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Competitive Cross-Subsidization*

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Abstract:
Cross-subsidization arises naturally when firms with different comparative advantages compete for consumers with diverse shopping patterns. Firms then face a form of co-opetition, being substitutes for one-stop shoppers and complements for multi-stop shoppers. Competition for one-stop shoppers then drives total prices down to cost, but firms subsidize weak products with the profit made on strong products. While firms and consumers would bene.t from cooperation limiting cross-subsidization (e.g., through price caps), banning below-cost pricing instead increases firms profits at the expense of one-stop shoppers; this calls for a cautious use of below-cost pricing regulations in competitive markets.

Keywords: cross-subsidization, shopping patterns, multiproduct competition, co-opetition.

JEL Classification Numbers: L11, L41.

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1 Introduction

Multi-product firms compete through a variety of pricing strategies, such as bundling[1] or cross-subsidization.[2] Interestingly, while competitive bundling has already been extensively studied,[3] cross-subsidization has instead been mostly studied in the context of regulated or monopolistic markets.[4] Indeed, according to conventional economic theory, cross-subsidization only arises when a firm has substantial market power,[5] even though, cross-subsidization is a common feature in unregulated, competitive markets. For instance, supermarkets often rely on "loss leaders" to lure consumers to their stores; likewise, retail banks offer zero account fees or free travel insurance to gain customers. The prevalence of these practices is such that it has led many countries to prohibit or restrict certain forms of below-cost pricing.[6] OECD (2007), however, argues that these laws are more likely to harm consumers than to benefit them.

Conventional economic theory is thus at odds with real-world practice, and, therefore, does not shed much light on this policy debate. This paper aims to fill this gap by taking into account the diversity of purchasing patterns, and studying its implications for pricing strategies.

As noted further below, the related literature on competitive multiproduct pricing typically assumes that customers engage in "one-stop shopping" and purchase all products from the same supplier. Yet in practice, many customers engage in multi-stop shopping

[1] Namely, offering a discount on some products, conditional on the purchase of others (mixed bundling), or offering selected products only as a bundle (pure bundling).

[2] Namely, pricing some products below cost, compensating the loss with the profits from other products.

[3] See, e.g., Matutes and Regibeau (1992), Armstrong and Vickers (2010), and Zhou (2014).

[4] For instance, cross-subsidization is a well-known feature in telecommunications, energy, and postal markets, where historical incumbents often subsidize their activity in newly liberalized segments with the profits achieved in protected segments.

[5] For instance, in a survey Faulhaber (2005, pp.442) asserts that "under competitive conditions, the issue of cross-subsidy simply does not arise."

[6] In the US, half of the states have adopted laws against below-cost resale, and additional states have adopted specific rules for gasoline markets; see Calvani (2001). In the EU, below-cost resale is banned in Belgium, France, Ireland, Luxembourg, Portugal, and Spain, and is restricted in Austria, Denmark, Germany, Greece, Italy and Sweden.

and rely on several suppliers to fulfill their needs.[7] The choice between these purchasing patterns is driven not only by the diversity and the relative merits of suppliers' offerings, but also by transaction costs. Klemperer (1992) mentions, for example, physical costs, such as transportation costs, as well as non-physical costs, such as the opportunity cost of time. Industrial customers may face many other costs, e.g., purchasing large and small aircraft from different suppliers may increase the costs that an airline faces for pilot training and certification, as well as for spare parts and maintenance services.[8] For the sake of exposition, we will refer to these costs as "shopping costs."[9] Obviously, these costs vary across customers. For example, some consumers may face tighter time constraints and/or dislike shopping, whereas others may be less time-constrained and/or enjoy shopping, and smaller airlines may face relatively higher transaction costs than larger ones. Other things being equal, customers with high transaction costs tend to favor 'one-stop shopping,' whereas others are more prone to 'multi-stop shopping.'

We first note that the diversity of purchasing patterns gives rise to a form of 'co-opetition:' On the one hand, firms offer substitutes for one-stop shoppers who look for the best assortment; on the other hand, they offer complements for multi-stop shoppers, who seek to combine suppliers' best products. We show that this duality drastically affects firms' pricing strategies and can lead to cross-subsidization, even in competitive markets.

Specifically, we consider a setting where two firms offer the same product line (which consists of two products for simplicity) to consumers with heterogeneous shopping costs. Consumers are perfectly informed about prices, and they have an inelastic demand for each product. These assumptions fit markets such as grocery retailing well, and allow us to abstract away from the motivations already highlighted in the literature on cross-subsidization (see the literature review below). Each firm enjoys a comparative advantage

[7] According to the marketing literature, patronizing multiple stores becomes an important pattern in the grocery retail business; see, *e.g.*, Gijsbrechts *et al.* (2008), who conclude that in the U.S. roughly 75% of grocery shoppers regularly shop at more than one store every week.

[8] This feature was mentioned by the European Commission in its decision to block a merger between regional aircraft manufacturers; see Decision No. IV/M053 - Aerospatiale-Alenia/de Havilland, available at http://ec.europa.eu/competition/mergers/cases/decisions/m53_en.pdf.

[9] This terminology is widely adopted in the literature of multiproduct competition – see, e.g., Klemperer and Padilla (1997) and Armstrong and Vickers (2010).

over one product – due, e.g., to lower costs and/or higher quality. For the sake of exposition, we initially assume that firms have similar comparative advantages; that is, their assortments generate the same total surplus, but each firm has a stronger product than its rival. In equilibrium, consumers with high shopping costs engage in one-stop shopping, and competition for these one-stop shoppers drives firms' aggregate prices down to cost. By contrast, consumers with low shopping costs engage in multi-stop shopping and buy each firm's strong product, on which the firm can make a profit. Hence, cross-subsidization arises naturally in this context: Each firm prices its weak product below cost and subsidizes the loss with the profit from its strong product.

This provides some insights on the outcome of co-opetition. On the one hand, aggregate price levels are "competitive:" One-stop shoppers are supplied at cost; if firms could coordinate their pricing strategies, they would raise total prices in order to exploit one-stop shoppers. At the same time, however, lack of coordination over multi-stop shoppers leads to 'double marginalization.' Here this takes the form of excessive cross-subsidization and results in too little multi-stop shopping; limiting cross-subsidization would therefore benefit both firms and consumers.

These insights are quite robust and remain valid in more general settings. We show, in particular, that the analysis applies when the dispersion of shopping costs is limited (as long as both shopping patterns remain present in equilibrium) or when one firm offers a better assortment than the other and thus enjoys market power over one-stop shoppers (as long as the other maintains a comparative advantage on one of the products). We also extend our framework to account for the development of online sales, which we capture as reducing the shopping costs for "internet-savvy" consumers. Interestingly, we find that this not only boosts firms' profit, but also leads to higher prices for multi-stop shoppers.

Finally, to shed some light on the policy debates mentioned above, we consider a variant where below-cost pricing is banned. The equilibrium then involves mixed strategies: Firms sell weak products at cost but randomize prices for their strong products. Banning below-cost pricing then results in higher prices for one-stop shoppers who are no longer supplied at cost, and greater profitability for firms – their (expected) profits are actually more than twice what they were in the absence of a ban. The impact on multi-stop shoppers is less clear-cut. However, when weak products offer relatively low value, there is not much one-stop shopping; in that case, firms are not too concerned about losing sales to one-stop shoppers, and charge higher prices to multi-stop shoppers as well. Depending on the distribution of shopping costs, this reduction in consumer surplus may exceed the increase in firms' profits and thus result in lower total welfare. This suggests that regulations on below-cost pricing in competitive markets should be carefully evaluated.[10]

Related literature. There is a small literature on competitive cross-subsidization spanning industrial organization and behavioral economics. In a setting where consumers are initially unaware of prices, Lal and Matutes (1994) show that firms then advertise a loss-leader product to attract consumers.[11] Ambrus and Weinstein (2008) study symmetric competition for one-stop shoppers, and show that below-cost pricing can arise when consumers have elastic demands exhibiting a specific form of complementarity.[12] Recently, Johnson (2014) considers one-stop shoppers who may underestimate their needs; below-cost pricing then arises when consumers have different biases across products.[13] By contrast, in our paper, cross-subsidization arises even with independent inelastic demands and perfectly informed consumers, and is driven by the diversity of shopping patterns.[14]

The paper is organized as follows. Section 2 illustrates the main intuition by way of a simple example. Section 3 develops our baseline framework, with symmetric comparative advantages and a wide range of transaction costs. Section 4 presents our main insights – in equilibrium, both shopping patterns coexist, and firms engage in cross-subsidization despite selling their assortments at cost. Section 5 shows that the insights remain valid

[10]By contrast, Chen and Rey (2012) show that banning below-cost pricing in concentrated markets can discipline the pricing behavior of a dominant firm competing with smaller firms; such a ban then benefits both consumers and smaller rivals, and enhances social welfare.

[11]In equilibrium consumers stop searching after the first visit, and are thus one-stop shoppers.

[12]By contrast, below-cost pricing does not arise when consumers have inelastic demands, or when consumers have sufficiently diverse preferences.

[13]Johnson also finds that banning below-cost pricing harms consumers, as is the case for one-stop shoppers in our setting, see the discussion at the end of Section 6.

[14]The literature on competitive bundling (see footnote 1) also accounts for the diversity of shopping patterns, there driven by heterogeneous preferences over product characteristics; mixed bundling then constitutes an effective price-discrimination device, but cross-subsidization does not arise. By focussing on homogeneous valuations, and by accounting instead for the heterogeneity of transaction costs, we offer a complementary approach, motivated by a different policy question.

when transaction costs are bounded (as long as both shopping patterns arise) and when firms have asymmetric comparative advantages. Section 6 studies the impact of a ban on below-cost pricing and Section 7 concludes.

2 A simple example

A numerical example illustrates the main intuition. Consumers wish to buy two goods, A and B, which can both be supplied by two firms, 1 and 2. Firm 1 enjoys a lower unit cost for good A whereas firm 2 does so for good B: $c_1^A = c_2^B = \$10 < c_2^A = c_1^B = \$30$. Finally, consumers face a shopping cost s, reflecting the opportunity cost of the time spent in traffic, parking, selecting products, checking out, and so forth; it may also account for consumers' taste or dislike for shopping.[15]

Suppose first that all consumers face a "high" shopping cost, larger than the efficiency gain: $s \geq \Delta c = \$20$. In equilibrium, consumers then behave as one-stop shoppers, i.e., they buy both products from the same firm, and thus only the total basket prices, P_1 and P_2, matter. As the firms face the same total cost of \$40, Bertrand-like competition drives the basket price down to this cost: $P_1 = P_2 = \$40$.

Suppose instead that the shopping cost s is sufficiently low such that, in equilibrium, consumers behave as multi-stop shoppers and purchase each product at the lowest available price. Asymmetric Bertrand competition then leads firms to sell weak products at cost, i.e., $p_2^A = p_1^B = \$30$, and strong products at a price equal to (or just below) the rival's cost minus consumers' shopping cost: $p_1^A = p_2^B = \$30 - s$.

Hence, in these two situations, where all consumers adopt the same shopping pattern, firms have no incentive to engage in cross-subsidization.[16] Suppose now that half of the consumers face a high shopping cost, $s_H = \$20$, whereas the others have a low shopping cost, $s_L = \$2$. As before, fierce price competition dissipates profits from one-stop shoppers, and drives basket prices down to total cost:

$$p_1^A + p_1^B = p_2^A + p_2^B = \$40.$$

[15] Consumers' values for A and B are assumed to be larger than production and shopping costs.

[16] In the first case, where only total basket prices matter, firms may as well offer each product at cost.

Yet, as each firm has a cost advantage in one market, it can sell its strong product at a lower price than its rival. Keeping the total price constant for one-stop shoppers, it suffices to undercut the rival's weak product by the amount of $s_L = \$2$ to attract multi-stop shoppers. It follows that the equilibrium prices are given by

$$p_1^A = p_2^B = \$19, \; p_1^B = p_2^A = \$21.$$

That is, each firm sells its weak product below cost ($\$21 < \$30$) and compensates the loss with the profit from the strong product ($\$19 > \$10$). This pricing strategy does not affect the shopping behavior of high-cost consumers (who still face a total price of \$40), but generates a positive profit from multi-stop shoppers, who buy A from 1 and B from 2 as $p_1^A + p_2^B = \$38 < \$40$, giving each firm a positive margin of $p_1^A - c_1^A = p_2^B - c_2^B = \$9$.

3 Baseline model

We now consider more general supply and demand conditions. Consumers are willing to buy one unit of A and one unit of B. Each firm $i \in \{1, 2\}$ can produce a variety of each good, A_i and B_i, at constant unit costs c_i^A and c_i^B. Consumers have homogeneous preferences, and derive utility u_i^j from firm i's variety of good $j = A, B$.[17]

Throughout the analysis we assume that firm 1 enjoys a comparative advantage in the supply of good A, whereas firm 2 enjoys a comparative advantage for good B. This may reflect a specialization in different product lines, and be driven by a better quality (i.e., $u_1^A > u_2^A$), a lower cost (i.e., $c_1^A < c_2^A$), or a combination of both. For the sake of exposition we initially focus on the case where firms enjoy the same comparative advantage on their strong products:

$$u_1^A - c_1^A - \left(u_2^A - c_2^A\right) = u_2^B - c_2^B - \left(u_1^B - c_1^B\right) \equiv \delta > 0, \tag{1}$$

implying that their baskets offer the same total value:[18]

$$u_1^A - c_1^A + u_1^B - c_1^B = u_2^A - c_2^A + u_2^B - c_2^B \equiv w > \delta. \tag{2}$$

[17]While we focus here on independent demands for A and B, the analysis carries over when there is partial substitution or complementarity, that is, when the utility derived from enjoying both A_i and B_h is either lower or higher than $u_i^A + u_h^B$.

[18]That $w > \delta$ reflects the assumption that A_2 and B_2, despite offering less value than A_1 and B_2,

Our key modelling feature is that consumers incur a shopping cost, s, to visit a firm, and that this cost varies across consumers, reflecting the fact they may be more or less time-constrained, or value the shopping experience in different ways. Intuitively, consumers with high shopping costs favor one-stop shopping, whereas those with lower shopping costs can take advantage of multi-stop shopping. Shopping patterns are however endogenous and depend on firms' prices. We allow for general distributions of this shopping cost s, characterized by a cumulative distribution function $F(\cdot)$ with a continuous, positive density function $f(\cdot)$ over $\mathbb{R}_+$ and a strictly increasing inverse hazard rate $h(\cdot) \equiv F(\cdot)/f(\cdot)$.

Finally, we assume that firms compete in prices; that is, the firms simultaneously set their prices, $\left(p_1^A, p_1^B\right)$ and $\left(p_2^A, p_2^B\right)$,[19] and then consumers, having observed all prices, make their shopping decisions. We will look for the subgame-perfect Nash equilibria of this game.

4 Competitive cross-subsidization

A consumer is willing to buy both products from firm i only if the value of the basket $A_i - B_i$ exceeds the consumer's shopping cost, that is, if:

$$v_i \equiv u_i^A + u_i^B - p_i^A - p_i^B = w - m_i \geq s,$$

where $m_i \equiv p_i^A - c_i^A + p_i^B - c_i^B$ denotes firm i's total margin. However, one-stop shoppers patronize firm i only if it offers better value than its rival, firm j; as the two baskets generate the same surplus, this amounts to charging a lower total margin than the rival:

$$v_i > v_j \Leftrightarrow m_i < m_j.$$

nevertheless generate a surplus:

$$u_1^A - c_1^A = u_2^B - c_2^B = \frac{w+\delta}{2} > u_2^A - c_2^A = u_1^B - c_1^B = \frac{w-\delta}{2} > 0.$$

[19] We show later on that allowing for pure or mixed bundling does not affect the analysis; see the remark at the end of Section 4.

