DICEing with sustainability: On reconciling climate economics and science

John C. V. Pezzey¹, Christopher M. Kellett² and Timm Faulwasser³

¹ Fenner School of Environment & Society, Australian National University (email: Jack.Pezzey@anu.edu.au)
² School of Electrical Engineering & Computing, University of Newcastle, Australia
³ Institute for Applied Computer Science, Karlsruhe Institute of Technology, Germany

6 December 2017

PRELIMINARY, INCOMPLETE DRAFT: DO NOT CITE OR CIRCULATE

A key but under-publicised result in Nordhaus's (2017, *PNAS*) DICE-2016R climate-economy model ('D16'), that 4.1°C peak global warming is optimal, is incompatible with climate scientists' recommendations and global climate policy aims. We explore here a set of pessimistic, implicitly risk-averse changes to D16's (deterministic) parameter assumptions that result in lower peak warming: as low as 2.6°C for optimal development; and 2.3°C for sustainable development, as defined by a constraint of non-declining wellbeing. Exogenous parameter changes investigated are: a higher climate damage function (of warming); slower exogenous decline in the difficulty of emission abatement; slower growth of total factor productivity (TFP); a higher utility discount rate; and a small part of climate damage affecting TFP instead of GDP. Less expected results from this limited study suggest that more difficult emissions control has little effect on global development; but reductions in TFP, whether exogenous or caused by climate damage are very important, and severely limit the options for sustainable development. All scenarios entail stringent emissions control, and assuming this is achieved, the focus for very long-term sustainability policy then shifts to greatly boosting saving rates, hence capital and gross output, in order to sustain consumption net of high climate damage.

1. Introduction

Important but as yet under-publicised results from DICE-2016R (hereafter D16), the latest version of Nordhaus's famous and influential global climate-economy model, is are its projections of peak global warming. D16 projects peak warming of 4.1°C in 2165 under "optimal" policies, including 100% participation and full efficiency by all emitters worldwide, and 7.2°C in 2270 under sub-optimal, Business-As-Usual policies. As Figure 1 illustrates, such projections entail dramatically unprecedented warming rates from the current 0.85°C above pre-industrial levels, with D16's projected optimal warming of 2.9°C from 2015 to 2115 being nearly 300 times the baseline cooling rate since 5000 BCE (Steffen et al. 2016). A key reason why DICE projects such unprecedentedly high and fast warming to be optimal is its climate damage function (CDF), which assumes only 4% and 12% damage to global GDP from 4.1°C and 7.2°C warming respectively. The small difference in damage is the main reason why the drop in projected growth during 2015-2400 of consumption per person net of climate damage (DICE's well-being measure) in Figure 2 is so small, from 53-fold growth on the optimal path to 40-fold growth on the Business-as-usual path.

When shown such projections, climate scientists typically express disbelief, derision or dismay that a 2.9°C/century approach to 4.1°C peak warming could ever be regarded as "optimal", but their published warnings about damage from high, fast warming almost never directly criticise any model's CDF. By contrast, some leading climate economists do criticise DICE's CDF, either implicitly or explicitly, for example:

"...the impacts of [4°C] warming on the natural environment, economies and societies could be severe, with reason to believe in the risk of vast movements of population and associated conflict, unrest and loss of life. Global mean temperatures regularly exceeding 4°C above pre-industrial have probably not been seen for at least 10 Myr..." (Dietz and Stern 2015, p582)
"[DICE's] damage function is made up out of thin air. It isn’t based on any economic (or other) theory or any data. Furthermore, even if this inverse quadratic function were somehow the true damage function, there is no theory or data that can tell us the values for [its] parameters, the correct probability distributions for those parameters, or even the correct means and variances." (Pindyck 2017, p.104)

Figure 1: Reconstructed global mean temperature anomalies for 0-2015 CE (from Marcott et al. 2013, Figure 1C), and DICE-2016R projections for 2015-2400 (author's calculation)

Figure 2: Global well-being measures: GDP/person estimates for 0-2000 (from Maddison 2007), and DICE-2016R projections of per-person consumption net of climate damage for 2015-2400 (author's calculation)
Another criticism of DICE's assumptions is that of excessive discounting of the distant future, with Stern (2007) suggesting alternative parameters that lower DICE's initial consumption discount rate from about 5%/yr to 2%/yr. This raises an obvious fear for those aware of Dasgupta and Heal's (1979, p.299) theoretical result that an "optimal" consumption path with a finite, non-renewable resource and a constant utility discount rate is single-peaked, with "later generations (should they exist) suffer[ing] incredibly as a result of the initial profligacy under the [optimal] programme".

The fear is that a combination of the nearly non-renewable nature of the global atmosphere's ability to absorb greenhouse gases, a much more pessimistic (many would say more realistic) CDF, and a high discount rate will indeed result in an unsustainable peak of well-being this century, followed by falling wellbeing for a long time afterwards, in stark contrast to the cornucopian future depicted by Figure 2.

This paper reports on experimental variations on D16 which try to bridge the plausibility gap between D16's projections in Figure 1 and the much more pessimistic view of future held by most climate scientists. What key parameters in D16 could be modified to bridge this gap? What implications would such parameter changes have for climate and other economic policies? Out of dozens of parameter variations, we have chosen just one set to illustrate some of the possibilities. In particular, we have deliberately sought a set of parameter changes which lead to the modified D16 optimal path being severely unsustainable, defined as having consumption per person net of climate damage (DICE's well-being measure) lower in 2115, a century after the model's present-day in 2015, thus to some extent fulfilling Dasgupta and Heal's theoretical prediction. Are these changes plausible? Do they open any useful avenues for debate between climate scientists and economists?

