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Young children's mathematics problem solving in playworld

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Abstract

In the literature in recent years, a number of developmental studies have demonstrated the importance of children entering the school environment with a solid foundation of mathematical content knowledge, and argued that problem solving, as an important mathematical process, should be acknowledged in early childhood mathematical education. However, there is less research on how children process mathematics information through problem solving in play-based early childhood education settings. This paper draws upon a cultural-historical concept of play, motives and pedagogical process of a playworld approach to investigate how Math@Imagination creates the conditions to support children to achieve meaningful learning of the mathematical concepts of repeating patterns. We argue that Math@Imagination, as a mathematical pedagogical approach within the worlds of imaginary situations, should be promoted, as it builds the motivating conditions that support meaningful learning of mathematical concepts in the double sense created in children. This study also contributes to understanding young children's mathematical problem-solving processes in the collective imaginary situation, considering how learning processes become personally meaningful for children and capturing teachers' role in play for supporting children's mathematisation.

Keywords: Meaningful learning, Patterning, Motives, Playworld

Declarations

This project was funded by Authors' University, faculty of education small grant research funding. There is no conflict of interest. The authors' university Human Research Ethics Committee (Project ID:18835) and the Department of Education and Early Childhood Development (Project ID: 2019-004013) granted ethic approval for the project. The informed consent was provided by family, teachers and directors of the early learning centre for the use of collected data for research and educational purpose.

Introduction

A large number of studies have demonstrated that young children have significant capacities to learn and master mathematical concepts at a very early age (Fisher et al. 2013; Ginsburg et al. 2008; Presser et al. 2015). Using six longitudinal datasets, Duncan et al. (2007) revealed that early mathematics learning is a powerful predictor of children's mathematics achievement in school. A large number of developmental studies have been undertaken to better understand young children's mathematical learning and development, and advocated that early childhood education in mathematics is vital (Docket and Goff 2013; Dumas et al. 2017; Presser et al. 2015). As Hachey (2013) argued, there is a need 'for intentional, high-quality early childhood mathematics education to help build young children's fragile intuitive knowledge into the robust and generalizable knowledge that marks more sophisticated mathematical thinking' (p. 444). This approach requires that early childhood education be better prepared to support young children in building foundational mathematics knowledge and skills.

Recent research has focused on children's achievement in mathematics, teachers' beliefs and attitudes in mathematics education, teachers' professional development and teachers' understanding of early childhood mathematics (Linder and Simpson 2018; MacDonald and Murphy 2019). There is less research on how children process the mathematics information and their process of problem solving in early childhood education settings. Further, previous studies have argued that problem solving, as an important mathematical process, should be acknowledged in early childhood mathematical education (Helenius et al. 2016; Johansson et al. 2016). In addition, recent research has highlighted the value of play-based mathematics learning and proposed that there is a need for increased play-based mathematics content in early childhood settings (Cohrssen et al. 2016; Moss et al. 2016). Therefore, in this paper, we investigate how to create the motivating conditions to support children's problem solving that helps achieve meaningful learning of mathematical concepts—specifically focusing on the development of algebraic reasoning of creating repeating patterns in play.

Imaginative play and mathematical learning

Vygotsky (1966) argued that imaginative play is the leading activity for preschool aged children, as it determines their development. He explained that '[a]s in the focus of a magnifying glass, play contains all developmental tendencies in a condensed form; in play, it is as though the child were trying to jump above the level of his normal behaviour' (Vygotsky 1966, p. 16). In play, children create the imaginary situation to meet their desires, where they change the meaning of the objects and actions, giving them a new sense. Further, 'from the point of view of development, the fact of creating an imaginary situation can be regarded as a means of developing abstract thought', as children see the world with meaning and sense (Vygotsky 1966, p. 18). The imagination in play acts as a bridge that enables children to imitate the roles of adults and explore cultural knowledge, such as mathematical concepts, and develop their conceptual thinking (Fleer 2011; Li 2020; Worthington and van Oers 2016).

Some empirical cultural-historical studies have provided evidence that Vygotsky's imaginary play acts as a source of development that positively affects children's mathematical learning and development. Poland and van Oers's (2007) large-scale longitudinal experiment showed strong evidence for the promotion of mathematical thinking through children's meaningful learning in play-based contexts. They argued that children in early childhood should learn to schematise to identify the relationships between objects, such as numbers and variables, because schematisations help children solve problems and communicate mathematical reasoning. Further, Worthington and van Oers's (2016) longitudinal and ethnographic studies have collectively found that children draw on their personal cultural knowledge in their imaginary play, which in turn influences their mathematical thinking. Therefore, it can be noted that imaginary play creates the source for young children's mathematical concept learning and development. A study conducted with 10 Scottish public nurseries by Munn and Schaffer (1993) found that there is a great need to maximise the adult–child interaction in play to support children's mathematics in play, as there is less mathematical content observed in role play without adult involvement. Through the lens of cultural-historical theory, van Oers (2010) argued that mathematical learning and problem solving can be performed in a meaningful way while children play with peers and adults in a play-based curriculum. He also concluded that there is an urgent need to detail how the learning process of mathematical operations and performance can become meaningful in the context of children's imaginative play. What we do not know is how the problem-solving process can be embedded in children's imaginative play and service to children's mathematic concept learning. Therefore, in this study, we are interested in the question of how the mathematical problem-solving process becomes personally meaningful for young children in the mathematical imaginative playworld.

