2	0.0	0.7	-2.6	-0.6	1.5	-1.0	-1.5	-0.1	0.5	-1.1	0.7	-4.2	-0.2	-1.4
INR	-1.2	0.4	-1.5	-0.7	-0.4	-0.8	-1.1	-0.5	-0.8	-0.1	-0.9	-0.7	0.0	-3.3	-1.3	0.8	-1.7	-2.2	-0.8	-0.2	-1.8	-0.0	-4.9	-0.9	-2.1
JPY	2.1	3.7	1.8	2.6	2.9	2.5	2.2	2.8	2.5	3.2	2.4	2.6	3.3	0.0	2.0	4.1	1.6	1.1	2.5	3.1	1.5	3.3	-1.6	2.4	1.2
KRW	0.1	1.7	-0.2	0.6	0.9	0.5	0.2	0.8	0.5	1.2	0.4	0.6	1.3	-2.0	0.0	2.1	-0.4	-0.9	0.5	1.1	-0.5	1.3	-3.6	0.4	-0.8
MXN	-2.0	-0.4	-2.3	-1.5	-1.2	-1.6	-1.9	-1.3	-1.6	-0.9	-1.7	-1.5	-0.8	-4.1	-2.1	0.0	-2.5	-3.0	-1.6	-1.0	-2.6	-0.8	-5.7	-1.7	-2.9
MYR	0.5	2.1	0.2	1.0	1.3	0.9	0.6	1.2	0.9	1.6	0.8	1.0	1.7	-1.6	0.4	2.5	0.0	-0.5	0.9	1.5	-0.1	1.7	-3.2	0.8	-0.4
NOK	1.0	2.6	0.7	1.5	1.8	1.4	1.1	1.7	1.4	2.1	1.3	1.5	2.2	-1.1	0.9	3.0	0.5	0.0	1.4	2.0	0.4	2.2	-2.7	1.3	0.1
NZD	-0.4	1.2	-0.7	0.1	0.4	-0.0	-0.3	0.3	-0.0	0.7	-0.1	0.1	0.8	-2.5	-0.5	1.6	-0.9	-1.4	0.0	0.6	-1.0	0.8	-4.1	-0.1	-1.3
PLN	-1.0	0.6	-1.3	-0.5	-0.2	-0.6	-0.9	-0.3	-0.6	0.1	-0.7	-0.5	0.2	-3.1	-1.1	1.0	-1.5	-2.0	-0.6	0.0	-1.6	0.2	-4.7	-0.7	-1.9
SEK	0.6	2.2	0.3	1.1	1.4	1.0	0.7	1.3	1.0	1.7	0.9	1.1	1.8	-1.5	0.5	2.6	0.1	-0.4	1.0	1.6	0.0	1.8	-3.1	0.9	-0.3
SGD	-1.2	0.4	-1.5	-0.7	-0.4	-0.8	-1.1	-0.5	-0.8	-0.1	-0.9	-0.7	-0.0	-3.3	-1.3	0.8	-1.7	-2.2	-0.8	-0.2	-1.8	0.0	-4.9	-0.9	-2.1
TRY	3.7	5.3	3.4	4.2	4.5	4.1	3.8	4.4	4.1	4.8	4.0	4.2	4.9	1.6	3.6	5.7	3.2	2.7	4.1	4.7	3.1	4.9	0.0	4.0	2.8
TWD	-0.3	1.3	-0.6	0.2	0.5	0.1	-0.2	0.4	0.1	0.8	-0.0	0.2	0.9	-2.4	-0.4	1.7	-0.8	-1.3	0.1	0.7	-0.9	0.9	-4.0	0.0	-1.2
ZAR	0.9	2.5	0.6	1.4	1.7	1.3	1.0	1.6	1.3	2.0	1.2	1.4	2.1	-1.2	0.8	2.9	0.4	-0.1	1.3	1.9	0.3	2.1	-2.8	1.2	0.0

Note: Numbers are presented in percentage. Estimates are as of September 30, 2023. A positive (negative) number indicates that the currency in the corresponding row is expected to appreciate (depreciate) against the currency in the corresponding column by the amount (in percentage) equal to the absolute value of the number.

AUD: Australian Dollar, USD: US Dollar, BRL: Brazilian Real, CAD: Canadian Dollar, CHF: Swiss Franc, CNY: Chinese Yuan Renminbi, DKK: Danish Krone, EUR: Euro, GBP: Pound Sterling, HKD: Hong Kong Dollar, IDR: Indonesian Rupiah, ILS: Israeli Shekel, INR: Indian Rupee, JPY: Japanese Yen, KRW: South Korean Won, MXN: Mexican Peso, MYR: Malaysian Ringgit, NOK: Norwegian Krone, NZD: New Zealand Dollar, PLN: Polish Zloty, SEK: Swedish Krona, SGD: Singapore Dollar, TRY: Turkish Lira, TWD: New Taiwan Dollar, ZAR: South African Rand.

By combining the forecasted real equity returns in local currencies, inflation and real exchange rates, readers, if desired, can derive the expected returns in any currency using Equation (20). Table 5 demonstrates the effects of foreign exchange rate changes applied particularly to Australian and U.S. investors.

Table 5: 10-year forecasted equity returns in AUD and USD

Equity Market	Expected Returns (Unhedged, in AUD)		Expected Returns (Unhedged, in USD)	
	Real	Nominal	Real	Nominal
Australia	6.0	9.3	7.6	11.0
Austria	9.1	12.5	10.7	14.1
Belgium	4.3	7.6	5.9	9.2
Brazil	9.9	13.2	11.5	14.8
Canada	4.9	8.2	6.5	9.9
China	8.8	12.1	10.4	13.8
Denmark	5.5	8.8	7.1	10.4
Emerging Markets	7.5	10.9	9.1	12.5
Europe	4.3	7.6	5.9	9.2
Finland	5.3	8.6	6.9	10.3
France	3.9	7.2	5.5	8.8
Germany	6.4	9.7	8.0	11.3
Hong Kong	7.8	11.2	9.4	12.8
India	2.1	5.5	3.7	7.1
Indonesia	5.6	8.9	7.2	10.5
Ireland	3.3	6.6	4.9	8.2
Israel	6.5	9.8	8.1	11.4
Italy	5.0	8.3	6.6	9.9
Japan	6.8	10.2	8.4	11.8
Malaysia	8.5	11.8	10.1	13.5
Mexico	4.2	7.5	5.8	9.1
Netherlands	3.6	6.9	5.2	8.6
New Zealand	4.2	7.5	5.8	9.2
Norway	7.5	10.8	9.1	12.5
Poland	2.9	6.2	4.5	7.9
Portugal	4.1	7.4	5.7	9.0
Singapore	7.8	11.1	9.4	12.7
South Africa	8.3	11.6	9.9	13.2
South Korea	8.7	12.1	10.3	13.7
Spain	6.4	9.7	8.0	11.4
Sweden	11.0	14.3	12.6	15.9
Switzerland	5.2	8.5	6.8	10.1
Taiwan	4.8	8.2	6.4	9.8
Turkey	11.3	14.6	12.9	16.3
UK	4.9	8.2	6.5	9.9
US	2.0	5.3	3.6	6.9
World	2.7	6.1	4.3	7.7

Note: Numbers are presented in percentage. Estimates are as of September 30, 2023.

5 Volatility and Correlation

To estimate volatility and correlation of asset returns, we first calculate a series of variance-covariance matrix of all assets using 10-year rolling windows of monthly data with complete pairwise observations. Within each 10-year window, we require an asset to have at least 60 observations (i.e., five years of data) to be included in the calculation. Next, we calculate an average variance-covariance matrix by employing an exponential weighted moving average of each matrix element (using 20 years of data and a half-life of 10 years). Forecasted volatility and correlation are finally derived from this average variance-covariance matrix, in which volatility is squared root of the corresponding diagonal element, and correlation is calculated as:

$$cor(i,j) = \frac{cov(i,j)}{\sigma_i \times \sigma_j} \tag{23}$$

where $cov(i,j)$ is the covariance of asset i and j, and σ_i and σ_j are respectively volatility of asset i and j.

Forecasted volatility and correlation of returns in a particular currency (e.g., AUD or USD) are estimated using the same method where the variance-covariance matrix is calculated using historical returns that are converted to the respective currency.

Apart from correlation of equity returns, we also forecast the correlation between equity returns and inflation. To be consistent with the equity data, we use month-over-month inflation for the correlation estimates. However, we do not use the variance-covariance matrix to estimate inflation volatility but rather stick to the method using year-over-year inflation described in Section 3. The reason is that the variance-covariance matrix results in monthly volatility which eventually needs to be converted to yearly volatility. However, as noted by Lo (2002), the conversion needs to take into account the auto-correlation structure of the series. While auto-correlation usually does not exist among public equity returns, it is significant for inflation. As we believe that an extra estimation step would add more noise to our forecasts, we opt for the simpler method of using year-over-year inflation.

We report our forecasted results as of September 30, 2023 for volatility in Table 6, and for correlation in Figure 7.[11]

[11] For the sake of space, we only presents the forecasted correlations of some key inflation/equity returns series. The full correlation matrix is available in our Dashboard at `https://mcfs.shinyapps.io/dashboard/`.

Table 6: 10-year forecasted equity volatility

Equity Market	Real			Nominal		
	Local	AUD	USD	Local	AUD	USD
Australia	13.4	13.4	21.7	13.4	13.4	21.8
Austria	23.6	21.9	27.9	23.6	21.9	28.0
Belgium	18.9	17.9	21.9	18.8	17.9	22.0
Brazil	23.5	28.5	35.0	23.7	28.5	35.1
Canada	13.6	14.2	19.6	13.7	14.2	19.8
China	27.4	23.4	28.3	27.4	23.3	28.3
Denmark	17.4	16.8	19.9	17.3	16.9	19.9
Emerging Markets	21.4	14.6	21.4	21.4	14.5	21.4
Europe	14.6	14.6	14.6	14.6	14.6	14.6
Finland	25.0	23.8	27.3	24.9	23.7	27.3
France	17.1	16.3	20.7	17.1	16.3	20.7
Germany	19.6	18.2	22.8	19.7	18.2	22.9
Hong Kong	21.3	16.9	21.3	21.3	16.9	21.4
India	22.9	23.4	27.5	22.8	23.4	27.5
Indonesia	25.6	29.5	33.9	25.1	29.4	33.9
Ireland	20.9	20.3	22.7	20.9	20.3	22.8
Israel	19.0	19.5	21.5	18.8	19.5	21.6
Italy	20.7	20.4	24.6	20.7	20.4	24.6
Japan	17.7	14.6	16.0	17.8	14.6	16.0
Malaysia	17.3	18.7	21.2	17.2	18.6	21.3
Mexico	17.5	19.9	24.6	17.5	19.9	24.7
Netherlands	17.4	16.4	20.4	17.4	16.4	20.4
New Zealand	14.3	15.8	21.1	14.3	15.7	21.1
Norway	19.6	20.0	26.1	19.4	20.0	26.3
Poland	23.3	26.0	31.7	23.3	26.0	31.8
Portugal	18.2	18.8	22.6	18.2	18.8	22.6
Singapore	19.4	16.8	22.5	19.2	16.7	22.5
South Africa	16.7	19.1	25.6	16.5	19.1	25.6
South Korea	22.6	24.0	29.0	22.6	23.9	29.1
Spain	20.7	20.3	24.9	20.7	20.4	25.0
Sweden	19.0	18.6	23.6	18.9	18.5	23.6
Switzerland	13.3	12.8	15.4	13.3	12.8	15.5
Taiwan	20.7	19.6	23.4	20.6	19.5	23.4
Turkey	34.3	38.2	42.2	34.3	38.1	42.2
UK	13.4	12.6	16.6	13.4	12.6	16.7
US	14.7	12.8	14.7	14.8	12.7	14.8
World	13.8	12.6	13.8	13.8	12.5	13.8

Note: Numbers are presented in percentage. Estimates are as of September 30, 2023.

Table 7: 10-year forecasted correlation matrix - Nominal returns in USD

	Australia Equity	Germany Equity	Japan Equity	UK Equity	US Equity	World Equity	Australia Inflation	Germany Inflation	Japan Inflation	UK Inflation	US Inflation
Australia Equity	1	0.76	0.61	0.8	0.77	0.79	−0.04	0.04	0.05	0.03	0.13
Germany Equity	0.76	1	0.59	0.85	0.84	0.87	−0.05	0.09	0.09	−0.01	0.06
Japan Equity	0.61	0.59	1	0.64	0.61	0.69	−0.08	0.07	0.08	−0.02	0.04
UK Equity	0.8	0.85	0.64	1	0.83	0.86	0	0.1	0.09	0.06	0.12
US Equity	0.77	0.84	0.61	0.83	1	0.97	−0.05	0.09	0.1	−0.01	0.07
World Equity	0.79	0.87	0.69	0.86	0.97	1	−0.06	0.1	0.09	−0.01	0.06
Australia Inflation	−0.04	−0.05	−0.08	0	−0.05	−0.06	1	0.29	0.14	0.37	0.39
Germany Inflation	0.04	0.09	0.07	0.1	0.09	0.1	0.29	1	0.13	0.43	0.41
Japan Inflation	0.05	0.09	0.08	0.09	0.1	0.09	0.14	0.13	1	0.12	0.26
UK Inflation	0.03	−0.01	−0.02	0.06	−0.01	−0.01	0.37	0.43	0.12	1	0.45
US Inflation	0.13	0.06	0.04	0.12	0.07	0.06	0.39	0.41	0.26	0.45	1

6 Arithmetic versus Geometric Average Returns

The methodology highlighted above delivers 10-year geometric annual average returns. The geometric mean return is important to investors because it express the periodic growth rate of their investments over time. However, for portfolio construction, arithmetic average return is also of importance because it describes the central tendency of a return distribution and thus is used widely in standard portfolio optimization theories (e.g., Markowitz, 1952). While we primarily present the geometric annual average returns in our CMA as the results of the above-mentioned models, geometric average returns can be easily computed using the following approximation:

$$\mu_a \approx \mu_g + \frac{1}{2}\sigma^2 \tag{24}$$

where μ_g is geometric mean, μ_a is arithmetic mean, and σ^2 is variance.