Consumers may, however, prefer buying both strong products, that is, purchasing A_1 from firm 1 and B_2 from firm 2,[20] rather than patronizing a single firm. Such multi-stop shopping involves an extra shopping cost s and yields a total value

$$v_{12} \equiv u_1^A - p_1^A + u_2^B - p_2^B = w + \delta - \rho_1 - \rho_2,$$

where $\rho_1 \equiv p_1^A - c_1^A$ and $\rho_2 \equiv p_2^B - c_2^B$ respectively denote firm 1 and 2's margins on strong products. Consumers favor multi-stop shopping over one-stop shopping if the additional value from mixing-and-matching exceeds the extra shopping cost, i.e., if

$$s \leq \tau \equiv v_{12} - \max\{v_1, v_2\}.$$

We first show that, in equilibrium, multi-stop and one-stop shopping patterns coexist, with multi-stop shoppers buying strong products and competition for one-stop shoppers driving firms' basket prices down to cost:

Lemma 1 *In equilibrium:*

- *(i) There are both multi-stop shoppers and one-stop shoppers.*
- *(ii) Multi-stop shoppers buy firms' strong products, A_1 and B_2.*
- *(iii) Firms sell their assortments at cost: their total margins are $m_1 = m_2 = 0$.*

Proof. See Appendix A. ■

The first two insights are intuitive. Consumers with very small shopping costs (s close to 0) are willing to visit both firms so as to combine products with better value. Conversely, consumers with high shopping costs (s close to w, and thus such that $s > \delta$) are willing to visit at most one firm. The last insight follows directly from Bertrand-like competition for one-stop shoppers, for which both firms are symmetric and offer the same value w by pricing at cost.

Thanks to Lemma 1, the equilibrium characterization is fairly simple. As one-stop shopping yields a consumer value $v_i = w$, multi-stop shoppers are those consumers with a shopping cost $s < \tau = \delta - \rho_1 - \rho_2$, whereas one-stop shoppers have a shopping cost

[20] In equilibrium, consumers never combine the weak products, A_2 and B_1; see Appendix A.

such that $\tau < s < w$. Furthermore, as firms only derive a profit from selling their strong products to multi-stop shoppers, firm i's profit can be expressed as

$$\pi_i (\rho_1, \rho_2) = \rho_i F(\tau) = \rho_i F(\delta - \rho_1 - \rho_2). \tag{3}$$

It is obviously optimal for firm i to charge a positive margin on its strong product, i.e., $\rho_i > 0$. As the basket is offered at cost (i.e., $m_i = 0$), this implies that firm i sells its weak product *below cost.* Letting $\mu_i \equiv m_i - \rho_i$ denote firm i's margin on its weak product, we thus have:

Corollary 1 *In equilibrium, firms sell their weak products below cost:* $\mu_1, \mu_2 < 0$.

The intuition is fairly simple. Obviously, it is not optimal for firm i to sell its strong product below cost, as this would yield a negative profit. Suppose now that firm i sells both products at cost, and consider the following "cross-subsidization" deviation: Keeping the total margin equal to zero, firm i slightly raises the margin on its strong product, reducing that on its weak product by the same amount. This deviation does not affect the profit generated by one-stop shoppers, but generates a profit from multi-stop shoppers, who now pay a higher price for the strong product. As the deviation decreases the value of multi-stop shopping, it may also induce some consumers to switch to one-stop shopping. But this does not affect firm i, which obtains zero profit from these consumers, regardless of which firm they go to. Hence, cross-subsidization is profitable.

The monotonicity of the inverse hazard rate $h(\cdot)$ ensures that the profit function given by (3) is strictly quasi-concave, and the "aggregative game" nature of this profit function then ensures that the equilibrium is unique and symmetric:[21] Both firms charge the same positive margin on their strong products, $\rho_i = \rho^*$, and the same negative margin on their weak products, $\mu_i = -\rho^*$, where the equilibrium margin ρ^* is characterized by the first-order condition

$$\rho^* = h(\delta - 2\rho^*).$$

The equilibrium threshold for multi-stop shopping, τ^*, thus satisfies

$$\tau^* = \delta - 2\rho^* = \delta - 2h(\tau^*),$$

[21] See Selten (1970). Here, firm i's profit is a function of its own margin ρ_i and of the threshold τ, which only depends on the sum of the two firms' margins, $\rho_1 + \rho_2$.

and is therefore given by $\tau^* = j^{-1}(\delta)$, where

$$j(x) \equiv x + 2h(x)$$

is strictly increasing. Finally, in equilibrium each firm earns a profit equal to

$$\pi^* = \rho^* F(\tau^*) = h(\tau^*) F(\tau^*).$$

The following proposition confirms the existence of this equilibrium:

Proposition 1 *There exists a unique equilibrium, in which both firms sell their weak products below cost and cross-subsidize them by their strong products. More precisely:*

(i) Consumers with a shopping cost $s < \tau^*$, where

$$0 < \tau^* \equiv j^{-1}(\delta) < \delta\,(< w),$$

engage in multi-stop shopping (they visit both firms and buy their strong products), whereas consumers with a shopping cost $\tau^* < s < w$ engage in one-stop shopping and buy both products from the same firm (either one).

(ii) Both firms offer their baskets at cost (i.e., $m_1^* = m_2^* = 0$), but charge the same margin $\rho^* = h(\tau^*) > 0$ on strong products and the same margin $-\rho^* < 0$ on weak products.

Proof. See Appendix B. ■

As mentioned in the introduction, here firms face a form of co-opetition: They offer substitute products to one-stop shoppers, but at the same time they offer complementary products to multi-shop shoppers. Indeed, the firms' assortments are perfect substitutes for one-stop shoppers; as is standard in such a case, fierce competition for these consumers drives total margins down to zero. Yet, firms make a profit on multi-stop shoppers, who visit both firms in order to buy their strong products; furthermore, a *reduction* in the margin of one firm's strong product encourages additional consumers to switch from one-stop to multi-stop shopping, thereby *increasing* the other firm's profit. As is usual with complements, the prices of strong products are subject to double marginalization problems: When contemplating an increase in the price of its strong product, firm i balances between the positive impact on its margin ρ_i and the adverse impact on multi-stop shopping, but ignores the negative effect of this reduction in multi-stop shopping activity on the other firm's profit; firms would therefore benefit from a mutual moderation of the margins charged on these products. Interestingly, while double marginalization is usually associated with excessively high price *levels*, here it yields excessively distorted price *structures*: Firms' total prices remain at cost, but they engage in excessive cross-subsidization, compared with what would maximize their joint profit; keeping total margins equal to zero, firms' joint profit when charging a margin ρ on strong products is given by

$$2\rho F(\tau) = 2\rho F(\delta - 2\rho),$$

and is maximal for some $\hat{\rho} < \rho^*$.[22]

Letting firms negotiate price cap agreements would enable them to alleviate double marginalization problems by limiting cross-subsidization.[23] If, for instance, they introduce a cap $\hat{\rho}$ on the margins charged on strong products, then in the resulting equilibrium: (i) competition for one-stop shoppers still drives total margins down to zero (i.e., $m_1 = m_2 = 0$); but (ii) price caps limit cross-subsidization: Firms charge $\rho_1 = \rho_2 = \hat{\rho}\,(< \rho^*)$ on strong products and $\mu_1 = \mu_2 = -\hat{\rho}$ on weak ones. Despite an increase in the prices of weak products, the adoption of price caps would nevertheless benefit both consumers and firms. Consumers opting for one-stop shopping would remain supplied at cost, and reducing the prices on strong products would not only benefit multi-stop shoppers but, by limiting double marginalization, would also increase the profit achieved on these multi-stop shoppers.

Remark: Shopping costs and complements. At first glance, that shopping costs generate complementarities in firms' products might not come as a surprise. Indeed, although consumers have independent demands for goods A and B, as one might expect, one-stop shopping introduces a complementarity between the products offered *within a firm*: Cutting the price of A_i, say, is likely to steer one-stop shoppers towards firm i, which in turn boosts the sales of the firm's other product, B_i. This form of complementarity is not specific to our setting and has already been documented, not only in marketing and

[22] A standard revealed preference argument yields $\hat{\rho}F(\delta - 2\hat{\rho}) > \rho^* F(\delta - 2\rho^*) > \hat{\rho}F(\delta - \rho^* - \hat{\rho})$, implying $\hat{\rho} < \rho^*$.

[23] For a discussion of such commercial cooperation and price cap agreements, see Rey and Tirole (2013); Lerner and Tirole (2015) also provide a discussion of price commitments in the context of standard setting.

retailing,[24] but in many other areas as well.[25] More interestingly, however, here multi-stop shopping introduces a complementarity *across firms*, namely, between their strong products: Cutting the price of one firm's strong product induces marginal consumers to switch from one-stop to multi-stop shopping, which boosts the sales of the other firm's strong product.[26]

Remark: Bundling. As consumers have homogeneous valuations, there is no scope here for tying and (pure or mixed) bundling. For instance, if one firm ties both products together physically, consumers are forced to engage in one-stop shopping, and price competition for one-stop shoppers leads to zero profit. A similar reasoning applies to pure bundling when products are costly, to such an extent that it does not pay to add one's favorite variety to a bundle. In principle, a firm may also engage in mixed bundling, and offer three prices: one for its strong product, one for the weak product, and one (involving a discount) for the bundle. However, as one-stop shoppers only purchase the bundle, and multi-stop shoppers only buy the strong product, no consumer will ever pick the weak product on a stand-alone basis. Hence only two prices matter here: the total price for the bundle, and the stand-alone price for the strong product. As these prices can be implemented using the stand-alone prices for the two products, offering a bundled discount, in addition to these stand-alone prices, cannot generate any additional profit.

5 Extensions

In our baseline model, below-cost pricing emerges as the result of two forces: Head-to-head competition for one-stop shoppers drives total basket prices down to aggregate costs, whereas market power over multi-stop shoppers yields positive margins on strong

[24] See, e.g., Messinger and Narasimhan (1997).

[25] These include public services (see, e.g., Dykman (1995) for a study of one-stop-shop-centers set-up by the US Department of Labor to provide employment and related social services), health care (see, e.g., Glick (2007) for multispecialty dental offices or Snow (1996) for long-term-care and managed-care organizations) and legal services (see, e.g., Bahls (1990)).

[26] A similar complementarity for multi-stop shoppers arises when shopping patterns are driven by heterogeneous preferences rather than transaction cost differences; see Armstrong and Vickers (2010).

products. We now show that these insights carry over as long as both shopping patterns arise in equilibrium.

5.1 Bounded shopping costs

The baseline model assumes a widespread dispersion of consumers' shopping costs, spanning the entire range from "pure multi-stop shoppers" (e.g., consumers with $s = 0$ will always choose the best value offered for each product) to "pure one-stop shoppers" (e.g., consumers with $s \geq \delta$ will never visit a second firm). To make a robustness check, suppose now that consumer shopping costs range from $\underline{s} \geq 0$ to $\overline{s} \leq w$,[27] say, and consider a candidate equilibrium with active one-stop and multi-stop shoppers. It is straightforward to check that, as before, (i) competition for one-stop shoppers drives total margins down to zero: $m_1^* = m_2^* = 0$, and (ii) multi-stop shoppers buy the strong products. Hence, firms derive their profits only from multi-stop shoppers, and firm i's profit is still given by (3). Ruling out local deviations then leads to the same characterization as before: $\rho_1^* = \rho_2^* = \rho^* = h(\tau^*)$, where $\tau^* = j^{-1}(\delta)$.

The following propositions confirm that this equilibrium exists whenever consumers' shopping costs are sufficiently diverse. By contrast, when shopping costs are all low enough, active consumers systematically visit both stores and only buy strong products, which firms price above cost. Conversely, when shopping costs are all high enough, consumers visit at most one firm, and symmetric Bertrand competition leads both firms to offer the basket at cost.

Formally, if shopping costs are bounded above by $\overline{s}$, we have:

Proposition 2 *Suppose that consumer shopping costs are distributed over* $[0, \overline{s}]$*, where* $\overline{s} > 0$*. Then:*

- *If* $\overline{s} > j^{-1}(\delta)$*, there exists a unique equilibrium, with both types of shopping patterns and the same prices as in the baseline model.*

- *If instead* $\overline{s} \leq j^{-1}(\delta)$*, there exist multiple equilibria. In each equilibrium: (i) only multi-stop shopping arises; and (ii) weak products are offered at below-cost prices,*

[27] Consumers with shopping costs exceeding w would never visit a firm anyway.

but firms only sell their strong products, with a positive margin ranging from $h(\bar{s})$ to $\delta - \bar{s} - h(\bar{s})$.

Proof. See Online Appendix A.1. ■

Hence, while firms always price their weak products below cost, it is only when some consumers have high enough shopping costs, namely, when $\bar{s} > j^{-1}(\delta)$, that cross-subsidization actually occurs. Otherwise, all consumers patronize both firms and only buy strong products – in the limit case $\bar{s} = 0$, where consumers incur no shopping costs, each firm earns a margin of up to δ on its strong product, reflecting its comparative advantage, as in standard asymmetric Bertrand competition.

If instead shopping costs are bounded below by $\underline{s} > 0$, we have:

Proposition 3 *Suppose that consumer shopping costs are distributed over $[\underline{s}, +\infty)$, where $\underline{s} < w$. Then:*

- *If $\underline{s} < \delta/3$, there exists a unique equilibrium, with both types of shopping patterns and the same prices as in the baseline model.*
- *If instead $\underline{s} > \delta$, there exist multiple equilibria in which: (i) only one-stop shopping arises, and (ii) firms make zero profit.*
- *Finally, if $\delta/3 \leq \underline{s} \leq \delta$, both types of equilibria coexist.*[28]

Proof. See Online Appendix A.2. ■

Thus, cross-subsidization arises in equilibrium as long as some consumers have a shopping cost lower than the extra value δ brought about by strong products, and it does arise for certain when some consumers have a low enough shopping cost (namely, lower than $\delta/3$).

5.2 Market power

Another feature of our analysis is that firms want to charge higher margins for multi-stop shoppers than for one-stop shoppers. We note here that this is likely to remain the

[28] In the limit case $\underline{s} = \delta$, however, only those consumers with a shopping cost equal to δ may opt for multi-stop shopping.

case when firms have market power over one-stop shoppers as well. This is because, as stressed above, firms offer substitute assortments to one-stop shoppers, and complements to multi-stop shoppers. Hence, even if limited, competition for one-stop shoppers still tends to curb total margins on firms' assortments, whereas double-marginalization tends, instead, to raise prices on strong products.

To see this more formally, suppose that firms have asymmetric comparative advantages, namely:

$$u_1^A - c_1^A - \left(u_1^B - c_1^B\right) \equiv \bar{\delta} > \underline{\delta} \equiv u_2^B - c_2^B - \left(u_2^A - c_2^A\right),$$

which implies that firm 1 is more efficient in serving one-stop shoppers:

$$w_1 - w_2 = \bar{\delta} - \underline{\delta} > 0,$$

where

$$w_1 \equiv u_1^A - c_1^A + u_1^B - c_1^B \text{ and } w_2 \equiv u_2^A - c_2^A + u_2^B - c_2^B$$

denote the surpluses generated by the firms' assortments. Firm 1 therefore enjoys some market power over one-stop shoppers: In equilibrium, firm 2 still offers its assortment at cost $(m_2 = 0)$ but firm 1 now attracts all one-stop shoppers and charges them a total margin reflecting its competitive advantage, $m_1 = \bar{\delta} - \underline{\delta}$. Hence, one-stop shoppers obtain a consumer value $v_1 = w_2$, and the multi-stop shopping threshold becomes[29]

$$\tau = v_{12} - v_1 = \left(w_2 + \bar{\delta} - \rho_1 - \rho_2\right) - w_2 = \underline{\delta} + \mu_1 - \rho_2,$$

where $\mu_1 = m_1 - \rho_1 = \bar{\delta} - \underline{\delta} - \rho_1$ denotes firm 1's margin on its weak product.

As firm 1 sells both products to one-stop shoppers, and sells its strong product to multi-stop shoppers as well, its profit can be expressed as

$$\begin{aligned} \pi_1 &= \rho_1 F(\tau) + m_1 \left[F(v_1) - F(\tau)\right] \\ &= \left(\bar{\delta} - \underline{\delta}\right) F(w_2) - \mu_1 F\left(\underline{\delta} + \mu_1 - \rho_2\right), \end{aligned}$$

[29] Combining A_1 and B_2 yields a surplus equal to

$$u_1^A - c_1^A + u_2^B - c_2^B = w_1 + \underline{\delta} = w_2 + \bar{\delta}.$$

which obviously leads firm 1 to subsidize its weak product: $\mu_1 < 0$.[30] To understand why cross-subsidization still arises when firm 1 enjoys market power over one-stop shoppers as well, consider again the following thought experiment: Increase ρ_1 by a small amount and decrease μ_1 by the same amount, so as to keep the total margin m_1 equal to $\bar{\delta}-\underline{\delta} > 0$. This alteration of the price structure does not affect the profit made on one-stop shoppers (who pay the same price for the basket) but increases the profit made on multi-stop shoppers (who pay a higher price for the strong product); in addition, this induces some multi-stop shoppers (those with s slightly below τ) to switch to one-stop shopping and buy firm 1's weak product as well (instead of buying only its strong product). It is therefore profitable for firm 1 to keep altering the price structure as long as it earns a non-negative margin on its weak product, which leads the firm to sell its weak product below cost.