Playworld approach

A playworld begins with a selected story, whereby children and adults build emotional connections with the story characters before they collectively enter the playworld (Fleer 2017, 2018; Hakkarainen et al. 2013; Li 2020; Lindqvist 2003). In the playworld, adults take active co-player roles and dramatise the emotionally charged problem for children and adults collectively to solve. By doing so, adults create the conditions to support children's learning in different areas. Empirical studies have identified the playworld approach as an effective pedagogy, including adults' active role and the collaboration between children and adults in the playworld (Hakkarainen and Bredikyte 2018; Marjanovic-Shane et al. 2011); the active participation of pre-service teachers and their play guidance in children's narrative playworld (Hakkarainen et al. 2013); the exploration of emotional-cognitive transformations in playworlds (Ferholt 2010); and the pedagogy of play in children's science, technology and engineering concept learning (Fleer 2017, 2018). Most recent studies have pointed out that the playworld approach is an important pedagogical practice to support children's problem solving in collective play (Fleer 2017), to transform children's engagement from passive to more active (Rainio 2010), and to form a transformative activity for children and adults (Ferholt 2009). However, these studies have mostly reported analysis of the collaborative relations between adults and children, which are beneficial for children's learning and development in imaginative playworlds, whereas little is known about how the problem-solving process becomes meaningful learning for young children in the imaginative playworld. Thus, this leads to the central research question of the present study by drawing upon the playworld approach: *How does Math@Imagination, as a new form of the playworld approach, create the conditions to support children to achieve meaningful learning of mathematical concepts?*

Meaningful learning in a double sense

Following Leont'ev's (1978) discussion of the two basic levels of meaning in human actions, van Oers (2010) elaborated that 'in order to be meaningful and to be stimulating for development, learning necessarily should be meaningful in this double sense: including both a cultural and a personal dimension at the same time' (p. 26). van Oers (2010) also further examplified, cultural meaning is related to the cultural tools that teachers provide to young children in play in a timely manner, such as mathematical concepts, expression and meanings for communication about the mathematical dimensions of everyday practices; while the personal dimension of meaning relates to development of the motives and involvement of personal sense in these cultural meanings.

In this study, we draw upon meaningful learning in a double sense to elucidate how children achieve meaningful learning by play with teachers in the shared imaginary situation in an imaginative playworld context. The improvement of children's mathematical thinking is achieved by the appropriation of cultural tools (mathematical meanings—the repeating pattern) in meaningful contexts. Mathematics, as culturally-based knowledge, should be achieved in the quality of the social situation, which contributes to children's capabilities for participation and enables children to communicate the mathematical meanings in culturally appropriate ways (Bishop 2016; van Oers 2010). Imagination in play enables children to imitate social roles and rules and contribute to the cultural contexts of mathematical learning, as 'everything the imagination creates is always based on elements taken from reality' (Vygotsky 2004, p. 13). That is, children's play as a social context creates the possibility to communicate mathematical meanings and contributes to the development of mathematical thinking. However, how to ensure that the process of performance for mathematical thinking becomes personally meaningful for children in play remains a challenge. To address this issue, we need to further understand the personal dimension of meaning, which refers to the development of children's motives in solving mathematical problems (van Oers 2010).

The development of motive

Cultural-historical theory is built on the idea that children's learning and development occur through their meaningful social interactions in cultural practices. This orients us to realise the dynamic relations between demands from cultural practices and children's motives in social interactions. According to Leont'ev's (1978) activity theory, as opposed to having an internal source, motives are determined by cultural practice, and subsequently influence how practice is structured. Hedegaard and Fleer (2013) built on this tradition by filling the gap to contribute to the theory of Leontiev that 'conceptualises the process of the transformation of primary biological needs into culturally valued motives as a straightforward process of collective activity. What is missing in this theory is the conceptualisation of the historical institutionalised demands that mediate this process' (p. 200). In other words, the demands from the practices play an important role in the social interaction.

Hedegaard (2014) further developed this theory by explaining the dynamic process of interchanges between the motives and demands a child encounters in different institutional practices, which influence learning and development. She argued that '[l]earning and development are conceptualised as progressing through conflictual relations between demands and motives that both change the child and his environment' (Hedegaard 2014, p. 193). Children develop their motives through participating in the activity settings in institutional practices, while contributing to the activity setting by placing demands on the cultural practice. When a young child joins the play activity setting, the teacher may apply his or her teaching agenda to place demands on children to learn particular concepts, such as a mathematical concept. However, if the child's motive orientation shows that his or her interests do not align with the demands of the teacher, a meaningful learning process will not be achieved. This means that pedagogical choices and demands must become 'emotionally meaningful and interesting as possible for the children' (Cecchin 2013, p. 59)—by doing so, children have opportunities to become involved themselves, and then will be able to try new possibilities (Hedegaard 2002). This requires teachers to value children's motives in play in their pedagogical agendas, which can motivate children's motives, build emotional experience related to the activity setting, and make personal sense in learning. In this paper, we explore children's motives in play and learning to illustrate how the process of solving a mathematical problem becomes personally meaningful for young children in play.

In our expanded theorisation of meaningful learning in a double sense in children's mathematical problem solving, we emphasise the psychological role of Math@Imagination in playworld activity settings. In Figure 1 below, we show the model that we conceptualised to explain the order in which children achieve meaningful learning in a double sense. Children need to develop their personal sense and motives in solving mathematical problems, and Math@Imagination in the playworld needs to be created to support this process of development.

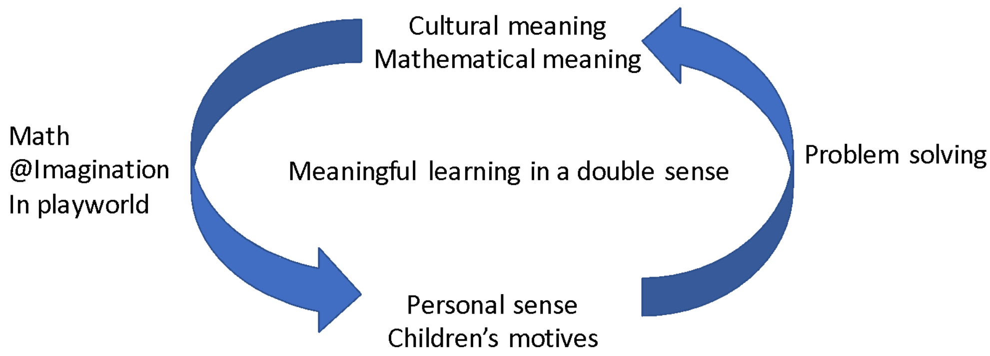

Figure 1. Mathematical meaning learning in a double sense

We draw upon the relational dimensions shown in Figure 1 when studying children's problem solving in mathematics at an early learning centre. The following section explains the research procedure and design.

