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Mathematics teaching: A study of early childhood teachers' pedagogical positioning in a playworld context

(Above, below, or equal? Exploring teachers' pedagogical positioning in a playworld context to teach mathematical concepts to preschool children)

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The research reported here is part of a larger study to investigate how early childhood teachers can use *Mathematical Playworld* to shift their pedagogical positions to enhance children's mathematical learning through the visual data analysis of an imaginative play scenario between two teachers and 11 children in an early learning centre. We argue that early childhood teachers consider using the playworld model to allow them to be inside children's play to align their learning intentions with children's imaginative play actions. This study frames how early childhood teachers can create the conditions for intentional mathematical teaching opportunities within play-based contexts.

Keywords: Imaginative play, Mathematical Playworld, Pedagogical positioning Play-based mathematical learning, Playworld

1. Introduction

There is a growing understanding that early childhood is a time for children to be exposed to and develop a range of mathematical ideas (MacDonald, 2018). In research conducted by Reid and Andrews (2016) on the numeracy abilities of 219 Australian children in the year prior to school, they demonstrated that children enter school settings with a broad range of mathematical knowledge. Indeed, there is a body of research that highlight children's prior to school experience allows them to enter school contexts having formed school-based mathematical expectations and that these foundations impact on subsequent later achievement in mathematics education (Clarke et al. , 2006; Clerkin & Gilligan, 2018; Gervasoni & Perry, 2015; MacDonald & Carmichael, 2018). While emphasising the significance of early math to young children's later academic achievement, there is an increasing concern of how to support early childhood children's mathematical concept learning in a playful way, without transforming these into school-based instructive approaches (Graue, Whyte & Karabon, 2015; Magnusson & Pramling, 2018). Play should be considered as a promising setting for early mathematics education, that takes children's perspective in teaching (Ginsburg, 2006). Helenius, et al. (2016, p. 154) argues that play can be mathematical for young children, while considering the interrelations among children's participatory, creativity and rule negotiation in play. Further they suggest that "a teacher's active participation in the play

could contribute to children learning more about mathematics" while confirming the role of teachers in children's play to help their mathematise as suggested by Lee and Ginsburg (2009).
The teachers' role in supporting children's mathematical learning in play has been emphasised in recent studies including noticing and understanding children's everyday mathematics and mathematical thinking in play (Anantharajan, 2020; Papandreou & Tsiouli, 2020); encouraging girls to engage with mathematical thinking (Simpson & Linder, 2016); creating a rich mathematical environment (Worthington & van Oers, 2016); providing the direct mathematical concept teaching (Carruthers & Worthington, 2011; Oppermann et al., 2016); posing mathematical problems to children to further their mathematical thinking (Fosse et al., 2020) and playing along with the children through the responsive engagement (Magnussan & Pramling, 2018). Although there is a shared argument in relation to the importance of teachers' role in promoting children's mathematical learning, there is no current research to explain the teachers' dynamic pedagogical positioning in supporting children's mathematical concept learning in play. In the present study, we will explore a kindergarten teacher's interaction with a small group of young children in a collective imaginary situation in play while they solve a mathematical problem in relation to the concept of measurement. More specifically, we will analyse the teacher's dynamic positioning as they become active play partners with the children, thus support children's mathematical thinking and problem solving in relation to the concept of measurement.

This paper starts with the literature review on the relations between mathematics education and play, then explains the conceptualisation of play and subject positioning by drawing upon cultural-historical theory, followed by an analysis of a teachers' interaction with a small group of children in play to solve a mathematical problem. The paper concludes with a discussion of the dynamic pedagogical positioning in imaginative play and implications for teaching.

2. Mathematics and play

Vygotsky (1966) makes clear that play is the leading activity of development in the preschool years. However, there appears a great tension in early childhood mathematics literature regarding the role of "play" within young children's mathematical learning. One of the key challenges in early childhood mathematics education is to make a meaningful mathematical learning environment for young children, one that avoids becoming a replica of school based structural instruction methods (Schwartz, 2005; van Oers, 2010).

In an extensive elaboration of how young children best learn mathematics, Salomonsen (2020) explains four types of play pedagogies, including direct instruction, guided adult-initiated play, guided child-initiated play, and free play. Similarly, Wickstrom et al. (2019) in their observational study in Ontario, Canada found the exact same four mathematical pedgagogies used in play-based kindergarten settings. Direct instruction led by teachers is effective when the instruction is aligned with children's everyday experiences and suitable to children's competence and skill level (Clements et al., 2013; Stockard & Engelmann, 2010). Some studies have criticized this approach as it doesn't encourage meaningful learning for young children, especially if the teachers' agenda doesn't match the children's interests and initiative (Clements et al, 2013; Gottfried, 2016).