Appendix A ARMA orders for the EPS growth model

Equity Market	AR Order (p)	MA Order (q)	EPS Start Date
Australia	1	6	Dec 31, 1969
Austria	1	5	Dec 31, 1969
Belgium	1	7	Dec 31, 1969
Brazil	1	4	Dec 31, 1987
Denmark	1	6	Dec 31, 1969
France	1	12	Dec 31, 1969
Germany	1	12	Dec 31, 1969
Mexico	1	2	Dec 31, 1969
Netherlands	1	12	Dec 31, 1969
New Zealand	1	5	Dec 31, 1969
Norway	1	7	Dec 31, 1969
Singapore	1	8	Dec 31, 1969
Sweden	1	1	Dec 31, 1969
Switzerland	1	11	Dec 31, 1969
Taiwan	2	6	Dec 31, 1969
Turkey	1	12	Dec 31, 1969
UK	1	11	Dec 31, 1969
US Large	1	12	Dec 31, 1969
World	1	12	Dec 31, 1969

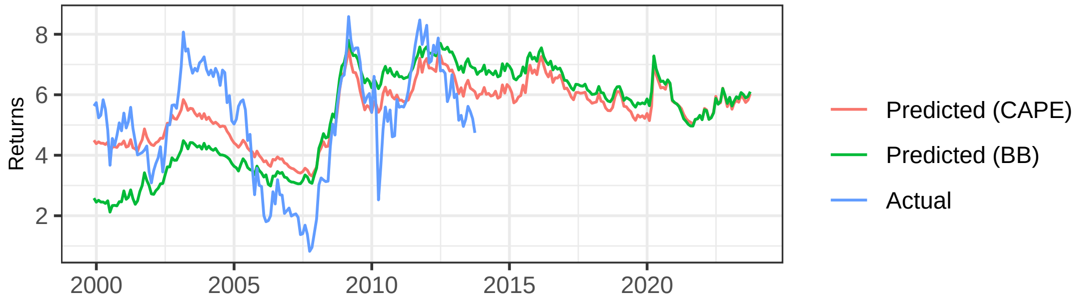
Appendix B 1/CAPE vs. Building Blocks

Equity Market	Building Block Model				1/CAPE
	Dividend Yield	EPS Growth	Valuation Change	Expected Returns	
Australia	4.4	1.5	0.6	6.4	6.0
Austria	3.1	3.6	2.5	9.3	9.8
Belgium	2.7	1.6	1.4	5.7	6.9
Brazil	5.9	4.6	2.6	13.1	9.6
Canada	3.0	1.4	1.1	5.6	5.4
China	2.2	2.2	4.2	8.5	9.2
Denmark	1.8	4.4	-1.3	4.9	2.7
Emerging Markets	2.7	3.2	2.7	8.7	9.1
Europe	3.2	1.8	-0.2	4.8	5.9
Finland	3.6	-0.0	1.6	5.2	6.0
France	2.8	2.5	-1.1	4.2	4.6
Germany	3.0	2.1	1.8	6.9	7.1
Hong Kong	3.0	3.7	3.8	10.6	8.9
India	1.3	3.5	-2.2	2.6	3.3
Indonesia	2.9	1.8	1.5	6.2	5.9
Ireland	1.6	1.1	-0.3	2.4	4.0
Israel	2.0	2.3	1.6	5.9	7.0
Italy	3.8	-1.4	-1.4	0.9	5.7
Japan	2.2	2.7	1.7	6.6	4.7
Malaysia	3.5	1.4	2.6	7.4	8.0
Mexico	2.7	3.6	2.3	8.6	6.2
Netherlands	2.1	2.1	-0.5	3.7	4.3
New Zealand	2.9	-3.0	-0.0	1.3	4.6
Norway	4.2	3.2	-0.6	6.8	6.5
Poland	2.8	-1.1	3.5	3.9	3.9
Portugal	3.9	-3.2	-3.5	-2.7	4.8
Singapore	3.8	2.4	2.4	8.5	9.0
South Africa	3.2	3.4	2.6	9.2	7.4
South Korea	1.9	5.6	1.8	9.3	8.6
Spain	4.1	3.0	-0.7	6.4	7.1
Sweden	3.0	6.4	1.9	11.3	6.1
Switzerland	3.0	2.5	0.9	6.4	4.7
Taiwan	3.6	4.3	0.8	8.7	5.1
Turkey	3.1	2.7	-1.9	3.8	7.6
UK	3.9	1.4	-0.1	5.3	7.1
US	1.8	2.7	-0.4	4.0	3.6
World	2.2	1.4	-0.3	3.3	4.3

Note: Numbers are presented in percentage. Estimates are as of September 30, 2023.

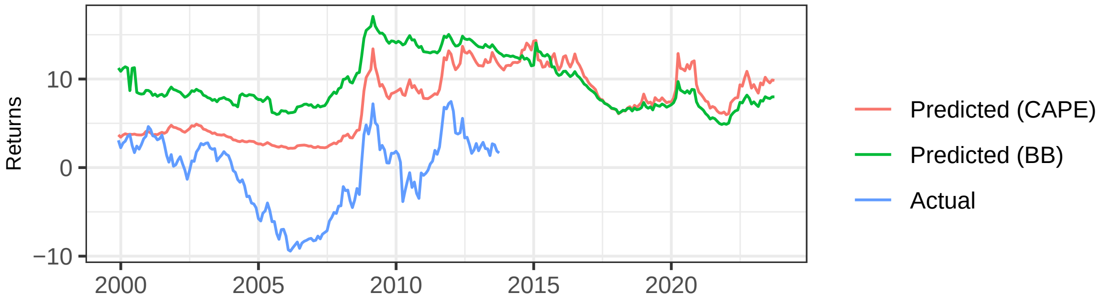
Appendix C CMA vs. Actual Series: Real Equity Returns

Australia



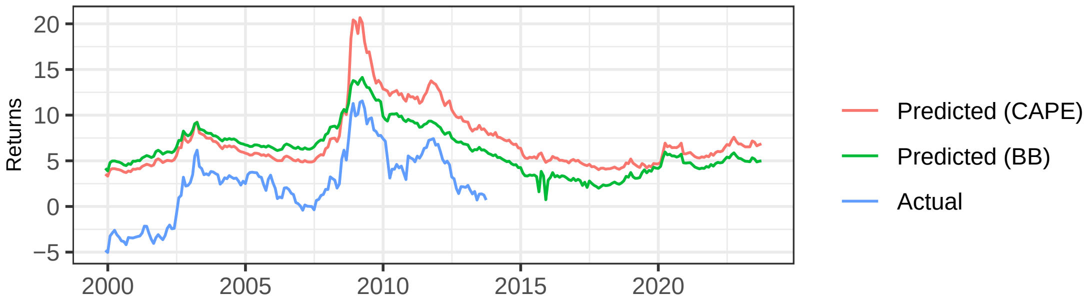
RMSE (CAPE) = 1.1% | RMSE (BB) = 1.7%
MAE (CAPE) = 0.97% | MAE (BB) = 1.4%

Austria



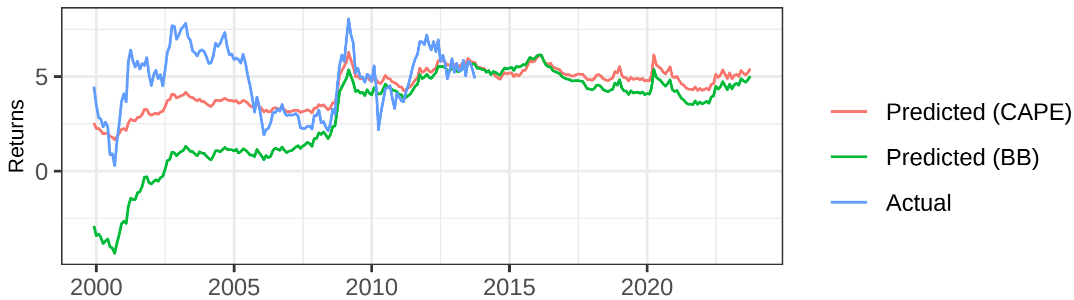
RMSE (CAPE) = 7.2% | RMSE (BB) = 11%
MAE (CAPE) = 6.4% | MAE (BB) = 11%

Belgium

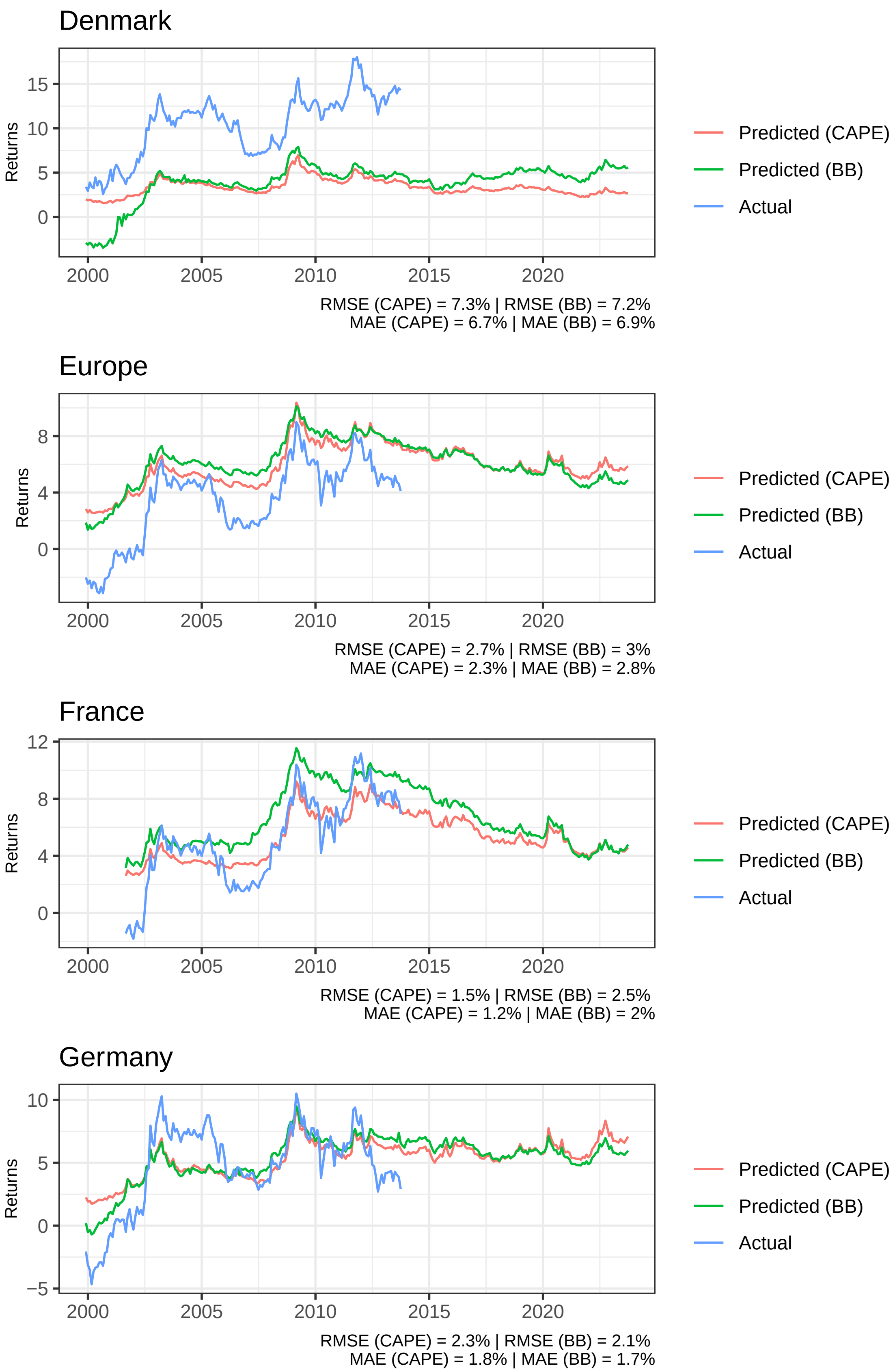


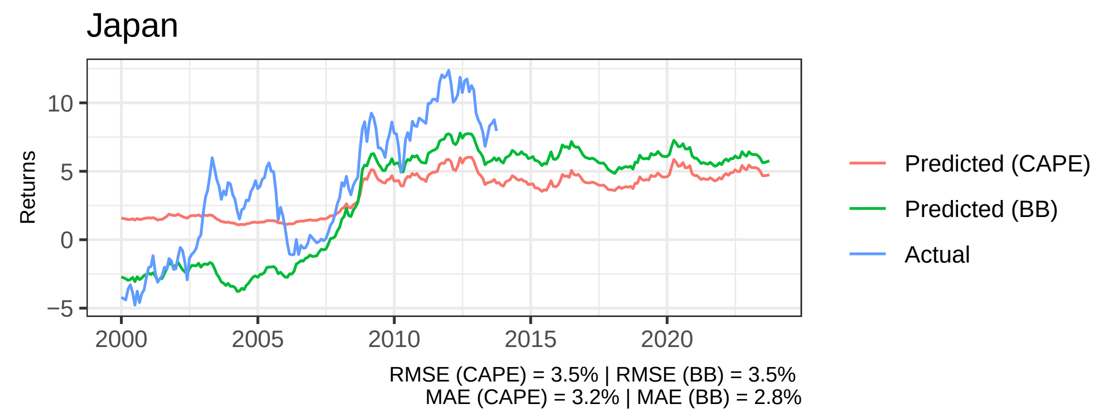
RMSE (CAPE) = 6.1% | RMSE (BB) = 5.6%
MAE (CAPE) = 5.8% | MAE (BB) = 5.2%

