Department of Economics
ISSN number 1441-5429
Discussion number 10/18

Bailout Stigma

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Abstract:
We develop a model of bailout stigma where accepting bailouts may signal firms' financial troubles and weaken their subsequent funding capabilities. Bailout stigma can lead to low or even no take-up of otherwise attractive bailout offers, the failure of market revival, or a government having to pay a hefty premium to support market revival. Nonetheless, the stigma has a salutary effect: by refusing to accept bailouts, firms may rehabilitate their market perceptions, thereby improving their subsequent financing. Secret bailouts may not eliminate bailout stigma, but secrecy accompanied by restrictions on early market revival removes the stigma and achieves constrained efficiency.

Keywords: Adverse selection, bailout stigma, secret bailout
JEL Codes: D82, G01, G18

1 Introduction

History is fraught with financial crises and large-scale government interventions, the latter often involving a highly visible and significant wealth transfer from taxpayers to banks and their creditors. According to an IMF estimate based on 124 systemic banking crises from around the world during the period 1970-2007, the average fiscal costs associated with crisis management were around 13 percent of GDP (Laeven and Valencia, 2008). More recently, during the 2007-2009 subprime mortgage crisis, the US government paid $125 billion for assets

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worth \$86-109 billion to nine largest banks under the Trouble Asset Relief Program (TARP) (Veronesi and Zingales, 2010).[1] The benefits of such interventions are difficult to measure since they depend on the unobservable counterfactual that would have played out in the absence of such interventions.

Philippon and Skreta (2012) and Tirole (2012) portray a plausible counterfactual in the form of market freeze and provide theoretical arguments for when and how government interventions may improve welfare. The essence of the argument is that the government can jump-start the market when severe adverse selection leads to market freeze. By cleaning up bad assets, or "dregs skimming," through public bailouts, the government can improve market confidence, thereby galvanizing transactions in healthier assets. However, this argument misses an important dynamic implication of bailouts. By signaling susceptibility to shocks, bailouts often attach stigma to their recipients and increase future borrowing costs. The fear of this stigma may in turn discourage financially distressed firms from accepting bailout offers.

In the wake of the *Great Recession*, policy makers were well-aware of such a fear and took efforts to alleviate the stigma. At the now-famous meeting held on October 13, 2008, Henry Paulson, then Secretary of the Treasury, "compelled" the CEOs of the nine largest banks to be the initial participants in the TARP, precisely to eliminate the stigma ("Eight days: the battle to save the American financial system," *The New Yorker*, September 21, 2009). The rates at the Fed's discount window, usually set above the federal funds rate, were cut half a percentage point to reduce the stigma that using the window would signal distress (Geithner, 2015, p. 129).[2]

Despite these efforts, the stigma remained real and significant. Defining a bid premium over the discount window rate as the discount window stigma, Armantier et al. (2011) find that the average stigma was 0.37 percent. Gauthier et al. (2015) further demonstrate that the banks that used the Term Auction Facility in 2008 paid approximately 0.31 percent less in interbank lending in 2010 than those that used the discount window. There are also anecdotes highlighting the presence of stigma. Ford refused rescue loans under the Auto Industry Program in the TARP, with a view to "legitimately portraying itself as the healthiest of Detroit's automakers" ("A risk for Ford in shunning bailout, and possibly a reward," *The*

[1] Congressional Budget Office (2012) estimates the overall cost of TARP at approximately \$32 billion, the largest part of which stems from assistance to AIG and the automotive industry while capital injections to financial institutions are estimated to have yielded a net gain. For detailed assessments of the various programs in TARP, see the *Journal of Economic Perspectives (2015)*. See also Fleming (2012) who discusses how the various emergency liquidity facilities provided by the Federal Reserve during the 2007-2009 crisis were designed to overcome the limitations of traditional policy instruments at the time of crisis. He also surveys the empirical literature that documents the effectiveness of those facilities.

[2] Such a concern is also echoed in a speech given by the former Federal Reserve chairman Ben Bernanke in 2009: "The banks' concern was that their recourse to the discount window, if it became known, might lead market participants to infer weakness—the so-called stigma problem."

New York Times, December 19, 2008).[3][4] In a similar vein, participants in the TARP were eager to exit the program early, often citing stigma as their main motivation. Signature Bank of New York was one of the first to repay its TARP debt of \$120 million for this reason.[5]

Examples such as these raise questions about whether public bailouts are effective in the first place and, if so, how such policy should be designed in the presence of the stigma associated with them. We address these questions by extending Tirole (2012) into two periods in the most parsimonious way. There is a continuum of firms, each with one unit of an asset in each period. For each firm, the quality of this asset in both periods is identical and is the firm's private information. In each period, an investment opportunity with positive NPV arrives for each firm. However, firms' liquidity-constraint and the lack of pledgeability of projects require the sale of their assets to fund the projects. Firms' first-period actions—whether they sell their assets, to whom, and at what terms—are observed publicly. Based on this observation, the market updates its belief on the cross-section of firms within each period and across the two periods. When the firms must sell their asset to fund their project in the second period, the market's offer is based on its revised belief.

This model involves not only within-period adverse selection, as in Tirole (2012), but also, and more interestingly, across-period adverse selection and signaling associated with accepting a bailout. Specifically, in the absence of government intervention, low-type firms (those with low-quality assets) are more likely to sell their assets earlier than high-type firms (i.e., those with high-quality assets), leading to what we call *the early sales stigma*: those selling early are stigmatized as low types and receive unfavorable market offers in the second period.[6] In turn, this stigma causes firms to delay sales and renders the market in the first period more prone to freeze than in the static adverse selection model. This problem further justifies the case for government intervention at an early stage, in addition to the within-period adverse selection recognized by Tirole (2012). However, a government bailout introduces its own stigma, which is even more severe than the early sales stigma. To the extent that low-type firms are more

[3] Such reluctance to receive government offers of recapitalization was also noted during the Japanese banking crisis of the 1990s (Corbett and Mitchell, 2000; Hoshi and Kashyap, 2010), which shares many commonalities with the subprime mortgage crisis in the US.

[4] The market initially perceived Ford's refusal to accept a bailout as a risky move, which was reflected in the rise in Ford's CDS spreads relative to Chrysler's. However, Ford's profit and stock price showed a remarkable turnaround in 2009, part of which is attributed to the respect Ford garnered with customers and investors by refusing a bailout. (`http://www.nasdaq.com/investing/ford-turns-a-profit-after-turning-down-bailout.aspx`, accessed Nov 17, 2015).

[5] Its chairman, Scott A. Shay, said, "We don't want to be touched by the stigma attached to firms that had taken money." ("Four small banks are the first to pay back TARP funds," *The New York Times*, April 1, 2009). It is also well known that Jamie Dimon, CEO of JP Morgan Chase, wanted to exit TARP to avoid the stigma ("Dimon says he's eager to repay 'Scarlet Letter' TARP," *Bloomberg*, April 16, 2009). Of course, the fear of stigma is not the only reason for an early exit. Wilson and Wu (2012) find that early exit by banks is also related to CEO pay, bank size, capital, and other financial conditions.

[6] This result is a consequence of the so-called single-crossing property: if a type θ finds it optimal to sell in $t = 1$, so must any lower types $\theta' < \theta$.

inclined to accept the bailout, those that accept the bailout are regarded by the market as being even less investment-worthy and receive strictly worse sales terms than those that refuse the bailout. We call this *the bailout stigma*.

We show that the bailout stigma affects bailout policies in several important ways. First, the stigma leads firms to reject otherwise attractive bailout terms, implying that even a moderately generous bailout policy might have no impact on the outcome. This means that a bailout offer has to be exceptionally generous to have any impact. Second, even such a generous offer may not sufficiently rehabilitate the market perception of the "remaining" firms—i.e., those refusing the bailout—to support immediate trade for them. In other words, unlike Tirole's one-period model, a bailout may not jump-start the market. Third, there is a multiplicity of equilibria due to the *endogeneity* of the bailout stigma: a severe bailout stigma could lead to the bailout recipients not being able to support trade in the subsequent period, which makes bailouts unacceptable for all but very low types of firms, which in turn validates and reinforces the severe stigma. Fourth, a sufficiently generous bailout offer can result in early market rejuvenation, but in such an equilibrium the government must pay a premium over the market price to compensate the recipients for their loss from the stigma. More strikingly, market sales act as an additional signaling instrument, the availability of which exacerbates the bailout stigma and ultimately reduces the overall market perception of *all* firms selling their assets in the early stage, to such an extent that suppressing the immediate market revival actually improves the effectiveness of the bailout policy.

Although the bailout stigma results in a low uptake and increased cost of bailouts, this does not necessarily mean that bailouts are ultimately ineffective. We show that a bailout policy can be effective, but the effect is *delayed and discontinuous*, and its mechanism is more nuanced than may be casually appreciated. The flip side of the stigma suffered by bailout recipients is the reputational gain enjoyed by those that refuse the bailout. An important, and paradoxical, way in which a bailout helps is by conferring firms an opportunity to boost their reputation by refusing the bailout offer. In other words, a firm's refusal to accept a bailout could rehabilitate its reputation and its ability to secure funding in a way not possible had there been no bailout (to refuse) in the first place. From this perspective, Ford's refusal to accept TARP rescue loans should not be taken as an evidence that the policy was not effective or not needed. This "salutary" effect of stigma is a very important lesson from our analysis, which has not been well appreciated in the literature or policy analysis. However, for such reputation building to be possible, some firms must accept the bailout offer. As noted above, the stigma means that no firms would accept the bailout unless its terms were sufficiently generous. In particular, we show that the effect may arise discontinuously; namely, the bailout has no effect as one progressively improves the terms of the bailout policy until, at some point, a small improvement in the terms produces a discretely large effect both in terms of the initial uptake and of the delayed effect through the reputational gain enjoyed by those that refuse the bailout. Our theory thus recognizes the need for bailout terms to be sufficiently generous to yield a tangible benefit. This implication, although departing from the classical Bagehot's rule,[7] is consistent with the approach taken by the policy makers in the recent crisis.

Since the bailout stigma stems from the transparency of the bailout program, a natural question is whether secrecy may mitigate the bailout stigma and encourage participation. Indeed, as noted above, many government programs withhold the identities of the bailout recipients. We study the implications of such a secret bailout program.[8] Given secrecy, the market observes only those that sell assets to the market in the first period but cannot distinguish those that accept the bailout from those that hold out. We find that secrecy eliminates the bailout stigma for high-type firms; thus, overall participation in a bailout program increases under secrecy. However, surprisingly, secrecy does not protect low-type recipients from the bailout stigma. These firms are exposed endogenously both by the presence of higher type firms that sell to the market in the first period (which the market observes) and self-selection by even higher types that receive the bailout but never participate in second-period sales (which would have provided a cover for low-type recipients). Thus, a stigma resurfaces for low-type bailout recipients even under secrecy, which also reduces the overall market perception of all firms selling in the second period. More importantly, secrecy deprives firms of the opportunity to improve their market perception by refusing the bailout. Both of these reduce the delayed effect of bailout. In short, a secret bailout stimulates early trade but dampens late trade compared with a transparent bailout. This trade-off means that the comparison between transparency and secrecy is generally ambiguous. However, we show that secrecy together with a restriction on early market sale can provide complete protection from the stigma, resulting in an increase in total trade relative to the transparent bailout.

Finally, we explicitly introduce the cost of a bailout into a model and investigate the welfare implications of alternative bailout policies. By casting the problem in the mechanism design framework, we develop a method for finding an optimal bailout policy and for comparing alternative policies, subject to some realistic constraints. Consistent with earlier findings, we show that a restriction on early market sales improves welfare under either a transparent or a secret bailout. This finding cautions against the prevailing view that appears to take early market revival as the barometer of the success of a bailout policy. We also find that, with the restriction on early market sale, a secret bailout welfare-dominates a transparent bailout. Indeed, the former with a carefully chosen term implements the (constrained) optimal outcome.

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[7] Bagehot's rule, orginating from the 1873 book, *Lombard Street*, by William Bagehot, prescribes that central banks should charge a higher rate than the markets to discourage banks from borrowing once the crisis subsides. Bailout stigma was not a serious issue in 1873, however, since the regulatory system in 1873 Britain ensured concealment of the identities of emergency borrowers, as Gorton (2015) points out.

[8] How such a policy may be implemented in practice is discussed in Section 5.

The remainder of the paper is organized as follows. Section 2 contains a review of the related studies. Section 3 presents our model and offers brief discussions of equilibria without government intervention. In Section 4, we study various equilibria under government intervention. Section 5 studies the case of secret bailouts. Section 6 provides the analysis of the optimal policy design while Section 7 concludes the paper. All the proofs, including characterization of all equilibria discussed in the paper, are provided by the Online Appendix.

2 Related Literature

While the broad theme of this paper is related to an extensive literature on the benefits and costs of government intervention in distressed banks,[9] our work is most closely related to Philippon and Skreta (2012) and Tirole (2012), who focus on adverse selection in asset markets as a primary reason for government intervention.[10] As mentioned previously, these studies rely on static models. As a result, although relatively low types accept bailouts, the resulting stigma does not have any adverse effect on subsequent financing. Our dynamic model not only explicitly captures the bailout stigma but also shows how the role of a bailout in a dynamic setting is qualitatively different from that in a static setting.

Banks' reputational concerns are explicitly considered in Ennis and Weinberg (2013), La'O (2014), and Chari, Shourideh and Zetlin-Jones (2014). In Ennis and Weinberg (2013), to meet their short-term liquidity needs, banks with high-quality assets use interbank lending while those with low-quality assets use the discount window. The resulting discount window stigma is reflected in the subsequent pricing of assets. In La'O (2014), financially strong banks use the Federal Reserve's Term Auction Facility since winning the auction at a premium signals financial strength, which protects them from predatory trading. The main focus in Chari, Shourideh and Zetlin-Jones (2014) is on how reputational concerns in secondary loan markets can result in persistent adverse selection. Since all three studies consider discrete types of banks and there is no government bailout, their results are not directly comparable to ours. However, the separating equilibrium in the first two studies roughly corresponds to a special

[9] The primary rationale for intervention is to prevent the contagion of bank runs whether it stems from depositor panic (Diamond and Dybvig, 1983), contractual linkages in bank lending (Allen and Gale, 2000), or aggregate liquidity shortages (Diamond and Rajan, 2005). The costs of anticipated bailouts due to the time-inconsistency of policy are discussed by, among others, Stern and Feldman (2004).

[10] Regarding the optimal form of bailouts, Philippon and Skreta (2012) show that optimal interventions involve the use of debt instruments when adverse selection is the main issue. With additional moral hazard but limits on pledgeable income, Tirole (2012) justifies asset purchases. When there is debt overhang due to lack of capital, Philippon and Schnabl (2013) find that optimal interventions take the form of capital injection in exchange for preferred stock and warrants. During the US subprime crisis, the EESA initially granted the Secretary of the Treasury authority to purchase or insure troubled assets owned by financial institutions. However, the Capital Purchase Program under TARP switched to capital injection against preferred stock and warrants.

case of our equilibria in which market is rejuvenated in the first period while the pooling equilibrium in the third study corresponds to our equilibrium in which government crowds out the market in the first period. We provide a full characterization of all possible equilibria in our model. In addition, these studies do not consider policy-related issues such as different disclosure rules.

Our paper is also related to studies on dynamic adverse selection in general (Inderst and Müller, 2002; Janssen and Roy, 2002; Moreno and Wooders, 2010; Camargo and Lester, 2014; Fuchs and Skrzypacz, 2015) and those with a specific focus on the role of information in particular (Hörner and Vieille, 2009; Daley and Green, 2012; Fuchs, Öry and Skrzypacz, 2016; Kim, 2017).[11] The key insight from the first set of studies is that dynamic trading generates sorting opportunities, which are not available in the static market setting. However, each seller has only one opportunity to trade in these studies, so signaling is not an issue. The second set of studies relates to different disclosure rules and how they affect dynamic trading. For example, Hörner and Vieille (2009) and Fuchs, Öry and Skrzypacz (2016) show that secrecy (private offers) tends to alleviate adverse selection but transparency (public offers) does not. Once again, each seller has only one trading opportunity in these studies. Hence, although past rejections can boost reputation, acceptance ends the game. In contrast, in our model, each seller has three distinct signaling opportunities, i.e., early sales, acceptance of the bailout offer, refusal to accept the bailout offer. Although our model also shows that secret bailouts dominate transparent bailouts, this is subject to the restriction on early market sales, which results in complete pooling in the first period. Most importantly, none of these papers studies government intervention in response to market failure.

There are several empirical studies that provide evidence on stigma in the financial market. As mentioned previously, Peristiani (1998) provides early evidence on the discount window stigma. Furfine (2001, 2003) finds similar evidence from the Federal Reserve's Special Lending Facility during the period 1999-2000 and the new discount window facility introduced in 2003. As mentioned earlier, Armantier et al. (2011) utilize the Federal Reserve's Term Auction Facility bid data from the 2007-2008 financial crisis to estimate the cost of stigma and its effect. Cassola, Hortaçsu and Kastl (2013) find evidence of stigma from the bidding data from the European Central Bank's auctions of one-week loans. Krishnamurthy, Nagel and Orlov (2014) find that in repo financing, dealer banks with higher shares of agency collateral repayments (implicitly) guaranteed by the government borrowed less from the Primary Dealer Credit Facility (PDCF) despite its attractive funding terms, which indeed supports there being a stigma attached to the users of the PDCF.

[11] Others include dynamic extensions of Spence's signaling model with public offers (Noldeke and Van Damme, 1990), private offers (Swinkels, 1999), and private offers with additional public information such as grades (Kremer and Skrzypacz, 2007).

3 Model and Preliminaries

Our model is a two-period extension of Tirole (2012). There is a continuum of firms each endowed with two units of legacy assets of the same value. The value of the asset θ is privately known to each firm and distributed on $[0, 1]$ according to distribution function F with density f. We assume that f satisfies log-concavity: $\frac{\partial^2 \log f(\theta)}{\partial \theta^2} < 0$ for all θ. Throughout, a truncated conditional expectation, $m(a, b) := \mathrm{E}[\theta | a \leq \theta \leq b]$, figures prominently in our analysis, and the log-concavity of f means that $0 < \frac{\partial m(a,b)}{\partial a}, \frac{\partial m(a,b)}{\partial b} < 1$, a property that we will use repeatedly throughout the paper (Bagnoli and Bergstrom, 2005). For convenience, we call a firm with legacy asset θ a type-θ firm.

In each of the two periods $t = 1, 2$, an investment project becomes available to each firm. The project requires cost I and yields strictly positive net return S and, hence, is socially valuable. However, the limited pledgeability of the project inhibits direct financing; the firm can only fund the project by selling its legacy asset to buyers in the competitive market. We assume that the firm sells at most one unit of its asset in each period,[12] and that the return from the $t = 1$ project cannot be used to fund the $t = 2$ project.

The government bailout may occur prior to the firms' investment decisions. Prior to period $t = 1$, the government offers to purchase firms' assets at a fixed price p_g. Firms then decide whether to accept this offer. Having observed the government offer and firms' responses, buyers make simultaneous offers to firms, and the firms decide whether to accept one of the offers. At the end of $t = 1$, all parties observe the set of firms, but not their individual types, that sold their assets in $t = 1$, whether the sale was to the government or to the market, and at what terms. The game up to this point is the same as that of Tirole (2012). Our model augments that game by introducing period $t = 2$, which repeats the $t = 1$ subgame. As will become clear, this simple extension brings significant new insights as well as new economic issues. The model also becomes more realistic: We can think of $t = 1$ as the time period that spans a possible government intervention to market recovery and $t = 2$ as the time period after market recovery. A crucial link between the two periods is the information that is generated in $t = 1$, which is reflected in the buyers' belief in $t = 2$.

Although our model is stylized, it introduces reputational considerations facing the firm in a simple way that allows for clear comparison with Tirole (2012). As will be seen, the main feature of this model is the inference that the market makes on the firm's type from its behavior in $t = 1$. Obviously, the inference is irrelevant in the one-shot model, but it now clearly affects the terms of trade in $t = 2$. Ultimately, our main focus is on how this reputational concern feeds back into the firm's decision to accept the government bailout in

[12] Any equilibrium that involves firms selling more than one unit in $t = 1$ can be implemented as an equilibrium in which each firm sells one unit in each period. Such an equilibrium can be supported if the market observes the number of units each firm sells, as it can attach an unfavorable belief to those selling two units.

$t = 1$. However, the reputational concern also affects the firm's decision in $t = 1$ even without the government intervention, as we discuss below.

3.1 One-Period Model à la Tirole (2012)

We begin by recapitulating the key insight from Tirole (2012) by considering the case with only one period. Suppose first that there is no government intervention. In this case, the model reduces to Akerlof's lemons market, described by Figure 1. A firm is willing to sell its asset if and only if the price offer p plus net investment return S (enabled by the asset sale) is no less than its asset value θ. Hence, the supply curve is given by $p = \theta - S$, where $\theta - S$ is the effective reservation value of the asset to the seller. Not surprisingly, firms with low values $\theta \leq p + S$ are willing to sell. Meanwhile, buyers must break even with respect to the average benefit $m(0, \theta)$ of the assets being sold, and thus, the demand is given by $p = m(0, \theta)$.

The equilibrium is given by the intersection $\theta_0^* := \sup\{\theta' | \theta' - S \leq m(0, \theta')\}$ of the two curves at price $p_0^* = m(0, \theta_0^*)$. The log-concavity of f ensures the uniqueness of the intersection point. (Note that the dependence on S will be suppressed, unless needed.) Since trade is always socially desirable,[13] a market failure arises, and its magnitude depends on S. There are two thresholds $0 < \underline{S}_0 < \overline{S}_0$ such that the market *freezes partially* ($\theta_0^* \in (0, 1)$) if $S \in (\underline{S}_0, \overline{S}_0)$; the market is *fully active* ($\theta_0^* = 1$) if $S > \overline{S}_0$. The lower threshold $\underline{S}_0$ is given by equation $m(0, \theta_0^*(\underline{S}_0)) = I$, suggesting that the break-even price that buyers offer must be at least equal to the investment cost. The upper threshold $\overline{S}_0$ is given by $1 + \overline{S}_0 = m(0, 1) = \mathrm{E}[\theta]$, meaning that even the highest type must be willing to sell at the breakeven price, when it equals the average value of all assets. When the market freezes partially or fully, some socially valuable projects are not undertaken, creating the scope for government intervention.

Suppose now that the government offers to purchase the legacy asset at some price p_g, before the market opens. Assume that $p_g \geq \max\{I, p_0^*\}$; otherwise, the bailout will have no effect. To sharpen prediction, Tirole (2012) makes a further refinement assumption whereby the market shuts down with a vanishingly small probability.[14] Given this, there exist two cutoffs $0 \leq \theta_g \leq \hat{\theta}_0 \leq 1$ such that types $\theta < \theta_g$ sell to the government at price p_g, types $\theta \in (\theta_g, \hat{\theta}_0)$ sell to the market at price p_1, and types $\theta > \hat{\theta}_0$ do not sell.

In equilibrium, with types $[0, \theta_g]$ removed by the bailout, the remaining market reduces to Akerlof's lemons problem on truncated types $[\theta_g, 1]$. Let $\gamma(\theta') := \sup\{\theta'' | \theta'' - S \leq m(\theta', \theta'')\}$

[13] This is seen by the fact that the marginal benefit θ (dashed line) is always above the supply curve.

[14] Formally, if the market collapses with probability $\varepsilon > 0$, a firm will sell to the market if and only if

$$(1 - \varepsilon)(p + S) + \varepsilon\theta \geq p_g + S \Leftrightarrow \theta \geq [p_g + S - (1 - \varepsilon)(p + S)]/\varepsilon,$$

where p is the price that prevails in the market.

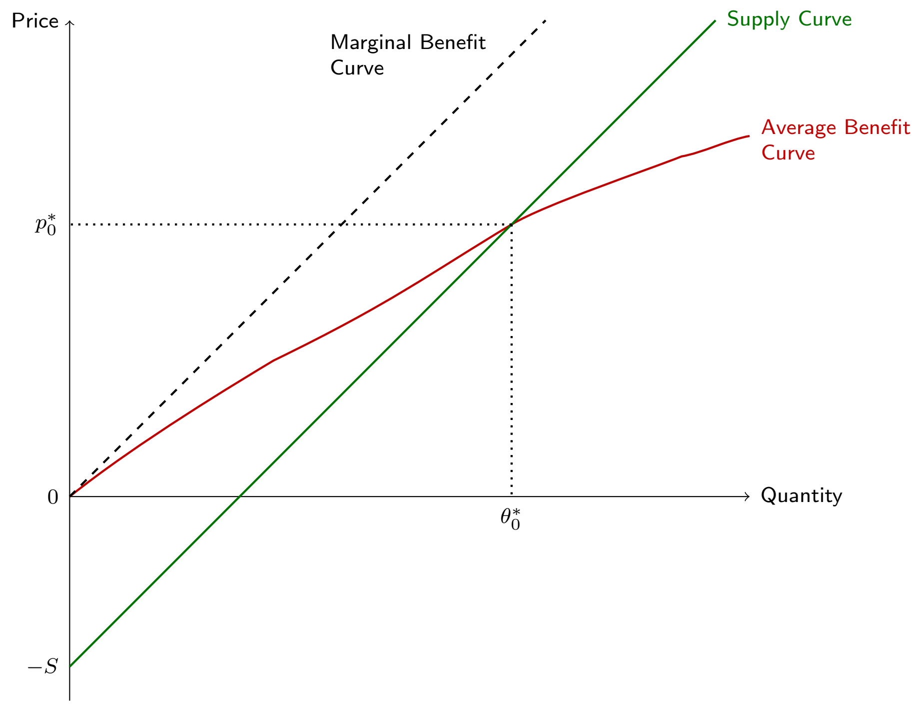


Figure 1 – One-shot model without bailouts

denote the lemon equilibrium on truncated types $[\theta', 1]$.[15] Then, the highest type $\hat{\theta}_0$ selling to the market must equal $\gamma(\theta_g)$. Further, if both the government and market offers are accepted by positive measures of firms, we must have $p_1 = p_g$; otherwise, the lower price offer will not be accepted. Hence,

$$p_g = m(\theta_g, \gamma(\theta_g)). \tag{1}$$

Given the log-concavity of f, the critical cutoff type θ_g satisfying (1) is well-defined for $p_g \in [p_0^*, 1]$. More importantly, whenever $p_g > \max\{I, p_0^*\}$, we have $\theta_g > 0$; hence, $\gamma(\theta_g) > \gamma(0) = \theta_0^*$. In other words, the bailout improves asset trading and therefore the financing of socially valuable projects whenever the market is not fully active absent the bailout—hence the welfare rationale for the bailout:

Theorem 1 (Tirole, 2012)**.** *If the government offers to purchase the legacy asset at price* $p_g \geq \max\{I, p_0^*\}$, *then types* $\theta < \theta_g$ *sell to the government and types* $\theta \in (\theta_g, \gamma(\theta_g))$ *sell to the market at the same price, where* θ_g *is given by (1) and satisfies* $\gamma(\theta_g) = p_g + S$*. Any offer* $p_g > \max\{I, p_0^*\}$ *increases the volume of trade and financing whenever* $S < \overline{S}_0$*.*

According to Theorem 1, the government purchases the most "toxic" assets; this improves

[15] In terms of Figure 1, the truncation shifts up the average benefit curve: its starting point moves along the marginal benefit curve by θ'. Consequently, the intersection point shifts out; that is, $\gamma(\theta')$ increases in θ'.

the market's perception of the remaining assets and increases the trading of assets beyond what is possible without the bailout. Through such dregs skimming, the government runs a deficit (per unit asset) equal to $p_g - m(0, \theta_g)$ and induces trade in additional assets $\theta \in (\theta_g, \gamma(\theta_g))$. While dregs skimming occurs in equilibrium, its role is not essential. As we observe next, market activation is outcome-irrelevant.[16]

Proposition 1 (Irrelevance of market rejuvenation)**.** *Suppose that the government's offer to purchase assets is accompanied by the prohibition of private sales to the market. Then, the resulting outcome—the total sales and government deficit—remains the same as if there were no prohibition of private sales. Specifically, given offer $p_g \geq \max\{I, p_0^*\}$, all types $\theta \in [0, p_g + S)$ accept the bailout and sell to the government.*

The idea is that, if the market shuts down, the firms $[\theta_g, \gamma(\theta_g)]$ that would have sold to private buyers simply sell to the government. Hence, the volume of total sales remains the same. The government deficit is also unaffected by the additional purchases since they do not entail any revenue loss due to (1).

3.2 Two-Period Model without Government Intervention

Unlike the one-shot problem, our two-period model introduces a signaling motive for firms, as their trading behavior in $t = 1$ affects the market's belief about their assets and thus the offers they receive in $t = 2$. As is well known, signaling games admit a multiplicity of equilibria. Accordingly, we focus on perfect Bayesian equilibria (in pure strategies) but impose several additional properties. First, we assume that firms discount their $t = 2$ payoff (arbitrarily) slightly.[17] Much in the same spirit of Tirole (2012), this assumption produces a natural sorting of firm types in terms of the timing of trading, with low types selling before high types. Second, we invoke the D1 refinement, which requires that, upon an off-the-path signal being sent, uninformed players (buyers in the market in our model) attribute the deviation to those types that have most to gain from that deviation (in terms of the set of responses that dominate equilibrium payoffs, following that deviation).[18] In addition to ruling out implausible equilibria, this refinement ensures that the equilibrium varies continuously with

[16] Although this point is not explicitly highlighted, irrelevance holds in both Tirole (2012) and Philippon and Skreta (2012). As will be seen, the irrelevance breaks down in our main model.

[17] Formally, we focus on the limit of the perfect Bayesian equilibria of a sequence of games in which players discount $t = 2$ by $\delta \in (0, 1)$ as $\delta \to 1$.

[18] The D1 refinement can be described formally as follows. Let $U^*(\theta)$ be the payoff for type θ in a 'putative' equilibrium, and let $u(r, s; \theta)$ be the payoff for type θ when it sends an 'off-the-path' signal s and elicits response r as a consequence. Let $D(\theta|s) := \{r | u(r, s; \theta) \geq U^*(\theta)\}$ be the set of possible responses that would yield a payoff for type θ that dominates its equilibrium payoff. Upon an off-the-path signal s being sent, the D1 refinement requires that the belief of the uninformed players be supported on the types θ for whom $D(\theta|s)$ is maximal, i.e., on types $\Theta(s) := \{\theta \in \Theta | \not\exists \theta' \text{ s.t. } D(\theta'|s) \supsetneq D(\theta|s)\}$. See Fudenberg and Tirole (1991).

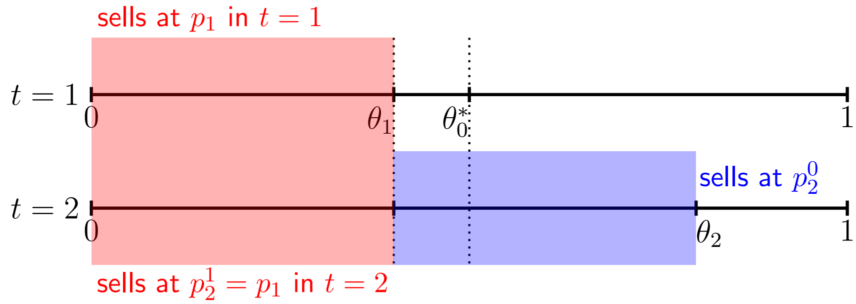


Figure 2 – Equilibrium without government intervention

parameter values. Third, we focus on an equilibrium in which buyers earn zero expected payoff. Unlike the one-shot model, zero profit is not a necessary implication of equilibrium even with the D1 refinement in our model, but any equilibrium violating this property rests on an unreasonable off-the-path belief.[19] We call a perfect Bayesian equilibrium with these properties simply an "equilibrium." We now show that any (such) equilibrium has a cutoff structure:

Lemma 1. *In any equilibrium without government intervention, there is a cutoff* $0 \leq \theta_1 \leq 1$ *such that all types* $\theta \leq \theta_1$ *sell in each of the two periods at price* $m(0, \theta_1)$ *and all types* $\theta > \theta_1$ *hold out in* $t = 1$ *and are offered price* $m(\theta_1, \gamma(\theta_1))$ *in* $t = 2$*, which types* $\theta \in [\theta_1, \gamma(\theta_1))$ *accept. If* $\theta_1 = 1$*, then* $S \geq 2(1 - \mathrm{E}[\theta])$*.*

A typical equilibrium is shown in Figure 2, where the $t = 1$ price is denoted by p_1, the $t = 2$ price for those that sold in $t = 1$ is denoted by p_2^1, and the $t = 2$ price for those that did not sell in $t = 1$ is denoted by p_2^0.[20] Intuitively, those selling in $t = 1$ have low values $\theta < \theta_1$, and those holding out have higher values $\theta > \theta_1$. Since firms' actions in $t = 1$ are observed by the market in $t = 2$, they are treated differently, with the holdouts receiving a higher offer than the early sellers. Specifically, the early sellers receive price $p_2^1 = m(0, \theta_1) = p_1$ (lemons equilibrium on types $[0, \theta_1]$), and the holdouts receive price $p_2^0 = m(\theta_1, \gamma(\theta_1))$ (lemons equilibrium on truncated types $[\theta_1, 1]$). The price difference $p_2^0 - p_2^1 = m(\theta_1, \gamma(\theta_1)) - m(0, \theta_1)$ constitutes the 'early sales stigma.'

How the early sales stigma affects firms' $t = 1$ incentives can be seen in Figure 3. As in Figure 1, the buyers' demand is given by the average benefit $m(0, \theta)$ of the assets being sold,

[19] In any such equilibrium, buyers offer a low price in $t = 1$ that leaves them with strictly positive profit. No buyer deviates to a higher price offer for fear that such an offer will be rejected since firms in turn believe that accepting it signals the worst type $\theta = 0$ to the market in $t = 2$. To overcome such an unfavorable belief, the deviation offer must be exceptionally generous, in fact so generous that it would result in a loss to the deviating buyer. Such a belief, while consistent with D1, is implausible since any price increase should be acceptable for firms with weakly higher types.

[20] The subscript refers to the period, and the superscript refers to whether trade occurred in $t = 1$, with 0 encoding 'no trade' and 1 encoding 'trade.'

when θ is the highest type asset being sold. The early sales stigma adds to firms' reservation value, and thus, the supply curve shifts up by the amount equal to the stigma and is given by $p = \theta - S + [m(\theta_1, \gamma(\theta_1)) - m(0, \theta_1)]$. Intuitively, those who sell in $t = 1$ would lose by $m(\theta_1, \gamma(\theta_1)) - m(0, \theta_1)$ in $t = 2$ and would thus require that much more to sell in $t = 1$. The presence of this stigma results in further market freeze: *the equilibrium trade at the intersection of the supply and average benefit curves shrinks relative to the one-shot model.* Let $\Delta(\theta; S) := m(0, \theta) + S - \theta - (m(\theta, \gamma(\theta)) - m(0, \theta))$ denote the buyers' average benefit minus the firms' reservation values, given a marginal type θ of firm selling in the market. Obviously, the equilibrium occurs when the average benefit equals the reservation value, or when $\Delta(\theta; S) = 0$, as depicted by the intersection of the two corresponding curves in Figure 3. The following assumption facilitates the analysis:

Assumption 1. *(i)* $\Delta(\theta; S) := m(0, \theta) + S - \theta - (m(\theta, \gamma(\theta)) - m(0, \theta))$ *is strictly decreasing in* θ*; (ii) if* $\Delta(0; S) \geq 0$*, then* $\Delta(0; S') > 0$ *for* $S' > S$*.*

Assumption 1-(i) ensures that the supply curve crosses the average benefit curve at most once. Assumption 1-(ii) simply means that $\Delta(0; S)$ satisfies the single-crossing property with respect to S. Both conditions are satisfied under standard distributions of F.[21] We now provide the main result of this section.