Remark: Collusion. The scope for below-cost pricing would, however, disappear if firms could coordinate their pricing decisions, e.g., through tacit or explicit collusion. Consider, for instance, our baseline setup but suppose now that firms interact repeatedly over time and are so "patient" (that is, their discount factors are close to 1) that they can perfectly collude and maximize their joint profits. Using the total margin, m, and the margin differential between strong and weak products, $t = \rho - \mu$, as decision variables, total industry profit can be expressed as

$$\Pi = mF(v) + tF(\tau) = mF(w-m) + tF(\delta - t).$$

It is thus separable in m and t and, as $w > \delta$, a revealed preference argument shows that the industry-profit maximizing margins satisfy $m = \rho + \mu > t = \rho - \mu$, and thus $\mu > 0$: There is no below-cost pricing. Hence, cross-subsidization arises here precisely when firms are strongly competing against each other for one-stop shoppers.

5.3 Online retailing

The last decade has seen established retailers developing their online activities. This offers consumers an alternative way of fulfilling their needs, but also has an impact on retail

[30] Firm 2 only sells its strong product – to multi-stop shoppers – and thus charges a positive margin on it; as $m_2 = 0$, firm 2 thus still prices its weak product below-cost, but consumers do not buy it in equilibrium.

competition and on retailers' pricing strategies. To explore these implications, consider the following variant of the baseline model, where a fraction λ of "internet-savvy" consumers see their shopping cost drop to zero. That is, the distribution of shopping costs can be characterized by a cumulative distribution function $F_\lambda(s)$ and a density $f_\lambda(s)$, where $F_\lambda(0) = \lambda$ and, for $s > 0$:

$$f_\lambda(s) = (1-\lambda) f(s) \text{ and } F_\lambda(s) = \lambda + (1-\lambda) F(s).$$

The inverse hazard rate becomes

$$h_\lambda(s) = h(s) + \frac{\lambda}{1-\lambda} \frac{1}{f(s)}$$

and thus increases with s if $f(s)$ does not increase with s; more generally, this inverse hazard rate remains an increasing function as long as λ is not too large,[31] in which case we have:

Proposition 4 *As long as $h'_\lambda(\cdot) > 0$, there exists a unique equilibrium, in which firms sell their baskets at cost, but charge a positive margin ρ^*_λ on strong products (and thus a negative margin $-\rho^*_\lambda < 0$ on weak products); furthermore, the equilibrium margin, ρ^*_λ, and the equilibrium profit, π^*_λ, both increase with λ.*

Proof. See Appendix D. ■

Proposition 4 points out that the development of online sales is not only profitable, but also consistent with an *increase* in the prices charged by firms on their strong products: While one-stop shoppers can still buy firms' baskets at cost, multi-stop shoppers (including those buying online) face higher prices as the proportion of online customers increases. The intuition is straightforward: An increase in the development on online activity, as measured by λ, boosts multi-stop shopping, which benefits the firms but also encourages them to take advantage of this shift in demand by raising the margins on their strong products – at the expense of the not so internet-savvy multi-stop shoppers.

[31] If $f'(s) > 0$, then $h_\lambda(s)$ still increases with s in the relevant range $s \in [0, \delta]$ if

$$\frac{\lambda}{1-\lambda} < \max_{s\in[0,\delta]} f^2(s) \frac{h'(s)}{f'(s)}.$$

6 Resale-below-cost laws

In regulated industries, cross-subsidization has been a well-recognized issue in both theory and practice,[32] and has prompted regulators to impose structural or behavioral remedies.[33] In contrast, in competitive markets the policy debate is more divided. Although below-cost pricing might be treated as predatory,[34] in many cases there is not such a thing as a "predatory phase" followed by a "recoupment phase" (e.g., once rivals have been driven out of the market), which constitute key features of predation scenarios.[35] As mentioned in the introduction, this has led many countries to adopt specific rules prohibiting or limiting below-cost pricing in retail markets. These rules, known as Resale-Below-Cost (RBC hereafter) laws, have been the subject of heated policy debates. In Ireland, for example, based on evidence that consumers pay more when grocery goods are subject to the prohibition of below-cost sales, in 2005 the Irish Competition Authority recommended terminating the RBC law.[36] However, the Irish Joint Committee on Enterprise and Small Business recommended keeping the RBC law due to concerns about increased concentration in grocery retailing and predatory pricing. The Irish example highlights the dilemma of antitrust authorities: RBC laws may prevent dominant retailers from engag-

[32] The seminal paper of Faulhaber (1975) rigorously defines the concept of cross-subsidy and introduces two tests for subsidy-free pricing, which have been widely applied in both regulation and antitrust enforcement. See Faulhaber (2005) for a recent survey.

[33] Such concerns led, for instance, to the break-up of *AT&T* and the imposition of lines of business restrictions on local telephone companies (*U.S. v. AT&T 1982*). More recently, the European Commission required the German postal operator to stop cross-subsidizing its parcel services with the profit derived from its legal monopoly on letter services (*Deutsche Post, 2001*).

[34] See e.g., Bolton, Brodley and Riordan (2000) and Eckert and West (2003) for detailed discussions of how predatory pricing tests should be designed. Rao and Klein (1992) and Berg and Weisman (1992) examine the treatment of cross-subsidization under U.S. antitrust laws.

[35] For instance, the feasibility of recoupment is a necessary condition for a case of predation in the U.S., since the Supreme Court decision in *Brooke Group Ltd. v. Brown & Williamson Tobacco Corp.*

[36] The Irish Competition Authority examined pricing trends under the Groceries Order (the RBC law introduced in Ireland in 1987). The authority found that prices for grocery items covered by the Order had been increasing, while prices for grocery items not covered by the Order had been decreasing; it concluded that, on average, Irish families were paying 500 euros more per year because of the Order. See OECD (2007).

ing in predatory pricing against smaller or more fragile rivals, but in competitive markets they may also lead to higher prices and thus harm consumers.

We now examine the impact, in our setting, of a ban on below-cost pricing. We first note that such a ban raises equilibrium total margins, which benefits firms at the expense of one-stop shoppers:

Proposition 5 *When below-cost pricing is prohibited, each firm obtains a profit at least equal to*

$$\bar{\pi} \equiv \max_{\rho} \rho F(\delta - \rho) > 2\pi^*.$$

It follows that, compared to the equilibrium that arises in the absence of a ban:

(i) Firms enjoy higher profits;

(ii) One-stop shoppers face higher prices.

Proof. See Appendix C. ■

The intuition is simple. If the rival offers both of its products at cost, a firm cannot make a profit on one-stop shoppers, but can still make a profit by selling its strong product to multi-stop shoppers. Indeed, charging a margin $\rho < \delta$ induces consumers with shopping cost $s < \tau = \delta - \rho$ to buy both strong products, thus generating a profit $\rho F(\delta - \rho)$; by choosing the optimal margin

$$\bar{\rho} \equiv \arg\max_{\rho} \rho F(\delta - \rho), \tag{4}$$

the firm can thus secure $\bar{\pi}$. To conclude the argument, it suffices to note that, in response to any prices set by the rival, the firm can still obtain at least $\bar{\pi}$ by charging a prohibitive margin on its weak product (so as to induce multi-stop shoppers to buy firms' strong products) and a margin set to $\bar{\rho}$ on its strong product.

Hence, in any equilibrium, each firm earns a profit at least equal to $\bar{\pi}$. Furthermore, as the rival can no longer subsidize its weak product, each firm now more than doubles its profit:[37]

$$\bar{\pi} = \max_{\rho} \rho F(\delta - \rho) > 2\rho^* F(\delta - 2\rho^*) = 2\pi^*.$$

[37]The strict inequality follows from $2\rho^* > \bar{\rho}$, or $\bar{\tau} = \delta - \bar{\rho} > \tau^*$. To see this, note that $\delta = j(\tau^*)$ whereas $\bar{\tau} = \delta - \bar{\rho} = \delta - h(\bar{\tau})$, which amounts to $\delta = l(\bar{\tau})$, where $l(\tau) \equiv \tau + h(\tau) < j(\tau) = \tau + 2h(\tau)$.

Finally, equilibrium total margins are positive, as weak products cannot be sold below cost, and strong products are sold with a positive margin. One-stop shoppers thus face higher prices than in the absence of the ban.

Intuitively, banning below-cost pricing should lead the firms to offer their weak products at cost (i.e., $\mu_1 = \mu_2 = 0$). Firm i's profit, as a function of the two firms' margins on their strong products, ρ_1 and ρ_2, is then given by:

$$\pi_i^b\left(\rho_i, \rho_j\right) \equiv \begin{cases} \rho_i F\left(w - \rho_i\right) & \text{if} \quad \rho_i < \rho_j, \\ \rho_i F\left(\delta - \rho_i\right) & \text{if} \quad \rho_i > \rho_j. \end{cases}$$

In the first case ($\rho_i < \rho_j$), firm i sells its strong product to both one-stop and multi-stop shoppers, whereas in the second case ($\rho_i > \rho_j$), it sells its strong product only to multi-stop shoppers. Note that, as a firm can obtain at least $\bar{\pi}$ by charging $\bar{\rho}$ to multi-stop shoppers, it will never charge so low a margin that it would obtain less than $\bar{\pi}$ even if it were to attract all shoppers. That is, no firm will ever charge $\rho < \underline{\rho}$, where $\underline{\rho}$ is the lower solution to

$$\underline{\rho} F\left(w - \underline{\rho}\right) = \bar{\pi}. \tag{5}$$

The next proposition shows that, while there is no pure-strategy equilibrium when below-cost pricing is banned, there exists an equilibrium in which firms indeed sell their weak products at cost, and obtain an expected profit equal to $\bar{\pi}$ by randomizing the margins on their strong products between $\underline{\rho}$ and $\bar{\rho}$:

Proposition 6 *When below-cost pricing is prohibited:*

(i) There exists no equilibrium in pure strategies.

(ii) There exists a symmetric mixed-strategy equilibrium in which firms obtain an expected profit equal to $\bar{\pi}$ by selling weak products at cost and randomizing the margins on strong products over $\left[\underline{\rho}, \bar{\rho}\right]$.

Proof. See Online Appendix B.1. ■

As in the sales model of Varian (1980), firms face a tension between exploiting "captive" customers (the uninformed consumers in Varian's model, and multi-stop shoppers here) and competing for "price-sensitive" customers (the informed consumers in Varian's model, and one-stop shoppers here). To see why there is no pure-strategy equilibrium, note that competition for one-stop shoppers would again drive *total* margins down to zero. But as below-cost pricing is banned, this would require selling *both* products at cost. Obviously, this cannot be an equilibrium, as a firm can make a profit on multi-stop shoppers by charging a small positive margin on its strong product.

The characterization of the mixed-strategy equilibrium is similar to the one proposed by Varian (1980) and Baye, Kovenock, and de Vries (1992).[38] In this equilibrium, *ex post* consumers with a shopping cost below

$$\tau^b(\rho_1, \rho_2) \equiv \delta - \max\{\rho_1, \rho_2\}$$

favor multi-stop shopping and buy both firms' strong products, whereas consumers with a shopping cost in the range

$$\tau^b(\rho_1, \rho_2) < s < v^b(\rho_1, \rho_2) \equiv w - \min\{\rho_1, \rho_2\}$$

are one-stop shoppers and buy from the firm that charges the lowest total margin.

Let us now examine the impact of a ban on consumers. We first note that $v^b > \tau^b$,[39] and so marginal consumers are one-stop shoppers. As banning below-cost pricing raises prices for one-stop shoppers, it follows that this reduces not only the number of one-stop shoppers, but also the total number of consumers – from $F(w)$ to $F\left(v^b(\rho_1, \rho_2)\right)$. Furthermore, the multi-stop shopping cost threshold τ^b satisfies:

$$\tau^b(\rho_1, \rho_2) \geq \bar{\tau} \equiv \tau^b(\bar{\rho}, \bar{\rho}) = \delta - \bar{\rho} > \tau^*.$$

Hence, banning below-cost pricing *fosters* multi-stop shopping.

This does not mean that multi-stop shoppers face lower prices, however. In particular, the upper bound $\bar{\rho}$ exceeds the margin ρ^* that arises in the absence of the ban,[40] implying that multi-stop shoppers face higher prices with at least some probability. The next Proposition shows that banning below-cost pricing actually harms multi-strop shoppers as well when weak products offer relatively little value, that is, when w is close to δ:[41]

[38] Using the analysis of the latter paper, it can moreover be shown that, conditional on pricing weak products at cost, the (mixed-strategy) equilibrium (for the prices of strong products) is unique.

[39] This follows directly from $w > \delta$ and $\max\{\rho_1, \rho_2\} \geq \min\{\rho_1, \rho_2\}$.

[40] To see this, it suffices to note that, from the first-order conditions, ρ^* and $\bar{\rho}$ satisfy respectively, $\rho = h(\delta - \rho^* - \rho)$ and $\rho = h(\delta - \rho)$, where $h(.)$ is an increasing function.

[41] From (1) and (2), the surplus generated by weak products is equal to $u_1^B - c_1^B = u_2^A - c_2^A = (w - \delta)/2$.

Proposition 7 *Keeping δ constant, for w close enough to δ:*

- *Every consumer' expected surplus is lower in the equilibrium characterized by Proposition 6 than in the equilibrium that arises in the absence of a ban.*

- *Total welfare can however be lower or higher, depending on the distribution of shopping costs. For instance, if $F(s) = s^k/k$, then there exists $\hat{k}(w,\delta) > 0$ such that total welfare is lower (resp., higher) when below-cost pricing is banned if $k < \hat{k}(w,\delta)$ (resp., $k > \hat{k}(w,\delta)$).*

Proof. See Online Appendix B.2. ■

The intuition is that, when weak products are "very" weak, there are relatively few one-stop shoppers. Firms can then raise the margin on their strong products so as to exploit multi-stop shoppers, without being too concerned about losing one-stop shoppers. Indeed, in the limit case where $w = \delta$, the lower bound $\underline{\rho}$ of the equilibrium margin distribution converges to the upper bound $\bar{\rho}\,(> \rho^*)$, and multi-stop shoppers thus face higher prices for certain. By continuity, multi-stop shoppers face higher *expected* prices as long as weak products are not too valuable.[42] As a ban on below-cost pricing increases firms' profits, the impact on total welfare remains, however, ambiguous, and depends in particular on the distribution of shopping costs.

Thus, in competitive markets, RBC laws increase firms' profits but hurt one-stop shoppers. When weak products offer relatively low value, multi-stop shoppers face higher prices as well, in which case banning below-cost pricing increases firms' profits at the expense of consumers. This finding gives support to the conclusion of the OECD (2007) report, which argues that RBC laws are likely to lead to higher prices and thus harm consumers. The reduction in consumer surplus may, moreover, exceed the increase in firms' profits and thus result in lower total welfare. However, when, instead, weak products offer high value, RBC laws may have a positive impact on multi-stop shoppers.[43]

[42]The upper bound $\bar{\rho}$ depends only on δ, whereas the lower bound $\underline{\rho}$ depends on both w and δ and, keeping δ constant, converges to $\bar{\rho} > \rho^*$ when w tends to δ. The lower bound $\underline{\rho}$ remains higher than ρ^* as long as w remains below some threshold $\hat{w} > \delta$; and for slightly larger values of w, $\underline{\rho}$ becomes lower than ρ^*, but the expected equilibrium value of ρ remains higher than ρ^*.

[43]However, RBC laws reduce total expected consumer surplus when, for instance, the density of the distribution of shopping costs does not increase between τ^* and δ; see Online Appendix B.3.

