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New Tests for Richness and Poorness: A Stochastic Dominance Analysis of Income Distributions in Hong Kong

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Abstract:

In this paper, we develop the theory of descending stochastic dominance for application to income distribution analysis. We show that conclusions of dominance obtained using our new tests of richness and poorness offer more accurate and more in-depth characterization of welfare inequality in any population. The empirical application of our proposed approach shows that, for Hong Kong, the distribution of total incomes in 2001 has less proportion of poor units in relatively lower income levels compared to that of 2006 at the same time that the distribution of total incomes in 2006 has a higher proportion of rich units in relatively higher income levels. Our analysis also suggests that there exist lower levels of household welfare in 2011 compared to both 2001 and 2006. In terms of age groups, the application of our new methods showed that the younger age cohorts tended to have lesser proportions of poor units in relatively lower income levels compared to those in the 65+ age group, while at the same time, those in the 65+ age group tended to have a higher proportion of rich units in the relatively higher income levels. These extreme concentrations of income units at the 'bottom end' for the younger households and at the 'top end' for the older households may help explain the overall high inequality level that has persisted in Hong Kong for several years now.

JEL Codes: C19, C44, I30

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1 Introduction

While society's concern for the poor is indisputable and enduring, the relative well-being of the rich has recently become the focus of increased attention in both research and policy circles. Increased pressure of global economic competition, and the problems of managing aging populations have both given rise to the renewed debate on whether the gap between rich and poor has widened, despite the enormous growth and economic progress achieved by countries around the world. Given that 'the rich' is universally acknowledged to be an important source of economic growth and have considerable economic and political power, the particular welfare question arising is whether the top end of the income distribution is pulling away from the poorer end over time, causing overall inequality to rise. Recent studies on the rich include Gonzales and Wen (2015), Jones (2015), Milanovic (2014), and Peichl, et.al. (2010).

In this context, this paper seeks to explain trends in inequality levels by providing a new procedure for measuring the contribution of the 'top-end' of the income distribution to observed changes in inequality. To this end, we introduce the theory of descending stochastic dominance to income distribution analysis, for which we devise tests for richness and poorness from there. The stochastic dominance (SD for short) approach is generally used in decision theory to refer to situations where one outcome (or a probability distribution over outcomes) can be ranked as superior to another. In the area of distributional analysis, the SD approach is useful when alternative inequality indices fail to provide unambiguous rankings of the same distributions, a situation which is not uncommon given the varying weights that different indices attach to different parts of a distribution.[1] In its 'standard' application to income distribution analysis, the SD approach is underlined by 'well-behaved' social welfare functions (SWF), meaning that the 'utility' in society is symmetric, increasing and concave in all its argument (e.g. income). The symmetry property implies that social welfare is not affected by two people switching income positions; SWF increasing implies that social welfare rises with income, while the concavity property implies that social welfare is more sensitive to a shift in the income of a

[1] For a more detailed discussion of SD approach in inequality analysis, see Valenzuela. et.al. (2014).

poorer individual than to the same shift affecting a richer individual. Mathematically, this characteristic is expressed by a negative second derivative.[2] Recent studies that have applied the technique to analyse welfare include: Maasoumi, Su and Heshmati (2013) which provides empirical results for China, Valenzuela, Lean and Athanasoupolous (2014) which analyses inequality and welfare among households in Australia, and Heshmati and Rudolf (2015) which investigates inequality in Korea.

In this paper, we contribute to the economic literature by introducing stochastic dominance techniques underlined by convex-shaped social welfare functions or more specifically social welfare functions with convex utility. We also apply improved tests of dominance relations between income distributions on that basis. Convexity in social welfare functions is akin to risk-seeking investor behavior at high levels of investment in investment theory; in similar fashion, we invoke convexity in social welfare functions to accommodate risk-seeking behavior at the highest income levels. This is a direct contrast to the risk-averse assumption implied by more commonly employed concave-shaped SWFs, but it is not without support in the economic literature. Friedman and Savage (1948), for example, was first to query the concave assumption in utility function, saying that "there is nothing to suggest that a dollar means less to a poor man than it does to a rich man." This was seconded by Markowitz (1952) who pointed out that people's willingness to purchase both insurance and lottery tickets is a strong indication that marginal utility is increasing over a range, thereby implying that utility functions of decision-makers are not everywhere concave, but have convex segments which make them accept some unfair risky gambles. Furthermore, a body of work by psychologists led by Kahneman and Tversky (1979) find experimental evidence that local risk-seeking behavior are consistent with the existence of convex utility function assumption. All in all, the requisite social welfare functions for direct welfare comparisons need not necessarily be concave all the time, and that by using convex-shaped SWFs in our stochastic dominance analysis, we can obtain more precise statements about welfare in income distribution analysis.

[2]Please see Blackorby and Donaldson (1980) for a detailed discussion of various ethically desirable criteria and the sorts of SWFs that respect these criteria.

On this basis, we apply new tests to determine conditions of dominance just like in the standard approach. We demonstrate, using data, that our proposed SD approach enables analysts to more confidently reach more conclusive statements about dominance relations, than if the welfare analysis was done by the standard SD approach alone. For example, using our new approach, finding a higher proportion of rich unit in relatively high income levels can conclusively indicate a higher social welfare under social welfare function with convex utility. However, if such findings have been obtained by using the standard SD approach alone, the inference would have been inconclusive.

To distinguish our contribution to the income distribution literature, we will hereon refer to the standard approach as the ascending stochastic dominance (ASD) approach, and to our contribution as the descending stochastic dominance (DSD) approach in this paper. ASD and DSD differ mainly their underlying social welfare functions – ASD works for SWFs that are increasing and concave-shaped, while DSD allows for preferences under SWFs that are also increasing but which can be convex in shape. As a consequence, ASD looks into the effect of varying degrees of poorness in income distributions while DSD could measure the impact of degrees of richness in the same. In general, both ASD and DSD approaches are well developed in investment theory, but, as far as we know, only the theory of ASD has been used for income distribution analysis. Thus, the main contribution of this paper to existing literature is the introduction of DSD theory to income distribution and the recommendation to use both ASD and DSD approaches to study the joint welfare effects of the poorness and richness of income distribution.

The theory of DSD has been well-developed in the finance area, see, for example, Hammond (1974), Meyer (1977), and Stoyan (1983). Readers may also refer to Levy (2015), Wong (2007), and Sriboonchitta, et al. (2009) for more recent work involving the theory of DSD in finance. In this paper, we build on this strand of the literature and develop DSD test procedures to test the significance of richness of income distribution. We illustrate the joint feasibility of our proposed approach with ASD analysis. Further, we make minor improvements in the SD testing procedure developed by Bai et al. (2015) so that results lend to easier economic interpretation.

That is, it allows us to compute the test statistics for a pre-described set of grids so that the tests could be used to measure different degrees of the poorness and richness, say, for example, the top 10 per cent of the richest income units as well as the bottom 20 per cent of the poorest income units.

To demonstrate the feasibility of our approach, we will apply the proposed technique to analyze the distribution of incomes in Hong Kong and study the impact of poorness and richness on welfare in Hong Kong over time and across demographic groups. Conclusions drawn from DSD implementation will be compared to those derived from the implementation of both singular measures and ASD approaches, which we also implement here for comparison purposes.

This paper has the following structure. After a brief introduction and overview in Section 1, Section 2 discusses ascending and descending stochastic dominance theory, each one's relationship to social welfare, and how they can be used and interpreted in practice. Section 3 derives and presents the new tests of stochastic dominance for richness and poorness. Section 5 presents results of the application of these new techniques to data, and discusses their interpretations for the Hong Kong economy. Section 6 concludes.

2 Definitions, Notations and Interpretations

All studies in welfare analysis have an underlying implicit welfare function. We here first define explicitly the social welfare function that underpins all foregoing analysis. Let F be a distribution function and u represent the corresponding utility of individual (or household), u is increasing in its argument x. Define a social welfare function (SWF) of the form

$$W(F) = \int u(x) dF(x) \qquad (1)$$

where $u: \mathbb{R}_+ \to \mathbb{R}$ is a continuous function. SWFs generally represent the collective welfare of all members in society, or the overall social state. We assume W in (1) to be symmetric and increasing in all its arguments so that various ethical criteria of desirable, well-behaved SWFs can be used. Different SWFs of the form in (1) give the same order of ranking as that of SD if one stochastically dominates the other of the first-order. If we impose an additional restriction that the second derivative of $u''(x)$ is negative, then all SWFs in this restricted class likewise give a unanimous ranking of two distributions if one dominates the other at second order.[3]

2.1 Definitions of ASD and its interpretations

To implement the stochastic dominance approach, we consider welfare outcomes X and Y defined over the real number space $\Omega = [a, b]$; that is, $X, Y \in \Omega = [a, b]$ with probability distribution functions F and G, respectively, where "a" is strictly non-negative.[4] To facilitate exposition, we let X and Y be income series.[5] For any x, we define the k-order cumulative distribution functions F_k^A and G_k^A of X and Y to be

$$F_j^A(x) = \int_a^x F_{j-1}^A(t)\, dt \quad \text{and} \quad G_j^A(x) = \int_a^x G_{j-1}^A(t)\, dt, \quad \text{for } j = 2, 3\,; \qquad (2)$$

$F_1^A(x) = \int_a^x f\,(t)dt$ and $G_1^A(x) = \int_a^x g\,(t)dt$ where f and g denote the probability density functions of X and Y. We now define ascending stochastic dominance (ASD).

Definition 1: X is said to first (second)-order dominate Y by ASD, denoted by $X \succ_1^A Y$ or $\mathrm{F} \succ_1^A \mathrm{G}$ ($X \succ_2^A Y$ or $F \succ_2^A G$) if and only if $F_1^A(x) \leq G_1^A(x)$ ($F_2^A(x) \leq G_2^A(x)$) for all x with strict inequality for at least one interval of x; and X is said to third-order dominate Y by ASD, denoted by $X \succ_3^A Y$ or $\mathrm{F} \succ_3^A \mathrm{G}$ if and only if $F_3^A(x) \leq G_3^A(x)$ for all x with a strong inequality for at least one x_0 with a strong inequality for at least one interval of x and $\mu_X \geq \mu_Y$ where μ_X and μ_Y denote the means of X and Y, respectively.