Theorem 2. *(i) There is an equilibrium in which firms with* $\theta \leq \theta_1^*$ *sell at price* $p_1^* := m(0, \theta_1^*)$ *in both periods: firms with* $\theta \in (\theta_1^*, \theta_2^*)$ *sell only in* $t = 2$ *at price* $p_2^* := m(\theta_1^*, \theta_2^*)$*, and firms with* $\theta > \theta_2^*$ *never sell, where* θ_1^* *and* θ_2^* *are defined by* $\Delta(\theta_1^*; S) = 0$ *and* $\theta_2^* = \gamma(\theta_1^*)$*, respectively, and satisfy* $\theta_1^* \leq \theta_0^* \leq \theta_2^*$*, and* $p_1^* \leq p_0^* \leq p_2^*$*. Given Assumption 1-(i), there is at most one such equilibrium with an interior* θ_1^**.*

(ii) Given Assumption 1-(ii), the $t = 1$ *market in equilibrium is fully active if* $S \geq \overline{S}^*$*, suffers from partial freeze if* $S \in (\underline{S}^*, \overline{S}^*)$*, and full freeze if* $S < \underline{S}^*$*, where* $\underline{S}^*$ *and* $\overline{S}^*$ *are defined by* $\Delta(0; \underline{S}^*) = 0$ *and* $\Delta(1; \overline{S}^*) = 0$*, respectively, and satisfy* $\underline{S}^* > \underline{S}_0$ *and* $\overline{S}^* > \max\{\overline{S}_0, \underline{S}^*\}$*.*

(iii) In addition, there is an equilibrium with full market freeze in $t = 1$ *for any* S*.*

The above proposition shows that the equilibrium trade is smaller in $t = 1$ but larger in $t = 2$ than in the one-period model. The reduced trade in $t = 1$ explains the increased range of S's for which the $t = 1$ market freezes. However, the flip side of the reputational loss from early sales is the reputational gain enjoyed by the firms that 'refuse to sell' in $t = 1$. This reputational gain leads to better terms for these firms and thus mitigates adverse selection in

[21] Examples include truncated normal distributions on $[0, 1]$, beta distributions with various values of the shape parameters, and the uniform distribution on $[0, 1]$. Equilibrium characterization without these assumptions is more cumbersome and adds no significant insight.

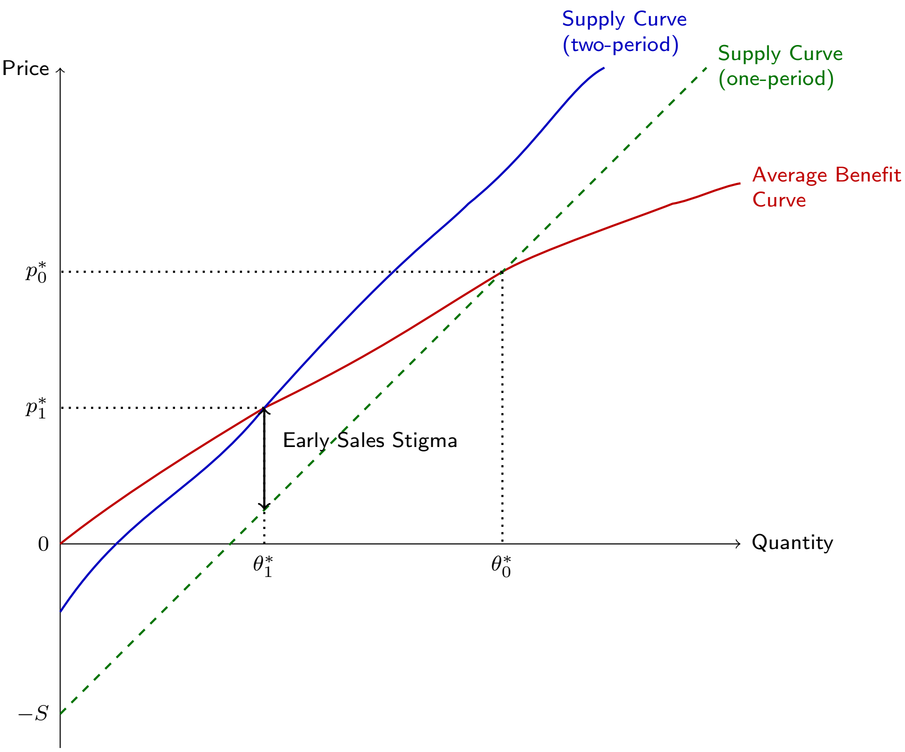


Figure 3 – The $t = 1$ market under early sales stigma

$t = 2$, resulting in greater trade than in the static model. The dynamic trading pattern is reminiscent of patterns found in dynamic adverse selection models (Fuchs, Öry and Skrzypacz, 2016; Fuchs and Skrzypacz, 2015); however, the signaling motive is absent from these models since informed players trade only once.

Strikingly, our model admits an equilibrium in which the market completely shuts down in $t = 1$, regardless of S (see Part (iii)). This phenomenon, which has no analogue in Tirole (2012) or in dynamic adverse selection models, results from an interaction between signaling and adverse selection. In this equilibrium, the buyers refrain from making *any* viable offer in $t = 1$, for fear that firms accepting that offer may suffer from extreme stigma (signaling of $\theta = 0$, say), meaning that either the offer is rejected altogether or that, if it is accepted *despite the extreme stigma,* this could only mean that the quality of assets sold must be too low to be profitable for the buyers (adverse selection). *Adverse selection* alone cannot support such an equilibrium for a large S; *adverse signaling* from $t = 1$ trade leads to such an equilibrium. Our dynamic model thus identifies a novel form of market failure that results from extreme stigma feeding extreme adverse selection.[22]

[22]Importantly, the extreme stigma is consistent with the D1 refinement, although its application in the current context is somewhat unusual; note that any signaling arises only off the path when a buyer deviates and makes an offer.

4 Dynamic Adverse Selection and Bailout Stigma

In this section, we study a government bailout of the firms via purchases of their assets. Specifically, we augment the game in the previous section by adding period $t = 0$ in which the government announces an offer to purchase at most one unit of the asset from each firm at price $p_g \in [I, 1]$,[23] and firms decide whether to accept that offer. The bailout offer is available only in $t = 0$.[24] The game described in the previous section is then played. Specifically, in the first period, the private buyers may make their offers, after observing the government's offer and firms' responses to that offer. In each period, firms selling assets at a price weakly greater than I finance their projects for that period and collect net surplus S. Whether a firm sells its asset to the government or to the market and on what terms are publicly observed at the end of $t = 1$.

For any bailout offer p_g, we consider perfect Bayesian equilibria of the ensuing subgame satisfying the aforementioned refinements, plus Tirole (2012)'s refinement—namely, that the private market shuts down with an arbitrarily low probability. Again, we call the resulting concept simply an "equilibrium."

4.1 Characterization of Equilibria

To characterize the equilibria given p_g, we begin by observing that the general structure of the equilibrium takes the form depicted in Figure 4 (with the possibility that some region may vanish depending on the parameter values).

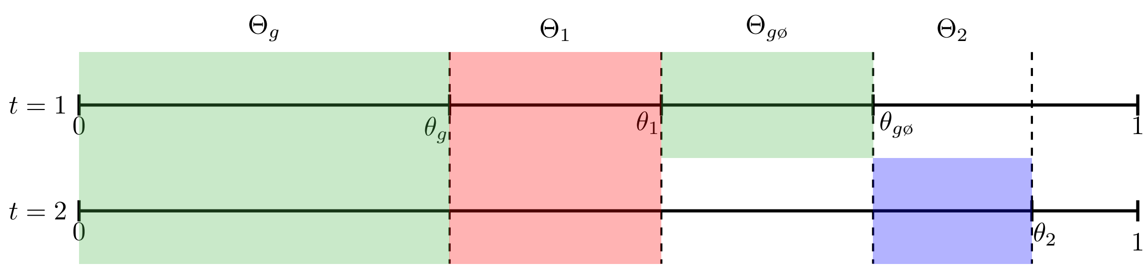


Figure 4 – A general equilibrium structure

[23]No $p_g < I$ has any impact on the outcome, and thus, we exclude such low offers. We also do not consider $p_g > 1$ on the practical basis that it would be politically infeasible for a government to pay an amount clearly in excess of asset values. In addition, any such offer is unlikely to affect the outcome since all firms would likely already trade.

[24]This is consistent with the observed practice: governments refrain from engaging in long-term bailouts and from complete 'nationalization' of distressed firms (which would be equivalent to purchasing two units of the asset in our model). Further, our goal is to study the reputational consequence of taking the government bailout, which can be studied most effectively when no government bailout is available in the second period.

Lemma 2. *In any equilibrium, there are four possible cutoffs* $0 \leq \theta_g \leq \theta_1 \leq \theta_{g\o} \leq \theta_2 \leq 1$ *such that types* $\theta \in \Theta_g := [0, \theta_g)$ *sell to the government in* $t = 1$ *and to the market in* $t = 2$*, types* $\theta \in \Theta_1 := (\theta_g, \theta_1)$ *sell to the market in both periods, types* $\theta \in \Theta_{g\o} := (\theta_1, \theta_{g\o})$ *sell only in* $t = 1$ *to the government, types* $\theta \in \Theta_2 := (\theta_{g\o}, \theta_2)$ *sell only in* $t = 2$ *to the market, and types* $\theta > \theta_2$ *sell in neither period.*

This lemma rests on several observations. First, the firms' preferences satisfy the single-crossing property, implying that across the two periods, a lower type has more incentives to sell its asset than does a higher type. This implies that the total quantity of trade must be non-increasing in θ in any equilibrium. Second, the fact that buyers (either the government or the market) never ration their purchasing means that the quantity traded for each firm must be either zero or one in each period. Third, an arbitrarily small discounting of the second-period payoff, along with the first two observations, implies that, among those that trade only in one period, early traders are of lower types than late traders. Finally, a low probability of market collapse, as in Tirole (2012), gives rise to a single-crossing property with the implication that lower types are more likely to sell to the government than higher types, all else being equal (i.e., if both are selling the same total quantity of the asset).

Using Lemma 2, we provide below a complete characterization of all possible equilibria, which are grouped into three types: (i) the **No Response** equilibrium in which no firm accepts the bailout offer, and hence, the bailout has no effect; (ii) the **No Market Rejuvenation** equilibrium in which a positive measure of firms sells to the government but no firm sells to the market in $t = 1$; and (iii) the **Market Rejuvenation** equilibrium in which positive measures of firms sell to the government and to the market in $t = 1$. The type (iii) equilibrium invokes a regularity condition:

Assumption 2. *(i) For every* $0 < \tilde{\theta} < \theta < 1$, $\Delta(\theta; \tilde{\theta}, S) := m(\tilde{\theta}, \theta) + S - \theta - (m(\theta, \gamma(\theta)) - m(\tilde{\theta}, \theta))$ *is decreasing in* θ*; (ii) if* $\Delta(0; \tilde{\theta}, S) \geq 0$*, then* $\Delta(0; \tilde{\theta}, S') \geq 0$ *for every* $S' > S$*; (iii) for every* $0 < \tilde{\theta} < \theta < 1$, $2m(\tilde{\theta}, \theta) - m(0, \tilde{\theta})$ *is decreasing in* $\tilde{\theta}$*; and (iv)* $\theta_0^* - m(\theta_0^*, \gamma(\theta_0^*)) + S > 0$ *for every* $S > 0$*.*

This assumption extends and strengthens Assumption 1 and has a similar interpretation: the $t = 1$ buyers' average benefit (the first term of Δ) minus the $t = 1$ sellers' reservation value (the negative of the remaining terms in Δ) is decreasing in θ and satisfies single-crossing with respect to S. As with Assumption 1, Assumption 2 holds for many natural distributions, including the uniform distribution. We next characterize all possible equilibria that may arise from different p_gs. Figure 5 describes the types of firms selling in $t = 1$ and $t = 2$ in these equilibria.

Theorem 3. *There exists an interval of bailout terms* $P^k \subset \mathbb{R}_+$ *that supports alternative equilibrium types* $k = NR, SBS, MBS, MR$*, described as follows.*

(i) ***No Response (NR)****: If $p_g \in P^{NR}$, then there exists an equilibrium in which no firm accepts the government offer, and the outcome in Theorem 2 prevails.*

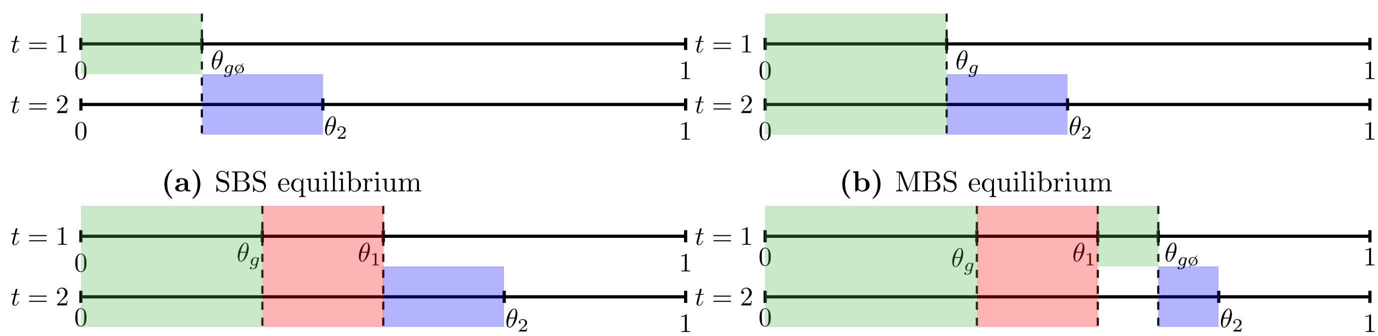
(ii) ***No Market Rejuvenation***

- ***Severe Bailout Stigma (SBS)****: If $p_g \in P^{SBS}$, then there exists an equilibrium with $\Theta_g = \Theta_1 = \emptyset, \Theta_{g\emptyset}, \Theta_2 \neq \emptyset$.*
- ***Moderate Bailout Stigma (MBS)****: If $p_g \in P^{MBS}$, then there exists an equilibrium with $\Theta_g, \Theta_2 \neq \emptyset, \Theta_1 = \Theta_{g\emptyset} = \emptyset$.*

(iii) ***Market Rejuvenation (MR)****: If $p_g \in P^{MR}$, then there exists an equilibrium with $\Theta_1 \neq \emptyset$.*

Specifically, $P^{NR} = [0, p_2^*]$, $\inf P^{SBS} = p_0^*$, and $\sup P^{MBS} \leq \inf P^{MR}$, meaning that an MR equilibrium requires a strictly higher p_g than does an MBS equilibrium.[25]

The specific type of equilibrium depends partly on the bailout term p_g, which Theorem 3 and Figure 5 describe roughly in ascending order of p_g. However, the bailout terms supporting the different types of equilibria are not strictly ranked. In addition, multiple equilibria may exist for the same term p_g, as we shall discuss.



(a) SBS equilibrium **(b)** MBS equilibrium

(c) MR1 equilibrium **(d)** MR2 equilibrium

Figure 5 – Different types of equilibria

Figure 6 describes the equilibrium volume of trade (represented on the horizontal axis) in each of the two periods as a function of bailout term p_g (represented on the vertical axis), assuming that $S = 1/3$, $I = 1/10$, and θ is uniformly distributed on $[0, 1]$. One may intuitively think of this as firms' supply response to the government's price commitment p_g. The colors correspond to alternative equilibrium types. Recall that in this case, $\underline{S}^* = 1/2 + 1/10 > S = 1/3$, and thus, without government intervention, the asset market freezes in $t = 1$ and the

[25] A more detailed characterization of the range of bailout terms supporting each equilibrium is provided in Online Appendix.

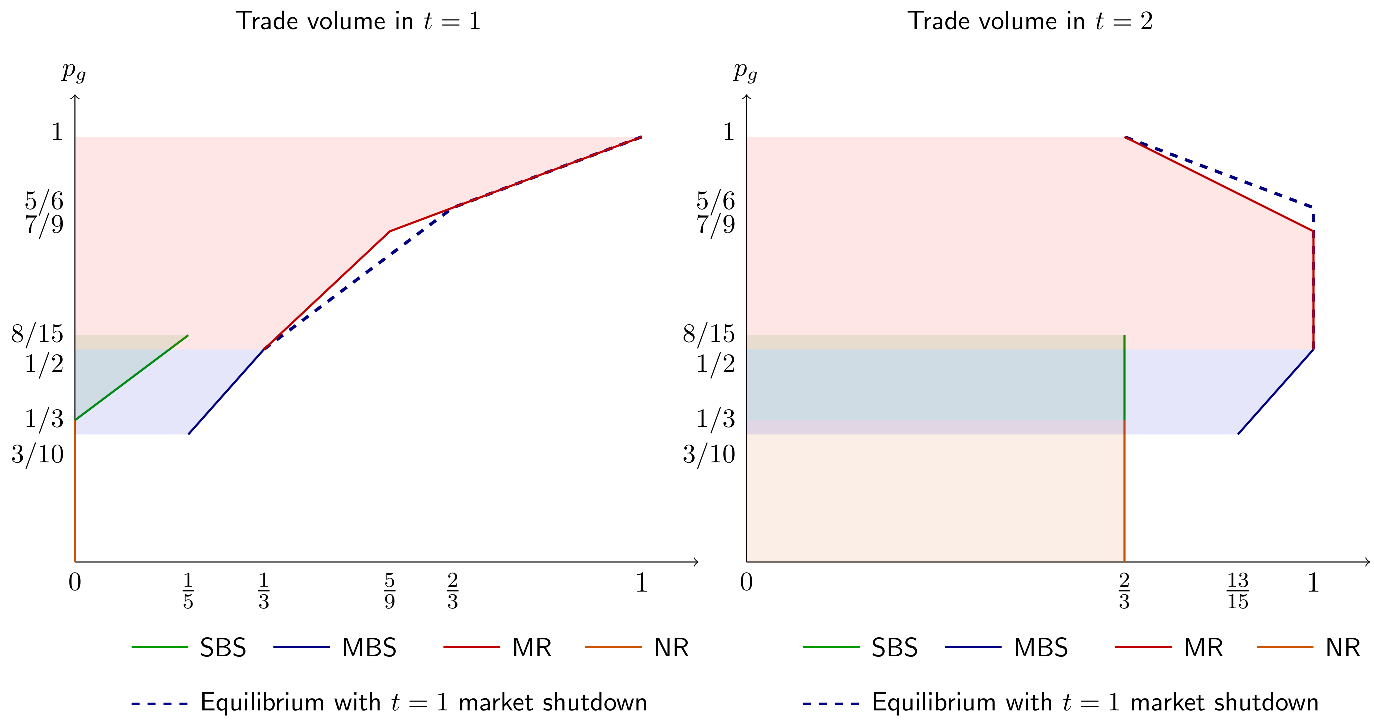


Figure 6 – Trade volume in each period in all equilibria, including that with market shutdown depicted by the dashed blue line ($S = \frac{1}{3}, I = \frac{1}{10}$).

$t = 2$ trade equals the one-shot equilibrium trade volume $\theta_0 = 2S = 2/3$. All other equilibria can be understood in comparison with this no-intervention benchmark.

We now describe each type of equilibrium in greater detail, referring to Figures 5 and 6 whenever relevant.

□ **No Response**: In this equilibrium, a government bailout elicits no response, meaning the laissez-faire equilibrium described in Theorem 2 prevails. 'No response' would make sense if the bailout term p_g were sufficiently unattractive, for instance, below the laissez-faire price p_1^* in $t = 1$. Surprisingly, however, this outcome could arise even when $p_g > p_1^*$. The reason is the extreme stigma associated with bailout recipients. If the $t = 2$ market assigns the worst belief $\theta = 0$ to these firms, then they will not have any profitable opportunity to sell their assets in $t = 2$. Thus, unless the bailout term is as high as p_2^* to compensate for the loss, no firm will accept the bailout offer. This explains why the NR equilibrium occurs for all $p_g \leq p_2^*$.[26]

[26]Specifically, consider the NR equilibrium (the same as that in Theorem 2). Suppose that a firm with any θ rejects the bailout offer $p_g \leq p_2^*$. Then, its payoff is no less than $\theta + p_2^* + S$, since the firm has an option not to sell its asset in $t = 1$ and sell it at p_2^* in $t = 2$. Suppose now that the firm deviates and accepts such an offer. Then, given the worst out-of-equilibrium belief, its payoff is at most $\theta + p_g + S$ since it sells at p_g and finances its project in $t = 1$, but it can at best consume its asset and realize θ in $t = 2$. Clearly the former is no less than the latter, so long as $p_g \leq p_2^*$. Importantly, the extreme stigma is not unreasonable in the sense that it does satisfy D1.

In the leading example, $p_2^* = 1/3$, and thus, no response is an equilibrium for any $p_g \leq 1/3$. While any $p_g \geq I = 1/10$ should be of interest for sufficiently low-type firms, they do not accept the bailout for fear of the stigma that will prove costly in the $t = 2$ market. Thus no firms accept $p_g \leq 1/3$ in equilibrium. In fact, in our example, this is the unique equilibrium for $p_g \leq 3/10$.

□ **No Market Rejuvenation**: In this equilibrium, some firms accept the bailout but no firm sells to the market in $t = 1$. Thus the bailout does not 'prop up' the market, as envisioned by Tirole (2012). This $t = 1$ market shutdown is attributed to an extreme early sales stigma. Suppose that buyers in the $t = 2$ market form an off-the-path belief that any firm selling to the market in $t = 1$ must be of type $\theta = \theta_g$, the lowest possible type that refuses the bailout. Given this belief and the ensuing $t = 2$ market offer, no firm will wish to accept an offer in the $t = 1$ market that could at least make the buyers break even unless the offer is so high to entail loss for the buyers. This equilibrium exists when p_g is not too high, less than 8/15 in the example.[27] In this equilibrium, relatively low types in $[0, \theta_g]$ accept the bailout and suffer a bailout stigma. Depending on the severity of this stigma, the equilibrium could be either of the two types.

First, there could be an equilibrium with **Severe Bailout Stigma (SBS)**, in which the $t = 2$ market attaches such a severe stigma to the bailout recipients that they never receive an offer of at least I for their $t = 2$ assets. Thus, the bailout recipients cannot fund their projects in $t = 2$. Therefore, as depicted in Figure 5-(a), lower type bailout recipients do not sell their assets in $t = 2$, whereas higher type holdouts sell in $t = 2$ at p_2. Remarkably, the latter price equals the bailout price p_g. This can be seen as follows. For the marginal recipient θ_g to accept the bailout at p_g and forgo the opportunity to sell at price p_2 offered to the holdouts in $t = 2$, we must have

$$p_g + S + \theta_g = \theta_g + p_2 + S \iff p_g = p_2. \tag{2}$$

Namely, θ_g is indifferent between accepting the bailout (which yields the payoff equal to the LHS of the first equality) and rejecting it (which yields the payoff equal to the RHS of the first equality). Since p_2 must allow those buying in $t = 2$ to break even, it follows that

$$p_g = m(\theta_g, \gamma(\theta_g)). \tag{3}$$

If θ_g determined by (3) is so low that the zero-profit offer $m(0, \theta_g)$ to the bailout recipients in $t = 2$ is less than I, no sale will occur for these firms, which in turn validates the severe stigma in equilibrium.

Note that the SBS equilibrium exists even when the bailout term p_g is strictly more

[27]If p_g is sufficiently high, then θ_g becomes high, which creates a profitable deviation for buyers in the market.

favorable than the laissez-faire $t = 1$ market price p_1^*. Suppose, for instance, that $p_g \approx p_0^* = m(0, \gamma(0)) \geq p_1^*$. Then, $\theta_g \approx 0$, and thus, $m(0, \theta_g) < I$ and the SBS equilibrium is supported. This feature is also illustrated in the leading example, where the SBS equilibrium arises for $p_g \in P^{SBS} := (1/3, 8/15)$.[28]

Second, there could be an equilibrium with **Moderate Bailout Stigma (MBS)** in which the $t = 2$ market assigns a more favorable belief to the bailout recipients and makes an offer higher than I. This induces the bailout recipients to sell their assets in $t = 2$. Figure 5-(b) depicts the pattern of trade for this equilibrium. Then, the indifference condition for the marginal type θ_g is given by

$$p_g + S + m(0, \theta_g) + S = \theta_g + m(\theta_g, \gamma(\theta_g)) + S, \tag{4}$$

where the LHS is its payoff from accepting the bailout and the RHS is the payoff from rejecting it.

One notable feature is that some bailout terms p_g can induce multiple equilibria with different degrees of severity of the bailout stigma. To see this, note that the RHS in (4) is the same as that (of the first equality) in (2), but the LHS is strictly larger than the corresponding LHS in (2), as long as $m(0, \theta_g) + S > \theta_g$ or $\theta_g < \theta_0^*$. This means that the marginal type θ_g given by (4) is strictly higher than the corresponding marginal type given by (2). In fact, θ_g under (4) can be so high that the equilibrium belief about the bailout recipients can support the investment – i.e., $m(0, \theta_g) \geq I$ – even though it cannot under (2). This means that both types of equilibria may exist for some range of p_gs. The multiplicity is due to the endogeneity of beliefs: even for the same p_g, different beliefs about the bailout recipients give rise to discretely different incentives for accepting the bailout, supporting (potentially drastically) different beliefs. In fact, if both types of equilibria exist for nonempty sets of p_gs (i.e., $P^{SBS} \neq \emptyset$ and $P^{MBS} \neq \emptyset$), the multiplicity exists for a range of p_gs (i.e., $P^{SBS} \cap P^{MBS} \neq \emptyset$). This is precisely the case in our leading example. In that example, the MBS equilibrium arises if $p_g \in P^{MBS} := [3/10, 1/2]$, which clearly overlaps with P^{SBS} for $p_g \in (1/3, 1/2]$ (see Figure 6).[29]

☐ **Market Rejuvenation**: In this equilibrium, a bailout induces a positive measure of firms to sell to the market in $t = 1$. Hence, consistent with Tirole (2012), the government takes out the worst assets and allows the market to buy better assets. This is particularly the case for the **MR1** equilibrium depicted in Figure 5-(c). In the leading example, the **MR1** equilibrium

[28] For any p_g in this region, types $\theta \leq p_g - \frac{1}{3}$ sell to the government in $t = 1$ but are excluded from the $t = 2$ market. Observe that the break-even price for these types in $t = 2$ is their average type, $m(0, p_g - S) = (p_g - \frac{1}{3})/2$. If $p_g < 8/15$, then the average type is strictly less than $1/10$, the required funds for investment. This illustrates why no market develops for these firms in equilibrium. Meanwhile, types $\theta \in [p_g - \frac{1}{3}, p_g + \frac{1}{3}]$ refuse to accept the bailout and consequently sell at $p_2 = m(p_g - \frac{1}{3}, p_g + \frac{1}{3}) = p_g$ in the $t = 2$ market.

[29] In this equilibrium, types $\theta \leq \frac{2}{3}p_g$ sell to the government in $t = 1$ and to the market in $t = 2$ at price $\frac{1}{3}p_g$, and types $\theta \in (\frac{2}{3}p_g, \frac{2}{3}p_g + \frac{2}{3}]$ refuse the bailout in $t = 1$ but sell in $t = 2$ at a higher price $\frac{2}{3}p_g + \frac{1}{3}$.

arises if $p_g \in (1/2, 7/9]$.[30]

Despite the resemblance to the one-shot model, there are some differences relative to Tirole (2012). Since the bailout attracts the worst types, its recipients are subject to a stigma in the $t = 2$ market. For the bailout to be acceptable, the offer p_g must compensate its recipients for the loss from the stigma. Thus, unlike Tirole (2012), the bailout term includes a premium over the market that makes up for the stigma loss $m(\theta_g, \theta_1) - m(0, \theta_g)$. In the leading example, the firms sell the assets in the $t = 1$ market at a discount of more than 1/6, or the government is paying a premium of at least 1/6 to offset the bailout stigma![31] This means that the cost of inducing trade is higher than in the one-shot model.

Interestingly, there could exist a different type of equilibrium, labeled **MR2**, in which some firms selling to the government have higher quality assets than those selling to the $t = 1$ market. As depicted in Figure 5-(d), this equilibrium exists when $p_g \in (7/9, 1]$ in the leading example. Strikingly, the types of firms accepting the bailout are non-contiguous in this equilibrium. The reason for this is again the stigma associated with the bailout. For a sufficiently high p_g, the bailout becomes attractive to firms, but bailout stigma remains a problem for them. Hence, some high-type firms accept the bailout but never sell their assets in $t = 2$. Consequently, unlike Tirole (2012), dregs skimming need not be the role of a bailout in the presence of the stigma.

4.2 The Effects of a Bailout

We have seen that the bailout stigma dampens firms' willingness to accept bailouts and thus increases the cost of a bailout for the government. The ultimate question is, could a bailout still be effective? If so, how does the benefit materialize, and why? We argue below that, despite the stigma and the associated cost, a bailout does stimulate trade and investment, but these effects may be slow to materialize and rely on a hitherto unrecognized reputational effect.

We first illustrate these points with the leading example, i.e., $I = 1/10$ and $S = 1/3$. Figure 6 summarizes the trade volume supported by different bailout terms p_g across the two periods. Again, it is intuitive to view this as firms' total supply response to the government's price commitment p_g. Consistent with our earlier discussions, there is a clear sense in which the bailout stigma discourages the uptake of a bailout. As noted previously, any offer $p_g \in [1/10, 3/10]$, despite covering the investment cost, is rejected outright. Even at a higher offer p_g, the types that would otherwise have accepted the bailout reject it for fear of the attached

[30] In this equilibrium, types $\theta \in [0, \frac{2}{5}p_g + \frac{2}{15}]$ sell to the government in $t = 1$ and to the market in $t = 2$ at price $\frac{1}{5}p_g + \frac{1}{15}$; types $\theta \in (\frac{2}{5}p_g + \frac{2}{15}, \frac{4}{5}p_g - \frac{1}{15}]$ sell to the market in both periods at price $\frac{3}{5}p_g + \frac{1}{30}$; and types $\theta \in (\frac{4}{5}p_g - \frac{1}{15}, 1]$ sell only in $t = 2$ at price $\frac{2}{5}p_g + \frac{7}{15}$.

[31] The marginal type to accept the bailout is $\frac{2}{5}p_g + \frac{2}{15}$, and the marginal type to sell to the $t = 1$ market is $\frac{4}{5}p_g - \frac{1}{15}$. Thus, the stigma loss $m(\theta_g, \theta_1) - m(0, \theta_g)$ is $\frac{2}{5}p_g - \frac{1}{30}$, which is at least $\frac{1}{6}$ for all $p_g \in (1/2, 7/9]$.

stigma. In other words, the initial/direct response to bailouts is dampened and lackluster due to the stigma.

Although dampened in its $t = 1$ effect, the bailout does stimulate trade and investment. This can be seen in Figure 7, which illustrates the increase in net trade across the two periods thanks to the bailout.[32] The net trade gain is positive except for a p_g that entails the NR equilibrium. This means that, even under the SBS equilibrium, the bailout increases the volume of trade.

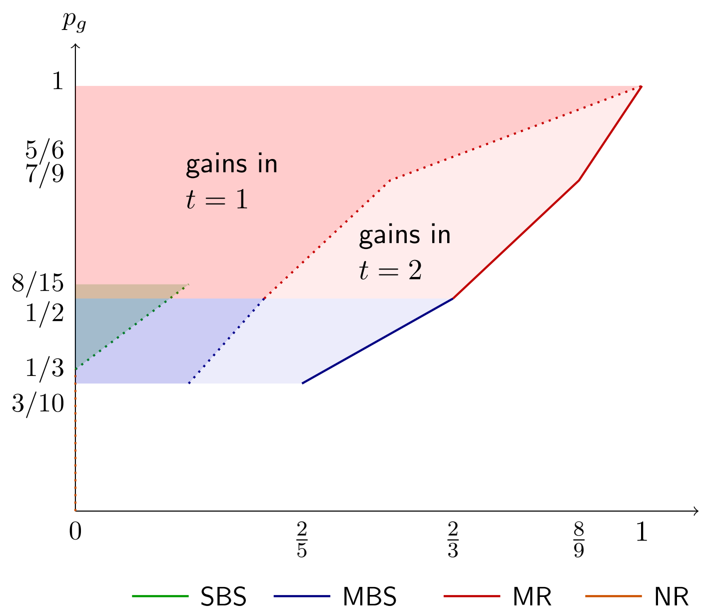


Figure 7 – Net gains in trade volume from a bailout across two periods.

Interestingly, as Figure 7 shows, the bailout effect is spread over two periods, meaning that a bailout has an indirect and delayed benefit. In the example, recall that without a bailout, no firms sell in $t = 1$ and only firms with $\theta \leq 2/3$ sell in $t = 2$. Suppose that the government offers $p_g \in [3/10, 1/2]$ and an MBS equilibrium arises. Now, a positive measure of firms sell in $t = 1$, but firms with $\theta > 2/3$ (that would not sell without a bailout) also sell in $t = 2$ (see the blue areas in Figures 6).[33] The reason for such a delayed trade increase has to do with the way a bailout affects the reputation of the firms rejecting it. In essence, the flip side of

[32]Since the trade volume without bailout is zero in $t = 1$ and $2/3$ in $t = 2$ in the example, the increase in net trade is the total trade under the bailout minus $2/3$.

[33]Even under an SBS equilibrium, firms with $\theta > 2/3$ never trade without a bailout, but some of them do with a bailout, although the total trade volume in $t = 2$ remains the same since firms with $\theta \leq p_g - 1/3$ now cannot sell their assets in $t = 2$.

the bailout stigma suffered by those accepting a bailout is the reputational boost enjoyed by those that refuse the bailout. Hence, a bailout creates an opportunity for those that could not otherwise credibly signal the quality of their assets to do so by refusing to accept the bailout. Consequently, more firms sell their assets and undertake investment in $t = 2$.[34] Since such an opportunity is absent without a bailout, one must view the delayed trade as a benefit of a bailout.

Another interesting result is that the effect of bailouts can be discontinuous, as can be seen in Figure 7. For instance, in our example, a bailout at any $p_g < 3/10$ results in the NR equilibrium. Hence, if the policy maker raises p_g within that range, the net trade gain from a bailout remains zero. However, when p_g reaches $3/10$, an MBS equilibrium arises, and the net trade gain from a bailout (across two periods) jumps from 0 to $2/5$.[35] That is, successive improvements in the bailout terms could initially be met with frustratingly little response, but at some point, a small further improvement in the bailout offer can result in a massive response.

These observations are generalized in the following proposition. For results (ii) and (iii), we assume that $\theta \geq 2m(0, \theta)$ for all θ. This assumption, which ensures that the MR2 equilibrium arises only for a sufficiently large p_g, is made to simplify the analysis. Importantly, the assumption is only sufficient for the results and is satisfied by many standard distribution functions.

Proposition 2. *(i) (Dampened initial responses) Fix $p_g \geq \max\{p_0^*, I\}$. In any equilibrium, the trade volume in $t = 1$ is (weakly) smaller than the trade volume $F(p_g + S)$ in the one-shot model.*

(ii) (Positive net gains) The total trade volume is higher with a bailout than without, if either MBS, MR1, or MR2 equilibrium would prevail under a bailout. The same holds even when an SBS equilibrium arises from a bailout if the $t = 1$ market fully freezes without a bailout.

(iii) (Delayed benefits) The $t = 2$ trade volume is higher with a bailout than without, if either MBS or MR1 equilibrium would prevail under a bailout.

(iv) (Discontinuous effects) Let $\Phi(p_g)$ denote the set of total trade volumes that would result from some equilibrium given bailout $p_g \in [I, 1]$. The correspondence $\Phi(\cdot)$ does not admit a selection that is continuous in p_g.

[34] In a sense, this can be seen as an "intertemporal" analogue of propping up trade, except that the propped-up trade is delayed. Just as a bailout in the one-shot model boosts the reputation of those selling to the market, the stigma boosts the reputation of those selling in $t = 2$ after having refused the bailout in $t = 1$.

[35] Similarly, as p_g increases toward $8/15$, an equilibrium may shift from an SBS to MR1, in which case the net gain from the bailout would jump up from $1/5$ to $52/75$.

Interestingly, Proposition 2-(ii) does not rule out the possibility that a seemingly attractive bailout could reduce overall trade when it triggers an SBS equilibrium. Indeed, one could confirm such a possibility.[36]

4.3 Is Early Market Rejuvenation Beneficial? The Effect of $t = 1$ Market Shutdown

A striking difference from the one-shot model is that a bailout may not immediately rejuvenate the market. This is the case when either an SBS or MBS equilibrium arises. However, even for MR equilibria, is having the government immediately prop up the market important or even beneficial? In the one-shot model, as Proposition 1 highlights, such a role is irrelevant: *even if the government shuts down the market, the outcome would be the same.* In other words, the exact types of firms that would sell their assets to the market will simply sell to the government in the case of market shutdown.

In our dynamic model, however, the early market rejuvenation is not payoff-irrelevant. Surprisingly, it discourages trade. To illustrate, recall from Figure 6 that an MR (either MR1 or MR2) equilibrium exists for $p_g \in [1/2, 1]$. Suppose now that the government shuts down the $t = 1$ market. In Figure 6, the resulting trade is depicted by the dashed line. One can see that trade volume would increase as a result of the shutdown of the $t = 1$ market. This observation holds more generally in our dynamic model.