It proves to be surprisingly hard to find such a parameter set which seems plausible, but bear in mind that DICE is a purely deterministic model of a world in which many of its parameters are highly uncertain. The natural risk aversion of those wishing to keep the only planet we have in good shape can therefore account for much of any plausibility gap revealed below. (Bear in mind also that such risk aversion many authors consider cost-benefit analysis of the far future, which is what DICE effectively attempts, to be simply an inappropriate tool for providing any guidance to climate policies, which should instead be based on cost-effective pathways to achieve socially agreed, physical climate targets; see for instance Dietz and Fankhauser 2010, Grubb et al. 2014, van den Bergh and Botzen 2015, and CPLC 2017!).

We start by outlining in section 2 which key parameters in D16 we will change. Section 3 then shows in stages how these parameter changes affect the model outcomes. Section 4 gives some very preliminary conclusions.

2. Key features of DICE-2016R. and the combined effect of pessimistic parameter changes

DICE is an "integrated assessment" or global climate-economy model, which chooses (endogenous) saving rates and emissions control rates over time in order to maximise welfare, defined as the discounted sum over 2015-2500 of total utility from per capita consumption net of the costs of climate damages (assumed to be a proportion, the CDF, of GDP that depends solely on global warming $T_{AT}$ at time $t$), and of carbon emissions abatement (assumed to be a

1 However, as illustrated in Pezzey and Burke (2014, Fig 4), the CDF in Stern (2007) is actually weaker than that in the DICE-2007 model current at the time.
complex function of the proportion of industrial emissions abated, subject to various exogenous efficiency changes over time). D16 is formally reported in Nordhaus (2017), and we have converted its computer code from GAMS (currently unavailable from the website cited in Nordhaus 2017) to Matlab, as described in Appendix 1. This is a copy of Kellett et al. (2017), which also lists all D16’s equations and parameter values, with the values we change in this paper highlighted in yellow. In the present paper we use the Kellett et al. model with one important change, namely that we use a CDF $D(T_{AT})$, for the proportion of GDP lost, that is

$$\tilde{D}(T_{AT}) = \frac{a_2 T_{AT} \alpha_3}{1 + a_2 T_{AT} \alpha_3}$$

instead of $D(T_{AT}) = a_2 T_{AT} \alpha_3$ as found in D16’s code so that $\tilde{D}(T_{AT}) \rightarrow 1$ as $T_{AT} \rightarrow \infty$. This is an insignificant change with D16’s parameters, where $D(7.2) \approx 12\%$ as already noted, but very significant if we consider much higher damage parameters, as we do in Table 1, where $\tilde{D}(7.2) \approx 99\%$ with our ultra-pessimistic parameters of $a_2 = 0.005, a_3 = 5$, but non-sensically $D(7.2)$ would be $\approx 9000\%$.

Table 1 Changes made in this paper to D16 parameters (changes are cumulative, i.e. each is made in addition to the changes above them in the Table)

<table>
<thead>
<tr>
<th>Parameter group</th>
<th>Individual parameter</th>
<th>Formula or symbol in Appendix 1</th>
<th>Value in DICE-2016R</th>
<th>Alternative, pessimistic change made in this paper</th>
<th>Version number used in this paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Climate damage function</td>
<td>Damage multiplier</td>
<td>$a_2$</td>
<td>0.00236</td>
<td>0.005</td>
<td>v05G40</td>
</tr>
<tr>
<td></td>
<td>Damage exponent</td>
<td>$a_3$</td>
<td>2</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>&quot;Adjusted backstop cost&quot; (emissions intensity times abatement cost)</td>
<td>Decline rate of backstop cost</td>
<td>$\delta_{pb}$</td>
<td>0.025</td>
<td>0.010</td>
<td>v05G41</td>
</tr>
<tr>
<td></td>
<td>Initial decline rate of emissions intensity</td>
<td>$g_\sigma$</td>
<td>0.0152</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Decline of decline rate</td>
<td>$\delta_\sigma$</td>
<td>0.001</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>Growth of Total Factor Productivity (TFP)</td>
<td>Initial TFP growth rate</td>
<td>$g_a$</td>
<td>0.076</td>
<td>0.04</td>
<td>v05G42</td>
</tr>
<tr>
<td></td>
<td>Decline rate of TFP growth</td>
<td>$\delta_A$</td>
<td>0.001</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>Discounting</td>
<td>Rate of social time preference</td>
<td>$\rho$</td>
<td>0.015</td>
<td>0.06</td>
<td>v05G39</td>
</tr>
<tr>
<td>Damage proportion occurring to TFP</td>
<td>(Not in D16)</td>
<td>0</td>
<td>0.05</td>
<td></td>
<td>v05T39</td>
</tr>
</tbody>
</table>

Table 1 lists four other major changes to D16’s parameters that we explore, all of which leave unchanged D16’s parameterisation of the *current* global economy, being changes to expected
(hence less "knowable") rates of parameter growth or decline over the very long run. First, to three parameters which determine how technical progress is assumed to reduce over time the "adjusted backstop cost", a measure of the difficulty of emissions control which in D16 is a product of emissions intensity (kg CO$_2$ gross emissions per $ of GDP) and abatement cost ($ per tCO$_2$ of emissions abated). Second, to two parameters which affect the assumed exogenous growth over time of Total Factor Productivity (TFP) $A(t)$, the "growth engine" of DICE, whereby gross output is $Y(t) = A(t) K(t)^γ L(t)^{1-γ}$, with $K$ and $L$ being total capital stock and population as usual. Third, to the rate of social time preference $ρ$ which determines the utility discount factor $1/(1 + ρ)^t$. Lastly, to an innovation by Dietz and Stern (2015) not found in D16, included because it proves to be important in the very long term, is to shift 5% of the incidence of climate damage from gross output to TFP, so that at each time-step $A(t)$ is $1 - 0.5D(T_{AT})$ times what it would otherwise have been.