[3] See Foster and Shorrocks (1988) for detail.
[4] We note that this is a strict condition that can be relaxed empirically to accommodate the kinds of welfare outcomes under study. For instance, "a" is strictly positive for income but could be negative for wealth.
[5] We will use incomes only for the exposition, but X and Y may refer to any chosen welfare outcome or indicator such as expenditures, wealth, well-being, etc. in continuously measurable units.

We denote the first-, second-, and third-order ascending stochastic dominance by FASD, SASD, and TASD, respectively. The j-order ASD can be defined similarly for any $j > 3$.

In empirical studies comparing income distributions, if all individuals with incomes equal to or below a specified value of *x* are considered poor, then findings of first-order dominance of *X* over Y $(X \succ_1^A Y)$ means that distribution *F* will always have less or equal proportion of poor income units compared to distribution *G* for any value of *x.* More simply, we say that FASD of X over Y implies that the proportion of poor units in X is less than the proportion of poor units in Y. On the other hand, findings of second-order ascending dominance of *X* over *Y* $(X \succ_2^A Y)$ means that the integral of the cumulative probability of *X* is less than that of *Y*. Unlike in FASD, this SASD finding does not necessarily imply that the income distribution of the units in *X* have less proportion of poor units compared to that in *Y* for any income level *x*; rather, it implies that income distribution *X* has less proportion of poor units compared to that in *Y for some relative lower income levels.* To be more specific, for income levels x_A and x_B where $x_A < x_B$, $X \succ_2^A Y$ means that *F* will always have less proportion of poor income units than distribution *G* for any *x* that is smaller or equal to x_A, but at the same time, *F* could also have a higher proportion of poor income units for any *x* where $x_A < x \leq x_B$. In other words, findings of SASD of *X* over *Y* $(X \succ_2^A Y)$ implies that the proportion of poor units in *X* is less than the proportion of poor units in *Y for relative lower income levels*. For this reason, we will call the test that could detect ASD relations as the "test for poorness" in this paper.

We note that SASD or $X \succ_2^A Y$ can only imply that income distribution *X* has less proportion of poor units compared to that in *Y* for some relative lower income levels. At the same time however, income distribution *X* could also have a smaller or greater proportion of poor units compared to that in *Y* for some relative higher income levels. If income distributions *X* and *Y* have the same mean, then $X \succ_2^A Y$ implies two things: (a) income distribution X has less proportion of poor units compared to that in *Y* for some relative lower income levels; and (b) income distribution X has a higher proportion of poor units compared to that in *Y* for some relative higher income levels. We note here that (b) is equivalent to saying that *X* has less proportion of rich units compared to that in *Y* for some relative higher income levels, but we will discuss this in more details after we define descending stochastic dominance in the next section.

Findings of ascending stochastic dominance is highly relevant to social welfare analysis. Foster and Shorrocks (1988) show that for the class of all monotonic, symmetric additively separable social welfare functions of the form in (1), the following statement holds:

$$F \succ_j^A G \text{ if and only if } W(F) \geq W(G) \text{ for all } u \in \mathcal{U}_j^A \text{ for } j = 1, 2, \text{ and } 3. \quad (3)$$

$\mathcal{U}_1^A \subset \mathcal{U}$ is defined by including the condition $u'(x) > 0$; $\mathcal{U}_2^A \subset \mathcal{U}_1^A$ is defined by including the additional condition $u''(x) < 0$; and $\mathcal{U}_3^A \subset \mathcal{U}_2^A$ is defined by including another additional condition $u'''(x) > 0$. This effectively implies that the condition (3) holds only for social welfare functions with concave utility. This result is also applied in inequality analysis by Atkinson (1970). In addition, Atkinson (1970) shows that second-order stochastic dominance is equivalent to the Lorenz dominance if the means of the compared income series are the same.

2.2 Definitions of DSD and its interpretations

We now set the notation for the introducing the concept of descending stochastic dominance (DSD) for income distributions. Let F_j^D and G_j^D be the j^{th}-order reverse cumulative distribution functions for observed outcomes X and Y. For any argument x, they are defined as follows:

$$F_j^D(x) = \int_x^b F_{j-1}^D(t)dt \text{ and } G_j^D(x) = \int_x^b G_{j-1}^D(t)dt \text{ for } = 2, 3 \text{ ;} \quad (4)$$

$F_1^D(x) = \int_x^b f(t)dt$ and $G_1^D(x) = \int_x^b g(t)dt$ where f and g denote the probability density functions of X and Y, respectively.

Definition 2: X is said to first (second)-order dominate Y by DSD, denoted by $X \succ_1^D Y$ or $F \succ_1^D G$ ($X \succ_2^D Y$ or $\succ_2^D G$) if and only if $F_1^D(x) \geq G_1^D(x)$ ($F_2^D(x) \geq G_2^D(x)$) for all x with strict inequality for at least one interval of x; and third-order dominates Y by DSD, denoted by

$X \succ_3^D Y$ or $F \succ_3^D G$ if and only if $F_3^D(x) \geq G_3^D(x)$ for all x with a strict inequality for at least one interval of x and $\mu_X \geq \mu_Y$ where μ_X and μ_Y denote the mean of *X* and *Y*, respectively.

We denote the first-, second-, and third-order descending stochastic dominance by FDSD, SDSD and TDSD, respectively. The j-order DSD can be similarly defined for any *j>3*.

If we consider all individuals with incomes equal to or above a specified value of *x* to be rich, then, findings of first-order descending stochastic dominance FDSD of X over Y ($X \succ_1^D Y$) implies that the reverse cumulative distribution of *X*, F_j^D, will always have a higher proportion of rich individuals than that of *Y*, G_j^D, for any income level *x*. On the other hand, findings second-order descending dominance SDSD of X over Y ($X \succ_2^D Y$) means that the integral of the reverse cumulative probability of *X* always lies above of that of *Y*. However, the income distribution of the units in *X* does not necessarily have higher proportion of rich units compared to that in Y for any income level. Instead SDSD means that the former has a higher proportion of rich units than that in *Y for some relatively higher income levels*. To be more specific, for income levels x_A and x_B where $x_A < x_B$, $X \succ_2^D Y$ means that F will always have higher proportion of rich income units than distribution *G* for any *x* that is higher or equal to x_B, but at the same time *F* could have smaller proportion of rich income units for some *x* where $x_A \leq x < x_B$. In other words, if tests conclude SDSD of *X* over *Y*, one can infer that the proportion of rich units in *X* is higher than the proportion of rich units in *Y for some relative higher income levels*. Because of the above properties, we will call the test that could detect DSD relations as "test for richness" in the rest of the paper.

The application of DSD to income distribution is new and so we formally discuss below its relationship with social welfare. Let W denotes the class of all monotonic, symmetric additively separable social welfare functions of the form $W(F) = \int u(x)dF(x)$, where $u: \mathbb{R}_+ \to \mathbb{R}$ is a continuous function. Let $\mathcal{U}_1^D \subset \mathcal{U}$ be defined by the condition $u'(x) > 0$; let $\mathcal{U}_2^D \subset \mathcal{U}_1^D$ be defined by $u''(x) > 0$; and let $\mathcal{U}_3^D \subset \mathcal{U}_2^D$ be defined by $u'''(x) > 0$. Under these settings, we can obtain the following result for income distribution analysis:

$$F \succ_j^D G \text{ if and only if } W(F) \geq W(G) \text{ for all } W \in \mathcal{U}_j^D \text{ for all } j = 1, 2, \text{ and } 3. \qquad (5)$$

Equation (5) shows that descending stochastic dominance implies welfare dominance and we show this below for second- and third-order stochastic dominance following on the approach of Wong and Li (1999) and Levy (2015) for convex stochastic dominance theory.[6]

To show that $\int_x^b [G(t) - F(t)]\, dt \geq 0 \Longrightarrow W(F) \geq W(G)$, we use the definition of the social welfare function and get:

$$W(F) - W(G) = \int_a^b u(x) dF(x) - \int_a^b u(x) dG(x)$$

$$= \int_a^b u(x)\, [f(x) - g(x)] dx. \qquad (6)$$

Integrating (6) by parts, we get

$$W(F) - W(G) = u'(b) \int_a^b [G(t) - F(t)]\, dt - \int_a^b u''(x) \left(\int_a^x [G(t) - F(t)] dt \right) dx. \qquad (7)$$

The second term in (7) can be rewritten as

$$-\int_a^b u'(x) \left(\int_a^x [G(t) - F(t)] dt \right) dx \qquad (8)$$

$$= -\int_a^b u''(x) \left(\int_a^b [G(t) - F(t)] dt \right) dx$$

$$+ \int_a^b u''(x) \left(\int_x^b [G(t) - F(t)] dt \right) dx.$$

Then, we have

$$W(F) - W(G) = u'(a) \int_a^b [G(t) - F(t)]\, dt + \int_a^b u''(x) \left(\int_x^b [G(t) - F(t)] dt \right) dx. \qquad (9)$$

[6] The first order case under convex social welfare function is equivalent to the first order case under concave social welfare function, so derivations are shown only for second- and third orders only.

Given that $W \in \mathcal{U}_2^D$, it follows that F dominates G by SDSD. Equivalently, we say that $\int_x^b [G(t) - F(t)]dt \geq 0$ implies that distribution F is preferred to G in term of social welfare (measured by social welfare functions with concave utility), that is, $(F) \geq W(G)$.
For the third order case we have

$$W(F) - W(G) = u'(a) \int_a^b [G(t) - F(t)]\, dt + \int_a^b u''(x) \left(\int_x^b [G(t) - F(t)dt] \right) dx. \tag{10}$$

Integrating the second right-hand side term in (10) by parts yields:

$$\begin{aligned} &\int_a^b u''(x) \left(\int_x^b [G(t) - F(t)dt] \right) dx \\ &\qquad = u''(x) \left(\int_a^x \int_t^b [G(z) - F(z)]\, dzdt \right) \Bigg|_a^b \\ &\qquad - \int_a^b u'''(x) \left(\int_a^x \int_t^b [G(z) - F(z)]\, dzdt \right) dx. \end{aligned} \tag{11}$$

and rewriting, we have

$$\begin{aligned} &u''(x) \left\{ \left(\int_a^b \int_t^b [G(z) - F(z)]\, dzdt \right) - \left(\int_x^b \int_t^b [G(z) - F(z)]\, dzdt \right) \right\} \Bigg|_a^b \\ &\qquad - \int_a^b u'''(x) \left\{ \left(\int_a^b \int_t^b [G(z) - F(z)]\, dzdt \right) \right. \\ &\qquad \left. - \left(\int_x^b \int_t^b [G(z) - F(z)]\, dzdt \right) \right\} dx. \end{aligned} \tag{12}$$

Using (12) to rewrite (10), we get the following result:

$$W(F) - W(G) = u'(a) \int_a^b [G(t) - F(t)]\, dt + u''(a) \left(\int_a^b \int_t^b [G(z) - F(z)]\, dzdt \right) + \int_a^b u'''(x) \left(\int_x^b \int_t^b [G(z) - F(z)]\, dzdt \right) dx. \tag{13}$$

From (13), we can see that for F to dominate G by TDSD, we require both $\mu_x - \mu_Y \geq 0$ and $\int_x^b \int_t^b [G(z) - F(z)]\, dzdt \geq 0$ for all $x \in [a, b]$ which imply $W(F) - W(G) \geq 0$ for any $W \in \mathcal{U}_3^D$.