Proposition 3 (Dampening effect of market rejuvenation)**.** *Suppose that an MR (either MR1 or MR2) equilibrium arises given p_g. In that case, offering a bailout at the same p_g,* but with the $t = 1$ market shut down*, would (at least weakly) increase the total trade volume.*

This result stands in stark contrast to Proposition 1. The simple intuition is that early market rejuvenation exacerbates adverse selection. Intuitively, accepting a bailout *in the presence of other (higher type) firms selling to the market* is a worse signal than accepting the same bailout *in the absence of such firms.* In other words, the stigma suffered by bailout

[36] Suppose that θ is uniformly distributed from $[0, 1]$, $I \in (0, 1/2)$, and $S \in [I + 1/2, 1)$. In this case, absent government intervention, there is an equilibrium in which firms with types $\theta \leq 2S - 1$ trade in $t = 1$ and all firms trade in $t = 2$. Suppose now the government offers a bailout at $p_g \in (1/2, (1 + I)/2)$. Note that this p_g exceeds the $t = 1$ market price without intervention, which is $m(0, 2S - 1) = S - 1/2 < 1/2$. Such a p_g supports an SBS equilibrium: the average type of bailout recipients is $m(0, 2p_g - 1) = p_g - 1/2 < I$, and thus, no market develops for these firms in $t = 2$, and given this, the $t = 2$ market price for hold-out firms is $m(2p_g - 1, \gamma(2p_g - 1)) = m(2p_g - 1, 1) = p_g$. Therefore, type $\theta_{g\o} = 2p_g - 1$ is indifferent in $t = 1$ between accepting the bailout and holding out. In this equilibrium, firms with types $\theta \leq 2p_g - 1$ trade in $t = 1$, and only firms with types $\theta \in [2p_g - 1, 1]$ trade in $t = 2$. Consequently, the bailout reduces overall trade from $2S > 1$ to 1. Strictly speaking, however, one could have selected a full-freeze equilibrium without government intervention, in which case SBS does not reduce overall trade. Incidentally, this example also illustrates why Proposition 2-(iii) does not include an SBS equilibrium: the $t = 2$ trade volume would be 1 without intervention but equals $2 - 2p_g < 1$ in the SBS equilibrium under the bailout.

recipients is worsened by the presence of firms selling to the market. Interestingly, even the latter firms do not fare any better than the former in equilibrium; otherwise, the former firms (those accepting the bailout in equilibrium) would all have sold to the market in $t = 1$. In short, the viability of market sale as *an additional signaling option* reduces the reputation of all firms that sell in $t = 1$, regardless of whom they sell to, and this in turn reduces the reputation of those that refuse to sell. This is a surprising insight, which to our knowledge has not been recognized in the literature, meaning that the government suppression of early market rejuvenation could increase trade volume.

The same insight provides a potentially useful lesson for the design of a bailout policy. In economic crises, policy makers often offer a menu of multiple bailout programs. Our result cautions against offering multiple bailout options in the presence of adverse selection. Just as a market option exacerbates adverse selection, offering an additional bailout option could also do so and diminish the effect of bailouts. This is shown formally below.

Proposition 4. *Suppose that the market remains closed in $t = 1$, and let p_g be a given bailout offer. The total trade volume decreases when the government adds another bailout offer $p'_g \in [I, p_g)$.*

5 Secret Bailouts

A natural policy response to bailout stigma is to conceal the identities of its recipients from the market. Indeed, it is not uncommon for governments to offer protection of privacy for firms seeking financial bailouts. For instance, the Federal Reserve conventionally runs the discount window as a measure to inject public liquidity to banks in need of short-term funding. The so-called discount window stigma is generated by the fact that borrowing banks can be easily identified because they no longer use the federal funds market, an alternative that banks usually rely on for short-term funding.[37] To reduce the stigma attached to the discount window, the Federal Reserve created the Term Auction Facility that is intended to conceal the identities of borrowing banks.[38]

To examine the implications of such a **secret bailout**, we consider the same model as before with the government running the asset purchase program at price $p_g \geq I$ in $t = 1$, but

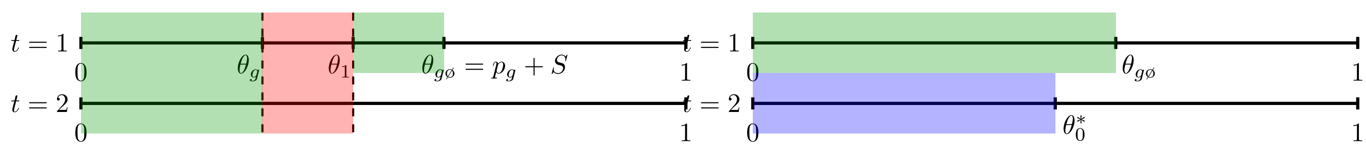
[37] The Dodd-Frank Wall Street Reform and Consumer Protection Act passed in 2010 further requires the Federal Reserve to publicly disclose, with a two-year lag, the names of banks borrowing from the discount window and the total amount of money they borrow. This regulation will further facilitate identification of the firms using the discount window.

[38] Gorton and Ordoñez (2014) supports such a policy. During crises, debt contracts lose "information insensitivity" as investors scrutinize the downside risk of underlying collaterals, leading to an adverse selection. They argue that withhoding information on whether borrowers borrow from discount windows of central banks can make debtors less information sensitive and alleviate adverse selection. As will be seen, secrecy has a more nuanced effect in our model.

suppose the firm's decision whether to accept the bailout can be concealed from the buyers in the market.[39] For comparison, we call the asset purchase program in the previous section a **transparent bailout**.

Under secrecy, the market cannot distinguish between the firms accepting the bailout and those that do not sell their assets in $t = 1$. The market observes only the set of firms selling assets to the market in $t = 1$. Thus, secrecy allows the bailout recipients to pool with the holdouts. Since the latter firms are likely to be of higher types, one could conjecture that a secret bailout mitigates the stigma suffered by recipients of a transparent bailout. In what follows, we analyze how secrecy affects the firm's decision to accept the bailout and trade assets across the two periods.

To begin, fix any purchase price $p_g \in [p_1^* \vee I, 1]$, where p_1^* is the $t = 1$ market price of assets without a bailout (defined in Theorem 2).[40] We argue that the only possible equilibrium is what we call the **Secret Bailout with Market Rejuvenation (SMR)**, depicted in Figure 8-(a). In this equilibrium, there are thresholds $0 < \theta_g < \theta_1 < \theta_{g\emptyset}$ such that low-type firms



(a) SMR equilibrium **(b)** Equilibrium under shutdown

Figure 8 – Equilibrium under secret bailout

$\theta < \theta_g$ sell to the government in $t = 1$ and to the market in $t = 2$, middle types $\theta \in (\theta_g, \theta_1)$ sell to the market in both periods, higher types $\theta \in (\theta_1, \theta_{g\emptyset})$ sell to the government in $t = 1$ but do not sell in $t = 2$, and even higher types $\theta > \theta_{g\emptyset}$ do not sell in either period. We formally state below the characterization of possible equilibria under secret bailouts.

Theorem 4. *There exists a nonempty interval $P^{SMR} \subset (I, 1]$ of bailout terms such that (i) an SMR equilibrium exists with cutoffs $\theta_1 < \theta_0^*$ and $\theta_{g\emptyset} = p_g + S$ if $p_g \in P^{SMR}$, and (ii) no other equilibria exist.*[41]

The theorem states that only an SMR is possible under a secret bailout. In particular, we do not have the analogues of no response or no early market rejuvenation for any $p_g \geq p_1^*$. First, it is easy to see why no response cannot be an equilibrium outcome. Were it an equilibrium,

[39] Legislation can improve such secrecy. For example, a special act, such as the Emergency Economic Stabilization Act, can explicitly incorporate a clause that conceals the identities of bailout recipients for a specified period of time to encourage the uptake of bailout offers but guarantee full disclosure of information after the specified period and criminal liabilities for those who are found guilty of any wrongdoing.

[40] We adopt the convention that $p_1^* \equiv 0$ when no firms sell in $t = 1$ without a bailout. If p_g is sufficiently lower than p_1^*, then an NR equilibrium arises, but this is uninteresting.

[41] The interval P^{SMR} is characterized more precisely in the Online Appendix.

the laissez-faire equilibrium described in Theorem 2 would prevail, but then types slightly higher than θ_1 would profitably deviate by accepting the bailout; they would do so since the $t = 2$ market cannot distinguish them from the $t = 1$ holdouts and thus would offer a discretely higher price than to those that sold to the market in $t = 1$. Second, an equilibrium without early market rejuvenation cannot be supported either. If such an equilibrium were to occur, then given secrecy, all firms with types up to $p_g + S$ would accept the bailout. The fact that any remaining firms must have higher θ then allows private buyers to make profitable offers that would be acceptable to the remaining firms. Hence, it is impossible for the market to be inactive in $t = 1$. Finally, an MR1 equilibrium cannot arise, as any potential sellers to the market in $t = 1$ would rather deviate and take the bailout because it involves a premium and hide behind the cloak of secrecy to act as a holdout and enjoy a higher price in $t = 2$. This leaves us with SMR as the only possible equilibrium.[42] Several features of the SMR equilibrium are worth noting. First, all firm types up to $p_g + S$ trade in $t = 1$. This is due to the secrecy conferred to the bailout recipients. In particular, the marginal type $\theta = p_g + S$ has nothing to lose from accepting, or to gain from refusing, a bailout.

Second, and more strikingly, secrecy does not eliminate the bailout stigma for some types of firms. As can be seen in Figure 8-(a), the low-type recipients ($\theta \leq \theta_g$) are exposed (as a group) in the $t = 2$ market to be distinct from those selling to the market in $t = 1$. Thus the former face a price $m(0, \theta_g)$ in $t = 2$, which is strictly lower than the price $m(\theta_g, \theta_1)$ the latter face. The persistence of the stigma under secrecy is attributed to both the endogeneity of the stigma and to early market revival. True to its intent, secrecy protects the identities of the bailout recipients, in particular keeping them indistinguishable from the holdouts. However, secrecy does not keep them indistinguishable from those selling to the market. Hence, the $t = 2$ market correctly infers those that do not sell to the $t = 1$ market but are "willing" to sell in $t = 2$ as bailout recipients whose types are worse than those that sell to the $t = 1$ market. High-type bailout recipients with $\theta \in (\theta_1, \theta_{g\o}]$ boycott the $t = 2$ market to avoid this stigma, which in turn defeats any hope on the part of the low-type recipients to "pool"

[42] In particular, this means that no equilibria exist for some values of p_g. In the leading example, nonexistence arises for $p_g < 8/15$. This nonexistence is to some extent a consequence of our refinements: D1 and the zero-profit requirement for buyers. For example, one can show that without D1, the full participation equilibrium exists even for some (large) $p_g < 1$. Similarly, relaxing the zero-profit requirement expands the region of p_gs for which the SMR equilibrium is supported. Nevertheless, existence cannot be restored for all $p_g \geq \max\{I, p_1^*\}$ by simply relaxing these two constraints. We suspect that the remaining culprits are our restriction to equilibria in pure strategies and/or infinite actions in our model, which are known to cause nonexistence of subgame perfect equilibria (of which perfect Bayesian equilibrium is a refinement) in an infinite game. See Harris et al. (1995) and Luttmer and Mariotti (2003) for examples of the nonexistence of subgame perfect equilibrium. As will be seen below, however, the existence of an equilibrium is fully restored under the optimal policy (i.e., shutdown of the $t = 1$ market).

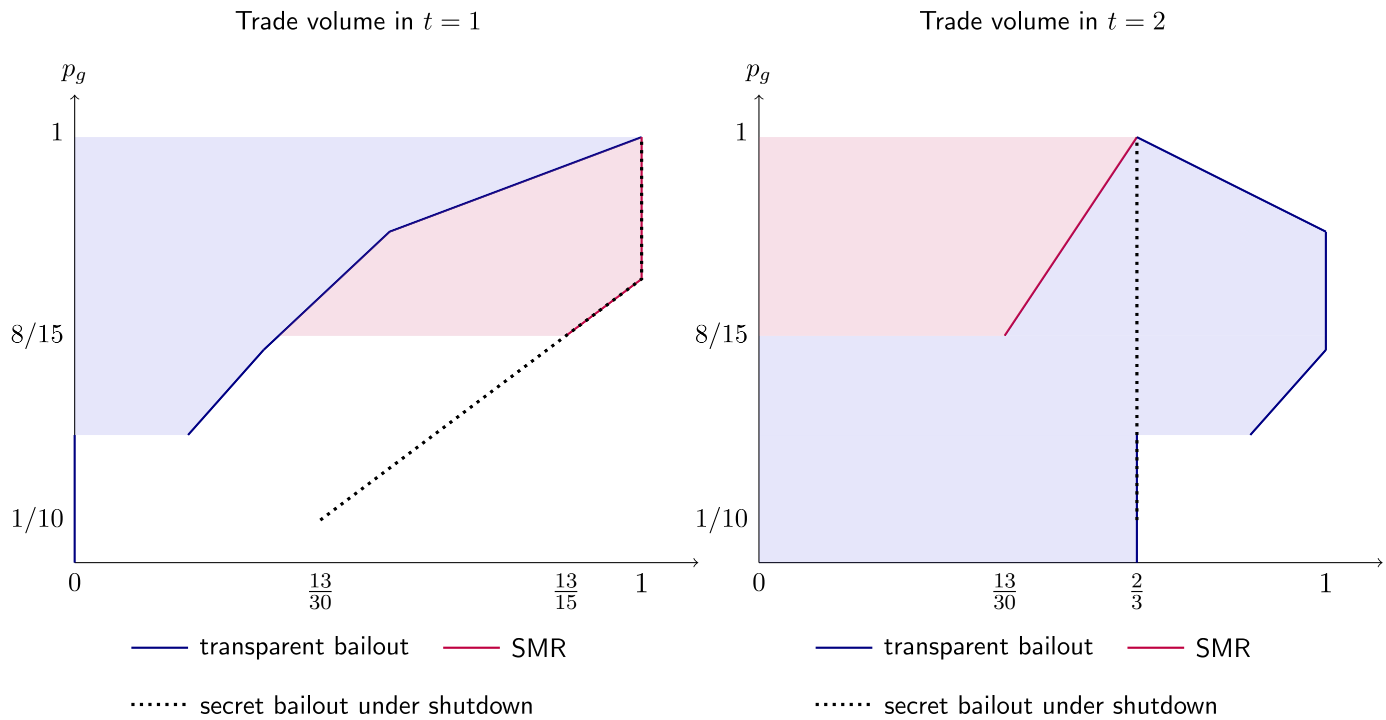


Figure 9 – Trade volume in each period in all equilibria including under a secret bailout ($S = \frac{1}{3}, I = \frac{1}{10}$).

with them.[43] Thus, the low-type bailout recipients continue to suffer the stigma. Moreover, incentive compatibility means that even those that sell to the market in $t = 1$ would also suffer indirectly in $t = 2$. This reduces the $t = 2$ trade volume relative to the one-shot model; one can readily see that the marginal trading type is $\theta_1 < \theta_0^*$.

Third, the presence of the bailout stigma means that, for a bailout to be acceptable, the government still needs to pay a premium relative to the market price. This can be seen from incentive compatibility. Since selling to the government and selling to the market in $t = 1$ must yield the same payoff, we must have that $p_g + S + m(0, \theta_g) + S = m(\theta_g, \theta_1) + S + m(\theta_g, \theta_1) + S$. This implies that the premium over the $t = 1$ market price is

$$p_g - m(\theta_g, \theta_1) = m(\theta_g, \theta_1) - m(0, \theta_g) > 0.$$

As before, this means that the stigma increases the cost of bailout for the government.

[43] High-type bailout recipients' unwillingness to pool with low-type bailout recipients and the bailout stigma suffered by the low types are mutually reinforcing. This feature does not mean, however, that there is an additional equilibrium, say, in which such pooling—and thus mitigation of stigma—occurs. Were such an equilibrium to exist, some bailout recipients that sell in both periods would have higher types than those that sell to the market in $t = 1$, which would violate the single-crossing property (which must hold due to Tirole's refinement).

Fourth, the SMR equilibrium has non-contiguous types of firms accepting the bailout. The reason is similar to that of the MR2 equilibrium under the transparent bailout. The high-type recipients $\theta \in (\theta_1, \theta_{g\o}]$, $\theta_{g\o} = p_g + S$, find the bailout attractive but do not wish to be subject to the stigma in $t = 2$, and thus, they boycott the $t = 2$ market. In fact, the incentives facing threshold types θ_g and θ_1 are exactly the same as those in the MR2 equilibrium. The only difference is that more firms accept the bailout in the SMR equilibrium. This is because secrecy eliminates the stigma for high-type bailout recipients, as discussed above. The increase in trade volume in $t = 1$ is shown in Figure 9. However, the gain from the "front-loading" of trade is precisely offset by the loss of reputation suffered by those that refuse the bailout. Recall that in the MR2 equilibrium, those that hold out in $t = 1$ could improve their reputation and sell at a higher price in $t = 2$. However, this effect disappears in the SMR equilibrium. In short, secrecy simply encourages early trade but at the expense of the delayed trade that the transparent bailout would have generated in the MR2 equilibrium. In fact, one can easily see that total trade volume is the same between the MR2 and SMR equilibria, provided that both are supported by the given p_g (this is true for $p_g \geq 7/9$ in the leading example). Figure 10 shows this for our leading example.

The SMR equilibrium exists under secrecy for any p_g that admits the MR2 equilibrium under transparency, namely any $p_g \in [7/9, 1]$ in the leading example. As can be seen in Figure 9, the SMR equilibrium may also exist outside that range, for instance at p_g that would entail the MR1 equilibrium under transparency. The comparison between the SMR and MR1 equilibria—and thus the effect of secrecy—involves the tradeoff mentioned above. On the one hand, the removal of the stigma for high types (relative to holdouts) clearly encourages early trade. On the other hand, secrecy removes the opportunity for firms to boost their reputation by refusing the bailout, thereby reducing trade in $t = 2$. The net effect is ambiguous. In the leading example, as can be seen in Figure 10, secrecy supports higher total trade for $p_g \in (13/21, 7/9)$ but lower total trade for $p_g \in (8/15, 13/21)$.

The above observations are generalized as follows.

Proposition 5. *(i) (Front-loading of trade) An SMR equilibrium, if it exists, supports a larger trade volume in $t = 1$ but a smaller trade volume in $t = 2$ than an MR equilibrium for the same p_g.*

(ii) Given $p_g \in P^{SMR}$, the total trade volume supported in the SMR equilibrium is the same as that in the MR2 equilibrium if p_g admits the MR2 equilibrium; however, the comparison is ambiguous if p_g admits the MR1 equilibrium.

As noted above, the reason that secrecy does not eliminate the stigma attached to the low-type bailout recipients is the presence of firms selling to the market in $t = 1$. These latter firms, together with the high-type bailout recipients that refuse to "pool" with low-type bailout recipients, remove the cloak of secrecy from low-type bailout recipients and expose

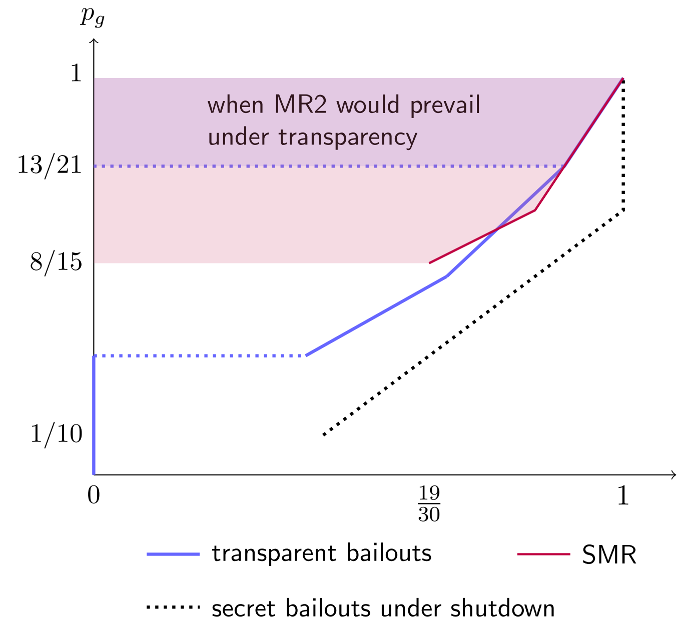


Figure 10 – Net gains in trade volume from secret bailouts relative to transparent bailouts

them in the $t = 2$ market. This argument suggests that the government may be able to strengthen the effect of secrecy by discouraging early market participation. Indeed, we can show that the bailout recipients can enjoy complete anonymity in the $t = 2$ market if the $t = 1$ market is shut down. Under the shutdown of the $t = 1$ market, buyers in $t = 2$ cannot update the belief about firms' types because no action taken by firms in $t = 1$ is observable. Given this, and hence the absence of stigma, the outcome in $t = 2$ is the same as that in the one-shot model, with all firms $\theta \leq \theta_0^*$ selling to the market. In $t = 1$, all types $\theta \leq p_g + S$ will accept the bailout since firms' action in $t = 1$ has no dynamic consequence.

Proposition 6 (Dampening effect of market rejuvenation under secrecy)**.** *Suppose that the government offers a secret bailout at $p_g \geq \max\{I, p_1^*\}$ and further shuts down the $t = 1$ market. Then, in equilibrium,*

(i) firms with types $\theta \leq p_g + S$ accept the bailout in $t = 1$ and those with $\theta \leq \theta_0^*$ sell to the market in $t = 2$;

(ii) the total trade volume in this equilibrium is larger than in the SMR equilibrium, whenever the latter exists for the same p_g.

6 Cost of Bailout and Optimal Policy Design

The preceding analysis has abstracted from the cost of bailouts. In practice, however, the cost of bailouts—the burden on the society of using public funds—is a crucial part of policy debates. In this section, we perform welfare analysis of alternative bailout programs, while explicitly accounting for the cost of operating them. To model the cost, we follow the literature (see Tirole (2012)) and assume that raising a dollar of public funds costs society $(1 + \lambda)$ dollars, where $\lambda \geq 0$ represents the deadweight loss of raising public funds, e.g., distortionary taxation. The welfare effect of a bailout policy would be then captured by the investment surplus generated by the policy minus the total cost of raising public funds that the policy would incur.

To accommodate a general class of bailout policies, we cast the welfare analysis in a mechanism design framework. Specifically, we invoke the revelation principle to consider a class of **direct mechanisms** or **outcomes**, $(q, t) : \Theta \to [0, 2] \times \mathbb{R}$, where $q(\theta) \in [0, 2]$ is total sales across the two periods for type-θ firms and $t(\theta)$ is the transfer paid to them in equilibrium. Without further restrictions, this class encompasses all possible outcomes that would arise when the social planner has total control over all aspects of firms' trading decisions. In practice, however, the policy maker enjoys only a limited scope of control. For the analysis to be relevant for realistic scenarios, we need to restrict the implementable set of mechanisms. In particular, in keeping with the previous analysis, we assume that (1) the scope of the government bailout is limited to the trading of one unit per firm in period $t = 1$; (2) the sale of the second unit must break even for the buyers, as it can occur only in a competitive market; and (3) the government never offers a stochastic policy and never rations a firm on the offered package. As argued before, these assumptions accord well with the empirical fact that bailouts are often confined to a limited duration (modeled in our paper by $t = 1$).

Despite these restrictions, our framework allows for a general set of bailout policies that the government may employ in terms of the bailout terms and disclosure options. For instance, the government may offer a menu of bailout packages with varying degrees of disclosure. Formally, the government may offer a menu of packages $\{(p_g^i, \gamma^i)\}_{i \in \mathcal{I}}$, where $\mathcal{I}$ is an arbitrary index set, such that firms choosing package i are allowed to sell their asset in $t = 1$ at price p_g^i and their identities are revealed with probability $\gamma^i \in \{0, 1\}$.[44] One simple example is that the government offers a menu of two packages $\{(p_g^1, 1), (p_g^2, 0)\}$ such that those firms that pick the first package can sell their assets in $t = 1$ at price p_g^1, which is revealed to the market, and those that choose the second package can sell their assets in $t = 1$ at price p_g^2, and their identities are concealed from the market. Our framework encompasses all such possibilities, in short, allowing for arbitrary bailout and disclosure policies that the government may employ.

[44] For instance, the government induces a set of types given by a sub-distribution F^i of F, where $0 \leq F^i(\theta) \leq F(\theta)$ for all θ, and both F^i and $1 - F^i$ are nondecreasing.

While the feasible set of outcomes consistent with the aforementioned restrictions is not easy to characterize, several observations prove helpful. First, assumption (3) restricts the range of mechanisms such that $q(\theta) \in \mathcal{Q} := \{0, 1, 2\}$. Since the only reason for a firm to sell its asset is to finance the project and enjoy the surplus, it is without loss to restrict attention to mechanisms in which $t(\theta) \geq q(\theta)I$. For any mechanism $M = (q, t)$, if a type-θ firm reports $\tilde{\theta}$, its payoff is

$$U^M(\tilde{\theta}|\theta) := t(\tilde{\theta}) + \theta(2 - q(\tilde{\theta})) + Sq(\tilde{\theta}),$$

since each unit of asset sold enables the financing of a unit of a project with net surplus S, and the remaining unsold units $(2 - q(\tilde{\theta}))$ yield value θ to the firm. For any outcome $M = (q, t)$ to be consistent with equilibrium, it must be incentive compatible:

$$u^M(\theta) := U^M(\theta|\theta) \geq U^M(\tilde{\theta}|\theta) \;\; \forall \theta, \tilde{\theta} \in [0, 1]. \tag{IC}$$

Next, each firm has the option of not participating and enjoying the payoff realized from its asset. In other words,

$$u^M(\theta) \geq 2\theta \;\; \forall \theta, \tilde{\theta} \in [0, 1]. \tag{IR}$$

Since a dollar deficit entails a deadweight loss of $\lambda > 0$, the social welfare from a mechanism $M = (q, t)$ is given by:

$$W(M) := \int_0^1 \left[u^M(\theta) + \theta q(\theta) - t(\theta) - \lambda(t(\theta) - \theta q(\theta))\right] f(\theta) d\theta,$$

where the first term is the surplus accruing to the firms, the next two terms $\theta q(\theta) - t(\theta)$ aggregate the surplus the government and the market enjoy, and finally, the last terms $\lambda(t(\theta) - \theta q(\theta))$ account for the deadweight loss associated with deficit the government runs. Recall that the market must break even in any equilibrium, and thus, the government may need to bear a net deficit to support asset trade.

To facilitate the welfare analysis, we further assume that (IR) is binding for the highest type, i.e., type $\theta = 1$ obtains a payoff of 2 in any mechanism.[45] In other words, type $\theta = 1$ will never receive a payment for its assets greater than the maximum possible value. This assumption can be justified by the cost of public funds ($\lambda > 0$); the government will not wish to offer strict rents to the highest type firm.[46]

[45] This implicitly assumes that, absent a government bailout, not all types can sell to the competitive market. Obviously, if all firms sell in $t = 1$, then there is no scope for government intervention.

[46] With these two assumptions, incentive compatibility can take the standard form of an envelope formula in the mechanism design literature. In fact, the envelope theorem applied to (IC), along with $u^M(1) = 2$, yields $u^M(\theta) = 2 - \int_\theta^1 (2 - q(s))ds$. Using this result, one can also show that (IR) holds for all types of firms: taking the derivative of $u^M(\theta)$ with respect to θ yields $\frac{d}{d\theta}(u^M(\theta) - 2\theta) = -q(\theta) \leq 0$, and thus $u^M(\theta) \geq 2\theta$ for all $\theta \leq 1$.

The next theorem characterizes the feasible mechanisms satisfying all the aforementioned restrictions and provides a method for comparing alternative mechanisms.

Theorem 5. *Let $\mathcal{M}$ denote the set of mechanisms that satisfy the restrictions imposed above. Then, the following holds:*

(i) If $M = (q, t) \in \mathcal{M}$, then $q(\cdot)$ is nonincreasing, and $q(\theta) \leq 1$ for all $\theta > \theta_0^*$, where θ_0^* is the highest type that sells its asset in the one-shot model without a bailout.

(ii) [Revenue Equivalence] If $M = (q, t)$ and $M' = (q', t')$ both in $\mathcal{M}$ have $q = q'$, then $W(M) = W(M')$. In other words, an equilibrium allocation pins down the welfare, expressed as follows:

$$\int_0^1 \left[J(\theta)q(\theta) - 2\lambda + 2\left((1+\lambda)\theta + \lambda \frac{F(\theta)}{f(\theta)} \right) \right] f(\theta) d\theta, \tag{5}$$

where

$$J(\theta) := (1+\lambda)S - \lambda \frac{F(\theta)}{f(\theta)}.$$

(iii) Consider two possible mechanisms, labeled A and B (possibly associated with different levels of p_g or different disclosure policies), such that equilibrium $i = A, B$ induces trade volume $q_i(\cdot)$ across the two periods. Suppose that

$$\int_0^1 q_A(\theta) f(\theta) d\theta = \int_0^1 q_B(\theta) f(\theta) d\theta$$

but there exists $\tilde{\theta} \in (0, 1)$ such that $q_A(\theta) \geq q_B(\theta)$ for $\theta \leq \tilde{\theta}$ and $q_A(\theta) \leq q_B(\theta)$ for $\theta \geq \tilde{\theta}$. Then, equilibrium A yields higher welfare than equilibrium B, strictly so if $q_A(\theta) \neq q_B(\theta)$ for a positive measure of θs.

Part (i) of Theorem 5 characterizes the set of possible allocations that are consistent with incentive compatibility and the government's laissez-faire approach in $t = 2$. In particular, it states that no firm with type greater than the one-shot threshold can sell in both periods. This captures the upper bound on trading across the two periods imposed by the underlying adverse selection and the government's limited involvement in the intervention.

Part (ii) identifies the social value of firms' asset trading. Specifically, the sale of a type-θ firm's asset generates the virtual value $J(\theta) = (1+\lambda)S - \lambda\frac{F(\theta)}{f(\theta)}$. This consists of two parts. The first is the social value $(1+\lambda)S$ of inducing a sale, namely the value of funding the investment project. The second is the deadweight loss $\lambda\frac{F(\theta)}{f(\theta)}$ required to incentivize the sale. The incentive cost is increasing in θ since higher types require stronger incentives to sell.

Part (ii) also describes the extent to which a bailout (of some form) can be cost-effective. Since the virtual value $J(\theta)$ is decreasing in θ, given the log-concavity assumption, we can define a cutoff type

$$\hat{\theta}^* := \sup\left\{\theta \in [0,1] \middle| (1+\lambda)S \geq \lambda \frac{F(\theta)}{f(\theta)}\right\}.$$

Let $\hat{\lambda} := \sup\{\lambda \geq 0 | (1+\lambda)S \geq \lambda \frac{F(\theta_0^*)}{f(\theta_0^*)}\}$. In words, $\hat{\lambda}$ is the highest shadow value that justifies government intervention in Tirole's one-shot model. It is easy to see that $\hat{\lambda} > 0$.[47] We focus on the case in which $\lambda \leq \hat{\lambda}$, thus capturing the case in which a bailout is sufficiently relevant. This condition ensures that the government finds it optimal to induce the firms to trade at least up to θ_1^* in each period.

Part (iii) suggests that, all else being equal, welfare will be enhanced when the respective marginal types that trade across the two periods are smoothed or equalized. The simple intuition is that the incentive cost of enabling trade increases with the firm's type θ, as was seen in part (ii). This means that an *unconstrained* optimal trading decision involves a single cutoff structure, i.e., firms of types $\theta \leq \hat{\theta}_t$ must sell in each period $t = 1, 2$, and moreover, the cutoffs must be identical across the two periods (otherwise reallocating trade from high types to low types improves welfare). However, the constraint identified in part (i) suggests that any single cutoff above θ_0^* is not implementable. For this reason, an equilibrium with non-identical cutoffs may be desirable, particularly in the cases in which the unconstrained optimal trading cutoff is strictly greater than θ_0^*. Nevertheless, part (iii) suggests that for such an equilibrium, smoothing/equalizing the trading cutoffs across the two periods economizes on the incentive costs and improves welfare.

Part (iii) can be also used as a method to facilitate the comparison of welfare among alternative equilibria studied in the previous sections. To economize on description, we say that an equilibrium of type A **dominates in welfare** an equilibrium of type B, *if for any equilibrium with property B that arises under any bailout term p_g, there is an equilibrium with property A arising from some bailout term p_g' which yields weakly higher welfare, and strictly higher welfare in some instances.*

Proposition 7. *The equilibria are compared as follows.*

(i) Given a transparent bailout policy, an equilibrium with the $t = 1$ market shutdown dominates in welfare an equilibrium without the $t = 1$ market shutdown.

(ii) Given a secret bailout policy, an equilibrium with the $t = 1$ market shutdown dominates in welfare an equilibrium without the $t = 1$ market shutdown.

[47] In particular, $\hat{\lambda} = \infty$ if $S \geq \frac{F(\theta_0^*)}{f(\theta_0^*)}$, which holds trivially if $\theta_0^* = 0$, or the market would freeze in the one-shot model.

(iii) With the $t = 1$ market shutdown, an equilibrium under secrecy dominates in welfare an equilibrium under transparency.

Parts (i) and (ii) of Proposition 7 suggest that the early revival of market trading reduces welfare under both transparent and secret bailouts. The reason is that early market revival exacerbates the bailout stigma, as shown by Propositions 3 and 6. The worsening of the bailout stigma expands the wedge between the volumes of trade implemented across the two periods, and according to part (iii) of Theorem 5, this is not desirable from a welfare perspective.

Part (iii) states that secrecy improves welfare relative to transparency when early market revival is suppressed. As noted previously, a secret bailout mitigates the stigma attached to the bailout recipients, which increases uptake of a bailout in $t = 1$. Further, the suppression of the $t = 1$ market sustains trade for firms up to θ_0^*. For an optimal p_g that is weakly greater than $\theta_0^* - S$ given $\lambda \leq \hat{\lambda}$, under secrecy, more trade occurs in $t = 1$ than in $t = 2$. By contrast, under transparency, more trade occurs in $t = 2$ than in $t = 1$, and the marginal trading type in $t = 1$ is no greater than θ_0^*. Given this, p_g can be chosen under secrecy to achieve a greater intertemporal smoothing of trading cutoffs than under an optimal transparent policy (with early market shutdown).

Combining the results, we argue that the secret bailout without immediate market revival performs best among all the equilibria studied thus far. Indeed, one can show that this regime implements a (constrained) optimum among the full set of outcomes. To show this formally, we use parts (i) and (ii) of Theorem 5 to formulate the following relaxed program:

$$[P] \qquad \max_{q:[0,1]\to\mathcal{Q}} \int_0^1 \left[J(\theta)q(\theta) - 2\lambda + 2\left((1+\lambda)\theta + \lambda \frac{F(\theta)}{f(\theta)} \right) \right] f(\theta) d\theta$$

subject to

$$q(\cdot) \text{ is nondecreasing;}$$

$$q(\theta) \leq 1 \text{ if } \theta > \theta_0^*.$$

Proposition 8. *The optimal bailout mechanism has*

$$q^*(\theta) = \begin{cases} 2 & \text{if } \theta \leq \theta_0^*, \\ 1 & \text{if } \theta \in (\theta_0^*, \hat{\theta}^*], \\ 0 & \text{if } \theta > \hat{\theta}^*. \end{cases}$$

The optimal policy is implemented by a secret bailout policy with $p_g = \hat{\theta}^* - S$ accompanied by the shutdown of the market in $t = 1$.

The logic of the above proposition is simple. Without any restriction, the unconstrained optimal policy would implement trade for all types $\theta \leq \hat{\theta}^*$ in both periods. As identified by

Theorem 5-(i), however, this is impossible given the adverse selection in the market and the government's limited intervention. The second-best solution is therefore to implement trade up to type $\hat{\theta}^*$ in one period but up to the one-period limit θ_0^* in the other period. As noted in the preceding section, such an outcome can be implemented by a secret bailout at $p_g = \hat{\theta}^* - S$ along with the shutdown of $t = 1$ market. Our analysis suggests that such a simple policy dominates all other policies, for instance those that may offer a menu of bailout options or offer partial or full disclosure of the identities of firms participating in the bailout.

7 Conclusion

This paper has studied a dynamic model of a government bailout in which firms have a continuing need to fund their projects by selling/collateralizing their assets. Asymmetric information about the quality of assets gives rise to adverse selection within each period and across periods, resulting in a market freeze, particularly in the early stage. This provides a rationale for a government bailout, just as in Tirole (2012). However, in contrast to the one-shot model of Tirole (2012), markets stigmatize bailout recipients, which jeopardizes their ability to fund their subsequent projects in the markets. The presence of this bailout stigma and other dynamic incentives yields a much more complex and nuanced portrayal of how bailouts impact the economy than have been recognized in the extant literature.