In a setting where consumers are one-stop shoppers who underestimate (some of) their needs, Johnson (2014) finds that banning below-cost pricing has an unambiguously negative impact: It increases the price for potential loss leaders (those products for which consumers do not underestimate their needs) and harms consumers, despite decreasing the prices for the other products. In our setting, a ban on below-cost pricing also raises the price of potential loss leaders (namely, the weak products), but can either increase or decrease the (expected) price of the other products (the strong ones). Also, while one-stop shoppers are worse-off under RBC laws, as in Johnson's paper, we allow for multi-stop shoppers as well, and they can either be worse-off or better-off. In spite of these discrepancies, both Johnson's and our paper call for the cautious use of below-cost pricing regulations in competitive markets; and where they are implemented, their impact should be carefully evaluated.

7 Conclusion

We consider a competitive environment where (i) multiproduct firms enjoy comparative advantages over different goods or services; and (ii) customers have heterogeneous shopping costs: Those with low costs tend to patronize multiple suppliers, whereas those with higher shopping costs are more prone to one-stop shopping. This gives rise to a form of co-opetition, as firms' assortments are substitutes for one-stop shoppers, but their strong products are complements for multi-stop shoppers. As a result, competition for one-stop shoppers drives total prices down to total cost but, to exploit their market power over multi-stop shoppers, firms price strong products above cost and weak products below cost. Furthermore, the complementarity of firms' strong products generates double marginalization problems, which here take the form of excessive cross-subsidization: Firms would benefit from mutual moderation, e.g., by agreeing to put a cap on the prices of strong products; such bilateral price cap agreements would benefit consumers (one-stop shoppers would remain supplied at cost, and multi-stop shoppers would benefit from lower prices) and would also increase profits by boosting multi-stop shopping.

The antitrust treatment of cross-subsidization in competitive markets has triggered hot debates. We find that banning below-cost pricing substantially benefits firms – their profits more than double – at the expense of one-stop shoppers, and can also reduce consumer surplus and total welfare, depending on the value offered by weak products and the distribution of shopping patterns. Our analysis thus calls for cautious use of resale-below-cost laws in competitive markets.

We have developed these insights using a simple setup, with individual unit demands and homogeneous consumer valuations for the goods. It would be interesting to extend our analysis to other environments, e.g., by allowing for more general demand or for correlation (e.g., due to underlying characteristics such as wealth) between customers' preferences and their transaction costs; we leave this task to future research.

Our framework can also be used as a building block to revisit classic issues such as product differentiation strategies or investment in quality (e.g., by endogenizing the characteristics of firms' "strong" products), or newer ones, such as the development of online sales: While our first exploration suggests that firms have indeed an incentive to reduce the transaction costs of their customers, it would be interesting to model explicitly the investment in online activities, and to study the implications of its impact on transaction costs.

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Appendix

Notation. Throughout the exposition:

- We will refer to the two firms as firms i and j, with the convention that $i \neq j \in \{1,2\}$.
- For each firm $i \in \{1,2\}$, we will denote the social value generated by its strong (resp., weak) product by $\bar{w}_i$ (resp., by $\underline{w}_i$), and we will denote the margin charged on its strong (resp., weak) product by ρ_i (resp., by μ_i). By assumption, we have $\bar{w}_i, \underline{w}_i > 0$ and

$$\bar{w}_i + \underline{w}_i = w$$

for $i \in \{1,2\}$ and, for $j \neq i \in \{1,2\}$:

$$\bar{w}_i - \underline{w}_j = \delta.$$

- The value offered by firm $i \in \{1,2\}$ is thus equal to

$$v_i \equiv \max\{\underline{w}_i - \mu_i, 0\} + \max\{\bar{w}_i - \rho_i, 0\},$$

whereas multi-stop shoppers obtain a value

$$v_{12} = \max\{\bar{w}_1 - \rho_1, 0\} + \max\{\bar{w}_2 - \rho_2, 0\}$$

if they buy both strong products, and obtain instead a value

$$\underline{v}_{12} = \max\{\underline{w}_1 - \mu_1, 0\} + \max\{\underline{w}_2 - \mu_2, 0\}$$

if they buy both weak products.

- Using the "adjusted" margins, defined as

$$\hat{\mu}_i \equiv \min\{\mu_i, \underline{w}_i\} \text{ and } \hat{\rho}_i \equiv \min\{\rho_i, \bar{w}_i\},$$

these values can be respectively expressed as

$$\begin{aligned} v_i &= \underline{w}_i - \hat{\mu}_i + \bar{w}_i - \hat{\rho}_i = w - \hat{\mu}_i - \hat{\rho}_i, \\ v_{12} &= \bar{w}_1 - \hat{\rho}_1 + \bar{w}_2 - \hat{\rho}_2 = w + \delta - \hat{\rho}_1 - \hat{\rho}_2, \\ \underline{v}_{12} &= \underline{w}_1 - \hat{\mu}_1 + \underline{w}_2 - \hat{\mu}_2 = w - \delta - \hat{\mu}_1 - \hat{\mu}_2. \end{aligned}$$

Note that a multi-stop shopper would buy both strong products only if $\rho_i < \bar{w}_i$ (that is, $\hat{\rho}_i = \rho_i$) for both firms (otherwise, the value from such multi-stop shopping, net of shopping costs, would be lower than the net value from one-stop shopping), implying that

$$v_{12} = w + \delta - \rho_1 - \rho_2.$$

Likewise, a multi-stop shopper will actually buy both weak products only if $\mu_i < \underline{w}_i$ (that is, $\hat{\mu}_i = \mu_i$) for both firms, implying that

$$\underline{v}_{12} = w - \delta - \mu_1 - \mu_2.$$

Moreover, for a firm that attracts one-stop shoppers, it is never optimal to charge a margin that exceeds the social value of the product, i.e., $\rho_i > \bar{w}_i$ and $\mu_i > \underline{w}_i$ cannot arise in equilibrium where firm i serves some one-stop shoppers. Suppose firm i charges $\mu_i > \underline{w}_i$ and $\rho_i \leq \bar{w}_i$, and one-stop shoppers only buy its strong product. Reducing μ_i such that $\tilde{\mu}_i = \underline{w}_i - \varepsilon > 0$ increases firm i's profit by selling its weak product as well to one-stop shoppers and by attracting more one-stop shoppers as $\tilde{v}_i > v_i$. Doing so may also transform some multi-stop shoppers (if there are any multi-stop shoppers buying strong products) into one-stop shoppers as now $\tilde{\tau} = \delta + \tilde{\mu}_i - \rho_j < \tau$, on which firm i earns a higher profit. Similarly, charging $\rho_i > \bar{w}_i$ is never optimal if firm i attracts some one-stop shoppers. Therefore, without loss of generality we focus on $\mu_i \leq \underline{w}_i$ and $\rho_i \leq \bar{w}_i$ if one-stop shoppers patronize firm i.

The shopping cost thresholds, below which consumers favor picking both strong products rather than patronizing only firm 1 or firm 2, are respectively $\tau_1 = v_{12} - v_1 = \delta + \mu_1 - \rho_2$ and $\tau_2 = v_{12} - v_2 = \delta - \rho_1 + \mu_2$, and $\tau = \min\{\tau_1, \tau_2\}$. Likewise, the thresholds for picking weak products are $\underline{\tau}_1 \equiv \underline{v}_{12} - v_1 = \rho_1 - \mu_2 - \delta$, $\underline{\tau}_2 \equiv \underline{v}_{12} - v_2 = \rho_2 - \mu_1 - \delta$, and $\underline{\tau} = \min\{\underline{\tau}_1, \underline{\tau}_2\}$. Note that $\underline{\tau}_1 = -\tau_2$, $\underline{\tau}_2 = -\tau_1$, and thus $\underline{\tau} = -\tau$. Therefore, in equilibrium, it cannot be the case that some multi-stop shoppers buy strong products, and other buy weak products.

A Proof of Lemma 1

To prove the lemma, we first establish the following claims.

Claim 1 *Some consumers are active in equilibrium.*

Proof. Suppose there is no active consumer. It must be the case that $\max\{v_1, v_2, v_{12}, \underline{v}_{12}\} \leq 0$, and firms make no profit. Consider the following deviation for firm 1: charge $\tilde{\mu}_1 > 0$ and $\tilde{\rho}_1 > 0$ such that $\tilde{m}_1 = \tilde{\rho}_1 + \tilde{\mu}_1 = w - \varepsilon$, for some $\varepsilon \in (0, w)$. Firm 1 then attracts consumers with shopping cost $s \leq \tilde{v}_1 = \varepsilon$ and earns a positive profit, a contradiction. Thus some consumers must be active in equilibrium. ■

Claim 2 *If there are active one-stop shoppers in equilibrium, then $m_1 = m_2 = 0$.*

Proof. Consider a candidate equilibrium in which some one-stop shoppers are active, which requires $\max\{v_1, v_2\} > 0$. We first show that no firm charges a negative total margin. To see this, suppose firm 1 sets $m_1 < 0$ (and thus $v_1 > w > 0$), say, then:

- If $m_1 < m_2$, firm 1 incurs a loss by attracting one-stop shoppers; then consider the following deviations:
  - If there is no multi-stop shopper, or if firm 1 does not make a profit on multi-stop shoppers, then firm 1 could avoid all losses by increasing both of its prices.
  - If some multi-stop shoppers buy the strong products, and firm 1 makes a profit on them (that is, $\rho_1 > 0$), then firm 1 would benefit from raising the margin on its weak product: Keeping ρ_1 constant, raising the margin on the weak product to $\tilde{\mu}_1 = -\rho_1 = \mu_1 - m_1 > \mu_1$ (i) yields $\tilde{m}_1 = 0$, thus avoiding the loss from one-stop shoppers, and (ii) moreover increases the demand from multi-stop shoppers (on which firm 1 makes a positive margin), as it reduces the value from one-stop shopping without affecting that of multi-stop shopping.
  - If some multi-stop shoppers buy the weak products, and firm 1 makes a profit on them (that is, $\mu_1 > 0$), then firm 1 could avoid the loss from one-stop shoppers by raising the margin on its strong product to $\tilde{\rho}_1 = -\mu_1$ (yielding $\tilde{m}_1 = 0$), which would also increase the demand from multi-stop shoppers by reducing the value from one-stop shopping without affecting that of multi-stop shopping.

- If instead $m_1 \geq m_2$ (and thus $m_2 < 0$), then the same argument applies to any firm that attracts one-stop shoppers (firm 2 if $m_1 > m_2$, and at least one of the firms if $m_1 = m_2$).

Next, we show that both firms charging a positive total margin cannot be an equilibrium. Suppose firms set $m_1, m_2 > 0$. Then:

- If any firm, say firm 1, charges a higher margin than its rival ($m_1 > m_2 > 0$ and $v_2 > \max\{v_1, 0\}$), it faces no demand from one-stop shoppers; then consider the following deviations:
  - If there is no multi-stop shopper, which requires $\max\{v_{12}, \underline{v}_{12}\} \leq v_2$, firm 1 can make a positive profit by undercutting both of its rival's "quality-adjusted" prices by $\varepsilon/2$: for ε positive but small enough, charging $\tilde{\rho}_1 = \mu_2 + \delta - \varepsilon/2$ and $\tilde{\mu}_1 = \rho_2 - \delta - \varepsilon/2$ profitably attracts one-stop shoppers (as $\tilde{v}_1 = v_2 + \varepsilon > v_2$ and $\tilde{m}_1 = m_2 - \varepsilon > 0$), without transforming them into multi-stop shoppers (as $\tilde{v}_1 = v_2 + \varepsilon > \max\{\tilde{v}_{12}, \tilde{\underline{v}}_{12}\} = v_2 + \varepsilon/2$).
  - If some multi-stop shoppers are active, which requires $\max\{v_{12}, \underline{v}_{12}\} > v_2 \ (> v_1)$, firm 1 can make a profit by keeping constant its margin on the product purchased by multi-stop shoppers, and reducing its other margin so as to yield $\tilde{m}_1 = m_2 - \varepsilon$, with $\varepsilon > 0$: Doing so attracts all one-stop shoppers (as $\tilde{v}_1 = v_2 + \varepsilon > v_2$ and $\tilde{m}_1 = m_2 - \varepsilon > 0$), at the cost of slightly reducing the demand of multi-stop shoppers (as it does not affect the value from multi-stop shopping, $\max\{v_{12}, \underline{v}_{12}\}$, and only increases the value of one-stop shopping by ε, from v_2 to $\tilde{v}_1 = v_2 + \varepsilon$), and is obviously profitable for ε small enough.
- If both firms charge the same total margin ($m_1 = m_2 > 0$), then $v_2 = v_1$ and $\tau_1 = \tau_2$. At least one firm, say firm 1, does not obtain more than half of the demand from one-stop shoppers; but then, this firm can attract all one-stop shoppers using the deviations described above for the case $m_1 > m_2$, and the gain from doing so offsets the loss from the slight reduction in demand, if any, from multi-stop shoppers.

Finally, we show that no firm charges a positive total margin in equilibrium. Suppose, for instance, that $m_1 > m_2 = 0$; firm 2 then makes zero profit from one-stop shoppers. If it makes a loss on multi-stop shoppers, then it could profitably deviate by raising all of its prices so as to avoid the loss. If instead it supplies multi-stop shoppers at or above cost, then it could profitably deviate by increasing by some $\varepsilon > 0$ the margin of the product *not picked* by multi-stop shoppers, keeping constant its other margin: For ε small enough, firm 2 still supplies all one-stop shoppers, but now makes a profit on them; and doing so moreover *increases* the demand from multi-stop shoppers and thus the profit on them.

We conclude that firms must charge $m_1 = m_2 = 0$ in any equilibrium with active one-stop shoppers. ■

Claim 3 *In equilibrium, active multi-stop shoppers buy the strong products.*

Proof. Suppose that some multi-stop shoppers buy the weak products. Each firm must then offer better value on its weak product than the rival's strong product; that is, each firm must sell its strong product with a margin that exceeds its rival's "quality-adjusted" margin: $\rho_2 \geq \mu_1 + \delta$ and $\rho_1 \geq \mu_2 + \delta$. We show that such a configuration cannot be an equilibrium. We consider two cases:

- Suppose first that there are only multi-stop shoppers (buying the weak products). To make profits, firms must charge non-negative margins on their weak products, i.e., μ_1, $\mu_2 \geq 0$. From the above, this implies that each firm sells its strong product with a margin that exceeds its comparative advantage δ: $\rho_2 \geq \delta$ and $\rho_1 \geq \delta$. But then, any firm could profitably undercut its rival. For instance, keeping μ_1 unchanged, by charging $\tilde{\rho}_1 = \mu_2 + \delta - \varepsilon > 0$ firm 1 would sell its strong product as well to all previously active consumers, as it now offers better value on A: $\tilde{v}_1^A = v_2^A + \varepsilon$; the deviation may also attract additional one-stop shoppers on which the firm makes a profit as $\tilde{\rho}_1 > 0$ and $\mu_1 \geq 0$.

- Suppose instead that there are both one-stop shoppers and multi-stop shoppers. From Claim 2, price competition for one-stop shoppers then leads to $m_1 = m_2 = 0$. As firms make no profit from one-stop shoppers, they must charge non-negative margins on their weak products, i.e., μ_1, $\mu_2 \geq 0$. But this implies that margins on strong products are non-positive, say, $\rho_1 = m_1 - \mu_1 \leq 0$, which contradicts the condition $\rho_1 \geq \mu_2 + \delta \geq \delta$.

Therefore multi-stop shoppers must buy strong products in equilibrium. ■

Claim 4 *Some multi-stop shoppers are active in equilibrium.*

Proof. Suppose all active consumers are one-stop shoppers, which requires $\max\{v_1, v_2\} > 0$ and $\max\{v_1, v_2\} \geq \max\{v_{12}, \underline{v}_{12}\}$. From Claim 2, price competition for one-stop shoppers then leads to $m_1 = m_2 = 0$, and thus firms make zero profit. We show that this configuration cannot be an equilibrium.