An educational experiment

This Math@Imagination project draws upon Hedegaard's (2008) educational experiment as a research methodology in the implementation of mathematics education for children aged three to four years in a kindergarten class. An education experiment refers to 'a form of action or intervention research where everyday situations are systematically intervened, combining an educational perspective with a research perspective' (Lindqvist 1995, p. 67). In this Math@Imagination project, we chose the method of educational experiment to collaborate with the teachers, as this shaped how we conducted the research over time. Through an educational experiment, the intervention was planned by the teachers and researchers to form interrelations between the teachers' pedagogy and cultural knowledge (e.g. the repeating pattern as a mathematical concept holding a cultural meaning in this study). Hedegaard (2008) emphasised that an educational experiment should involve group activity, and should consider how collective engagement influences children's problem solving (changing children's conceptual model), active exploration (children's personal thinking) and development of motives. In this study, two researchers and two focus teachers collaboratively developed a mathematical playworld, where a group of 11 children and two teachers shared the collective imaginary situation. In this paper, we particularly focus on how children's problem-solving processes in collective play motivate their active exploration and motives in learning mathematical concepts, and thus create a meaningful learning experience.

Participants and research context

This paper focuses on one class of 22 children aged three to four years, and two teachers, Mayer and Sue (pseudonyms), at an early learning centre in Melbourne, Australia. At the time of the research, teacher Mayer had a qualification at Bachelor level and had completed four years of early childhood education study. The other teacher, Sue, held a diploma and had completed 18 months of full-time early childhood study. Both teachers had worked for about 10 years in this field. The early learning centre was selected randomly in the urban area. For the purpose of this paper, a 35-minute video of 10 children and two teachers entering the mathematical playworld was selected for analysis.

Ethical approval was received from the researchers' university and the Department of Education and Training (DET), Victoria. In alignment with ethical principles at the researchers' university and DET, informed consent was provided by family, teachers and directors of the early learning centre for the use of collected data for research and educational purpose.

Overview of data collection

In the Math@Imagination project, 26 hours of video observations were generated in the early learning centre, displaying teachers' practice with young children and the educational experiment (five visits) related to implementing the mathematical playworld. Six visits in total were made to the early learning centre, with two to three hours of filming per visit. One video camera was organised during each visit to focus on the teachers' interactions with the children in the class, while the other camera captured children's explorations of the whole class.

Pre- and post-intervention pair group interviews with the two teachers were arranged for one hour each, prompted by using a selected video clip. An interview with both teachers was completed before the workshop and after the first video observation at the early learning centre, lasting one hour. Post-intervention pair group interviews lasting one hour were conducted after the video observation of the planned implementation of the mathematical playworld to capture their perspectives on the affectiveness and effectiveness of the Math@Imagination pedagogical approach. A total of two hours of filmed interview data were collected.

The focus group discussion and workshop with two teachers were undertaken to explore the play and pedagogies, and to introduce the playworld approach to the teachers. The teachers and researchers discussed how to create a mathematical playworld environment that enabled children's joint play and mathematical conceptual learning.

In addition to the researchers generating visual data, the ongoing cooperation between the researchers and teachers was captured with the medium of email. The researchers responded to questions and discussed the planning with the teachers. The researcher–teacher dialogue was generated as part of the data collection during the process of the educational experiment. The ongoing conversation mainly challenges the ideas of the pedagogies related to the innovative approach of teaching mathematics in play-based contexts.

Data analysis

In alignment with the educational experiment, when studying children's mathematical learning in play-based contexts, we drew upon a dialectical-interactive and wholeness approach (Hedegaard 2008; Li 2014), including the societal, institutional and personal, to analyse the data by using the spirals of the analytical framework. The spiral of analysis, as the progress of the analytical model, enabled us to interpret the data in a dialectical way, as the process of interpretation was not linear (Li 2014).

The first spiral of analysis focused on the common-sense interpretation of the visual data to understand the multiple perspectives of the interactions in the activity setting. In this Math@Imagination project, the digital video data also helped us capture the teachers' and children's perspectives and intentions in the activity setting. Further, through the educational experiment, we collaborated with the teachers and discussed the filmed activity settings with the teachers during the pre- and post-interview in order to validate the observation (Hedegaard 2008).

Based on the common-sense interpretation, the second spiral of analysis involved interpreting the single activity settings—called 'situated practice interpretation' (Hedegaard 2008). In this stage, the interactive patterns were identified in relation to the conceptual framework. In the Math@Imagination project, we invested efforts into the interactions between the teachers and children while they entered the playworld environment by capturing the conflicts that arose while children tried to solve the dramatised mathematical problem and transition to the playworld activity setting, and how the teachers coped with these conflicts. This enabled us to identify the children's competence and motives in the activity setting.

The third spiral of analysis directly linked to the research aim—called 'thematic interpretation'. The theoretical relations were generalised by using the concepts to identify the meaningful patterns based on the situated interpretation (Hedegaard 2008). In this Math@Imagination project, we conceptualised the findings by using the concepts of motives and meaningful learning in a double sense in the playworld. By doing so, we attained deeper insights into the development of children's mathematical thinking.

Findings and discussion

One of the aims of the Math@Imagination project was to explore how the mathematical playworld creates conditions that support children's mathematical thinking and conceptual learning through drama. This paper reports how children's problem-solving process became personally meaningful for young children in the mathematical playworld activity setting.

In this section, four vignettes are gathered from the fourth week of the educational experiment, where the children and teachers solved the dramatised mathematical problem created in the playworld. This problem related to the development of algebraic reasoning through a repeating patterns problem. As emphasised by McGarvey (2013), '[e]xploring patterns in the early years is seen as an important introduction to algebraic thinking as children begin to notice similarities and differences between and among patterns, create rules to describe relationships, and eventually represent those relationships using symbols' (p. 564). Therefore, this study analysed children's active exploration of the repeating patterns in the playworld. These data revealed how the children experienced the tensions during the playworld and how the teachers actively responded to the conflicts, which motivated the children's motives to identify which part of the pattern was being repeated. The visual narratives created allowed the researchers to explain the process of using patterns as a problem-solving technique, and how children and educators achieved this during active exploration in the mathematical playworld.