First, bailout stigma leads to low or no take-up of otherwise attractive bailout offers. Further, a bailout need not immediately revive the market, and even when it does, it requires the government to pay a premium over the market terms to compensate for the stigma that would attach to its recipients. Immediate market revival also exacerbates adverse selection to such an extent that it is desirable to initially keep the market closed. Despite the bailout stigma and the associated cost, bailout can be effective in stimulating trade and investment, but its effects are delayed and discontinuous, suggesting that stimuli are frustratingly slow to obtain initially but may gain rapid momentum after a long-sustained rescue effort. Delayed benefits materialize as bailouts provide firms with opportunities to boost their reputation in the market by rejecting bailout offers, and this improves their ability to trade in the market in the subsequent periods. A discontinuous effect arises since the severity of the stigma suffered by bailout recipients depends on endogenously formed market beliefs, which could change rather abruptly with a change in bailout terms.

We also analyzed the implications of secret bailouts, in which the identities of the bailout recipients are concealed from the market. Secrecy keeps bailout recipients indistinguishable from those who refuse to sell their assets in the early stage but not from those who sell to the market. Hence, although more firms accept bailouts, the stigma remains for those bailout recipients that seek to sell their assets in the later stage, which increases early trade but decreases later trade, compared with the case of transparent bailouts. Consequently, the overall comparison is ambiguous. Nevertheless, when the shadow cost of the bailout is sufficiently low, we show that a secret bailout welfare dominates a transparent bailout and is constrained-efficient, provided that the early market revival is suppressed to minimize the stigma.

The central lesson from the current work is that, compared with the static setting, the effects of bailouts are very different due to the interplay of the bailout stigma, the early sales stigma, and the market's belief on and off the equilibrium path. To the best of our knowledge, the insights we develop and the forces we identify are novel and have not been recognized in the previous literature. While a careful empirical assessment is needed to quantify the importance of our findings, they should be considered in policy discussions.

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Online Appendix for "Bailout Stigma"

(Not for Publication)

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August 1, 2018

A Technical Preliminaries

In this section, we derive technical results that will be used frequently in the proofs.

Definition A.1. *Define the following:*

(i) $m(a,b) := \mathrm{E}[\theta \,|\, \theta \in [a,b]\,] \quad \forall\, 0 \le a \le b \le 1$*;*

(ii) $\gamma(a) := \max\{\theta \ge a \,|\, \theta \le m(a,\theta) + S\} \quad \forall\, a \in [0,1]$*;*

(iii) $\mathring{m}(a,b,c,d) := \mathrm{E}[\theta \,|\, \theta \in [a,b] \cup [c,d]\,] \quad \forall\, 0 \le a \le b < c \le d \le 1$*;*

(iv) $\mathring{\gamma}(a,b,c) := \max\{\theta \ge a \,|\, \theta \le \mathring{m}(a,b,c,\theta) + S\} \quad \forall\, 0 \le a \le b < c \le 1.$

Given that θ is a continuous random variable, m and $\mathring{m}$ are well-defined and differentiable with respect to every argument. Next, recall our log concavity assumption:

Assumption A.1 (Log-concavity)**.** $\frac{d^2}{d\theta^2} \log f(\theta) < 0$ *for all* $\theta \in [0,1]$.

Based on the above, we obtain the following results.

Lemma A.1. *(i)* $\frac{\partial}{\partial a} m(a,b),\ \frac{\partial}{\partial b} m(a,b) \le 1 \ \ \forall\, 0 \le a \le b \le 1$*; (ii)* γ *is a well-defined function and* $\frac{\partial}{\partial a}\gamma(a) \le 1 \ \ \forall\, a \in [0,1]$*; (iii)* $\frac{\partial}{\partial d}\mathring{m}(a,b,c,d) \le 1 \ \ \forall\, 0 \le a \le b < c \le d \le 1$*; (iv)* $\mathring{\gamma}$ *is a well-defined function.*

Proof. See Bagnoli and Bergstrom (2005) for the proof of part (i).

For part (ii), note that (i) implies $\theta - m(a,\theta)$ is increasing and continuous in θ for all $\theta \geq a$. Thus $\gamma(a)$ defined in Definition A.1-(ii) exists and is uniquely determined for every $a \in [0,1]$ as the minimum of the unique solution to $\theta - m(a,\theta) = S$ and 1. To prove the second part of (ii), first suppose $\gamma(a) = m(a,\gamma(a)) + S$. Applying the implicit function theorem, we have $\frac{d}{da}\gamma(a) = \frac{\frac{\partial}{\partial a}m(a,\gamma(a))}{1-\frac{\partial}{\partial\gamma}m(a,\gamma(a))}$. From Corollary 1 of Szalay (2012), we have $\frac{\partial}{\partial a}m(a,b) + \frac{\partial}{\partial b}m(a,b) \leq 1$ for all $0 \leq a \leq b \leq 1$ if $f(\cdot)$ is log-concave. This implies $\frac{d}{da}\gamma(a) = \frac{\frac{\partial}{\partial a}m(a,\gamma(a))}{1-\frac{\partial}{\partial\gamma}m(a,\gamma(a))} \leq 1$. If $\gamma(a) = 1$, then $\gamma(x) = 1$ for all $x > a$, implying that the right derivative of $\gamma(a)$ is 0 for all $a \in [0,1]$ such that $\gamma(a) = 1$.

To prove part (iii), define $\check{F}(\theta) := F(\theta) - [F(c) - (F(b) - F(a))]$ for every $\theta \geq c$. $\check{F}(\theta)$ is log-concave as shown below:

$$\begin{aligned}
\frac{d^2 \log \check{F}(\theta)}{d\theta^2} &= \frac{d}{d\theta}\left(\frac{f(\theta)}{F(\theta) - [F(c) - (F(b) - F(a))]}\right)\\
&= \frac{d}{d\theta}\left(\frac{f(\theta)}{F(\theta)} \times \frac{F(\theta)}{F(\theta) - [F(c) + (F(b) - F(a))]}\right)\\
&= \frac{d}{d\theta}\left(\frac{f(\theta)}{F(\theta)} \times \left(1 + \frac{F(c) + (F(b) - F(a))}{F(\theta) - [F(c) + (F(b) - F(a))]}\right)\right)\\
&\leq 0.
\end{aligned}$$

In the above, the last inequality follows since $1 + \frac{F(c)+(F(b)-F(a))}{F(\theta)-[F(c)+(F(b)-F(a))]}$ is positive and decreasing in θ, and $\frac{f(\theta)}{F(\theta)}$ is decreasing in θ by the log-concavity $f(\theta)$. Using the log-concavity of $\check{F}(\theta)$, we now show $\frac{\partial \mathring{m}}{\partial d} \leq 1$. Differentiating $\mathring{m}$ with respect to d, we have

$$\begin{aligned}
\frac{\partial \mathring{m}}{\partial d} &= -\frac{f(d)}{[(F(d) - F(c)) + (F(b) - F(a))]^2}\int_a^b \theta dF(\theta) + \frac{\partial}{\partial d}\left(\frac{1}{(F(d) - F(c)) + (F(b) - F(a))}\int_c^d \theta dF(\theta)\right)\\
&< \frac{\partial}{\partial d}\left(\frac{1}{(F(d) - F(c)) + (F(b) - F(a))}\int_c^d \theta dF(\theta)\right).
\end{aligned}$$

To prove $\frac{\partial \mathring{m}}{\partial d} \leq 1$, it suffices to show $\frac{\partial}{\partial d}\left(\frac{1}{(F(d)-F(c))+(F(b)-F(a))}\int_c^d \theta dF(\theta)\right) \leq 1$. Define a map $\check{\delta} : [c,1] \to \mathbb{R}$ as

$$\check{\delta}(\theta) := \theta - \frac{1}{(F(\theta) - F(c)) + (F(b) - F(a))}\int_c^\theta u dF(u),$$

which is a variant of $\delta(\theta)$ introduced by Bagnoli and Bergstrom (2005). Since $\mathring{m}(a,b,c,\theta) = \frac{1}{(F(\theta)-F(c))+(F(b)-F(a))}\int_c^\theta u dF(u)$, we need to show $\check{\delta}(\theta)$ is increasing in θ. Since $\check{F}(\theta) = (F(\theta) - F(c)) + (F(b) - F(a))$, we have $dF = d\check{F}$ for all $\theta \geq c$. Thus $\check{\delta}(\theta)$ can be rewritten as $\check{\delta}(\theta) =$

$\theta - \frac{1}{\check{F}(\theta)}\int_c^\theta u d\check{F}(u)$. Integrating $\int_c^\theta u d\check{F}(u)$ by parts yields

$$\check{\delta}(\theta) = \theta - \frac{\theta\check{F}(\theta) - \int_c^\theta \check{F}(u)du}{\check{F}(\theta)} = \frac{\int_c^\theta \check{F}(u)du}{\check{F}(\theta)}.$$

From Theorem 1-(ii) in Bagnoli and Bergstrom (2005), $\int_c^\theta \check{F}(u)du$ is log-concave for all $\theta \geq c$ if $\check{F}(u)$ is log-concave for all $\theta \geq c$. Thus $\frac{\int_c^\theta \check{F}(u)du}{\check{F}(\theta)}$ is increasing in θ, which implies $\check{\delta}(\theta)$ is also increasing in θ.

Part (iv) follows from part (iii): since $\theta - \mathring{m}(a,b,c,\theta)$ is increasing and continuous in θ, $\mathring{\gamma}(a,b,c)$ is the minimum of the unique solution to $\theta - \mathring{m}(a,b,c,\theta) = S$ and 1. *Q.E.D.*

For future reference, we reproduce our regularity conditions below.

Assumption A.2 (Assumption 2 in the main text)**.** *(i) $\forall\ 0 < \tilde{\theta} < \theta < 1$, $\Delta(\theta;\tilde{\theta},S) := m(\tilde{\theta},\theta) + S - \theta - (m(\theta,\gamma(\theta)) - m(\tilde{\theta},\theta))$ is decreasing in θ;*

(ii) If $\Delta(0;\tilde{\theta},S) \geq 0$, then $\Delta(0;\tilde{\theta},S') \geq 0$ for every $S' > S$ and every $\tilde{\theta} \in (0,1)$;

(iii) For every $0 < \tilde{\theta} < \theta < 1$, $2m(\tilde{\theta},\theta) - m(0,\tilde{\theta})$ is decreasing in $\tilde{\theta}$;

(iv) $\theta_0^* - m(\theta_0^*,\gamma(\theta_0^*)) + S > 0$ for every $S > 0$, where $\theta_0^* = \gamma(0)$;

(v) $\theta \geq 2m(0,\theta)$ for all $\theta \in [0,1]$.

B Proofs for Section 3

Lemma 1. *In any equilibrium without government interventions, there is a cutoff $0 \leq \theta_1 \leq 1$ such that all types $\theta \leq \theta_1$ sell in each of the two periods at price $m(0,\theta_1)$ and all types $\theta > \theta_1$ hold out in $t = 1$ and are offered price $m(\theta_1,\gamma(\theta_1))$ in $t = 2$, which types $\theta \in [\theta_1,\gamma(\theta_1))$ accept. If $\theta_1 = 1$, then $S \geq 2(1 - \mathrm{E}[\theta])$.*

Proof of Lemma 1. Fix a game with discount factor $\delta \in (0,1)$ and fix an equilibrium of the required form. Let $(q_1(\theta), q_2(\theta))$ be the units of the asset a type-θ firm sells in each of the two periods and $(t_1(\theta), \theta_2(\theta))$ be the corresponding transfers.

Step 1. There exists $0 \leq \hat{\theta} \leq \check{\theta} \leq \tilde{\theta} \leq 1$ such that $q_1(\theta) = q_2(\theta) = 1$ for $\theta < \hat{\theta}$; $q_1(\theta) = 1, q_2(\theta) = 0$ for any $\theta \in (\hat{\theta},\check{\theta})$; $q_1(\theta) = 0, q_2(\theta) = 1$ for any $\theta \in (\check{\theta},\tilde{\theta})$; and $q_1(\theta) = q_2(\theta) = 0$ for any $\theta > \tilde{\theta}$.

Proof. In pure strategy equilibrium, we have $q_i(\theta) \in \{0,1\}$ for each θ, $i = 1,2$. The expected discounted payoff for type-θ when imitating type-θ' is $u(\theta';\theta) := q_1(\theta')[S+t_1(\theta')]+[1-q_1(\theta')]\theta+\delta[q_2(\theta')(S+t_2(\theta'))+(1-q_2(\theta'))\theta]$. Let $Q(\cdot) := q_1(\cdot)+\delta q_2(\cdot)$ and $t(\cdot) := q_1(\cdot)t_1(\cdot)+\delta q_2(\cdot)t_2(\cdot)$. Since we must have $u(\theta;\theta) \geq u(\theta';\theta)$ for all θ,θ' in equilibrium, it follows that $(S-\theta)[Q(\theta)-Q(\theta')] \geq t(\theta')-t(\theta)$. Similarly we must have $u(\theta';\theta') \geq u(\theta;\theta')$, which leads to $(S-\theta')[Q(\theta')-Q(\theta)] \geq t(\theta)-t(\theta')$. Combining these two inequalities, we have $(\theta-\theta')[Q(\theta)-Q(\theta')] \leq 0$. This implies monotonicity: $Q(\theta) \leq Q(\theta')$ for any $\theta \geq \theta'$. Since $q_i(\theta) \in \{0,1\}$ for each θ, $i = 1,2$, the monotonicity implies the desired property.

Step 2. If $\check{\theta} < 1$, then those that hold out in $t = 1$ must be offered $m(\check{\theta},\gamma(\check{\theta}))$ in equilibrium, which is accepted by types $\theta \in (\check{\theta},\gamma(\check{\theta}))$ where $\gamma(\check{\theta}) > \check{\theta}$.

Proof. The belief in $t = 2$ for the holdouts is the truncated distribution of F on $[\check{\theta},1]$. This is essentially a one-shot problem with a truncated support. Thus the stated result follows from the definition of γ.

Step 3. $\hat{\theta} = \check{\theta}$.

Proof. Suppose to the contrary that $\hat{\theta} < \check{\theta}$. Let p be the price offered in $t = 1$. By the zero profit condition, p must be a break-even price for the types that accept it. Since type-$\check{\theta}$ firm must weakly prefer accepting p in $t = 1$ to not selling in either periods, we have

$$p + S + \delta\check{\theta} \geq (1+\delta)\check{\theta} \Leftrightarrow p + S \geq \check{\theta}. \tag{B.1}$$

This means that all firms accepting p in $t = 1$ will accept the same price p in $t = 2$ if that price were offered in $t = 2$, with strict incentive for all firms with $\theta < \check{\theta}$. The fact that types $\theta \in (\hat{\theta},\check{\theta})$ choose not to sell in $t = 2$ means that the price offered in $t = 2$ to those that accept p in $t = 1$, denoted by p^-, is strictly less than p.

Suppose first $\check{\theta} < 1$. Then, by Step 2, an offer $p_2 := m(\check{\theta},\gamma(\check{\theta}))$ must be made in equilibrium to those that hold out in $t = 1$, which is accepted by types $\theta \in (\check{\theta},\gamma(\check{\theta}))$. Since type-$\check{\theta}$ must be indifferent between selling at p in $t = 1$ only and selling at p_2 in $t = 2$ only, we must have

$$p + S + \delta\check{\theta} = \check{\theta} + \delta(p_2 + S). \tag{B.2}$$

In particular, $p_2 + S > \check{\theta}$, so $p + S > \check{\theta}$. This means that, if a buyer deviates by offering $p - \varepsilon > p^-$ for sufficiently small $\varepsilon > 0$ in $t = 2$ to those that accepted p in $t = 1$, then all of them must accept that offer. The buyer makes a strictly positive profit with such a deviation since p is the break-even price. We thus have a contradiction.

Suppose next $\check{\theta} = 1$, meaning that all types sell in $t = 1$. Here we invoke the D1 refinement to derive a contradiction. In the candidate equilibrium, the payoff for type $\theta > \hat{\theta}$ is $p+S+\delta\theta$, and the payoff for all types $\theta < \hat{\theta}$ is some constant u^*, which must equal $p+S+\delta\hat{\theta}$ since type-$\hat{\theta}$ must be indifferent. Let $U^*(\theta)$ be the equilibrium payoff type-θ enjoys when the candidate equilibrium is played: $U^*(\theta) := \max\{u^*, p+S+\delta\theta\}$. Suppose a type θ firm deviates by refusing the bailout in $t = 1$, and suppose a buyer offers p_2' to that deviating firm in $t = 2$. Type-θ's deviation payoff is then $\theta + \delta \max\{p_2' + S, \theta\}$. Thus, when type-$\theta$ deviates by choosing holdout, the set of market's offers in $t = 2$ that dominate the candidate equilibrium for type-θ is

$$D(\text{holdout}, \theta) := \{p_2' | \theta + \delta \max\{p_2' + S, \theta\} \geq \max\{u^*, p + S + \delta\theta\}\}.$$

Note that for fixed p_2', the payoff difference $\theta + \delta \max\{p_2' + S, \theta\} - \max\{u^*, p + S + \delta\theta\}$ is strictly increasing in θ, so the set $D(\text{holdout}, \theta)$ is nested in the sense that $D(\text{holdout}, \theta) \subset D(\text{holdout}, \theta')$ for $\theta' > \theta$. In other words, $D(\text{holdout}, 1)$ is maximal, and more importantly, $D(\text{holdout}, \theta)$ is not maximal if $\theta < 1$. Given this, the D1 refinement entails that the belief by the market must be supported on $\theta = 1$ in case of holdout. Thus following the deviation, the market's offer must be $p_2' = 1$. This means that, for the market's offer in the candidate equilibrium to satisfy D1, type $\check{\theta} = 1$ must enjoy the payoff of at least $1 + \delta(1 + S)$ in case of deviation to holdout from the candidate equilibrium. Since type $\check{\theta} = 1$ chooses to sell in $t = 1$ in the candidate equilibrium, we must have

$$p + S + \delta \geq 1 + \delta(1 + S), \tag{B.3}$$

which implies $p + S > 1$. This in turn implies that in $t = 2$, a buyer can deviate by offering a price slightly below p and induce acceptance from all types that accepted p in $t = 1$. Once again, the buyer makes a strictly positive profit with such a deviation, hence a contradiction.

Step 4. All types $\theta < \hat{\theta} = \check{\theta}$ (if is nonempty) are offered a single price in both periods equal to $m(0, \hat{\theta})$. All types $\theta > \hat{\theta}$ are offered price $m(\hat{\theta}, \gamma(\hat{\theta}))$, which is accepted by types $\theta \in (\hat{\theta}, \gamma(\hat{\theta}))$.

Proof. Suppose there are two distinct prices p, p' that are accepted by positive measures of firms. By the zero profit condition, both prices must be breaking even for the types that accept them. But then, no type will accept the lower price, hence a single price is offered to all types $\theta < \hat{\theta}$. The market's break-even condition then pins down the price to $m(0, \hat{\theta})$. The second statement follows from Step 2.

Step 5. If $\hat{\theta} = 1$, then $S \geq 2(1 - \mathrm{E}[\theta])$.

Proof. Applying the D1 argument as in Step 3, type-$\hat{\theta}$'s equilibrium payoff $(1+\delta)(p+S)$ should not be smaller than $1+\delta(1+S)$ where $p = m(0,1) = \mathrm{E}[\theta]$ by Step 4. From $(1+\delta)(p+S) \geq 1+\delta(1+S)$ follows the stated condition as $\delta \to 1$. *Q.E.D.*

Theorem 2.

(i) There is an equilibrium in which firms with $\theta \leq \theta_1^*$ sell at price $p_1^* := m(0, \theta_1^*)$ in both periods, firms with $\theta \in (\theta_1^*, \theta_2^*)$ sell only in $t = 2$ at price $p_2^* := m(\theta_1^*, \theta_2^*)$, and firms with $\theta > \theta_2^*$ never sell, where θ_1^* and θ_2^* are defined by $\Delta(\theta_1^*; S) = 0$ and $\theta_2^* = \gamma(\theta_1^*)$, respectively. We have $\theta_1^* \leq \theta_0^* \leq \theta_2^*$, hence $p_1^* \leq p_0^* \leq p_2^*$, where the inequalities hold strictly if the cutoff in the one-period model satisfies $\theta_0^* \in (0, 1)$. Given Assumption 1-(i), there is at most one such equilibrium with an interior θ_1^*.

(ii) Given Assumption 1-(ii), the $t = 1$ market in equilibrium is fully active if $S \geq \overline{S}^*$, suffers from partial freeze if $S \in (\underline{S}^*, \overline{S}^*)$, and full freeze if $S < \underline{S}^*$, where $\underline{S}^*$ and $\overline{S}^*$ are defined by $\Delta(0; \underline{S}^*) = 0$ and $\Delta(1; \overline{S}^*) = 0$, respectively, and satisfy $\underline{S}^* > \underline{S}_0$ and $\overline{S}^* > \max\{\overline{S}_0, \underline{S}^*\}$.

(iii) In addition, there is an equilibrium with full market freeze in $t = 1$ for any S.

Proof of Theorem 2. To prove (i), note first that the existence of cutoffs θ_1 and θ_2 was established by Lemma 1. Thus it suffices to show $\theta_1 \leq \theta_0 \leq \theta_2$ with strict inequalities if $\theta_0^* \in (0, 1)$. Consider the $t = 1$ cutoff θ_1. If $\theta_1 < 1$, then it satisfies $\Delta(\theta_1, S) \leq 0$. Thus $\theta_1 \leq 2m(0, \theta_1) - m(\theta_1, \gamma(\theta_1)) + S < m(0, \theta_1) + S$ since $m(0, \theta_1) < m(\theta_1, \gamma(\theta_1))$. Since $\theta_0^* := \sup\{\theta | \theta \leq m(0, \theta) + S\}$, we have $\theta_1 \leq \theta_0^*$ with strict inequality if $\theta_0^* < 1$. If $\theta_1 = 1$, then $\Delta(\theta_1, S) \geq 0$ by the D1 refinement, which in turn implies $\theta_0^* = 1$. Next, the $t = 2$ cutoff $\theta_2 = \gamma(\theta_1)$ satisfies $\theta_2 \leq m(\theta_1, \gamma(\theta_1)) + S$. Since $\theta_1 \geq 0$, we have $\theta_2 = \gamma(\theta_1) \geq \gamma(0) = \theta_0^*$, where the inequality is strict for $\theta_0^* < 1$ and $\theta_1 > 0$.

For (ii), note that, by Assumption 1, $\theta_1 - 2m(0, \theta_1) + m(\theta_1, \gamma(\theta_1))$ is strictly increasing in θ_1 for each S. Since $\theta_1 < 1$ satisfies $\Delta(\theta_1, S) \leq 0$, a unique θ_1 can be found, which is increasing in S. From this and the first claim follows the second claim.

For (iii), consider the candidate equilibrium in which the market in $t = 1$ completely freezes. This means that, in $t = 2$, we have one period equilibrium with cutoff given by θ_0^* and price p_0. In this equilibrium, the payoff for type $\theta \leq \theta_0^*$ is $\theta + \delta(p_0 + S) = \theta + \delta\theta_0^*$ and the payoff for type $\theta > \theta_0^*$ is $(1 + \delta)\theta$. Thus the equilibrium payoff for type θ is $U^*(\theta) = \max\{\theta + \delta\theta_0^*, (1 + \delta)\theta\}$. Suppose a buyer deviates and offers p_1 in $t = 1$. Let p_2 be the market's offer in $t=2$ to those that accept the deviation offer p_1. Then the payoff to type θ from accepting p_1 is $p_1+S+\delta\max\{p_2+S,\theta\}$. As in the proof of Lemma 1, define the set

$$D(\text{sell},\theta) := \{p_2 | p_1+S+\delta\max\{p_2+S,\theta\} \geq \max\{\theta+\delta\theta_0^*, (1+\delta)\theta\}\}\,.$$

For fixed p_2, the payoff difference is strictly decreasing in θ, hence $D(\text{sell},0)$ is maximal. Then by the D1 refinement, the market's belief must be supported on $\theta=0$ when a firm accepts a deviation offer p_1 in $t=1$. Given this belief, no firm will accept p_1 if $p_1 \leq p_0$. If $p_1 > p_0$, then all types $\theta \leq p_1+S$ accept the deviation offer p_1. Then the buyer will lose money given $p_1 > p_0$. *Q.E.D.*

C Proofs for Section 4

In Section C.1, we establish necessary conditions for various equilibria. Section C.2 presents the main characterization of equilibria. To appreciate the main points, readers can skip Section C.1 and jump directly to Section C.2.

C.1 Necessary Conditions for Equilibria

We first characterize the various types of equilibria and derive necessary conditions for the existence of each type of equilibria.

C.1.1 Equilibrium Cutoff Structure

Lemma 2. *In any equilibrium, there are four possible cutoffs $0 \leq \theta_g \leq \theta_1 \leq \theta_{g\varnothing} \leq \theta_2 \leq 1$ such that types $\theta \in \Theta_g := [0,\theta_g)$ sell to the government in $t=1$ and to the market in $t=2$, types $\theta \in \Theta_1 := (\theta_g,\theta_1)$ sell to the market in both periods, types $\theta \in \Theta_{g\varnothing} := (\theta_1,\theta_{g\varnothing})$ sell only in $t=1$ to the government, types $\theta \in \Theta_2 := (\theta_{g\varnothing},\theta_2)$ sell only in $t=2$ to the market, and types $\theta > \theta_2$ sell in neither period.*

Proof of Lemma 2. Similar to the proof of Lemma 1, fix a game with discount factor $\delta \in (0,1)$ and the probability of market collapse $\varepsilon \in (0,1)$. Also, fix any equilibrium of the required form. Let $q_g(\theta)$ be the unit of the asset a type-θ firm sells to the government, $(q_1(\theta), q_2(\theta))$ be the units of the asset the firm sells in each of the two periods, and $(t_g(\theta), t_1(\theta), t_2(\theta))$ be the corresponding transfers. The expected payoff for a type-θ firm when playing as if it is type-θ' is

$$\begin{aligned}
u(\theta';\theta) = {} & q_g(\theta')\big[[S+t_g(\theta')]+\varepsilon\delta\theta\big] + (1-q_g(\theta'))\varepsilon(1+\delta)\theta \\
& + (1-\varepsilon)\{1-q_g(\theta')\}\big[q_1(\theta')\{S+t_1(\theta)\}+\{1-q_1(\theta')\}\theta\big] \\
& + (1-\varepsilon)\delta\big[q_2(\theta')\{S+t_2(\theta')\}+\{1-q_2(\theta')\}\theta\big].
\end{aligned}$$

Let $Q(\cdot) := q_g(\cdot)+(1-\varepsilon)\{1-q_g(\cdot)\}q_1(\cdot)+(1-\varepsilon)\delta q_2(\cdot)$ and $T(\cdot) := q_g(\cdot)t_g(\cdot)+(1-\varepsilon)\{1-q_g(\cdot)\}q_1(\cdot)t_1(\cdot)+(1-\varepsilon)\delta q_2(\cdot)t_2(\cdot)$. Since $u(\theta;\theta)-u(\theta';\theta)\geq 0$ and $u(\theta';\theta')-u(\theta;\theta')\geq 0$ for every $\theta\neq\theta'$ in equilibrium, it follows that $(S-\theta)[Q(\theta)-Q(\theta')]\geq T(\theta')-T(\theta)$ and $(S-\theta')[Q(\theta')-Q(\theta)]\geq T(\theta)-T(\theta')$. Combining these inequalities, we have $(\theta'-\theta)[Q(\theta)-Q(\theta')]\geq 0$, which implies that $Q(\theta)$ is decreasing in θ. Since $1>(1-\varepsilon)>(1-\varepsilon)\delta$ and $q_j(\theta)\in\{0,1\}$ for every $j=g,1,2$ in pure-strategy equilibrium, there exist cutoffs $0\leq\theta_g\leq\theta_1\leq\theta_{g\varnothing}\leq\theta_{1\varnothing}\leq\theta_2\leq 1$ such that: *(i)* $(q_g(\theta),q_1(\theta),q_2(\theta))=(1,0,1)$ if $\theta\in[0,\theta_g]$; *(ii)* $(q_g(\theta),q_1(\theta),q_2(\theta))=(0,1,1)$ if $\theta\in(\theta_g,\theta_1]$; *(iii)* $(q_g(\theta),q_1(\theta),q_2(\theta))=(1,0,0)$ if $\theta\in(\theta_1,\theta_{g\varnothing}]$; *(iv)* $(q_g(\theta),q_1(\theta),q_2(\theta))=(0,1,0)$ if $\theta\in(\theta_{g\varnothing},\theta_{1\varnothing}]$; *(v)* $(q_g(\theta),q_1(\theta),q_2(\theta))=(0,0,1)$ if $\theta\in(\theta_{1\varnothing},\theta_2]$; *(vi)* $(q_g(\theta),q_1(\theta),q_2(\theta))=(0,0,0)$ if $\theta>\theta_2$. Applying the same logic used for the proof of Step 3 in the proof of Lemma 1, it can be shown that $\theta_{g\varnothing}=\theta_{1\varnothing}$. Thus any equilibrium must be characterized by the cutoff structure $0\leq\theta_g\leq\theta_1\leq\theta_{g\varnothing}\leq\theta_2\leq 1$. *Q.E.D.*

In what follows, we describe the incentive compatibility constraints for firms in each type of equilibria. Since the equilibria are characterized by a set of cutoff types by Lemma 2, we derive conditions characterizing these cutoffs in each type of equilibria. Lastly, we find bailout terms compatible with each type of equilibria.

C.1.2 Necessary Conditions for SBS Equilibrium

In this equilibrium, there exists $\theta_{g\varnothing}\in(0,\theta_2)$ and $\theta_2\leq 1$ such that: types $\theta\in[0,\theta_{g\varnothing}]$ sell to the government at price p_g in $t=1$ but cannot sell in $t=2$ due to $m(0,\theta_{g\varnothing})<I$; types $\theta\in(\theta_{g\varnothing},\theta_2]$ sell only in $t=2$ at price $m(\theta_{g\varnothing},\theta_2)$; the market fully freezes in $t=1$ so no trade occurs.

We derive necessary conditions for each type to play the above equilibrium strategies. The payoffs from playing the equilibrium strategies across the two periods are $p_g+S+\theta$ for all $\theta\in[0,\theta_{g\varnothing}]$, and $\theta+m(\theta_{g\varnothing},\theta_2)+S$ for all $\theta\in(\theta_{g\varnothing},\theta_2]$. Since type $\theta_{g\varnothing}$ is indifferent between these choices, we must have $m(\theta_{g\varnothing},\theta_2)=p_g$. Meanwhile, type θ_2 must be the highest type that will be induced to sell in $t=2$. It must follow that $\theta_2=\gamma(\theta_{g\varnothing})$, where γ is defined in Definition A.1-(ii).

Thus we have the following conditions that determine the marginal types $\theta_{g\o}$ and θ_2:

$$\theta_2 = \gamma(\theta_{g\o}); \tag{C.1}$$

$$p_g = m(\theta_{g\o}, \gamma(\theta_{g\o})). \tag{C.2}$$

By Lemma A.1, $\theta_{g\o}$ and θ_2 are uniquely determined by p_g. Moreover, since both $\gamma(\theta)$ and $m(\theta, \gamma(\theta))$ are differentiable and increasing in θ, we have $\frac{d}{dp_g}\theta_{g\o} > 0$. The case in which all firms sell to the government in $t = 1$ (i.e., $\theta_{g\o} = 1$) can be supported if and only if $p_g \geq m(1, \gamma(1)) = 1$. The reason is the following. Applying the same argument used to prove Lemma 1, one can easily see that the worst off-the-path belief consistent with D1 for a deviator (one who holds out) is $\theta_{g\o} = 1$. Given the belief, no firm deviates if and only if $p_g \geq m(1, \gamma(1)) = 1$. Since we restrict $p_g \leq 1$, such a case ($\theta_{g\o} = 1$) can be observed only if $p_g = 1$.

In addition, the SBS equilibrium requires that types $\theta \in [0, \theta_{g\o}]$ cannot sell in $t = 2$, hence

$$m(0, \theta_{g\o}) < I. \tag{C.3}$$

Furthermore, either of the following constraints on $\theta_{g\o}$ must hold:

$$\theta_{g\o} < I, \tag{C.4}$$

$$\theta_{g\o} - m(\theta_{g\o}, \gamma(\theta_{g\o})) + S \leq 0. \tag{C.5}$$

To see why, suppose the $t = 1$ market opens after the bailout and a buyer deviates and offers $p_1' \geq I$. Since the belief in $t = 1$ is that only the types $\theta > \theta_{g\o}$ are available for asset sales, the firms accepting p_1' are assigned the off-the-path belief as being the worst of the available types (i.e., $\theta = \theta_{g\o}$).

First, suppose $\theta_{g\o} < I$. Given the off-the-path belief, firms that deviate and sell at price p_1' cannot sell in $t = 2$, hence the total payoff from the deviation equals $p_1' + S + \theta$. Clearly $p_1' > m(\theta_{g\o}, \gamma(\theta_{g\o}))$ for, otherwise, the deviating firms will not sell at p_1'. But if $p_1' > m(\theta_{g\o}, \gamma(\theta_{g\o}))$, then all types $\theta \in (\theta_{g\o}, (p_1' + S) \wedge 1]$ will sell at p_1', and $m(\theta_{g\o}, (p_1' + S) \wedge 1) - p_1' > 0$ from the definition of $\gamma(\cdot)$. Thus no buyers in $t = 1$ deviate if $\theta_{g\o} < I$.

Second, suppose $\theta_{g\o} \geq I$. Given the off-the-path belief above, firms selling at p_1' can sell at price $\theta_{g\o}$ in $t = 2$. Thus the total payoff to the deviating firm is $p_1' + S + \max\{\theta, \theta_{g\o} + S\}$. In the above, we showed that $p_1' > m(\theta_{g\o}, \gamma(\theta_{g\o}))$ is not possible. If $p_1' \leq m(\theta_{g\o}, \gamma(\theta_{g\o}))$, then types $\theta > \theta_{g\o} + S$ do not sell at p_1' since $p_1' + S + \max\{\theta, \theta_{g\o} + S\} = p_1' + S + \theta < \theta + \max\{m(\theta_{g\o}, \gamma(\theta_{g\o})) + S, \theta\}$. On the contrary, types $\theta \in (\theta_{g\o}, \theta_{g\o} + S]$ sell at p_1' if and only if $p_1' + \theta_{g\o} + 2S \geq \theta + m(\theta_{g\o}, \gamma(\theta_{g\o})) + S$, or equivalently $p'_1 \geq \theta + m(\theta_{g\emptyset}, \gamma(\theta_{g\emptyset})) - \theta_{g\emptyset} - S$. Let $\theta'_1 := p'_1 + (\theta_{g\emptyset} - m(\theta_{g\emptyset}, \gamma(\theta_{g\emptyset})) + S)$. Then types $\theta \in (\theta_{g\emptyset}, \theta'_1]$ sell to the deviating buyer, so the deviating buyer gets the expected payoff $m(\theta_{g\emptyset}, \theta'_1) - p'_1$. Since $\frac{d\theta'_1}{dp'_1} = 1$ and hence $\frac{d}{dp'_1}(m(\theta_{g\emptyset}, \theta'_1) - p'_1) < 0$, we have $m(\theta_{g\emptyset}, \theta'_1) - p'_1 < 0$ for any $p'_1 \leq m(\theta_{g\emptyset}, \gamma(\theta_{g\emptyset}))$ if and only if $m(\theta_{g\emptyset}, \theta'_1) - p'_1 \leq 0$ at $p'_1 = m(\theta_{g\emptyset}, \gamma(\theta_{g\emptyset})) - S$. Since $m(\theta_{g\emptyset}, \theta'_1) - p'_1 = \theta_{g\emptyset} - m(\theta_{g\emptyset}, \gamma(\theta_{g\emptyset})) + S$, we must have $\theta_{g\emptyset} - m(\theta_{g\emptyset}, \gamma(\theta_{g\emptyset})) + S \leq 0$.

Let P^{SBS} denote the range of p_gs for which SBS equilibrium exists.