Guided play includes two types of play, including adult-initiated and child-initiated play. Guided adult-initiated play encourages teacher initiated play activities that support their mathematical teaching agenda, progressively leading to self-initiated activities by the child. Guided play has been shown to provide significant higher mathematical learning outcomes for preschool aged children than direct instruction (Vogt et al., 2018). In contrast, child-initiated play means the adults become the participants in the child's own play without interrupting children's play, while guiding children to play with other children in mathematical problem solving (Salomonsen, 2020). In order to achieve a high-quality learning in mathematical free play, there is a great need for the teachers to introduce interesting mathematical problems into their play and giving the relevant feedbacks in relation to the children's mathematical performance (Gottfried, 2016; van Oers & Poland, 2012). Magnussan and Pramling (2018) recommend that guided play needs to be promoted to enhance children's mathematical thinking as it is associated with children's play interests rather than the teacher-directed play approach. Furthermore, there is growing literature that speaks to an increase in children's mathematical learning outcomes through guided play (Eason & Ramani, 2020; Fisher et al., 2013; Jemutai & Webb, 2019; Nakawa, 2020).

Another type of play pedagogy in relation to children's mathematics education is free play, which is defined as the child's own play and without intervention from adults (Salomonsen, 2020). There is a great debate in relation this play approach. Some studies explain that children's own exploration through play allows them to transfer and develop their everyday mathematical concepts (Carruthers, 2015; Ginsberg, et al., 2008). However, others argue that children apply mathematics in their free play in a limited degree and children achieve less in math through free play due to less involvement with adults who possess deeper mathematical conceptual knowledge (Chien te al., 2010; Gifford, 2005).

Kinnear and Wittmann (2018) explain that for children to use mathematics with purpose, mathematical elements and rules underpinned by mathematical pattern and structure must be learned. They propose that the teaching of such knowledge cannot be done purely through everyday interaction or in an ad-hoc manner. Instead, there is a specific need for these mathematical concepts to be taught, including within early childhood settings. Without mathematical conceptual knowledge , children will not have the mathematical tools to draw upon to be creative in their play. Suggesting that for children to be mathematically creative, they must understand mathematical elements and rules to put into action. Sumpter (2016) suggests that young children that are guided can expand their mathematical thinking further than without a guide and in Ryoo's et al. (2018) study when teachers play an active role in children's mathematucal learning, positive outcomes occur for children. Such results promote the role of the teacher as a facilitator of culturally specific mathematical knowledge. However, as Fleer (2015, p. 1812) argues, when children are engaged within play, teachers should not try "smuggling in content, or bolting onto an existing play event a learning goal", therefore the role of the facilitator should not compromise children's creative imagination. Kravtsov and Kravtsova (2010) further articulate this point by suggesting that children, peers, and adults need to be involved in the collective imaginative scenario to align with the shared collective play experience. Otherwise, there is potential for the players to be misaligned within their objectives. Magnusson and Pramling (2018) also suggest that early childhood pedagogy should promote and build a mutual ground between teachers and children in play, rather than simply following children's interests, or offering just direct instruction.

Furthermore, Kinnear and Wittmann (2018, p. 29) postulate that in order to bring the pure aspects of mathematics to bear, it is necessary "that children also experience mathematics as an 'artificial' world, that is, an abstract mathematical world that has been created by humans and that is governed by internal rules integrally connected to the pattern and structure inherent in mathematics". It is giving rise to the notion that teachers can create conditions for this to occur. Thus, the ultimate tension regarding 'play' and its role within children's mathematical development appears less 'if' play is a suitable vehicle, instead 'how' can it be utilized within pedagogical practice to fulfil children's interests and mathematical development. However, there is currently a dearth of research investigating how teachers can balance their pedagogical roles and reconsider their pedagogical positioning within play-based activity setting to support children's initiative and mathematical content knowledge learning. This paper will provide an evidence that demonstrates how the teachers dynamically position themselves in play to support children's mathematical learning.

Using a cultural-historical understanding, it can be said that when developing children's mathematical knowledge, teachers need to consider their role within children's play. Considering how they can create the conditions in the learning environment that motivate children to act and build upon prior experience to facilitate mathematical knowledge growth through co-created imaginary situations within play. To understand the pedagogical decision making of early childhood teachers in the context of play-based settings, the paper will draw upon a cultural-historical concept of play to investigate how teachers take the dynamic pedagogical positions in supporting children's mathematics learning (the concept of measurement in this case) in an imaginative play-based activity setting.

3. Theoretical framework

3.1. A cultural-historical conceptualisation of play

Play has been variously defined by different theories in the past (Dewey, 1900; Parten, 1933; Grusec & Lytton, 1988). Instead of reviewing the different perspectives on play, this study draws upon a cultural-historical theoretical understanding of play to theorize how mathematical knowledge can be developed within play-based learning contexts (Vygotsky, 1966). The two key characteristics of Vygotsky's concept of play (1966) has emerged to help position our understanding of children's mathematical learning within the current study. Children create the imaginary situation in play (Vygotsky, 1966), which in our research was central to children's mathematical conversations and learning. Imaginative play does not come about due to biological maturation; instead, it is highlighted by the conditions created for children to grow and learn (Elkonin, 2005). Thus, this study argues that children and adults collectively create the imaginary situation, which motivates children's mathematical problem-solving processes, thus supporting their mathematical learning.