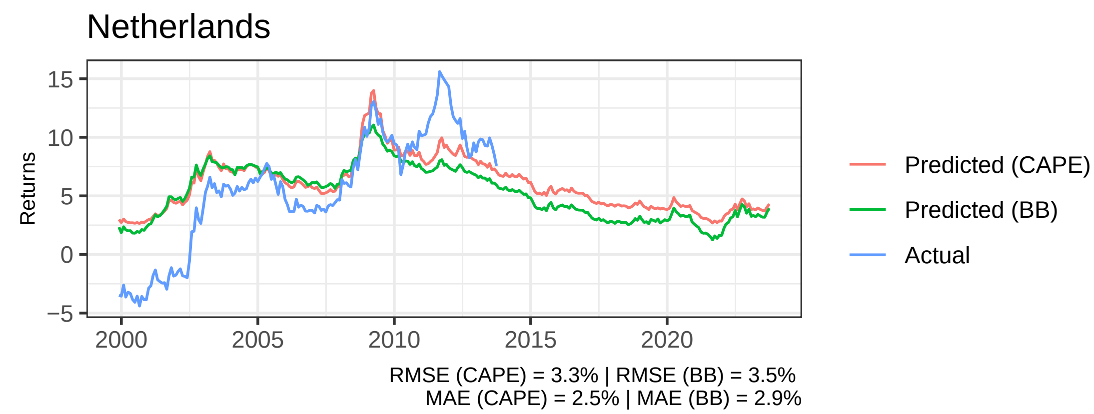
Canada

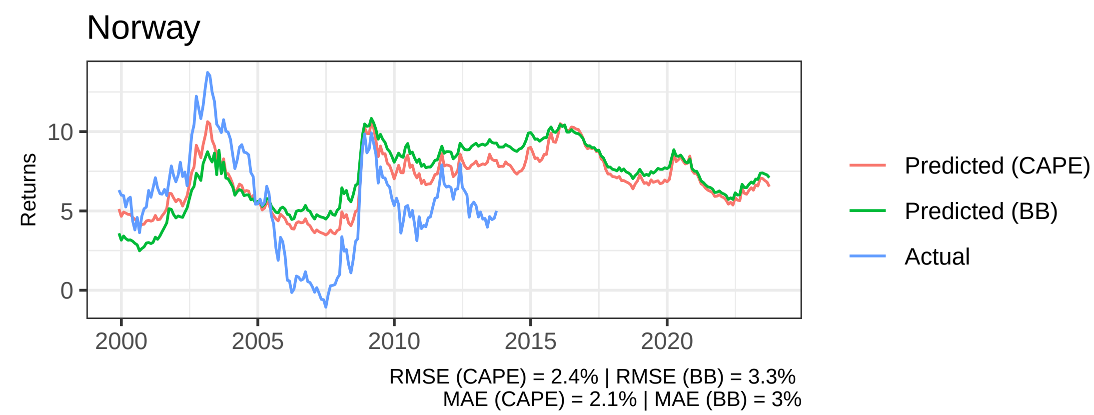


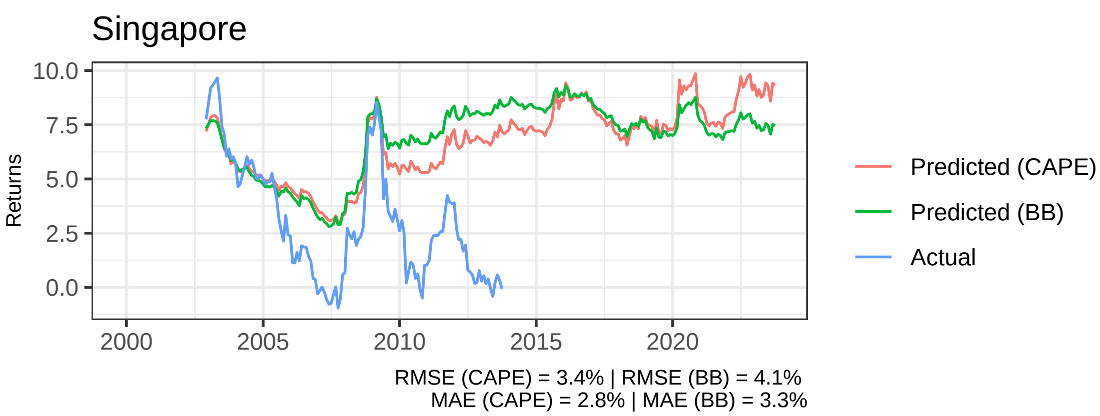
RMSE (CAPE) = 1.7% | RMSE (BB) = 3.9%
MAE (CAPE) = 1.3% | MAE (BB) = 3%

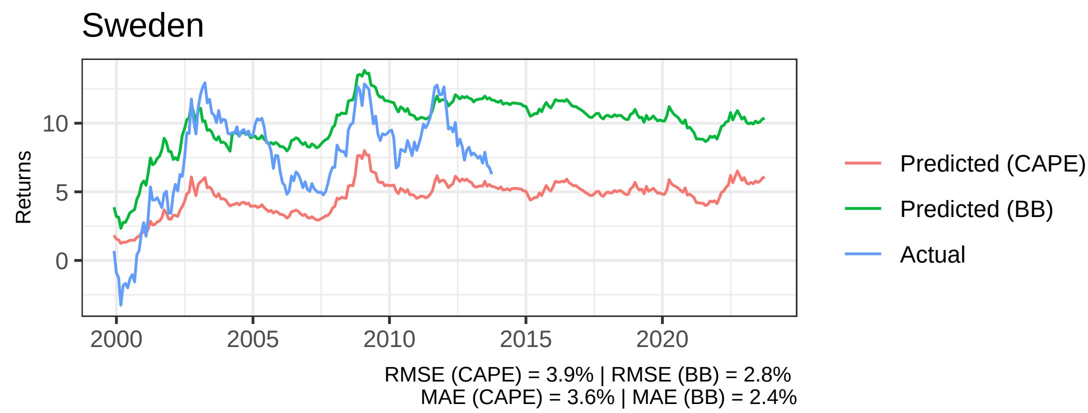

Denmark
Returns
15
10
5
0
2000
2005
2010
2015
2020
Predicted (CAPE)
Predicted (BB)
Actual
RMSE (CAPE) = 7.3% | RMSE (BB) = 7.2%
MAE (CAPE) = 6.7% | MAE (BB) = 6.9%
Europe
Returns
8
4
0
2000
2005
2010
2015
2020
Predicted (CAPE)
Predicted (BB)
Actual
RMSE (CAPE) = 2.7% | RMSE (BB) = 3%
MAE (CAPE) = 2.3% | MAE (BB) = 2.8%
France
Returns
12
8
4
0
2000
2005
2010
2015
2020
Predicted (CAPE)
Predicted (BB)
Actual
RMSE (CAPE) = 1.5% | RMSE (BB) = 2.5%
MAE (CAPE) = 1.2% | MAE (BB) = 2%
Germany
Returns
10
5
0
−5
2000
2005
2010
2015
2020
Predicted (CAPE)
Predicted (BB)
Actual
RMSE (CAPE) = 2.3% | RMSE (BB) = 2.1%
MAE (CAPE) = 1.8% | MAE (BB) = 1.7%

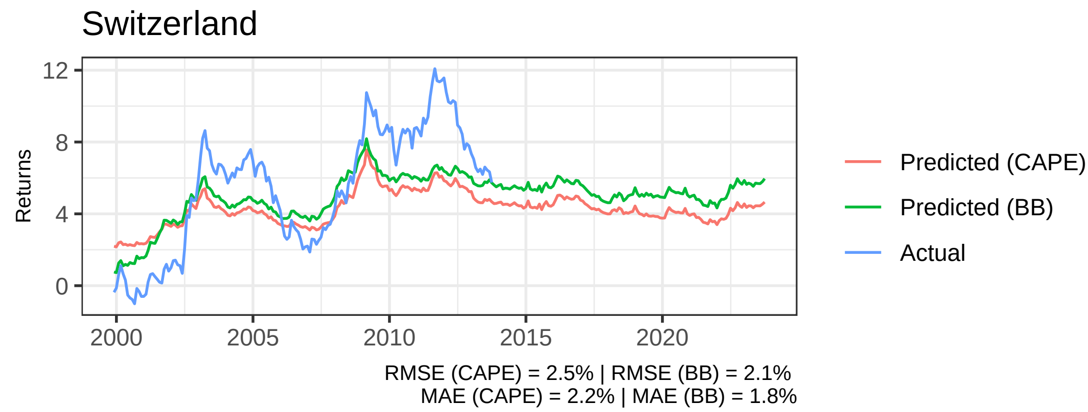

Japan
Returns
10
5
0
−5
2000
2005
2010
2015
2020
Predicted (CAPE)
Predicted (BB)
Actual
RMSE (CAPE) = 3.5% | RMSE (BB) = 3.5%
MAE (CAPE) = 3.2% | MAE (BB) = 2.8%

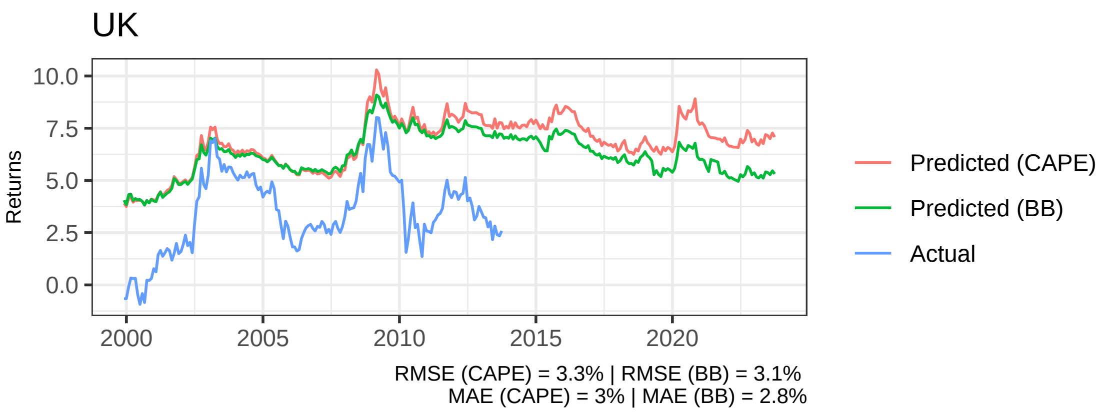

Netherlands
Returns
15
10
5
0
−5
2000
2005
2010
2015
2020
Predicted (CAPE)
Predicted (BB)
Actual
RMSE (CAPE) = 3.3% | RMSE (BB) = 3.5%
MAE (CAPE) = 2.5% | MAE (BB) = 2.9%

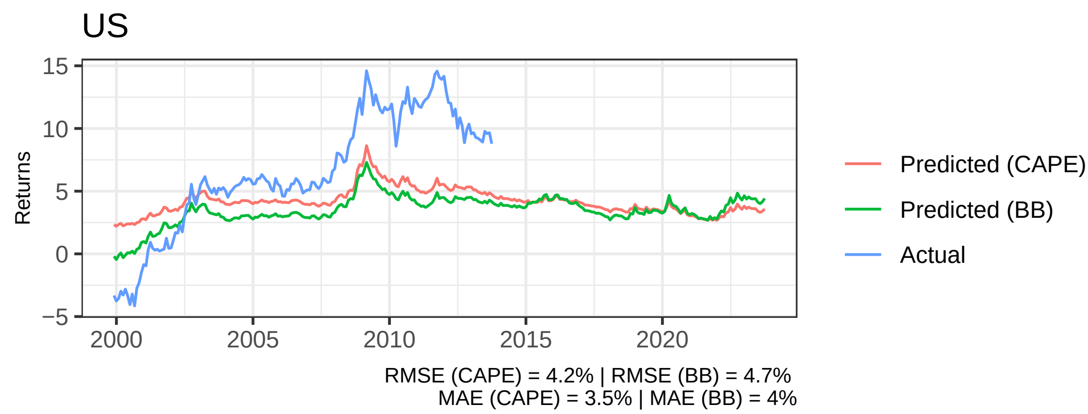

Norway
Returns
10
5
0
2000
2005
2010
2015
2020
Predicted (CAPE)
Predicted (BB)
Actual
RMSE (CAPE) = 2.4% | RMSE (BB) = 3.3%
MAE (CAPE) = 2.1% | MAE (BB) = 3%

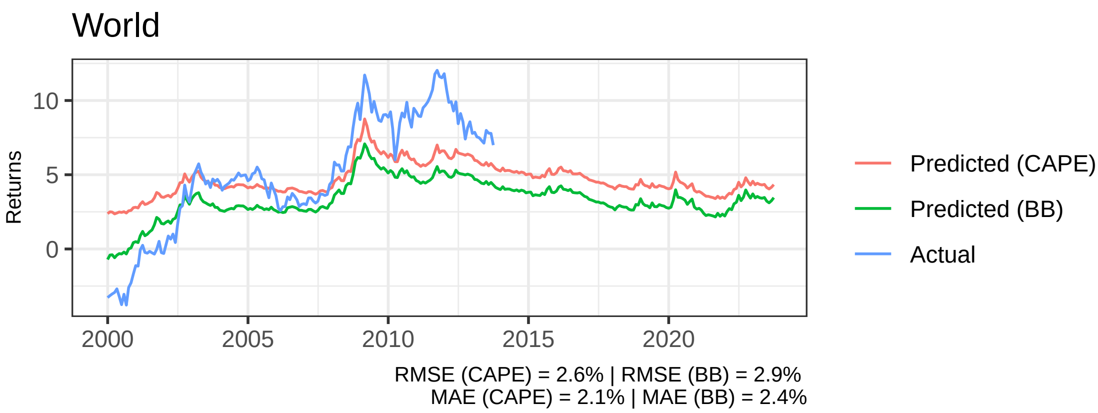

Singapore
Returns
10.0
7.5
5.0
2.5
0.0
2000
2005
2010
2015
2020
Predicted (CAPE)
Predicted (BB)
Actual
RMSE (CAPE) = 3.4% | RMSE (BB) = 4.1%
MAE (CAPE) = 2.8% | MAE (BB) = 3.3%


