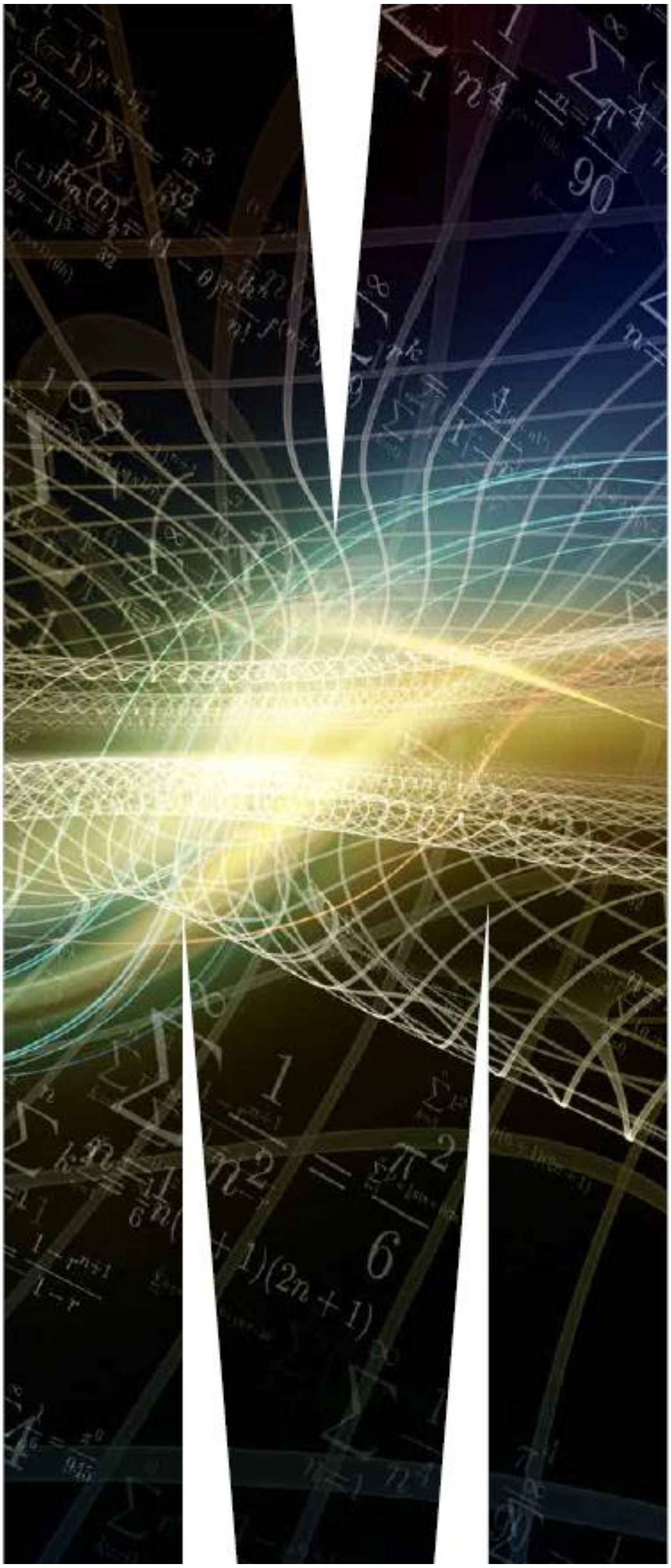
MONASH University

School of Mathematics

Honours and Masters research projects 2025-2026



MONASH SCIENCE

List of Supervisors and projects

(A) Applied mathematics, (P) Pure mathematics, (S) Stochastic Processes & Statistics, (F) Financial Mathematics

This booklet provides an overview of the research activities within the School of Mathematics to give you an indication of the Honours and Masters projects that are on offer.

Students should discuss prospective projects with at least two supervisors before choosing their preferred project. Please note that the project descriptions are quite short, and more details can be obtained when speaking to supervisors. Develop a project title and outline with your project supervisor.

Do not hesitate to contact a potential supervisor even if he/she has nothing offered officially. Oftentimes, academics have many projects available.

Prof Santiago Badia

Approximation of PDEs on dynamic surfaces

In this project, we aim at designing structure-preserving numerical methods for the approximation of PDEs on dynamic active surfaces. Active surfaces are common in biology and are useful in the simulation of cell dynamics. In order to do this, we want to combine advances in numerical analysis with the exterior calculus formalism. Stating the problem using differential forms, exterior derivatives, Lie derivatives, etc, we aim to design schemes that preserve at the discrete level the conservation properties that hold at the continuous level. One driving example in this project will be the simulation of nematic fluids on dynamic surfaces.