Furthermore, Vygotsky (1966) also discusses the appearance of symbolic objects through imagination as one of characteristics of children's play. His now-famous example of a child using a stick as a horse explains how and why children change the meanings of objects, thereby giving a new sense to actions while meeting the desires of play. That is, these objects are then used symbolically to assist the child in acting out their understanding of societal conditions. In this study, we will explore how the teachers positioning themselves in supporting children to use symbolic objects to solve mathematical problems in the collective imaginative play.

Drawing upon cultural-historical concept of play, a pedagogical approach, playworld is developed within the collective imaginary situation created by children and teachers in play, based on a selected storybook (Lindqvist, 1995). Over the past 25 years, the playworld approach has found resonance within the early childhood community in several different ways, including enhancing working with infants and toddlers (Authors, 2020) and detailing methods of connecting the personal and professional self within children's play (Ferholt et al., 2019). Playworlds have also been used to make learning personally meaningful for young children (Ferholt & Rainio, 2016), enhancing executive functioning (Fleer, 2019a; Fleer et al., 2017; Fleer et al., 2020) and used to teach such curricula areas as science (Fleer, 2019b, 2019a), engineering (Fleer, 2020) and mathematics (Authors, 2021). By extending Lindqvist's playworld approach, Fleer (2018a) developed a Conceptual PlayWorld model, where children and teachers create meaningful, emotionally changed movements by taking on character roles from the selected story and through the imaginative process, create and solve conceptual problems that arise. The essence is for both the children and teachers to be 'inside' of the play scenario, whereby the conceptual problem can arise from the shared collective moment (Fleer, 2019a; Authors, 2020). To date, using a playworld model for the specific enhancement of children's mathematical development has only just been undertaken (Authors, 2021), which has focused on children's personal meaningful learning experience in a *Mathematical Playworld*. However, how the teachers dynamically position themselves within the *Mathematical Playworld*, to support children's mathematics learning has not been explored, which is the focus of this paper.

3.2. Subject positioning in play

In considering the adults' role in children's play, Kravtsov and Kravtova (2010) have conceptualised the pair pedagogues as subject positioning in play, where the teachers work in pairs to positioning themselves in relation to the other teacher or child(ren). The dynamic positioning can be explained including "above the child", "equal with the child", "below the child", "child is independent of the teacher", and in a "primordial we" position. When taking "above" position, the teacher guides the children to connect the content knowledge with their play through the suggestions, modelling and explanation. When we see the child take the initiative in play and solve their own problem, then the teacher takes the "below" position. When the teacher takes an "equal" position, we might see the child and the teachers equally negotiate their play ideas on what to play, how to play or solving the problem together (Author, 2012). The "primordial we" means the teacher actively takes the lead to model the play performance as the child needs more help, although he or she is part of the play, but not essentially understanding and contributing to the play. For instance, when a younger child joins an imaginative play scenario about dragons, they may enter as if a baby dragon, but not really understand the storyline or purpose of the play, the teacher might be as a mummy dragon, holding the child's hands to engage with the play narrative. When the child is independent of the teacher, we might see the child is much more capable to play, work out the play narrative and solve the "problem" without the support of the teacher. Meanwhile, the child will still need the teachers' social references from a distance to confirm what they have done (Fleer, 2015). By drawing upon Kravtsov and Kravtova (2010)'s subject positioning, in this paper, we argue teachers and child(ren) positioning in play is a dynamic process of play interactions and can be a diving force to support children's mathematical learning.

3. Method

3.1 Educational Experiment

In order to understand the teachers' pedagogical positioning, drawing upon cultural-historical theory, the larger study was an educational experiment in which *Mathematical Playworld* is a pedagogical intervention in the early learning centre (Hedegaard, 2008b; Fleer, Fragkiadaki, & Rai, 2020). The educational experiment is a process whereby the researcher and the teachers collaborate to facilitate a pedagogical intervention that is systematically observed (Hedegaard, 2008b). This study implemented a Fleer's Conceptual PlayWorld approach to facilitate mathematical learning for children aged 3-4 years old within an early learning centre. The two researchers worked with two teachers to develop a playworld over an extended period. Hedegaard (2008b, p. 185) explains that the educational experiment "is that the intervention is planned in relation to a theoretical system and not simply from agendas of practice". In this study, the theoretical system was framed within cultural-historical research, seeing mathematical learning in imagination and play. The theoretical problem for change in relation to the pedagogical positioning to teaching and learning in play has been shaped within the educational experiment.