Sweden
Returns
10
5
0
2000
2005
2010
2015
2020
Predicted (CAPE)
Predicted (BB)
Actual
RMSE (CAPE) = 3.9% | RMSE (BB) = 2.8%
MAE (CAPE) = 3.6% | MAE (BB) = 2.4%

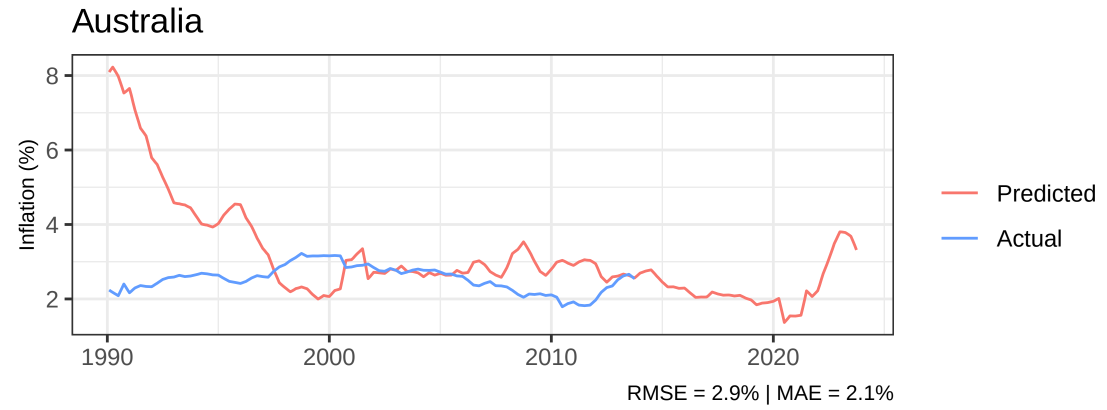

Switzerland
Returns
12
8
4
0
2000
2005
2010
2015
2020
Predicted (CAPE)
Predicted (BB)
Actual
RMSE (CAPE) = 2.5% | RMSE (BB) = 2.1%
MAE (CAPE) = 2.2% | MAE (BB) = 1.8%

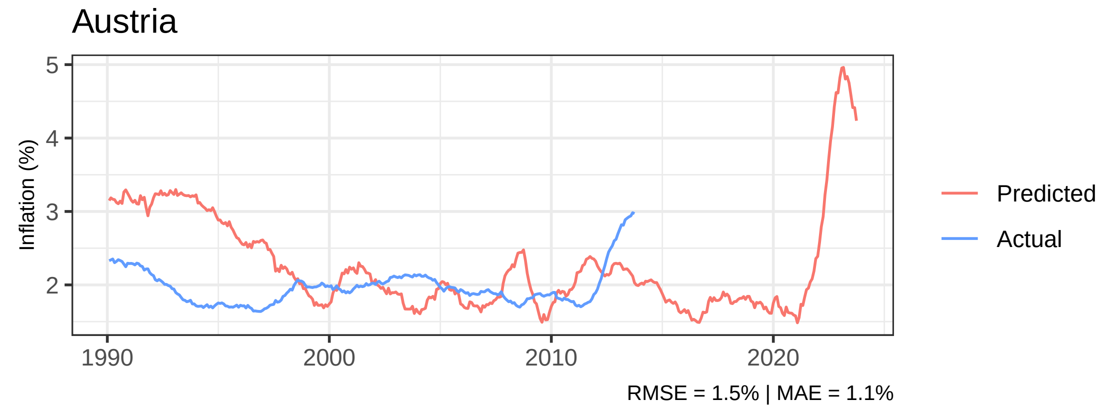

UK
Returns
10.0
7.5
5.0
2.5
0.0
2000
2005
2010
2015
2020
Predicted (CAPE)
Predicted (BB)
Actual
RMSE (CAPE) = 3.3% | RMSE (BB) = 3.1%
MAE (CAPE) = 3% | MAE (BB) = 2.8%

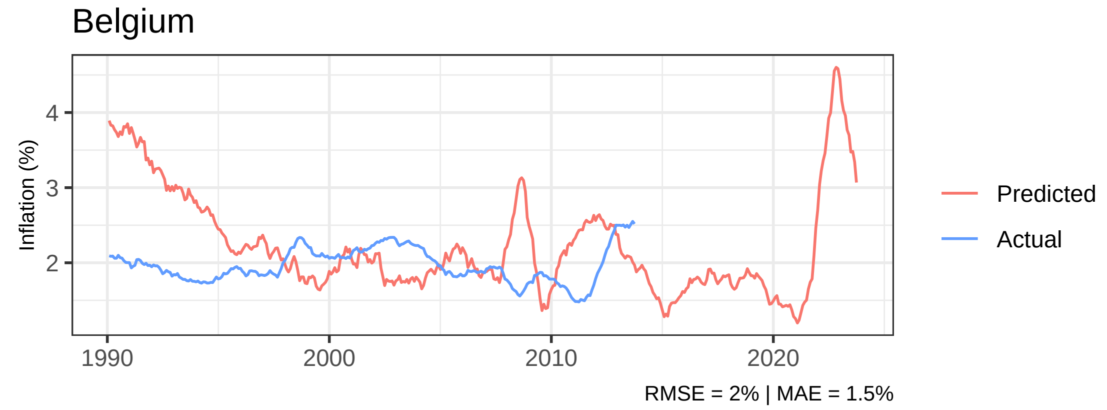

US
Returns
15
10
5
0
−5
2000
2005
2010
2015
2020
Predicted (CAPE)
Predicted (BB)
Actual
RMSE (CAPE) = 4.2% | RMSE (BB) = 4.7%
MAE (CAPE) = 3.5% | MAE (BB) = 4%

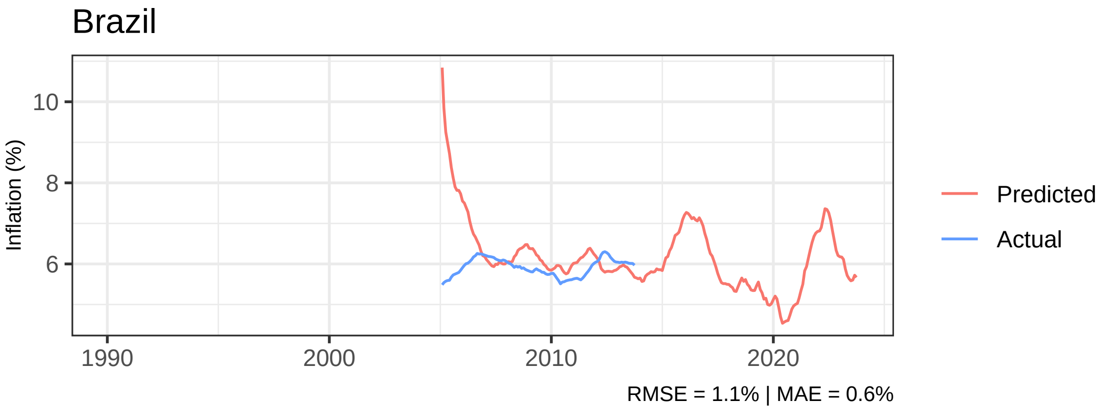

World
Returns
10
5
0
2000
2005
2010
2015
2020
Predicted (CAPE)
Predicted (BB)
Actual
RMSE (CAPE) = 2.6% | RMSE (BB) = 2.9%
MAE (CAPE) = 2.1% | MAE (BB) = 2.4%

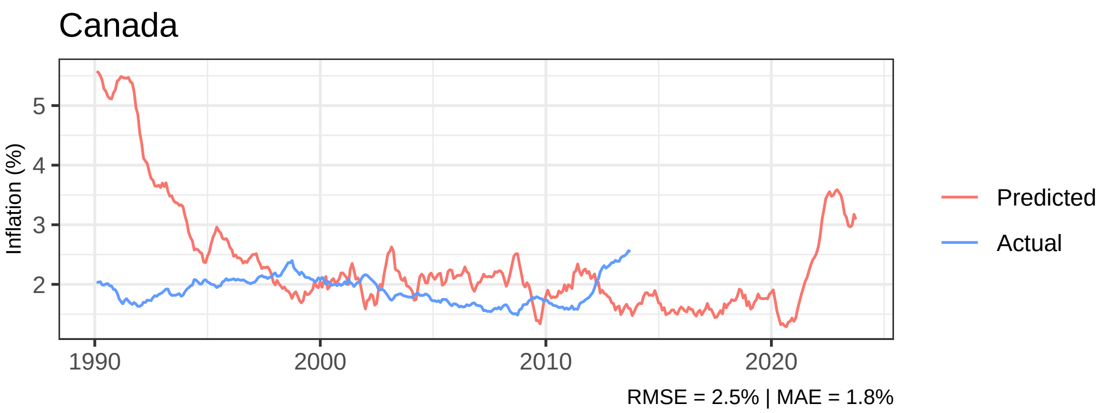
Appendix D CMA vs. Actual Series: Inflation