Background in numerical analysis, PDEs, functional analysis, differential geometry and coding skills will be helpful.

For more details, please contact Prof Badia.

Nonlinear model reduction

The numerical approximation of partial differential equations for complex nonlinear phenomena usually requires huge computational resources. Besides, there exist many applications in which one is interested in solving the same physical problem for different values of a set of parameters (e.g., physical properties, boundary conditions, etc) and extract quantities of interest out of the solution (e.g., flow rates, drag in a wing, etc). In these scenarios, it would be excellent to approximate the map from the space of parameters to the space of quantities of interest by making use of the high-fidelity simulations in a “training set”. Unfortunately, linear techniques like singular value decomposition fail to approximate these maps for highly nonlinear phenomena (e.g., turbulent flows), since the solution map is a highly nonlinear manifold. In this project, we want to explore encoder-decoder neural networks to approximate these maps for the numerical approximation of incompressible fluid flows.

Background in numerical analysis, PDEs and coding skills will be helpful.

For more details, please contact Prof Badia.

Dr Yann Bernard

The analysis of Willmore surfaces (P)

Erythrocytes (also called red blood cells) are the body's principal mean of transporting vital oxygen to the organs and tissues. The "inside" of an erythrocyte is rich in haemoglobin, which chemically tends to bind to oxygen molecules and retain them. To maximize the possible intake of oxygen by each cell, erythrocytes – unlike all the other types of cells of the human body – have no nuclei. The membrane of a red blood cell is such that it isolates the "inside" from the "outside". One might be tempted to conclude that in order to maximize volume as "densely" as possible, red blood cells should be spherical. This is however not at all true (having spherical red blood cells is a serious disease). In fact, when incorporating into the problem that the membrane of a red blood cell is an elastic surface, one is led to the so-called the Willmore energy. It appears in various areas of science: cell biology, but also in elasticity theory, in optics, and even in general relativity. I am offering three reading projects on the analytical aspects of the Willmore energy with different focus points. Each project will consist in reading and understanding two articles from the recent literature, as well as to reformulate certain crucial proofs. At the end of the reading project, the student will be familiar with Willmore surfaces and with the various analytical techniques involved in the study of the 4th order nonlinear Willmore equation.

References:
[1] Y. Bernard, G. Wheeler, V. Mira-Wheeler "Local analysis of the inhomogeneous Willmore equation", preprint to appear.
[2] Y. Bernard, T. Rivière "Singularity removability at branch points for Willmore surfaces", Pacific J. Math. 265 (2013), no. 2, 257-311.

Research Project No. 1: Variational considerations. The student will learn how to apply Noether's theorem to the Willmore functional as well as to other important functionals appearing in geometric analysis in order to derive conservation laws. These laws will be used to understand various analytical and geometric properties of Willmore surfaces.

Research No. 2: Local analytical aspects. The student will learn in details how to study the regularity of the solutions to the Willmore equation, as well as to inspect the asymptotic behaviour of a branched solution (i.e. one with point singularities).

Research Project No. 3: The compactness question. In this reading project, the point of focus will be to understand how one can extract from a given sequence of solutions to the Willmore equation a subsequence that converges in an appropriate sense, and to understand the analytical and geometric properties of the limit.

Variational methods in shape optimisation (A,P)

(co-supervised with Dr Janosch Rieger)

Shape optimisation has various applications in theory as well as in industry. The minimisation of the drag of a vehicle, the maximisation of the effectiveness of a magnet with given weight in a wind turbine and the reconstruction of the most likely shape of an object by tomography are particular examples of shape optimisation problems.

From a mathematical perspective, a shape optimisation problem is an optimisation problem, where the optimisation variable is a subset of a Euclidean vector space. As the collection of these subsets has no linear structure, standard approaches to optimisation problems do not work in this setting.

The aim of this project is to work through the first chapters of the books *Introduction to Shape Optimisation* by Sokolowski and Zolesio and *Variational Methods in Shape Optimisation Problems* by Bucur and Buttazzo to obtain a thorough understanding of the problem as such. The second step will be a numerical treatment of a selected shape optimisation problem.