The combined effect of all the parameters changes in Table 1 can be seen by comparing Figures 3a-c, which shows (up to 2250 rather than 2500) D16’s paths of capital $K(t)$, global warming $T_{AT}(t)$ and well-being, which we will denote $c(t)$, with Figures 4a-c, which shows the same paths given Table 1’s parameter changes, using the same scales. To aid later comparisons, the vertical scale used for well-being in Figure 3c (and 4c) is vastly amplified compared to that in Figure 2, but the numbers for optimal well-being are the same.

**Figures 3a-c.** D16 standard: Paths during 2015-2250 of capital, global warming and well-being (consumption per capita net of climate damage) for D16’s standard parameters

**Figures 4a-c.** Our pessimistic scenario: Paths during 2015-2250 of capital, global warming and well-being for our combined parameter set (v05T39 in Table 1). Red dashed curves show optimal paths, green solid curves show "sustainable" paths, defined by a constraint of non-declining well-being.

By design, the optimal wellbeing path in Figure 4c is "severely unsustainable", peaking in 2045 at ~7% above its 2015 level, and falling by 2115 to ~80% of its 2015 level. By contrast,
the "sustainable" path, computed in Matlab by requiring that welfare is maximised subject to a constraint that wellbeing never declines (as originally proposed by Pezzey 1989) is constant at ~95% of the initial optimal level. Such sustainability is partly achieved by more stringent emissions control, but only partly because despite the 6% utility discount rate, industrial emissions are already zero by 2055 on the optimal path, in response to the hugely higher CDF we assume in Table 1 (shown below in Figure 5). So even though emissions control is (unrealistically) 100% (with 100% global participation) by 2020 on the sustainable path in Figure 4, this only lowers peak warming from ~2.6°C to ~2.3°C (both reached in about 2160). Instead, the main endogenous response is vastly higher savings rates, peaking at ~55% in 2090, to accumulate the huge peak in capital stock shown by the green curve in Figure 4a, and hence a huge peak in gross output, in order so sustain consumption per capita after deducting the assumed ~15% damage done to gross output from ~2°C warming then.

3. Step-by-step analysis of our pessimistic scenario

We now show visually both each change in parameter assumption made in Table 1 – in turn to the CDF, adjusted backstop cost, growth of TFP, discounting, and damage to TFP – and then its dynamic effects on the paths of capital, warming and wellbeing, in addition to the effects of the previous parameter changes.

Figure 5 shows the vastly higher CDF we assume in going from version G00 to G40, and Figures 6a-c show its dynamics effects. Whether or not climate damage is ~50% of GDP at ~3°C, as shown by

![Figure 5](image)

**Figure 5.** Much higher climate damage function (CDF) assumed in Table 1 (for 0-8°C T_Ar)

![Figures 6a-c](image)

**Figures 6a-c.** Paths during 2015-2250 of capital, global warming and well-being, given our higher CDF
Figure 5 – which is higher damage than Dietz and Stern's (2015) High case, which is in turn higher than Weitzman's (2012) guesstimate – is highly debatable and ultimately highly unknowable (Pezzey and Burke 2014), but we use it as a risk-averse justification of the UN's (2009, 2015) 2°C maximum warming target. Figures 6a-c show that the main result of the world accepting a much higher CDF is much greater emission control (assumed by D16 to be very cheap), hence much lower warming (compare Figures 6b and 3b), but almost no diminution in the prediction of sustained, steep growth wellbeing (compare Figures 6c and 3c). Hence the non-declining wellbeing constraint is non-binding, and the optimal and sustainable paths are identical as shown (both as green lines).

Next, Figure 7 shows the much slower decline over time in "adjusted backstop cost" – \( \theta(t) \) in equation (7f) in Appendix 1, the product of backstop (carbon-free technology) cost and (carbon) emissions intensity, to which the total abatement cost, the last term in equation (4), is proportional – made by our pessimistic change in Table 1, compared to D16's standard assumption (still labelled as case G00, as throughout this paper). As shown in Figures 8a-c,

**Figure 7.** Comparison of our pessimistic, and D16's standard, adjusted backstop cost, 2015-2250.

**Figures 8a-c.** Paths during 2015-2250 of capital, global warming and well-being, given our higher adjusted backstop cost in Figure 7 (in addition to the change in Figure 5)

the main result is counter-intuitive: much greater emission control after about 2150 (even though it is now assume to much dearer), hence much lower warming, almost no difference to wellbeing. This is perhaps explained as an unintended consequence of D16's assumption (not mentioned until now, and discussed remarkably little in either Nordhaus 2017 or Nordhaus and Sztorc 2013, the manual for DICE-2013R, which is almost identical to DICE-2016R except for parameter values) that negative emissions technologies (such as bio-energy with carbon capture
and storage or BECCS, as discussed extensively in IPCC 2014), allow net industrial emissions to be –20% of gross industrial emissions from 2165 onwards. Further investigation of this anomaly is clearly warranted!