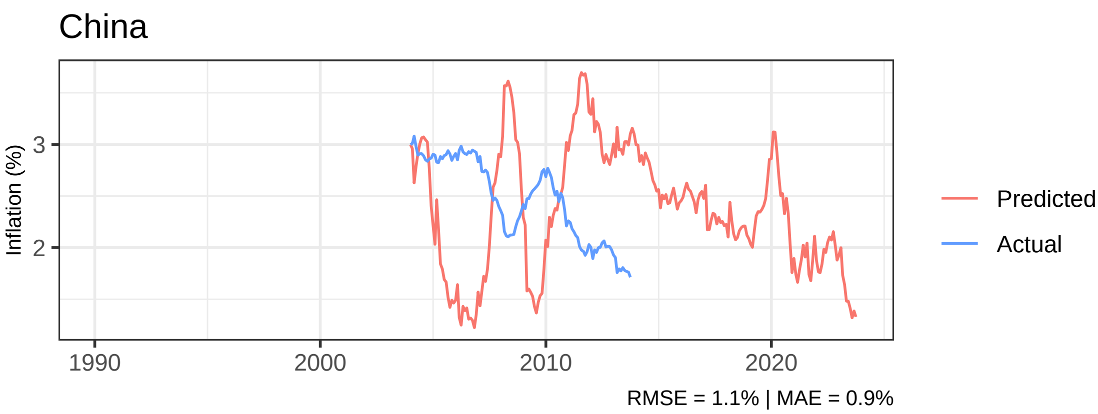


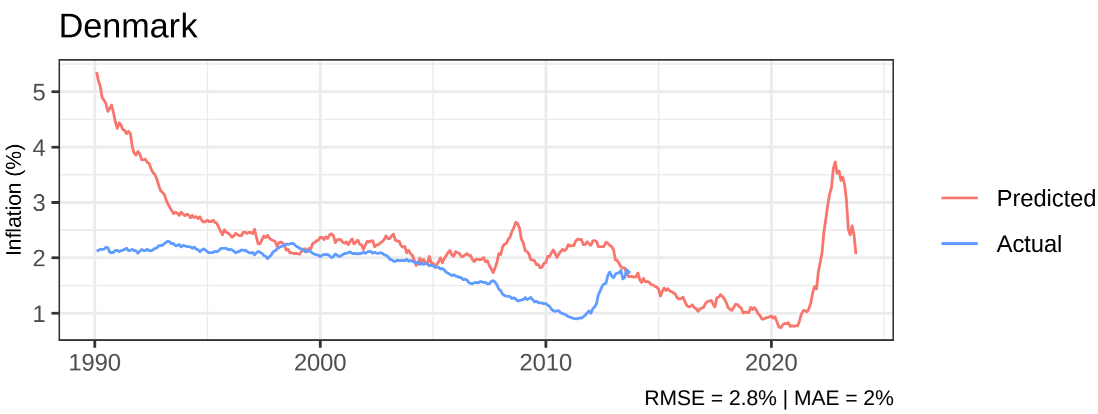


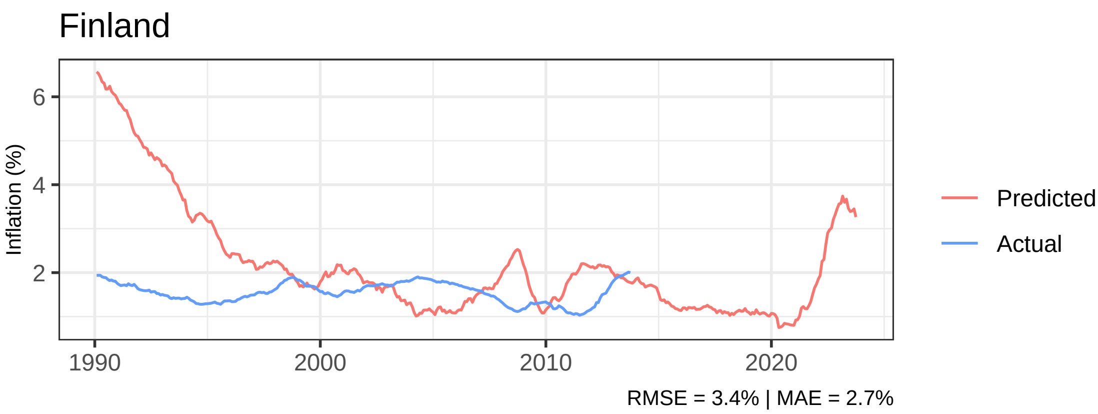


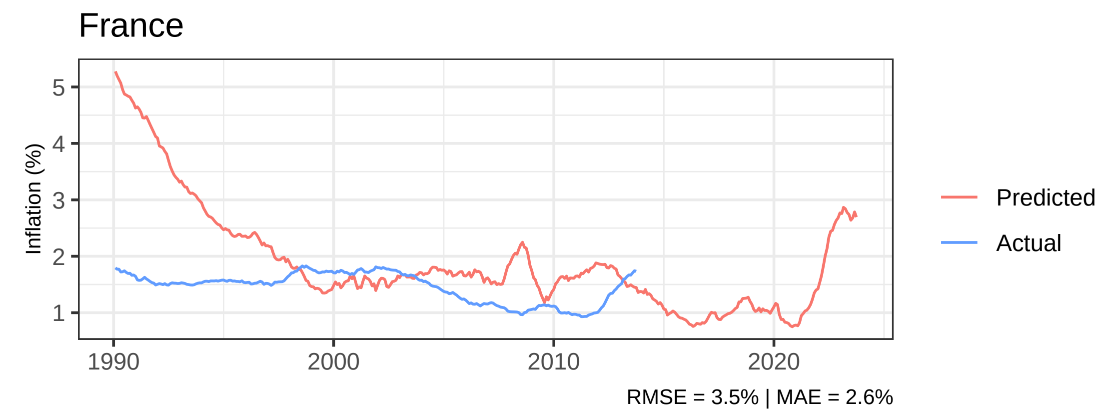


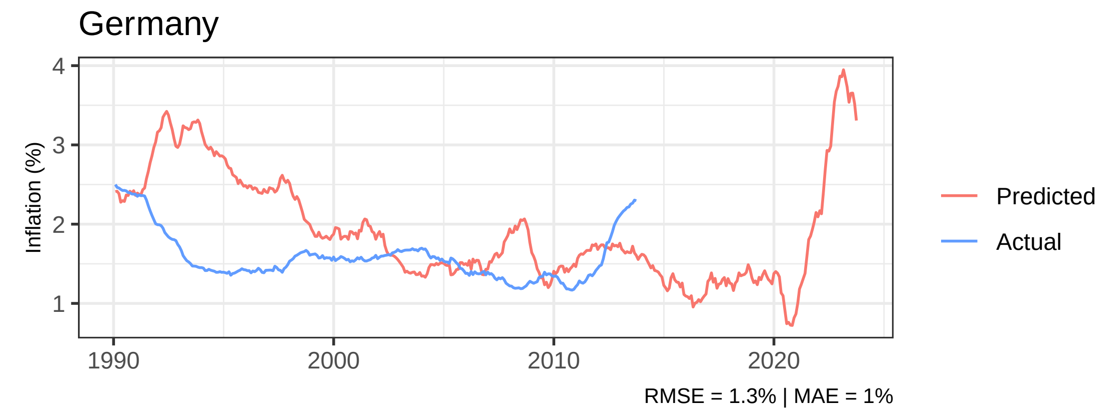

Canada
Inflation (%)
5
4
3
2
1990
2000
2010
2020
Predicted
Actual
RMSE = 2.5% | MAE = 1.8%

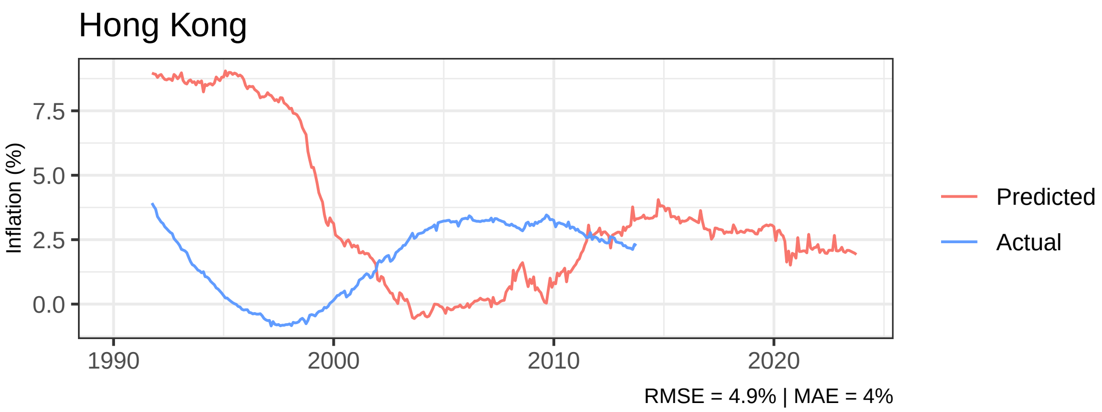

China
Inflation (%)
3
2
1990
2000
2010
2020
Predicted
Actual
RMSE = 1.1% | MAE = 0.9%

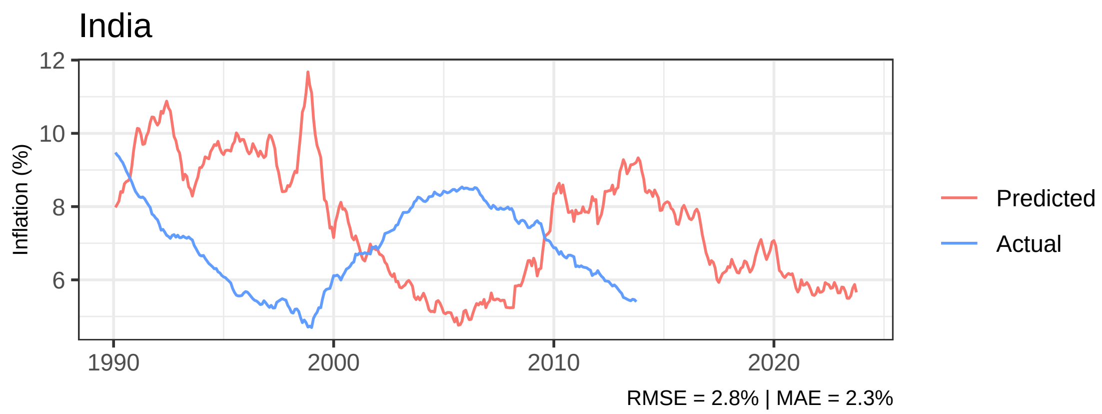

Denmark
Inflation (%)
5
4
3
2
1
1990
2000
2010
2020
Predicted
Actual
RMSE = 2.8% | MAE = 2%

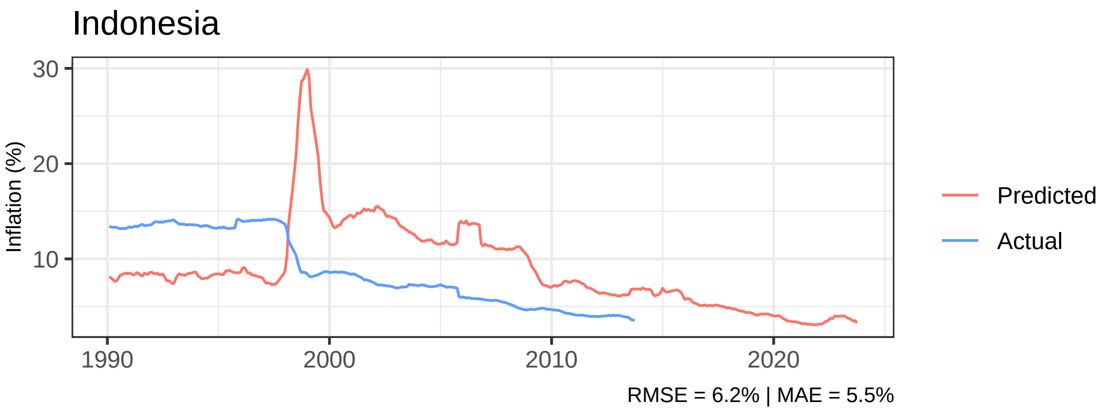

Finland
Inflation (%)
6
4
2
1990
2000
2010
2020
Predicted
Actual
RMSE = 3.4% | MAE = 2.7%

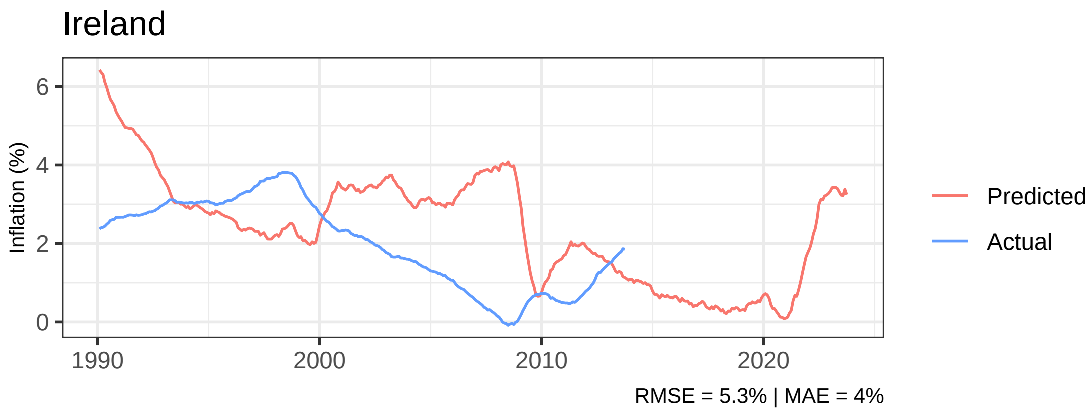

France
Inflation (%)
5
4
3
2
1
1990
2000
2010
2020
Predicted
Actual
RMSE = 3.5% | MAE = 2.6%

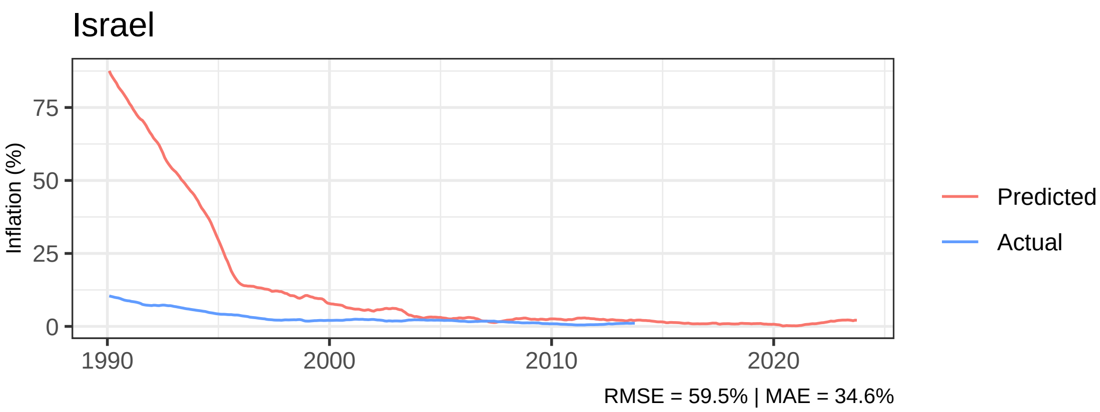

Germany
Inflation (%)
4
3
2
1
1990
2000
2010
2020
Predicted
Actual
RMSE = 1.3% | MAE = 1%

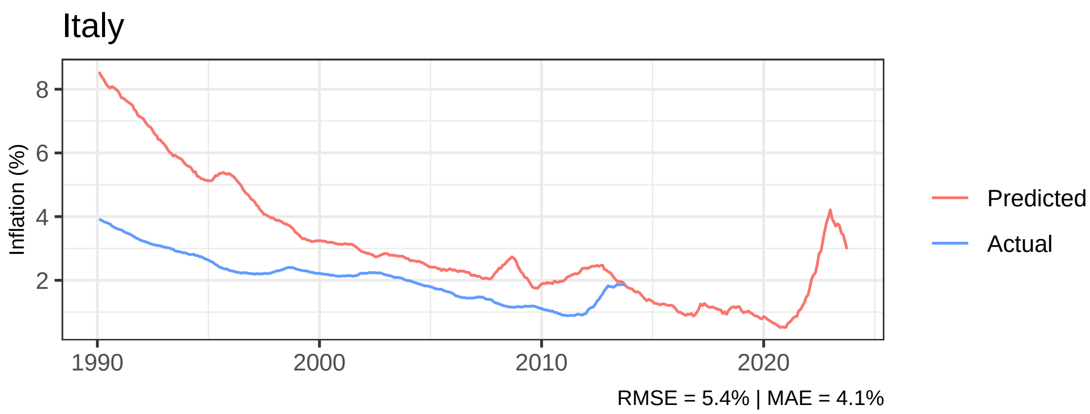

Hong Kong
Inflation (%)
7.5
5.0
2.5
0.0
1990
2000
2010
2020
Predicted
Actual
RMSE = 4.9% | MAE = 4%

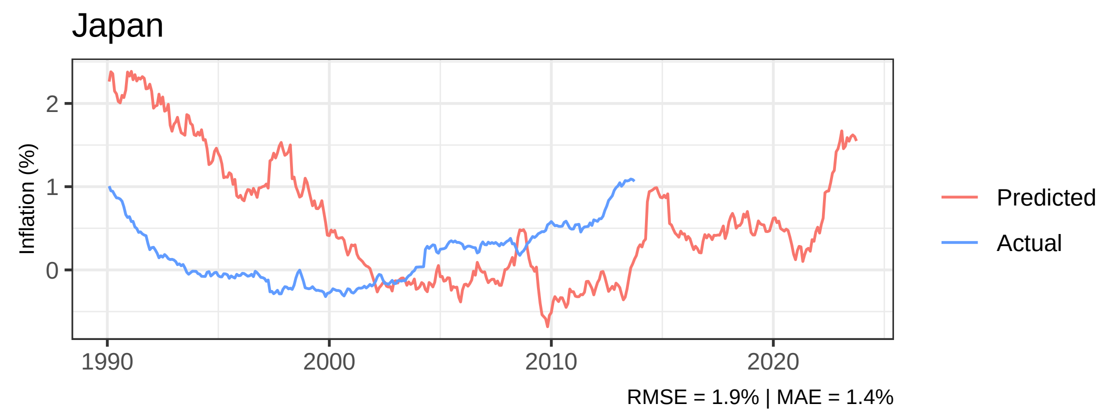

India
Inflation (%)
12
10
8
6
1990
2000
2010
2020
Predicted
Actual
RMSE = 2.8% | MAE = 2.3%