The prerequisites for this project are a solid knowledge in real analysis and linear algebra. Familiarity with functional analysis, partial differential equations, nonlinear optimisation and computational mathematics is beneficial, but not required.

Dr Anne Borschmann

Computational phase-field modelling (A)

Evolving interfaces are central to many engineering and biological applications, including multi-phase fluid flows, crack propagation in solids, vesicle dynamics and biological growth phenomena. Classically, the study of these moving boundary problems requires the numerical approximation of a set of partial differential equations on a moving domain, where the motion of the domain is also unknown. This becomes especially complex in applications where the domain undergoes topological changes (such as the break-up or coalescence of droplets).

The phase-field method treats the interface in a diffuse manner, allowing for the continuous transisiton of fields across the boundary. This is achieved by the introduction of a phase-field variable, acting as an indicator function for the diffuse interface, as well as a set of partial differential equations that govern its evolution. This way certain discretization challenges from sharp-interface models can be overcome - whilst giving rise to interesting new ones. Moreover, phase-field models can be derived from thermodynamic principles, thereby providing a valuable tool for simulating interfacial phenomena.

Based on the student's interest and background, this project could focus on (a combination of) the following

- the study of the fundamental laws of phase-field methods, to subsequenlty develop a phase-field model for a particular interfacial phenomenom, (e.g. chemotaxis, tumour growth, adhesive interactions, vesicle dynamics);
- the connection between the phase-field model and their sharp-interface counterpart;
- the numerical implentation using the Finite Element Method (e.g. in Gridap);
- the numerical analysis, e.g. time-discretization schemes and their energy-stability.

Reference:

Gomez, H. and van der Zee, K.G. (2017). Computational Phase-Field Modeling. In Encyclopedia of Computational Mechanics Second Edition (eds E. Stein, R. Borst and T.J.R. Hughes).

Prof Paul Cally

Does Magnetohydrodynamics work when the ionization fraction is very low? (A)

Classical magnetohydrodynamics (MHD) is a description of the interaction of magnetic fields and highly ionized fluids used widely in astrophysics and space physics and in many industrial contexts. It postulates that the field and fluid are tightly bound together because of the free charges in the latter. However, in cool stars such as the Sun the ionization fraction at the surface (photosphere) is very low; only about 10^{-4} for our star. So, does MHD still apply, and is it possible to generate Alfvén waves at the surface widely invoked to heat the corona high above to million-degree temperatures?

In this project you will explore and extend an eigenvalue decomposition method for describing partially ionized plasmas (Cally, 2023). In this way you will assess the accuracy and applicability of ray-based methods to MHD.

References

Cally, P. S. “Efficiency of magnetohydrodynamic wave generation in weakly ionized atmospheres”, Astrophys. J. 954:85 (2023)

Khomenko, E., Collados, M., Díaz, A., and Vitas, N. “Fluid description of multi-component solar partially ionized plasma”, Physics of Plasmas 21, 092901 (2014)

Soler, R. “Magnetohydrodynamic waves in the partially ionized solar plasma”, Phil. Trans. Roy. Soc. A 382:20230223 (2024)

Vranjes, J., Poedts, S., Pandey, B., and De Pontieu, B. “Energy flux of Alfvén waves in weakly ionized plasma”.

A/Prof Julie Clutterbuck

The mountain pass technique in modern geometry (P)

In many natural physical and mathematical situations, we want to not only minimise an energy, but also find its critical points-- places where may be only local maxima or minima, or even saddle points. One strategy to address this delicate problem is the mountain pass technique, where we look for points that have minimise energy in one direction, but maximise it in others--- like finding the lowest point of a mountain range in order to cross it.

This project will study the evolution of the technique, from the classic variational techniques it developed from, to its use in geometric problems such as finding closed geodesics on manifolds, and recent developments such as Coda Marques and Neves' proof of the Willmore conjecture.

Prerequisites: solid background in analysis and PDE (MTH3011 Partial differential equations, MTH3140 – Real analysis). Metric spaces MTH3160 and Differential Geometry 3110 would be useful.

A/Prof Andrea Collevecchio

Topics on random walks (S)

See supervisor for details.

Topics on large deviations (S)

See supervisor for details.