The foregoing implies that conclusions of descending stochastic dominance DSD, say of *F* over *G* for *j*=1, 2 or 3, could be applied to social welfare functions that are increasing and convex. Furthermore, if the social welfare function is composited by convex utility, the foregoing implies that the DSD approach to analysis is deemed the more appropriate one to use and that the ASD approach is totally irrelevant. In practice, the true form of the social welfare function is unknown, which leaves the choice between ASD and DSD approaches indeterminate. As in this paper, we recommend the use the ASD and DSD approaches simultaneously, with clear advise for caution in interpretation of results.

2.3 Hierarchal orders within and between ASD and DSD

Hierarchy exists in both ASD and DSD where these are stated in detail for the general case in Levy and Levy (2002), Chan *et al*. (2012), and Levy (2015). In what follows, we state below the key relationships between different orders of ASD and DSD which are critical for correctly interpreting results in income distribution analysis. For each one, we provide a guide to interpretation or an example to demonstrate the case.

Given two income distributions *X* and *Y*, the following ordered outcomes can be obtained:

i. *If X is found to j-order dominate Y by ASD, then X will j+1-order dominate Y by ASD. Similarly, if X is found to j-order dominate Y by DSD, then X will j+1-order dominate that of Y by DSD.*

This implies that it will be sufficient to report the lowest dominance order of ASD and DSD being found.

ii. *If X is found to dominate Y by FASD, then X will dominate Y by FDSD.*

This implies that if *X* has less proportion of poor units compared to *Y* for any level of income, then *X* has higher proportion of rich individuals than *Y* for any level of income.

iii. *X can dominate Y in SASD and SDSD at the same time even if there is no FSD relationship between X and Y.*

This is best illustrated by a hypothetical example: Let F and G be two income distributions and their associated probabilities f and g have the following values for each income level (x). One could easily see that there is no FASD or FDSD between F and G, it is easy to verify that $F \succ_j^A G$ and $F \succ_j^D G$ for $j = 2$ and 3 at the same time.

x	1	2	3	4	5	6
f	0.1	0.2	0.25	0.15	0.1	0.2
g	0.2	0.2	0.15	0.25	0.2	0.1

iv. If X and Y have the same mean, then X dominates Y in SASD if and only if Y dominates X in SDSD.[7]

This implies that the social welfare of *X* is higher than *Y* if the utility function is concave but the social welfare of *Y* is higher than *X* if the utility function is convex. Note here that this result bears special significance to inequality analysis if we assume that the mean value of income is irrelevant to social welfare, that is, when one only uses the mean divided income series to compare.

v. X and Y have the same direction of domination for both ASD and DSD if the dominance is of the first order. However, if dominance is of the second order and their means are equal (or not rejected to be equal), then the directions of ASD and DSD dominance are reversed. However, if dominance is of the second-order and their means differ significantly, then the directions of ASD and DSD dominance could be the same.

These are results commonly observed in empirical applications. With the third order, Chan *et. al.* (2012) shows that no singular direction dominates. That is, under some conditions, the directions of the dominance of TASD and TDSD are the same but under other situations, the directions are reversed.

3. Improved SD Tests for Richness and Poorness

In the economic literature, tests of stochastic dominance are of two types – those that make inferences based on the comparison of the object *x* (e.g. income or wealth) at all points in the

[7] This statement can be relaxed to "not rejected to be equal".

support (e.g. Barrett and Donald (2003), Linton et al. (2005)), or those that make inferences based on the comparison of objects at arbitrarily chosen, and fixed numbers of values along the ordered distribution (e.g. Anderson (1996), Davidson and Duclos (2000), and Bai, et.al. (2015). The former are variants of the Kolmogorov-Smirnov tests and are highly desired because of their consistency property (Barrett and Donald 2003) but they are also noted for difficulty in constructing appropriate rejection regions (McFadden 1989). The latter type, on the other hand, are strongly preferred in practice because of their flexibility in the number comparison points required, although it also has greater tendency to introduce test inconsistency (Davidson and Duclos 2000). Given that, other studies such as Wei and Zhang (2003), Tse and Zhang, (2004), and more recently, Bai et al (2015) have focused on introducing methodology that can help decide critical points and prove consistency for this type of SD tests. Lean et al. (2008) also show that this type of SD tests are robust to non-*i.i.d*. data, including heteroscedastic data and is convenient for comparing any part of distributions under study. In this paper, these build on the works of Davidson and Duclos (2000) and Bai, et al. (2015) and propose new and improved stochastic dominance tests for richness and poorness based on a pre-defined points on our distribution. We introduce them in this section below and illustrate their feasibility in analysing real-life income distributions in the next.

To present our tests, we first define some notations. Assume $\{f_i\}(i = 1,2,\cdots N_f)$ and $\{g_i\}\left(i = 1,2,\cdots N_g\right)$ are observations drawn from the income distributions X and Y, with distribution functions F and G, respectively, and with their integrals $F_j^A(x)$ and $G_j^A(x)$ defined in (2) for $j = 1,2,3$, and their reverse integrals $F_j^D(x)$ and $G_j^D(x)$ defined in (4) for $j = 1,2,3$. We further set a grid of pre-selected points on our distribution $x_1,\ x_2,\dots,x_k$ for the testing.

3.1 Test for Poorness

To test for poorness, we apply ASD principle to test the following set of null hypotheses[8] for a pre-designed finite numbers of values *x*:

[8] This follows from Bishop (1992).

$H_0\colon F_j^A(x_i) = G_j^A(x_i)$ for all x_i;
$H_A\colon F_j^A(x_i) \neq G_j^A(x_i)$ for some x_i;
$H_{A1}\colon F_j^A(x_i) \leq G_j^A(x_i)$ for all $x_i, F_j^A(x_i) < G_j^A(x_i)$for some x_i;
$H_{A2}\colon F_j^A(x_i) \geq G_j^A(x_i)$ for all $x_i, F_j^A(x_i) > G_j^A(x_i)$for some x_i;

for all $i = 1,2,\dots,k$ and j = 1, 2, and 3. We note that in the above hypotheses, H_A is set to be exclusive of both H_{A1} and H_{A2}. This means that if the test does not reject H_{A1} or H_{A2}, it will not be classified as H_A. The jth-order ASD test statistic is:

$$T_j^A(x) = \frac{\hat{F}_j^A(x) - \hat{G}_j^A(x)}{\sqrt{\hat{V}_j^A(x)}} \tag{6}$$

where $\hat{V}_j^A(x) = \hat{V}_{F_j}^A(x) + \hat{V}_{G_j}^A(x) - 2\hat{V}_{FG_j}^A(x)$;

$$\hat{V}_{H_j}^A(x) = \frac{1}{N_h}\left[\frac{1}{N_h((j-1)!)^2}\sum_{i=1}^{N_h}(x-h_i)_+^{2(j-1)} - \hat{H}_j^A(x)^2\right], H = F, G;\ h = f, g;$$

$$\hat{V}_{FG_j}^A(x) = \frac{1}{N_h}\left[\frac{1}{N_h((j-1)!)^2}\sum_{i=1}^{N_h}(x-f_i)_+^{j-1}(x-g_i)_+^{j-1} - \hat{F}_j^A(x)\hat{G}_j^A(x)\right].$$

and $\hat{H}_j^A(x) = \frac{1}{N_h(j-1)!}\sum_{i=1}^{N_h}(x-h_i)_+^{j-1}$.

Following Bai et al. (2015), we apply the following decision rules:

$\max_{1\le k\le K}\left|T_j^A(x_k)\right| < M_\alpha^j$, accept $H_0\colon X =_j Y$
$\max_{1\le k\le K} T_j^A(x_k) > M_\alpha^j$and $\min_{1\le k\le K} T_j^A(x_k) < -M_\alpha^j$, accept $H_A\colon X \neq_j Y$
$\max_{1\le k\le K} T_j^A(x_k) < M_\alpha^j$and $\min_{1\le k\le K} T_j^A(x_k) < -M_\alpha^j$, accept $H_{A1}\colon X \succeq_j Y$
$\max_{1\le k\le K} T_j^A(x_k) > M_\alpha^j$and $\min_{1\le k\le K} T_j^A(x_k) > -M_\alpha^j$, accept $H_{A2}\colon Y \succeq_j X$

where M_α^j is the bootstrapped critical value of the j-order ASD statistic.

The test statistic is compared with M_α^j at each point of the combined sample. However, it is empirically difficult to do so when the sample size is very large. In order to make the computation easy, we specify K equal-distance grid points $\{x_k, k = 1,2,\dots,K\}$ which cover the common support of random samples $\{X_i\}$ and $\{Y_i\}$. Simulations show that the performance of the modified ASD statistic is not sensitive to the number of grid points for some reasonably large number. In practice, we follow Fong et al. (2005), Gasbarro et al. (2007) and choose K = 100. We note that Bai et al. (2015) improved the ASD test by deriving the limiting process of the ASD statistic $T_j^A(x)$ so that the ASD test can be performed by using $\max_x|T_j^A(x)|$ to take care of the dependency of the partitions. In this paper, we suggest to apply this ASD test by using both limited number of grids and $\max_x|T_j^A(x)|$ comparison. Fong et al. (2005), Valenzuela, et al. (2014), and others used the former while Bai et al. (2015) adopted the latter but neither studies used both, while we do. Further, we follow Bai et al. (2015) and use simulation to obtain the critical value M_α^j in our analysis.

