STOCHASTIC INVENTORY PROBLEM

Lilian Chan
February 2015
THE PROBLEM

- A manufacturing company faces an inventory problem in which a monthly decision needs to be made on the order size requested from suppliers.
- There are many probabilistic uncertainties, such as varying demand, material consumption rate and delivery time.
- There is also a chance of faulty parts, and the materials’ perishability increases with time.
- Upon stock-out of materials, an express delivery request for parts can be sent to the supplier at a much greater cost (undesirable).
- There is also an ongoing cost for the storage of materials at the company.
The following information is currently known:

- Parameters such as the number of pallets available
- Historical data on the probability distribution of delivery times
- Data relating to the spread of the variable consumption rate
- Probability of encountering faulty items and how this increases with increasing age
- Information about the daily, weekly and monthly variation of demand for the items
The main sources of uncertainty are:

- Monthly order arrival time: $35 \pm 5$ days
- Consumption rate of material: $\pm 5\%$
- Uncertainty of demand from Company: $\sim 20\%$

There are four different parts which are produced by the company, each on different, non-interchangeable manufacturing lines, and with different demands. The following discussion focuses on a single part. A balance is to be found between avoiding stock-out, and wastage of materials due to deterioration.
MODELLING THE SYSTEM

- The AnyLogic simulation software was used to model this problem.
- Two variations of the model were developed: a stock-based model and an agent-based model.
- The stock-based model offered the advantages of simplicity and accessibility. It may be easily modified and used by people with no experience of the software. Additionally, optimisations can be run to determine the optimal order size.
- The agent-based model offered a more accurate representation of all the uncertainties involved, but requires more work when modifying the values and more computing memory.
Figure 1: Image showing the stock-based model of ‘inventory problem’
STOCK-BASED MODEL

- A simple stock-based model was developed as a preliminary basis of the final version.
- This involved an inventory ‘inv’ with initial stock size 1000.
- The variable demand was modelled by ‘flow’, which was a uniform, random distribution of between 80 – 100 items per day.
- The events ‘order’ and ‘order_exp’ represent the cases when orders are made to the supplier for parts.
- It was decided that an express order would be made whenever the inventory had less than 100 items, and that this would be 2.5 times more expensive than a normal order.

Figure 2: Image showing the stock-based model running
STOCK-BASED MODEL (Optimisation)

- The order sizes for the monthly and express orders were parameters which could be changed.
- The variable, ‘cost’ shows the current expenses from the orders and express orders.
- Optimization was run to determine the optimal order sizes to minimise the cost.
- This was found to be a monthly order size of ‘2699’.
- The optimal express order size was found to be 0 (as it is undesirable to use an express order).
- The optimal order size may vary slightly from run to run, due to the stochastic nature of the problem.

Figure 3: Image showing optimisation of the stock-based model.
STOCK-BASED MODEL (Optimisation)

- These optimal settings were then used to modify the original parameters of order sizes.
- As can be seen from the stock-time graph, the inventory size sharply increases at the arrival of each batch of stock (modelled to be in intervals of a random, linear probability between 30 – 40 days).
- The inventory size then decreases throughout the month in a mostly linear fashion.
- This graph lacks the flat regions at the bottom seen on the graph with smaller order size (Figure 2), indicating that express orders have not been needed in this case.

**Figure 4**: Image showing stock-based model running using optimum order size.
MODIFIED STOCK-BASED MODEL

- Although the previous model offered a good starting point, there were several considerations which had not been taken into account.
- These were the perishability of the items, and the ongoing unit-holding cost.
- New events were now added to the model: ‘inv_cost’ uses the current number of items in the inventory and adds costs daily, ‘aging’ causes the variable ‘age’ to increase daily, and ‘decaying’ is an event which occurs whenever stock is more than 42 days old, discarding old stock.

Figure 5: Image showing the improved stock-based model.
MODIFIED STOCK-BASED MODEL (Optimisation)

- As the model has now been changed, optimisation needs to be run again, to determine the new optimal order size.
- The total cost to be optimised is now the addition between inventory holding cost and order costs.
- Here, inventory cost was modelled to be $0.2 per item, per day.
- The optimal order size was found to be '1044'. However, this size would change depending on the specifics of the order, express order and holding costs (not disclosed by company).
- These cost values can easily be changed by Company to find the actual optimum size.

**Figure 6:** Image showing optimisation of the improved stock-based model.
MODIFIED STOCK-BASED MODEL

- It is evident that there are still some limitations with this model. This is because the items in ‘inv’ are considered as one entity, thus individual stocks cannot be made to have different ages.
- However, in the actual scenario, there would be stock with a range of different ages.
- Thus, an agent-based model is constructed, using the optimum values obtained from the stock-based model as a guideline.

**Figure 7:** Image showing the improved stock-based model running
AGENT-BASED MODEL (GIS)

Figure 8: Image showing the agent-based model of 'inventory problem'
AGENT-BASED MODEL

- The agent-based model defines each component of the system as an agent.
- The ‘company’ is an agent which orders supplies from the ‘distributor’, and the end products are sent upon demand to the retailers in 10 different locations.
- The carriers used to distribute supplies from the distributor to the company are ships and planes.
- The carriers used to distribute products from the company to the retailers are the trucks.
- The stocks in the inventory are now individual agents in a population.

**Figure 9:** Image showing the agents involved in the agent-based model.
AGENT-BASED MODEL

The same variables as in the stock-based model are used:

- Orders are received with a uniform probability of between 30 – 40 days
- Express orders are requested when there are less than 100 items in stock
- A standard order size of 2699 is used for monthly orders and 10 for express orders
- Express orders are 2.5 times more expensive than normal orders
- Stock is deemed to be unusable after 42 days
- Inventory holding cost is $0.2 per item per day

Figure 10: Image showing the agent-based model running
AGENT-BASED MODEL

- The demand is 8 – 10 items per day among each of the 10 retail stores (or 80 – 100 items per day altogether)
- The order size can be changed monthly, using the edit box, as would be the case in practice

As can be seen from the graph, the number of stock increases as each shipment arrives. This time, the decrease throughout the month is non-linear, due to item deterioration effects. Express orders have not been needed, as the stock does not decrease below 100 items

**Figure 11:** Image showing the stock-time graph for the running agent-based model
Although the simulations can provide a good estimate of the optimum, balanced order size, several simplifications have been utilised:

- The material deterioration rate is actually a function of time, rather than a definite cut between useable and unusable at 42 days
- Defects may still be found upon stock arrival (age at day 1)
- Stock is delivered randomly to retailers with no regard of their age, though in reality, older items would be preferentially delivered
- Only the delivery of one part is considered, but there are four parts in the actual situation
- Probability distributions are assumed to be uniform
SUMMARY

However, to further develop the model, more specific information regarding the different costs (the ratio between normal order and express order costs, the inventory holding costs) need to be known.

Using historical and experimental data, an equation for the rate of decay in relation to item age can be developed and incorporated in the model.

Modifications can be made using more accurate distribution functions, such as triangular, using data on the variable demand and transit times.

Different initial inventory size and order size can be experimented with, due to changes from when the optimisation tests were performed.

These modifications will improve the current model, which may prove to be a valuable asset to Company in helping to solve the inventory problem.
REFERENCES

- Nazari A, Dunstall S, Ernst A, ‘Stochastic Inventory and related container packing problem’, CSIRO Computational Informatics, Australia