Dr Kengo Deguchi

Topics on linear and nonlinear analyses of fluid dynamics problems (A)

Option 1: Numerical exploration of Navier-Stokes phase space (A)

In numerical simulations of fluid flows, changing the initial conditions can lead to different results, making it challenging to obtain general conclusions. According to dynamical systems theory, there are invariant solutions in phase space. Those solutions often clarify the mechanisms underlying the dynamics observed in simulations. In this project, we will use simple model equations to learn the concepts of dynamical systems theory while exploring new solutions to the Navier-Stokes equations through computational tools.

Option 2: Perturbation analysis of hydrodynamic stability (A)

Determining the stability of a given fluid flow is essential in many practical applications. However, the governing equations of fluid dynamics are often complex, making it challenging to assess stability across a wide range of parameters. Perturbation theory is a powerful tool for simplifying these governing equations and has played a crucial role in stability theory. The student may choose and analyse a model flow, such as blood flow through a pipe, magnetohydrodynamic flows, free-surface problems, or flows over rough walls, among others.

Option 3: Mathematical proof of hydrodynamic stability or upper bound of turbulent transport (A)

In some cases, stability can be rigorously determined without relying on numerical calculations. Establishing simple sufficient and necessary conditions for stability is valuable, as the actual stability boundary should lie between them. Mathematical analysis can also be applied to nonlinear problems. A typical example of this is determining the upper bounds of turbulent transport, such as wall drag and heat transfer. We will build on existing published work and, where possible, aim to establish new results. A better model?

Prof Heiko Dietrich

Topics in (computational) algebra (P)

If you have completed Algebra and Number Theory 1 (& 2), and if you are interested in the area of algebra, then you can discuss possible projects with Heiko, heiko.dietrich@monash.edu

Depending on background and interests, topics can be chosen from the broad areas of:

- Group Theory,
- Lie algebras,
- Coding Theory / Cryptography,

or from any other suitable area in algebra; feel free to discuss your preferences.

If interested, then a project could be tailored to involve the computer algebra systems GAP (gap-system.org).

Skills required: Interest in algebra and understanding of pure maths proofs.
Prerequisites: Algebra 1: Group Theory and Algebra 2: Rings and Fields

A/Prof Norman Do

Tropical enumerative geometry (P)

Background: How many lines pass through two randomly chosen points? OK, so that's an easy one. But how many degree two curves in the plane pass through five randomly chosen points? And what happens when you consider higher degree curves? This problem from the realm of *enumerative geometry* was studied for centuries before being resolved by the Fields medallist Maxim Kontsevich in 1995 using ideas from theoretical physics. In the last several years, a new and relatively simple approach to such problems has arisen from the world of *tropical geometry*.

Project outline: There are many possible projects, depending on the preferences and strengths of the student. The initial goal would be to understand the basic notions of tropical enumerative geometry. From that point, a student could explore the notion of tropical Hurwitz numbers or the rich algebraic structures that arise in tropical enumerative geometry,

References

[1] G. Mikhalkin. What is… a tropical curve? (2007)
[2] R. Cavalieri, P. Johnson and H. Markwig. Tropical Hurwitz numbers (2008)

The algebra of knots (P)

Background: A knot is made by taking a piece of string, tying it up in some fashion, and then gluing the ends together. For well over a century, mathematicians and scientists have been preoccupied with the question of how to distinguish two given knots. Over recent decades, inspirations from algebra and theoretical physics have led to a vast theory of knot polynomials, which help to answer this question. A particularly important example is the sequence of coloured Jones polynomials.

Project outline: There are many possible projects, depending on the preferences and strengths of the student. One is to examine the AJ conjecture, which relates the recursive structure of coloured Jones polynomials with the A-polynomial of a knot. Another is to consider modern approaches to the volume conjecture, which relates coloured Jones polynomials with the volume of a space related to the knot. Physicists have recently proposed that these conjectures can be investigated using a theory known as topological recursion.

References

[1] C. C. Adams. The knot book. (2004)
[2] H. Murakami. An introduction to the volume conjecture. (2010)

Plane partitions and the topological vertex (P)

Background: How many ways are there to push N unit cubes into the corner of a room? Such an arrangement is called a plane partition and there is a beautiful formula to enumerate them that has been known for over a century. However, about twenty years ago, it was realised that plane partitions are closely related to the topological vertex, which is a procedure to perform certain calculations in topological string theory. Although these topics have been studied intensely, many mathematical mysteries still remain.

Project outline: There are many possible projects, depending on the preferences and strengths of the student. The initial goal would be to understand the MacMahon's formula for the enumeration of plane partitions and its proof via the algebra of operators on the infinite wedge space. From that point, a student could explore the underlying geometry of the topological vertex, consider plane partitions and the tropological vertex in other "types", or calculate quantum mirror curves for the topological vertex.