Lemma C.1. *If P^{SBS} in non-empty, it is a convex set such that $p_g > p_0^*$ for all $p_g \in P^{SBS}$.*

Proof. Since $m(\theta, \gamma(\theta))$ is increasing in θ, $\theta_{g\emptyset}^{SBS}(p_g)$ is increasing in p_g if $\theta_{g\emptyset}^{SBS}$ is well-defined. Hence there exists $\tilde{p}_g^{SBS}$ such that $\theta_{g\emptyset}^{SBS}(p_g)$ satisfies (C.3) if and only if $p_g < \tilde{p}_g^{SBS}$. Furthermore, since $p_0^* = m(0, \theta_0^*) = m(0, \gamma(0))$, we have $p_g > p_0^*$ for $\theta_{g\emptyset}^{SBS}(p_g) > 0$. Lastly, since $\theta - m(\theta, \gamma(\theta)) + S$ is increasing in θ, there exists $\hat{p}_g^{SBS}$ such that: *(i)* if (C.4) binds but (C.5) does not, then $\theta_{g\emptyset}^{SBS}(p_g)$ satisfies (C.4) if and only if $p_g < \hat{p}_g^{SBS}$; *(ii)* if (C.5) binds but (C.4) does not, then $\theta_{g\emptyset}^{SBS}(p_g)$ satisfies (C.5) if and only if $p_g \leq \hat{p}_g^{SBS}$. Therefore, $P^{SBS} = (p_0^*, \tilde{p}_g^{SBS})$ if (C.3) binds, $P^{SBS} = (p_0^*, \hat{p}_g^{SBS}]$ if (C.4) binds, and $P^{SBS} = (p_0^*, \hat{p}_g^{SBS})$ if (C.5) binds. *Q.E.D.*

For every $p_g \in P^{SBS}$, we let $\theta_{g\emptyset}^{SBS}(p_g)$ denote the marginal type determined by (C.2). For expositional convenience, we may occasionally abbreviate $\theta_{g\emptyset}^{SBS}(p_g)$ to $\theta_{g\emptyset}^{SBS}$.

C.1.3 Necessary Conditions for MBS Equilibrium

In this equilibrium, there exist $\theta_g \in (0, \theta_2)$ and $\theta_2 \leq 1$ such that: types $\theta \in [0, \theta_g]$ sell to the government at price p_g in $t = 1$ and to the market at price $m(0, \theta_g)$ in $t = 2$; types $\theta \in (\theta_g, \theta_2]$ sell only in $t = 2$ at price $m(\theta_g, \theta_2)$; no asset trade occurs in the $t = 1$ market.

In equilibrium, the total payoffs for the firms are $p_g + m(0, \theta_g) + 2S$ for all $\theta \in [0, \theta_g]$, $\theta + m(\theta_g, \theta_2) + S$ for all $\theta \in (\theta_g, \theta_2]$, and 2θ for all $\theta > \theta_2$. For expositional ease, we treat the case in which θ_2 is interior (so it is characterized by an indifference condition). As one can see clearly, our characterization also works for the boundary case $\theta_2 = 1$. From type θ_g's indifference condition, we have $p_g + m(0, \theta_g) + S - \theta_g - m(\theta_g, \theta_2) = 0$. Similarly, type θ_2's indifference condition leads to $\theta_2 = m(\theta_g, \theta_2) + S = \gamma(\theta_g)$. Thus we have the following conditions that determine θ_g and θ_2:

$$\theta_2 = \gamma(\theta_g), \tag{C.6}$$

$$p_g = \theta_g - m(0, \theta_g) - S + m(\theta_g, \gamma(\theta_g)). \tag{C.7}$$

Since $\theta - m(0, \theta) + m(\theta, \gamma(\theta))$ is continuous and increasing in θ, the marginal type θ_g is uniquely determined by and increasing in p_g. If p_g is very large in that $p_g \geq 1 - m(0, 1) - S + m(1, \gamma(1)) = 2 - \mathrm{E}[\theta] - S$, then we assign $\theta_g = 1$. Such an assignment is supported by the off-the-path belief at the $t = 2$ market that the holdouts in $t = 1$ are perceived as the highest type $\theta = 1$. Once again, one can check this is the only belief that satisfies the D1 refinement.

In addition, the bailout recipients also sell in $t = 2$. Thus, we have

$$m(0, \theta_g) \geq I. \tag{C.8}$$

Furthermore, the $t = 1$ market freezes completely. From the analysis of SBS equilibrium (recall (C.4) and (C.5)), we showed that the $t = 1$ market freezes fully if and only if either $\theta_g < I$ or $\theta_g - m(\theta_g, \gamma(\theta_g)) + S \leq 0$. Since $\theta_g > m(0, \theta_g) \geq I$ from (C.8), the latter condition must hold:

$$\theta_g - m(\theta_g, \gamma(\theta_g)) + S \leq 0. \tag{C.9}$$

Lastly, all types $\theta \in [0, \theta_g]$ sell assets at price $m(0, \theta_g)$ in $t = 2$. Thus we must have $\theta_g \leq m(0, \theta_g) + S$, or equivalently

$$\theta_g \leq \theta_0^*. \tag{C.10}$$

Lemma C.2. *There exist* $\underline{p}_g^{MBS} \leq \overline{p}_g^{MBS}$ *such that* (C.7) *admits a unique* θ_g *that satisfies* (C.8) *–* (C.10) *if and only if* $p_g \in [\underline{p}_g^{MBS}, \overline{p}_g^{MBS}]$.

Proof. Since $\theta - m(0, \theta) + m(\theta, \gamma(\theta)) - S$ is increasing in θ, $\theta_g^{MBS}(p_g)$ is increasing in p_g if well-defined. Moreover, since $m(0, \theta)$ is increasing in θ, there exists $\underline{p}_g^{MBS}$ such that $\theta_g^{MBS}(p_g)$ is well-defined and satisfies (C.8) if and only if $p_g \geq \underline{p}_g^{MBS}$. Furthermore, since $\theta - m(0, \theta) + S$ is increasing in θ, there exists $\overline{p}_g^{MBS}$ such that $\theta_g^{MBS}(p_g)$ (if well-defined) satisfies (C.9) and (C.10) if and only if $p_g \leq \overline{p}_g^{MBS}$. Putting all these results together, we have $P^{MBS} = [\underline{p}_g^{MBS}, \overline{p}_g^{MBS}]$. *Q.E.D.*

For every $p_g \in [\underline{p}_g^{MBS}, \overline{p}_g^{MBS}]$, we let $\theta_g^{MBS}(p_g)$ denote the marginal type θ_g determined by (C.7). For expositional convenience, we may abbreviate $\theta_g^{MBS}(p_g)$ to θ_g^{MBS}. We also let $P^{MBS} := [\underline{p}_g^{MBS}, \overline{p}_g^{MBS}]$.

C.1.4 Necessary Conditions for MR Equilibrium

In this equilibrium, a positive measure of firms sell to the market in $t = 1$ (in addition to a positive measure of firms selling to the government). Bailout rejuvenates the $t = 1$ market.

There are two possible types of MR equilibria: **MR1** in which $\Theta_g, \Theta_1, \Theta_2 \neq \emptyset$, but $\Theta_{g\emptyset} = \emptyset$, and **MR2** in which $\Theta_g, \Theta_1, \Theta_{g\emptyset}, \Theta_2 \neq \emptyset$.

MR1 equilibrium: In this equilibrium, there exist $0 < \theta_g < \theta_1 \leq \theta_2 \leq 1$ such that: types $\theta \in [0, \theta_g]$ sell to the government in $t = 1$, and to the market in $t = 2$ at price $m(0, \theta_g)$; types $\theta \in (\theta_g, \theta_1]$ sell to the market at price $m(\theta_g, \theta_1)$ in both periods; types $\theta \in (\theta_1, \theta_2]$ sell to the market at price $m(\theta_1, \theta_2)$ only in $t = 2$; types $\theta > \theta_2$ do not sell in either period.

In equilibrium, firms' total payoffs are $p_g + m(0, \theta_g) + 2S$ for all $\theta \in [0, \theta_g]$, $2m(\theta_g, \theta_1) + 2S$ for all $\theta \in (\theta_g, \theta_1]$, $\theta + m(\theta_1, \theta_2) + S$ for all $\theta \in (\theta_1, \theta_2]$, and 2θ for all $\theta > \theta_2$. From these payoffs, it is straightforward to see the three marginal types must satisfy relevant incentive constraints (i.e., indifference in the case of an interior solution):

$$p_g = 2m(\theta_g, \theta_1) - m(0, \theta_g); \tag{C.11}$$

$$\theta_1 = \max\{\theta \in [\theta_g, 1] \,|\, m(\theta_g, \theta) + S - \theta - (m(\theta, \gamma(\theta)) - m(\theta_g, \theta)) \geq 0\}; \tag{C.12}$$

$$\theta_2 = \gamma(\theta_1). \tag{C.13}$$

(C.11) is the indifference condition for type-θ_g firm, (C.12) that for type-θ_1 firm, and (C.13) that that for type-θ_2 firm. Assumption A.2-(i) ensures that θ_1 is uniquely determined by θ_g. As before (C.12) allows for the possibility that $\theta_1 = 1$. The D1 refinement suggests that if $\theta_1 = 1$, the worst off-the-path belief for a deviating holdout firm is $\theta_1 = 1$, so (C.12) ensures that given that belief, no firm wishes to deviate.

There are additional necessary conditions for the MR1 equilibrium. First, types $\theta \in [0, \theta_g]$ should be able to finance their projects in $t = 2$ (an implication of Lemma 2):

$$m(0, \theta_g) \geq I. \tag{C.14}$$

Second, types $\theta \in (\theta_1, \gamma(\theta_1)]$ must prefer selling only in $t = 2$ to either selling to the market or selling to the government in $t = 1$, which requires $m(\theta_1, \gamma(\theta_1)) \geq m(\theta_g, \theta_1)$ and $m(\theta_1, \gamma(\theta_1)) \geq p_g$. Since the first inequality holds trivially, we only state the second condition:

$$m(\theta_1, \gamma(\theta_1)) \geq p_g. \tag{C.15}$$

The conditions (C.11) – (C.15) will be used later when characterizing the set of bailout terms that support the MR1 equilibrium. One can also see that the same conditions are also sufficient for MR1 to arise.

Lemma C.3.

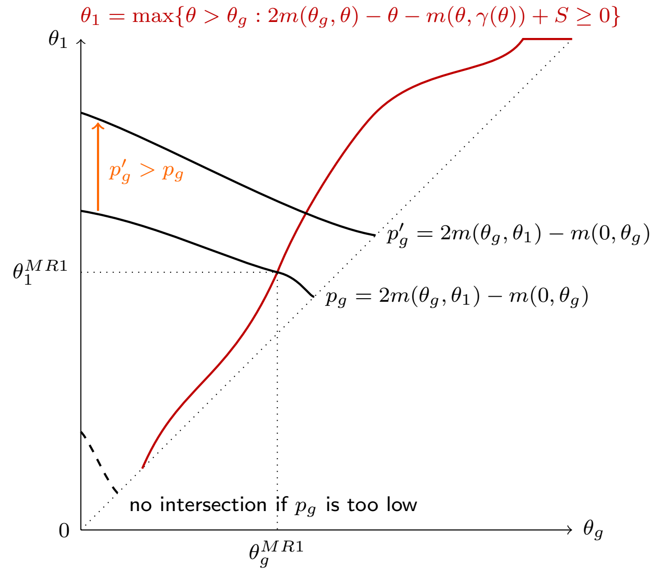


Figure 1 – Characterization of θ_g^{MR1} and θ_1^{MR1}

(i) There exists $\check{p}_g^{MR1}$ *such that* (C.11) *and* (C.12) *admit a unique* (θ_g, θ_1) *that satisfies* $0 < \theta_g < \theta_1 \leq 1$ *if and only if* $p_g > \check{p}_g^{MR1}$.

(ii) There exists $\underline{p}_g^{MR1} \geq \check{p}_g^{MR1}$ *such that* θ_g *determined by* (C.11) *and* (C.12) *satisfies* (C.14) *if and only if* $p_g \geq \underline{p}_g^{MR1}$.

Proof. We first prove part (i). Given Assumption A.2-(iii), (C.11) defines θ_1 as a decreasing function of θ_g, labelled $\tilde{\theta}_1(\theta_g)$, whenever well-defined. Furthermore, given Assumption A.2-(i), (C.12) defines θ_1 as an increasing function of θ_g, labelled $\theta_1^I(\theta_g)$, whenever well-defined. In fact, there exists $\underline{\theta}_g < 1$ such that $\theta_1^I(\theta_g)$ is well-defined if and only if $\theta_g > \underline{\theta}_g$.[1] Therefore, (θ_g, θ_1) satisfying (C.11) and (C.12), if well-defined, is characterized by a unique point of intersection between $\tilde{\theta}_1(\theta_g)$ and $\theta_1^I(\theta_g)$. Moreover, it can be shown that $\tilde{\theta}_1(\theta_g)$ shifts up as p_g increases, which is also illustrated in Figure 1. Lastly, one can find that there exists a $p_g \leq 1$ such that $\tilde{\theta}_1(\theta_g)$ and $\theta_1^I(\theta_g)$ intersect.[2] Putting all results together, there exists $\check{p}_g^{MR1}$ such that two curves $\tilde{\theta}_1(\theta_g)$ and $\theta_1^I(\theta_g)$ intersect at a unique point if and only if $p_g > \check{p}_g^{MR1}$.

[1] From Assumption A.2-(i), $\theta_1^I(\theta_g)$ is well-defined if and only if $\theta_g - m(\theta_g, \gamma(\theta_g)) + S > 0$. Since $\theta - m(\theta, \gamma(\theta))$ is increasing in θ and $1 - m(1, \gamma(1)) + S = S > 0$, there exists $\underline{\theta}_g < 1$ such that $\theta_g - m(\theta_g, \gamma(\theta_g)) + S > 0$ if and only if $\theta_g > \underline{\theta}_g$.

[2] To show this, we prove that there exists p_g at which $\tilde{\theta}_1(\underline{\theta}_g) = \theta_1^I(\underline{\theta}_g)$. From the definitions of $\tilde{\theta}_1(\cdot)$ and $\theta_1^I(\cdot)$, $\tilde{\theta}_1(\underline{\theta}_g) = \theta_1^I(\underline{\theta}_g)$ is equivalent to

$$p_g = (\underline{\theta}_g - m(\underline{\theta}_g, \gamma(\underline{\theta}_g)) - S) + m(\underline{\theta}_g, \gamma(\underline{\theta}_g)). \tag{C.16}$$

Since $\underline{\theta}_g < \theta_0^*$ from Assumption A.2-(iv) and $\underline{\theta}_g < 1$, there exists $p_g < 1$ that solves (C.16).

We next prove part (ii). Since $\tilde{\theta}_1(\theta_g)$ shifts up as p_g increases, θ_g determined by (C.11) and (C.12), is increasing in p_g for all $p_g > \underline{p}_g^{MR1}$. Therefore, there exists $\underline{p}_g^{MR1} \geq \check{p}_g^{MR1}$ such that θ_g determined by (C.11) and (C.12) satisfies $m(0, \theta_g) \geq I$ if and only if $p_g \geq \underline{p}_g^{MR1}$. *Q.E.D.*

For any $p_g > \check{p}_g^{MR1}$, let $\theta_g^{MR1}(p_g)$ and $\theta_1^{MR1}(p_g)$ denote the marginal types θ_g and θ_1 determined by (C.11) and (C.12). For expositional convenience, we may abbreviate $\theta_g^{MR1}(p_g)$ and $\theta_1^{MR1}(p_g)$ to θ_g^{MR1} and θ_1^{MR1}, respectively.

MR2 equilibrium: In this equilibrium, there exist $0 < \theta_g < \theta_1 < \theta_{g\o} \leq \theta_2 \leq 1$ such that: types $\theta \in [0, \theta_g]$ sell to the government in $t = 1$ and to the market in $t = 2$ at price $m(0, \theta_g)$; types $\theta \in (\theta_g, \theta_1]$ sell to the market at price $m(\theta_g, \theta_1)$ in both periods; types $\theta \in (\theta_1, \theta_{g\o}]$ sell to the government in $t = 1$ but do not sell in $t = 2$; types $\theta \in (\theta_{g\o}, \theta_2]$ sell at price $m(\theta_{g\o}, \theta_2)$ only in $t = 2$; types $\theta > \theta_2$ do not sell in either period.

As before, $(\theta_g, \theta_1, \theta_{g\o}, \theta_2)$ must satisfy the following conditions:

$$p_g = 2m(\theta_g, \theta_1) - m(0, \theta_g); \tag{C.11}$$

$$\theta_1 = m(0, \theta_g) + S; \tag{C.17}$$

$$p_g = m(\theta_{g\o}, \gamma(\theta_{g\o})); \tag{C.18}$$

$$\theta_2 = \gamma(\theta_{g\o}). \tag{C.19}$$

Similar to the boundary case of the SBS equilibrium, the case in which all firms sell in $t = 1$ (i.e., $\theta_{g\o} = 1$) can be supported if and only if $p_g \geq m(1, \gamma(1)) = 1$. Indeed, the worst off-the-path belief consistent with D1 for a deviator (one who holds out) is $\theta_{g\o} = 1$. Given this belief, no firm deviates if and only if $p_g \geq 1$. Since we restrict $p_g \leq 1$, such a case ($\theta_{g\o} = 1$) can be observed only for $p_g = 1$.

As before, the marginal type θ_g must also satisfy (C.14). Note that (C.14) implies

$$m(\theta_{g\o}, \gamma(\theta_{g\o})) > m(\theta_g, \theta_1) > I.$$

Second, since $\Theta_{g\o} \neq \emptyset$, we must have

$$\theta_1 < \theta_{g\o}. \tag{C.20}$$

Lemma C.4. *There exist* $\underline{p}_g^{MR2}$ *and* $\overline{p}_g^{MR2}$ *such that* (C.11) *and* (C.17) *admit a unique* (θ_g, θ_1) *that satisfies* $0 < \theta_g < \theta_1$ *and* (C.14) *if and only if* $p_g \in [\underline{p}_g^{MR2}, \overline{p}_g^{MR2})$.

Proof. Similar to the proof of Lemma C.3, (C.11) defines θ_1 as a decreasing function of θ_g, labelled

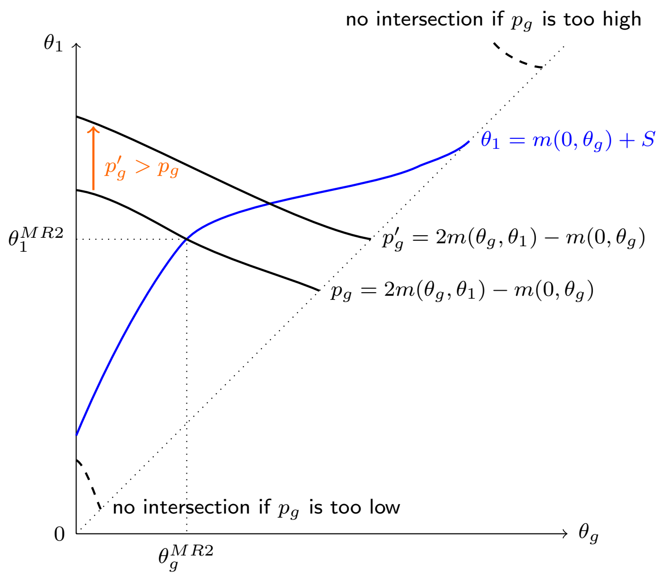


Figure 2 – Characterization of θ_g^{MR2} and θ_1^{MR2}

$\tilde{\theta}_1(\theta_g)$, whenever well-defined. Moreover, (C.17) defines θ_1 as an increasing function of θ_g, labelled $\theta_1^{II}(\theta_g)$, whenever well-defined. Therefore, (θ_g, θ_1) satisfying (C.11) and (C.17), if well-defined, is characterized by a unique point of intersection between $\tilde{\theta}_1(\theta_g)$ and $\theta_1^{II}(\theta_g)$. Moreover, one can find that there exists $p_g \leq 1$ at which $\tilde{\theta}_1(\theta_g)$ and $\theta_1^{II}(\theta_g)$ intersect.[3] Since $\tilde{\theta}_1(\theta_g)$ shifts up as p_g increases, there exists $\check{p}_g^{MR2}$ such that (C.11) and (C.17) admit a unique (θ_g, θ_1) satisfying $\theta_g > 0$ if and only if $p_g > \check{p}_g^{MR2}$. Furthermore, as depicted in Figure 2, θ_g determined by (C.11) and (C.17) is increasing in p_g for all $p_g > \check{p}_g^{MR2}$. Since $\theta_1^{II}(\theta_g) \leq \theta_g$ for all $\theta_g \geq \theta_0^*$, there exists $\overline{p}_g^{MR2} > \check{p}_g^{MR2}$ such that (θ_g, θ_1) determined by (C.11) and (C.17) satisfies $0 < \theta_g < \theta_1$ if and only if $p_g \in (\check{p}_g^{MR2}, \overline{p}_g^{MR2})$. Lastly, there exists $\underline{p}_g^{MR2} > \check{p}_g^{MR2}$ such that θ_g determined by (C.11) and (C.17) satisfies (C.14) if and only if $p_g \geq \underline{p}_g^{MR2}$. *Q.E.D.*

For any $p_g > \check{p}_g^{MR2}$, let $\theta_g^{MR2}(p_g)$, $\theta_1^{MR2}(p_g)$, and $\theta_{g\o}^{MR2}(p_g)$ denote the marginal types θ_g, θ_1, and $\theta_{g\o}$ determined by (C.11), (C.17), and (C.18). For expositional convenience, we may abbreviate $\theta_g^{MR1}(p_g)$, $\theta_1^{MR1}(p_g)$, and $\theta_{g\o}^{MR2}(p_g)$ to θ_g^{MR1}, θ_1^{MR1}, and $\theta_{g\o}^{MR2}$, respectively.

Lemma C.5. *Fix p_g. Suppose both $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$ and $(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g))$ are well-defined at that p_g. If the former violates (C.15), then the latter satisfies (C.20). Conversely, if*

[3] We first show that if MR2 equilibrium exists, then it requires $S < 1$. Suppose to the contrary MR2 equilibrium exists for some $S \geq 1$. Then $\theta_1^{II}(\theta_g) = 1$ for all $\theta_g \geq 0$, so any (θ_g, θ_1) satisfying (C.17) violates (C.20), a contradiction. We next show that there exists p_g at which $\tilde{\theta}_1(0) = \theta_1^{II}(0)$. From the definitions of $\tilde{\theta}_1(\cdot)$ and $\theta_1^{II}(\cdot)$, $\tilde{\theta}_1(0) = \theta_1^{II}(0)$ is equivalent to $p_g = 2m(0, S)$. Since $1 > S > 2m(0, S)$ from Assumption A.2-(v), $\tilde{\theta}_1(0) = \theta_1^{II}(0)$ at $p_g = 2m(0, S) < 1$. Thus, there is a unique point of intersection between $\tilde{\theta}_1(\theta_g)$ and $\theta_1^{II}(\theta_g)$ for any $p_g \in (2m(0, S), 1)$, as is also seen in Figure 2.

the former satisfies (C.20)*, then the latter violates* (C.15)*.*

Proof. We first establish some technical results used in the proof.

Claim 1. *Suppose θ_g and θ_1 satisfy* (C.11) *and* (C.12)*. Then θ_g and θ_1 satisfy* (C.15) *if and only if $\theta_1 \leq m(0, \theta_g) + S$.*

Proof. Suppose there exist θ_g and θ_1 determined by (C.11) and (C.12) that satisfy (C.15). Then we have

$$\begin{aligned}\theta_1 &\leq 2m(\theta_g, \theta_1) - m(\theta_1, \gamma(\theta_1)) + S \\ &= p_g + m(0, \theta_g) - m(\theta_1, \gamma(\theta_1)) + S \\ &= m(0, \theta_g) + S + (p_g - m(\theta_1, \gamma(\theta_1))) \\ &\leq m(0, \theta_g) + S,\end{aligned}$$

where the first inequality follows from (C.12), the second equality follows from (C.11), and the last inequality follows from (C.15). Conversely, one can show that if θ_g and θ_1 satisfy (C.11) and (C.12), then $\theta_1 \leq m(0, \theta_g) + S$ is sufficient for them to satisfy (C.15). *Q.E.D.*

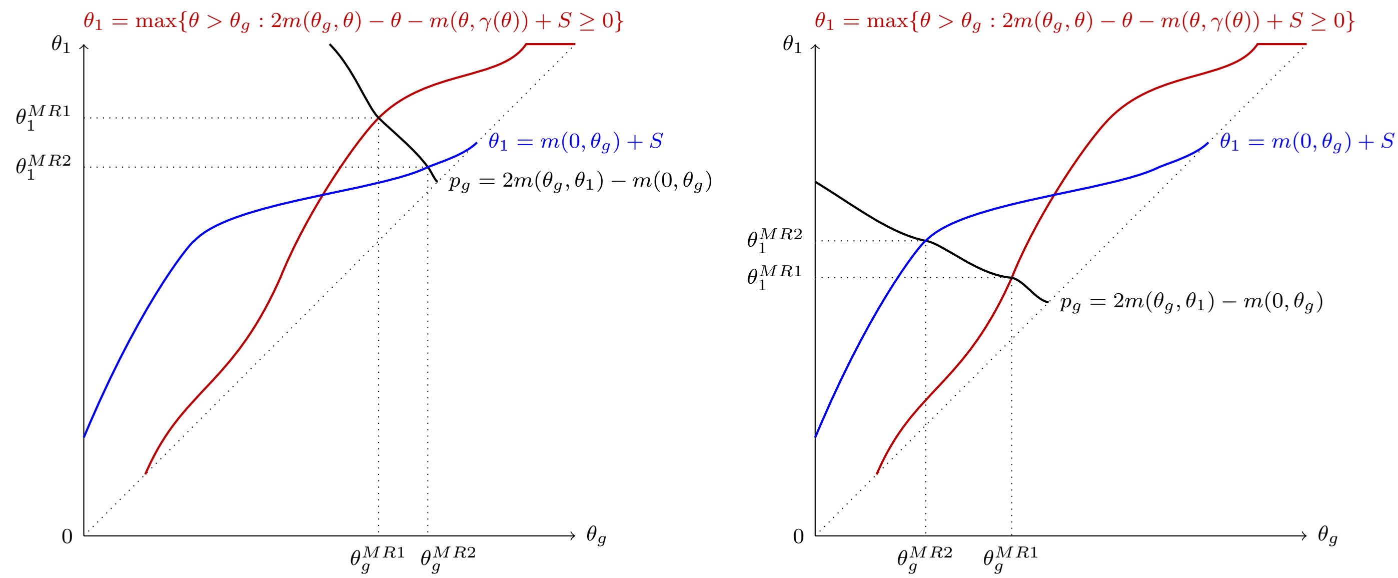
Claim 2. *Suppose θ_g, θ_1, and $\theta_{g\o}$ satisfy* (C.11)*,* (C.17)*, and* (C.18)*. Then θ_1 and $\theta_{g\o}$ satisfy* (C.20) *if and only if $2m(\theta_g, \theta_1) - \theta_1 - m(\theta_1, \gamma(\theta_1)) + S > 0$.*

Proof. Suppose there exist θ_g, θ_1, and $\theta_{g\o}$ that satisfy (C.11) – (C.18) and also (C.20). Then we have

$$\begin{aligned}0 &= m(0, \theta_g) - \theta_1 + S \\ &= 2m(\theta_g, \theta_1) - \theta_1 - p_g + S \\ &= 2m(\theta_g, \theta_1) - \theta_1 - m(\theta_{g\o}, \gamma(\theta_{g\o})) + S \\ &< 2m(\theta_g, \theta_1) - \theta_1 - m(\theta_1, \gamma(\theta_1)) + S,\end{aligned}$$

where the first equality follows from (C.17), the second equality follows from (C.11), the third equality follows from (C.18), and the last inequality follows from (C.20). Conversely, one can also show that if θ_g, θ_1, and $\theta_{g\o}$ satisfy (C.11) – (C.18), then $2m(\theta_g, \theta_1) - \theta_1 - m(\theta_1, \gamma(\theta_1)) + S > 0$ is sufficient for them to satisfy (C.20). *Q.E.D.*

Fix a p_g at which both $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$ and $(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g))$ are well-defined. Recall $\tilde{\theta}_1(\theta_g)$, $\theta_1^I(\theta_g)$, and $\theta_1^{II}(\theta_g)$ from the proofs of Lemma C.3 and Lemma C.4, which are the functions of θ_g corresponding to (C.11), (C.12), and (C.17), respectively.



(a) $\tilde{\theta}_1(\theta_g^{MR1}) > \theta_1^{II}(\theta_g^{MR1})$, or equivalently, $\tilde{\theta}_1(\theta_g^{MR2}) < \theta_1^{I}(\theta_g^{MR2})$

(b) $\tilde{\theta}_1(\theta_g^{MR1}) \leq \theta_1^{II}(\theta_g^{MR1})$, or equivalently, $\tilde{\theta}_1(\theta_g^{MR2}) \geq \theta_1^{I}(\theta_g^{MR2})$

Figure 3 – $p_g \in P^{MR}$ supports only one type of MR equilibria.

If $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$ violates (C.15), then we have $\theta_1^{MR1}(p_g) > m(0, \theta_g^{MR1}(p_g)) + S$ from Claim 1. This implies $\tilde{\theta}_1(\theta_g^{MR1}(p_g)) > \theta_1^{II}(\theta_g^{MR1}(p_g))$. Furthermore, we have shown in Lemma C.4 that $\tilde{\theta}_1(\theta_g)$ is decreasing in θ_g and $\theta_1^{II}(\theta_g)$ is increasing in θ_g if they are well-defined. Thus we have $\theta_g^{MR1}(p_g) < \theta_g^{MR2}(p_g)$ and $\theta_1^{MR1}(p_g) > \theta_1^{MR2}(p_g)$ as illustrated by Figure 3a. Moreover, we have shown in the proof of Lemma C.3 that $\theta_1^{I}(\theta_g)$ is increasing in θ_g if it is well-defined. Hence, we have $\theta_1^{I}(\theta_g^{MR2}(p_g)) > \theta_1^{I}(\theta_g^{MR1}(p_g)) = \theta_g^{MR1}(p_g) > \theta_1^{MR2}(p_g)$, which implies

$$2m(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g)) - \theta_1^{MR2}(p_g) - m(\theta_1^{MR2}(p_g), \gamma(\theta_1^{MR2}(p_g))) + S > 0.$$

Therefore, $(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g))$ satisfies (C.20) by Claim 2.

Conversely, If $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$ satisfies (C.15), we have $\theta_1^{MR1}(p_g) \leq m(0, \theta_g^{MR1}(p_g)) + S$ from Claim 1. This implies $\tilde{\theta}_1(\theta_g^{MR1}(p_g)) \leq \theta_1^{II}(\theta_g^{MR1}(p_g))$. Since $\tilde{\theta}_1(\theta_g)$ is decreasing in θ_g and $\theta_1^{II}(\theta_g)$ is increasing in θ_g if $\tilde{\theta}_1$ and θ_1^{II} are well-defined, we have $\theta_g^{MR1}(p_g) \geq \theta_g^{MR2}(p_g)$ and $\theta_1^{MR1}(p_g) \leq \theta_1^{MR2}(p_g)$ as illustrated by Figure 3b. When $\theta_1^{MR1}(p_g) < 1$, we have $2m(\theta_g^{MR1}, \theta_1^{MR1}) - \theta_1^{MR1} - m(\theta_1^{MR1}, \gamma(\theta_1^{MR1})) + S = 0$. Then, by Assumption A.2-(i), we have

$$2m(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g)) - \theta_1^{MR2}(p_g) - m(\theta_1^{MR2}(p_g), \gamma(\theta_1^{MR2}(p_g))) + S \leq 0,$$

which implies that $(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g))$ violates (C.20) by Claim 2. When $\theta_1^{MR1}(p_g) = 1$, we must have $\theta_1^{MR2}(p_g) = 1$: otherwise, we have $\theta_1^{MR1}(p_g) > m(0, \theta_g^{MR2}(p_g)) + S$. This implies $1 = \theta_1^{MR2}(p_g) \geq \theta_{g\o}^{MR2}(p_g)$. Then $(\theta_1^{MR2}(p_g), \theta_{g\o}^{MR2}(p_g))$ violates (C.20). *Q.E.D.*

The following observation follows immediately from the above.

Corollary C.1. *At most one of MR1 and MR2 exists for any given* p_g*.*

In what follows, we characterize the range of bailout terms that admit either type of MR equilibria, denoted by P^{MR}. Specifically, we show that P^{MR} is a convex set.

Lemma C.6. P^{MR} *is a convex set, whenever it is non-empty.*

Proof.

Step 1. MR equilibrium cannot exist for any $p_g \geq \overline{p}_g^{MR2}$.

Fix $p_g \geq \overline{p}_g^{MR2}$. Clearly, MR2 cannot exist by Lemma C.4. Suppose $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$ exists and satisfies $\theta_1^{MR1}(p_g) \leq m(0, \theta_g^{MR1}(p_g)) + S$, which is equivalent to $\tilde{\theta}_1(\theta_g^{MR1}(p_g)) \leq \theta_1^{II}(\theta_g^{MR1}(p_g))$, where $\tilde{\theta}_1$ and θ_1^{II} are functions of θ_g corresponding to (C.11) and (C.17) respectively, as in the proof of Lemma C.4. Recall also that $\tilde{\theta}_1(\theta_g)$ is decreasing in θ_g and $\theta_1^{II}(\theta_g)$ is increasing in θ_g. These properties imply that $(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g))$ also exists at the same p_g as depicted by Figure 3b, a contradiction to Corollary C.1. Therefore, if $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$ exists, we must have $\theta_1^{MR1}(p_g) > m(0, \theta_g^{MR1}(p_g)) + S$. By Lemma C.5, the last inequality implies $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$ violates (C.15), so MR1 cannot exist, a contradiction.

Step 2. MR equilibrium exists for every $p_g \in (\underline{p}_g^{MR1}, 1] \cap [\underline{p}_g^{MR2}, \overline{p}_g^{MR2})$.

Fix $p_g \in (\underline{p}_g^{MR1}, 1] \cap [\underline{p}_g^{MR2}, \overline{p}_g^{MR2})$. By Lemma C.3 and C.4, both $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$ and $(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g))$ are well-defined. Furthermore, both $\theta_g^{MR1}(p_g)$ and $\theta_g^{MR2}(p_g)$ satisfy (C.14). If $\theta_1^{MR1}(p_g) \leq m(0, \theta_g^{MR1}(p_g)) + S$, then by Claim 1, $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$ satisfies (C.15), and MR1 exists. Otherwise, by Lemma C.5, $(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g))$ satisfies (C.20), and MR2 exists.

Step 3. If $\underline{p}_g^{MR1} < \underline{p}_g^{MR2}$, then MR1 exists for every $p_g \in (\underline{p}_g^{MR1}, \underline{p}_g^{MR2})$.

Fix $p_g \in (\underline{p}_g^{MR1}, \underline{p}_g^{MR2})$. Since $p_g > \underline{p}_g^{MR1}$, it follows from Lemma C.3 that there exists $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$ that satisfies $m(0, \theta_g^{MR1}(p_g)) \geq I$. Suppose $\theta_1^{MR1}(p_g) > m(0, \theta_g^{MR1}(p_g)) + S$, or equivalently, $\tilde{\theta}_1(\theta_g^{MR1}(p_g)) > \theta_1^{II}(\theta_g^{MR1}(p_g))$. Since $p_g < \overline{p}_g^{MR2}$, the inequality $\tilde{\theta}_1(\theta_g^{MR1}(p_g)) >$

$\theta_1^{II}(\theta_g^{MR1}(p_g))$ and Lemma C.4 ensure that $(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g))$ is well-defined. The same inequality also implies $\theta_g^{MR2}(p_g) > \theta_g^{MR1}(p_g)$, as is seen in Figure 3a. However, $\theta_g^{MR2}(p_g) > \theta_g^{MR1}(p_g)$ implies $m(0, \theta_g^{MR2}(p_g)) > m(0, \theta_g^{MR1}(p_g)) \geq I$, which contradicts $p_g < \underline{p}_g^{MR2}$. We thus conclude that $\theta_1^{MR1}(p_g) \leq m(0, \theta_g^{MR1}(p_g)) + S$. By Lemma C.5, $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$ satisfies (C.15), so MR1 exists.

<u>Step 4.</u> Suppose $\underline{p}_g^{MR2} \leq \underline{p}_g^{MR1} < \overline{p}_g^{MR2}$, then MR equilibrium exists for every $p_g \in [\underline{p}_g^{MR2}, \underline{p}_g^{MR1}]$ if and only if

$$2m(\theta_g^{MR2}(\underline{p}_g^{MR2}), \theta_1^{MR2}(\underline{p}_g^{MR2})) - \theta_1^{MR2}(\underline{p}_g^{MR2}) - m(\theta_1^{MR2}(\underline{p}_g^{MR2}), \gamma(\theta_1^{MR2}(\underline{p}_g^{MR2}))) + S \geq 0 \tag{C.21}$$

The proof is tedious and therefore omitted, but it is available from the authors.