3.2 Test for Richness

To test for richness, we apply the DSD principles on the following null hypotheses:

$$H_0: F_j^D(x_i) = G_j^D(x_i) \text{ for all } x_i;$$
$$H_D: F_j^D(x_i) \neq G_j^D(x_i) \text{ for some } x_i;$$
$$H_{D1}: F_j^D(x_i) \geq G_j^D(x_i) \text{ for all } x_i, F_j^D(x_i) > G_j^D(x_i) \text{ for some } x_i;$$
$$H_{D2}: F_j^D(x_i) \leq G_j^D(x_i) \text{ for all } x_i, F_j^D(x_i) < G_j^D(x_i) \text{ for some } x_i;$$

$i = 1,2,\ldots,k$ and j = 1, 2, and 3. Not rejecting either H_0 or H_A or H_D implies the non-existence of any SD relationship between X and Y, and that neither of these distributions is preferred to the other. If $H_{A1}(H_{A2})$ of order one is accepted, $X(Y)$ stochastically dominates $Y(X)$ at first order, while if $H_{D1}(H_{D2})$ of order one is accepted, distribution $X(Y)$ stochastically dominates $Y(X)$ at first order. If H_{A1} (H_{A2}) $[H_{D1}$ $(H_{D2})]$ is accepted at order two (three), a particular distribution stochastically dominates the other at second- (third-) order.

For our test of richness, the j-order DSD test statistic, T_j^D is:

$$T_j^D(x) = \frac{\hat{F}_j^D(x) - \hat{G}_j^D(x)}{\sqrt{\hat{V}_j^D(x)}} \quad (7)$$

where $\hat{V}_j^D(x) = \hat{V}_{F_j}^D(x) + \hat{V}_{G_j}^D(x) - 2\hat{V}_{FG_j}^D(x)$;

$$\hat{H}_j^D(x) = \frac{1}{N_h(j-1)!}\sum_{i=1}^{N_h}(h_i - x)_+^{j-1},$$

$$\widehat{V}_{H_j}^D(x) = \frac{1}{N_h}\left[\frac{1}{N_h((j-1)!)^2}\sum_{i=1}^{N_h}(h_i - x)_+^{2(j-1)} - \hat{H}_j^D(x)^2\right], H = F, G;\ h = f, g;$$

$$\widehat{V}_{FG_j}^D(x) = \frac{1}{N_h}\left[\frac{1}{N_h((j-1)!)^2}\sum_{i=1}^{N_h}(f_i - x)_+^{j-1}(\mathrm{g}_i - x)_+^{j-1} - \hat{F}_j^D(x)\hat{G}_j^D(x)\right].$$

Following Bai et al. (2015), we apply the following decision rules:

$$\max_{1\le k\le K}\left|T_j^D(x_k)\right| < M_\alpha^j, \text{accept } H_0: X =_j Y$$
$$\max_{1\le k\le K} T_j^D(x_k) > M_\alpha^j \text{and} \min_{1\le k\le K} T_j^D(x_k) < -M_\alpha^j, \text{accept } H_D: X \neq_j Y$$
$$\max_{1\le k\le K} T_j^D(x_k) > M_\alpha^j \text{and} \min_{1\le k\le K} T_j^D(x_k) > -M_\alpha^j, \text{accept } H_{D1}: X \succeq_j Y$$
$$\max_{1\le k\le K} T_j^D(x_k) < M_\alpha^j \text{and} \min_{1\le k\le K} T_j^D(x_k) < -M_\alpha^j, \text{accept } H_{D2}: Y \succeq_j X$$

where M_α^j is the bootstrapped critical value of the j-order DSD statistic. The test statistic is compared with M_α^j at each point of the combined sample.[9] As in the ASD tests, we follow Fong et al. (2005, 2008) and Valenzuela, et al. (2014) and make 100 partitions in the common support for the distributions X and Y, use simulation to obtain the critical value M_α^j, and use $\max_x\left|T_j^D(x)\right|$ to test for the convex preference assumption of income units in the upper end of the income distributions.

We note here that the bootstrap method we use for identifying critical points ensure that our estimated critical value will be closer to the true critical values. In the analysis of income distributions and inequality, this allows for a more accurate and reliable conclusions about relative welfare levels or living standards. Furthermore, the welfare conclusions from our test for some relative poorness does not require the specification of a poverty line. Since the second

[9] Refer to Bai et al. (2015) for the construction of the bootstrapped critical value M_α^j.

order ascending stochastic dominance is equivalent to Lorenz dominance, our test for some relative poorness in second-order (using our test in mean divided income series) could also be considered as an improved test for Lorenz dominance.

4 Empirical Illustration: Data, Results and Discussion

For our empirical analysis, we use the 5 per cent sample data set obtained from 2001, 2006 and 2011 Population Census conducted by Hong Kong Census and Statistics Department. Our unit of analysis is the person, so there is no need for application of equivalence scales as is done when using households as the unit of analysis. Three particular welfare data series are used for our analysis: Total personal income (*Total*), Earned income ($Einc$) and Other cash income ($Oinc$). $Einc$ pertains to the amount a person has earned monthly from employment including salary or wage, bonus, commission, overtime, housing allowance, tips and other cash allowances.[10] $Oinc$, on the other hand, refers to total recurrent cash incomes received by a person each month. This includes all income received which are not remuneration for work e.g. rent income, interest, dividend, education grants (excluding loan), regular/monthly pensions, social security payment, old age allowance, disability allowance, comprehensive social security assistance, scholarships, regular contribution from persons outside the household and contribution from charities. We note that *Total* is generated by adding income from all employment and other cash income together. That is, *Total* = *Einc* + *Oinc*. All data used in the paper are adjusted by inflation rate.[11]

5.1 Descriptive statistics and singular measures of poverty and inequality

Table 1 presents some descriptive statistics from our sample population. From here we can see that the population of the under 35s decreased steadily from 39.9 to 34.6 per cent between 2001 and 2011; this is a decline of 12.6 percentage points over 10 years or a 1.3 per cent per annum reduction rate. In contrast, the population share of the 35-64 group grew by 0.8 per cent per

[10] New Year bonus and double pay are excluded.
[11] CPI is used for this purpose. Also some modifications have also been applied to the data set: 1) All the data that is label as N.A in any of the two series are deleted. 2) All the data that is label as 0 in their corresponding age column are deleted.

year, increasing from 58.7 per cent in 2001 to 63.6 per cent in 2011. The share of the 65+ group remained small at less than 2 per cent and is virtually unchanged between 2001 and 2011. These changes are small but are consistent in the direction of a slowly ageing population.

Tables 2, 3and 4 present singular measures of poverty and inequality for various age groups in different years. Table 2 presents the computed Head Count Ratios as well as the poverty intensity sensitive measure Sen-Shorrocks-Thon (SST) index.[12] We can see that across all in the population, the proportion of poor in the total population stood at 15.85 per cent in 2001, declined marginally to 15.14 per cent in 2006 and went back up to its 15.85 percent in 2011. For earned income, the poverty rate was low at 14.25 rate in 2001 but it appears to have increased significantly in 2006 and 2011. Computed age-group specific rates show that those under 35 years old experienced the largest increases in poverty rates between 2001 and 2011 – for both total and earned income –while poverty rates among those in the age 35-64 age band declined, and that of the 65+ group either increased slightly (using total income) or decreased slightly (using earned income). Using the distribution-sensitive SST index, we can see that total income poverty increased by 20 per cent between 2001 to 2011 (SST Index values rose from 0.1017 in 2001 to 0.1219 in 2011) while earned income poverty increased by about 11 per cent between 2001 to 2011 (SST Index rose from 0.104 in 2001 to 0.1154 in 2011). Continuing on with the SST indices calculated, we find that those under 35 years old experienced significant increases in poverty rates between 2001 and 2011, while poverty rates amongst those in the age bands 65 and over age declined. This trend may be a cause for concern, not just in the context of Hong Kong's older cohorts but also for the younger ones as they take a greater proportion of the whole population with the passing of time.

Gini indices shown in Tables 3 show that, in general, inequality levels have steadily risen between 2001 to 2011 for 15-34 and 35-64 age groups and for overall results. Our findings show that the Gini indices tended to rise with age for both *Total* and *Einc* but not for *Oinc*.

[12] The Sen-Shorrocks-Thon (SST) index of poverty intensity (Shorrocks, 1995) can be calculated as I = (rate)*(gap)*(1+G(x)) where "rate" is the percentage of the population with incomes below the poverty line (sometimes called the head count ratio), "gap" is the average percentage gap between the incomes of the poor and the poverty line and G(x) is the Gini index of inequality of the poverty gap among all people. As such, it combines consideration of the poverty rate, average poverty gap ratio and inequality among the poor.

Further, our results indicate extremely high levels of inequality for *Oinc* – something that is not unexpected given the very small proportion of persons receiving dividends and other non-earned income. The age-income profiles of *Einc* and *Oinc* are also reversed as expected, that is, we find that inequality based on earned income is positively related with age, while inequality based on other income is inversely related with age.

5.2 Results of stochastic dominance tests

SD tests results for comparing distributions over time are summarized in Table 4. For *Total* income, we find that the distribution in 2001 ($Total^{01}$) dominated that of 2006 ($Total^{06}$) by SASD, but the reverse holds when testing for SDSD. This suggests that $Total^{01}$ has a lower proportion of poor units in relatively lower income levels but $Total^{06}$ has a higher proportion of rich units in relatively higher income levels. There is no stochastic dominance relationship when we compare $Total^{01}$ and $Total^{11}$ and compare $Total^{06}$ to $Total^{11}$. This simply means $Total^{01}$ has a higher social welfare if utility is concave and $Total^{06}$ has a higher social welfare if utility is convex. These findings demonstrate the important insights that can be obtained by the application of our new DSD approaches. In particular, the results show that by relying on the ASD test alone, one may be tempted to believe that the social welfare was lower in 2006 compared to that of 2001, because the latter dominated the former by SASD. However, with the new DSD tests of richness incorporated in the analysis, and finding that 2006 dominated 2011 by SDSD, we know that in fact we are uncertain about whether the social welfare has increased or decreased between 2006 and 2011, and that the conclusion reached about social welfare rankings from SASD alone can be misleading.