3.2 Contexts and Participants

As fitting an educational experiment, the researchers worked directly with pseudonym named teachers Mayer (bachelor qualification, 10+ years teaching experience) and Sue (diploma qualification, between 5-10 years teaching experience) at a randomly selected early learning centre in Melbourne, Australia, focusing on one class of 22 children aged 3-4 years of age. Working within a Victorian early childhood service, Mayer and Sue are mandated to implement the Victorian Early Years Learning and Development Framework [VEYLDF] (Department of Education and Training Victoria [DET], 2016). Within the VEYLDF (DET, 2016), they recommend three integrated approaches to teaching and learning, being 1) adult led learning, 2) child-directed play and learning, and 3) guided play and learning, which explains the dynamic adult-child interactions in play and teachers' positioning in play.

Ethical consent for this study was obtained through both researchers' university Research Ethics Committee and the Performance and Evaluation Division, Department of Education and Training (DET). In alignment with ethical principles, informed full consent was given from family, teachers and directors of the early learning centre for the use of collected data for research and educational purpose. The participants' pseudonym has been used and their personal information such as facial expression and school logo on the uniform were deidentified.

Before beginning the study, the researchers, in collaboration with the teachers introduced themselves and explained the study to the children. It was also explained to the children that the researchers would be filming them during their play. The researchers agree with Parks and Schmeichel (2014) and their analysis that digital recording mathematical interactions has the potential to provide great insight into children's thinking. However, the reseacrhers are also hightently aware of the tensions of collecting data via visual methods and retaining ethical consent at all times (Robson, 2011; Rutanen et al., 2018). For example, in our study it was explained to children and teachers and that if they did not want the researchers to record them, they could ask them to stop verbally or through hand gestures (i.e. holding up their palm in a stop sign). More than children's verbalisations or outward gestures to stop, the researchers were aware to observe for other signs, such as staring at the camera or walking away as signs to stop recording. Furthermore, researchers were also sensitive to interpersonal moments between teachers and children that did not need to be recorded for the study, for example not following participants into bathrooms or movements of behaviour management not pertinent to children's mathematical learning.

3.3 Data Collection

Pre-study surveys were conducted with both teachers to ascertain demographic details and to consider their mathematical pedagogical practices in the early learning centre. The survey addressed confidence and competence to teach mathematics intentionally.

Before implementing the playworld, the teachers and researchers conducted a focus group discussion as a professional development workshop with the participating teachers to discuss the theoretical background, practical implementation of playworlds and mathematical concept knowledge in the early years. Within the larger study, there was ongoing collaboration between the researchers and teachers in implementing the playworld and discussing specific mathematical questions and pedagogical inquires the teachers had. This dialogue was captured using the medium of email and onsite discussion after each filming, that served as both a planning tool and reflection platform.

In the larger study, the researchers used video observation to record the implementation of the Mathematical Playworlds (see Table 1). As teachers implemented the playworld with children, the two researchers positioned themselves, so one video recorder was focused on teacher interactions, the other camera was focused on the whole group setting. The researchers visited the early learning service on six occasions, with recording lasting between 1.5-3 hours per visit, totalling 26 hours of video data in total. The first three sessions of the Mathematical Playworld focussed on the concept of measurement. In this paper, we only selected one session (Mathmatical Playworld session #3) from the six visits with two cameras each capturing 1.5 hours of video data that focused on concepts related to measurement. To contextualise children's engagement with measurement concepts, Van de Walle et al. (2018, p. 332) explains that in order to measure something, children must 1) decide on the attribute to be measured 2) select a unit that has that attribute 3) and compare the units. The vignettes shown in the results section detail how children went about measuring their "broom" within the Mathematical Playworld.

Pre-and post-pair group interviews were conducted before and after the playworld implementation (see Table 1). Each interview was video recorded and lasted 1 hour in duration. The pre-interview focused on teachers' perception of playworlds heading into implementation and the post-interview aimed to find out the affectiveness and effectiveness of a *Mathematical Playworld* pedagogical approach, identified by the teachers.

Table 1. *Chronological summary of visual data*

Focus of data recording	**Research activities**	**Hours of data**
2 cameras (1 focussed on teacher, 1 focussed on children during observation of regular play activities)	Visual baseline data collection: Visit #1 (2x3 hours) Visit #2 (2x3 hours)	12
1 camera focussed on interview	Pre-educational experiment pair interview	1
2 cameras (1 focussed on teacher, 1 focussed on children during observation of Mathematical Playworld implementation)	Visual Mathematical Playworld data collection: Session 1 (2x1.5 hours) Session 2 (2x2 hours) Session 3 (2x1.5 hours) Session 4 (2x2 hours)	14
1 camera focussed on interview	Post-educational experiment pair interview	1
Total hours of data collected (26 hours of visual data, 2 hours of interview data)		28

3.4 Data analysis

To align with the educational experiment, the study drew upon a dialectical-interactive and wholeness approach (Hedegaard, 2008a; Authors, 2014), which includes the societal, institutional and personal to analyse the data by using the spirals of analytical framework. After being heavily involved in the research setting, the analysis allows the researchers to remove themselves from the research setting and interpret the research protocols to explicate the theoretical concepts (Hedegaard, 2008a). The spirals of visual data analysis allow for a non-linear interpretation of data, explaining how teachers engage with children in the imaginative play to promote children's mathematical concept learning (Authors, 2014).