<u>Step 5.</u> Suppose $\underline{p}_g^{MR1} \geq \overline{p}_g^{MR2}$. Proceeding similarly as in Step 4 and applying Step 1 and 2, one can find that every $p_g \in [\underline{p}_g^{MR2}, \overline{p}_g^{MR2})$ supports MR equilibrium if (C.21) holds, but no $p_g \in [\underline{p}_g^{MR2}, \overline{p}_g^{MR2})$ supports MR equilibrium otherwise.

Combining all the results above together, we conclude that: $P^{MR} = (\underline{p}_g^{MR1}, \overline{p}_g^{MR2})$ if $\underline{p}_g^{MR1} < \underline{p}_g^{MR2}$; $P^{MR} = [\underline{p}_g^{MR2}, \overline{p}_g^{MR2})$ if $\underline{p}_g^{MR1} \in [\underline{p}_g^{MR2}, \overline{p}_g^{MR2})$ and (C.21) holds; $P^{MR} = (\underline{p}_g^{MR1}, \overline{p}_g^{MR2})$ if $\underline{p}_g^{MR1} \in [\underline{p}_g^{MR2}, \overline{p}_g^{MR2})$ but (C.21) does not hold; $P^{MR} = [\underline{p}_g^{MR2}, \overline{p}_g^{MR2})$ if $\underline{p}_g^{MR1} \geq \overline{p}_g^{MR2}$ and (C.21) holds; $P^{MR} = \emptyset$ if $\underline{p}_g^{MR1} \geq \overline{p}_g^{MR2}$ but (C.21) does not hold. *Q.E.D.*

The following lemma describes an important property of P^{MR}, which will be used in the proof of Theorem 3.

Lemma C.7. *If $P^{MBS} \neq \emptyset$ and $P^{MR} \neq \emptyset$, then $\sup P^{MBS} = \inf P^{MR}$.*

Proof. Recall from Lemma C.2 that $\sup P^{MBS} = \overline{p}_g^{MBS}$. Since the condition (C.10) binds at $p_g = \overline{p}_g^{MBS}$, we have

$$\theta_g^{MBS}(\overline{p}_g^{MBS}) - m(\theta_g^{MBS}(\overline{p}_g^{MBS}), \gamma(\theta_g^{MBS}(\overline{p}_g^{MBS}))) + S = 0. \tag{C.22}$$

It follows that $\theta_g^{MBS}(\overline{p}_g^{MBS}) = \underline{\theta}_g$, where $\underline{\theta}_g$, as defined in the proof of Lemma C.3-(i), is a lower bound of θ_g such that (C.12) is well-defined. Since $\theta_g^{MBS}(p_g)$ satisfies (C.7), (C.22) also implies

$$\overline{p}_g^{MBS} = 2\theta_g^{MBS}(\overline{p}_g^{MBS}) - m(0, \theta_g^{MBS}(\overline{p}_g^{MBS})) = 2m(\theta_g^{MBS}(\overline{p}_g^{MBS}), \theta_g^{MBS}(\overline{p}_g^{MBS})) - m(0, \theta_g^{MBS}(\overline{p}_g^{MBS})). \tag{C.23}$$

Since $\theta_g^{MBS}(\overline{p}_g^{MBS}) = \underline{\theta}_g$, (C.23) implies that $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$, the unique intersection between two curves $\tilde{\theta}_1(\theta_g)$ and $\theta_1^I(\theta_g)$, exists if and only if $p_g > \overline{p}_g^{MBS}$, and thus $\overline{p}_g^{MBS} = \check{p}_g^{MR1}$.

Since $\theta_1^I(\theta_g)$, if well-defined, is increasing in θ_g, $\theta_g^{MR1}(p_g) - m(\theta_g^{MR1}(p_g), \gamma(\theta_g^{MR1}(p_g))) + S > 0$ for all $p_g > \check{p}_g^{MR1}$. Since $\theta - m(\theta, \gamma(\theta))$ is increasing in θ, we have $\theta_g^{MR1}(p_g) > \theta_g^{MBS}(\overline{p}_g^{MBS})$ for all $p_g > \overline{p}_g^{MBS} = \check{p}_g^{MR1}$ from (C.22). By (C.8), we have $m(0, \theta_g^{MR1}(p_g)) > I$ for all $p_g > \check{p}_g^{MR1}$, which implies $\overline{p}_g^{MBS} = \underline{p}_g^{MR1}$ by Lemma C.3. Furthermore, that $\overline{p}_g^{MBS} = \check{p}_g^{MR1}$ implies that even if $(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g))$ exists for some $p_g \leq \overline{p}_g^{MBS}$, we have $\theta_g^{MR2}(p_g) - m(\theta_g^{MR2}(p_g), \gamma(\theta_g^{MR2}(p_g))) + S \leq 0$. Thus

$$2m(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g)) - \theta_1^{MR2}(p_g) - m(\theta_1^{MR2}(p_g), \gamma(\theta_1^{MR2}(p_g))) + S < 0,$$

where the strict inequality follows from Assumption A.2-(i). Hence, $(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g))$ violates (C.20) by Claim 2. Therefore, we have $\inf P^{MR} = \underline{p}_g^{MR1}$, and thus $\overline{p}_g^{MBS} = \underline{p}_g^{MR1} = \inf P^{MR}$. *Q.E.D.*

C.1.5 Necessary Conditions for Equilibria under the Market Shutdown in $t = 1$

In equilibrium, there exist $\theta_g \in (0, \theta_2)$ and $\theta_2 \leq 1$ such that types $\theta \in [0, \theta_g]$ sell to the government at price p_g in $t = 1$ and sell to the market at price $m(0, \theta_g \wedge \theta_0^*)$ in $t = 2$ if $m(0, \theta_g) \geq I$; and types $\theta \in (\theta_g, \theta_2)$ sell only in $t = 2$ at price $m(\theta_g, \theta_2)$.

As before, we derive conditions for such an equilibrium to exist. There are three types of equilibria: *(i)* $m(0, \theta_g) < I$; *(ii)* $m(0, \theta_g) \geq I$ and $\theta_g \leq \theta_0^*$; *(iii)* $\theta_g > \theta_0^*$.

In case *(i)*, types $\theta \in [0, \theta_g]$ cannot sell in $t = 2$. Thus, as in SBS, we must have:

$$\begin{aligned} \theta_2 &= \gamma(\theta_g), \\ p_g &= m(\theta_g, \gamma(\theta_g)), \\ \text{s.t. } & m(0, \theta_g) < I, \end{aligned}$$

which are same the as (C.1) – (C.3). However, the conditions (C.4) and (C.5) are no longer necessary since $t = 1$ market is shut down. With the D1 refinement, the case $\theta_g = 1$ arises if and only if $p_g \geq m(1, \gamma(1)) = 1$ and $\mathrm{E}[\theta] = m(0,1) < I$.

In case *(ii)*, all types $\theta \in [0, \theta_g]$ sell in $t = 2$ at price $m(0, \theta_g)$. Since this equilibrium has the same structure as MBS equilibrium, θ_g and θ_2 satisfy

$$\theta_2 = \gamma(\theta_g),$$
$$p_g + m(0, \theta_g) + S = \theta_g + m(\theta_g, \gamma(\theta_g)),$$

subject to

$$m(0, \theta_g) \geq I,$$
$$\theta_g \leq \theta_0^*,$$

which are the same as (C.6) – (C.8) and (C.10). Given that the $t = 1$ market is shut down, the condition (C.9) is no longer necessary. With the D1 refinement, the case $\theta_g = 1$ can be supported if and only if $p_g \geq 1 + m(1, \gamma(1)) - m(0,1) - S = 2 - \mathrm{E}[\theta] - S$ and $\theta_0^* = 1$.

Lastly, consider case *(iii)*, where types $\theta \in [0, \theta_g]$ sell to the government in $t = 1$, but only types $\theta \in [0, \theta_0^*]$ sell at price $m(0, \theta_0^*)$ in $t = 2$. In this equilibrium, θ_g and θ_2 satisfy

$$\theta_2 = \gamma(\theta_g),$$
$$p_g = m(\theta_g, \gamma(\theta_g)),$$
$$\text{s.t. } \theta_g > \theta_0^*.$$

With the D1 refinement, the case $\theta_g = 1$ can be supported if and only if $p_g \geq m(1, \gamma(1)) = 1$.

Using the conditions above and proceeding similarly as in the proof of Theorem 3-(ii) below, one can show that the above conditions on θ_g and θ_2 are also sufficient to support each type of the equilibria. For later use, we let $\theta_g^{sd}(p_g)$ denote the marginal types satisfying the conditions for each alternative type of equilibria. To avoid expositional complexity, $\theta_g^{sd}(p_g)$ can be occasionally abbreviated to θ_g^{sd}. Note that $\theta_g^{sd}(p_g)$ is continuous and increasing in p_g if $\theta_g^{sd}(p_g)$ is well-defined at such p_g.

C.2 Proofs of Theorem 3 and Proposition 2 – 4

Theorem 3. *There exists an interval of bailout terms $P^k \subset \mathbb{R}_+$ that supports alternative equilibrium types $k = NR, SBS, MBS, MR$, described as follows:*

(i) ***No Response (NR)****: If $p_g \in P^{NR}$, then there exists an equilibrium in which no firm accepts the government offer and the outcome in Theorem 2 prevails.*

(ii) ***No Market Rejuvenation***

- ***Severe Bailout Stigma (SBS)****: If $p_g \in P^{SBS}$, then there exists an equilibrium with $\Theta_g = \Theta_1 = \emptyset, \Theta_{g\o}, \Theta_2 \neq \emptyset$.*
- ***Moderate Bailout Stigma (MBS)****: If $p_g \in P^{MBS}$, then there exists an equilibrium with $\Theta_g, \Theta_2 \neq \emptyset, \Theta_1 = \Theta_{g\o} = \emptyset$.*

(iii) ***Market Rejuvenation (MR)****: If $p_g \in P^{MR}$, then there exists an equilibrium with $\Theta_1 \neq \emptyset$.*

Specifically, $P^{NR} = [0, p_2^*]$, $\inf P^{SBS} = p_0^*$, and $\sup P^{MBS} \leq \inf P^{MR}$, meaning an MR equilibrium requires a strictly higher p_g than does an MBS equilibrium.

Proof of Theorem 3. We first state a lemma that will be used in the proof.

Lemma C.8. *Suppose buyers in $t = 2$ believe that types $\theta \in [a, b]$ offer assets for sale. Then buyers offer the price $m(a, \gamma(a) \wedge b)$. If $m(a, \gamma(a) \wedge b) \geq I$, then types $\theta \in [a, \gamma(a) \wedge b]$ sell their assets. If $m(a, \gamma(a) \wedge b) < I$, then the $t = 2$ market fully freezes.*

Proof. See Proposition 1 of Tirole (2012). *Q.E.D.*

Proof of Theorem 3-(i).

Suppose $p_g \leq p_2^*$. Recall p_2^* is defined in Theorem 2-(i). Consider the strategies specified in Theorem 3-(i). The beliefs on the equilibrium path are given by the Bayes' rule. The off-the-path belief for firms which accept the bailout in $t = 1$ is $\theta = 0$, and such a belief satisfies D1. Given these beliefs, type θ's payoff from accepting the bailout is $p_g + S + \theta$. If $\theta \in [0, \theta_1^*]$, then we have $2p_1^* + 2S \geq \theta + p_2^* + S \geq p_g + S + \theta$. If $\theta \in (\theta_1^*, 1]$, then we have $\theta + \max\{\theta, p_2^* + S\} \geq p_g + S + \theta$. In either case, it is not profitable to accept the bailout. Hence the NR equilibrium exists for all $p_g \in [0, p_2^*]$.

Proof of Theorem 3-(ii).

Consider first the SBS equilibrium. As we saw in Section C.1.2, given $p_g \in P^{SBS}$, there exist $\theta_{g\o} = \theta_{g\o}^{SBS}(p_g)$ and $\theta_2 = \gamma(\theta_{g\o}^{SBS}(p_g))$ that satisfy (C.3) – (C.5).

We show that the prescribed equilibrium strategies are optimal for all types of firms. Consider $t = 2$. For bailout recipients, the $t = 2$ market fully freezes since $m(0, \theta_{g\o}) < I$ from (C.3). For $t = 1$ holdouts, the $t = 2$ market offer is $m(\theta_{g\o}, \gamma(\theta_{g\o}))$. Given this, types

$\theta \in (\theta_{g\varnothing}, \gamma(\theta_{g\varnothing})]$ sell in $t = 2$, but types $\theta > \gamma(\theta_{g\varnothing})$ do not since $\theta \leq m(\theta_{g\varnothing}, \gamma(\theta_{g\varnothing})) + S \iff \theta \leq \gamma(\theta_{g\varnothing})$. Consider now $t = 1$. Type-θ firm receives payoff $p_g + S + \theta$ from accepting the bailout and $\theta + \max\{m(\theta_{g\varnothing}, \gamma(\theta_{g\varnothing})) + S, \theta\}$ from rejecting it. Since $p_g = m(\theta_{g\varnothing}, \gamma(\theta_{g\varnothing}))$ from (C.2), playing the prescribed equilibrium strategy in $t = 1$ is optimal for every type $\theta \in [0, 1]$. Given the equilibrium strategies chosen by firms, it is straightforward to see that the equilibrium price offers are also optimal and buyers break even.

Next consider the MBS equilibrium. As we saw in Section C.1.3, for every $p_g \in P^{MBS}$, the marginal types $\theta_g = \theta_g^{MBS}(p_g)$ and $\theta_2 = \gamma(\theta_g^{MBS}(p_g))$ satisfy (C.8) – (C.10). Using these conditions and proceeding similarly as in the SBS equilibrium, it is easy to show that the prescribed equilibrium strategies are optimal for all types of firms, and the equilibrium price offers are optimal for buyers in the market. What remains to show is that it is optimal for types $\theta \in [0, \theta_g]$ to sell at price $m(0, \theta_g)$ in $t = 2$ after accepting the bailout, and for buyers in $t = 2$ to offer the price $m(0, \theta_g)$ to the bailout recipients. But these follow immediately from (C.8), (C.10), and Lemma C.8.

Proof of Theorem 3-(iii).

Consider the MR1 equilibrium first. Consider $p_g \in P^{MR}$ such that there exist $\theta_g = \theta_g^{MR1}(p_g)$, $\theta_1 = \theta_1^{MR1}(p_g)$, and $\theta_2 = \gamma(\theta_1^{MR1}(p_g))$ that satisfy (C.14) and (C.15).

We first show that it is optimal for each type of firms to play the prescribed equilibrium strategies. Consider $t = 2$ on the equilibrium path. Since $m(0, \theta_g) \geq I$ from (C.14), $\theta_g < \theta_1 \leq m(0, \theta_g) + S < m(\theta_g, \theta_1) + S$ from (C.15) and Claim 1, and $\gamma(\theta_1) \leq m(\theta_1, \gamma(\theta_1)) + S$ from Definition A.1-(ii), it is optimal for types $\theta \in [0, \theta_g]$ to sell at price $m(0, \theta_g)$, types $\theta \in (\theta_g, \theta_1]$ to sell at price $m(\theta_g, \theta_1)$, and types $\theta \in (\theta_1, \gamma(\theta_1)]$ to sell at price $m(\theta_1, \gamma(\theta_1))$. However, types $\theta \in (\gamma(\theta_1), 1]$ do not sell since $\theta > m(\theta_1, \gamma(\theta_1)) + S$ for all $\theta > \gamma(\theta_1)$ from Lemma A.1-(i). Next consider $t = 1$. Accepting the bailout is optimal for types $\theta \in [0, \theta_g]$ since

$$p_g + m(0, \theta_g) + 2S = 2m(\theta_g, \theta_1) + 2S \geq \theta + m(\theta_1, \gamma(\theta_1)) + S, \tag{C.24}$$

where the first equality follows from (C.11) and the second inequality is from (C.12). From (C.24), it is also optimal for types $\theta \in (\theta_g, \theta_1]$ to sell at price $m(\theta_g, \theta_1)$. Lastly, since $2m(\theta_g, \theta_1) + 2S < \theta + \max\{\theta, m(\theta_1, \gamma(\theta_1)) + S\}$ for all $\theta > \theta_1$, it is optimal for types $\theta \in (\theta_1, 1]$ not to sell in $t = 1$.

Next, we show that the equilibrium price offers are optimal for buyers in each period. The optimality of $t = 2$ prices directly follows from Lemma C.8, so we consider only $t = 1$. We show below that no buyer benefits by deviating from offering $m(\theta_g, \theta_1)$. In $t = 1$, buyers believe that only types $\theta > \theta_g$ are available for asset sales to the market. Suppose a buyer deviates and offers

$p' \neq m(\theta_g, \theta_1)$. Any firm that accepts this offer is assigned the off-the-path belief that it is the worst available type, i.e., $\theta = \theta_g$. This belief is consistent with D1. Given this belief, any such firm will be offered price $p_2 = \theta_g$ in $t = 2$. Since $\theta_g > m(0, \theta_g) \geq I$, the deviating firm enjoys the total payoff $p' + S + \max\{\theta, \theta_g + S\}$.

There are two possibilities, either $2m(\theta_g, \theta_1) - \theta_g \geq m(\theta_1, \gamma(\theta_1))$ or $2m(\theta_g, \theta_1) - \theta_g < m(\theta_1, \gamma(\theta_1))$. Consider the former case. If $p' \leq m(\theta_1, \gamma(\theta_1)) \leq 2m(\theta_g, \theta_1) - \theta_g$, we have $2m(\theta_g, \theta_1) + 2S \geq \theta_1 + p' + S$, $2m(\theta_g, \theta_1) + 2S \geq p' + \theta_g + 2S$, and $p' + S + \theta \leq \theta + m(\theta_1, \gamma(\theta_1)) + S$. Therefore, no type $\theta \in (\theta_g, 1]$ will sell at such p'. If $p' > m(\theta_1, \gamma(\theta_1))$, then we have $2m(\theta_g, \theta_1) + 2S < \theta_1 + p' + S$. Letting $\tilde{\theta} := 2m(\theta_g, \theta_1) + S - p' < \theta_1$, all types $\theta \in (\tilde{\theta}, (p' + S) \wedge 1]$ sell at p'. However, by definition of $\gamma(\cdot)$, we have $m(\tilde{\theta}, (p' + S) \wedge 1) - p' < m(\theta_1, (p' + S) \wedge 1) - p' < 0$, and thus the deviating buyer will make a loss by offering $p' \neq m(\theta_g, \theta_1)$.

Consider the latter case next. If $p' \in (2m(\theta_g, \theta_1) - \theta_g, m(\theta_1, \gamma(\theta_1))]$, then types $\theta \leq (\theta_g, \theta' \wedge 1]$ will sell at p', where $\theta' := p' - m(\theta_1, \gamma(\theta_1)) + \theta_g + S$. Since $\lim_{p' \to 2m(\theta_g, \theta_1) - \theta_g} \theta' = \theta_1$, we have

$$\lim_{p' \to 2m(\theta_g, \theta_1) - \theta_g} (m(\theta_g, \theta' \wedge 1) - p') = m(\theta_g, \theta_1) - (m(\theta_g, \theta_1) + (m(\theta_g, \theta_1) - \theta_g)) < 0.$$

Moreover, since $\frac{d\theta'}{dp'} = 1$ and $\frac{\partial}{\partial b} m(a, b) < 1$ for any $0 \leq a \leq b \leq 1$ from Lemma A.1-(i), we have $\frac{d}{dp'}(m(\theta_g, \theta' \wedge 1) - p') \leq 0$ for all $p' \in (2m(\theta_g, \theta_1) - \theta_g, m(\theta_1, \gamma(\theta_1))]$. As a result, it is not profitable for the buyers to offer any $p' \in (2m(\theta_g, \theta_1) - \theta_g, m(\theta_1, \gamma(\theta_1))]$. If $p' > m(\theta_1, \gamma(\theta_1))$, then all types $\theta \in (\theta_g, (p' + S) \wedge 1]$ will sell at p'. Since $p' > m(\theta_1, \gamma(\theta_1)) > m(\theta_g, \gamma(\theta_g))$, we have $m(\theta_g, (p' + S) \wedge 1) - p' < 0$ for all $p' > m(\theta_1, \gamma(\theta_1))$. Hence, the deviating buyer will make a loss by offering p'. If $p' \leq 2m(\theta_g, \theta_1) - \theta_g$, then, as shown before, types $\theta > \theta_g$ will not sell at such p'. Consequently, all buyers in $t = 1$ optimally offer $m(\theta_g, \theta_1)$.

We now turn to the MR2 equilibrium. Suppose MR2 exists at $p_g \in P^{MR}$. Then, by Lemma C.6, there exist $\theta_g = \theta_g^{MR2}(p_g), \theta_1 = \theta_1^{MR1}(p_g), \theta_{g\o} = \theta_{g\o}^{MR2}(p_g)$, and $\theta_2 = \gamma(\theta_{g\o}^{MR2}(p_g))$ satisfying (C.14) and (C.20).

First, we show that it is optimal for each type of firms to play the prescribed equilibrium strategies. Consider $t = 2$ first. Since $\theta_1 = m(0, \theta_g) + S$ from (C.17), we have $\theta_g < \theta_1 = m(0, \theta_g) + S$, $\theta_1 = m(0, \theta_g) + S < m(\theta_g, \theta_1) + S$, and $\theta > m(0, \theta_g) + S$ for all $\theta > \theta_1$. Therefore it is optimal for types $\theta \in [0, \theta_g]$ to sell at price $m(0, \theta_g)$, but types $\theta \in (\theta_1, \theta_{g\o}]$ not to sell at that price. Furthermore, it is optimal for types $\theta \in (\theta_g, \theta_1]$ to sell at price $m(\theta_g, \theta_1)$. Finally, from the definition of $\gamma(\cdot)$, it is optimal for types $\theta \in (\theta_{g\o}, \gamma(\theta_{g\o})]$ to sell at price $m(\theta_{g\o}, \gamma(\theta_{g\o}))$, but types $\theta \in (\gamma(\theta_{g\o}), 1]$ not to sell at that price. Consider next $t = 1$. From (C.11), (C.17), and (C.18), we have

$$p_g + m(0, \theta_g) + 2S = 2m(\theta_g, \theta_1) + 2S \geq \theta + p_g + S = \theta + m(\theta_{g\emptyset}, \gamma(\theta_{g\emptyset})) + S,$$

where the inequality holds strictly if and only if $\theta \in [0, \theta_1)$. This inequality implies the prescribed equilibrium strategies are optimal for all types $\theta \in [0, \theta_1]$. For all $\theta > \theta_1$, the above inequality is reversed. Thus it is optimal for types $\theta \in (\theta_1, \theta_{g\emptyset}]$ to accept the bailout and for types $\theta \in (\theta_{g\emptyset}, 1]$ not to sell in $t = 1$.

Next, we show that the equilibrium price offers are also optimal for buyers in the market. Consider the $t = 2$ market first. On the equilibrium path, buyers believe that types $\theta \in [0, \theta_g] \cup (\theta_1, \theta_{g\emptyset}]$ accept the bailout in $t = 1$, types $\theta \in (\theta_g, \theta_1]$ sell to the $t = 1$ market, and types $\theta \in (\theta_{g\emptyset}, 1]$ do not sell in $t = 1$. By Lemma C.8, it is optimal for the $t = 2$ buyers to offer $m(\theta_g, \theta_1)$ to the second types and $m(\theta_{g\emptyset}, \gamma(\theta_{g\emptyset}))$ to the the third types. It remains to show that it is optimal for the $t = 2$ buyers to offer $m(0, \theta_g)$ to types $\theta \in [0, \theta_g] \cup (\theta_1, \theta_{g\emptyset}]$. Suppose a buyer deviates and offers $p' \neq m(0, \theta_g)$. By Lemma C.8, it can be shown that $p' < m(0, \theta_g)$ cannot be an equilibrium strategy. Suppose now $p' > m(0, \theta_g)$. If $p' \leq \theta_1 - S$, then p' can attract types $\theta \in [0, \theta_g]$ at most, resulting in a loss to the deviating buyer. If $p' > \theta_1 - S$, then types $\theta \in [0, \theta_g] \cup (\theta_1, \theta']$ sell assets at p', where $\theta' := (p' + S) \wedge \theta_{g\emptyset}$. Thus the deviating buyer's payoff is $\mathring{m}(0, \theta_g, \theta_1, \theta') - p'$ (see Definition A.1-(iii) for the definition of $\mathring{m}(a, b, c, d)$). From the definition of θ', we have $\lim_{p' \to m(0,\theta_g)} \theta' = \theta_1$. Since $\frac{\partial}{\partial \theta'} \mathring{m}(0, \theta_g, \theta_1, \theta') < 1$ from Lemma A.1-(iii), we have $\mathring{m}(0, \theta_g, \theta_1, \theta') - p' < 0$ for any $p' > m(0, \theta_g)$. Thus any deviation $p' \neq m(0, \theta_g)$ results in a loss to the deviating buyer, and thus the offer $m(0, \theta_g)$ to the firms accepting the bailout in the previous period is optimal for buyers.

To complete the proof, we need to show that it is optimal for buyers to offer $m(\theta_g, \theta_1)$ in $t = 1$. Suppose a buyer deviates to $p' \neq m(\theta_g, \theta_1)$. The off-the-path belief assigned to the firms accepting p' (consistent with the D1 refinement) is that they are the type $\theta = \theta_g$ with probability 1. Since $\theta_g > m(0, \theta_g) \geq I$ from (C.14), these firms can sell in $t = 2$ at price $\theta_g > I$ after selling at the deviation price p' in $t = 1$, and thus get the payoff $p' + S + \max\{\theta, \theta_g + S\}$.

If $p' > m(\theta_{g\emptyset}, \gamma(\theta_{g\emptyset}))$, then all types $\theta \in (\theta_{g\emptyset}, (p' + S) \wedge 1]$ will sell at price p' in $t = 1$ since $\theta + p' + S > \theta + m(\theta_{g\emptyset}, \gamma(\theta_{g\emptyset})) + S$. Moreover, some types $\theta \in (\theta_g, \theta_1]$ may also sell at p'. Since $(2m(\theta_g, \theta_1) + 2S) - (\theta + p' + S)$ is decreasing in θ, there exists a $\tilde{\theta}$ such that types $\theta \in (\tilde{\theta}, \theta_1]$ will sell at p'. The deviating buyer's payoff is then $\mathring{m}(\tilde{\theta}, \theta_1, \theta_{g\emptyset}, (p' + S) \wedge 1) - p'$. However, we have $\mathring{m}(\tilde{\theta}, \theta_1, \theta_{g\emptyset}, (p' + S) \wedge 1) - p' < m(\theta_{g\emptyset}, (p' + S) \wedge 1) - p' < 0$, where the last inequality follows from the definition of $\gamma(\cdot)$ and the condition $p' > m(\theta_{g\emptyset}, \gamma(\theta_{g\emptyset}))$. If $p' \in (2m(\theta_g, \theta_1) - \theta_g, m(\theta_{g\emptyset}, \gamma(\theta_{g\emptyset}))]$ (and if such an interval is not empty), we have $p' + \theta_g + 2S > 2m(\theta_g, \theta_1) + 2S$. This inequality implies that types $\theta \leq (\theta_g, \theta_1] \cup (\theta_{g\o}, \theta']$ sell at p' in $t = 1$, where $\theta' := p' + \theta_g + S - m(\theta_{g\o}, \gamma(\theta_{g\o}))$. Thus the deviating buyer's payoff is $\mathring{m}(\theta_g, \theta_1, \theta_{g\o}, \theta') - p'$. However, since $\lim_{p' \to 2m(\theta_g, \theta_1) - \theta_g} \theta' = \theta_1$, $\frac{d\theta'}{dp'} = 1$, and $\frac{\partial}{\partial \theta'} \mathring{m}(\theta_g, \theta_1, \theta_{g\o}, \theta') < 1$, we have $\mathring{m}(\theta_g, \theta_1, \theta_{g\o}, \theta') - p' < 0$ for any $p' \in (2m(\theta_g, \theta_1) - \theta_g, m(\theta_{g\o}, \gamma(\theta_{g\o}))]$, resulting in a loss to the deviating buyer. If $p' \leq 2m(\theta_g, \theta_1) - \theta_g$, then we have $p' + \theta_g + 2S \leq 2m(\theta_g, \theta_1) + 2S$, so p' will not be accepted by any type. Put together, we conclude that it is optimal for the $t = 1$ market buyers to offer $m(\theta_g, \theta_1)$.

The proof of $\sup P^{MBS} \leq \inf P^{MR}$ for the case $P^{MR}, P^{MBS} \neq \emptyset$ is referred to Lemma C.7. Finally, it follows immediately from Corollary C.1 that any $p_g \in P^{MR}$ can support at most one type of MR equilibrium. *Q.E.D.*

Proposition 2.

(i) (Dampened initial responses) Fix $p_g \geq \max\{p_0^*, I\}$. In any equilibrium, the trade volume in $t = 1$ is (weakly) smaller than the trade volume $F(p_g + S)$ in the one-shot model.

(ii) (Positive net gains) The total trade volume is higher with a bailout than without, if either MBS, MR1, or MR2 equilibrium would prevail under a bailout. The same holds even when an SBS equilibrium arises from a bailout if the $t = 1$ market fully freezes without a bailout.

(iii) (Delayed benefits) The $t = 2$ trade volume is higher with a bailout than without, if either MBS or MR1 equilibrium would prevail under a bailout.

(iv) (Discontinuous effects) Let $\Phi(p_g)$ denote the set of total trade volumes that would result from some equilibrium given bailout $p_g \in [I, 1]$. The correspondence $\Phi(\cdot)$ does not admit a selection that is continuous in p_g.

Proof of Proposition 2.

Proof of Proposition 2-(i).

First consider the SBS equilibrium. Since $\theta_{g\o}^{SBS} < \gamma(\theta_{g\o}^{SBS}) \leq m(\theta_{g\o}^{SBS}, \gamma(\theta_{g\o}^{SBS})) + S = p_g + S$ from (C.2), we have $\theta_{g\o}^{SBS} < (p_g + S) \wedge 1$. The statement thus follows. Next consider the MBS equilibrium. Since $\theta_g^{MBS} > 0$, we have $m(0, \theta_g^{MBS}) < m(\theta_g^{MBS}, \gamma(\theta_g^{MBS}))$, which implies $\theta_g^{MBS} < (p_g + S) \wedge 1$ by (C.7). Third, consider the MR1 equilibrium. By Claim 1 within the proof of Lemma C.5, we have $\theta_1^{MR1} \leq m(0, \theta_g^{MR1}) + S$. Moreover, it follows from (C.11) that $p_g = m(\theta_g^{MR1}, \theta_1^{MR1}) + (m(\theta_g^{MR1}, \theta_1^{MR1}) - m(0, \theta_g^{MR1})) > m(\theta_g^{MR1}, \theta_1^{MR1}) > m(0, \theta_g^{MR1})$. Therefore,

$\theta_1^{MR1} \le m(0, \theta_g^{MR1}) + S < (p_g + S) \wedge 1$, as was to be shown. Lastly, in the MR2 equilibrium, $\theta_{g\o}^{MR2} < (p_g + S) \wedge 1$ since $\theta_{g\o}^{MR2} < m(\theta_{g\o}^{MR2}, \gamma(\theta_{g\o}^{MR2})) + S$.

Proof of Proposition 2-(ii).

Recall from Theorem 2 that θ_1^* is the marginal type selling in $t = 1$ in the equilibrium without bailout.

First, consider the MBS equilibrium. By (C.9), the MBS equilibrium can exist only if $\theta - m(\theta, \gamma(\theta)) + S < 0$ for some $\theta > 0$, thereby implying $0 - m(0, \gamma(0)) + S < 0$ since $\theta - m(\theta, \gamma(\theta)) + S$ is increasing in θ. Thus we have $2m(0, \theta) - \theta - m(\theta, \gamma(\theta)) + S < \theta - m(\theta, \gamma(\theta)) + S < 0$ for all $\theta > 0$, hence $\theta_1^* = 0$ from Assumption A.2-(i) (i.e., $t = 1$ freezes absent bailout). Thus the total volume of trade in the absence of bailout is $F(\gamma(0)) = F(\theta_0^*)$ (i.e., the trade occurs only in $t = 2$). Since $F(\theta_g^{MBS}) + F(\gamma(\theta_g^{MBS})) > F(0) + F(\gamma(0)) = F(\theta_0^*)$, the MBS equilibrium, if it exists, yields larger total trade than without bailout.

Second, consider the MR1 equilibrium which yields the total trade volume$F(\theta_1^{MR1}) + F(\gamma(\theta_1^{MR1}))$. Since θ_1^{MR1} is determined by (C.12), we have $\Delta(\theta_g^{MR1}, \theta_1^{MR1}; S) = m(\theta_g^{MR1}, \theta_1^{MR1}) - \theta_1^{MR1} + (m(\theta_g^{MR1}, \theta_1^{MR1}) - m(\theta_1^{MR1}, \gamma(\theta_1^{MR1}))) + S \ge 0$. Furthermore, θ_1^* must satisfy $\Delta(0, \theta_1^*; S) \ge 0$ since $\theta_1^* = \max\{\theta > 0 : \Delta(0, \theta; S) = 2m(0, \theta) - \theta - m(\theta, \gamma(\theta)) + S \ge 0\}$. Since $\theta_g^{MR1} > 0$, we have $\Delta(0, \theta; S) < \Delta(\theta_g^{MR1}, \theta; S)$, so we have $\theta_1^{MR1} \ge \theta_1^*$, where the equality holds for the case $\theta_1^* = 1$. Therefore, $F(\theta_1^{MR1}) + F(\gamma(\theta_1^{MR1})) \ge F(\theta_1^*) + F(\gamma(\theta_1^*))$, as was to be shown, where the equality holds for the case $\theta_1^* = 1$.

Next, consider the MR2 equilibrium, which yields the total trade volume $F(\theta_1^{MR2}) + F(\gamma(\theta_{g\o}^{MR2}))$. If $\theta_1^* = 0$, then the MR2 equilibrium yields larger total trade than without bailout since $\theta_1^* = 0 < \theta_1^{MR2}$ and $\gamma(\theta_1^*) = \gamma(0) < \gamma(\theta_{g\o}^{MR2})$. Suppose $\theta_1^* > 0$. If $S \ge 1$, then it follows from (C.17) that $\theta_1^{MR2} = 1$, implying $\Theta_{g\o} = \emptyset$. Thus, $S < 1$ for the MR2 equilibrium to exist. From Assumption A.2-(v), we have

$$
\begin{aligned}
& m(0, \theta) \le \theta - m(0, \theta) \text{ for all } \theta \in [0, 1], \\
\Longrightarrow\ & m(0, S) \le S - m(0, S), \\
\Longrightarrow\ & 2m(0, S) \le m(S, \gamma(S)), \\
\Longrightarrow\ & 2m(0, S) - S - m(S, \gamma(S)) + S \le 0, \\
\Longrightarrow\ & \theta_1^* \le S.
\end{aligned}
$$

Therefore, we have $m(0, \theta) + S > \theta_1^*$ for all $\theta \in (0, \theta_0^*)$. By (C.17) and (C.20), we have $\theta_{g\o}^{MR2} > \theta_1^{MR2} > \theta_1^*$, and thus $F(\theta_1^{MR2}) + F(\gamma(\theta_{g\o}^{MR2})) > F(\theta_1^*) + F(\gamma(\theta_1^*))$.

Lastly, consider the SBS equilibrium, which yields the total trade volume $F(\gamma(\theta_{g\o}^{SBS}(p_g)))$. If $\theta_1^* = 0$, then the total trade volume without bailout is $F(\gamma(0))$. Since $\theta_{g\o}^{SBS}(p_g) > 0$ for all $p_g \in P^{SBS}$, we have $F(\gamma(\theta_{g\o}^{SBS}(p_g))) > F(\gamma(0))$.

Proof of Proposition 2-(iii).

First, consider the MBS equilibrium, which yields the trade volume $F(\gamma(\theta_g^{MBS}))$ in $t = 2$. As shown in the proof of Proposition 2-(ii), the MBS equilibrium exists only if $\theta_1^* = 0$, and thus the trade volume in $t = 2$ in the absence of bailout is $F(\gamma(0)) = F(\theta_0^*)$. This implies $F(\gamma(\theta_g^{MBS})) > F(\gamma(0)) = F(\theta_0^*)$.