For earned income, no ascending dominance is detected between different years, but the 2001 ($Einc^{01}$) and 2006 ($Einc^{06}$) distributions appear to have dominated 2011 ($Einc^{11}$) by TDSD. Between $Einc^{01}$ and $Einc^{06}$, $Einc^{06}$ is shown to have dominated $Einc^{01}$ by TDSD. Lastly, for *Oinc* distributions, the income distribution of 2006 ($Oinc^{06}$) dominated that of 2001 ($Oinc^{01}$) by FASD and FDSD; 2006 distribution also dominated 2011 ($Oinc^{11}$) in the TASD and TDSD sense. These results indicate that the $Oinc^{06}$ distribution had a lower proportion of poor units in relatively lower income levels, and at the same time had a higher proportion of rich units in relatively higher income levels compared to the than $Oinc^{01}$ distribution. These lead us to conclude that the $Oinc^{06}$ distribution had a higher social welfare compared to the $Oinc^{01}$ distribution. Between $Oinc^{01}$ and $Oinc^{11}$, meanwhile, we find that $Oinc^{01}$ dominated $Oinc^{11}$ by SASD. This implies that the 2001 distribution $Oinc^{01}$ had a higher social welfare level compared to the $Oinc^{11}$ distribution only if utility is concave.

To obtain a deeper insight into the results obtained above and achieve a better characterization of the income distribution in Hong Kong, we partitioned the population into age groups, 15-34, 35-64 and 65+, and compared income distributions within and between them over the years 2001, 2006 and 2011. The results, found in Tables 5 to 7 are quite revealing. Additionally, they demonstrate the benefits obtained from using the new DSD tests of richness and poorness we propose in this paper alongside the standard ASD tests employed in the income distribution literature.

To facilitate the discussion of each table, we partition the each into blocks, as marked. Blocks A, C and F contain results for the pairwise comparisons of the three age groups, 15-34, 35-64 and 65+, respectively, with the same age cohorts across the three comparison years, 2001, 2006 and 2011. Blocks B, D and E meanwhile contain results which compares distributions between different age groups, for all possible pairs across the three comparison years.

In Table 5, results in Block A show that among the 15-34 age groups, the distribution of *Total* income in 2001 dominated both the 2006 and 2011 *Total* distributions, under both SASD and SDSD tests. This contemporaneous result suggests that the distribution in 2001 had comparatively less proportion of poor persons in relatively lower income levels, at the same time that the 2001 *Total* distribution had a higher proportion of rich persons in relatively higher income levels compared to those for 2006 and 2011. This implies that across the three year amongst persons aged 15-34, social welfare was higher in 2001 than in both 2006 and 2011 no matter utility is concave or convex.

For the within 35-64 group comparisons, in Block C, we can see that the income distribution for 2006 dominated the distributions of both 2001 and 2011 by SDSD. However, in the sense of SASD, the 2006 distribution was found to dominate the 2001 distribution only, at the same time that it is dominated by 2011 distribution. These results imply that in 2006, the 35-64 age group had both less proportion of poor units in relatively lower income levels, and a higher proportion of rich units in relatively higher income levels compared to the same age group in 2001. Further, this same 2006 age group, appears to have had only a higher proportion of rich units in relatively higher income levels compared to the same age cohort in 2011. In terms of social welfare ranking, these findings suggest that for this 35-64 age group, those in 2006 enjoyed a higher level of welfare compared to those 2001 no matter utility is concave or convex, but results are different between 2006 and 2011 such that 2011 has a higher social welfare if utility is concave and 2006 has a higher social welfare if utility is convex.

For the 65+ age group comparisons, in Block F, we find first-order dominance of 2011 distribution over both 2006 and 2001 distributions. And between 2006 and 2001, we also find strong first-order dominance of the 2006 distribution over the 2001 distribution. These results imply the income distribution of 2011 had the highest social welfare across time for any increasing utility, and that the social well-being of the elderly group had steadily improved between 2001 and 2011.

For results comparing inter-age distributions, Block B reveals that the *Total* distributions for the 15-34 age group were dominated by nearly all *Total* distributions of the 35-64 age group, in both the SASD and SDSD sense. In Block D, we likewise find that the *Total* distributions of the 15-34 age group were also dominated by all *Total* distributions for the 65+ age group, in the SDSD sense, and that they are mostly equivalent in the sense of FASD. The first finding implies that compared to their younger age cohort, the 35-64 age group had less proportion of poor units in relatively lower income levels, at the same time that they had a higher proportion of rich units in relatively higher income levels. The second finding meanwhile implies that the 65+ age group also had a higher proportion of rich units in relatively higher income levels compared to those in the 15-34 age group. If we use ASD results alone, These findings would only allow us to conclude that social welfare levels (with concave utility) are the same for both age groups, 15-34 and 65+ ,and that this level lower than the social welfare level in the 35-64 age group. But given the additional results on SDSD, we are able to conclude further on the existence of higher welfare levels (with convex utility) in the 65+ group compared to the 15-34 age group.

Block E contains the results for the pairwise comparisons of the 35-64 age distributions to the 65+ distributions over the years. We find that the younger cohort's income distributions dominated nearly all the distributions of the 65+ age group, in both the FASD and FDSD sense. This suggests that those in the 35-64 age group were generally better off than the 65+ group, having both lesser proportions of poor units, and a higher proportions of rich units.

Table 6 presents ASD and DSD results for *Einc* while Table 7 presents results for *Oinc*. In From Block A in Table 6, we find that amongst the 15-34 age group, the 2001 *Einc* distribution dominated the corresponding *Einc* distribution of 2006 by SASD, and that of 2011 by both SASD and SDSD. For this age group, these imply that those in 2001 had a less proportion of poor units in relatively lower income levels than those in 2006, at the same time that it had both a less proportion of poor units in relatively lower income levels and a higher proportion of rich units in relatively higher income levels compared to those in 2011. These suggest that the 2001 distribution enjoyed higher social welfare compared to 2006 and 2011 no matter utility is concave or convex – which is the same conclusion obtained from the analysis of *Total* for this age group.

Results for the 35-64 age group contained in Block C of Table 6 show that the *Einc* distribution for 2011 dominated all other *Einc* distributions in both the 2001 and 2006 by SASD, at the same time that the 2006 distribution dominated all others by SDSD. This combination of results lead us to conclude that that the 2011 *Einc* distribution had a higher social welfare compared to 2001 in we use the tradition approach, but the relative social welfare rankings of 2006 and 2011 become reverse given the additional SDSD results. Again, this demonstrates the advantage of using our test for both poorness and richness which is, in this case, avoiding misleading conclusions on relative welfare rankings based on SASD alone.

For the 65+ age group comparisons in Block F, we find that the 65+ *Einc* distribution for 2011 dominated the one of 2001 by FASD and FDSD; the same also dominated the distributions of 2006 by SASD but was dominated by 2006 in the sense of TDSD. These results suggest that for this older age group, 2011 had a higher social welfare to 2001 .

Test results comparing *Einc* distributions across age groups are found in Blocks B, D & E of Table 6. From B, we can see that between the *Einc* distributions of the 15-34 and the 35-64 age groups, six of the nine comparison tests conducted yielded no stochastic dominance between them. The exceptions found were the first-order ascending and descending dominance of the 35-64 group's 2006 distribution over the *Einc* distributions for the 15-34 age group for both 2006 and 2011. Additionally, the 2011 distribution for the 35-64 age group was also found to FASD and FDSD its younger counterpart in the same year. For these pairs, the results imply that those in the 35-64 age group had both a lower proportion of poor units, and a higher proportion of rich units – resulting in a higher social welfare.

Meanwhile in Block D, we can see that the 15-34 *Einc* distributions generally dominated those of the 65+ age group by SASD, but the reverse is observed for the SDSD results. This implies that the younger cohort generally had a lesser proportion of poor units in relatively lower income levels compared to those in their older counterpart; at the same time, those in the 65+ group are shown to have had a higher proportion of rich units in relatively higher income levels compared to the 15-34 group. In Block E, we further find that the *Einc* distributions of the 35-64 group dominated all the distributions of the 65+ in both FASD and FDSD sense. This suggests that the income distributions of age group 35-64 had a higher social welfare for any increasing utility.

All in all, these results are provide new insights into the relatively welfare levels in Hong Kong as measured by the distribution of incomes. First, we find that the 35-64 age tended to enjoy higher social welfare levels compared to their younger (15-34 group) and older (65+) counterparts. Secondly, we find high concentrations of poor income units for amongst the younger (15-34) cohort, at the same time that there are high concentrations of richer income units in the older 65+ cohorts. These results are clearly plausible and appear consistent with life-cycle theories of income and savings, which essentially predict low average incomes for the young, highest income levels in middle-age and lower incomes, but highest levels of savings and accumulated wealth into retirement. Our results show that such outcomes for the life cycle can be more accentuated in Hong Kong, where the economy is vibrant, dynamic and highly diversified in terms of the economic opportunities it presents for its constituents. Early estimates of the Gini coefficient were 0.43 in 1971, rising to 0.453 in 1986, and further up to 0.476 in 1991 (Chow and Papanek, 1981). The most recent government statistics indicate Hong Kong Gini coefficient to be 0.525 in 2001, 0.533 in 2006, 0.537 in 2011, placing the State alongside the poorest African and Latin American countries in terms of being the most income-unequal societies in the world. Indeed, the high concentrations of young poor and old rich that we found at the tails of the income distributions in Hong Kong across the years may help explain the overall high inequality level that has persisted in the economy since its rapid growth years of the 1970s until today.[13]

5.3 Further Discussion on SASD and SDSD

In the SD literature, it is more common to observe the reverse relationship between SASD and SDSD, that is, F dominates G by SASD at the same time that G dominates F by SDSD. In the last section however, the result where one income distribution both SASD and SDSD another distribution turns out to have appeared quite often in this paper. Levy (2015) has mentioned that it is possible that X both SASD and SDSD dominates Y but he has not given any real data example to illustrate this possibility and so far we have not read any real example report such finding. Thus, we believe that our multiple findings of such effect is the first to illustrate this point in the SD literature.