**Figure 9** shows the much slower rise, and ultimate stagnation, of TFP that we pessimistically assume in Table 1, and **Figures 10a-c** the further effects this has on global development. The

![Figure 9](image1.png)

**Figure 9.** Comparison of our pessimistic, and D16's standard, TFP, 2015-2250.

**Figures 10a-c.** Paths during 2015-2250 of capital, global warming and well-being, given our much lower TFP in Figure 9 (in addition to the changes in Figures 5 and 7).

changes compared to Figures 8a-c are dramatic. Much lower TFP growth means much lower growth of capital stock (hence output) and wellbeing, but (perversely) less negative emissions after 2165, hence a slower decline then in global warming. Wellbeing now has a shallow local peak in 2085, followed by a shallow local trough in 2155, after which growth resumes. Both are smoothed out on the sustainable wellbeing path by a different saving path, as reflected by the slightly different capital path shown in Figure 10a, while Figure 10b reflects negligible difference between emissions on the optimal and sustainable paths.

**Figure 11a** shows the much higher pure time discounting we assume in Table 1. **Figure 11b** gives another interpretation of this parameter change, by showing that high discounting can be an approximation for instead assuming a slow growth of global participation in emission control (not yet programmed in our Matlab code, and computed laboriously here from the original GAMS code). In Figure 11b, the effect on an already severely unsustainable variant of D16 of raising the utility discount rate $\rho$ from 4%/yr to 6%/yr is very similar to the effect of
assumming global participation in emission control growing from only 10% in 2015 to 100% in 2120. Figures 12a-c show further dramatic effects on global develop paths: the optimal capital stock barely rises, the peak in optimal wellbeing at 2050 is 35 years earlier (2050) and much lower than in Figure 10c, and it declines into a much longer, deeper trough throughout the 22nd century. To prevent wellbeing from declining, much more extra saving is needed on the sustainable path, as reflected by the dome in capital during about 2100-2150 (the green curve in Figure 12a). Peak global warming is slightly lowered from 2.64°C on the optimal path to 2.51°C on the sustainable path, which lowers the CDF from ~39% to ~33%, given its steepness in this warming range as shown in Figure 5.

Lastly, Figure 13 shows the effect of assuming, after Dietz and Stern (2015), that just 5% of damage is done to TFP instead of to gross output. Because the TFP path then depends on global warming which is endogenous, TFP differs between the optimal and sustainable paths, with both declining forever after about 2090, as shown. Figures 14a-c show the further, dramatic effects of this perpetual decline in TFP. Optimal wellbeing declines forever after its 2045 peak. The long-run level of sustainable well-being is lowered only slightly, from 12.4 $k/person.year in Figure 12c to 11.3 $k/person.year in Figure 14c, but this achieved only by an implausible surge in the saving rate to a 56% peak in 2090, reflected in the huge peak in capital in Figure 14a. Peak global warming is 2.61°C optimally (causing 38% climate damage, of which 1.9% points happens to TFP), and 2.31°C sustainably (25% climate damage).
4. Conclusions [highly provisional]

The above results, selected from only a few dozen variant runs of D16, suggest some interesting hypotheses to pursue; but countless plausible or semi-plausible parameter sets would need to be investigated before these hypotheses could in any way be confirmed. From just the above results, it appears that much more difficult emissions control (Figure 7) has less effect on global development under a pessimistic climate-economy scenario than one might at first expect. Conversely, reductions in total factor productivity, whether resulting from non-climate causes (Figure 9) or climate damage (Figure 13) are hugely important, and severely limit the options for sustainable development. If climate damage is as severe as suggested in Figure 5, all scenarios entail very stringent emissions control, in almost all cases reaching zero industrial emissions by 2050. Assuming such control is achieved, the focus for very long-term sustainability policy then shifts to greatly boosting saving rates, hence capital and gross output, in order to sustain consumption net of high climate damage. Whether or not such substitution of material consumption for climate damage – a future with pervasive, highly efficient airconditioning powered by renewable electricity? – works is another matter for debate.
References


DICE2013R-mc (v2.1): A Matlab / CasADi Implementation of Vanilla DICE2013R and 2016R

Christopher M. Kellett¹, Timm Faulwasser², and Steven R. Weller¹

¹School of Electrical Engineering and Computing, University of Newcastle, Callaghan, New South Wales 2308, Australia, email: {Chris.Kellett, Steven.Weller}@newcastle.edu.au
²Institute for Applied Computer Science, Karlsruhe Institute of Technology, 76344 Eggenstein-Leopoldshafen, Germany, e-mail: timm.faulwasser@kit.edu

November 8, 2017

Abstract

This brief document provides a description of how to use DICE2013R-mc [2], a Matlab and CasADi-based implementation of the Dynamic Integrated model of Climate and Economy (DICE). DICE2013R-mc provides the same basic functionality as the GAMS code¹ for DICE2013R as available at [4] and for DICE2016R as available at [5]. DICE2016R essentially represents an update of model parameters from DICE2013R and either version can be run in DICE2013R-mc by setting an appropriate parameter in the main file.

1 Software Requirements

This implementation of DICE2013R (DICE2016R) makes use of the CasADi framework for algorithmic differentiation and numeric optimization [1] in conjunction with Matlab². Version 3.2.1 of CasADi is used and, hence, Matlab 2014a or later is generally required. Appropriate binaries³ of CasADi v.3.2.1 are available at [1].

Similar to CasADi, DICE2013R-mc is distributed under the GNU Lesser General Public License (LGPL), and hence the code can be used royalty-free even in commercial applications.