Three different forms of interpretation within the spirals of analytical framework shape and unite in analysis by using the digital video technologies (Fleer, 2014). The *common-sense interpretation* as the first spiral of analysis of the raw data explains the multiple perspectives and understandings of the interactions within the activity setting. Within this project, the different perspectives including teachers' perspective, children's perspective and researchers' views on the visual data have been identified through the common-sense interpretation. The researchers selected video clips (such as the presented data in this paper) in terms of the research aims as a prompt to share with the participating teachers in the pre- and post-interviews to gain the teachers' perspectives in their pedagogical decision. Through this spiral of analyses, the researchers also generate a basic understanding of the social situation, such as the playworld activity settings (Sørensen, 2019).

Situated practice interpretation, as the second spiral of analysis focuses on the interactions between children and adults by using the theoretical concepts. In this study, the two researchers watched the video data, made comments to the data, then used the theoretical concepts (in this case, the concept of play and positioning) to identify the patterns of interaction between teachers and children, in particular the teacher's dynamic pedagogical positions within the mathematical playworld activity setting were sought.

The third spiral of analysis is *interpretation on a thematic level* by using the theoretical concepts to answer the research questions. Within this study, a cultural-historical theoretical understanding of play guided the interpretation of teacher and child interactions within the playworld context to answer the research questions. It allowed researchers to conceptualize how teachers used different positions while being inside children's imaginative playworld in order to meet children's demands, thus achieve the pedagogical agenda in supporting children's mathematical problem solving in relation to measurement.

4. Findings

4.1 Pedagogical positioning in teaching and learning

As explained earlier, playworld starts with a selection of the storybook, that orients children and teachers' collective imagination. In this study, the teachers chose to use the children's book *Room On The Broom* (Donaldson, 2003) as they observed how children in their class are interested in Witches and they also believed this book is closely relevant to the mathematical concept of measurement.

After reading the storybook with children, the teachers asked the children to reflect what they like in the storybook and most children expressed that they like the new broom as they can fly on it to see the world. Both teachers (Teacher Sue as a witch, and Teacher Mayer as a dog) and children (acting as a frog, a dog, a cat etc.) took the character roles of the book and collectively entered the playworld activity setting.

4.1.1 Vignette 1: Teachers' below positioning

By taking children's perspective, the teachers dramatized a mathematical problem by reading a letter from the Witch (Teacher Sue)'s sister, Britchy. The letter invited the children to 'the witchy-Britchy birthday party', with the primary stipulation that they all come on one newly designed broom to fly to the party. The children and teachers had collectively counted the number of seats they would need on the broom based on how many characters were involved, which was 13. After entering the playworld, children initially play with materials (e.g. blocks) to create a broom that can carry 13 people to fly to the party (see Figure 1). The teachers, in this case, both held the below position to observe children's play, while children constructed the "broom" as if they were the friends of the witches.

[Figure 1 near here].

Figure 1. Teachers watching on as children co-construct a broom during a child initiated play moment.

During a several-minute period, all children were engaged with constructing a straight-line structure that resembles the basic shape of a broom, as represented in Figure 1. During this time, children collectively discussed the practical nature of constructing the broom in terms of where the blocks needed to go. Also, the children were motivated by acting as if they were frog, cat or dog; for example, in Figure 1, there are three frogs standing together, deciding where to sit as a group of frogs and what they would need. Also, Samuel (taking the role of a frog) had a detailed conversation about his character needing a water tap to keep his skin moist. However, what they appeared more focused on, was their own individual seat for their acting characters, instead of working towards one broom for all the animal friends. Therefore, they lost the chance to discuss and use mathematical language to create a broom that could fit 13 people. Instead, children randomly placed blocks to create the broom without any child appearing to count how many seats there were altogether or considering the new broom for all the animal friends.

This vignette has shown children engaged in play and exploring the mathematical problem, with teachers acting primarily as observers of children's play and positioning themselves as below to primarily see how children constructed the 'broom'. In this case, both teachers were observers while being character roles as if they were witch or dog. This allowed both teachers to build up their understanding of children's intention, initiation and exploration of this dramatized conceptual problem. It was observed that children concentrated on their role within the constructed imaginary situation, going to the witch's birthday party. In the post playworld interview, Mayer credited the imaginary processes involved in playworlds for motivating children to participate in the problem solving when stating, "*I think the opportunity to engage in that role play, to actually be able to take on a character*" and Sue finished "*They really took that on board*". The teachers sat themselves with below positions, which allowed children to become embedded within the moment and affectively engaged within the play scenario and their character roles. It can be seen that collectively the children are building a broom symbolically based on their prior knowledge and understanding of what a broom is and what they would need for the journey to the witch's party.