The playworld started with a story book, *Room on the Broom* (Donaldson 2003), telling the story of a kind witch who always has space on her flying broom for her new friends—a collection of animals. After sharing this story with the children, the teacher asked them about their favourite part of the story. Most of the children said that they loved the new broom. When the teacher further asked the children 'why do you like the broom?', a couple of children said that it was new and strong, and they could fly the broom to have an adventure. Therefore, the first couple of playworld activity settings involved creating the new broom, with the children and teachers playing different character roles in the story. The children were told by the kind witch (played by teacher Sue) that her sister, Witchy Britchy, had invited them to attend her birthday party; thus, they needed to create a new broom to fly to her house, which was far away. In this paper, we will focus on the fourth occasion of entering the witch's playworld, where the children and teachers were ready to 'fly' to Witchy Britchy's house. Vignette 1 below occurred towards the end of the fourth session of playworld implementation, showing how the teachers dramatised the emotionally charged mathematical conceptual problem and how the children responded to this situation.

Dramatising the emotionally charged mathematical problem

This vignette occurred in the late morning, just before the children's lunch time. Two teachers and 10 children played their character roles and transitioned into the playworld. Teacher Sue played the witch, which had been her role since the beginning of this educational experiment, and teacher Mayer played a frog. Previously, the children had been able to choose their own roles within the playworld experience based on characters from the book; however, as a result of the planned intervention to enhance the children's understanding of repeating patterns, five children were frogs and five children were dogs. Through the educational experiment, an A–B repeating pattern was intentionally planned as a mathematical concept to solve the dramatised problem. During the day of the implementation, as a result of one child being absent, 10 children and one teacher played the role of six dogs and five frogs, and one teacher played a witch who entered the playworld. The children and teachers wore name badges displaying images of their respective characters.

Vignette 1: Motivating children's emotional engagement in play—'One doggy ate others' cakes'

To begin the playworld, the children and teachers went through the 'magical door and curtain' to change to their play role as they transitioned to the playworld space. The teacher Sue (witch) used her 'wand' to help the children and the other teacher (Mayer) transition to the playworld activity setting. Teacher Mayer (who was a frog) and the children were very excited and animated their roles by making sounds like their characters, while sitting in a circle. Each child was sitting or kneeling on a square mat of their own, waiting for the playworld to begin, as teacher Sue, the witch, held a yellow 'wand' and sat side by side with the children, stating that they had received another letter from the witch's sister, Witchy Britchy.

Teacher Mayer produced the letter, confirming that they had collected it from the letter box, and read aloud in a dramatic voice:

> Dear Witchy, today is my party day. Please fly over. I need to go out to get some cakes [here, teacher Mayer stopped and asked the children if they would like some cakes; the children answered in their roleplay voices: 'woof' or 'ribbit'], but you can help me by setting the table with plates and chairs for all of us. But it is important that the dogs are separated. I do not want two dogs sitting next to each other. I'm worried they'll behave like the dog in this video clip and eat all the cakes. I've sent you the video to watch. Love, Witchy Britchy.

She held the letter with one hand, while gesturing with the other according to the content of the letter. The children were all quiet and listened to her.

Then, teacher Sue, the witch, showed the children the video clip and said, 'this is what Britchy doesn't want to happen'. The video clip showed three dogs sitting together and ready to eat cakes; however, the dog in the middle quickly finished his cake and then ate the other dogs' cakes. The other dogs appear shocked and sad in the video. After watching this, teacher Sue stated, 'oh, no, the two dogs were very sad. Why do they feel sad?'. All children displayed very sad faces. One child answered, 'one doggy ate others' cakes'. Teacher Mayer asked, 'Do you think it is fair?'. The children answered, 'no'. Teacher Mayer stated, 'So, we need to make sure no dogs are sitting together in the table'. Teacher Sue said to the group, 'It is time to fly our broom to the party'.

While observing the transition to the playworld and the dramatisation of the mathematical problem, we could see that the children had initially developed an emotional connection with the play roles. The magical curtain and witch's wand helped the teacher and children collectively enter the playworld, as they imitated the voices of their selected roles of frog or dog. In addition, the letter from Witchy Britchy introduced the play plot and created a shared playworld of fiction. The teachers acted out the characters (witch and frog) through dramatic actions and playful sounds, which brought the story to life, encouraged the imaginative play and supported children to express their emotions and thoughts. The children were closely attuned with the sad experience of the dogs in the video. Both teachers used their play role to support the children's interest in the unknown and facilitate a common understanding and meaning making about why two dogs could not sit together. Through appropriate crafting by the teachers, a new adventure was created and a shared playful moment arose collectively. This meant that the playworld activity setting was 'charged with meaning' to explore the mathematical conceptual problem (Lindqvist 2003, p. 71). Through dramatising the emotionally charged mathematical problem, the children were oriented to eagerly fly to Britchy's birthday party.

Stimulating children's meaning making in collective play

Vignette 2 shows the two teachers sensitively and dramatically helping the children revisit the dramatised problem that 'no two dogs can sit together', and the way the children responded to the emotionally charged situation using mathematical thinking about patterns to solve the problem.

Vignette 2: Revisiting the mathematical problem—'No two dogs sitting together'

The children and teachers 'flew' to Britchy's house for the birthday party. Two tables, each covered with a tablecloth, were placed together to form a long table. Around the table were

13 chairs and a plate with one colour (either green or orange) for each chair. The children and teachers (still in their role of dog, frog or witch) chose a seat around the square table (see Figure 2). They then together started counting the number of seats and people around the table to ensure they had the correct number to include everyone, plus an extra seat for Witchy Britchy.