References:

[1] B. Young. Counting colored boxes. (2008)
[2] M. Mariño. Lectures on the topological vertex. (2008)

Dr Alina Donea

Local Helioseismology: Listening to the Sun (A)

(co-supervised with Prof Paul Cally)

Background: We will apply mathematical techniques such as Fourier Decomposition and Power spectra to Analyse the sounds interaction with the magnetic field in sunspots. Visualisation techniques such as Paraview will be used to trace field lines of magnetic field and follow acoustic waves paths bouncing on field lines.

Aims: Generating visualisation tools for research purposes
Tools: Mathematica, unix scripts, unix friendly environment, easy to learn
Prerequisite: MTH2032 or equivalent, love for solar physics

References: ADS/NASA: search for the supervisor's name.

The comprehensive statistical analysis of the most boring part of the Sun (A)

Who said that the Quiet Sun is boring? Our understanding of the quiet Sun magnetic fields has turned up-side-down during the last decade. The quiet Sun was thought to be basically non-magnetic, whereas according to the current views, it is fully magnetized.
In this project we will summarize the main observational properties of the quiet sun magnetic fields. Magnetograms from the Helioseismic and Magnetic Imager (HMI) instrument on board of the Solar Dynamical Observatory will be analysed.
We then address specific properties of the quiet Sun magnetic fields: the distribution of magnetic field strengths and unsigned magnetic flux, the distribution of magnetic field inclinations, and the time evolution of the signals on short time-scales.

Skills: Apply mathematical algorithms for processing Satellite Images of the quiet sun.
Tools: unix scripts, unix friendly environment, easy to learn
Prerequisite: MTH2032 or equivalent, love for solar physics

References
[1] J. S´anchez Almeida and M. Mart´ınez Gonz´alez, arXiv:1105.0387v1 [astro-ph.SR] 2 May 2011.

Machine Learning Algorithms in Solar Physics and Space Weather (P)

The volume of data being collected in solar physics has exponentially increased over the past decade and with the introduction of the Daniel K. Inouye Solar Telescope (DKIST) we will be entering the age of petabyte solar data. Feature detection will be an invaluable tool for post-processing of solar images to create catalogues of helioseismic relevant targeted data, ready for helioseismologists to use. The algorithm we will work with allows the user to easily identify the images of most importance to the study they are carrying out. Furthermore, having a pre-trained CNN that understands the solar features can be very beneficial for "transfer learning". We will be working on generating data-driven solar physics-based simulations, followed by its statistics and improving the algorithms. What is needed? a deep convolutional neural network adapts at feature extraction and processing images quickly. We train our network using only data from satellites.

You will feel comfortable with doing research on line, searching GitHub for possible solutions to our imaging tasks.

References:
Conference on Machine Learning in Heliophysics http://bit.ly/ml-helio19
New Book: Machine Learning Techniques for Space Weather http://tiny.cc/ML_spaceweather_book
Journal of Space Weather and Space Climate www.swsc-journal.org

The Mathematics of Sound Field Recording (P)

We will analyse sounds and wave fields. We examine ways in which various recording techniques encode the information from sound fields in two and three dimensions into discrete signals. A microphone will pick up some or all of the information that is contained in the Fourier coefficients. The aim of this work is to study recording with coincident microphone arrays in which we employ various types of microphones. Cardioids and Hypercardioids is our language. A mathematical model for the sound field and background noise will be presented. This is mathematics, but sound recording is also part of the fun.
References: talk to the supervisor.

Prof Andreas Ernst

A range of projects are available in discrete optimisation ranging from mostly abstract/theoretical to the very applied. Below are some examples of projects, but there is a much longer list available upon request. Please contact Andreas Ernst if you are interested in doing a project in optimisation in some other application area to discuss some possible alternative projects.

The Bee Benders Algorithm (A)

Project Outline: This project will look at combining the Benders Decomposition Algorithm for integer linear programs with the Artificial Bee Colony Optimisation algorithm used in Artificial Intelligence. The combined hybrid algorithm will be applied to an abstract mixed integer optimisation problem. The proposed test problem for the algorithm is the arc-disjoint path problem of finding the maximum number of paths between a given set of origin-destination pairs in a graph so that each edge is only used once. An initial implementation of the hybrid algorithm has been tested and shown to be promising on another application. The aim in the honours research will be to develop a strong understanding of the theory underpinning Benders Decomposition and to use this to build a more effective Bee Benders hybrid method.