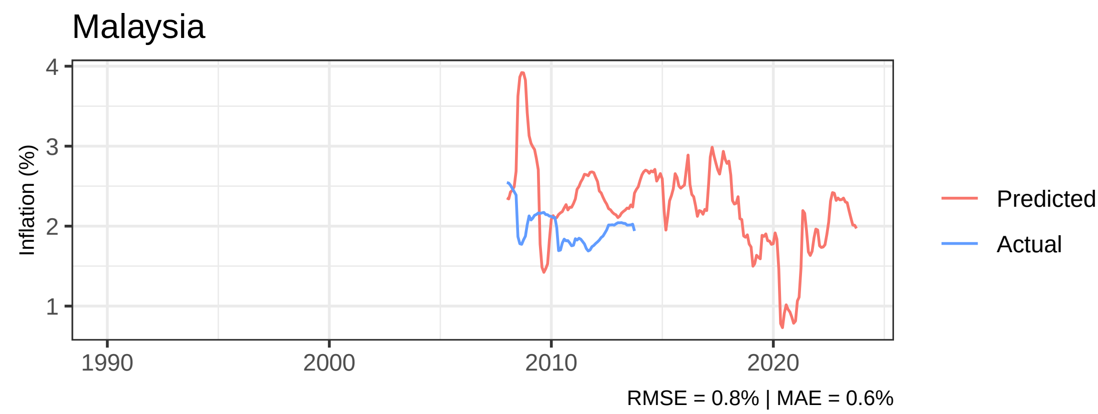

Indonesia
Inflation (%)
30
20
10
1990
2000
2010
2020
Predicted
Actual
RMSE = 6.2% | MAE = 5.5%

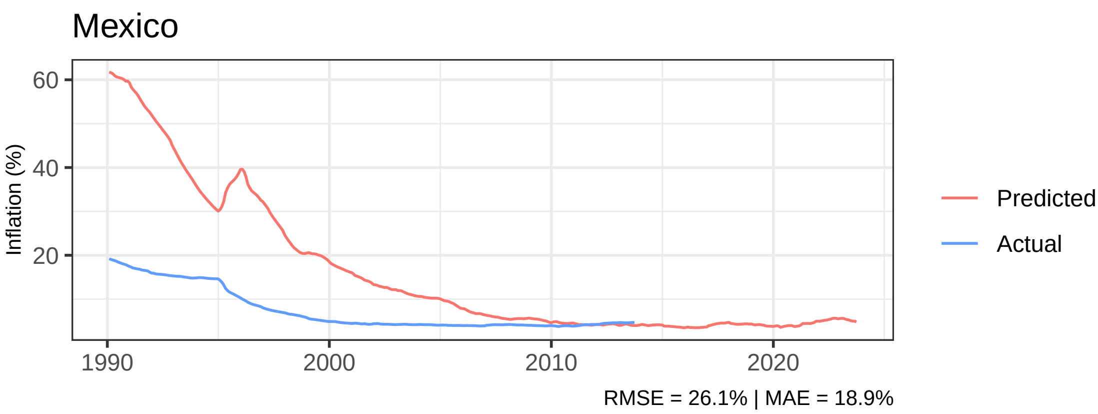

Ireland
Inflation (%)
6
4
2
0
1990
2000
2010
2020
Predicted
Actual
RMSE = 5.3% | MAE = 4%

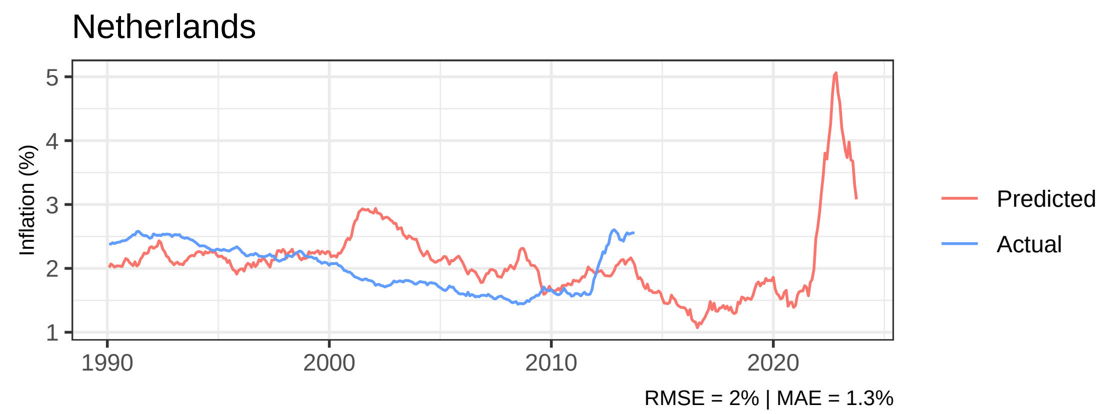

Israel
Inflation (%)
75
50
25
0
1990
2000
2010
2020
Predicted
Actual
RMSE = 59.5% | MAE = 34.6%

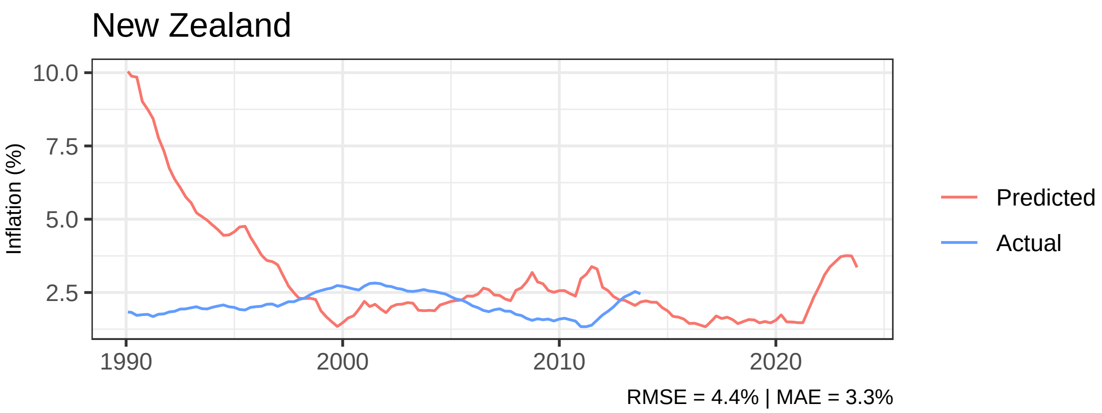

Italy
Inflation (%)
8
6
4
2
1990
2000
2010
2020
Predicted
Actual
RMSE = 5.4% | MAE = 4.1%

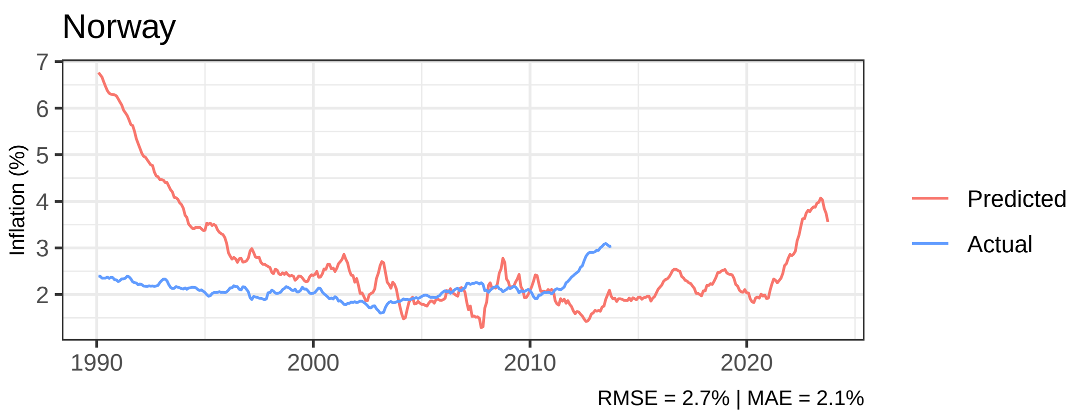

Japan
Inflation (%)
2
1
0
1990
2000
2010
2020
Predicted
Actual
RMSE = 1.9% | MAE = 1.4%

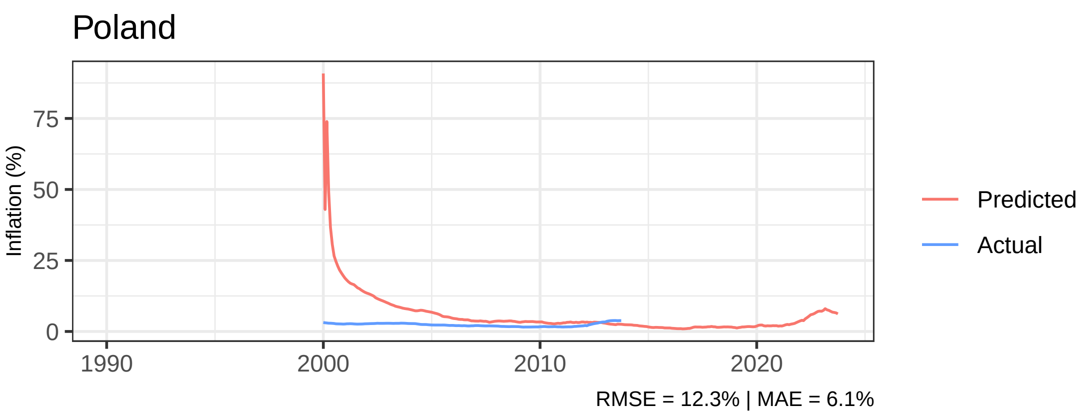

Malaysia
Inflation (%)
4
3
2
1
1990
2000
2010
2020
Predicted
Actual
RMSE = 0.8% | MAE = 0.6%

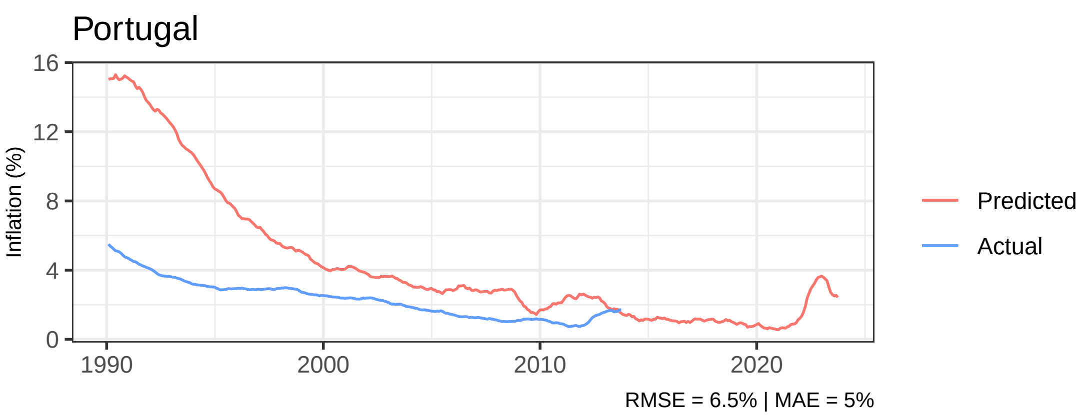

Mexico
Inflation (%)
60
40
20
1990
2000
2010
2020
Predicted
Actual
RMSE = 26.1% | MAE = 18.9%


Netherlands
Inflation (%)
5
4
3
2
1
1990
2000
2010
2020
Predicted
Actual
RMSE = 2% | MAE = 1.3%


New Zealand
10.0
7.5
5.0
2.5
Inflation (%)
1990
2000
2010
2020
Predicted
Actual
RMSE = 4.4% | MAE = 3.3%


Norway
7
6
5
4
3
2
Inflation (%)
1990
2000
2010
2020
Predicted
Actual
RMSE = 2.7% | MAE = 2.1%

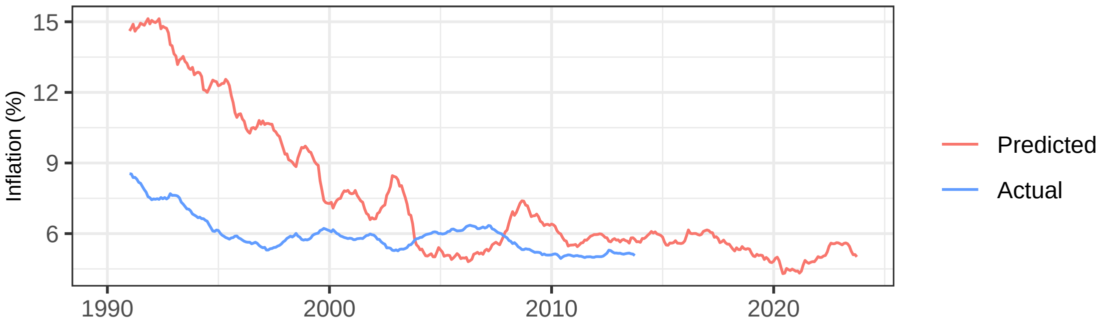

Poland
75
50
25
0
Inflation (%)
1990
2000
2010
2020
Predicted
Actual
RMSE = 12.3% | MAE = 6.1%


Portugal
16
12
8
4
0
Inflation (%)
1990
2000
2010
2020
Predicted
Actual
RMSE = 6.5% | MAE = 5%