Next, consider the MR1 equilibrium, which yields the trade volume $F(\gamma(\theta_1^{MR1}))$ in $t = 2$. Similar to the proof of Proposition 2-(ii), we have $\theta_1^{MR1} \geq \theta_1^*$ where the equality holds for the case $\theta_1^* = 1$. This inequality implies $F(\gamma(\theta_1^{MR1})) \geq F(\gamma(\theta_1^*))$ with the equality when $\theta_1^* = 1$.

Proof of Proposition 2-(iv).

Let $\phi : [I, 1] \to \mathbb{R}_+$ be a function such that $\phi(p_g) \in \Phi(p_g)$. In what follows, we show that $\phi(\cdot)$ cannot be continuous at every $p_g \in [I, 1]$ if $\theta_1^* > 0$. The argument for the case $\theta_1^* = 0$ is similar but with a slight modification.[4]

Step 1. For $p_g = I$, the only possible equilibrium is NR-type and thus $\Phi(I) = \{F(\theta_1^*) + F(\gamma(\theta_1^*))\}$.

Suppose either an MR1 or MR2 equilibrium exists for $p_g = I$. By (C.11), (C.14), and (C.15), we have $I \leq \max\{m(0, \theta_g^{MR1}), m(0, \theta_g^{MR2})\} < p_g = I$, a contradiction. Moreover, since $2m(0, \theta_1^*) - \theta_1^* - m(\theta_1^*, \gamma(\theta_1^*)) + S \geq 0$, it follows from Assumption A.2-(i) that $2m(0, 0) - 0 - m(0, \gamma(0)) + S = 0 - m(0, \gamma(0)) + S > 0$, which implies $\theta - m(\theta, \gamma(\theta)) + S > 0$ for all $\theta \in [0, 1]$. From (C.9), MBS equilibrium cannot exist for any $p_g \in [I, 1]$. Lastly, since $I \leq m(0, \theta_1^*) \leq p_0^* = m(0, \gamma(0))$, we have $p_0^* = \inf P^{SBS} \geq I$. Since $p_g > \inf P^{SBS}$ for all $p_g \in P^{SBS}$ by Lemma C.1, SBS equilibrium cannot exist for $p_g = I$.

Step 2. If $\phi(\cdot)$ is a continuous selection from $\Phi(\cdot)$, then $\phi(p_g) = F(\theta_1^*) + F(\gamma(\theta_1^*))$ for all $p_g \in [I, p_2^*]$ (Recall $p_2^* = m(\theta_1^*, \gamma(\theta_1^*))$ from Theorem 2).

Suppose there exists an SBS equilibrium for some $p_g \in (I, p_2^*]$. We have $m(0, \theta_{g\o}^{SBS}(p_g)) < I \leq m(0, \theta_1^*)$ by (C.3), which implies $\theta_{g\o}^{SBS}(p_g) < \theta_1^*$. Since the total trade volume is $F(\theta_1^*) + F(\gamma(\theta_1^*))$ under the NR equilibrium and the equilibrium absent bailout and $F(\gamma(\theta_{g\o}^{SBS}(p_g)))$ under the SBS equilibrium, the NR equilibrium yields strictly larger overall trade than the SBS type. Suppose next there exists either an MR1 or MR2 equilibrium for some $p_g \in (I, p_2^*]$. By

[4] In the case $\theta_1^* = 0$, we have to take into account the possibility that the MBS equilibrium can exist for some $p_g \in [I, 1]$. But the MBS equilibrium cannot exist in the case $\theta_1^* > 0$, in which we have $\theta - m(\theta, \gamma(\theta)) + S > 0$ for any $\theta > 0$, so any $\theta_g > 0$ cannot satisfy (C.9).

Proposition 2-(ii), either of the two yields strictly larger overall trade than the NR equilibrium. Since $\phi(p_g) = F(\theta_1^*) + F(\gamma(\theta_1^*))$ at $p_g = I$ from Step 1, continuity of ϕ requires that $\phi(p_g) = F(\theta_1^*) + F(\gamma(\theta_1^*))$ for all $p_g \in [I, p_2^*]$.

Step 3. Any $\phi \in \Phi$ such that $\phi(p_g) = F(\theta_1^*) + F(\gamma(\theta_1^*))$ for all $p_g \in [I, p_2^*]$ is discontinuous at $p_g = p_2^*$.

Suppose there exists an SBS equilibrium for some $p_g \geq p_2^* = m(\theta_1^*, \gamma(\theta_1^*))$. Then, it follows from (C.2) that $\theta_{g\emptyset}^{SBS}(p_g) \geq \theta_1^*$, which implies $m(0, \theta_{g\emptyset}^{SBS}(p_g)) \geq m(0, \theta_1^*) \geq I$. However, this inequality contradicts (C.3). Therefore, there cannot exist SBS equilibrium for any $p_g \geq p_2^*$, and thus $\phi(p_g)$ must be equal to the total trade volume under either MR1 or MR2 equilibrium for any $p_g \in [p_2^*, 1]$. By Proposition 2-(ii), both MR1 and MR2 equilibria yield strictly larger total trade than $F(\theta_1^*) + F(\gamma(\theta_1^*))$ for any $p_g \in [p_2^*, I]$. Thus we must have $\phi(p_g) > F(\theta_1^*) + F(\gamma(\theta_1^*))$ for all $p_g \in [p_2^*, 1]$. By Step 2, the last inequality implies $\lim_{p_g \to p_2^*+} \phi(p_g) > \lim_{p_g \to p_2^*-} \phi(p_g)$.

Combining Step 2 and 3, we conclude that Φ does not admit a continuous selection. *Q.E.D.*

Proposition 3. *Suppose that an MR (either MR1 or MR2) equilibrium arises given p_g. In that case, offering a bailout at the same p_g,* but with the $t = 1$ market shut down, *would (at least weakly) increase the total trade volume.*

Proof of Proposition 3. First, suppose p_g admits an MR1 equilibrium. By Claim 1 within the proof of Lemma C.5, we have $\theta_1^{MR1} \leq m(0, \theta_g^{MR1}) + S$. Since $\theta_1^{MR1}(p_g) > \theta_g^{MR1}(p_g)$, we also have $\theta_1^{MR1}(p_g) \leq \theta_0^*$, where the equality holds when $\theta_1^{MR1}(p_g) = 1$. Since $p_g \leq m(\theta_1^{MR1}(p_g), \gamma(\theta_1^{MR1}(p_g)))$ from (C.15), we have $p_g \leq m(\theta_0^*, \gamma(\theta_0^*))$. This result implies there exists $\theta_g^{sd}(p_g) \leq \theta_0^*$ such that $p_g = (\theta_g^{sd}(p_g) - m(0, \theta_g^{sd}(p_g)) - S) + m(\theta_g^{sd}(p_g), \gamma(\theta_g^{sd}(p_g)))$. Since $p_g + m(0, \theta_g^{MR1}(p_g)) + S \geq \theta_1^{MR1}(p_g) + m(\theta_1^{MR1}(p_g), \gamma(\theta_1^{MR1}(p_g)))$ from (C.11) and (C.12), we have $p_g > \theta_1^{MR1}(p_g) - m(0, \theta_1^{MR1}(p_g)) - S + m(\theta_1^{MR1}(p_g), \gamma(\theta_1^{MR1}(p_g)))$. By Lemma A.1, $\theta - m(0, \theta) + m(\theta, \gamma(\theta))$ is increasing in θ, so we have $\theta_g^{sd}(p_g) \geq \theta_1^{MR1}(p_g)$, which implies $m(0, \theta_g^{sd}(p_g)) \geq I$ from (C.14). Therefore, when the market is shut down in $t = 1$, the same p_g admits an equilibrium which yields the total trade volume $F(\theta_g^{sd}(p_g)) + F(\gamma(\theta_g^{sd}(p_g))) \geq F(\theta_1^{MR1}(p_g)) + F(\gamma(\theta_1^{MR1}(p_g)))$, as was to be shown, where the equality holds for the case $\theta_1^{MR1}(p_g) = \theta_0^* = \theta_g^{sd}(p_g) = 1$.

Next, suppose p_g admits an MR2 equilibrium. Since $\theta_1^{MR2}(p_g) = m(0, \theta_g^{MR2}(p_g)) + S$ from (C.17) and $\theta_g^{MR2}(p_g) < \theta_1^{MR2}(p_g)$, we have $\theta_1^{MR2}(p_g) < m(0, \theta_1^{MR2}(p_g)) + S$, which implies $\theta_1^{MR2}(p_g) < \theta_0^*$. If $\theta_{g\emptyset}^{MR2}(p_g) > \theta_0^*$, then there exists $\theta_g^{sd}(p_g)$ such that $p_g = m(\theta_g^{sd}(p_g), \gamma(\theta_g^{sd}(p_g)))$.

By (C.18), we have $\theta^{MR2}_{g\o}(p_g) = \theta^{sd}_g(p_g)$. Hence, under the market shutdown in $t = 1$, p_g admits the equilibrium which yields the total trade volume $F(\theta^*_0) + F(\gamma(\theta^{sd}_g(p_g))) > F(\theta^{MR2}_1(p_g)) + F(\gamma(\theta^{MR2}_{g\o}(p_g)))$, as was to be shown. If $\theta^{MR2}_{g\o}(p_g) \le \theta^*_0$, we have $p_g \le m(\theta^*_0, \gamma(\theta^*_0))$ by (C.18). Thus there exists $\theta^{sd}_g(p_g) \le \theta^*_0$ such that $p_g + m(0, \theta^{sd}_g(p_g)) + S = \theta^{sd}_g(p_g) + m(\theta^{sd}_g(p_g), \gamma(\theta^{sd}_g(p_g)))$. Since $\theta^{sd}_g(p_g) \le \theta^*_0 \iff \theta^{sd}_g(p_g) \le m(0, \theta^{sd}_g(p_g)) + S$, we have $p_g \le m(\theta^{sd}_g(p_g), \gamma(\theta^{sd}_g(p_g)))$. Since $p_g = m(\theta^{MR2}_{g\o}(p_g), \gamma(\theta^{MR2}_{g\o}(p_g)))$ and $\theta^{MR2}_{g\o}(p_g) > \theta^{MR2}_g(p_g)$, we have $m(0, \theta^{sd}_g(p_g)) \ge I$ from (C.14). Therefore, under the market shutdown in $t = 1$, p_g admits the equilibrium which yields the total trade volume $F(\theta^{sd}_g(p_g)) + F(\gamma(\theta^{sd}_g(p_g)) > F(\theta^{MR2}_1(p_g) + F(\gamma(\theta^{MR2}_{g\o}(p_g))$, as was to be shown. *Q.E.D.*

Proposition 4. *Suppose that the market remains closed in $t = 1$, and let p_g be a given bailout offer. The total trade volume decreases when the government adds another bailout offer $p'_g \in [I, p_g)$.*

Proof of Proposition 4. Fix p_g and p'_g such that $p_g > p'_g \ge I$. For the bailout that has two options $\{p'_g, p_g\}$, there are two possibilities. First, only one of the offers is accepted by a positive measure of firms. Second, both offers are accepted by a positive measure of firms. Since the first possibility is identical to the case with a single offer, we hereafter restrict our focus on the second possibility.

Let θ^{do}_g denote the highest type selling to the government at either p_g or p'_g. In this equilibrium, types $\theta \in (\theta^{do}_g, \gamma(\theta^{do}_g)]$ sell at price $m(\theta^{do}_g, \gamma(\theta^{do}_g))$ in $t = 2$ by Lemma C.8. Since both offers are accepted by positive measures of types, we have the following observations. First, all types choosing p'_g must also sell in $t = 2$; otherwise, they will get a higher payoff by selling at price $p_g > p'_g$ in $t = 1$. Second, the $t = 2$ price for types choosing p_g (denoted by p_2) should be strictly lower than the $t = 2$ price for types choosing p'_g (denoted by p'_2), which follows from the fact that types $\theta \in (\theta^{do}_g, \gamma(\theta^{do}_g)]$ are indifferent between selling at p_g and selling at p'_g, i.e., $p'_g + p'_2 + 2S = p_g + p_2 + 2S$. These observations imply that there are two types of equilibria: all types selling at price p_g in $t = 1$ also sell in $t = 2$; a positive measure of types selling at price p_g in $t = 1$ do not sell in $t = 2$. We show below that any type of equilibria given the bailout with a single offer p_g yields larger overall trade than both types of equilibria above given the bailout with double offers.

First, consider the equilibrium in which all firms choosing p_g in $t = 1$ also sell in $t = 2$, which yields the total trade volume $F(\theta^{do}_g) + F(\gamma(\theta^{do}_g))$. Since buyers in $t = 2$ earn zero profit and $p_2 < p'_2$, there exists $\underline{\theta}^{do}_g < \theta^{do}_g$ such that $p_2 = m(0, \underline{\theta}^{do}_g)$. Since type-$\theta^{do}_g$ firm prefers selling at price p_2 in $t = 2$ to not selling, we have $\theta_g^{do} \le m(0, \underline{\theta}_g^{do}) + S$, thereby implying $\theta_g^{do} \le \theta_0^*$. Furthermore, since type-θ_g^{do} firm prefers selling at price p_g in $t = 1$ to not selling, we have

$$p_g + p_2 + 2S = p_g + m(0, \underline{\theta}_g^{do}) + 2S \ge \theta_g^{do} + m(\theta_g^{do}, \gamma(\theta_g^{do})) + S.$$

Since $\underline{\theta}_g^{do} < \theta_g^{do}$, we have $p_g > \theta_g^{do} - m(0, \theta_g^{do}) - S + m(\theta_g^{do}, \gamma(\theta_g^{do}))$. Therefore, there always exists θ_g^{sd} determined by either $p_g \ge \theta_g^{sd} - m(0, \theta_g^{sd}) - S + m(\theta_g^{sd}, \gamma(\theta_g^{sd}))$ (the inequality holds strictly for the case $\theta_g^{sd} = 1$) subject to $\theta_g^{sd} \le \theta_0^*$ or $p_g = m(\theta_g^{sd}, \gamma(\theta_g^{sd}))$ subject to $\theta_g^{sd} > \theta_0^*$. If $\theta_g^{sd} \le \theta_0^*$, then $\theta_g^{sd} \ge \theta_g^{do}$ since $\theta - m(0, \theta) + m(\theta, \gamma(\theta))$ is increasing in θ; if $\theta_g^{sd} > \theta_0^*$, then $\theta_g^{sd} > \theta_g^{do}$ since $\theta_g^{do} \le \theta_0^*$. These observations imply that there exists an equilibrium given the bailout with the single offer p_g, which yields the overall trade $F(\theta_g^{sd} \wedge \theta_0^*) + F(\gamma(\theta_g^{sd})) \ge F(\theta_g^{do}) + F(\gamma(\theta_g^{do}))$, as was to be shown, where the equality holds for the case $\theta_g^{sd} = \theta_g^{do} = \theta_0^* = 1$.

Next, consider the equilibrium where a positive measure of types selling at price p_g in $t = 1$ do not sell in $t = 2$. By playing this strategy, a type-θ firm earns the total payoff $p_g + S + \theta$, which is increasing in θ. Hence, θ_g^{do} is determined by $p_g + S + \theta_g^{do} = \theta_g^{do} + m(\theta_g^{do}, \gamma(\theta_g^{do})) + S$, which is equivalent to $p_g = m(\theta_g^{do}, \gamma(\theta_g^{do}))$. Let $\hat{\theta}_g^{do} \in [\underline{\theta}_g^{do}, \theta_g^{do})$ be the highest type that sells in both periods. Then the total trade volume under this equilibrium is $F(\hat{\theta}_g^{do}) + F(\gamma(\theta_g^{do}))$. Since type $\hat{\theta}_g^{do}$ must be indifferent between selling in both periods and selling only in $t = 1$, we have

$$p_g + p_2 + 2S = \hat{\theta}_g^{do} + p_g + S \iff \hat{\theta}_g^{do} = m(0, \underline{\theta}_g^{do}) + S.$$

Since $\hat{\theta}_g^{do} \ge \underline{\theta}_g^{do}$, the condition above implies $\hat{\theta}_g^{do} \le \theta_0^*$. Furthermore, the same condition also implies $p_g > \hat{\theta}_g^{do} - m(0, \hat{\theta}_g^{do}) - S + m(\hat{\theta}_g^{do}, \gamma(\hat{\theta}_g^{do}))$ since $\underline{\theta}_g^{do} \le \hat{\theta}_g^{do} < \theta_g^{do}$. From these results, one can find that the bailout with the single offer p_g admits one of the following types of equilibria: either $\theta_g^{sd} \le \theta_0^*$ and $m(0, \theta_g^{sd}) \ge I$ or $\theta_g^{sd} > \theta_0^*$. First, suppose $\theta_g^{sd} \le \theta_0^*$ and $m(0, \theta_g^{sd}) \ge I$. If $\theta_g^{sd} < 1$, then $p_g = (\theta_g^{sd} - m(0, \theta_g^{sd}) - S) + m(\theta_g^{sd}, \gamma(\theta_g^{sd})) \le m(\theta_g^{sd}, \gamma(\theta_g^{sd}))$, which implies $\theta_g^{do} \le \theta_g^{sd}$. If $\theta_g^{sd} = \theta_0^* = 1$, then $\theta_g^{do} \le \theta_g^{sd}$. Next, suppose $\theta_g^{sd} > \theta_0^*$, then we have $\theta_g^{do} = \theta_g^{sd} > \theta_0^* \ge \hat{\theta}_g^{do}$. Putting all the results altogether, we have shown that $F(\theta_g^{sd} \wedge \theta_0^*) + F(\gamma(\theta_g^{sd})) \ge F(\hat{\theta}_g^{do}) + F(\gamma(\theta_g^{do}))$, that is, the bailout with the single offer p_g always yields (weakly) larger total trade than the bailout with double offers $\{p_g', p_g\}$. *Q.E.D.*

D Proofs for Section 5

Proceeding similarly as in Section C.1, we derive conditions characterizing the marginal types θ_g, θ_1, and $\theta_{g\emptyset}$ of SMR equilibrium. Next, we find the set of bailout terms that support the SMR equilibrium, denoted by P^{SMR}.

D.1 Necessary Conditions for SMR Equilibrium

In this equilibrium, types $\theta \in \Theta_g = [0, \theta_g]$ sell to the government in $t = 1$ and to the market in $t = 2$ at price $m(0, \theta_g)$, types $\theta \in \Theta_1 = (\theta_g, \theta_1]$ sell to the market at price $m(\theta_g, \theta_1)$ in both periods, types $\theta \in \Theta_{g\o} = (\theta_1, \theta_{g\o}]$ sell only in $t = 1$ to the government, and the rest do not sell in either period. The firms' total payoffs are $p_g + m(0, \theta_g) + 2S$ if $\theta \in [0, \theta_g]$, $2m(\theta_g, \theta_1) + 2S$ if $\theta \in (\theta_g, \theta_1]$, $p_g + S + \theta$ if $\theta \in (\theta_1, \theta_{g\o}]$, and 2θ if $\theta \in (\theta_{g\o}, 1]$. From these payoffs, the marginal types must satisfy the following indifference conditions:

$$p_g + m(0, \theta_g) = 2m(\theta_g, \theta_1), \tag{D.1}$$

$$\theta_1 \leq m(0, \theta_g) + S, \tag{D.2}$$

$$\theta_{g\o} = (p_g + S) \wedge 1. \tag{D.3}$$

Note that the inequality in (D.2) is strict for the case $\theta_1 = 1$.

In addition to the above conditions, we need the conditions that guarantee that the $t = 2$ price covers the investment cost and that the sets Θ_g, Θ_1, and $\Theta_{g\o}$ must be non-empty, except for the boundary case. Thus

$$m(0, \theta_g) \geq I, \tag{D.4}$$

$$\theta_g < \theta_1 \leq \theta_{g\o}. \tag{D.5}$$

From (D.1), we have $m(0, \theta_g) < p_g$. Applying this to (D.3) shows that $\theta_1 < \theta_{g\o}$ always holds for the interior case $\theta_1 < 1$. The weak inequality in (D.5) holds as equality for the boundary case $\theta_1 = 1$.

Lemma D.1. *There exist $\underline{p}_g^{SMR} \leq \overline{p}_g^{SMR}$ such that (D.1) – (D.3) admit a unique $(\theta_g, \theta_1, \theta_{g\o})$ that satisfies (D.4) and (D.5) if and only if $p_g \in [\underline{p}_g^{SMR}, \overline{p}_g^{SMR})$.*

Proof. Note that (D.1) and (D.2) are equivalent to (C.11) and (C.17), respectively. Thus, using these conditions and proceeding similarly to the proof of Lemma C.4, one can show that there exist $\underline{p}_g^{SMR} \leq \overline{p}_g^{SMR}$ such that (D.1) and (D.2) define a unique (θ_g, θ_1) that satisfies $0 < \theta_g < \theta_1$ and (D.4) if and only if $p_g \in [\underline{p}_g^{SMR}, \overline{p}_g^{SMR})$. Furthermore, since $\theta_1 = m(0, \theta_g) + S < m(\theta_g, \theta_1) + S < p_g + S$, where the second inequality follows from (D.1), $\theta_{g\o}$ determined by (D.3) satisfies (D.5) for all $p_g \in [\underline{p}_g^{SMR}, \overline{p}_g^{SMR})$. *Q.E.D.*

For any $p_g \in [\underline{p}_g^{SMR}, \overline{p}_g^{SMR})$, let $\theta_g^{SMR}(p_g)$, $\theta_1^{SMR}(p_g)$, and $\theta_{g\o}^{SMR}(p_g)$ denote the marginal types θ_g, θ_1, and $\theta_{g\o}$ determined by (D.1) – (D.3). For expositional convenience, we may abbreviate $\theta_g^{SMR}(p_g)$, $\theta_1^{SMR}(p_g)$, and $\theta_{g\o}^{SMR}(p_g)$ to θ_g^{SMR}, θ_1^{SMR}, and $\theta_{g\o}^{SMR}$, respectively. In addition, define $P^{SMR} := [\underline{p}_g^{SMR}, \overline{p}_g^{SMR})$.

D.2 Proofs of Theorem 4 and Proposition 5 and 6

Theorem 4. *There exists a nonempty interval $P^{SMR} \subset (I, 1]$ of bailout terms such that (i) an SMR equilibrium exists with cutoffs $\theta_1 < \theta_0^*$ and $\theta_{g\o} = p_g + S$ if $p_g \in P^{SMR}$; and (ii) no other equilibria exist.*

Proof of Theorem 4.

Proof of Theorem 4-(i).

Consider $p_g \in P^{SMR}$ such that there exist $\theta_g = \theta_g^{SMR}(p_g), \theta_1 = \theta_1^{SMR}(p_g)$, and $\theta_{g\o} = \theta_{g\o}^{SMR}(p_g)$ that satisfy (D.4) and (D.5).

First, we show that it is optimal for each type of firms to play the prescribed equilibrium strategies. Consider $t = 2$ first. For firms accepting the bailout or holding out in $t = 1$, the price offer in $t = 2$ is $m(0, \theta_g)$. By (D.2), we have $\theta_1 \leq m(0, \theta_g) + S$, where the inequality holds for the case $\theta_1 = 1$. Thus, after accepting the bailout in $t = 1$, types $\theta \in [0, \theta_g]$ sell at price $m(0, \theta_g)$ in $t = 2$, but types $\theta > \theta_1$ do not sell in $t = 2$. For types selling to the market in $t = 1$, the price offer in $t = 2$ is $m(\theta_g, \theta_1)$. Since $\theta_1 \leq m(0, \theta_g) + S < m(\theta_g, \theta_1) + S$, types $\theta \in (\theta_g, \theta_1]$ sell in $t = 2$ after selling in $t = 1$. Consider $t = 1$ next. By (D.1), we have $p_g + m(0, \theta_g) + 2S = 2m(\theta_g, \theta_1) + 2S$. If $\theta_1 < 1$, then, by (D.2), we have $p_g + m(0, \theta_g) + 2S = 2m(\theta_g, \theta_1) + 2S < \theta + p_g + S \iff \theta > \theta_1$. If $\theta_1 = 1$, then $\theta + p_g + S \leq p_g + m(0, \theta_g) + 2S = 2m(\theta_g, \theta_1) + 2S$ for all $\theta \in [0, 1]$, where the inequality follows from (D.2). Thus, it is optimal for types $\theta \in [0, \theta_g] \cup (\theta_1, 1]$ to accept the bailout and types $\theta \in (\theta_g, \theta_1]$ to sell to the market. Lastly, since $\theta_{g\o} = (p_g + S) \wedge 1$ from (D.3), it is optimal for types $\theta > \theta_{g\o}$ not to sell.

Second, we show that it is optimal for buyers to make the equilibrium price offers. Consider $t = 2$ first. For firms not selling at price $m(\theta_g, \theta_1)$ in $t = 1$, buyers believe that their types are $\theta \in [0, \theta_g] \cup (\theta_1, 1]$. By Lemma C.8, any $p' < m(0, \theta_g)$ cannot be an equilibrium price offer in $t = 2$. Next, suppose a buyer in $t = 1$ deviates and offers $p' > m(0, \theta_g)$. Since $p' + S > \theta_1$ from (D.2), types $\theta \in [0, \theta_g] \cup (\theta_1, (p' + S) \wedge 1]$ will sell at price p'. Then, the deviating buyer's payoff is $\mathring{m}(0, \theta_g, \theta_1, (p' + S) \wedge 1) - p'$. However, since $\lim_{p' \searrow (\theta_1 - S)} \mathring{m}(0, \theta_g, \theta_1, (p' + S) \wedge 1) = m(0, \theta_g)$ and $\frac{\partial}{\partial \theta} \mathring{m}(0, \theta_g, \theta_1, \theta) \leq 1$ for all $\theta \geq \theta_1$ from Lemma A.1-(iii), we have $\mathring{m}(0, \theta_g, \theta_1, (p' + S) \wedge 1) - p' < 0$

for any $p' > \theta_1 - S$. Therefore, buyers optimally offer $m(0, \theta_g)$ to all firms that do not sell to the market in $t = 1$. For firms selling to the market in $t = 1$, buyers believe their types are $\theta \in (\theta_g, \theta_1]$. Since $\theta_1 \leq m(0, \theta_g) + S < m(0, \theta_1) + S$, we have $\theta_1 \leq \theta_0^* = \gamma(0) < \gamma(\theta_g)$. Hence, by Lemma C.8, buyers optimally offer $m(\theta_g, \theta_1)$. Lastly, proceeding similarly as in the characterization of MR2 equilibrium in the proof of Theorem 3, one can show that it is optimal for buyers to offer $m(\theta_g, \theta_1)$ in $t = 1$.

Proof of Theorem 4-(ii). We show below that no cutoff structure but the SMR type is possible in equilibrium.

Case 1. $\Theta_g \neq \emptyset$, $\Theta_1 \neq \emptyset$, $\Theta_{g\emptyset} \neq \emptyset$, *and* $\Theta_2 \neq \emptyset$

This equilibrium is characterized by $0 < \theta_g < \theta_1 < \theta_{g\emptyset} < \theta_2 \leq 1$. Let p_2 be the $t = 2$ price offer to the firms that do not sell to the market in $t = 1$. Under the secret bailout, such p_2 will be offered to types $\theta \in [0, \theta_g] \cup (\theta_1, 1]$. Since types $\theta \in (\theta_1, \theta_{g\emptyset}]$ choose not to sell in $t = 2$, we must have $\theta_{g\emptyset} \geq p_2 + S$. But types $\theta \in (\theta_{g\emptyset}, \theta_2]$ prefer selling in $t = 2$, hence $\theta_2 \leq p_2 + S$, a contradiction.

Case 2. $\Theta_g \neq \emptyset$, $\Theta_1 = \emptyset$, $\Theta_{g\emptyset} = \emptyset$, *and* $\Theta_2 \neq \emptyset$

This equilibrium is characterized by $0 < \theta_g < \theta_2 \leq 1$. In this case, types $\theta \in [0, \theta_g]$ accept the bailout in $t = 1$ and types $\theta \in [0, \theta_2]$ sell to the $t = 2$ market at price $m(0, \theta_2)$. When the $t = 1$ market is open after the bailout, buyers believe that types $\theta \in (\theta_g, 1]$ are available for asset trade. That is, a buyer at the $t = 1$ market can make a positive profit by offering $p' = m(\theta_g, \gamma(\theta_g)) - \varepsilon$ for some $\varepsilon > 0$, a contradiction.

Case 3. $\Theta_g \neq \emptyset$, $\Theta_1 \neq \emptyset$, $\Theta_{g\emptyset} = \emptyset$, *and* $\Theta_2 = \emptyset$

This equilibrium is characterized by $0 < \theta_g < \theta_1 \leq 1$, where types $\theta \in [0, \theta_g]$ accept the bailout in $t = 1$ and sell at price $m(0, \theta_g)$ in $t = 2$, and types $\theta \in (\theta_g, \theta_1]$ sell at price $m(\theta_g, \theta_1)$ in both periods. For these strategies to be optimal, we must have $p_g + m(0, \theta_g) + 2S = 2m(\theta_g, \theta_1) + 2S$, or equivalently $p_g = 2m(\theta_g, \theta_1) - m(0, \theta_g)$. Since type θ_1 firm weakly prefers selling in both periods to not selling in either period, we must have $2m(\theta_g, \theta_1) + 2S = 2\theta_1$, or equivalently, $\theta_1 = m(\theta_g, \theta_1) + S$. However, since $p_g > m(\theta_g, \theta_1)$, we have $p_g + S > m(\theta_g, \theta_1) + S \geq \theta_1$. That is, type θ_1 firm will get a strictly higher payoff by accepting the bailout, a contradiction.

Case 4. $\Theta_g \neq \emptyset$, $\Theta_1 = \emptyset$, $\Theta_{g\emptyset} \neq \emptyset$, *and* $\Theta_2 \neq \emptyset$

This case is similar to Case 1.

Case 5. $\Theta_g \neq \emptyset$, $\Theta_1 = \emptyset$, $\Theta_{g\emptyset} \neq \emptyset$, *and* $\Theta_2 = \emptyset$

This equilibrium is characterized by $0 < \theta_g < \theta_{g\emptyset} < 1$, where types $\theta \in [0, \theta_{g\emptyset}]$ accept the bailout, and only a subset of these types $\theta \in [0, \theta_g]$ sell to the market at price $m(0, \theta_g)$ in $t = 2$. When the market is open in $t = 1$ after the bailout, types $\theta \in (\theta_{g\o}, 1]$ are available for asset trade. Thus, a buyer can make a positive profit by deviating and offering a price $p' = m(\theta_{g\o}, \gamma(\theta_{g\o})) - \varepsilon$ for some $\varepsilon > 0$, a contradiction.

Case 6. $\Theta_g \neq \emptyset$, $\Theta_1 \neq \emptyset$, $\Theta_{g\o} = \emptyset$, *and* $\Theta_2 \neq \emptyset$*:*

This equilibrium is characterized by $0 < \theta_g < \theta_1 < \theta_2 \leq 1$, where types $\theta \in [0, \theta_g]$ accept the bailout in $t = 1$ and sell at price $\mathring{m}(0, \theta_g, \theta_1, \theta_2)$ in $t = 2$, types $\theta \in (\theta_g, \theta_1]$ sell to the market at price $m(\theta_g, \theta_1)$ in both periods, and types $\theta \in (\theta_1, \theta_2]$ sell only in $t = 2$ at price $\mathring{m}(0, \theta_g, \theta_1, \theta_2)$. Thus the indifference conditions for the marginal types are

$$p_g + \mathring{m}(0, \theta_g, \theta_1, \theta_2) = 2m(\theta_g, \theta_1), \tag{D.6}$$

$$2m(\theta_g, \theta_1) + 2S = \theta_1 + \mathring{m}(0, \theta_g, \theta_1, \theta_2) + S, \tag{D.7}$$

$$\theta_2 = \mathring{\gamma}(0, \theta_g, \theta_1). \tag{D.8}$$

From (D.6) and (D.7), we have

$$\theta_1 = p_g + S. \tag{D.9}$$

To show that this equilibrium cannot exist, we prove that there exists $p_1' \neq m(\theta_g, \theta_1)$ which gives a positive profit to market buyers in $t = 1$. When the market is open in $t = 1$, types $\theta \in (\theta_g, 1]$ are available for asset sales. Thus firms accepting p_1' offered by a deviating buyer are assigned the off-the-path belief consistent with D1 equal to $\theta = \theta_g$. This implies that, after selling at p_1' in $t = 1$, these firms can sell at price θ_g in $t = 2$ if $\theta_g \geq I$, leading to the total payoff $p_1' + S + \max\{\theta, \theta_g + S\}$. If $\theta_g < I$, then they do not sell in $t = 2$, hence the total payoff $p_1' + S + \theta$.

There are three possibilities: $\theta_g < I$, $\theta_g \in [I, \theta_1 - S)$, or $\theta_g \geq \theta_1 - S$. First, suppose $\theta_g < I$. Since type-θ firm sells at price p_1' if and only if $p_1' + S + \theta \geq p_g + \mathring{m}(0, \theta_g, \theta_1, \theta_2) + 2S$, types $\theta \in [\theta_1', (p_1' + S) \wedge 1]$ sell at price p_1', where

$$\theta_1' := (\mathring{m}(0, \theta_g, \theta_1, \theta_2) - p_1' + (p_g + S)) \vee \theta_g = (\mathring{m}(0, \theta_g, \theta_1, \theta_2) - p_1' + \theta_1) \vee \theta_g.$$

By offering p_1', the buyer earns $m(\theta_1', (p_1' + S) \wedge 1) - p_1'$. Since $m(\theta_1, \theta_2) - \mathring{m}(0, \theta_g, \theta_1, \theta_2) > 0$, $\lim_{p_1' \to \mathring{m}(0, \theta_g, \theta_1, \theta_2)+} (p_1' + S) \wedge 1 = \theta_2$ from (D.8), and $\lim_{p_1' \to \mathring{m}(0, \theta_g, \theta_1, \theta_2)+} \theta_1' = \theta_1$ from (D.9), there exists $p_1' > \mathring{m}(0, \theta_g, \theta_1, \theta_2)$ such that $m(\theta_1', (p_1' + S) \wedge 1) - p_1' > 0$.

Next, suppose $\theta_g \geq I$ but $\theta_g + S < \theta_1$. Since $\theta_g + S < \theta_1 = p_g + S$, we have $p_1' + S + \theta \geq$

$p_g + \mathring{m}(0, \theta_g, \theta_1, \theta_2) + 2S \iff \theta \geq (\mathring{m}(0, \theta_g, \theta_1, \theta_2) - p_1') + \theta_1$. Proceeding similarly as in the previous case, one can show that there exists $p_1' > \mathring{m}(0, \theta_g, \theta_1, \theta_2)$ that gives a positive profit to the deviating buyer.

Lastly, suppose $\theta_g + S \geq \theta_1$, which is equivalent to $\theta_g \geq p_g$ by (D.9). Types $\theta \in (\theta_g, \theta_1]$ sell at p_1' if and only if

$$\mathring{m}(0, \theta_g, \theta_1, \theta_2) + p_g + 2S \leq p_1' + \theta_g + 2S \iff p_1' \geq \mathring{m}(0, \theta_g, \theta_1, \theta_2) - (\theta_g - p_g).$$

Similarly, types $\theta \in (\theta_1, \theta_g + S]$ sell at p_1' if and only if

$$\mathring{m}(0, \theta_g, \theta_1, \theta_2) + S + (\theta_g + S) \leq p_1' + \theta_g + 2S \iff p_1' \geq \mathring{m}(0, \theta_g, \theta_1, \theta_2).$$

Furthermore, types $\theta > \theta_g + S$ sell at p_1' if and only if

$$p_1' + S + \theta \geq \theta + \mathring{m}(0, \theta_g, \theta_1, \theta_2) + S \iff p_1' \geq \mathring{m}(0, \theta_g, \theta_1, \theta_2).$$

Thus, by offering $p_1' \geq \mathring{m}(0, \theta_g, \theta_1, \theta_2)$, the buyer's expected payoff is $m(\theta_g, (p_1' + S) \wedge 1) - p_1'$. Since $\mathring{m}(0, \theta_g, \theta_1, \theta_2) < m(\theta_g, \theta_2)$ and $\lim_{p_1' \to m(0,\theta_g,\theta_1,\theta_2)+} (p_1' + S) \wedge 1 = \theta_2$ from (D.8), there exists $p_1' \geq \mathring{m}(0, \theta_g, \theta_1, \theta_2)$ such that $m(\theta_g, (p_1' + S) \wedge 1) - p_1' > 0$. Thus the prescribed equilibrium strategy cannot be optimal for buyers in $t = 1$. *Q.E.D.*

Proposition 5.