On the consistency of the results, stochastic dominance theory does not restrict the possibility that F dominates G by SASD at the same time that F also dominates G by SDSD. Levy (2015)

[13] These Gini coefficients come from Census and Statistic Department, Hong Kong SAR Thematic Report Household Income Distribution in Hong Kong, 2011 Population Census
http://www.statistics.gov.hk/pub/B11200572012XXXXB0100.pdf

gives out an example which shows why it is not necessary to have an opposite dominance by SASD and why it might be the case that F dominates G by SASD and at the same time F also dominates G by SDSD. In this paper, such results also appear, particularly in the comparisons between income distributions of different age group. A further illustration of this point is provided in both Tables 8 and 9. Table 8 reports the corresponding first-, second- and third-order ASD test statistics (i.e., T1, T2 and T3) over 100 grid points. We find that 24 per cent of T1 (i.e., grid 8 to grid 15) is significantly positive and 2 per cent of T1 (grid 3 to grid 4) is significantly negative. Therefore, we conclude that $Total_{35\sim64}^{06}$ does not dominate $Total_{35\sim64}^{01}$ by FASD . However, we find that 96 per cent of T2 (i.e., grid 7 to grid 100) and 95 per cent of T3 (i.e., grid 8 to grid 100) are significantly positive, while no T2 or T3 is significantly negative. Thus, we conclude that $Total_{35\sim64}^{06}$ dominates $Total_{35\sim64}^{01}$ by SASD and one could easily check from our tables that at the same time $Total_{35\sim64}^{06}$ dominates $Total_{35\sim64}^{01}$ by SDSD, suggesting that $Total_{35\sim64}^{06}$ has less proportion of poor income units than $Total_{35\sim64}^{01}$ in some relative lower income levels at one hand and on the other hand, $Total_{35\sim64}^{06}$ has higher proportion of rich income units than $Total_{35\sim64}^{01}$ in some relative higher income levels. This, in turn, implies that $Total_{35\sim64}^{06}$ has higher social welfare if utility on income is both concave and convex.

Table 9 reports the corresponding first-, second- and third-order DSD test statistics (i.e., T1, T2 and T3) over 100 grid points. We find that 2 per cent of T1 (grid 3 to grid 4) is significantly positive and 21 per cent of T1 (grid 8 to grid 15) is significantly negative. Therefore, we conclude that $Total_{35\sim64}^{06}$ does not dominate $Total_{35\sim64}^{01}$ by FDSD . However, we find that 18 per cent of T2 (i.e., grid 1 to grid 15) and 11 per cent of T3 (i.e., grid 1 to grid 7) are significantly negative, while no T2 or T3 is significantly positive. Thus, we conclude that $Total_{35\sim64}^{06}$ dominates that of $Total_{35\sim64}^{01}$ by SDSD, suggesting that $Total_{35\sim64}^{06}$ has a higher proportion of rich income units than $Total_{35\sim64}^{01}$ in some relative higher income levels and $Total_{35\sim64}^{06}$ has higher social welfare if utility or welfare on income is convex. The graph of this example will be further illustrated in the Appendix 2.

6. Conclusion

In this paper, we propose new stochastic dominance tests to achieve a more robust analysis of relative welfare levels in the study of income distributions. Our new tests of richness and poorness provided greater capacity for achieving sharper inference for welfare analysis. In particular, when comparing two income distributions X and Y, we showed the achievement of the following new findings not previously achievable from using standard (ascending) stochastic dominance analysis alone:

a) X could have both "less poor" and "more rich" than Y, *for any income levels*, which implies that X has a higher social welfare for all increasing well-behaved SWFs
b) X could have "less poor" than Y in *some relative lower income levels* AND have "more rich" than Y in *some relative higher income levels*, which implies that X has a higher social welfare for well-behaved SWFs with concave or convex utility function.
c) X could have "less poor" than Y in *some relative lower income levels* at the same time that Y could have "more rich" than X in *some relative higher income levels*, which implies that X has a higher social welfare for well-behaved SWFs with concave utility function and has a less social welfare for well-behaved SWFs with convex utility function.

Such depth of inference that can be achieved with our proposed approach – incorporating dominance results from both the lower and upper tails of welfare distributions - bear important implications for policy. Conclusions of welfare dominance therefrom offer better characterization and understanding of current levels of inequality, which can lead to more appropriately designed social and economic programs for achieving the desired welfare levels in their respective societies.

Our empirical application provides results that are theoretically consistent with life-cycle theory of income and savings. As well, they provide new insights into relatively welfare of age groups in Hong Kong in the period between 2001 and 2011. Using our new tests of richness and poorness to Hong Kong census data, we find, among other things, that there exists a very high concentration of poor individuals among the younger age cohorts, as well as high concentration of rich individuals among the older, retired cohorts. This result can be a viable explanation to the overall high inequality level that has persisted in the economy since its rapid growth years of the 1970s until today.

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Table 1. Descriptive statistics

	2001	2006	2011
Persons by Age Group			
% share 15-34	39.6	37.3	34.6
% share 35-64	58.7	60.9	63.6
% share 65+	1.7	1.7	1.8
Mean Incomes (HK$)			
Total Personal Income	16,561	16,651	17,013
	(140.66)	(143.17)	(143.34)
Earned Income	15,934	15,704	16,094
	(165.16)	(135.31)	(135.68)
Other Income	627	947	919
	(72.01)	(86.58)	(83.44)

Table 2: Poverty Measures, Hong Kong

	$Total$			$Einc$			$Oinc$		
Age	2001	2006	2011	2001	2006	2011	2001	2006	2011
Head Count Ratio									
Overall	0.1586	0.1514	0.1585	0.1425	0.1607	0.1568	0	0	0
15-34	0.1450	0.1682	0.1877	0.1484	0.1716	0.1897	0	0	0
35-64	0.1671	0.1586	0.1581	0.1643	0.1686	0.1397	0	0	0
65->	0.1364	0.1419	0.1396	0.1825	0.1616	0.1703	0	0	0
Sen-Shorrocks-Thon Poverty Index									
Overall	0.1017	0.1061	0.1219	0.1040	0.1169	0.1154	0	0	0
15-34	0.1234	0.1058	0.1261	0.1261	0.1098	0.1234	0	0	0
35-64	0.1118	0.1077	0.1079	0.1079	0.1128	0.1118	0	0	0
65->	0.1134	0.1164	0.1134	0.1595	0.1562	0.1595	0	0	0

Table 3: Gini Index for broad age groups

	$Total$			$Einc$			$Oinc$		
Age	2001	2006	2011	2001	2006	2011	2001	2006	2011
15-34	0.3935	0.4029	0.4262	0.3904	0.3935	0.4177	0.9870	0.9874	0.9893
35-64	0.4905	0.4959	0.4957	0.4865	0.4868	0.4848	0.9695	0.9688	0.9716
65->	0.5837	0.5858	0.5669	0.5703	0.5593	0.5514	0.909	0.9295	0.8988
Overall	0.4651	0.4793	0.4859	0.4602	0.4682	0.4742	0.9759	0.9753	0.9767

Table 4. Results of Stochastic Dominance Tests of Richness and Poorness

Total income [$F\backslash G$]	$Total^{01}$	$Total^{06}$
$Total^{06}$	$F \prec_2^A G$ $F \succ_2^D G$	
$Total^{11}$	$F \equiv^A G$ $F \equiv^D G$	$F \equiv^A G$ $F \equiv^D G$
Employment income [$F\backslash G$]	$Einc^{01}$	$Einc^{06}$
$Einc^{06}$	$F \equiv^A G$ $F \succ_3^D G$	
$Einc^{11}$	$F \equiv^A G$ $F \prec_3^D G$	$F \equiv^A G$ $F \prec_3^D G$
Other income [$F\backslash G$]	$Oinc^{01}$	$Oinc^{06}$
$Oinc^{06}$	$F \succ_1^A G$ $F \succ_1^D G$	
$Oinc^{11}$	$F \succ_2^A G$ $F \equiv^D G$	$F \prec_3^A G$ $F \prec_3^D G$

Note: The results are based on SD statistics for the first three SD orders, related SD theory and related income distribution function

Table 5. Stochastic Dominance tests of Richness and Poorness for total income, *Total*.

$F \backslash G$	$Total^{01}_{15\sim34}$	$Total^{06}_{15\sim34}$	$Total^{11}_{15\sim34}$	$Totalh^{01}_{35\sim64}$	$Total^{06}_{35\sim64}$	$Total^{11}_{35\sim64}$	$Total^{01}_{\geq65}$	$Total^{06}_{\geq65}$	$Total^{11}_{\geq65}$
$Total^{01}_{15\sim34}$		**A**							
$Total^{06}_{15\sim34}$	$F \prec^A_2 G$ $F \prec^D_2 G$								
$Total^{11}_{15\sim34}$	$F \prec^A_2 G$ $F \prec^D_2 G$	$F \equiv^A G$ $F \equiv^D G$							
$Total^{01}_{35\sim64}$	$F \equiv^A G$ $F \succ^D_2 G$	$F \equiv^A G$ $F \succ^D_2 G$ **B**	$F \succ^A_2 G$ $F \succ^D_2 G$		**C**				
$Total^{06}_{35\sim64}$	$F \equiv^A G$ $F \succ^D_2 G$	$F \succ^A_2 G$ $F \succ^D_2 G$	$F \succ^A_2 G$ $F \succ^D_2 G$	$F \succ^A_2 G$ $F \succ^D_2 G$					
$Total^{11}_{35\sim64}$	$F \succ^A_3 G$ $F \succ^D_2 G$	$F \succ^A_1 G$ $F \succ^D_1 G$	$F \succ^A_2 G$ $F \succ^D_2 G$	$F \succ^A_2 G$ $F \prec^D_3 G$	$F \succ^A_2 G$ $F \prec^D_2 G$				
$Total^{01}_{\geq65}$	$F \prec^A_2 G$ $F \succ^D_2 G$	$F \prec^A_2 G$ $F \succ^D_2 G$ **D**	$F \prec^A_3 G$ $F \succ^D_2 G$	$F \prec^A_1 G$ $F \prec^D_1 G$	$F \prec^A_1 G$ $F \prec^D_1 G$ **E**	$F \prec^A_1 G$ $F \prec^D_1 G$		**F**	
$Total^{06}_{\geq65}$	$F \equiv^A G$ $F \succ^D_2 G$	$F \equiv^A G$ $F \succ^D_2 G$	$F \equiv^A G$ $F \succ^D_2 G$	$F \prec^A_1 G$ $F \prec^D_1 G$	$F \prec^A_2 G$ $F \prec^D_2 G$	$F \prec^A_1 G$ $F \prec^D_1 G$	$F \succ^A_1 G$ $F \succ^D_1 G$		
$Total^{11}_{\geq65}$	$F \equiv^A G$ $F \succ^D_2 G$	$F \equiv^A G$ $F \succ^D_2 G$	$F \equiv^A G$ $F \succ^D_2 G$	$F \prec^A_1 G$ $F \prec^D_1 G$	$F \prec^A_1 G$ $F \prec^D_1 G$	$F \prec^A_1 G$ $F \prec^D_1 G$	$F \succ^A_1 G$ $F \succ^D_1 G$	$F \succ^A_1 G$ $F \succ^D_1 G$	

Note: The result is based on SD statistics for the first three SD orders, related SD theory and related income distribution function.