By construction, $v_1 + v_2 = v_{12} + \underline{v}_{12}$, as it corresponds to the total value of buying one unit of both products from both firms. Here, we moreover have $v_1 = v_2 \geq \max\{v_{12}, \underline{v}_{12}\}$, and it follows that $v_1 = v_2 = v_{12} = \underline{v}_{12}$; that is, firms must offer the same value on both products, by charging a margin $\delta/2$ on strong products, and subsidizing weak products by the same amount. It follows that it is profitable for any firm to encourage some consumers to buy only its strong product: For instance, increasing μ_1 by $\varepsilon > 0$ and decreasing ρ_1 by the same amount raises both τ_1 and τ_2 by ε, which triggers some multi-stop shopping as $\tau = \varepsilon > 0$; and as $\tilde{\rho}_1 = \delta/2 - \varepsilon > 0$ for ε small enough, firm 1 now makes a positive profit on these multi-stop shoppers. ■

Claim 5 *Some one-stop shoppers are active in equilibrium.*

Proof. Suppose there are only multi-stop shoppers who, from Claim 3, buy the strong products. Consumers are willing to visit both firms if $2s \leq v_{12}$ (i.e., $s \leq v_{12}/2$), but would prefer one-stop shopping if $s > \tau = v_{12} - \max\{v_1, v_2\}$; hence, we must have:

$$\frac{v_{12}}{2} \leq \tau = v_{12} - \max\{v_1, v_2\},$$

which implies $\max\{v_1, v_2\} \leq v_{12}/2$, and the demand from multi-stop shoppers is $F(v_{12}/2)$. As consumers only buy strong products, firms must charge non-negative margins on these products. Without loss of generality, suppose $\rho_2 \geq \rho_1$ (≥ 0), and consider the following deviation for firm 1: Keeping ρ_1 constant, change μ_1 to

$$\tilde{\mu}_1 = \frac{w - \delta + \rho_2 - \rho_1}{2} - \varepsilon \geq \frac{w - \delta}{2} - \varepsilon > 0,$$

so as to *increase* the value offered to one-stop shoppers to

$$\tilde{v}_1 = w - \rho_1 - \tilde{\mu}_1 = \frac{w + \delta - \rho_1 - \rho_2}{2} + \varepsilon = \frac{v_{12}}{2} + \varepsilon.$$

This deviation does not affect v_{12} nor τ_2 (which only depends on ρ_1, ρ_2 and μ_2), but it decreases τ_1 to $\tilde{\tau}_1 = \delta + \tilde{\mu}_1 - \rho_2 = v_{12}/2 - \varepsilon$; as initially $\tau \geq v_{12}/2$, it follows that the multi-stop shopping threshold becomes $\tilde{\tau} = \tilde{\tau}_1 \, (< v_{12}/2) < \tilde{v}_1$. This adjustment thus induces some of the initial multi-stop shoppers to buy both products from firm 1 (those whose shopping cost lies between $\tilde{\tau}_1$ and $v_{12}/2$), on which firm 1 earns an extra profit from selling its weak product (as $\tilde{\mu}_1 > 0$), and it moreover attracts some additional one-stop shoppers (those whose shopping cost lies between $v_{12}/2$ and $\tilde{v}_1$), which generates additional profit (as $\rho_1 \geq 0$ and $\tilde{\mu}_1 > 0$). ■

Claims 4 and 5 establish part (i) of the Lemma. Part (ii) then follows from Claim 3, whereas part (iii) follows from Claim 2.

B Proof of Proposition 1

Thanks to Lemma 1, the equilibrium is interior and consumers whose shopping cost lies below $\tau^* > 0$ patronize both firms, whereas those whose shopping cost lies between τ^* and w patronize a single firm. The monotonicity of the inverse hazard rate $h(\cdot)$ furthermore ensures that the first-order conditions characterize a unique candidate equilibrium, satisfying $m_1^* = m_2^* = 0$ and $\rho_1^* = \rho_2^* = \rho^*$, such that

$$\rho^* = h(\tau^*),$$

where

$$\tau^* = j^{-1}(\delta).$$

We show now that firms cannot benefit from any deviation. Suppose firm 1 charges ρ_1 and μ_1 instead of $\rho_1^* = \rho^*$ and $\mu_1^* = -\rho^*$. Then:

- It cannot make a profit from one-stop shoppers, as it would have to charge $m_1 \leq m_2^* = 0$ to attract them.
- It cannot make a profit either by offering the weak product to multi-stop shoppers, as it would have to charge $\mu_1 \leq \rho_2^* - \delta = \rho^* - \delta < 0$ (as $\rho^* < \delta$) to attract them.

- Thus, it can only make a profit from multi-stop shoppers, and this profit is equal to $\rho_1 F(\tau)$, where $\tau = \min\{\delta + \mu_2^* - \rho_1, \delta + \mu_1 - \rho_2^*\}$; but then

$$\rho_1 F(\tau) \leq \rho_1 F\left(\delta + \mu_2^* - \rho_1\right) = \rho_1 F(\delta - \rho^* - \rho_1) \leq \pi^*,$$

where the inequality comes from the fact that the profit function $\rho_1 F(\delta - \rho^* - \rho_1)$ is quasi-concave, from the monotonicity of $h(\cdot)$, and by construction maximal for $\rho_1 = \rho^* = \rho_1^*$.

C Proof of Proposition 5

We now derive the minmax profit that each firm can earn when below-cost pricing is not allowed. Consider first firm i's response when firm j sets both of its margins to zero, that is, $\mu_j = \rho_j = 0$. Firm i cannot make a profit from one-stop shoppers who can obtain both products at cost from firm j, and thus it can only make a profit by selling its strong product to multi-stop shoppers. The threshold for multi-stop shopping is $\tau = \delta - \rho_i$, and thus the profit from multi-stop shoppers is given by $\rho_i F\left(\delta - \rho_i\right)$. Choosing ρ_i so as to maximize this profit gives firm i

$$\bar{\pi} \equiv \max_{\rho} \rho F\left(\delta - \rho\right) > 0,$$

where the inequality stems from $\delta > 0$. The associated margin is given by

$$\bar{\rho} \in \arg\max_{\rho} \rho F\left(\delta - \rho\right).$$

Note that this margin satisfies $\bar{\rho} < (\delta \leq) \bar{w}_i$ for $i \in \{1, 2\}$.[44]

To conclude the argument, it suffices to note that, in response to any rival's margins $\mu_j \geq 0$ and $\rho_j \geq 0$, firm i can always secure at least $\bar{\pi}$ by charging $\mu_i \geq \underline{w}_i$ and $\rho_i = \bar{\rho}$. Choosing $\mu_i \geq \underline{w}_i$ ensures that any multi-stop shoppers will buy both firms' strong products. In addition:

If $v_j \geq v_i$, then the threshold for multi-stop shopping is given by

$$\tau = v_{12} - v_j$$

[44] In case there are multiple solutions, then any solution satisfies the properties in the proof below.

and thus satisfies

$$\begin{aligned}\tau &= w+\delta-\bar{\rho}-\hat{\rho}_j-\left(w-\hat{\mu}_j-\hat{\rho}_j\right)\\ &= \delta+\hat{\mu}_j-\bar{\rho}\\ &\geq \delta-\bar{\rho},\end{aligned}$$

where the inequality stems from $\hat{\mu}_j = \min\left\{\mu_j, \underline{w}_j\right\} \geq 0$. It follows that firm i obtains at least $\bar{\pi}$:

$$\pi_i = \rho_i F(\tau) = \bar{\rho} F(\tau) \geq \bar{\rho} F(\delta-\bar{\rho}) = \bar{\pi}.$$

If instead $v_j < v_i$, then firm i sells its strong product to both one-stop and multi-stop shoppers and thus again obtains at least $\bar{\pi}$:

$$\pi_i = \rho_i F\left(\max\left\{v_i, \frac{v_{12}}{2}\right\}\right) \geq \rho_i F(v_i) = \bar{\rho} F(\bar{w}_i-\bar{\rho}) \geq \bar{\rho} F(\delta-\bar{\rho}) = \bar{\pi},$$

where the second inequality stems from $\bar{w}_i > \delta$.

It follows that, in any candidate equilibrium, firms must obtain a positive profit $\pi_i \geq \bar{\pi}$, and thus charge a positive total margin $m_i > 0$ (as $m_i = 0$ would imply $\mu_i = \rho_i = 0$, and thus $\pi_i = 0$).

D Proof of Proposition 4

We consider here the impact of an increase in the proportion λ of internet-savvy consumers with zero shopping costs. The analysis developed for the baseline model carries over as long as the inverse hazard rate $h_\lambda(\cdot)$ remains an increasing function of the shopping cost s. The equilibrium margin, ρ_λ^*, and the associated multi-stop shopping threshold, $\tau_\lambda^* = \delta - 2\rho_\lambda^*$, are now such that

$$\rho_\lambda^* = h_\lambda(\tau_\lambda^*) = h_\lambda(\delta - 2\rho_\lambda^*).$$

As $h_\lambda(\cdot)$ increases with λ, it follows that ρ_λ^* increases as well with λ. Conversely, the threshold τ_λ^* is such that

$$\frac{\delta-\tau_\lambda^*}{2} = h_\lambda(\tau_\lambda^*).$$

Hence, as $h_\lambda(\cdot)$ increases with λ, τ_λ^* decreases as λ increases. Finally, the equilibrium profit satisfies

$$\pi_\lambda^* = \max_\rho \rho F_\lambda (\delta - 2\rho),$$

and thus, as F_λ increases with λ, π_λ^* increases as well with λ.

Online Appendix
(Not for Publication)

This online Appendix first analyzes the case of bounded shopping costs considered in Section 5.1, before studying the mixed-strategy equilibrium generated by RBC laws (6).

A Bounded shopping costs

We consider here the case where shopping costs are bounded either above ($s \leq \overline{s}$) or below ($s > \underline{s}$).

A.1 Proof of Proposition 2

Suppose that consumers' shopping costs are distributed over $[0, \overline{s}]$, where $\overline{s} > 0$. It is straightforward to check that the first four claims in the proof of Lemma 1 still hold; that is, in any equilibrium, there exist active multi-stop shoppers who buy the strong products; in addition, if there are active one-stop shoppers, then $m_1 = m_2 = 0$.

We first note that the equilibrium identified in the baseline model still exists when $\overline{s}$ is large enough:

Claim 6 *When $\overline{s} > j^{-1}(\delta)$, then there exists an equilibrium with both types of shoppers: Consumers with a shopping cost lower than $\tau^* = j^{-1}(\delta)$ engage in multi-stop shopping, and face a margin $\rho^* = h(\tau^*)$ on each strong product, whereas those with a higher cost favor one-stop shopping.*

Proof. As shown in the text, there is a unique candidate equilibrium where both types of shopping patterns arise, and it is as described in the Claim. The existence of one-stop shopping, however, requires $\overline{s} > \tau^* = j^{-1}(\delta)$. Conversely, when this condition holds, the margins $m_1^* = m_2^* = 0$ and $\rho_1^* = \rho_2^* = h(\tau^*)$ do support an equilibrium: Indeed the reasoning of the proof of Proposition 1 ensures that no deviation is profitable. ■

Next, we show that one-stop shopping cannot arise if $\overline{s}$ is too low:

Claim 7 *When $\overline{s} \leq j^{-1}(\delta)$, then one-stop shopping does not arise in equilibrium.*

Proof. Suppose there exist some one-stop shoppers, which requires $\tau < \min\{\max\{v_1, v_2\}, \overline{s}\}$. Competition for these one-stop shoppers leads to $m_1 = m_2 = 0$, and thus $\tau_1 = \tau_2 = \delta - \rho_1 - \rho_2 < \overline{s}$, which implies $\rho_1 + \rho_2 > \delta - \overline{s} > 2h(\overline{s})$. Therefore, at least one of the margins on strong products must exceed $h(\overline{s})$. Suppose $\rho_1 > h(\overline{s})$; then $\rho_1 > h(\overline{s}) > h(\tau)$, as $\overline{s} > \tau$ and $h(\cdot)$ is strictly increasing. Consider now the following deviation: decrease ρ_1 to $\tilde{\rho}_1$ and increase μ_1 by the same amount, so as to maintain the total margin. This does not affect the profit from one-stop shoppers (which remains equal to zero), but yields a profit from multi-stop shoppers, equal to $\tilde{\pi}_1 = \tilde{\rho}_1 F(\tilde{\tau})$, where $\tilde{\tau} = \delta - \tilde{\rho}_1 - \rho_2$. As $d\tilde{\pi}_1/d\tilde{\rho}_1|_{\tilde{\rho}_1=\rho_1} = -f(\tau)(\rho_1 - h(\tau))$, which is strictly negative as $\rho_1 > h(\tau)$, such a deviation is profitable. Hence, one-stop shopping does not arise in equilibrium. ■

Claims 6 and 7 together establish the first part of the Proposition. We now characterize the equilibria where all consumers are multi-stop shoppers.

Claim 8 *When $\overline{s} \leq j^{-1}(\delta)$, any margin profile such that $\rho_1 \in [h(\overline{s}), \delta - \overline{s} - h(\overline{s})]$, $\mu_2 = \rho_1 - \delta + \overline{s}$ and $\mu_1 = \rho_2 - \delta + \overline{s}$, constitutes an equilibrium in which all active consumers are multi-stop shoppers.*

Proof. Suppose there are only multi-stop shoppers who, from Claim 3, buy the strong products. Consumers are willing to visit both firms if $2s \leq v_{12}$ (i.e., $s \leq v_{12}/2$), but would prefer one-stop shopping if $s > \tau = v_{12} - \max\{v_1, v_2\}$; hence, we must have $\tau \geq \min\{v_{12}/2, \overline{s}\}$, and the demand from multi-stop shoppers is $F(\min\{v_{12}/2, \overline{s}\})$. As consumers only buy strong products, firms must charge non-negative margins on these products: $\rho_1, \rho_2 \geq 0$.

If $\overline{s} < \min\{v_{12}/2, \tau\}$, each firm can profitably deviate by slightly raising the price for its strong product: This increases the margin without affecting the demand, equal to $F(\overline{s})$. Hence, without loss of generality, we can assume $\overline{s} \geq \min\{v_{12}/2, \tau\}$. The condition $\tau \geq \min\{v_{12}/2, \overline{s}\}$ then implies that either $v_{12}/2 \leq \min\{\tau, \overline{s}\}$, or $v_{12}/2 \geq \tau = \overline{s}$. We consider these two cases in turn.

Consider the first case, and note that the condition

$$\frac{v_{12}}{2} \leq \tau = v_{12} - \max\{v_1, v_2\}$$

then implies $\max\{v_1, v_2\} \leq v_{12}/2$. Without loss of generality, suppose $\rho_2 \geq \rho_1$ (≥ 0), and consider the following deviation for firm 1: Keeping ρ_1 constant, reduce μ_1 so as to offer

$\tilde{v}_1 = v_{12}/2 + \varepsilon$, which amounts to charging

$$\tilde{\mu}_1 = \frac{w - \delta + \rho_2 - \rho_1}{2} - \varepsilon \geq \frac{w - \delta}{2} - \varepsilon > 0.$$

This deviation does not affect v_{12} or $\tau_2 = v_{12} - v_2$, but it decreases τ_1 to $\tilde{\tau}_1 = v_{12} - \tilde{v}_1 = v_{12}/2 - \varepsilon$; as initially $\tau_2 \geq \tau \geq v_{12}/2$, it follows that the multi-stop shopping threshold becomes $\tilde{\tau} = \tilde{\tau}_1 \,(< v_{12}/2) < \tilde{v}_1$. This adjustment thus induces some multi-stop shoppers to buy everything from firm 1 (those whose shopping cost lies between $\tilde{\tau}_1$ and $v_{12}/2$), on which firm 1 earns an extra profit from selling its weak product (as $\tilde{\mu}_1 > 0$), and it moreover attracts some additional one-stop shoppers (those whose shopping cost lies between $v_{12}/2$ and $\tilde{v}_1$), generating additional profit (as $\rho_1 \geq 0$ and $\tilde{\mu}_1 > 0$).

Hence, we cannot have an equilibrium of the type $v_{12}/2 \leq \min\{\tau, \overline{s}\}$.

Consider now the second case: $\overline{s} = \tau \leq v_{12}/2$. Note first that, if $\tau = \tau_i = v_{12} - v_i < \tau_j = v_{12} - v_j$, then firm i could again profitably deviate by increasing the margin on its strong product without affecting the demand (as τ_i does not depend on ρ_i). Hence, we must have $\overline{s} = \tau = \tau_1 = \tau_2$, and thus $v_1 = v_2$, or $m_1 = m_2 = m$.