2 Model and Optimal Control Problem

The DICE2013R model operates on five year time steps beginning from 2010 (DICE2016R starts from 2015). To formalize this, let \( t_0 = 2010 \), \( \Delta = 5 \), and \( i = 1, 2, 3, \ldots \) be the discrete time index. Then

\[
t = t_0 + \Delta \times i
\]

yields \( t = 2010, 2015, 2020, \ldots \) as desired.

The DICE2013R model has six endogenous state variables: two variables to model the global climate in the form of atmospheric and oceanic temperatures (\( T_{AT} \) and \( T_{LO} \), respectively, in units

¹ A manual is available for DICE2013R [6]. However, the description of the model in the manual [6] differs in several respects from the available code [4]. As our aim is replicate the functionality of [4, 5], the description of the model herein is in reference to the implementation in [4, 5] rather than the description in [6].
²For those new to Matlab, MathWorks has several online tutorial resources available at [3].
³After downloading an appropriate binary, be sure to add CasADi to your Matlab path as described at [1].
of °C), three variables to model the global carbon cycle in the form of carbon concentrations in the atmosphere, upper ocean, and lower ocean (\(M_{AT}, M_{UP}, \text{and } M_{LO}\), respectively, in units of GtC), and one state for global capital (\(K\), in units of trillions 2005USD\(^4\)). Decision variables or control inputs are the emissions mitigation rate (\(\mu\)) and the savings rate (\(s\)) where the latter is the ratio of investment to net economic output. Finally, the model is also driven by several exogenous, time-varying terms such as population and total factor productivity. The full dynamics are given by:

\[
\begin{align*}
\begin{bmatrix}
T_{AT}(i + 1) \\
T_{LO}(i + 1)
\end{bmatrix} &= \begin{bmatrix}
\phi_{11} & \phi_{12} \\
\phi_{21} & \phi_{22}
\end{bmatrix}\begin{bmatrix}
T_{AT}(i) \\
T_{LO}(i)
\end{bmatrix} + \begin{bmatrix}
\xi_1 \\
0
\end{bmatrix} R_F(i) \\
\begin{bmatrix}
M_{AT}(i + 1) \\
M_{UP}(i + 1) \\
M_{LO}(i + 1)
\end{bmatrix} &= \begin{bmatrix}
\zeta_{11} & \zeta_{12} & 0 \\
\zeta_{21} & \zeta_{22} & \zeta_{23} \\
0 & \zeta_{32} & \zeta_{33}
\end{bmatrix}\begin{bmatrix}
M_{AT}(i) \\
M_{UP}(i) \\
M_{LO}(i)
\end{bmatrix} + \begin{bmatrix}
\xi_2 \\
0 \\
0
\end{bmatrix} E(i) \\
K(i + 1) &= (1 - \delta)K(i) \\
&\quad + \Delta \left(1 - a_2 T_{AT}(i)^{\alpha_3} - \theta_1(i)\mu(i)^{\theta_2}\right) A(i) K(i)^{\gamma} \left(\frac{L(i)}{1000}\right)^{1 - \gamma} s(i),
\end{align*}
\]

where emissions (\(E\) in units of GtCO\(_2\)) and radiative forcing\(^5\) (\(R_F\)) are given by

\[
E(i) = \sigma(i)(1 - \mu(i))A(i)K(i)^{\gamma} \left(\frac{L(i)}{1000}\right)^{1 - \gamma} + E_{\text{Land}}(i)
\]

\[
R_F(i) = \eta \log_2 \left(\frac{\zeta_{11}M_{AT}(i) + \zeta_{12}M_{UP}(i) + \zeta_{22}E(i)}{M_{AT,1750}}\right) + F_{\text{EX}}(i).
\]

Parameter values can be found in the table at the end of this document.

The exogenous, time-varying signals are given by\(^6\):

\[
\begin{align*}
\sigma(i + 1) &= \sigma(i) \exp \left(-g_\sigma \cdot (1 - \delta_\sigma)\Delta i \cdot \Delta\right), \quad \sigma(1) = \sigma_0 \\
L(i + 1) &= L(i) \left(\frac{L_0}{L(i)}\right)^{\ell_a}, \quad L(1) = L_0 \\
A(i + 1) &= \frac{1 - g_\sigma \exp(-g_{A(i)} \Delta A(i - 1)) - \Delta A(i - 1)}{A(i)}, \quad A(1) = A_0 \\
E_{\text{Land}}(i) &= E_{L0} \cdot (1 - \delta_{EL})^{(i-1)} \\
F_{\text{EX}}(i) &= f_0 + \min \left\{ \frac{f_1 - f_0}{t_f} (i - 1), f_1 - f_0 \right\} \\
\theta_1(i) &= \frac{p_b}{1000 \cdot \theta_2} (1 - \delta_{\theta_2})^{i-1} \cdot \sigma(i).
\end{align*}
\]

Utility is given by

\[
U(C(i), L(i)) = L(i) \left(\frac{\left(\frac{1000C(i)}{L(i)}\right)^{1 - \alpha} - 1}{1 - \alpha} - 1\right)
\]

where the consumption (\(C\)) is

\[
C(i) = (1 - a_2 T_{AT}(i)^{\alpha_3} - \theta_1(i)\mu(i)^{\theta_2}) A(i) K(i)^{\gamma} \left(\frac{L(i)}{1000}\right)^{1 - \gamma} (1 - s(i)).
\]

\(^4\)DICE2016R measures capital in units of trillions 2010USD.

\(^5\)The form of the radiative forcing given here is due to the use of an inconsistent discretization of a continuous-time climate model, mixing forward and backward Euler discretizations for the two states, that leads to \(T_{AT}(i + 1)\) depending on \(M_{AT}(i + 1)\) instead of \(M_{AT}(i)\). Since the aim of this release is to replicate the functionality of \([4, 5]\), we have not corrected this inconsistency.