Table 6. Stochastic Dominance tests of Richness and Poorness for employment income, *Einc*.

$F \backslash G$	$Einc^{01}_{15\sim34}$	$Einc^{06}_{15\sim34}$	$Einc^{11}_{15\sim34}$	$Einc^{01}_{35\sim64}$	$Einc^{06}_{35\sim64}$	$Einc^{11}_{35\sim64}$	$Einc^{01}_{\geq65}$	$Einc^{06}_{\geq65}$	$Einc^{11}_{\geq65}$
$Einc^{01}_{15\sim34}$			**A**						
$Einc^{06}_{15\sim34}$	$F \prec^A_2 G$ $F \equiv^D G$								
$Einc^{11}_{15\sim34}$	$F \prec^A_2 G$ $F \prec^D_2 G$	$F \equiv^A G$ $F \equiv^D G$							
$Einc^{01}_{35\sim64}$	$F \equiv^A G$ $F \succ^D_2 G$	$F \equiv^A G$ $F \equiv^D G$ **B**	$F \equiv^A G$ $F \succ^D_2 G$			**C**			
$Einc^{06}_{35\sim64}$	$F \equiv^A G$ $F \succ^D_2 G$	$F \succ^A_1 G$ $F \succ^D_1 G$	$F \succ^A_1 G$ $F \succ^D_1 G$	$F \succ^A_2 G$ $F \succ^D_2 G$					
$Einc^{11}_{35\sim64}$	$F \equiv^A G$ $F \equiv^D G$	$F \succ^A_2 G$ $F \equiv^D G$	$F \succ^A_1 G$ $F \succ^D_1 G$	$F \succ^A_2 G$ $F \prec^D_3 G$	$F \succ^A_2 G$ $F \prec^D_2 G$				
$Einc^{01}_{\geq65}$	$F \prec^A_2 G$ $F \succ^D_3 G$	$F \prec^A_1 G$ $F \prec^D_1 G$ **D**	$F \prec^A_2 G$ $F \succ^D_2 G$	$F \prec^A_1 G$ $F \prec^D_1 G$	$F \prec^A_1 G$ $F \prec^D_1 G$ **E**	$F \prec^A_1 G$ $F \prec^D_1 G$			**F**
$Einc^{06}_{\geq65}$	$F \prec^A_2 G$ $F \succ^D_2 G$	$F \prec^A_3 G$ $F \succ^D_2 G$	$F \prec^A_2 G$ $F \succ^D_2 G$	$F \prec^A_1 G$ $F \prec^D_1 G$	$F \prec^A_1 G$ $F \prec^D_1 G$	$F \prec^A_1 G$ $F \prec^D_1 G$	$F \succ^A_1 G$ $F \succ^D_1 G$		
$Einc^{11}_{\geq65}$	$F \prec^A_3 G$ $F \equiv^D G$	$F \equiv^A G$ $F \equiv^D G$	$F \equiv^A G$ $F \succ^D_2 G$	$F \prec^A_1 G$ $F \prec^D_1 G$	$F \prec^A_1 G$ $F \prec^D_1 G$	$F \prec^A_1 G$ $F \prec^D_1 G$	$F \succ^A_2 G$ $F \prec^D_3 G$	$F \succ^A_2 G$ $F \prec^D_3 G$	

Note: The result is based on SD statistics for the first three SD orders and related income distribution function.

Table 7. Stochastic Dominance tests of Richness and Poorness for other income, *Oinc*

$F\backslash G$	$Oinc_{15\sim34}^{01}$	$Oinc_{15\sim34}^{06}$	$Oinch_{15\sim34}^{11}$	$Oinc_{35\sim64}^{01}$	$Oinc_{35\sim64}^{06}$	$Oinc_{35\sim64}^{11}$	$Oinc_{\geq65}^{01}$	$Oinc_{\geq65}^{06}$	$Oinc_{\geq65}^{11}$
$Oinc_{15\sim34}^{01}$		**A**							
$Oinc_{15\sim34}^{06}$	$F \succ_1^A G$ $F \succ_1^D G$								
$Oinc_{15\sim34}^{11}$	$F \succ_2^A G$ $F \succ_2^D G$	$F \prec_1^A G$ $F \prec_1^D G$							
$Oinc_{35\sim64}^{01}$	$F \succ_1^A G$ $F \succ_1^D G$	$F \succ_2^A G$ $F \equiv^D G$ **B**	$F \succ_1^A G$ $F \succ_1^D G$		**C**				
$Oinc_{35\sim64}^{06}$	$F \succ_1^A G$ $F \succ_1^D G$	$F \succ_1^A G$ $F \succ_1^D G$	$F \succ_1^A G$ $F \succ_1^D G$	$F \succ_1^A G$ $F \succ_1^D G$					
$Oinc_{35\sim64}^{11}$	$F \succ_1^A G$ $F \succ_1^D G$	$F \succ_2^A G$ $F \equiv^D G$	$F \succ_1^A G$ $F \succ_1^D G$	$F \succ_2^A G$ $F \equiv^D G$	$F \prec_2^A G$ $F \prec_2^D G$				
$Oinc_{\geq65}^{01}$	$F \succ_1^A G$ $F \succ_1^D G$	$F \succ_1^A G$ $F \succ_1^D G$ **D**	$F \succ_1^A G$ $F \succ_1^D G$	$F \succ_1^A G$ $F \succ_1^D G$	$F \succ_2^A G$ $F \prec_3^D G$ **E**	$F \succ_1^A G$ $F \succ_1^D G$		**F**	
$Oinc_{\geq65}^{06}$	$F \succ_1^A G$ $F \succ_1^D G$	$F \succ_1^A G$ $F \succ_1^D G$	$F \succ_1^A G$ $F \succ_1^D G$	$F \succ_1^A G$ $F \succ_1^D G$	$F \succ_1^A G$ $F \succ_1^D G$	$F \succ_1^A G$ $F \succ_1^D G$	$F \succ_2^A G$ $F \succ_2^D G$		
$Oinc_{\geq65}^{11}$	$F \succ_1^A G$ $F \succ_1^D G$	$F \succ_1^A G$ $F \succ_1^D G$	$F \succ_1^A G$ $F \succ_1^D G$	$F \succ_2^A G$ $F \succ_2^D G$	$F \succ_2^A G$ $F \equiv^D G$	$F \succ_1^A G$ $F \succ_1^D G$	$F \succ_1^A G$ $F \succ_1^D G$	$F \succ_2^A G$ $F \prec_2^D G$	

Note: The result is based on SD statistics for the first three SD orders, related SD theory related income distribution function.

Table 8. ASD test statistics for comparing $Total^{01}_{35\sim64}$ and $Total^{06}_{35\sim64}$

grid	T1	T2	T3	grid	T1	T2	T3	grid	T1	T2	T3	grid	T1	T2	T3
1	8.259	0.000	0.000	26	-0.171	5.651	5.866	51	4.507	5.241	5.536	76	-1.651	5.030	5.399
2	5.235	6.617	6.581	27	2.221	5.563	5.860	52	3.378	5.252	5.531	77	-1.780	5.020	5.393
3	-4.361	3.119	4.631	28	0.467	5.494	5.850	53	2.690	5.257	5.526	78	-1.661	5.010	5.387
4	-6.158	0.351	2.490	29	3.445	5.473	5.838	54	0.971	5.255	5.521	79	-1.702	5.000	5.381
5	2.291	1.305	1.603	30	2.092	5.411	5.824	55	1.155	5.250	5.517	80	-1.724	4.991	5.375
6	4.296	2.324	1.661	31	0.662	5.367	5.809	56	1.359	5.245	5.512	81	-0.934	4.981	5.369
7	2.529	3.044	2.087	32	2.769	5.331	5.793	57	1.443	5.240	5.507	82	-0.787	4.974	5.363
8	8.588	4.210	2.573	33	5.415	5.286	5.776	58	-0.243	5.230	5.503	83	-0.787	4.967	5.357
9	10.371	4.606	3.102	34	1.235	5.323	5.761	59	-0.459	5.217	5.498	84	-1.540	4.959	5.351
10	6.049	5.156	3.577	35	1.245	5.283	5.746	60	-0.995	5.205	5.493	85	-1.540	4.951	5.346
11	8.237	5.716	3.984	36	0.965	5.243	5.731	61	-1.425	5.191	5.487	86	-1.540	4.943	5.340
12	6.273	5.966	4.366	37	5.113	5.261	5.716	62	-0.440	5.176	5.482	87	-2.378	4.934	5.334
13	3.577	6.112	4.676	38	1.160	5.246	5.702	63	-0.901	5.165	5.476	88	-1.811	4.926	5.328
14	9.022	6.170	4.921	39	0.862	5.215	5.688	64	-1.474	5.152	5.471	89	-2.007	4.917	5.323
15	4.586	6.242	5.129	40	2.338	5.195	5.674	65	-0.355	5.139	5.465	90	-2.007	4.909	5.317
16	1.982	6.405	5.304	41	1.185	5.175	5.660	66	-1.159	5.128	5.459	91	-1.856	4.901	5.311
17	8.592	6.261	5.448	42	0.851	5.144	5.647	67	-0.590	5.118	5.453	92	-2.273	4.893	5.305
18	2.445	6.319	5.566	43	1.381	5.114	5.632	68	-1.269	5.108	5.447	93	-2.121	4.884	5.300
19	7.198	6.301	5.659	44	0.844	5.090	5.618	69	-1.298	5.098	5.441	94	-2.189	4.876	5.294
20	-0.202	6.169	5.735	45	3.694	5.090	5.604	70	-1.289	5.087	5.435	95	-2.189	4.868	5.289
21	0.830	6.087	5.787	46	1.773	5.087	5.591	71	-1.733	5.076	5.429	96	-2.425	4.860	5.283
22	4.325	5.983	5.822	47	1.603	5.072	5.578	72	-0.989	5.066	5.423	97	4.461	4.852	5.278
23	-0.300	5.905	5.849	48	0.842	5.049	5.565	73	-0.989	5.057	5.417	98	4.461	4.850	5.272
24	2.038	5.807	5.862	49	17.137	5.025	5.552	74	-1.244	5.048	5.411	99	4.461	4.849	5.267
25	3.198	5.694	5.867	50	17.877	5.163	5.542	75	-1.253	5.039	5.405	100	0.000	4.847	5.261

Notes: This table reports ASD test statistics over 100 grid points for comparison between $Total^{01}_{35\sim64}$ and $Total^{06}_{35\sim64}$.