We now show that firms' margins on weak products must satisfy $\mu_1, \mu_2 \leq -h(\overline{s})$, and margins on strong products must satisfy $\rho_1, \rho_2 \geq h(\overline{s})$. To see this, note that firm 1, say, could induce some multi-stop shoppers to buy its weak product B as well, by reducing the margin on its weak product, so that $\tilde{\tau}_1 = \delta + \tilde{\mu}_1 - \rho_2 < \tau_1 \,(= \delta + \mu_1 - \rho_2) = \overline{s}$, keeping the total margin constant: $\tilde{\rho}_1 + \tilde{\mu}_1 = m_1$. By so doing, firm 1 would earn a profit equal to

$$\begin{aligned} \pi_1 &= \tilde{\rho}_1 F(\tilde{\tau}_1) + m_1(F(\overline{s}) - F(\tilde{\tau}_1)) \\ &= m_1 F(\overline{s}) - \tilde{\mu}_1 F(\delta + \tilde{\mu}_1 - \rho_2). \end{aligned}$$

To rule out such a deviation, μ_1 must satisfy

$$\mu_1 \in \arg\max_{\tilde{\mu}_1 \leq \mu_1} -\tilde{\mu}_1 F(\delta + \tilde{\mu}_1 - \rho_2),$$

which, given the monotonicity of $h(\cdot)$, amounts to

$$\mu_1 \leq -h(\overline{s}).$$

Alternatively, firm 1 could discourage some multi-stop shoppers by increasing $\tilde{\rho}_1$, so that $\tilde{\tau}_2 = \delta + \mu_2 - \tilde{\rho}_1 < \tau_2 \,(= \delta + \mu_2 - \rho_1) = \overline{s}$, keeping $\tilde{\mu}_1$ unchanged. Doing so yields a profit equal to

$$\pi_1 = \tilde{\rho}_1 F(\tilde{\tau}_2).$$

Ruling out this deviation thus requires

$$\rho_1 \in \arg\max_{\tilde{\rho}_1 \geq \rho_1} \tilde{\rho}_1 F(\delta + \mu_2 - \tilde{\rho}_1),$$

or:

$$\rho_1 \geq h(\bar{s}).$$

The conditions $\mu_2 \leq -h(\bar{s})$ and $\rho_2 \geq h(\bar{s})$ can be derived using the same logic.

Therefore, the margins for any candidate equilibria must satisfy (using $\tau = \delta + \mu_1 - \rho_2 = \bar{s}$): $-h(\bar{s}) \geq \mu_1 = \rho_2 - \delta + \bar{s} \geq h(\bar{s}) - \delta + \bar{s}$, implying $\bar{s} + 2h(\bar{s}) \leq \delta$. Hence, an equilibrium with only multi-stop shopping exists only when $\bar{s} \leq j^{-1}(\delta)$. Conversely, when this condition holds, any margins satisfying $\rho_1, \rho_2 \in [h(\bar{s}), \delta - \bar{s} - h(\bar{s})]$, $\mu_2 = \rho_1 - \delta + \bar{s}$ and $\mu_1 = \rho_2 - \delta + \bar{s}$ constitute an equilibrium in which all consumers are multi-stop shoppers.
■

Claims 7 and 8 together establish the second part of the Proposition.

A.2 Proof of Proposition 3

Suppose that consumers' shopping costs are distributed over $[\underline{s}, +\infty)$, where $\underline{s} < w$. We first show that part of Lemma 1 still applies:

Lemma 2 *Suppose that consumer shopping costs are distributed over $[\underline{s}, +\infty)$, where $\underline{s} < w$. Then, in equilibrium:*

- *(i) Some one-stop shoppers are active;*
- *(ii) $m_1 = m_2 = 0$;*
- *(iii) Active multi-stop shoppers buy strong products.*

Proof. It is straightforward to check that the first three claims of the proof of Lemma 1 remain valid: In equilibrium, some consumers are active (Claim 1); $m_1 = m_2 = 0$ whenever there are active one-stop shoppers (Claim 2), and active multi-stop shoppers buy the strong products (Claim 3). Furthermore, Claim 3 establishes part (iii) of the Lemma, whereas Claim 2 implies that part (ii) follows from part (i). Finally, to complete the proof, it suffices to note that the proof of Claim 5 remains valid, which yields part (i). ■

We now proceed to establish the proposition. We first note that multi-stop shopping must arise when some consumers have low enough shopping costs:

Lemma 3 *If $\underline{s} < \delta/3$, some multi-stop shoppers are active in equilibrium.*

Proof. Suppose all active consumers are one-stop shoppers. From Claim 2, price competition for one-stop shoppers then leads to $m_1 = m_2 = 0$. Ruling out multi-stop shopping requires $v = w \geq \underline{v}_{12} - \underline{s} = w - \delta - \mu_1 - \mu_2 - \underline{s}$, or (using $m_1 = m_2 = 0$) $\rho_1 + \rho_2 \leq \delta + \underline{s}$. If firm 2, say, is the one that charges less on its strong product (i.e., $\rho_2 \leq \rho_1$), then we must have $\rho_2 \leq (\delta + \underline{s})/2$. Consider the following deviation for firm 1: charge $\tilde{\rho}_1 = \varepsilon > 0$ and $\tilde{\mu}_1 = -\varepsilon$ such that the total margin remains zero. The multi-stop shopping threshold becomes

$$\tilde{\tau} = \delta - \tilde{\rho}_1 - \rho_2 \geq \delta - \varepsilon - \frac{\delta + \underline{s}}{2} = \frac{\delta - \underline{s}}{2} - \varepsilon.$$

As $\delta > 3\underline{s}$ (implying $(\delta - \underline{s})/2 > \underline{s}$), it follows that $\tilde{\tau} > \underline{s}$ for ε sufficiently small. Hence, firm 1 can induce some consumers to engage in multi-stop shopping and make a profit on them. ■

Next, we show that there indeed exist an equilibrium with multi-stop shopping as long as some consumers' shopping costs are not too large:

Lemma 4 *If $\underline{s} < \delta$, there exists an equilibrium exhibiting both types of shopping patterns, in which firms' total margins are zero ($m_i^* = 0$) and the margins on their strong products are equal to $\rho_i^* = \rho^* = h(\tau^*)$, where $\tau^* = j^{-1}(\delta)$.*

Proof. Suppose $\underline{s} < \delta$. As discussed in the text, the unique candidate equilibrium exhibiting both types of shopping patterns is such that: (i) both firms charge zero total margins ($m_i^* = 0$) and a positive margin on their strong products equal to $\rho_i^* = \rho^* = h(\tau^*)$, where $\tau^* = j^{-1}(\delta)$; and (ii) consumers with a shopping cost lying between $\underline{s}$ and τ^* engage in multi-stop shopping, whereas those with a shopping cost lying between τ^* and w are one-stop shoppers. Therefore, this type of equilibrium exists when $\underline{s} < \tau^* = j^{-1}(\delta)$. As the function $j(\cdot)$ is strictly increasing and satisfies $j(\underline{s}) = \underline{s} + 2h(\underline{s}) = \underline{s}$, the condition $\underline{s} < \tau^*$ amounts to $\underline{s} < \delta$.

Conversely, these margins indeed constitute an equilibrium. By construction, given the equilibrium prices charged by the other firm, a firm cannot make a profit on one-stop shoppers, and charging ρ^* on the strong product maximizes the profit that a firm earns from multi-stop shoppers. ■

It follows that the analysis of the baseline model still applies when the lower bound is small enough, namely, when $\underline{s} < \delta/3$. From Lemmas 2 and 3, both types of shopping patterns must arise in equilibrium; Lemma 4 then ensures that the unique candidate identified in the text is indeed an equilibrium. This establishes the first part of the Proposition.

We now turn to the second part of the Proposition, and first note that multi-stop shopping cannot arise when all consumers have high shopping costs:

Lemma 5 *If $\underline{s} > \delta$, there are no multi-stop shoppers in equilibrium.*

Proof. Suppose, to the contrary, there are some active multi-stop shoppers. From Lemma 2, $m_1 = m_2 = 0$ and multi-stop shoppers must buy strong products; hence, $\tau = \delta - \rho_1 - \rho_2 > \underline{s}$. As $\underline{s} > \delta$, it follows that $\rho_1 + \rho_2 < 0$; hence, at least one firm must charge a negative margin on its strong product and incur a loss from serving multi-stop shoppers. But this cannot be an equilibrium, as that firm could avoid the loss by increasing its prices. ■

Finally we show that, when all consumers have large enough shopping costs, there exists equilibria with no multi-stop shoppers.

Lemma 6 *There exist equilibria with one-stop shopping if and only if $\underline{s} \geq \delta/3$. In these equilibria, margins satisfy (i) $\rho_1 + \mu_1 = \mu_2 + \rho_2 = 0$, (ii) $\delta - \underline{s} \leq \rho_1, \rho_2, \rho_1 + \rho_2 \leq \delta + \underline{s}$, and (iii) $-\underline{w}_1 \leq \rho_1 \leq \bar{w}_1$ and $-\underline{w}_2 \leq \rho_2 \leq \bar{w}_2$.*

Proof. Consider a candidate equilibrium with only one-stop shopping. From Lemma 2, $m_1 = m_2 = 0$ and thus $\tau = \delta - \rho_1 - \rho_2$. For firm 1, say, it cannot be profitable to deviate by attracting one-stop shoppers, as this would require a negative total margin $\tilde{m}_1 < 0$. Firm 1 could, however, deviate so as to induce some consumers to engage in multi-stop shopping; more specifically:

- (i) It could induce some consumers to buy both strong products by charging $\tilde{\rho}_1$ such that $\tilde{\tau}_2 = \delta - \tilde{\rho}_1 + \mu_2 = \delta - \tilde{\rho}_1 - \rho_2 > \underline{s}$, or $\tilde{\rho}_1 < \delta - \underline{s} - \rho_2$.
- (ii) Alternatively, it could induce some consumers to buy both weak products by charging $\tilde{\mu}_1$ such that $\underline{\tilde{\tau}}_2 = -\delta + \rho_2 - \tilde{\mu}_1 > \underline{s}$, or $\tilde{\mu}_1 < \rho_2 - \delta - \underline{s}$.

Ruling out the first type of deviation requires $\rho_2 \geq \delta - \underline{s}$, while preventing the second type of deviation requires $\rho_2 \leq \delta + \underline{s}$. Therefore, the equilibrium margin ρ_2 must lie between $\delta - \underline{s}$ and $\delta + \underline{s}$. Applying the same logic to rule out firm 2's deviations requires the equilibrium margin ρ_1 to lie between $\delta - \underline{s}$ and $\delta + \underline{s}$ as well. Moreover, the margins cannot exceed the social values, which requires $-\underline{w}_1 \leq \rho_1 \leq \bar{w}_1$ and $-\underline{w}_2 \leq \rho_2 \leq \bar{w}_2$.

Conversely, any margins that satisfy (i) $\rho_1 + \mu_1 = \mu_2 + \rho_2 = 0$, (ii) $\delta - \underline{s} \leq \rho_1, \rho_2, \rho_1 + \rho_2 \leq \delta + \underline{s}$, and (iii) $-\underline{w}_1 \leq \rho_1 \leq \bar{w}_1$ and $-\underline{w}_2 \leq \rho_2 \leq \bar{w}_2$ constitute an equilibrium in which all active consumers are one-stop shoppers and both firms earn zero profits.

The above analysis shows that equilibrium margins must satisfy: (i) $\delta - \underline{s} \leq \rho_1, \rho_2$, implying $\rho_1 + \rho_2 \geq 2\delta - 2\underline{s}$; and (ii) $\rho_1 + \rho_2 \leq \delta + \underline{s}$. These two conditions then lead to $2\delta - 2\underline{s} \leq \delta + \underline{s}$, which amounts to $\delta/3 \leq \underline{s}$. It thus follows that such an equilibrium exists if and only if $\delta/3 \leq \underline{s}$. ■

Combining Lemmas 5, 6 and 2 yields the second part of the Proposition, whereas Lemmas 4 and 6 together yield the last part.

B RBC laws

We now turn to the mixed-strategy equilibrium that arise under RBC laws.

B.1 Proof of Proposition 6

We show first that there is no pure strategy Nash equilibrium under RBC laws. We first note that in any pure strategy Nash equilibrium each firm $i = 1, 2$ would have to charge $\rho_i, \mu_i \geq 0$, so as to satisfy the RBC laws, and from Proposition 5 we have:

Corollary 2 *Under RBC laws, in any equilibrium each firm must obtain a positive profit; therefore, each firm should attract some consumers and sell them at least one product with a positive margin.*

Proof. This follows directly from Proposition 5, which implies that under RBC laws, in any equilibrium each firm i must obtain a profit at least equal to $\pi_i \geq \bar{\pi} > 0$. ■

It follows that, in any equilibrium with pure strategies, some consumers must be active; we successively consider below the cases where one-stop shoppers would be supplied by both firms, one firm, or none (that is, only multi-stop shoppers would be active).

Case (1): Both firms supply one-stop shoppers. This case can only arise when the two firms offer one-stop shoppers the same positive value: $v_1 = v_2 > 0$, implying $\hat{m}_1 = \hat{m}_2$. By construction, at least one firm, say firm i, attracts only a fraction of these one-stop shoppers; and from Corollary 2, firm i must sell at least one good with a positive margin. Suppose firm i deviates by reducing that margin by ε:

- This deviation enables firm i to attract all active one-stop shoppers.

- In addition, the relevant thresholds for multi-stop shopping, which can initially be expressed as

$$\begin{aligned} \tau &= v_{12} - \max\{v_1, v_2\} = v_{12} - v_i = \delta - \hat{\rho}_j + \hat{\mu}_i, \\ \underline{\tau} &= \underline{v}_{12} - \max\{v_1, v_2\} = \underline{v}_{12} - v_i = -\delta - \hat{\mu}_j + \hat{\rho}_i, \end{aligned}$$

  can only be lowered by the reduction of firm i's margin.[45] Therefore:

  - If initially there are only one-stop shoppers, then the deviation does not transform any of them into multi-stop shoppers.

  - If instead there are initially multi-stop shoppers as well, then the deviation can only transform marginal multi-stop shoppers into one-stop shoppers, on which firm i makes a higher profit.

[45] More precisely, as $\tilde{v}_i > v_j$, these thresholds either become $\tilde{\tau} = \tilde{v}_{12} - \tilde{v}_i = \tau - \varepsilon$ and $\tilde{\underline{\tau}} = \tilde{\underline{v}}_{12} - \tilde{v}_i = \underline{\tau}$ (if $\tilde{\mu}_i = \mu_i - \varepsilon$ and $\tilde{\rho}_i = \rho_i$), or $\tilde{\tau} = \tau$ and $\tilde{\underline{\tau}} = \tilde{\underline{\tau}} - \varepsilon$ (if $\tilde{\rho}_i = \rho_i - \varepsilon$ and $\tilde{\mu}_i = \mu_i$).

It follows that, for ε small enough, the deviation is profitable.