However, despite the collective imaginative play that children were engaged in whilst creating the broom, the mathematical focus of measurement (how many seats needed and how long/short are the seats) was not discussed by children. They did not collectively solve the conceptual problem of making *one* broom for all the animal friends, instead focused on their character role. However, relatively positioning children as above position led to children making emotional connection with their character roles, which motivated them to have an imagined solution for their role. As both teachers were inside the play, paying attention to children's thought processes, this built the foundation for them to guide children's exploration to use mathematical concepts to solve the problem of how to create a broom with 13 seats, as seen in Vignette 2.

4.1.2 Vignette 2: Teachers' equal positioning with children

After observing children's engagement in making the new broom for a short period, the teachers began to shift their teaching and learning approach and positioned themselves to explore the problem with children together. Mayer asks the group, “I've noticed that there are bigger seats and small seats”, holding up her hands in respective poses. Sue then asks the question, “so, does everyone have the same amount of room”? Both teachers, still using their play-roles and dramatized voices, began asking children how many friends can fit on their respective blocks. Some of the blocks are square and can fit one friend, whilst other blocks are rectangular (twice the single blocks) and can fit two friends; therefore, the units are not the same size. Mayer places three blocks in front of the group, two small and one big and asks, “how many friends can fit there?”. One child, Joan, [playing the role of a witch], calls out that “three people can sit on these three blocks”.

[Figure 2 near here].

Figure 2. Teacher and child engaged in guided play, discussing the attributes of length.

Teacher Mayer (see Figure 2) then asks, “are they all the same size [children call out “no”]…what's different about these blocks”? Samuel [a frog] then stands up and points at each block and says, “That one’s long [pointing at the rectangular block] and that one’s short and that one’s short [pointing at the two square blocks]”. Mayer asks him to repeat what he said, he points at the two smaller blocks and says, “these two are the same”, the teachers in unison say, “they're the same size”, and Mayer adds, “but this one is long [pointing to the longer one]”. Mayer then asks Samuel, “how many people do you think can fit on there”? Rose answers for Samuel and says, “two could sit there (pointing at the two smaller blocks and *one could sit there* (pointing at the larger block)”. Rose [a witch] then stands up and takes two smaller blocks, adds them to the broom length, and says, “two more could sit here”. The teacher removes Rose's block and says, “Thank you witchy [Rose's character role in playworld], but firstly we're going to test our theory…let's have a think about this”.

As the playworld continued to evolve, the teachers involved themselves within the children’s play and learning, building on their initial interest of using the blocks to make the witch’s new broom and responding to learning opportunities as they arose. Through the in-depth analysis, we found that Teacher Mayer has intentionally made the responses to Joan’s thoughts on three people sitting on the three blocks, and directly asked “Are they [three blocks] the same size?” to negotiate with the children. Through guided approach positioning herself as equally with children was applied by Mayer to build children's understanding of measurement attributes. Within Vignette 2, both teachers continually used their character roles to explore the problem with children together, asking guiding questions and discuss with children on the units of measurement. The teachers had shifted their positioning from below to be equal while they negotiate with children on how to measure a broom within the collective imaginary situation as if they were all friends of Witchy to create a new broom. In so doing, they were able to have mathematical conversations with children and responding directly to children’s challenges in relation to the measurement concepts as they played out their narrative in making a new broom.

Whilst the teachers were focused on the attributes of measurement, Rose [a witch] did not quite understand the attributes and was thinking in terms of the units being equal, demonstrated when stating that the long block could sit one person, the same as the short blocks. As shown in Vignette 3, the teachers realised a need to consolidate the mathematical conceptual understanding of measurement, which corresponds to a shift to position themselves as above to lead the learning moment.

4.1.3 Vignette 3: Teachers' above positioning

There were three blocks positioned in front of the group with four children sitting on the three blocks, one each on the two small blocks and two children on the larger block; all other children were sitting watching. Mayer asks, "so there were three blocks, but how many people have fitted on"? Rose [a witch], previously sitting on a block, stands up and counts by tapping children on the head and gets the correct sequence and correspondence saying "four". Mayer says, "three blocks, but four friends fitted, how come that happened"? then goes on to question, "the idea was that three friends would fit, but now we have four, how did that happen"? Mayer then explains that the smaller blocks have one 'friend' and that the larger blocks have two, because they are longer. Sue then asked another child to come and sit with the group [the child has a small block], asking the entire group how many the broom can now fit, and the children in unison say "five". This is repeated with varying sizes of blocks until all children and teachers are now sitting on the broom. Mayer says to the group, "we might need to write this down how many blocks do we have, how many short blocks…", before finishing, Rose says, "how many long blocks", to which Mayer echoes, "how many long blocks"? The children stand up and move behind the broom; Sue asks Belinda [A frog] to help count the blocks (see Figure 3).