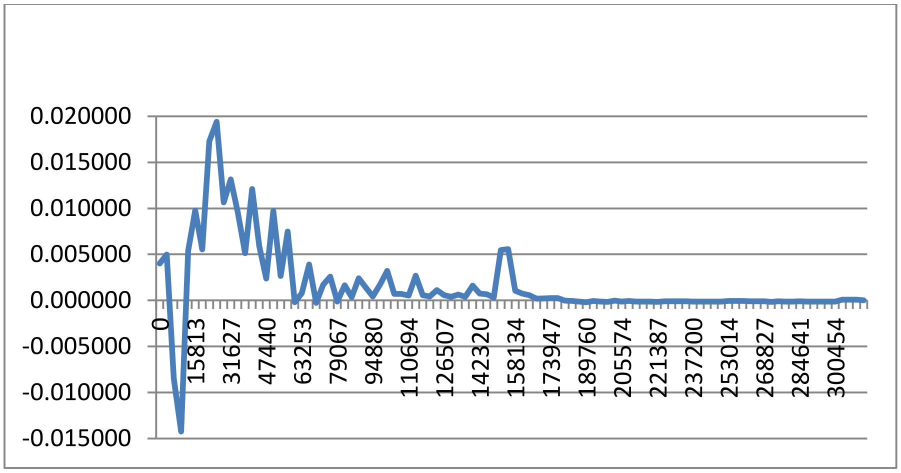
Table 9. DSD test statistics for comparing $Total_{35\sim64}^{01}$ and $Total_{35\sim64}^{06}$

grid	T1	T2	T3	grid	T1	T2	T3	grid	T1	T2	T3	grid	T1	T2	T3
1	0.000	-5.284	-3.386	26	0.171	-2.236	-1.265	51	-4.478	0.379	1.328	76	1.542	1.511	1.327
2	-5.369	-5.177	-3.283	27	-2.225	-2.295	-1.199	52	-3.355	0.634	1.368	77	1.680	1.497	1.306
3	4.485	-5.173	-3.173	28	-0.468	-2.330	-1.127	53	-2.674	0.837	1.398	78	1.566	1.479	1.283
4	6.328	-5.406	-3.049	29	-3.450	-2.268	-1.051	54	-0.959	0.986	1.421	79	1.594	1.454	1.259
5	-2.350	-5.395	-2.911	30	-2.088	-2.314	-0.971	55	-1.138	1.097	1.438	80	1.609	1.429	1.235
6	-4.399	-5.275	-2.768	31	-0.660	-2.323	-0.887	56	-1.345	1.212	1.450	81	0.859	1.393	1.210
7	-2.587	-5.128	-2.626	32	-2.762	-2.316	-0.796	57	-1.423	1.330	1.457	82	0.723	1.430	1.181
8	-8.775	-4.738	-2.489	33	-5.388	-2.344	-0.702	58	0.239	1.392	1.458	83	0.723	1.477	1.139
9	-10.60	-4.538	-2.363	34	-1.230	-2.136	-0.608	59	0.448	1.424	1.457	84	1.457	1.500	1.083
10	-6.178	-4.209	-2.247	35	-1.239	-2.168	-0.513	60	0.965	1.446	1.454	85	1.457	1.493	1.013
11	-8.406	-3.798	-2.141	36	-0.961	-2.206	-0.413	61	1.379	1.443	1.451	86	1.457	1.481	0.927
12	-6.399	-3.525	-2.050	37	-5.094	-2.039	-0.310	62	0.424	1.432	1.449	87	2.271	1.449	0.821
13	-3.639	-3.288	-1.967	38	-1.151	-1.994	-0.211	63	0.863	1.453	1.446	88	1.720	1.384	0.695
14	-9.179	-3.097	-1.890	39	-0.855	-2.018	-0.108	64	1.408	1.455	1.442	89	1.899	1.320	0.541
15	-4.659	-2.889	-1.822	40	-2.316	-2.002	0.002	65	0.336	1.437	1.438	90	1.899	1.231	0.350
16	-2.009	-2.577	-1.762	41	-1.176	-1.982	0.113	66	1.097	1.453	1.434	91	1.753	1.124	0.109
17	-8.706	-2.562	-1.709	42	-0.845	-2.021	0.230	67	0.560	1.475	1.429	92	2.128	0.978	-0.205
18	-2.472	-2.346	-1.661	43	-1.371	-2.055	0.354	68	1.203	1.477	1.423	93	1.981	0.751	-0.611
19	-7.274	-2.204	-1.615	44	-0.838	-2.062	0.483	69	1.218	1.488	1.415	94	2.029	0.420	-1.141
20	0.204	-2.219	-1.574	45	-3.669	-1.920	0.616	70	1.229	1.484	1.406	95	2.029	-0.117	-1.813
21	-0.835	-2.184	-1.530	46	-1.757	-1.783	0.744	71	1.659	1.467	1.398	96	2.235	-1.148	-2.559
22	-4.353	-2.191	-1.483	47	-1.586	-1.729	0.875	72	0.939	1.473	1.389	97	-3.000	-3.000	-3.000
23	0.302	-2.175	-1.436	48	-0.832	-1.723	1.010	73	0.939	1.492	1.378	98	-3.000	-3.000	-3.000
24	-2.047	-2.203	-1.384	49	-16.83	-1.710	1.152	74	1.172	1.509	1.364	99	-3.000	-3.000	-3.000
25	-3.214	-2.277	-1.326	50	-17.55	-0.449	1.269	75	1.178	1.520	1.347	100	-3.000	0.000	0.000

Notes: This table reports DSD test statistics over 100 grid points for comparison between $Total_{35\sim64}^{01}$ and $Total_{35\sim64}^{06}$.

Appendix 2. Interpreting DSD outcomes – An Illustration.

Figure 1. The difference between CDFs of $\boldsymbol{Total^{01}_{35\sim64}}$ and $\boldsymbol{Total^{06}_{35\sim64}}$



We assume that F and G are two different income distributions. If F dominates G by SASD, we can conclude that F has less proportion of poor units in some relative lower income levels and F could have more or less proportion of rich in some relative higher income levels. On the other hand, if F dominates G by SDSD, we can conclude that F has higher proportion of rich units in some relative rich income levels and F could have more or less proportion of poor units in relative some lower income levels. Thus, F both SASD and SDSD dominates G implies that F has less proportion of poor units in some relative lower income levels at the same time F has more proportion of rich in some relative higher income levels.

Figure 1 gives an example for such results. Here we plot the difference between cumulative distribution functions of $Total^{01}_{35\sim64}$ and $Total^{06}_{35\sim64}$, that is, plot out the value of cumulative distribution function of $Total^{01}_{35\sim64}$ minus cumulative distribution function of $Total^{06}_{35\sim64}$ for all income levels.

If $Total^{06}_{35\sim64}$ dominates $Total^{01}_{35\sim64}$ in the sense of FSD, then it should be found that the difference between CDFs of $Total^{01}_{35\sim64}$ and $Total^{06}_{35\sim64}$ should be negative for all income levels. We can see that distribution of $Total^{06}_{35\sim64}$ have less proportion of poor income units than distribution of $Total^{01}_{35\sim64}$ for almost all income levels. However with some of the exceptions, distribution of $Total^{06}_{35\sim64}$ now cannot dominates distribution of $Total^{01}_{35\sim64}$ by FASD and FDSD. We can see that the higher proportion in income levels around income level 6325 drive up the cumulative distortion function of $Total^{06}_{35\sim64}$, as a result, making the cumulative distribution of $Total^{06}_{35\sim64}$ higher than the cumulative distribution of $Total^{01}_{35\sim64}$ between income levels 6325 and 9488, making distribution of $Total^{06}_{35\sim64}$ dominates $Total^{01}_{35\sim64}$ only by SASD and SDSD but not FASD and FDSD. [14]

[14] We are just giving an example to let one understand what might happen. We suggest that one might not want to rely too much on the graph in the analysis because this approach totally ignore the statistical properties of the data. For more details about the relationship between SASD and SDSD, one could refer to Levy (2015).

Appendix 3. A Further Guide to interpretation of observed results.

To recap in general terms, we could have the following observations in a study for any two income distributions F and G:

Observation 1: F stochastically dominates G in both first-order ASD and first-order DSD.

Observation 2: F does not stochastically dominate G in both first-order ASD and first-order DSD, but F dominates G by both SASD and SDSD,

Observation 3: F does not stochastically dominate G in both first-order ASD and first-order DSD, but F dominates G by SASD while and G dominates F by SDSD,

Observation 4: $F = G$.

Observation 5: None of the observations in the above. In this situation, there could be higher (than 2) order dominance or there is no dominance between F and G.

If Observation 1 is found such that income distribution F stochastically dominates income distribution G in both first-order ASD and first-order DSD, then we can conclude that F is preferred to G for any increasing social welfare functions. If income distribution F does not stochastically dominate income distribution G in both first-order ASD and first-order DSD (that is, Observation 1 is not observed) and if F and G are different (that is, Observation 4 is not observed), then we have three different situations: Observations 2, 3, and 5. However, in this paper, we do not observe Observation 5. So, there are only two different situations left: Observations 2 and 3. We explain these 2 situations below: if Observation 2 is observed such that income distribution F dominates income distribution G by both SASD and SDSD, then we can safely conclude that F is preferred to G in terms of social welfare functions with concave or convex utility. On the other hand, if Observation 3 is observed such that F dominates G by SASD at the same time that G dominates F by SDSD,[15] we conclude that F is preferred to G in terms of social welfare with concave utility and G is preferred to in terms of social welfare with convex utility.

[15] For a linear social welfare function, one could compare the mean of different distribution. This is a very common practice in the society. There are many people always try to compare the GDP per capita and average personal income between countries or time periods and say something about the change on living standard or well-being. To making these kinds of inference valid, it will be necessary to assume the social welfare functions of the society are monotonic, symmetric additively separable and linear.