Case (2): One firm supplies one-stop shoppers. This case arises when, for instance, $v_i > v_j\,(> 0)$, implying $\hat{m}_j > \hat{m}_i$, in which case firm i attracts all one-stop shoppers. From Corollary 2, firm j must also obtain a profit, implying that some multi-stop shoppers must be active as well. For this to be the case, firm i must offer a positive value, $v_i^{ms} > 0$, on the product they target. It is moreover straightforward to see that firm i must offer a positive value, $v_i^{os} \equiv v_i - v_i^{ms} > 0$, on its other product as well. Starting from a situation where it would offer no value on this other product, reducing its margin so as to offer a slightly positive value on that product (e.g., $\tilde{v}_i^{os} = \varepsilon > 0$) would not only enable firm i to sell both of its products to one-stop shoppers (with an almost "full" margin on the other product) but, by slightly increasing its overall value, from v_i to $\tilde{v}_i = v_i + \varepsilon$, it would also transform marginal multi-stop shoppers into (more profitable) one-stop shoppers, buying both products from firm i. Therefore, we can restrict attention to firm i's margins such that $\rho_i < \bar{w}_i$ and $\mu_i < \underline{w}_i$. As from Corollary 2, firm i must sell at least one good with a positive margin, we thus have $(m_j \geq)\, \hat{m}_j > \hat{m}_i = m_i > 0$ and firm i's profit can be expressed as

$$\begin{aligned}\pi_i &= m_i \left[F\left(v_i\right) - F\left(\tau\right)\right] + m_i^{ms} F\left(\tau^{ms}\right). \\ &= m_i F\left(v_i\right) - m_i^{os} F\left(\tau^{ms}\right),\end{aligned}$$

where τ^{ms} denotes the threshold for multi-stop shopping, whereas m_i^{ms} and $m_i^{os} = m_i - m_i^{ms}$ respectively denote firm i's margins on the product bought by multi-stop shoppers (as well as by one-stop shoppers), and on the other product (bought only by one-stop shoppers).[46] Note that charging zero margin on the product bought by multi-stop shoppers is never optimal: Starting from $m_i^{ms} = 0$, deviating to $\tilde{m}_i^{ms} = \varepsilon$ (where ε is positive but "small") and $\tilde{m}_i^{os} = m_i - \varepsilon$ allows firm i to earn the same profit from one-stop shoppers, but in addition it now derives a positive profit from multi-stop shoppers; moreover, the deviation keeps v_i unchanged but reduces the multi-stop shopping threshold τ^{ms} to $\tilde{\tau}^{ms} = \tau^{ms} - \varepsilon$, and thus transforms marginal multi-stop shoppers into one-stop shoppers,

[46] If multi-stop shoppers buy strong products, we thus have $\tau^{ms} = \tau = v_{12} - v_i$, $m^{ms} = \rho_i$ and $m^{os} = \mu_i$; if instead multi-stop shoppers buy weak products, we have $\tau^{ms} = \underline{\tau} = \underline{v}_{12} - v_i$, $m^{ms} = \mu_i$ and $m^{os} = \rho_i$.

on which firm i makes more profit. Thus, in what follows we focus on $m_i^{ms} > 0$ and distinguish two cases, depending on whether firm i charges the monopoly profit-maximizing margin $m^* \equiv \arg\max_m mF(w-m)$ (which, given the monotonicity of the inverse hazard rate $h(\cdot)$, is uniquely defined by $h(w-m^*) = m^*$):

- If $m_i \neq m^*$, then suppose that firm i adjusts its margin on the product bought by multi-stop shoppers to $\tilde{m}_i^{ms} = m_i^{ms} + \varepsilon(m^* - m_i)$, where $\varepsilon > 0$ is small enough to ensure that $\tilde{m}_i < \hat{m}_j$ and $\tilde{m}_i^{ms} > 0$. Such a deviation does not change the threshold τ^{ms} (which depends on firm i's prices only through m_i^{os}) and firm i's profit becomes

$$\tilde{\pi}_i = (m_i + \varepsilon(m^* - m_i)) F(v_i - \varepsilon(m^* - m_i)) - m_i^{os} F(\tau^{ms}).$$

  The monotonicity of the inverse hazard rate $h(\cdot)$ ensures that the first term increases with ε as long as $\tilde{m}_i < m^*$, implying that such a deviation is profitable.

- If $m_i = m^*$, then firm j can benefit from undercutting its rival. Firm j's profit is given by

$$\pi_j = m_j^{ms} F(\tau^{ms}),$$

  where m_j^{ms} denotes firm j's margin on the product bought by multi-stop shoppers. Using

$$\tau^{ms} = v_i^{ms} + v_j^{ms} - v_i \leq v_j^{ms} < w - m_j^{ms},$$

  where the first inequality stems from $v_i = v_i^{os} + v_i^{ms} \geq v_i^{ms}$,[47] and the second one follows from the fact that the surplus generated by any single product cannot exceed w, we have:

$$\pi_j = m_j^{ms} F(\tau^{ms}) < m_j^{ms} F\left(w - m_j^{ms}\right) \leq \pi^* \equiv m^* F(w - m^*). \tag{6}$$

  That is, the maximum profit that firm j can earn from multi-stop shoppers is strictly lower than the monopoly profit on one-stop shoppers. Consider now firm j's deviation to $\tilde{\mu}_j = \max\{\rho_i - \delta - \varepsilon/2, 0\}$ and $\tilde{\rho}_j = m^* - \varepsilon - \tilde{\mu}_j$, for some $\varepsilon > 0$:

  - If $\rho_i > \delta$, then for ε small enough, $\tilde{\mu}_j = \rho_i - \delta - \varepsilon/2 \left(< \bar{w}_i - \delta = \underline{w}_j\right)$ and $\tilde{\rho}_j = \mu_i + \delta - \varepsilon/2 \,(< \underline{w}_i + \delta = \bar{w}_j)$, implying $\tilde{v}_{12} = \underline{\tilde{v}}_{12} = v_i + \varepsilon/2 < \tilde{v}_j = v_i + \varepsilon$,

[47] From the remarks above, this inequality is actually strict, as $v_i^{os} > 0$.

and thus $\tilde{\tau} = \underline{\tilde{\tau}} = -\varepsilon/2 < 0$. Therefore, firm j transforms all multi-stop shoppers into one-stop shoppers, and it attracts all one-stop shoppers to which it charges a total margin of $\tilde{m}_j = m^* - \varepsilon$.

– If instead $\rho_i \leq \delta$, which implies that multi-stop shoppers buy strong products,[48] then $\tilde{\mu}_j = 0$ and $\tilde{\rho}_j = m^* - \varepsilon$ (note that $\rho_i \leq \delta$ then implies $\tilde{\rho}_j < m^* = \rho_i + \mu_i \leq \delta + \underline{w}_i = \bar{w}_j$); firm j then attracts all one-stop shoppers and serves as well any remaining multi-stop shoppers (who still buy strong products, as $\tilde{\tau} = \tilde{v}_{12} - \tilde{v}_j = \delta - \rho_i \geq 0$), but makes the same margin $\tilde{m}_j = \tilde{\rho}_j = m^* - \varepsilon$ on both types of shoppers.

- In both cases, the deviation yields a profit

$$\tilde{\pi}_j = \tilde{m}_j F(\tilde{v}_j) = (m^* - \varepsilon) F(w - m^* + \varepsilon),$$

which, from (6), makes the deviation profitable for ε small enough.

Case (3): There only exist multi-stop shoppers. This case arises when $v^{ms} \equiv v_1^{ms} + v_2^{ms} \geq 2\max\{v_1, v_2\}$,[49] where as before v_i^{ms} denotes the value offered by firm i on the product targeted at multi-stop shoppers. By construction, however, $v_i = v_i^{ms} + v_i^{os}$, where, as before, v_i^{os} denotes the value offered by firm i on its other product. The first condition therefore implies[50]

$$v_1^{ms} = v_2^{ms} = \frac{v^{ms}}{2} > v_1^{os} = v_2^{os} = 0.$$

[48] Multi-stop shoppers would buy weak products only if $\underline{v}_{12} > v_i$, or $\underline{\tau} = \underline{v}_{12} - v_i = -\delta - \hat{\mu}_j + \hat{\rho}_i > 0$, which (using $\hat{\rho}_i = \rho_i$, as noted above) implies $\rho_i > \delta + \hat{\mu}_j \geq \delta$.

[49] We must have

$$v^{ms} - 2s \geq 0 \Longrightarrow v^{ms} - 2s \geq \{v_1, v_2\} - s,$$

which amounts to

$$s \leq v^{ms}/2 \Longrightarrow s \leq v^{ms} - \max\{v_1, v_2\},$$

or $\max\{v_1, v_2\} \leq v^{ms} - v^{ms}/2 = v^{ms}/2$.

[50] To see this, note that the condition $v^{ms} \geq 2v_j$ amounts to:

$$\begin{aligned} v_1^{ms} + v_2^{ms} &\geq 2\left(v_j^{ms} + v_j^{os}\right) \\ &\Longleftrightarrow v_i^{ms} - v_j^{ms} \geq 2v_j^{os}. \end{aligned}$$

But then any firm i can profitably deviate by charging a positive but non-prohibitive margin on its other product, leaving a positive value $\tilde{v}_i^{os} > 0$. This deviation does not affect the value offered to multi-stop shoppers, v^{ms}, but it *increases* the value offered to one-stop shoppers to

$$\tilde{v}_i = v_i^{ms} + \tilde{v}_i^{os} = \frac{v^{ms}}{2} + \tilde{v}_i^{os} > \frac{v^{ms}}{2}.$$

This deviation thus induces some of the initial multi-stop shoppers (namely, those whose shopping cost lies between $\tilde{\tau}^{ms} = v^{ms} - \tilde{v}_i$ and $v^{ms}/2$) to buy both products from firm i, enabling firm i to earn an extra profit from selling its other product, and it moreover attracts additional one-stop shoppers (those whose shopping cost lies between $v^{ms}/2$ and $\tilde{v}_i$), generating yet another profit.

To summarize, no pure-strategy satisfying $\rho_i \geq 0$ and $\mu_i \geq 0$ for $i \in \{1, 2\}$ can form a Nash equilibrium in any of the above configurations; hence there is no pure-strategy Nash equilibrium when below-cost pricing is prohibited.

We now characterize the mixed-strategy equilibrium. Consider a candidate equilibrium in which each firm i: (i) sells its weak product at cost; (ii) randomizes the margin ρ_i on its strong product according to a distribution $G(\rho)$ over some interval with continuous density $g(\rho)$; and obtains an expected profit equal to the minmax, $\bar{\pi}$. By construction, the bounds of the support of the distribution must be given by (4) and (5).

Consider consumers' responses to given margins ρ_i and ρ_j:

- Consumers buy both goods from firm i if
  - firm i undercuts its rival:
    $$\rho_j \geq \rho_i;$$
  - one-stop shopping is valuable:
    $$s \leq v_i = w - \rho_i;$$

As v_j^{os} cannot be negative (consumers can always opt out), and the condition $v^{ms} \geq 2v_j$ must hold for $j \in \{1, 2\}$, it follows that $0 \geq v_1^{ms} - v_2^{ms} \geq 0$, or $v_1^{ms} = v_2^{ms}$; this, in turn, implies $0 \leq v_j^{os} \leq 0$, or $v_j^{os} = 0$, for $j \in \{1, 2\}$.

– and more so than multi-stop shopping:

$$s \geq v_{12} - v_i = \delta - \rho_j.$$

- Consumers instead engage in multi-stop shopping if

$$s \leq v_{12} - \max\{v_1, v_2\},$$

which boils down to

$$s \leq \delta - \rho_i \text{ and } s \leq \delta - \rho_j.$$

Figure 1 depicts consumers' response.

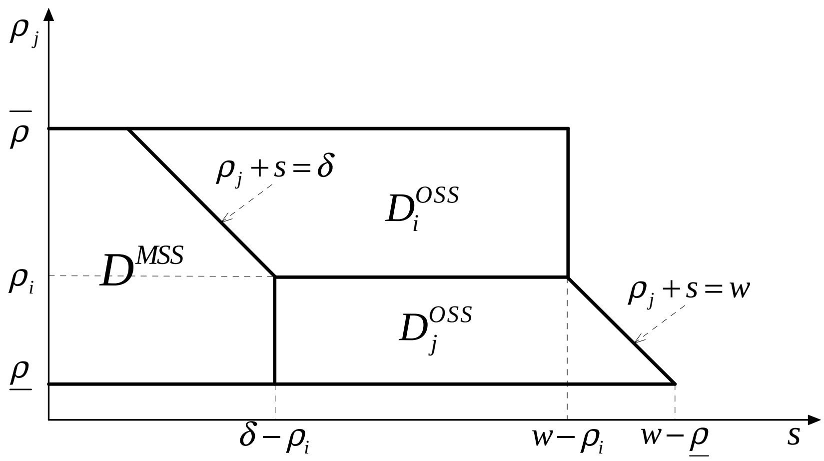


Figure 1

Firm i's expected profit can then be expressed as

$$\rho_i E\left(D_i^{OSS} + D^{MSS}\right),$$

where D_i^{OSS} represents the demand from one-stop shoppers going to firm i and D^{MSS} is the demand from multi-stop shoppers. As firm j's margin is distributed according to the distribution function $G\left(\rho_j\right)$, firm i's expected profit can be written as:

$$\begin{aligned}
\pi(\rho_i) &= \rho_i\left[(1 - G(\rho_i)) F(w - \rho_i) + G(\rho_i) F(\delta - \rho_i)\right] \\
&= \rho_i\left\{F(w - \rho_i) - G(\rho_i)\left[F(w - \rho_i) - F(\delta - \rho_i)\right]\right\}.
\end{aligned}$$

Hence, for a firm to obtain its minmax profit $\bar{\pi}$ we must have, for all ρ:

$$\rho\left\{F(w-\rho)-G\left(\rho\right)\left[F(w-\rho)-F\left(\delta-\rho\right)\right]\right\}=\bar{\pi},$$

or

$$G\left(\rho\right)\equiv\frac{\rho F\left(w-\rho\right)-\bar{\pi}}{\rho F\left(w-\rho\right)-\rho F\left(\delta-\rho\right)}. \tag{7}$$

By construction, the function $G\left(\cdot\right)$ defined by (7) is such that $G\left(\underline{\rho}\right)=0$ and $G\left(\bar{\rho}\right)=1$; it remains to check that it is increasing in ρ in the range $\left[\underline{\rho},\bar{\rho}\right]$. Differentiating (7) with respect to ρ, we have

$$G'\left(\rho\right)=\frac{\left[\bar{\pi}-\rho F\left(\delta-\rho\right)\right]\left[F(w-\rho)-\rho f(w-\rho)\right]+\left[\rho F(w-\rho)-\bar{\pi}\right]\left[F\left(\delta-\rho\right)-\rho f\left(\delta-\rho\right)\right]}{\left[\rho F(w-\rho)-\rho F\left(\delta-\rho\right)\right]^{2}}.$$

As $w>\delta$, and given (4) and (5), the functions $\rho F(w-\rho)$ and $\rho F(\delta-\rho)$ are both increasing in the range $\left[\underline{\rho},\bar{\rho}\right]$, and moreover satisfy $\underline{\rho}F(w-\underline{\rho})=\bar{\rho}F(\delta-\bar{\rho})=\bar{\pi}$ and $\rho F(w-\rho)>\bar{\pi}>\rho F(\delta-\rho)$ for $\underline{\rho}<\rho<\bar{\rho}$. It follows that $G'\left(\bar{\rho}\right)=0$ and $G'\left(\rho\right)>0$ for $\underline{\rho}\leq\rho<\bar{\rho}$.

We now show that the function $G\left(\cdot\right)$ supports a symmetric mixed strategy equilibrium. To see this, consider firm i's best response when its rival, firm j, adopts the above strategy. If firm i were to charge a total margin $m_i>\bar{\rho}$, one-stop shoppers would go to the rival and multi-stop shoppers are those consumers whose shopping cost is lower than $v_{12}-v_j=\delta-\rho_i$; hence, firm i would earn a profit equal to $\rho_i F\left(\delta-\rho_i\right)\leq\bar{\pi}$. Thus, without loss of generality, we can restrict attention to the deviations such that $m_i\leq\bar{\rho}$.

Suppose first that firm i prices its weak product above cost (i.e., its total margin satisfies $m_i>\rho_i$), and consider the impact of an increase in the margin on the strong product, ρ_i, keeping constant the total margin m_i:

• For realizations of the rival's margins such that $m_j\left(=\rho_j\right)>m_i$, one-stop shoppers (if any) favor firm i, and thus the multi-stop shopping threshold is $\tau=v_{12}-v_i=\delta+m_i-\rho_i-\rho_j$; two cases may then arise:

- If $\tau=v_{12}-v_i\leq v_i$, which amounts to $v_i\geq v_{12}/2$, consumers whose shopping cost lies below τ engage in multi-stop shopping and buy strong products, whereas those with s between τ and v_i buy both products from firm i. Hence, increasing ρ_i:

– Increases the profit earned from selling the strong product to all active consumers (that is, those with $s \leq v_i = w - m_i$);

– and also induces some multi-stop shoppers to buy firm i's weak product as well, which further enhances firm i's profit.

- If instead $v_i < v_{12}/2$, consumers whose shopping cost lies below $v_{12}/2$ engage in multi-stop shopping and buy strong products, and all other consumers are inactive. Hence, firm i's profit is equal to

$$\pi_i(\rho_i) = \rho_i F\left(\frac{v_{12}}{2}\right) = \rho_i F\left(\frac{w + \delta - \rho_1 - \rho_2}{2}\right),$$

which increases with ρ_i: The derivative is equal to

$$\pi_i'(\rho_i) = F\left(\frac{v_{12}}{2}\right) - \frac{\rho_i f\left(\frac{v_{12}}{2}\right)}{2} = \left[2h\left(\frac{v_{12}}{2}\right) - \rho_i\right] \frac{f\left(\frac{v_{12}}{2}\right)}{2},$$

where the term in brackets is positive, as $v_i < v_{12}/2$ implies $2h(v_{12}/2) > h(v_{12}/2) > h(v_i) = h(w - m_i) > m_i > \rho_i$ (where the penultimate inequality stems from $m_i \leq \bar{\rho}$, the function $m_i F(w - m_i)$ being increasing in m_i in that range).