[Figure 3 near here].

Figure 3. Belinda counting the number of long blocks, other children looking on attentively during an adult guided learning moment.

Belinda firstly counts the big blocks, up to four with correct sequence and correspondence, the group counts along. Mayer, holding a pen and writing pad, then says, "so the measurement is four long blocks", and writes the fact down. Another child, Kate, is then asked to count the small blocks, doing so with correct sequence and correspondence up to five. The teachers in unison say, "four long blocks and five short blocks", and Mayer finishes the sentence by saying, "fit how many people, how many friends"? One of the children calls out "13". Mayer then says, "13 friends, I think we've got a measurement, give ourselves a clap", to which the children give a loud clap. Mayer then says, "I like how this broom is very close, no spaces in between [linking to the notion of tiling], but I think we need to zoom for lunch now". The children then transition for lunch.

Vignette 3 has demonstrated an example of adult guided learning while teachers took above positions, which occurs when adults introduce the concept, direct the learning and ask questions that provide structure for the learning task. Teachers, building on children's learning from Vignette 2, supported children's mathematical understanding of measurement by introducing the concept of short and long blocks to seat people on to the broom. During Vignette 3, teachers modelled appropriate mathematical language (e.g., *so the measurement is four long blocks?*), explain the smaller blocks can fit one 'friend' and that the larger blocks have two, asked appropriate questions to challenge children's understanding about the attributes of length (*so there were three blocks, but how many people have fitted on?)* and confirm the outcomes *(four long blocks and five* *short blocks*). Sumpter (2016) describes that preschool-aged children need to be supported to explore and produce their mathematical reasoning, both individually and collectively, which is shown in Vignette 3 during the adult guided moment of adults taking above position. When Teacher Mayer introduced the concept of units by demonstrating the differences between small and long blocks, Rose, who had shown some confusion in Vignette 2, drew upon the mathematical experiences and responded with correct answers, demonstrating her newly developed conceptual understanding.

5. Discussion

This paper has drawn upon a cultural-historical concepts of play and subject positioning in play to demonstrate how teachers position themselves dynamically within a *Mathematical Playworld* activity setting to enhance children's mathematical learning. This research found that in order to meet the societal demands to the integrated teaching and learning approaches (DET, 2016), the teachers were able to dynamically position themselves in ways that allowed for children's initiative and content learning in play, all of which were highlighted within the shared play experience to enhance children's mathematical learning, in this case, a focus on measurement. The results from this study highlight the importance of positioning the teacher inside of children's play so that children were motivated to solve the mathematical problem and thus enhancing their mathematical learning. We conceptualise that shaping teachers' dynamic pedagogical positioning enables young children to transform their present moments of play to achieve mathematical learning within a collective imagination situation of playworld. This will be discussed forthwith.

We found that by using a playworld approach, teachers developed a keen awareness of children's play and mathematical learning needs, as they were inside of the play as play partners, thus shifting their pedagogical positions to meet those needs. Teachers were transitioning from one position to the next, dependent on the children's identified needs and aligning with the conceptual mathematical ideas they wanted to teach. The adult and child positioning within the *Mathematical Playworld* are essential as a driving force to support children's mathematical engagement. For example, the relative positioning of children as "above" in Vignette 1 appeared to unite the children in a collective imaginary play experience where they were motivated to build the broom and enrich their play. However, they were not using mathematical ideas or language to solve the problem of constructing a broom that could fit 13 people. This aligns with other studies (Edwards & Cutter-Mackenzie, 2013; Sliogeris & Almeida, 2019), that free play is the least likely form of play to prompt teachers to develop conceptual knowledge in children. In this case, the teachers were both 'hovering' around the children and watching the children's initiation. After observing and getting to know children's play intentions, the teachers repositioned themselves and moved to an "equal" subject positioning to negotiate with children, to find out the problem and challenge children's mathematical thinking in alignment with children's needs. Therefore, we argue that the teachers' below positioning [shown in Vignette 1] in the playworld activity setting appears to motivate children and support their initiative to make personally meaningful play and learning in relation to problem solving [e.g., making a new broom]. As the teachers are active inside of the imaginary situation supports them to make a clear understanding of children's concerns and challenges [e.g., how to make a broom for all 13 friends], in doing so, teachers are able to shift their pedagogical positions to actively support children's mathematical learning by taking children's perspective.

The equal positioning in the interaction were favoured by our participating teachers, as teacher Mayer suggested in her pre-playworld interview, "*…we might be stepping in to stretch that learning"*. By taking equal positions with children, the teachers are able to negotiate the play ideas and discuss the conceptual problem, thus *stretching* children's learning. As Van de Walle et al. (2018) describe, children need an opportunity to compare the attributes of measurement, in this case, shorter and longer, which Samuel [a frog] in Vignette 2 demonstrated to the group through shared conversation with the teachers and role modelled to the class using those specific mathematical concepts. This level of thinking and dialogue with teachers illustrate the point made by Sumpter (2016) that children can expand their mathematical thinking further with guidance than without.