Singapore

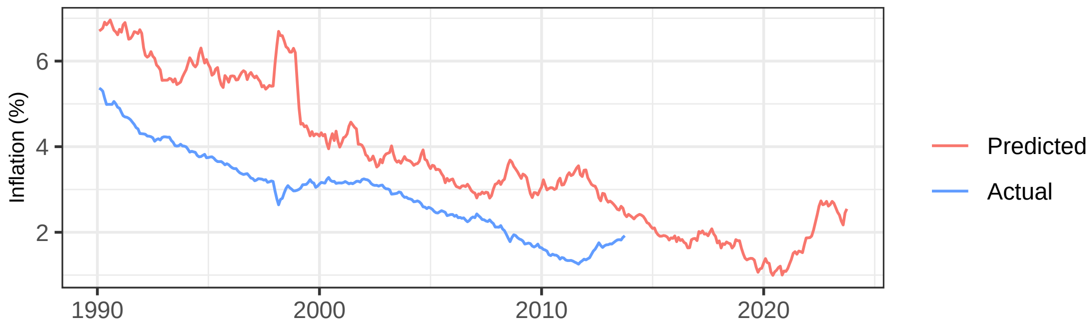
RMSE = 2.4% | MAE = 1.8%

South Africa



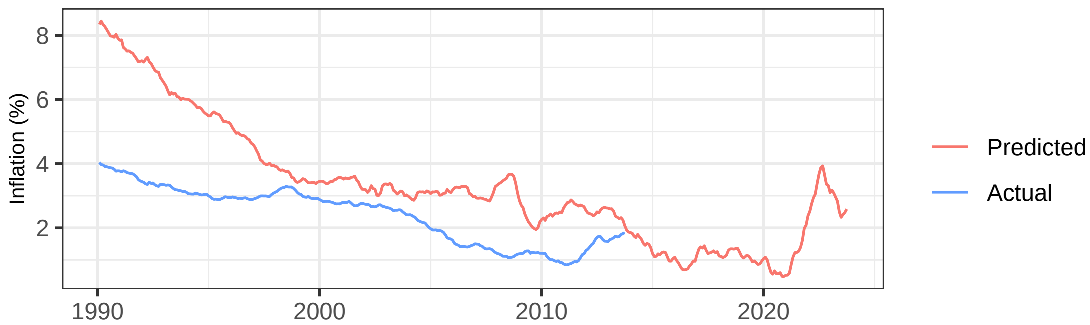
RMSE = 3.6% | MAE = 2.8%

South Korea

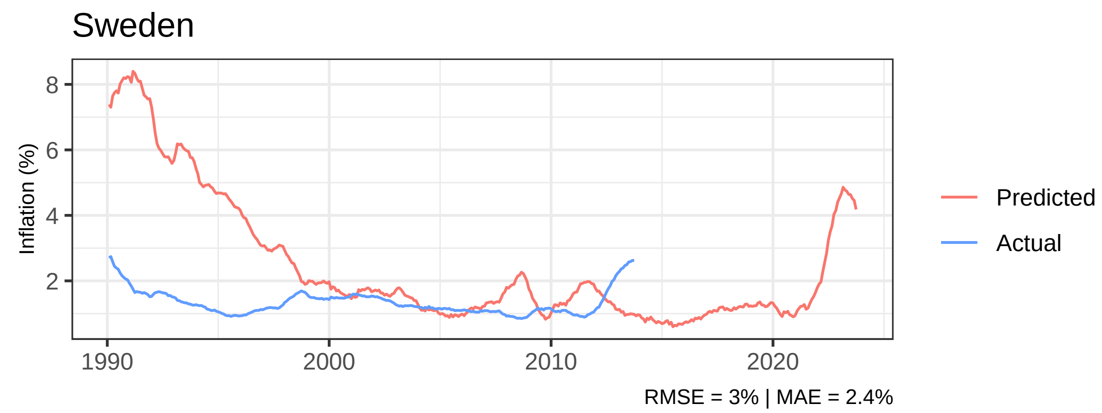


RMSE = 4.6% | MAE = 3.1%

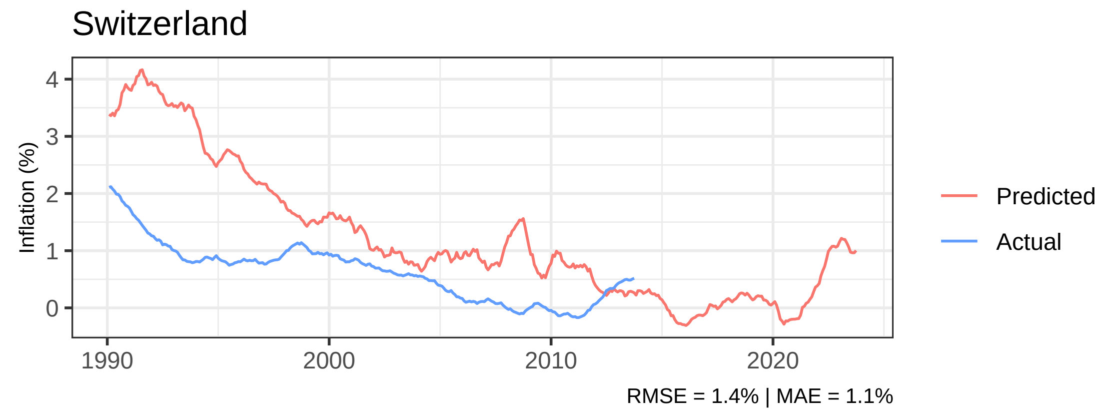
Spain



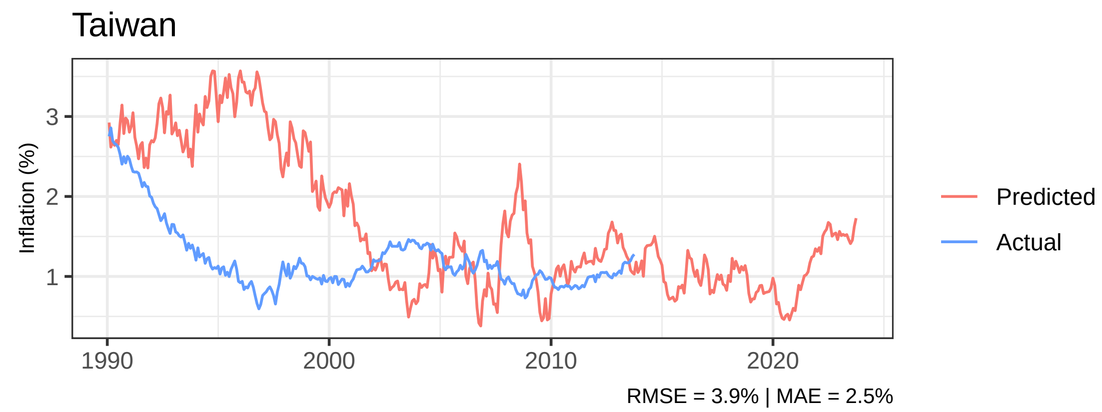
RMSE = 5% | MAE = 4%



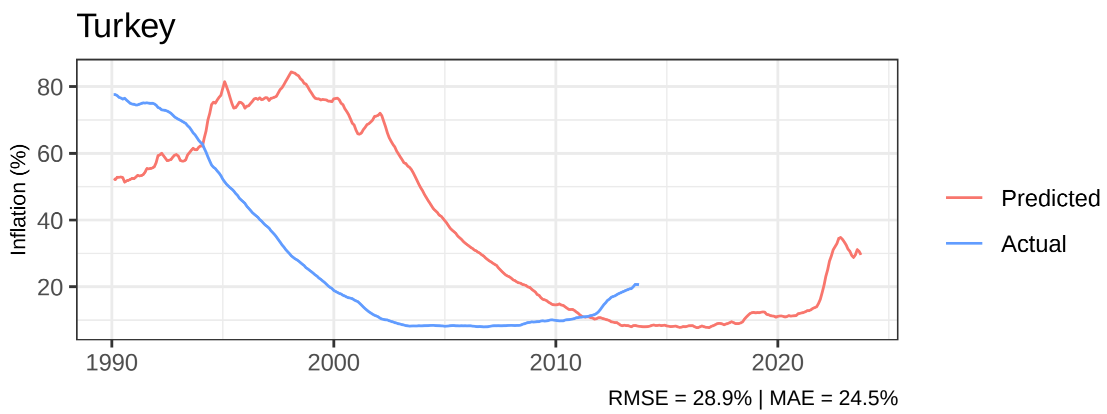
Sweden
Inflation (%)
8
6
4
2
1990
2000
2010
2020
Predicted
Actual
RMSE = 3% | MAE = 2.4%



Switzerland
Inflation (%)
4
3
2
1
0
1990
2000
2010
2020
Predicted
Actual
RMSE = 1.4% | MAE = 1.1%

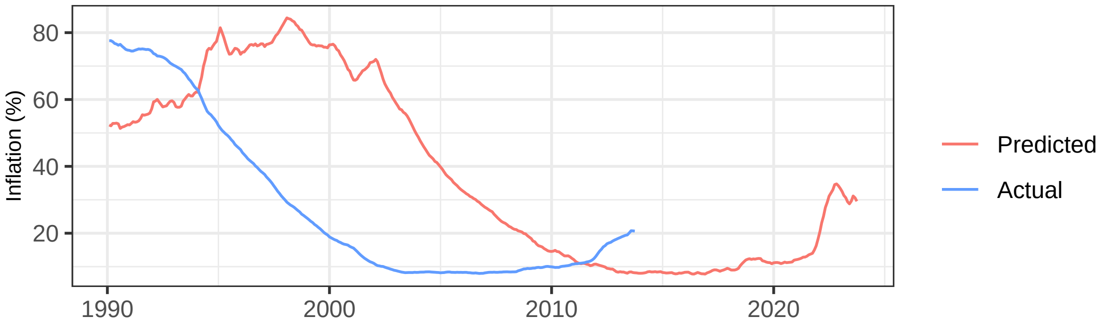


Taiwan
Inflation (%)
3
2
1
1990
2000
2010
2020
Predicted
Actual
RMSE = 3.9% | MAE = 2.5%



Turkey
Inflation (%)
80
60
40
20
1990
2000
2010
2020
Predicted
Actual
RMSE = 28.9% | MAE = 24.5%

Turkey

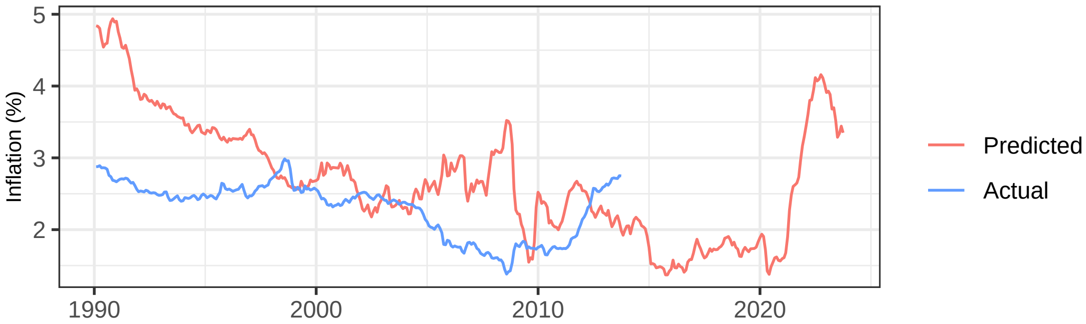


RMSE = 28.9% | MAE = 24.5%

United Kingdom

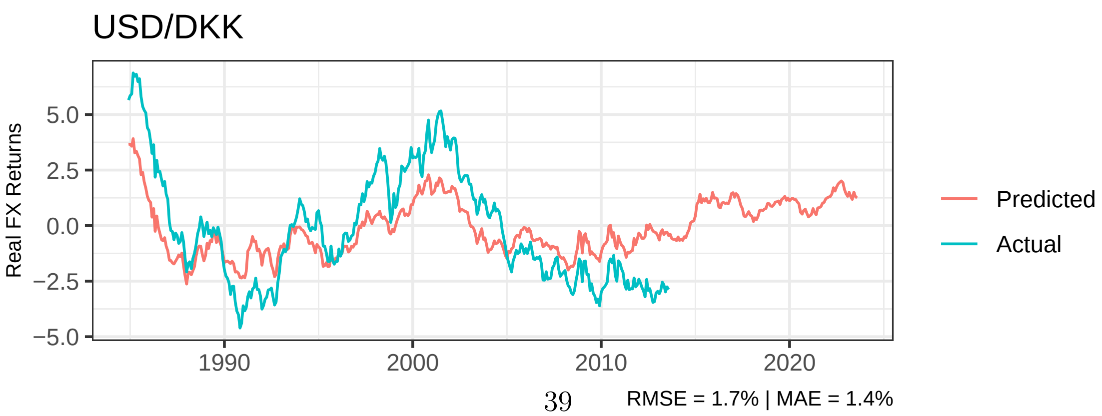
RMSE = 4.6% | MAE = 3.5%

United States of America



RMSE = 2.1% | MAE = 1.5%

Appendix E CMA vs. Actual Series: Real Foreign Exchange Returns



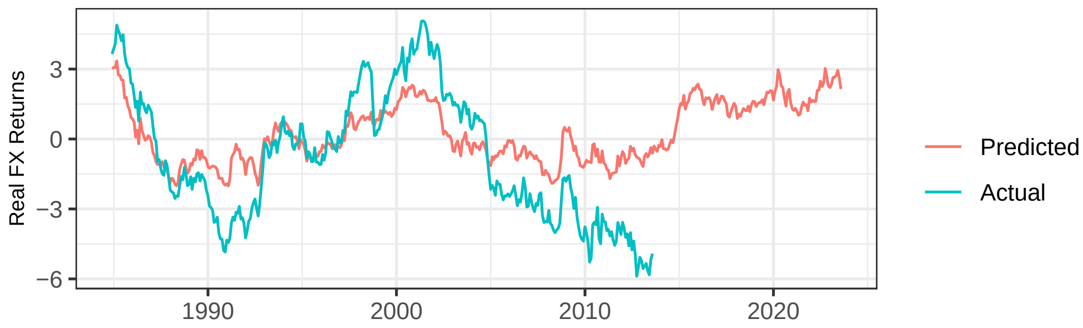

USD/GBP
Real FX Returns
5.0
2.5
0.0
−2.5
1990
2000
2010
2020
Predicted
Actual
RMSE = 1.5% | MAE = 1.2%