\(^6\)In \([4, 5]\), \(\theta_1\) is called cost1.
Optimal pathways are then derived by maximizing the social welfare:

\[
\max_{s, \mu} \Delta \ast \text{scale1} \ast \sum_{i=1}^{N} \frac{U(C(i), L(i))}{(1 + \rho)^{\Delta(i-1)}} - \text{scale2} \quad (10)
\]

subject to \( (2) - (4) \)

\[
\mu(1) = \mu_0 \quad (11)
\]

\[
\mu(i) \geq 0, \quad i = 2, \ldots, N \\
\mu(i) \leq 1, \quad i = 2, \ldots, 29 \\
\mu(i) \leq 1.2, \quad i = 30, \ldots, N \\
0 \leq s(i) \leq 1, \quad i = 1, \ldots, N - 10 \\
s(i) = s^*, \quad i = N - 9, \ldots, N.
\]

The social cost of carbon is given by the ratio of the marginal welfare with respect to emissions and with respect to consumption\(^7\):

\[
\text{SCC}(i) = -1000 \times \frac{\partial W/\partial E(i)}{\partial W/\partial C(i)}. \quad (12)
\]

3 Description of Code

DICE2013R-mc consists of three main files:

- **DICE2013R.mc.m** is the top-level file and calls the subsequent two files.

- **set.DICE.parameters.m** is a function that takes the desired version (2013, 2016, or 1 for a custom version of parameters), as a parameter and returns all other required parameters\(^8\), including exogenous signals, in the structure **Params**.

- **dice.dynamics.m** is a function that calculates the dynamic states (endogenous signals) of DICE2013R. In addition to the dynamic states, it also calculates the value of the objective function and the quantities required for the social cost of carbon computation as a ratio of marginals; namely the emissions and consumption.

Running DICE2013R-mc in the Matlab command window yields the DICE endogenous states (capital \(K\), temperatures \(T_{ATM}\) and \(T_{LO}\), and carbon concentrations \(M_{ATM}\), \(M_{UP}\), and \(M_{LO}\)) and the input values for the mitigation rate (\(\mu\)) and savings rate (\(s\)). Additionally, the marginals with respect to emissions (\(\lambda E\)) and with respect to consumption (\(\lambda C\)) are used to calculate the Social Cost of Carbon (SCC) and the optimal welfare is given by \(J\).

A **clear** command removes many of the variables and other objects used in the solution of the optimal control problem from the workspace. This command can be commented out if these items are required.

As well as the three core component files listed above, two hopefully useful utility files are provided:

- **plot_results.m** generates plots of the exogenous and endogenous signals, as well as the control inputs and social cost of carbon.

---

\(^7\)Note that in [5] the denominator includes an additive factor of 0.00001. This does not change the dollar value of the SCC until after about 2150.

\(^8\)One minor change in notation has been made in DICE2013R-mc from DICE2013R and this is the indexing into the climate and carbon matrices. DICE2013R uses a non-standard “column-row” numbering for matrices, whereas DICE2013R-mc uses standard “row-column” indexing.
• compute_auxiliary_quantities.m computes several additional quantities that are available as outputs of the GAMS code [4]. The selected quantities are described below. This file should provide a template for those wishing to define additional quantities of interest.

The additional quantities calculated by compute_auxiliary_quantities.m are: industrial emissions (IE), net economic output (NEO), per capita consumption (PCC), the damages fraction (DF), atmospheric carbon in parts per million (ACppm), and the marginal cost of abatement (MCA), where

\[\text{IE}(i) = \sigma(i)(1 - \mu(i))A(i)K(i)^\gamma \left( \frac{L(i)}{1000} \right)^{1-\gamma}\]  
\[\text{NEO}(i) = \left(1 - a_2 T_{AT}(i)^{a_3} - \theta_1(i)\mu(i)^{\theta_2}\right)A(i)K(i)^\gamma \left( \frac{L(i)}{1000} \right)^{1-\gamma}\]  
\[\text{PCC}(i) = \frac{1000 \times C(i)}{L(i)}\]  
\[\text{DF}(i) = a_2 T_{AT}(i)^{a_3}\]  
\[\text{ACppm}(i) = \frac{M_{AT}(i)}{2.13}\]  
\[\text{MCA}(i) = p_b \times (1 - \delta_{pb})^{i-1} \times \mu(i)^{\theta_2-1}.\]

3.1 GAMS Data and Verification Plots

Three additional files are provided with this release for the purpose of demonstrating that DICE2013R-mc replicates the functionality of the publicly available DICE2013R GAMS code [4] and the DICE2016R GAMS code [5] or for quickly seeing how proposed changes yield different results from the default settings of [4, 5]. These files are

• plot_gams_verification.m,
• GAMS_Results_2013.csv, and
• GAMS_Results_2016.csv.

The latter two contain the output generated by [4] and [5], respectively, while the former is an extended version of plot_results.m that loads and plots the data from DICE2013R or DICE2016R against the results of DICE2013R-mc.

The call to plot_gams_verification.m is commented out in the release. To view these plots, uncomment the call to plot_gams_verification.m.