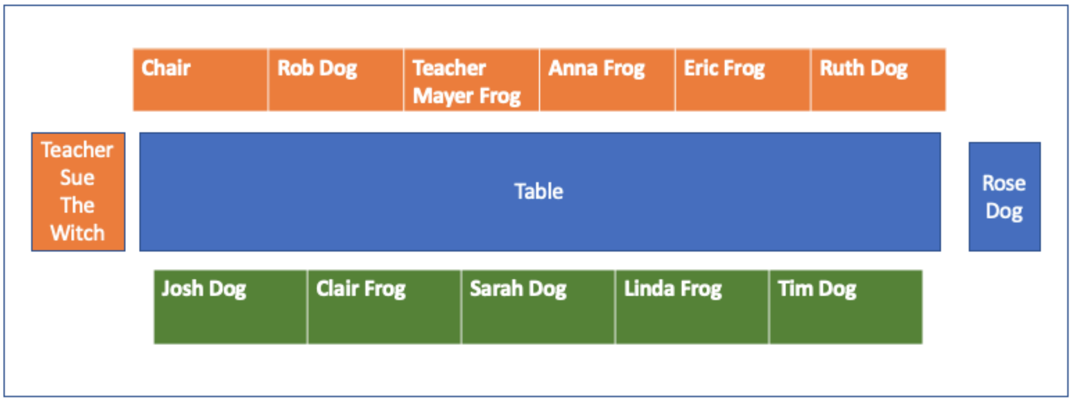

Figure 2. The children and teachers in their role of dog, frog or witch choosing a seat around the square table

Following this, teacher Sue, the witch, held her wand and stated, 'See, hold on! Hang on! The party can't start yet!'. Teacher Mayer (the frog) added, 'Remember what they say?'. She had a confused look on her face, touched her lip with her left hand and wondered, 'What does the letter say? Can I still remember?'. Linda, who was playing the role of a frog, eagerly stated, 'No!'. Teacher Mayer pointed to Linda, who continued, 'No two dogs sitting together!'.

Teacher Sue walked around the room identifying who was a dog and who was a frog. The children made the play character sounds when teacher Sue pointed to them. Teacher Sue continued pointing and speaking with a very slow voice when she found two dogs sitting together: 'Frog. Frog. Dog. Dog. Dog'. At this moment, teacher Mayer asked the children, while placing her hand on her mouth, with a wondering voice, 'Are there any dogs sitting together?'. Linda stood up and gestured with her two arms at children sitting at the end of the table, 'These three are sitting together. They are all dogs'. Teacher Sue asked, 'So what we need to do? What could happen if we have Eric dog, Anna dog and Rob dog sitting beside each other?'. Teacher Mayer added, 'What happens to the cake?'. Rose, playing the role of a dog, replied, 'They will be all gone!'. Teacher Sue asked, 'So what … How are we going to fix the problem?'.

In observing and analysing this vignette, we can see that the two teachers used their character roles inside the play to help make children aware of their situated problem. Dramatically, teacher Mayer presented a confused face and touched her left hand to her lip in wonder, saying, 'What does the letter say?'. This dramatic action and wonder drew the children's attention to the rules of setting up the party table. In particular, when teacher Sue was counting the three dogs together with a very slow voice, teacher Mayer captured that moment to remind the children, 'Are there any dogs sitting together?'. This drew the children's attention to the problematic situation that 'three dogs are sitting together', and they explored the sequence of sitting together, and Rose pointed out that 'the cake will be all gone'. Through stimulating children's motives in making meaning of the situation, children were oriented to fix the problem, using the concept of patterns as a method for solution. Given that the two dogs could not sit together, the pattern needed to change. This suggests that the mathematical playworld brought the party together and became personally meaningful for the children. This aligns with Vygotsky's (1997) argument that '[f]unctions initially are formed in the group in form of relation of the children, then they become mental functions of the individual' (p. 107). The collective imaginative play created a new social situation that oriented towards solving the mathematical problem. Learning must be culturally situated as 'the quality of the social situation that enables the leaners to socially co-construct their new cultural knowledge' (Bishop 2016, p. 47).

Mathematical demands placed on children in collective imaginary activity setting

The following vignette explores the collective thinking in solving the mathematical problem. The teachers used their play roles to guide children to analyse the mathematical problem and started teaching about repeating patterns. The children become active explorers *as if* they were the frog or dog and worried that 'the cake would be all gone'. Thus, they had to solve this emotionally charged problem.

Vignette 3: Solving the mathematical problem— 'Dog–frog–dog–frog–dog?'

Tim, playing the role of a dog, stated, 'No! No! No! No! No! I can fix it!'. Meanwhile, Linda quickly stood and pointed at different areas of the table, saying, 'put one there, put one there … and I'll sit there'. Following Linda's suggestion, Rose moved to the other side of the table and Linda sat at the far end of the table, where Rose used to sit: 'I will sit here' (between Tim [dog] and Ruth [dog]) (see Figure 3).

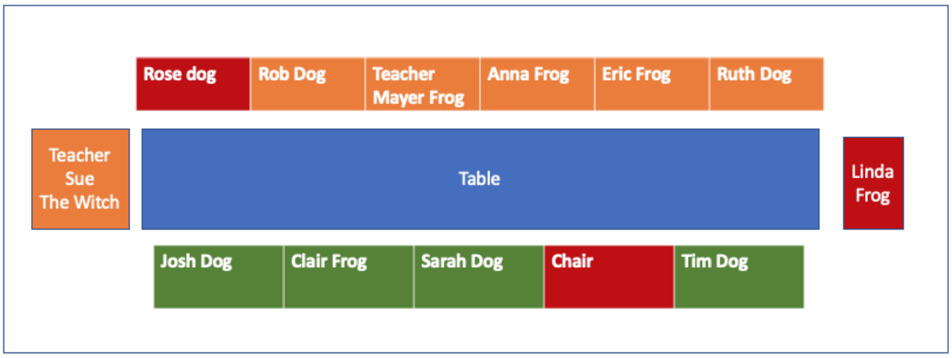

Figure 3. Rose dog and Linda frog sitting in the new place

Teacher Sue pointed to a corner of the table and asked, 'But then what happens here?'. The children were quiet and observed what happened (Rose also stood to watch what occurred on the other side of the table). Teacher Mayer and Sue started pointing and naming: 'So we go dog–frog–dog … dog'. While teacher Sue spoke, all the children were attentively listening and observing, and then joined in to name the characters. When they reached the end, they found that one dog had moved to sit beside another dog.

Linda ran to the other end of the table, patted a vacant place [The Witch seat] and spoke eagerly to ask Rose dog to sit the Witch seat. Teacher Sue responded, 'So you want to try and sit here? Rose dog'. Rose dog disagreed to move by shaking her head. Linda replies, 'If you [Rose dog] can sit here [The Witch seat], then I will sit here [referring to the other end of the table]'. She pointed to the other end of the table and moved over. Teacher Mayer commented again, 'I can hear a little bit of a pattern. Dog–frog–dog…'. Teacher Sue moved as she pointed and named dogs and frogs. The next in line was an empty seat, and teacher Mayer asked, ‘What should be there?’. Teacher Sue pointed to the empty seat and said to Linda, ‘If you were there, Linda frog’. Linda suddenly jumped up and ran over. ‘Frog!’, she exclaimed.