Prerequisites: MTH3330 (Linear programming duality, shortest path algorithms) and some basic computer programming skills

References:

[1] Rahmaniani, R., Crainic, T. G., Gendreau, M., & Rei, W. (2017). The Benders decomposition algorithm: A literature review. *European Journal of Operational Research*, *259*(3), 801-817.

[2] Blesa, M. J., & Blum, C. (2018). On Solving the Maximum Disjoint Paths Problem with Ant Colony Optimization. *Handbook of Approximation Algorithms and Metaheuristics: Contemporary and Emerging Applications, Volume 2*, 1.

Taking the fast train to Makassar (A)

Project outline: This project will look at the railway line between Makassar & Pare-Pare in the province of South Sulawesi, Indonesia. This railway line is currently being built to carry both slow freight trains and fast (150km/hr) passenger trains. However, many parts of the railway are just single track with passing loops at occasional stations. This means both the overtaking of trains in the same direction and the passing of trains in opposite directions has to be scheduled very carefully. This project will develop optimisation models and scheduling algorithms to try to maximise the number of trains that can be scheduled on this new railway. This project combines discrete mathematics (directed graphs), computational mathematics (optimisation) and a practical application focus.

Prerequisites: At least one of MTH3330 or MTH3310. Basic programming skills

References:

[1] Harbering, J., Ranade, A., Schmidt, M., & Sinnen, O. (2019). Complexity, bounds and dynamic programming algorithms for single track train scheduling. *Annals of Operations Research*, *273*(1), 479-500.

Optimal classification trees in machine learning (A)

Brief Description: Most forms of machine learning involves solving an optimisation problem to find the best fit between a mathematical model and a collection of training data. Typically, the optimisation problem is solved heuristically (that is in a somewhat ad-hoc manner). A recent paper showed that for a class of models called classification trees, the fitting of the model to the data can be optimally using integer programming approaches. The advantage is with a better fitting, models of the same complexity tend to produce a more accurate classification. However, the integer programs that arise in this application are challenging to solve, particularly for big data sets. This project will look at improving the performance through better integer programming formulations and smarter algorithms.

References:

[1] Bertsimas, D., & Dunn, J. (2017). Optimal classification trees. Machine Learning, 106(7), 1039-1082.

[2] Verwer, S., & Zhang, Y. (2019, July). Learning optimal classification trees using a binary linear program formulation. In Proceedings of the AAAI Conference on Artificial Intelligence (Vol. 33, pp. 1625-1632).

Dr Hasan Fallahgoul

Portfolio allocation and risk measures: which measure? (F)

One of the main questions in portfolio allocation is related to allocation investor's wealth across different assets for gaining highest profit in a specific time horizon. The earliest idea for this allocation was introduced by Markowitz in (1952). He proposed to maximize the returns according to some level of variance. The variance is controlling the amount of investor's risk. He optimized the allocation of wealth based on a constraint, i.e., a risk function.

After his pioneering paper, a lot of risk measures have been introduced. Surprisingly, to the best of our knowledge, there is no comprehensive empirical study to compare the performance of a portfolio allocation under all risk measures. Of course, there are some studies which only compare a few of them together.

The objective of this project is twofold: first, constructing a portfolio allocation strategy based on empirical regularities of financial time series such as heavy tails, asymmetry, and stochastic volatility. Second, provide a comprehensive empirical study of portfolio allocation for all existing risk measures such as standard deviation, value at risk, expected loss, expected shortfall, shortfall deviation risk, maximum loss, deviation entropic, and etc.

Reference:
Markowitz, H., 1952. Portfolio selection. J. Finance 7 (1), 77–91.
Rachev, S., Ortobelli, S. , Stoyanov, S. , Fabozzi, F.J. , Biglova, A. , 2008. Desirable properties of an ideal risk measure in portfolio theory. Int. J. Theor. Appl. Finance 11 (01), 19–54.

Application of AI/ML in Economic and Finance (F)

See supervisor for details.

Explainability of AI/ML: variable significance tests and model misspecification (F)

See supervisor for details.

Unstructured datasets and climate change (F,S)

See supervisor for details.