Nevertheless, within our study, it was shown that only taking the equal positioning through the guided play and learning was not enough to support children's conceptual thinking. Rose, who was engaged within the *Mathematical Playworld* experience, was struggling in Vignette 2 (guided play and learning by equal positioning in the interaction) to understand that the blocks differing sizes could allow more people to fit on them; instead, assuming that one block meant one person. In Vignette 3 by taking "above" positions through guided adult-initiated play, teachers demonstrated through a concrete representation the difference in size of the two blocks, asking every child to sit on a space, thereby showing a physical representation of measurement that the small blocks could fit one person and the larger blocks could fit two. By doing this, teachers take what could potentially be an abstract concept for the child and make it concrete by getting children to learn through the process of *doing mathematics*. As was subsequently observed from Rose in Vignette 3, this form of learning led to continued high levels of engagement from Rose as well as demonstrated correct responses to the teacher's questions. Our study highlights that to support conceptual thinking; teachers need to shift their pedagogical positioning to be above, to support children's conceptual thinking when taking the below or equal positions are not enough. Therefore, we argue that the playworlds model provides the possibilities for the teacher to shift their positions to meet their teaching agenda and enhance children's mathematical conceptual knowledge in alignment with children's intention.

In this paper, we echo the argument by Fleer (2015), explaining the diversity of pedagogical play practices observed within our study that featured the teachers being inside of the imaginary situation within the playworld. To begin, as seen in Vignette 1, the teachers acting as character roles [a dog and a witch], taking the below position while applying the guided child-initiated play to consider children's intention and challenges in solving the problem of making a new broom with 13 seats that could fit all of their friends. Through being inside the play and sharing the collective imaginary situation, the teachers were able to capture the learning moments as well as identifying the challenges children faced. Which, in this example, was a focus purely on their own seat without consideration of the collective learning task of creating a broom for all of their friends. Therefore, there was a distinct lack of mathematical thinking or language observed. Yet, owing to their collective imagining with children, the teachers shifted the position to "equal" interactions to guide children thinking mathematically, as can be seen in Vignette 2. The teachers shared the collective imaginary situation with children's play intent (Fleer, 2015), which was as if they were the friends of the witch, to create the broom through the symbolic use of blocks (Elkonin, 2005; Vygotsky, 1966). By entering the imaginary situation with children together, we argue that the playworld experience allowed for enacting mathematical concepts and problem-solving. Within the *Mathematical Playworld*, both the children and adults alike were engaged in sustained mathematical conversations whereby they had a shared collective experience, and their play was conceptually realised (Kravtsov & Kravtsova, 2010). Fleer (2015) explains the importance of sustained conversations between adults and children, and this occurred within the playworld experience when teachers shifted their pedagogical positions from below (allowing children taking initiation to make connections with the play problem) to equal (negotiating with children on how to solve the problem) and above (introducing the mathematical concepts). This ultimately allowed the teachers to recognize a need for mathematical conceptual knowledge support, demonstrated in the guided adult-initiated play moment in Vignette 3. Within this process, due to the nature of being *inside* children's play, adult agendas are aligned with children's play intentions.

That early childhood teachers are mandated to utilise play-based learning as a pedagogical approach (DET, 2019) drives the need for a successful play-based approach that supports mathematical teaching and learning. Some researchers have addressed differing forms of play, generally along a continuum of play types (Edwards et al., 2017; Wood, 2014) and Fleer's (2015) work has highlighted the importance of the adult being inside of children's play. Our paper builds upon Fleer's (2015) work, validating the importance of teachers being inside children's play. Moreover, we have extended on Fleer's work by giving substantive examples of how teachers can shift dynamic pedagogical positioning while being inside children's play to support mathematical learning within the *Mathematical Playworld* activity setting. We suggest that by taking different pedagogical positioning interwoven within the playworld environment, supports teachers in developing children's mathematical problem-solving actions.

6. Conclusion

The findings drawn from this in-depth qualitative study over a sustained seven-month period is limited due to the results being based on one early learning centre and two focus teachers. Therefore, the conclusions drawn from the study need to be tempered. However, there is no doubt that the findings point to the potential benefits of using a playworld approach to support children's mathematical learning in play-based settings. We argue that the playworld conditions created by the teachers allowed the adjustment of dynamic pedagogical positions to shape the way children and teachers engage with mathematical concepts and the conceptual problem. The findings of the study informs early years mathematical teaching and learning in play-based settings. Further research could be done to investigate how the adults' different pedagogical positions can be used in diverse early years settings to teach different mathematical concepts.

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