Teacher Mayer asked all the children, with her index finger in the air, ‘Oh, should we try again?’. Teacher Sue also stated, ‘Let’s try again! You ready?’. The two teachers stated, ‘Dog–frog–dog–frog–dog’. They pointed and named at the same time. They reached an empty chair at the end. Teacher Sue pointed to it and asked, ‘What should be next?’. Linda burst out, ‘Dog!’. Teacher Sue stated, ‘Let’s try that again’. Teacher Mayer added to the children, ‘You can say it’. This time, the children and two teachers named together: ‘Dog–frog–dog–frog–dog’. They reached the empty seat at the end of the table again, and this time, Josh, playing the role of a dog, said quietly, ‘Frog’. He pointed to the seat with excitement, with his mouth wide open and large, bright eyes. Teacher Mayer heard him and pointed to him. Linda also agreed, ‘We need a frog there!’. Teacher Mayer stood up and moved over: ‘I’m a frog!’. Rose stood up to look at the empty seat. Teacher Sue said, ‘We have a frog there!’ (see Figure 4).

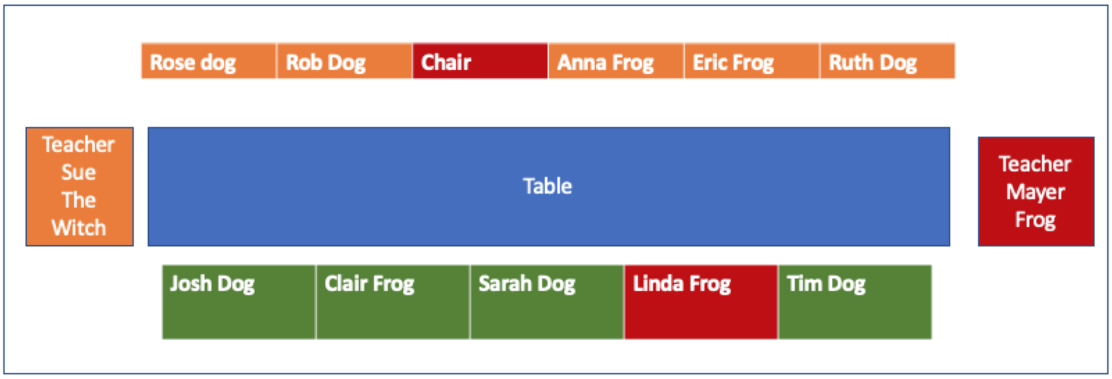

Figure 4. Teacher Mayer frog and Linda Frog moving to the new place

Teacher Mayer then asked the group, ‘Can we say the pattern together? … Dog–frog–dog–frog–dog–frog–dog–frog–frog–dog–dog’ (the group called out in unison). Teacher Sue said, ‘What happened?’. Linda immediately stood and pointed at the two frogs sitting together, and said, ‘They’re both sitting together’. Teacher Mayer asked Anna, playing the role of a frog, ‘Should you be next to a frog?’. Linda again stood and said, ‘No, sit next to a dog’. Anna stood and sat between the two dogs. Teacher Sue then said, ‘Should we check and do it again, see if we’ve got the pattern?’.

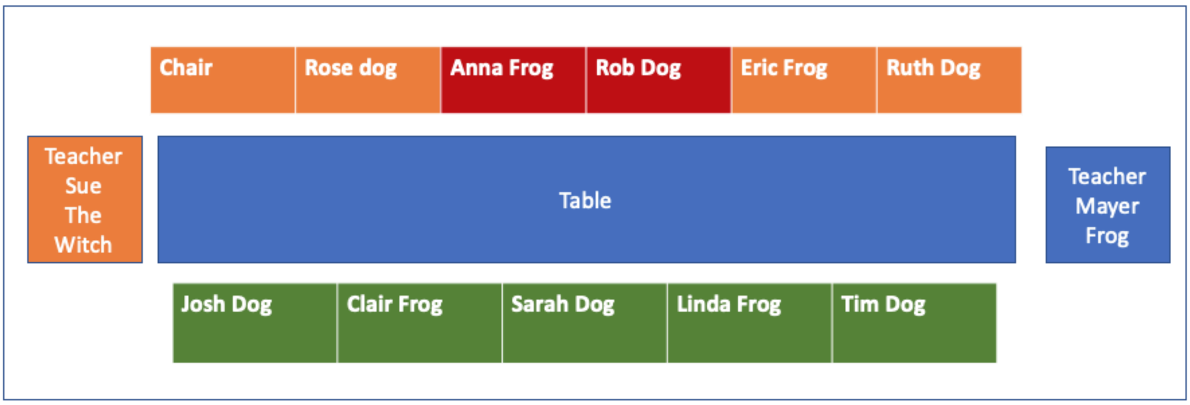

Figure 5. Anna frog and Rob Dog sitting in the new place

Linda stood and pointed, ‘No! She needs to … two dogs!’. Teacher Sue sought to clarify what Linda had said, and Anna stood up. ‘Anna frog needs to sit between Rose dog and Rob dog?’. Linda said, ‘Yeah’. Teacher Sue helped Anna sit between Rose and Rob (see Figure 5).

Teacher Sue asked, ‘Shall we try and do it again? Are we ready to do it together?’. The children responded, ‘Yeah’. Teacher Mayer said, ‘Take a deep breath’. She closed her eyes and breathed in deeply, and several children imitated her. Teacher Sue said: ‘Ready?’. The teachers started and the children joined together: ‘Dog–frog–dog–frog–dog–frog–dog–frog–dog–frog–dog’. Teacher Mayer asked with an excited voice: ‘Does that work?’. All the children cheered, ‘Yeah!’.