• For realizations of the rival's margins such that $m_j \left(= \rho_j\right) < m_i$, one-stop shoppers (if any) favor firm j; hence, firm i only sells (its strong product) to multi-stop shoppers, and the multi-stop shopping threshold is $\tau = v_{12} - v_j = \delta - \rho_i$; two cases may again arise:

- If $\tau = v_{12} - v_j \leq v_i$, which amounts to $v_j \geq v_{12}/2$, all consumers whose shopping cost lies below τ engage in multi-stop shopping, and so firm i's profit is equal to:

$$\pi_i(\rho_i) = \rho_i F(\tau) = \rho_i F(\delta - \rho_i),$$

which increases with ρ_i on the relevant range $\rho_i \leq \bar{\rho}$.

- If instead $v_j < v_{12}/2$, only those consumers with s below $v_{12}/2$ engage in multi-stop shopping, and so firm i's profit is equal to $\pi_i(\rho_i) = \rho_i F\left(\frac{v_{12}}{2}\right)$. The same reasoning as above then shows that this profit again increases with ρ_i.

Therefore, it is never optimal for a firm to price its weak product above cost: Starting from $\rho_i < m_i$, raising ρ_i would always increase firm i's *ex post* profit, and would thus increase its expected profit as well.

Suppose now that firm i sells its weak product at cost: $m_i = \rho_i$. By construction, choosing any ρ_i in the range $\left[\underline{\rho}, \bar{\rho}\right]$ yields the same expected profit, $\bar{\pi}$. It remains to check that it cannot be profitable to pick a margin ρ_i outside the support of G:

- Choosing $\rho_i < \underline{\rho}$ attracts all one-stop shoppers and thus yields an expected profit equal to $\pi_i(\rho_i) = \rho_i F(w - \rho_i)$, which increases in ρ_i for $\rho_i \leq \bar{\rho}$, and is thus lower than $\pi_i\left(\underline{\rho}\right) = \bar{\pi}$.
- Choosing $\rho_i > \bar{\rho}$ attracts no one-stop shoppers, and thus the expected profit must be lower than $\rho_i F(\delta - \rho_i) \leq \max_\rho \rho F(\delta - \rho) = \bar{\pi}$;

This establishes the first part of the proposition; the rest has been established in the main text.

B.2 Proof of Proposition 7

We now analyze the impact of banning below-cost pricing on consumer surplus. When below-cost pricing is not prohibited, the equilibrium consumer surplus can be expressed as

$$\begin{aligned} S^* &= \int_0^w (w - s) f(s) ds + \int_0^{\tau^*} (\tau^* - s) f(s) ds \\ &= \int_0^w F(s) ds + \int_0^{\tau^*} F(s) ds, \end{aligned}$$

where the second expression relies on integration by parts. The first term in that expression represents the surplus that would be generated if all consumers were one-stop shoppers (and thus bought the bundle at cost) and the second term represents the extra surplus from multi-stop shopping. When instead below-cost pricing is banned, *ex post* (i.e., for a given realization of the margins ρ_1 and ρ_2), consumer surplus can be written as

$$\begin{aligned} S^b(\rho_1, \rho_2) &= \int_0^{v^b(\rho_1,\rho_2)} \left[v^b(\rho_1, \rho_2) - s\right] f(s) ds + \int_0^{\tau^b(\rho_1,\rho_2)} \left[\tau^b(\rho_1, \rho_2) - s\right] f(s) ds \\ &= \int_0^{v^b(\rho_1,\rho_2)} F(s) ds + \int_0^{\tau^b(\rho_1,\rho_2)} F(s) ds. \end{aligned}$$

Thus, the resulting change in *ex post* consumer surplus is given by

$$\Delta S(\rho_1, \rho_2) = S^b(\rho_1, \rho_2) - S^* = \int_{\tau^*}^{\tau^b(\rho_1,\rho_2)} F(s) ds - \int_{v^b(\rho_1,\rho_2)}^{w} F(s) ds.$$

Banning below-cost pricing generates two opposite effects on consumer surplus. On the one hand, the increase in multi-stop shopping (recall that $\tau^b > \tau^*$) has a positive effect, represented by the first term in the above expression; on the other hand, one-stop shoppers face higher prices than before, causing a loss of consumer surplus represented by the second term. The net effect depends on the value of w, δ, and the distribution of shopping costs, which contribute to determining equilibrium prices.

To explore this further, we fix the parameter δ and examine the sign of ΔS as a function of the social value w. Note that τ^* and $\bar{\rho}$ do not depend on w, whereas $\underline{\rho}(w)$ is the lower solution to $\underline{\rho} F\left(w - \underline{\rho}\right) = \bar{\pi} = \bar{\rho} F(\delta - \bar{\rho})$, and thus decreases in w.

In the limit case where $w = \delta$, the lower bound $\underline{\rho}(w)$ coincides with $\bar{\rho}$; that is, both firms charge $\rho = \bar{\rho}$ with probability one. As $\bar{\rho} > \rho^*$ (and weak products are priced at cost, instead of being subsidized), all prices are higher than before, and thus every consumer's (expected) surplus goes down. By continuity, this remains the case as long as weak products offer sufficiently low value (i.e., as long as w is close enough to δ).

We now consider the impact of a ban on total welfare, that is, on the sum of consumer surplus and firms' profits. When w is close to δ, the equilibrium margin distribution tends to assign a probability mass of 1 on $\bar{\rho}$, and the impact of a ban on *expected* welfare then boils down to:

$$\begin{aligned}
\Delta W &= \Delta S(\bar{\rho}, \bar{\rho}) + 2(\bar{\pi} - \pi^*) \\
&= \int_{\tau^*}^{\tau^b(\bar{\rho}, \bar{\rho})} F(s)ds - \int_{v^b(\bar{\rho}, \bar{\rho})}^{w} F(s)ds + 2(\bar{\pi} - \pi^*) \\
&= 2\Phi(\delta - \bar{\rho}) - \Phi(\delta - 2\rho^*) - \Phi(\delta) + 2(\bar{\pi} - \pi^*),
\end{aligned}$$

where

$$\Phi(x) \equiv \int_0^x F(s)ds.$$

The sign of ΔW can be either positive or negative, depending on the distribution of shopping costs. To see this, in what follows we consider the case where shopping costs are distributed according to $F(s) = s^k/k$. The hazard rate assumption is satisfied for any $k > 0$, and:

$$f(s) = s^{k-1},\ \Phi(s) = \frac{s^{k+1}}{k(k+1)},\ h(s) = \frac{F(s)}{f(s)} = \frac{s}{k}.$$

When below-cost pricing is not prohibited, the equilibrium is characterized by

$$\begin{aligned}
\rho^* &= h\left(\delta - 2\rho^*\right) = \frac{\delta - 2\rho^*}{k} \Leftrightarrow \rho^* = \frac{\delta}{2+k}, \\
\tau^* &= \delta - 2\rho^* = \frac{k\delta}{2+k}, \\
\pi^* &= 2\rho^* F\left(\delta - 2\rho^*\right) = 2\rho^* \frac{\left(\delta - 2\rho^*\right)^k}{k} = 2k^{k-1}\left(\frac{\delta}{2+k}\right)^{k+1}, \\
v^* &= w = \delta.
\end{aligned}$$

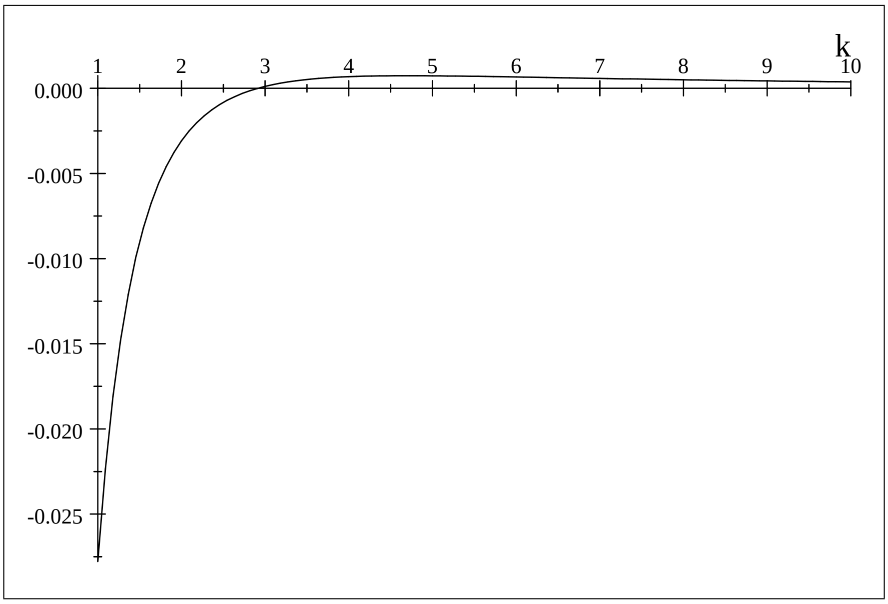
Instead, when below-cost pricing is banned, the equilibrium is characterized as follows

$$\begin{aligned}
\bar{\rho} &= h\left(\delta - \bar{\rho}\right) = \frac{\delta - \bar{\rho}}{k} \Leftrightarrow \bar{\rho} = \frac{\delta}{1+k}, \\
\bar{\tau} &= v^b\left(\bar{\rho}, \bar{\rho}\right) = \delta - \bar{\rho} = \frac{k\delta}{1+k}, \\
\bar{\pi} &= 2\bar{\rho} F\left(\delta - \bar{\rho}\right) = 2\bar{\rho} \frac{\left(\delta - \bar{\rho}\right)^k}{k} = 2k^{k-1}\left(\frac{\delta}{1+k}\right)^{k+1}.
\end{aligned}$$

Thus, banning below-cost pricing results in a change of total welfare

$$\Delta W(k) = 2k^{k-1}\left(\left(\frac{\delta}{1+k}\right)^{k+1} - \left(\frac{\delta}{2+k}\right)^{k+1}\right) + \frac{1}{k\left(1+k\right)}\left(2\left(\frac{k\delta}{1+k}\right)^{k+1} - \delta^{k+1} - \left(\frac{k\delta}{2+k}\right)^{k+1}\right).$$

This expression is continuous in k and tends to $-\infty$ when k goes to 0; hence banning below-cost pricing reduces total welfare when the distribution is not too convex. The following graph represents $\Delta W(k)/\delta^{k+1}$ and shows that banning below-cost pricing increases instead total welfare when the distribution of shopping cost is sufficiently convex (namely, for $k > \hat{k} \simeq 2.9$):



By continuity, for w close enough to δ, there exists $\hat{k}(w,\delta)$ such that banning below-cost pricing reduces total welfare when $k < \hat{k}(\delta)$.

B.3 On the impact of RBC laws on consumer surplus

We conclude by noting that RBC laws necessarily decrease (expected) consumer surplus when the density of the distribution of shopping costs does not increase between $\tau^*$and δ.

The impact of RBC laws on total expected consumer surplus can be expressed as the impact on expected social welfare, minus the impact on expected industry profit:

$$E[\Delta S(\rho_1,\rho_2)] = E[\Delta W(\rho_1,\rho_2)] - E[\Delta \Pi(\rho_1,\rho_2)],$$

where

$$E[\Delta \Pi(\rho_1 \rho_2)] = 2(\bar{\pi} - \pi^*),$$

and $\Delta W(\rho_1 \rho_2)$ can be obtained by comparing the two regimes:

- When firms are allowed to price below-cost, social welfare is equal to

  $$W^* = \int_0^w (w-s)\, dF(s) + \int_0^{\tau^*} (\delta - s)\, dF(s),$$

  where the first term is the social welfare that would be generated if all consumers were one-stop shoppers, and the second term represents the additional welfare from multi-stop shopping.

- Under RBC laws, *ex post* social welfare is equal to

  $$W^b(\rho_1,\rho_2) = \int_0^{v^b(\rho_1,\rho_2)} (w-s)\, dF(s) + \int_0^{\tau^b(\rho_1,\rho_2)} (\delta - s)\, dF(s),$$

  where

  $$v^b(\rho_1,\rho_2) = w - \min\{\rho_1,\rho_2\} \text{ and } \tau^b(\rho_1,\rho_2) = \delta - \max\{\rho_1,\rho_2\}.$$

Hence, the impact of a ban on *ex post* social welfare is given by:

$$\Delta W(\rho_1,\rho_2) = \int_{\tau^*}^{\tau^b(\rho_1,\rho_2)} (\delta - s)\, dF(s) - \int_{v^b(\rho_1,\rho_2)}^{w} (w-s)\, dF(s), \tag{8}$$

and the impact of RBC laws on total expected consumer surplus can thus be expressed as:

$$
\begin{aligned}
E\left[\Delta S\left(\rho_1,\rho_2\right)\right] &= E\left[\Delta W\left(\rho_1,\rho_2\right)\right]-2\left(\bar{\pi}-\pi^*\right)\\
&= E\left[\Delta W\left(\rho_1,\rho_2\right)-2\left(\bar{\pi}-\pi^*\right)\right]\\
&= E\left[\int_{\tau^*}^{\tau^b(\rho_1,\rho_2)}\left(\delta-s\right)dF\left(s\right)-\int_{v^b(\rho_1,\rho_2)}^{w}\left(w-s\right)dF\left(s\right)-2\left(\bar{\pi}-\pi^*\right)\right]\\
&\leq E\left[\int_{\tau^*}^{\tau^b(\rho_1,\rho_2)}\left(\delta-s\right)dF\left(s\right)-2\left(\bar{\pi}-\pi^*\right)\right]\\
&= E\left[\int_{\delta-2\rho^*}^{\delta-\max\{\rho_1,\rho_2\}}\left(\delta-s\right)dF\left(s\right)-2\left(\bar{\pi}-\pi^*\right)\right]\\
&= E\left[\phi\left(\max\left\{\rho_1,\rho_2\right\}\right)\right],
\end{aligned}
$$

where

$$\phi\left(\rho\right)\equiv\int_{\delta-2\rho^*}^{\delta-\rho}\left(\delta-s\right)dF\left(s\right)-2\left(\bar{\pi}-\pi^*\right).$$

It follows that RBC laws reduce expected consumer surplus whenever $E\left[\phi\left(\rho\right)\right]<0$, where the function $\phi\left(\rho\right)$ decreases as ρ increases:

$$\phi'\left(\rho\right)=-\rho f\left(\delta-\rho\right)<0.$$

We have:

Proposition 8 *If $f\left(s\right)$ is non-increasing for $s\in\left[\tau^*,\delta\right]$, then RBC laws reduce total expected consumer surplus.*

Proof. It suffices to show that $\phi\left(0\right)\leq 0$. Using $\tau^*=\delta-2\rho^*$, we have:

$$
\begin{aligned}
\phi\left(0\right) &= \int_{\tau^*}^{\delta}\left(\delta-s\right)f\left(s\right)ds-2\left(\bar{\pi}-\pi^*\right)\\
&\leq \int_{\tau^*}^{\delta}\left(\delta-s\right)f\left(\tau^*\right)ds-2\pi^*\\
&= \left[-\frac{\left(\delta-s\right)^2}{2}\right]_{\tau^*}^{\delta}\times\frac{F\left(\tau^*\right)}{\rho^*}-2\rho^*F\left(\tau^*\right)\\
&= \left[\frac{\rho^2}{2}\right]_0^{2\rho^*}\times\frac{F\left(\tau^*\right)}{\rho^*}-2\rho^*F\left(\tau^*\right)\\
&= 0,
\end{aligned}
$$

where the first inequality stems from the assumed monotonicity of $f(\cdot)$ on the range $[\tau^*, \delta]$ and from the fact that

$$\bar{\pi} = \max_{\rho} \rho F(\delta - \rho) \geq 2\rho^* F(\delta - 2\rho^*) = 2\pi^*,$$

and the equality that follows uses the first-order condition characterizing ρ^*, namely:

$$\rho^* f(\tau^*) = F(\tau^*).$$

It follows that $\phi(\rho) < 0$ for any $\rho > 0$, and thus

$$E[\Delta S(\rho_1, \rho_2)] \leq E[\phi(\max\{\rho_1, \rho_2\})] < 0.$$

■