4 Release notes

1.0 Initial release, requires casadi-matlabR2014b-v3.0.0 or casadi-matlabR2014b-v3.1.1.

1.1 Fixes compatability issue with casadi-matlabR2014b-v3.2.1.

2.0 Adds parameters for DICE2016R and switch statement to use 2013R, 2016R, or custom parameter values.

2.1 Minor parameter corrections for DICE2016R. Fixed time index error in \(\sigma(\cdot)\) and radiative forcing. Fixed scaling parameters for 2016R objective function. Added verification against available GAMS code for DICE2016R. Improved default plotting scripts. The authors would like to thank Professor Jack Pezzey (Australian National University) for beta testing v2.0, which led to several of the corrections in v2.1.
References


<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value DICE2013R</th>
<th>Value DICE2016R</th>
<th>Notes</th>
<th>GAMS Line No. (2013R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta )</td>
<td>5</td>
<td></td>
<td>Time step in years (tstep)</td>
<td>16</td>
</tr>
<tr>
<td>( t_0 )</td>
<td>2010</td>
<td>2015</td>
<td>Initial year</td>
<td></td>
</tr>
<tr>
<td>( N )</td>
<td>60</td>
<td>100</td>
<td>Horizon length</td>
<td>11</td>
</tr>
<tr>
<td>( \mu_0 )</td>
<td>0.039</td>
<td>0.03</td>
<td>Initial mitigation rate (miu0)</td>
<td>43</td>
</tr>
<tr>
<td>Climate diffusion parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \phi_{11} )</td>
<td>0.8630</td>
<td>0.8718</td>
<td>( 1 - c_1 \left( \frac{f_{co2}}{t_{2xco2}} + c_3 \right) )</td>
<td>261</td>
</tr>
<tr>
<td>( \phi_{12} )</td>
<td>0.0086</td>
<td>0.0088</td>
<td>( c_1 \cdot c_3 )</td>
<td>261</td>
</tr>
<tr>
<td>( \phi_{21} )</td>
<td>0.025</td>
<td>0.025</td>
<td>( c_4 )</td>
<td>78</td>
</tr>
<tr>
<td>( \phi_{22} )</td>
<td>0.975</td>
<td>0.975</td>
<td>( 1 - c_4 )</td>
<td>262</td>
</tr>
<tr>
<td>Carbon cycle diffusion parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \zeta_{11} )</td>
<td>0.912*</td>
<td>0.88</td>
<td>( b_{11} = 1 - b_{12} )</td>
<td>139</td>
</tr>
<tr>
<td>( \zeta_{12} )</td>
<td>0.03833*</td>
<td>0.196</td>
<td>( b_{21} = b_{12} \cdot MATEQ/MUEQ (= b_{12} \cdot 588/1350) )</td>
<td>140</td>
</tr>
<tr>
<td>( \zeta_{21} )</td>
<td>0.088</td>
<td>0.12</td>
<td>( b_{12} )</td>
<td>55</td>
</tr>
<tr>
<td>( \zeta_{22} )</td>
<td>0.9592*</td>
<td>0.797</td>
<td>( b_{22} = 1 - b_{21} - b_{23} )</td>
<td>141</td>
</tr>
<tr>
<td>( \zeta_{23} )</td>
<td>0.0003375*</td>
<td>0.001465</td>
<td>( b_{32} = b_{23} \cdot mueq/mleq (= b_{23} \cdot 1350/10000) )</td>
<td>142</td>
</tr>
<tr>
<td>( \zeta_{32} )</td>
<td>0.00250</td>
<td>0.007</td>
<td>( b_{23} )</td>
<td>56</td>
</tr>
<tr>
<td>( \zeta_{33} )</td>
<td>0.9996625*</td>
<td>0.99853488</td>
<td>( b_{33} = 1 - b_{32} )</td>
<td>143</td>
</tr>
<tr>
<td>Other parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \eta )</td>
<td>3.8</td>
<td></td>
<td>Forcings of equilibrium CO2 doubling (Wm-2)</td>
<td>79</td>
</tr>
<tr>
<td>( \xi_1 )</td>
<td>0.098</td>
<td>0.1005</td>
<td>Multiplier for ( \eta (c1) )</td>
<td>76</td>
</tr>
<tr>
<td>( \xi_2 )</td>
<td>5/3.666</td>
<td>588</td>
<td>Conversion factor for emissions (GtC / GtCO2)</td>
<td>258</td>
</tr>
<tr>
<td>( M_{AT,1750} )</td>
<td></td>
<td></td>
<td>Pre-industrial carbon in atmosphere (mateq)</td>
<td>250</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.3</td>
<td></td>
<td>Capital elasticity in production function (gama)</td>
<td>26</td>
</tr>
<tr>
<td>( \theta_2 )</td>
<td>2.8</td>
<td>2.6</td>
<td>Exponent of control cost function (expcost2)</td>
<td>89</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>0.00267</td>
<td>0.00236</td>
<td>Damage multiplier term (a2)</td>
<td>85</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>2</td>
<td></td>
<td>Damage exponent (a3)</td>
<td>86</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.1</td>
<td></td>
<td>Depreciation rate on capital (per year) (dk)</td>
<td>30</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>1.45</td>
<td></td>
<td>Elasticity of marginal utility of consumption (elasmu)</td>
<td>22</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.015</td>
<td></td>
<td>Initial rate of social time preference per year (prstp)</td>
<td>23</td>
</tr>
<tr>
<td>scale1</td>
<td>0.016408662</td>
<td>0.030245527</td>
<td>Utility multiplier</td>
<td>107</td>
</tr>
<tr>
<td>scale2</td>
<td>3855.106895</td>
<td>10993.704</td>
<td>Utility offset</td>
<td>108</td>
</tr>
<tr>
<td>Exogenous signal parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( L_0 )</td>
<td>6838</td>
<td>7403</td>
<td>Global population in 2010 (pop0)</td>
<td>27</td>
</tr>
<tr>
<td>( L_a )</td>
<td>10500</td>
<td>11500</td>
<td>Asymptotic global population (popasym)</td>
<td>29</td>
</tr>
<tr>
<td>( t_g )</td>
<td>0.134</td>
<td></td>
<td>Population growth rate (popadj)</td>
<td>28</td>
</tr>
<tr>
<td>( E_{L0} )</td>
<td>3.3</td>
<td>2.6</td>
<td>Initial land use emissions (eland0)</td>
<td>40</td>
</tr>
<tr>
<td>( \delta_{EL} )</td>
<td>0.2</td>
<td>0.115</td>
<td>Land use emissions decrease rate (deland)</td>
<td>41</td>
</tr>
<tr>
<td>( A_0 )</td>
<td>3.80</td>
<td>5.115</td>
<td>Initial total factor productivity (a0)</td>
<td>33</td>
</tr>
<tr>
<td>( g_0 )</td>
<td>0.079</td>
<td>0.076</td>
<td>Initial TFP growth (ga0)</td>
<td>34</td>
</tr>
<tr>
<td>( \delta_A )</td>
<td>0.006</td>
<td>0.005</td>
<td>Decline rate of TFP (delta)</td>
<td>35</td>
</tr>
<tr>
<td>( p_b )</td>
<td>344</td>
<td>550</td>
<td>Cost of backstop 2005$ per tCO2 2010 (pback)</td>
<td>90</td>
</tr>
<tr>
<td>( \delta_{pb} )</td>
<td>0.025</td>
<td></td>
<td>Decline rate of backstop cost (gbback)</td>
<td>91</td>
</tr>
<tr>
<td>( \sigma_0 )</td>
<td>0.5491</td>
<td>0.3503</td>
<td>Initial emissions intensity, calculated as ( \frac{\text{global emissions}}{33.61} )</td>
<td>38</td>
</tr>
<tr>
<td>( g_{\sigma} )</td>
<td>0.01</td>
<td>0.0152</td>
<td>Emissions intensity base rate (gsigma1)</td>
<td>39</td>
</tr>
<tr>
<td>( \delta_{\sigma} )</td>
<td>0.001</td>
<td></td>
<td>Emissions intensity decline rate (dsig)</td>
<td>39</td>
</tr>
<tr>
<td>( f_0 )</td>
<td>0.25</td>
<td>0.5</td>
<td>2010 forcings of non-CO2 GHG (Wm(^2)) (fex0)</td>
<td>68</td>
</tr>
<tr>
<td>( f_1 )</td>
<td>0.70</td>
<td>1.0</td>
<td>2100 forcings of non-CO2 GHG (Wm(^2)) (fex1)</td>
<td>69</td>
</tr>
<tr>
<td>( t_f )</td>
<td>18</td>
<td>17</td>
<td>Number of periods that exogenous forcings increase</td>
<td></td>
</tr>
</tbody>
</table>
• Initial conditions