Vignette 3 presents an example of a playworld activity setting acting as a stimulating motive to develop algebraic reasoning by creating a repeating pattern. This aligns well with Van de Walle et al.’s (2018) explanation that, when teaching patterns to children, educators should not ‘have children do patterns just to do patterns, rather, children should be engaged in looking for, describing, and extending patterns to help them develop the skills to look for structure to express regularity in all mathematical situations’ (p. 294). This was the case within this mathematical playworld, where the children used a repeating pattern to help them solve the mathematical problem of not allowing two dogs to sit together. Both teachers valued children’s dominating motives in play, and they collectively ‘flew’ to the witch’s sister’s house for the birthday party, where they acted as if they were friends as frogs and dogs under the guidance of the witch. An analysis of the data illustrated that the five sequence times of naming and pointing ‘dog–frog–dog–frog–dog’ supported the children to meet the new mathematical demands, and the teachers used questions such as ‘what should be next?’ and ‘what happened?’ to support children’s mathematical thinking while they observing the regulatory and detecting the pattern, as if they were at the witch’s birthday party, where no two dogs were allowed to sit together.

In addition, in the analysis of children’s active exploration, the children were fully motivated and inspired each other to keep moving with the problem solving, and their active interaction stimulated others to see the new potential in them (Parker-Rees 2007). For instance, Linda frog started to lead this problem-solving process and other children followed what she contributed to the imaginary situation by providing the relevant response through action and prediction. For instance, when reaching the empty seat at the end of the table, Josh dog said quietly, ‘frog’, and pointed to the seat with excitement. Linda also agreed that ‘we need a frog there’. This showed their analysis of the repeating pattern of dog–frog–dog–frog. They were able to predict what should be next. This indicates that the children were able to analyse the relationships between frogs and dogs and express generalisation of the repeated units, while they investigated the organisation of the table for frogs and dogs as if they were frogs and dogs at the witch’s party. Thus, they finally determined how the frog and dog seats repeated in mathematically predictable ways. This situation echoes Fleer’s (2020) argument that the children would act ‘as if’ they were in the imaginary situation, and solve this social problem with conceptual solutions. Further, it was noted that Linda frog led the children’s active thinking in solving the problem. As argued by Vygotsky (1987), ‘[w]hat the child can do in collaboration today he will be able to do independently tomorrow’ (p. 211). Therefore, we argue that the peer relations and interaction in the playworld also stimulated the motivating conditions to inspire the whole group of children to engage in problem-solving and enhance their collective thinking.

Vignette 4: Testing the repeating pattern—‘Orange–green–orange–green’

After they solved the problem of ‘no dogs sitting together’, teacher Mayer asked, ‘Can we make a pattern out of our plates?’. Claire, playing the role of a frog, replied ‘Yes’. The children then stated that frogs should have green plates. Linda walked to the other end of the table and helped teacher Sue sort the green and orange plates. Claire did not have a plate and teacher Sue asked Anna, playing the role of a frog, ‘What colour does Claire frog need?’. Anna said ‘Green’. Once all the children had plates, teacher Mayer asked, ‘What do you notice about the plates … have we made another pattern?’. The children cried out ‘Yes’, and Linda began to say ‘Orange–green–orange–green–orange’. Teacher Sue said, ‘Shall we say our colour pattern?’. She pointed to the children and they called out their plate colours (see Figure 6).

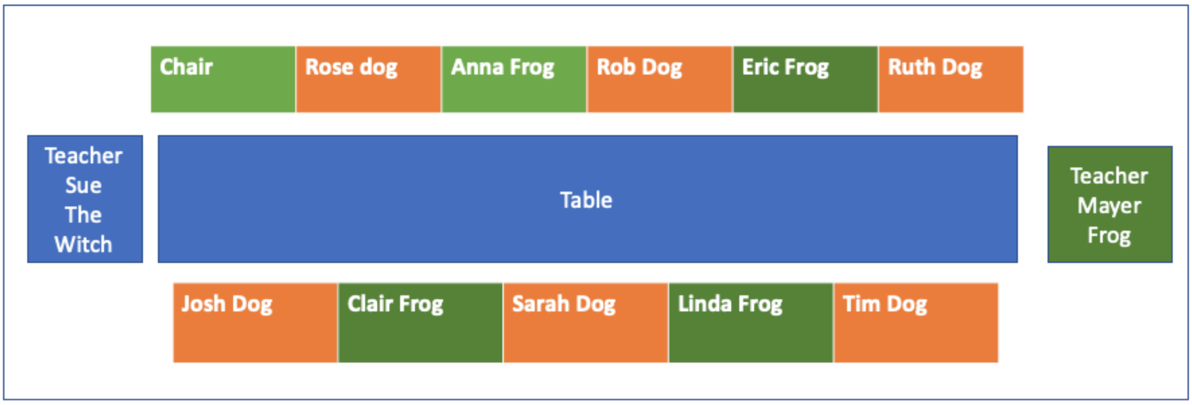

Figure 6. Testing their plate color patterning of orange-green-orange

Teacher Mayer asked, ‘Is that the right pattern?’. The children clapped and said, ‘Yeah’, and Anna frog said, ‘Now we can have the cake!’. Teacher Sue then picked up a large plate of cakes that she had prepared earlier and, using her magic wand, said the ‘magical words’ to reveal the cakes to the children.

The children’s conceptual play continued in Vignette 4 when teacher Mayer asked, ‘Can we make a pattern out of our plates?’. The children agreed with that and identified that the green plates should be for frogs and the orange plates for dogs. Allowing children to do this through a physical representation of pattern is the recommended approach for teaching patterns to young children (Van de Walle et al. 2018). The teacher’s question presented new demands and oriented the children to again use the mathematical concept of repeating pattern to test their answers, which supported their play development. Now their collective focus moved to examine their previous repeating pattern of dog–frog–dog–frog. When a particular problem emerged, one of the frogs did not have the plate and Anna, also a frog, helped find the correct plate, as if they were the frogs and dogs at the birthday party. This illustrates the dynamic interrelations between imagination and children’s abstract thoughts, such as repeating pattern in this case (Lindqvist 2003). Children made personal sense of the emotionally charged situated problems through their collective imagination. Thus, they developed meaningful learning of the mathematical concept, as if they were at the witch’s party, to solve the problem of ‘no two dogs sitting together’. Therefore, we found that the collective mathematical playworld created stimulating motivating conditions, as it placed new demands on children to solve and test the situated problem, which required children to use repeating patterns through identification of the structural similarities and differences between patterns. This helped the children achieve meaningful learning of a mathematical concept, and they were able to do this in an environment that allowed them to use the concept of patterning to solve the mathematical problem.