(i) (Front-loading of trade) An SMR equilibrium, if it exists, supports a larger trade volume in $t = 1$ but a smaller trade volume in $t = 2$ than an MR equilibrium for the same p_g.

(ii) Given $p_g \in P^{SMR}$, the total trade volume supported in the SMR equilibrium is the same as that in the MR2 equilibrium if p_g admits the MR2 equilibrium; but the comparison is ambiguous if p_g admits the MR1 equilibrium.

Proof of Proposition 5.

Proof of Proposition 5-(i).

First, suppose $p_g \in P^{SMR}$ admits the MR1 equilibrium under transparency. Since $\theta_1^{MR1}(p_g) \leq m(0, \theta_g^{MR1}(p_g)) + S$ from Claim 1 within the proof of Lemma C.5 and $p_g > m(0, \theta_g^{MR1}(p_g))$ from (C.11), we have $\theta_1^{MR1}(p_g) < p_g + S$, and thus $F(\theta_1^{MR1}(p_g)) \leq F(\theta_{g\o}^{SMR}(p_g))$. Therefore, the

SMR equilibrium arising from the same p_g yields a larger total trade volume in $t = 1$ than the MR1 equilibrium. Furthermore, since $p_g \leq m(\theta_1^{MR1}, \gamma(\theta_1^{MR1}))$ from (C.15), we have

$$\theta_1^{SMR}(p_g) < \theta_{g\o}^{SMR}(p_g) = (p_g + S) \wedge 1 \leq (m(\theta_1^{MR1}(p_g), \gamma(\theta_1^{MR1}(p_g))) + S) \wedge 1 = \gamma(\theta_1^{MR1}(p_g)).$$

Hence, we have $F(\theta_1^{SMR}(p_g)) < F(\gamma(\theta_1^{MR1}(p_g)))$, so the SMR equilibrium yields a smaller trade volume in $t = 2$ than the MR1 equilibrium.

Second, suppose $p_g \in P^{SMR}$ admits the MR2 equilibrium under transparency. Since (C.11) and (C.17) are equivalent to (D.1) and (D.2), respectively, we have $(\theta_g^{SMR}(p_g), \theta_1^{SMR}(p_g)) = (\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g))$. Since $p_g = m(\theta_{g\o}^{MR2}(p_g), \gamma(\theta_{g\o}^{MR2}(p_g)))$ from (C.18) and $\theta_{g\o}^{SMR}(p_g) = (p_g + S) \wedge 1$ from (D.3), we have $\theta_{g\o}^{SMR}(p_g) = \gamma(\theta_{g\o}^{MR2}(p_g))$. Hence, we have $F(\theta_{g\o}^{SMR}) \geq F(\theta_{g\o}^{MR2})$ and $F(\theta_1^{SMR}) \leq F(\theta_1^{MR2}) + (F(\gamma(\theta_{g\o}^{MR2})) - F(\theta_{g\o}^{MR2}))$, which is the desired result.

Proof of Proposition 5-(ii).

First, suppose $p_g \in P^{SMR}$ admits the MR2 equilibrium under transparency. As shown in the proof of Proposition 5-(i), we have $(\theta_g^{SMR}(p_g), \theta_1^{SMR}(p_g)) = (\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g))$. Moreover, $\theta_{g\o}^{SMR}(p_g) = \gamma(\theta_{g\o}^{MR2}(p_g))$ by (C.18) and (D.3). Since the total trade volume is $F(\theta_{g\o}^{SMR}(p_g)) + F(\theta_1^{SMR}(p_g))$ in SMR equilibrium and $F(\gamma(\theta_{g\o}^{MR2}(p_g))) + F(\theta_1^{MR2}(p_g))$ in MR2 equilibrium, we have the desired result.

Second, suppose some $p_g \in P^{SMR}$ admits the MR1 equilibrium. Recall the functions $\tilde{\theta}_1(\theta_g)$, $\theta_1^I(\theta_g)$, and $\theta_1^{II}(\theta_g)$ corresponding to (C.11), (C.12), and (C.17), respectively. As shown in the proof of Lemma C.3, $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$ is determined as a unique point of intersection between two curves $\tilde{\theta}_1(\theta_g)$ and $\theta_1^I(\theta_g)$. Given the equivalence of (D.1) and (D.2) with (C.11) and (C.17), $(\theta_g^{SMR}(p_g), \theta_1^{SMR}(p_g))$ is defined as a unique point of intersection between two curves $\tilde{\theta}_1(\theta_g)$ and $\theta_1^{II}(\theta_g)$, as seen in the proof of Lemma C.4. Since p_g supports the MR1 equilibrium, we have $\theta_1^{MR1}(p_g) \leq \theta_1^{II}(\theta_g^{MR1}(p_g)) = \theta_1^{SMR}(p_g)$, as shown in the proof of Lemma C.5. Moreover, by (C.15), we have $\theta_{g\o}^{SMR}(p_g) \leq (p_g + S) \wedge 1 \leq (m(\theta_1^{MR1}(p_g), \gamma(\theta_1^{MR1}(p_g))) + S) \wedge 1 \leq \gamma(\theta_1^{MR1}(p_g))$. From these observations, the comparison of the total trade volume is ambiguous. *Q.E.D.*

Proposition 6. *Suppose that the government offers a secret bailout at $p_g \geq \max\{I, p_1^*\}$ and further shuts down the $t = 1$ market. Then in equilibrium,*

(i) firms with types $\theta \leq p_g + S$ accept the bailout in $t = 1$ and those with $\theta \leq \theta_0^*$ sell to the market in $t = 2$;

(ii) the total trade volume in this equilibrium is larger than in the SMR equilibrium, whenever the latter exists for the same p_g.

Proof of Proposition 6.

Proof of Proposition 6-(i).

Fix any $p_g \geq \max\{p_1^*, I\}$. Since the market is shut down in $t = 1$, firms have only two choices available in $t = 1$: either accepting the bailout or rejecting it. In $t = 2$, buyers have the same belief as the prior regardless of the firms' action taken in $t = 1$, and thus they offer the price p_0^* in $t = 2$. Given p_0^*, only types $\theta \leq (p_0^* + S) \wedge 1 = \theta_0^*$ sell in $t = 2$. Since this $t = 2$ price is independent of firms' actions in $t = 1$, firms accept the bailout if and only if $\theta \leq (p_g + S) \wedge 1$.

Proof of Proposition 6-(ii).

Fix any $p_g \in P^{SMR} \cap [p_1^* \vee I, 1]$. Since $\theta_{g\o}^{SMR}(p_g) = (p_g + S) \wedge 1$ from (D.3), the SMR equilibrium yields the same trade volume in $t = 1$ as that under the market shutdown in $t = 1$. On the other hand, since $\theta_1^{SMR}(p_g) \leq \theta_0^*$ from (D.2) and (D.5), the SMR equilibrium yields smaller total trade volume in $t = 2$ than when the market is shut down in $t = 1$. *Q.E.D.*

E Proofs for Section 6

Theorem 5. *Let $\mathcal{M}$ denote the set of mechanisms that satisfy the restrictions imposed above. Then, the following holds:*

(i) If $M = (q, t) \in \mathcal{M}$, then $q(\cdot)$ is nonincreasing, and $q(\theta) \leq 1$ for all $\theta > \theta_0^*$, where θ_0^* is the highest type that sells its asset in the one-shot model without a bailout.

(ii) [Revenue Equivalence] If $M = (q, t)$ and $M' = (q', t')$ both in $\mathcal{M}$ have $q = q'$, then $W(M) = W(M')$. In other words, an equilibrium allocation pins down the welfare, expressed as follows:

$$\int_0^1 \left[J(\theta)q(\theta) - 2\lambda + 2\left((1+\lambda)\theta + \lambda\frac{F(\theta)}{f(\theta)}\right)\right] f(\theta)d\theta, \tag{E.1}$$

where

$$J(\theta) := (1+\lambda)S - \lambda\frac{F(\theta)}{f(\theta)}.$$

(iii) Consider two possible mechanisms, labeled A and B, (possibly associated with different levels of p_g or by different disclosure policies) such that equilibrium $i = A, B$ induces trade

volume $q_i(\cdot)$ across the two periods. Suppose

$$\int_0^1 q_A(\theta)f(\theta)d\theta = \int_0^1 q_B(\theta)f(\theta)d\theta$$

but there exists $\tilde{\theta} \in (0,1)$ such that $q_A(\theta) \geq q_B(\theta)$ for $\theta \leq \tilde{\theta}$ and $q_A(\theta) \leq q_B(\theta)$ for $\theta \geq \tilde{\theta}$. Then, equilibrium A yields higher welfare than equilibrium B, strictly so if $q_A(\theta) \neq q_B(\theta)$ for a positive measure of θ's.

Proof of Theorem 5.

Proof of Theorem 5-(i). As is standard, the monotonicity of $q(\cdot)$ follows from (IC). Fix any mechanism $\mathcal{M}$ that satisfies (IC) and (IR) for every $\theta \in [0,1]$ and all restrictions stated in the main text. For convenience of exposition, let $t_1(\theta)$ and $t_2(\theta)$ be respective transfers given to the firms in period 1 and 2 if they report their types as θ. Recall that $t_1(\cdot)$ can be made by the government or the private buyers, but $t_2(\cdot)$ is made only by the buyers. Moreover, notice that $\mathrm{E}[\theta|t_2(\theta) = t] = t$ for any $t \geq I$ such that $Pr(\theta|t_2(\theta) = t) > 0$, which follows from the zero-profit condition.

Define $\hat{\theta} := \sup\{\theta : q(\theta) = 2\}$ and $\check{\theta} := \sup\{\theta : q(\theta) = 1\}$. In addition, define $\bar{t} \geq I$ as total transfer offered to the firms reporting their types as $\theta \in (\hat{\theta}, \check{\theta}]$. By the non-rationing restriction, $q(\theta) = 1$ for all $\theta \in (\hat{\theta}, \check{\theta}]$. Hence, from ($IC$), $t(\theta) = \bar{t}$ for all $\theta \in (\hat{\theta}, \check{\theta}]$. Moreover, define $\underline{t} := \hat{\theta} - S$. Since $\check{\theta} \leq \bar{t} + S$ in equilibrium, we have $\bar{t} \geq \underline{t}$.

Before proving $q(\theta) \leq 1$ for all $\theta \geq \theta_0^*$, we first observe

$$t_1(\theta) \leq \bar{t} \text{ for all } \theta \in [0, \hat{\theta}]. \tag{E.2}$$

To prove this, suppose there exists $\tilde{\theta} \in [0, \hat{\theta}]$ such that $t_1(\tilde{\theta}) > \bar{t}$. Since $t_1(\tilde{\theta}) + S + \theta > \bar{t} + S + \theta$, every type $\theta \in (\hat{\theta}, \check{\theta}]$ will misreport its type as $\tilde{\theta}$, a contradiction.

We now prove $q(\theta) \leq 1$ for all $\theta \geq \theta_0^*$. By definition of $\hat{\theta}$, it suffices to show $\hat{\theta} \leq \theta_0^*$, or equivalently, $\hat{\theta} \leq m(0, \hat{\theta}) + S$. To show $\hat{\theta} \leq \theta_0^*$, we decompose the set $[0, \hat{\theta}] = \{\theta : q(\theta) = 2\}$ into two disjoint subsets $\hat{\Theta}_l$ and $\hat{\Theta}_h$ such that types $\theta \in \hat{\Theta}_l$ receive $t_2(\theta) < \bar{t}$ in $t = 2$, and types $\theta \in \hat{\Theta}_h$ receive $t_2(\theta) \geq \bar{t}$.

<u>Step 1.</u> $\underline{t} \leq \mathrm{E}[\theta|\theta \in \hat{\Theta}_l]$.

Fix any $\theta \in \hat{\Theta}_l$. By truthful reporting, type θ earns the payoff $t_1(\theta) + t_2(\theta) + 2S$. Since this type can get the payoff $t_1(\hat{\theta}) + t_2(\hat{\theta}) + 2S$ by reporting its type as $\hat{\theta}$, (IC) requires $t_1(\theta) + t_2(\theta) + 2S \geq t_1(\hat{\theta}) + t_2(\hat{\theta}) + 2S$. Since type $\hat{\theta}$'s (IC) implies the reverse inequality, we have

$t_1(\theta) + t_2(\theta) = t_1(\hat{\theta}) + t_2(\hat{\theta})$ for any $\theta \in \hat{\Theta}_l$. Since type $\hat{\theta}$ is indifferent between $q = 2$ and $q = 1$, we must have $t_1(\hat{\theta}) + t_2(\hat{\theta}) + 2S = \hat{\theta} + \bar{t} + S$. Given $\hat{\theta} = \underline{t} + S$, we have $t_1(\hat{\theta}) + t_2(\hat{\theta}) = \underline{t} + \bar{t}$, which implies $t_1(\theta) + t_2(\theta) = \underline{t} + \bar{t}$. From (E.2), it then follows $t_2(\theta) = \underline{t} + (\bar{t} - t_1(\theta)) \geq \underline{t}$. Furthermore, the zero-profit condition for the buyers in $t = 2$ implies $\mathrm{E}[\theta|\theta \in \hat{\Theta}_l] = \mathrm{E}[t_2(\theta)|\theta \in \hat{\Theta}_l]$. Hence, we have $\mathrm{E}[\theta|\theta \in \hat{\Theta}_l] = \mathrm{E}[t_2(\theta)|\theta \in \hat{\Theta}_l] \geq \mathrm{E}[\underline{t}|\theta \in \hat{\Theta}_l] = \underline{t}$.

Step 2. $\underline{t} \leq \mathrm{E}[\theta|\theta \in \hat{\Theta}_h]$.

We can decompose $\hat{\Theta}_h$ further into the two disjoint subsets $\hat{\Theta}_h^{>\bar{t}} := \{\theta \in \hat{\Theta}_h | t_2(\theta) > \bar{t}\}$ and $\hat{\Theta}_h^{=\bar{t}} := \{\theta \in \hat{\Theta}_h | t_2(\theta) = \bar{t}\}$. We show below $\underline{t} \leq \mathrm{E}[\theta|\theta \in \hat{\Theta}_h^{>\bar{t}}]$ and $\underline{t} \leq \mathrm{E}[\theta|\theta \in \hat{\Theta}_h^{=\bar{t}}]$.

First, consider $\hat{\Theta}_h^{>\bar{t}}$. Since $t_2(\theta) \leq \bar{t}$ if $\theta \in \hat{\Theta}_l \cup \hat{\Theta}_h^{=\bar{t}}$ and $t_2(\theta) = \bar{t}$ if $\theta \in (\hat{\theta}, \check{\theta}]$ (if such θ sells in $t = 2$), only types $\theta \in \hat{\Theta}_l^{>\bar{t}}$ will sell at a price higher than $\bar{t}$ in $t = 2$. From the zero-profit condition for the buyers in $t = 2$, it then follows $\mathrm{E}[\theta|\theta \in \hat{\Theta}_h^{>\bar{t}}] = \mathrm{E}[t_2(\theta)|\theta \in \hat{\Theta}_h^{>\bar{t}}]$. Since $t_2(\theta) > \bar{t} \geq \underline{t}$ for all $\theta \in \hat{\Theta}_h^{>\bar{t}}$, we have $\mathrm{E}[\theta|\theta \in \hat{\Theta}_h^{>\bar{t}}] = \mathrm{E}[t_2(\theta)|\theta \in \hat{\Theta}_h^{>\bar{t}}] > \bar{t} \geq \underline{t}$.

Next, consider $\hat{\Theta}_h^{=\bar{t}}$. Suppose to the contrary $\mathrm{E}[\theta|\theta \in \hat{\Theta}_h^{=\bar{t}}] < \underline{t}$. Let $\check{\Theta}$ be the set of types $\theta \in (\hat{\theta}, \check{\theta}]$ selling in $t = 2$. Since these types receive $t_2(\theta) = \bar{t}$, we have $\mathrm{E}[t_2(\theta)|\hat{\Theta}_h^{=\bar{t}} \cup \check{\Theta}] = \bar{t}$. Moreover, since $0 \leq Pr(\theta \in \check{\Theta}) \leq Pr(\theta \in (\hat{\theta}, \check{\theta}])$, there exists $\overline{\theta} \in (\hat{\theta}, \check{\theta}]$ such that $Pr(\theta \in \check{\Theta}) = Pr(\theta \in [\overline{\theta}, \check{\theta}])$ and $\mathrm{E}[\theta|\theta \in [\overline{\theta}, \check{\theta}]] \geq \mathrm{E}[\theta|\theta \in \check{\Theta}]$. By Lemma A.1-(i), we have

$$
\begin{aligned}
\frac{\partial}{\partial y}\mathrm{E}[\theta|\theta \in \hat{\Theta}_h^{=\bar{t}} \cup [\overline{\theta}, y]] &= \frac{f(y)}{\{F(y) - F(\overline{\theta})\} + Pr(\theta \in \hat{\Theta}_h^{=\bar{t}})}(y - m(\overline{\theta}, y)) \\
&\quad - \frac{f(y)}{[\{F(y) - F(\overline{\theta})\} + Pr(\theta \in \hat{\Theta}_h^{=\bar{t}})]^2} \int_{\theta \in \hat{\Theta}_h^{=\bar{t}}} \theta dF(\theta) \\
&\leq \frac{f(y)}{F(y) - F(\overline{\theta})}(y - m(\overline{\theta}, y)) \\
&= \frac{\partial}{\partial y} m(\overline{\theta}, y) \\
&< 1.
\end{aligned}
\tag{E.3}
$$

Since $\mathrm{E}[\theta|\theta \in \hat{\Theta}_h^{=\bar{t}}] < \underline{t}$ and $\underline{t} + S = \hat{\theta} \leq \overline{\theta}$, we have $\mathrm{E}[\theta|\theta \in \hat{\Theta}_h^{=\bar{t}}] + S < \overline{\theta}$. Combining this inequality with (E.3), we have $\mathrm{E}[\theta|\theta \in \hat{\Theta}_h^{=\bar{t}} \cup [\overline{\theta}, \check{\theta}]] + S < \check{\theta}$. Since $\check{\theta} \leq \bar{t} + S$, we have $\bar{t} + S \geq \check{\theta} > \mathrm{E}[\theta|\theta \in \hat{\Theta}_h^{=\bar{t}} \cup [\overline{\theta}, \check{\theta}]] + S \geq \mathrm{E}[\theta|\theta \in \hat{\Theta}_h^{=\bar{t}} \cup \check{\Theta}] + S$, where the last inequality follows from $\mathrm{E}[\theta|\theta \in \check{\Theta}] \leq \mathrm{E}[\theta|\theta \in [\overline{\theta}, \check{\theta}]]$. This implies $\bar{t} = \mathrm{E}[t_2(\theta)|\theta \in \hat{\Theta}_h^{=\bar{t}} \cup \check{\Theta}] > \mathrm{E}[\theta|\theta \in \hat{\Theta}_h^{=\bar{t}} \cup \check{\Theta}]$, which contradicts the zero-profit condition for the buyers in $t = 2$.

Step 3. $\hat{\theta} \leq \theta_0^*$.

Since $\underline{t} \leq \mathrm{E}[\theta|\theta \in \hat{\Theta}_l]$ from Step 1, $\underline{t} \leq \mathrm{E}[\theta|\theta \in \hat{\Theta}_h]$ from Step 2, and $\hat{\Theta}_l \cup \hat{\Theta}_h = [0, \hat{\theta}]$, we have $\underline{t} \leq \mathrm{E}[\theta|\theta \leq \hat{\theta}] = m(0, \hat{\theta})$, and thus $\hat{\theta} = \underline{t} + S \leq m(0, \hat{\theta}) + S$, or equivalently, $\hat{\theta} \leq \theta_0^*$.

Proof of Theorem 5-(ii). First, recall

$$t(\theta) = u^M(\theta) - \theta(2 - q(\tilde{\theta})) - Sq(\tilde{\theta}). \tag{E.4}$$

Next, the envelope theorem applied to (IC) along with $u^M(1) = 2$ gives us

$$u^M(\theta) = u^M(1) - \int_\theta^1 (2 - q(s))ds = 2 - \int_\theta^1 (2 - q(s))ds. \tag{E.5}$$

Substituting (E.4) and (E.5) into the welfare and integrating by parts leads to

$$\begin{aligned}
W(M) &= \int_0^1 \left[u^M(\theta) + (1+\lambda)\theta q(\theta) - (1+\lambda)t(\theta)\right] f(\theta)d\theta. \\
&= \int_0^1 \left[u^M(\theta) + (1+\lambda)\theta q(\theta) - (1+\lambda)(u^M(\theta) - \theta(2 - q(\theta)) - Sq(\theta))\right] f(\theta)d\theta \\
&= \int_0^1 \left[(1+\lambda)Sq(\theta) - \lambda u^M(\theta) + 2(1+\lambda)\theta\right] f(\theta)d\theta \\
&= \int_0^1 \left[\left((1+\lambda)S - \lambda\frac{F(\theta)}{f(\theta)}\right) q(\theta) - 2\lambda + 2\left((1+\lambda)\theta + \lambda\frac{F(\theta)}{f(\theta)}\right)\right] f(\theta)d\theta.
\end{aligned}$$

This gives us (E.1). Revenue equivalence follows also from the observation that the welfare depends only on the allocation rule q.

Proof of Theorem 5-(iii). The welfare difference between the two equilibria is

$$\begin{aligned}
W_A - W_B &= \int_0^1 J(\theta)[q_A(\theta) - q_B(\theta)]f(\theta)\theta \\
&= \int_0^{\tilde{\theta}} J(\theta)[q_A(\theta) - q_B(\theta)]f(\theta)\theta + \int_{\tilde{\theta}}^1 J(\theta)[q_A(\theta) - q_B(\theta)]f(\theta)\theta \\
&> \int_0^{\tilde{\theta}} J(\tilde{\theta})[q_A(\theta) - q_B(\theta)]f(\theta)\theta + \int_{\tilde{\theta}}^1 J(\tilde{\theta})[q_A(\theta) - q_B(\theta)]f(\theta)\theta \\
&= J(\tilde{\theta}) \int_0^1 (q_A(\theta) - q_B(\theta))f(\theta)d\theta = 0,
\end{aligned}$$

where the inequality follows from the fact that $q_A(\theta) \geq q_B(\theta)$ for $\theta \leq \tilde{\theta}$ and $q_A(\theta) \leq q_B(\theta)$ for $\theta \geq \tilde{\theta}$ and that J is decreasing. The inequality must be strict if $q_A(\theta)$ and $q_B(\theta)$ differ on a positive measure of θ's, since J is strictly decreasing. *Q.E.D.*

Proposition 7. *The equilibria are compared as follows.*

(i) Given a transparent bailout policy, an equilibrium with $t = 1$ market shutdown dominates in welfare an equilibrium without $t = 1$ market shutdown.

(ii) Given a secret bailout policy, an equilibrium with $t = 1$ market shutdown dominates in welfare an equilibrium without $t = 1$ market shutdown.

(iii) With $t = 1$ market shutdown, an equilibrium under secrecy dominates in welfare an equilibrium under transparency.

Proof of Proposition 7.

Proof of Proposition 7-(i).

Let $q_T(\theta) \in \{0, 1, 2\}$ denote the total quantity of assets sold by type-θ firm across the two periods in each alternative type of equilibria $T \in \{SBS, MBS, MR1, MR2, sd\}$ (we use notation $T = sd$ to label each alternative type of equilibria under the shutdown of the market in $t = 1$).

We first show that the $t = 1$ market shutdown yields at least the same welfare as either SBS or MBS does. From Section C.1.5, any $p_g \in P^{SBS}$ admits an equilibrium with the $t = 1$ market shutdown characterized by the marginal type $\theta_g^{sd}(p_g) = \theta_{g\emptyset}^{SBS}(p_g)$, where we have $q_{sd}(\theta) = q_{SBS}(\theta)$ for all $\theta \in [0, 1]$. Similarly, any $p_g \in P^{MBS}$ yields an equilibrium with the $t = 1$ market shutdown characterized by $\theta_g^{sd}(p_g) = \theta_g^{MBS}(p_g)$, where $q_{sd}(\theta) = q_{MBS}(\theta)$ for all $\theta \in [0, 1]$.

We next compare welfare under either an MR1 or MR2 equilibrium with that under the $t = 1$ market shutdown. First, fix $p_g \in P^{MR}$ that admits an MR1 equilibrium. As shown in the proof of Proposition 3, the same p_g admits an equilibrium with the $t = 1$ market shutdown characterized by $\theta_g^{sd}(p_g) \geq \theta_1^{MR1}(p_g)$, where (C.7) defines $\theta_g^{sd}(p_g)$. Since $\theta_g^{sd}(p_g)$ is increasing in p_g, there exists $p_g' \leq p_g$ such that $\theta_g^{sd}(p_g') = \theta_1^{MR1}(p_g)$, and thus $q_{sd}(\theta) = q_{MR1}(\theta)$ for all $\theta \in [0, 1]$.

Next, fix $p_g \in P^{MR}$ that admits an MR2 equilibrium, which yields

$$q_{MR2}(\theta) = \begin{cases} 2 & \text{if } \theta \leq \theta_1^{MR2}, \\ 1 & \text{if } \theta \in (\theta_1^{MR2}, \gamma(\theta_{g\emptyset}^{MR2})], \\ 0 & \text{if } \theta > \gamma(\theta_{g\emptyset}^{MR2}). \end{cases}$$

As shown in the proof of Proposition 3, the same p_g admits the equilibrium with the $t = 1$ market shutdown characterized by $\theta_g^{sd}(p_g) \geq \theta_{g\emptyset}^{MR2}(p_g)$, where $\theta_g^{sd}(p_g)$ is determined by either (C.7) or

$p_g = m(\theta_g, \gamma(\theta_g))$. This equilibrium yields

$$q_{sd}(\theta) = \begin{cases} 2 & \text{if } \theta \leq \theta_g^{sd} \wedge \theta_0^*, \\ 1 & \text{if } \theta \in (\theta_g^{sd} \wedge \theta_0^*, \gamma(\theta_g^{sd})], \\ 0 & \text{if } \theta > \gamma(\theta_g^{sd}). \end{cases}$$

As shown in Section C.1.5, $\theta_g^{sd}(p_g)$ is increasing in p_g. This implies that there exists an equilibrium under the $t = 1$ market shutdown for some $p'_g \leq p_g$ such that $\int_0^1 q_{sd}(\theta)dF(\theta) = \int_0^1 q_{MR2}(\theta)dF(\theta)$. Since $\theta_1^{MR2}(p_g) < \theta_0^*$ and $\theta_1^{MR2}(p_g) < \theta_{g\emptyset}^{MR2}(p_g)$ from (C.17) and (C.20), we have $\theta_1^{MR2}(p_g) < \theta_g^{sd}(p'_g) < \theta_{g\emptyset}^{MR2}(p_g)$. These inequalities imply that $q_{MR2}(\theta) \leq q_{sd}(\theta)$ if $\theta \leq \theta_g^{sd}(p'_g)$, where $q_{MR2}(\theta) < q_{sd}(\theta)$ for all $\theta \in (\theta_1^{MR2}(p_g), \theta_g^{sd}(p'_g)]$; and $q_{MR2}(\theta) \geq q_{sd}(\theta)$ otherwise, where $q_{MR2}(\theta) > q_{sd}(\theta)$ for all $\theta \in (\gamma(\theta_g^{sd}(p'_g)), \gamma(\theta_{g\emptyset}^{MR2}(p_g))]$. Hence, by Theorem 5-(iii), the equilibrium with the $t = 1$ market shutdown arising from p'_g yields higher welfare than the MR2 equilibrium arising from p_g.

Proof of Proposition 7-(ii).

Similar to the proof of Proposition 7-(i), let $q_T(\theta) \in \{0, 1, 2\}$ denote the total quantity of assets sold by type-θ firm over the two periods in each alternative type of equilibria $T \in \{SMR, sd\}$.

First, suppose $p_g \geq p_0^*$ admits an SMR equilibrium. In this equilibrium, we have $q_{SMR}(\theta) = 2$ for all $\theta \in [0, \theta_1^{SMR}(p_g)]$, $q_{SMR}(\theta) = 1$ for all $\theta \in (\theta_1^{SMR}(p_g), \theta_{g\emptyset}^{SMR}(p_g)]$, and $q_{SMR}(\theta) = 0$ otherwise. When the market is shut down in $t = 1$, the same p_g admits an equilibrium which yields $q_{sd}(\theta) = 2$ for all $\theta \in [0, \theta_0^*]$, $q_{sd}(\theta) = 1$ for all $\theta \in (\theta_0^*, (p_g + S) \wedge 1]$, and $q_{sd}(\theta) = 0$ otherwise. Since $\theta_{g\emptyset}^{SMR}(p_g) = (p_g + S) \wedge 1$ from (D.3) and $\theta_1^{SMR}(p_g) \leq \theta_0^*$ from (D.2), there exists a $p'_g \leq p_g$ which makes the total trade volume with the $t = 1$ market shutdown equal to that in the SMR equilibrium arising from p_g. Since $\theta_1^{SMR}(p_g) \leq \theta_0^* \leq (p'_g + S) \wedge 1 \leq \theta_{g\emptyset}^{SMR}(p_g)$, Theorem 5-(iii) implies that the equilibrium with the $t = 1$ market shutdown arising from p'_g yields higher welfare than the SMR equilibrium arising from p_g.

Next, suppose $p_g < p_0^*$ admits an SMR equilibrium. Since $\theta_1^{SMR}(p_g) < \theta_{g\emptyset}^{SMR}(p_g) < \theta_0^*$, the same p_g admits an equilibrium with the $t = 1$ market shutdown, which yields $q_{sd}(\theta) > q_{SMR}(\theta)$ if $\theta \in (\theta_1^{SMR}, \theta_0^*]$ and $q_{sd}(\theta) = q_{SMR}(\theta)$ otherwise. By the assumption $J(\theta) > 0$ if and only if $\theta < \hat{\theta}^*$ and $\hat{\theta}^* > \theta_0^*$, the equilibrium with the $t = 1$ market shutdown yields strictly higher welfare than the SMR equilibrium, as was to be shown.

Proof of Proposition 7-(iii).

First, suppose p_g admits an equilibrium under transparency characterized by $\theta_g^{sd}(p_g)$, where (C.2) defines $\theta_g^{sd}(p_g)$ subject to $m(0, \theta_g) < I$. In this equilibrium, we have $q(\theta) = 1$ if $\theta \in [0, \gamma(\theta_g^{sd}(p_g))]$ and $q(\theta) = 0$ if $\theta > \gamma(\theta_g^{sd}(p_g))$. Under secrecy, the same p_g admits an equilibrium, in which $q(\theta) = 2$ if $\theta \in [0, \theta_0^*]$, $q(\theta) = 1$ if $\theta \in (\theta_0^*, (p_g + S) \wedge 1]$, and $q(\theta) = 0$ if $\theta > (p_g + S) \wedge 1$. Since $(p_g + S) \wedge 1 = \gamma(\theta_g^{sd}(p_g))$ and $J(\theta) > 0$ for all $\theta \leq \theta_0^*$, the equilibrium under secrecy yields higher welfare than the equilibrium under transparency.

Next, suppose p_g admits an equilibrium under transparency characterized by $\theta_g^{sd}(p_g)$, where (C.7) defines $\theta_g^{sd}(p_g)$ subject to (C.8) and (C.10). In this equilibrium, we have $q(\theta) = 2$ if $\theta \in [0, \theta_g^{sd}(p_g)]$, $q(\theta) = 1$ if $\theta \in (\theta_g^{sd}(p_g), \gamma(\theta_g^{sd}(p_g))]$, and $q(\theta) = 0$ if $\theta > \gamma(\theta_g^{sd}(p_g))$. Since $\theta_g^{sd}(p_g) \leq \theta_0^* < \gamma(\theta_g^{sd}(p_g))$ and $p_g \leq m(\theta_g^{sd}(p_g), \gamma(\theta_g^{sd}(p_g)))$ from (C.7) and (C.10), there exists an equilibrium under secrecy for some $p_g' \in (\theta_0^* - S, m(\theta_g^{sd}(p_g), \gamma(\theta_g^{sd}(p_g))))$, which yields the same total trade volume as that under transparency with p_g. In this equilibrium, we have $q(\theta) = 2$ if $\theta \in [0, \theta_0^*]$, $q(\theta) = 1$ if $\theta \in (\theta_0^*, (p_g + S) \wedge 1]$, and $q(\theta) = 0$ if $\theta > (p_g + S) \wedge 1$. Hence, by Theorem 5-(iii), the equilibrium under secrecy arising from p_g' yields higher welfare than that under transparency arising from p_g.

Lastly, suppose p_g admits an equilibrium under transparency characterized by $\theta_g^{sd}(p_g)$, where $\theta_g^{sd}(p_g)$ is determined by $p_g = m(\theta_g, \gamma(\theta_g))$ subject to $\theta_g > \theta_0^*$. In this equilibrium, we have $q(\theta) = 2$ if $\theta \in [0, \theta_0^*]$, $q(\theta) = 1$ if $\theta \in (\theta_0^*, \gamma(\theta_g^{sd}(p_g))]$, and $q(\theta) = 0$ if $\theta > \gamma(\theta_g^{sd}(p_g))$. Under secrecy, the same p_g admits an equilibrium in which $q(\theta) = 2$ if $\theta \in [0, \theta_0^*]$, $q(\theta) = 1$ if $\theta \in (\theta_0^*, (p_g + S) \wedge 1]$, and $q(\theta) = 0$ if $\theta > (p_g + S) \wedge 1$. Since $(p_g + S) \wedge 1 = \gamma(\theta_g^{sd}(p_g))$, each type θ sells the exactly same quantity of assets under secrecy as it would under transparency. Hence, by Theorem 5-(iii), the equilibrium under secrecy induces the same welfare as the equilibrium under transparency. *Q.E.D.*

Proposition 8. *The optimal bailout mechanism has*

$$q^*(\theta) = \begin{cases} 2 & \text{if } \theta \leq \theta_0^*, \\ 1 & \text{if } \theta \in (\theta_0^*, \hat{\theta}^*], \\ 0 & \text{if } \theta > \hat{\theta}^*. \end{cases}$$

The optimal policy is implemented by a secret bailout policy with $p_g = \hat{\theta}^* - S$ accompanied by the shutdown of the market in $t = 1$.

Proof of Proposition 8. Let q be an arbitrary feasible allocation rule satisfying $[P]$. Then,

$$
\begin{aligned}
& W(q^*) - W(q) \\
&= \int_0^1 J(\theta)[q^*(\theta) - q(\theta)]f(\theta)d\theta \\
&= \int_0^{\min\{\hat{\theta}^*, \theta_0^*\}} J(\theta)[q^*(\theta) - q(\theta)]f(\theta)d\theta + \int_{\min\{\hat{\theta}^*, \theta_0^*\}}^{\max\{\hat{\theta}^*, \theta_0^*\}} J(\theta)[q^*(\theta) - q(\theta)]f(\theta)d\theta \\
&\quad + \int_{\max\{\hat{\theta}^*, \theta_0^*\}}^1 J(\theta)[q^*(\theta) - q(\theta)]f(\theta)d\theta.
\end{aligned}
$$

The first integral is nonnegative since $J(\theta) \geq 0$ and $q(\theta) \leq 2 = q^*(\theta)$ for $\theta < \min\{\hat{\theta}^*, \theta_0^*\} \leq \hat{\theta}^*$. The last integral is also nonnegative since $J(\theta) \leq 0$ and $q(\theta) \geq 0 = q^*(\theta)$ for $\theta > \max\{\hat{\theta}^*, \theta_0^*\} \geq \hat{\theta}^*$. Finally, we show below that the middle integral is also nonnegative. Suppose first $\hat{\theta}^* < \theta_0^*$. Then, for any $\theta \in (\min\{\hat{\theta}^*, \theta_0^*\}, \max\{\hat{\theta}^*, \theta_0^*\}] = (\hat{\theta}^*, \theta_0^*]$, $J(\theta) \leq 0$ and $q(\theta) \geq 1 = q^*(\theta)$, so the middle integral is nonnegative. Suppose next $\hat{\theta}^* < \theta_0^*$. Then, for $\theta \in (\min\{\hat{\theta}^*, \theta_0^*\}, \max\{\hat{\theta}^*, \theta_0^*\}] = (\theta_0^*, \hat{\theta}^*]$, $J(\theta) \geq 0$ and $q(\theta) \leq 1 = q^*(\theta)$, so the middle integral is nonnegative. Since all three integrals are nonnegative, the allocation rule q^* is optimal.

The last statement follows from Proposition 6-(i). *Q.E.D.*

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