<table>
<thead>
<tr>
<th></th>
<th>$T_{AT}(0)$</th>
<th>$T_{LO}(0)$</th>
<th>$M_{AT}(0)$</th>
<th>$M_{UP}(0)$</th>
<th>$M_{LO}(0)$</th>
<th>$K(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013R</td>
<td>0.8</td>
<td>0.0068</td>
<td>830.4</td>
<td>1527</td>
<td>10010</td>
<td>135</td>
</tr>
<tr>
<td>2016R</td>
<td>0.85</td>
<td>0.0068</td>
<td>851</td>
<td>460</td>
<td>1740</td>
<td>223</td>
</tr>
</tbody>
</table>

• Carbon cycle parameters to compute diffusion coefficients

<table>
<thead>
<tr>
<th></th>
<th>$b_{12}$</th>
<th>$b_{23}$</th>
<th>$mateq$</th>
<th>$mueq$</th>
<th>$mleq$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013R</td>
<td>0.088</td>
<td>0.0025</td>
<td>588</td>
<td>1350</td>
<td>10000</td>
</tr>
<tr>
<td>2016R</td>
<td>0.12</td>
<td>0.007</td>
<td>588</td>
<td>360</td>
<td>1720</td>
</tr>
</tbody>
</table>

• Climate parameters to compute diffusion coefficients

<table>
<thead>
<tr>
<th></th>
<th>$c_{1}$</th>
<th>$c_{3}$</th>
<th>$c_{4}$</th>
<th>$fco2_{2x}$</th>
<th>$t2xco2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013R</td>
<td>0.098</td>
<td>0.088</td>
<td>0.025</td>
<td>3.8</td>
<td>2.9</td>
</tr>
<tr>
<td>2016R</td>
<td>0.1005</td>
<td>0.088</td>
<td>0.025</td>
<td>3.6813</td>
<td>3.1</td>
</tr>
</tbody>
</table>

• Parameters for calculating $\sigma_0 = \frac{e_0}{q_0(1 - \mu_0)}$:

<table>
<thead>
<tr>
<th></th>
<th>$e_0$</th>
<th>$q_0$</th>
<th>$\mu_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013R</td>
<td>33.61</td>
<td>63.69</td>
<td>0.039</td>
</tr>
<tr>
<td>2016R</td>
<td>35.85</td>
<td>105.5</td>
<td>0.03</td>
</tr>
</tbody>
</table>

• Optimal savings rate calculated as

\[ s^* = \frac{\delta + 0.004}{\delta + 0.004\alpha + \rho\gamma} \]