Conclusions

By drawing upon cultural-historical concepts of play and motives, and a playworld approach, this paper has reported on a study that explored Math@Imagination, as a new form of playworld, in supporting children's mathematical meaningful learning, and investigated children's mathematical problem solving in the collective imaginative play context through analysing the double sense of meaningful learning. We found that the Math@Imagination approach appeared to be a driving force to stimulate children's motives in solving the mathematical problem in the collective imaginary situation, as well as advancing children's mathematical concept learning in a double sense.

The findings of this study highlight the need for the transformation of young children's meaningful learning of mathematical concepts in the playworld. First, children need to be motivated to solve problems that become personally meaningful. In this study, the children and adults played different character roles (e.g., witch, frog and dog) adopted from the storybook and shared common knowledge of the birthday party, which enabled them to encounter the same emotionally charged 'no two dogs sitting together' issue. Thus, children were motived to solve this situated problem by drawing upon the concept of repeating patterns and making these patterns personally meaningful while playing. Therefore, the meaningful learning process was shaped *as if* they were at the witch's birthday party. This confirms the importance of mathematical learning in situated practice, such as the playworld activity setting in this case (Bishop 2016). Children were able to achieve a high level of conceptual understanding through situated contexts (MacDonald and Lowrie 2011). Therefore, we argue that Math@Imagination, as a form of playworld, created the motivating conditions to stimulate children emotionally, related to problem solving (Fleer 2020) and the emergence of mathematical actions, as the mathematical meanings were interactively appointed to the children's actions in the imaginary situation (van Oers 2010).

Second, the dramatised problem needed to be emotionally charged, but also embed the mathematical means in order to achieve the cultural meanings in learning. Solving the problem was the starting points for schematising. Children affectively engaged with the schematisation that should be involved with the dynamic symbolic representations (Poland and van Oers 2007). In this case, as the children entered the witch's playworld, they were motivated to set up the party table by following the rule of 'no dogs sitting together'. The dynamic representations of the repeating patterns (A–B–A–B–A–B) were symbolically embedded in a collective imaginary situation that improved the children's conceptual understanding of mathematics. This finding appears to align with the argument by van Oers (2010) that '[d]ynamic representations are fundamental for the development of mathematical thinking' (p. 33).

Third, consistent with previous research of van Oers (2010), this study found that playworlds appear to be a situated practice developing children's mathematical competence and enriching children's interactions with others in the collective imaginary situation, where the children's mathematical thinking and actions (using a repeated pattern) begin to emerge. The children and teachers collectively entered the playworld *as if* they were the witch's friends, frogs and dogs, who flew to the birthday party on their new broom. The playworld activity setting created new mathematical demands for children as it oriented and stimulated the children to analyse and discuss the mathematical principle, such as repeating patterns, as they did not want the two dogs sitting together.

In contrast to previous research, in this study, the adults not only reacted in a mathematical way to the children's initiatives and actions, but also valued the children's dominant motives in play. They played their character roles as a play partner to dramatise the mathematical problem and guided the children to solve this problem by using the mathematical concepts. By doing so, the children's motive orientation was changed to solve the mathematical problem in this shared imaginary situation. As suggested by Hedegaard (2008), '[a]n important way to support the development of the learning motive is to create engagement and shared experiences among the students by letting the class participate together in events' (p. 193). Therefore, this paper illustrates that children transform mathematical thinking into action by playing with each other and adults in a shared imaginary situation. This finding supports the pedagogical implications that teachers need to acknowledge children's dominant motives and create the motivation to build on children's mathematical conceptions and capacities in play. Moreover, the teachers also used their play role and asked children 'what should be next?', 'what happened?' and 'what should be there?' to orient the children to identify the repeating units (dog–frog). This also aligns with McGarvey's (2013) identification of teaching practices in early childhood educational setting that support early algebraic thinking, which should include patterns in the instruction and orient children's attention to the repeating unit.

Finally, children's meaningful learning process can be transformed through a spiral approach to problem solving, where they analyse and discuss mathematical concepts. In the playworld, children were provided the opportunity to access and analyse the situated problem of 'no two dogs sitting together'. They were not told directly to complete this task by making a dog–frog–dog–frog (A–B) pattern, but had to analyse this problem again and again to test what did and did not work in the collective imaginary situation. Learning in the playworld is not a straightforward process, but a spiral progress of thinking and problem solving. As aforementioned, the teachers assumed play partner roles in play to support the children to reason mathematically, and created the motivating condition to stimulate the children to affectively and critically engage with the mathematical ideas (dog–frog repeating pattern). This aligns with Cheeseman's (2018) study that suggested that highly effective mathematics teachers use questioning to extend children's mathematical thinking, instead of telling them what to do.

What is new to previous research is the teachers' co-playing and co-modelling in the collective playworld with the children. By doing so, possibilities were created to support children's critical process of mathematical thinking, including naming (e.g., dog–frog pattern), describing (e.g., where they should sit), abstracting (e.g., identifying the unit of repeat: dog–frog or orange–green), generalising (e.g., what works to solve the problem, and realising the unit of repeating through recognition of the structural similarities and differences) and evaluating (e.g., using colourful plates to test if it worked), thus transforming their meaningful learning. This finding has implications that the research of a larger sample of Math@Imagination would enrich further understanding of children's meaningful learning experience aligned with the development of mathematics.

The conclusions drawn from this in-depth study, conducted over seven months, are limited because the results of this study were based on only one early learning centre. However, we argue that Math@Imagination, as a mathematical pedagogical approach of the playworld, within the world of imaginary situations, should be promoted because it builds the motivating conditions that support meaningful learning of the mathematical concepts in the double sense created in children. By closely examining the co-created playworld activity setting between teachers and children, this study contributes to understanding children's mathematical problem-solving process in collective playworld contexts, how the learning process becomes personally meaningful for children, and teachers' role in play for supporting children's mathematisation.

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