USD/INR
Real FX Returns
3
0
−3
1990
2000
2010
2020
Predicted
Actual
RMSE = 3.7% | MAE = 3.2%


USD/JPY
Real FX Returns
10
5
0
−5
1990
2000
2010
2020
Predicted
Actual
RMSE = 2.9% | MAE = 2.3%


USD/MXN
Real FX Returns
5.0
2.5
0.0
−2.5
−5.0
1990
2000
2010
2020
Predicted
Actual
RMSE = 1.9% | MAE = 1.7%

USD/NOK



RMSE = 2% | MAE = 1.6%

USD/NZD

RMSE = 2.3% | MAE = 1.9%

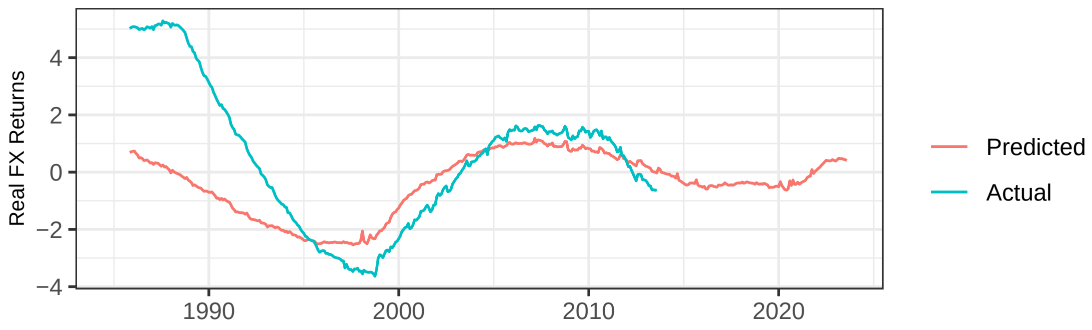
USD/SEK

RMSE = 2.5% | MAE = 2.1%

USD/ZAR

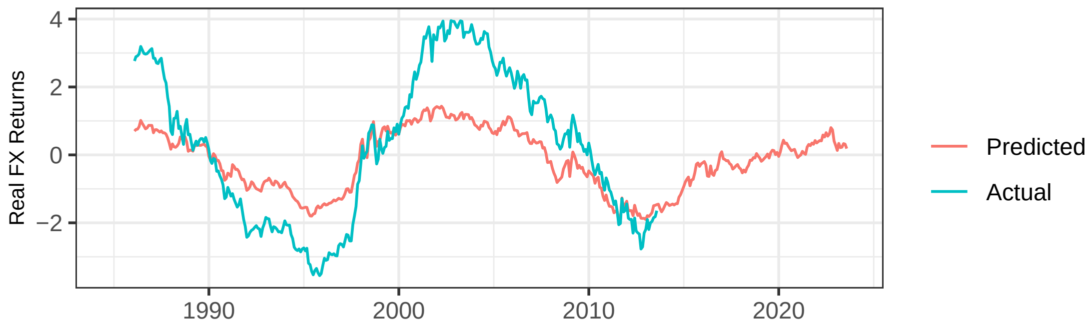
RMSE = 2.9% | MAE = 2.5%

USD/HKD



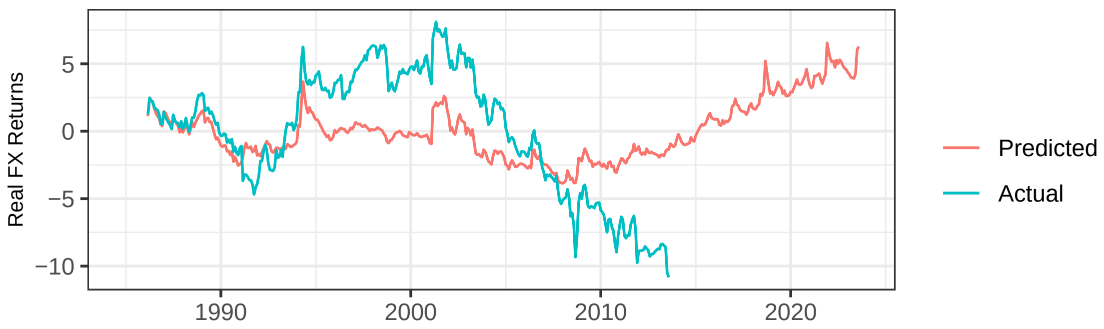
RMSE = 2.1% | MAE = 1.4%

USD/SGD



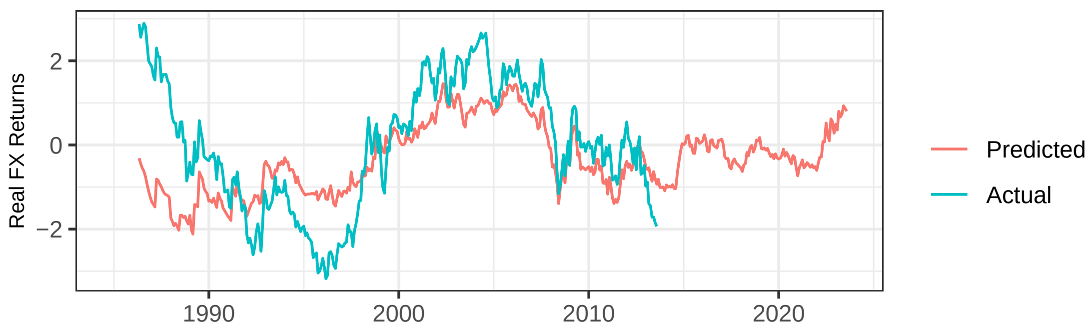
RMSE = 1.4% | MAE = 1.2%

USD/TRY



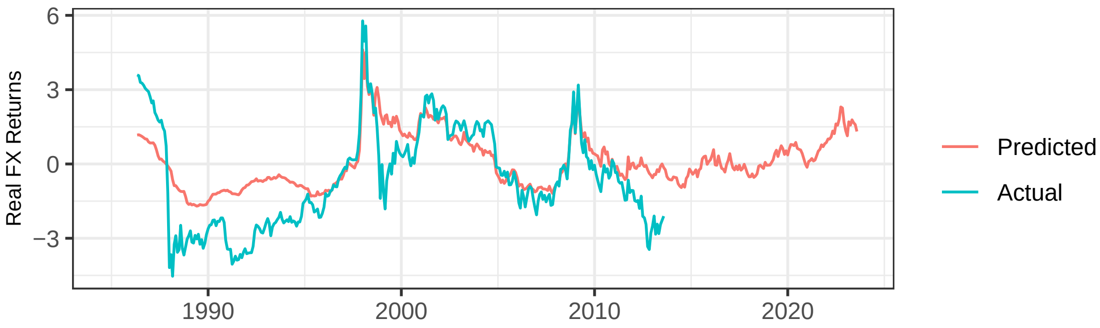
RMSE = 3.7% | MAE = 3.1%

USD/ILS



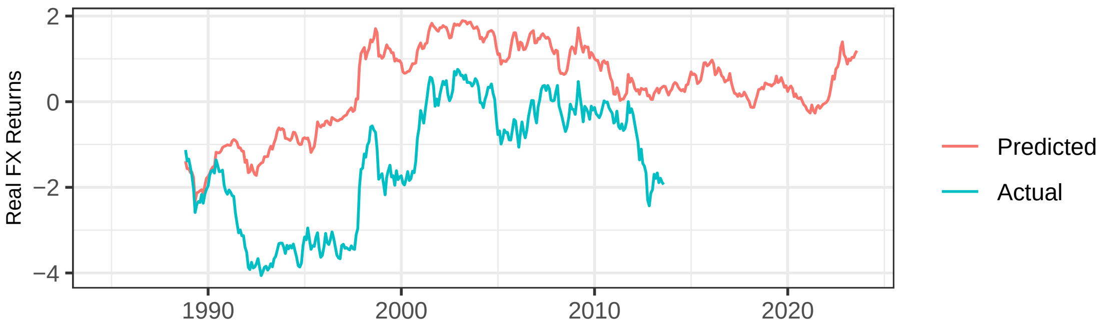
RMSE = 1.2% | MAE = 0.9%

USD/KRW



RMSE = 1.4% | MAE = 1.1%

USD/TWD

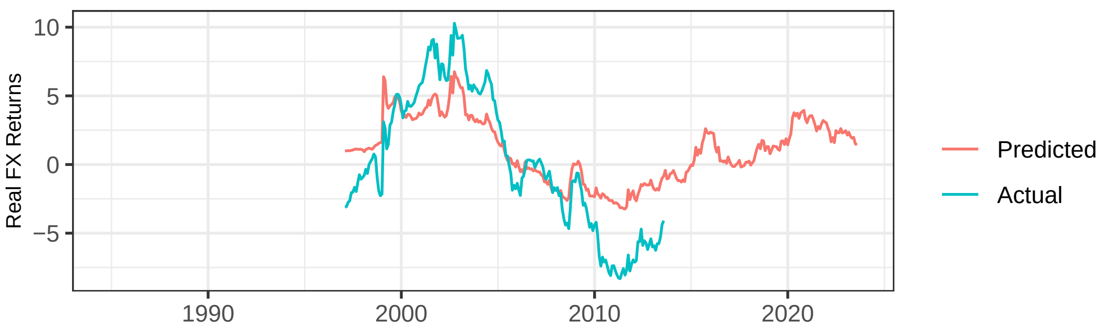


RMSE = 2% | MAE = 1.8%

USD/IDR

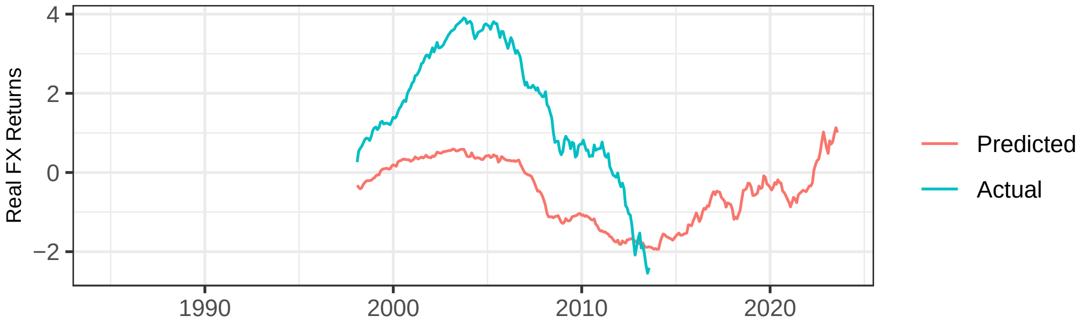
RMSE = 1.8% | MAE = 1.5%

USD/BRL



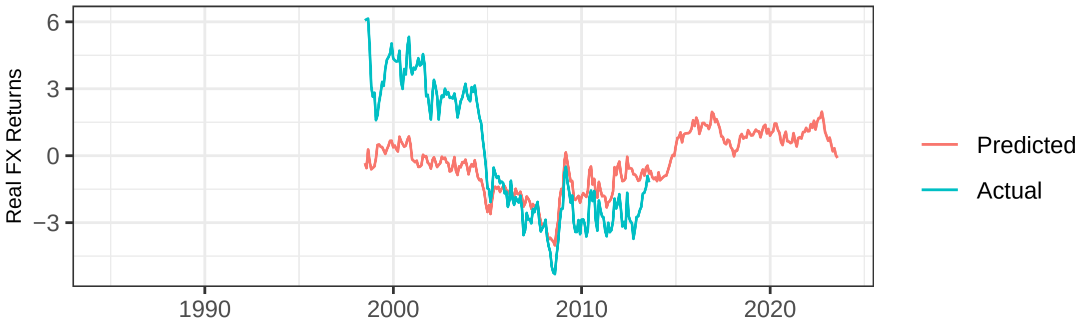
RMSE = 2.8% | MAE = 2.4%

USD/CNY



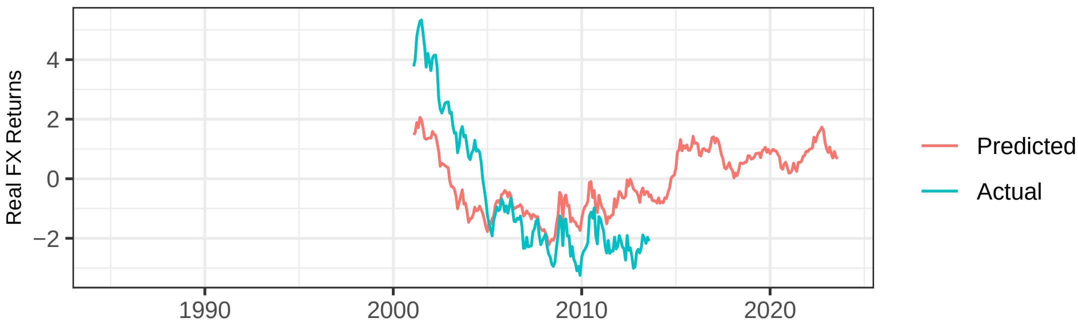
RMSE = 2.2% | MAE = 2.1%

USD/PLN



RMSE = 2.5% | MAE = 2%

USD/EUR



RMSE = 1.6% | MAE = 1.4%

USD/MYR

RMSE = 1.7% | MAE = 1.4%

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