Dr Mark Flegg

Stochastic-deterministic hybrid modelling of reaction-diffusion processes in biology (A)

Background: Describing spatially heterogeneous reacting systems mathematically is typically done with the use of the reaction-diffusion partial differential equation (PDE). In systems with sparsely distributed molecules the intrinsic stochasticity becomes important. These types of systems are the norm rather than the exception in intracellular environments, for example. In biology in particular, spatially heterogeneous reacting chemical systems are modelled using one of two approaches; compartment-based or molecular-based methods.

Molecular-based methods (often referred to as microscopic methods) trace the trajectories of all molecules in the system. Modelling a system in this way is only computationally possible if the copy numbers are very low but will provide the highest level of accuracy in a simulation. Compartment-based methods (often referred to as mesoscopic methods) discretise the domain into regions which are considered to be well-mixed. Diffusion is simulated by means of jumping events between compartments. Since it is only necessary to store the copy number per compartment, mesoscopic methods are much quicker but considered to be less accurate.

In order to deal with the complexity of biochemical systems, a vast number of hybrid methods have been proposed which utilise the most appropriate algorithm in various regions of the domain. The coupling technique has to be carefully constructed to join the modelling regimes seamlessly. This has been done in the case of coupling PDEs to molecular-based methods [1] and compartment-based methods to molecular-based methods [2] but not for PDEs and compartment-based methods.

The aim of this project will be to devise an accurate coupling algorithm for coupling a reaction-diffusion PDE with compartment-based simulation. The ultimate goal of this work would be to write an algorithm which would adaptively use the correctly modelling algorithm when and if it is needed within the domain for maximum efficiency and accuracy.

References:
[1] Franz B, Flegg MB, Chapman SJ, Erban R (2012) Multiscale reaction-diffusion algorithms: PDE-assisted Brownian dynamics. SIAM Journal on Applied Mathematics 73(3): 1224-1247.
[2] Flegg MB, Chapman SJ, Erban R (2012) The two-regime method for optimizing stochastic reaction-diffusion simulations. Journal of the Royal Society Interface 9(70): 859-868.

Growing kidneys (A)

How a single cell can undergo many generations of cell division and differentiation to create a fully functional human being at the moment of birth is a miracle as much as it is a mystery. Occasionally, the process is not completed properly. These congenital abnormalities can be devastating for the health of the future child. The most common anatomical site for congenital abnormalities is the kidney. The kidney, alongside other organs such as the heart and the brain, is essential to human life and many kidney abnormalities can be fatal.

In this project we will be generating theoretical and computational aspects of the morphology of kidneys in the developing embryo. We will be investigating how small changes in cellular responses to kidney forming stimuli may easily lead to severe congenital disorders. In particular, we will be interested in how the initial kidney precursor (the ureteric bud) is induced. Failure of this process to occur leads to kidney agenesis (and subsequent death of the individual).

The primary aim of this project is to develop a model of the developing ureteric bud. This may be done using a couple of different approaches continuum or discrete. A continuum approach would involve writing partial differential equations for inducing morphogens which allow for determination of the budding site on a one-dimensional domain. The discrete approach would be to make a cellular automaton simulation of budding process. The discrete approach would allow for significantly more complexity at the cost of tractability. The references [1,2] are good background reading to introduce kidney morphogenesis.

References:
[1] M. Little, A. P. McMahon, “Mammalian Kidney Development: Principles, Progress and Projections”, Cold Spring Harb Perspect Biol, 4:a008300, 2012
[2] F. Costanitini, R. Kopan, “Patterning a Complex Organ: Branching Morphogenesis and Nephron Segmentation in Kidney Development”, Dev Cell Rev, 18, 2010

Uncrossing the signals of cancer (A)

In 1971, Richard Nixon declared war on cancer by signing the National Cancer Act. Over 40 years of intense worldwide fundraising and research later and cancer is still developed by one in three people and kills around one in five. Why is cancer such a difficult disease to eradicate? The answer is because cancer is not one disease: every cancer is different. Cancer is caused by a normal cell undergoing a mutation. This mutation can be any one of a many of mutations but the end result is the same. The cell starts to hallucinate: it perceives chemical signals which do not exist. If that signal is an instructional signal for the cell to divide (or to live longer than it should) the result is a tumorous growth.

In this project, we will be investigating a particular aberrant signal which is common to many cancers. This signal is usually propagated by “Wnt” proteins. In many cancers, cells behave as though they are receiving this signal when they are not and, as a result, multiply out of control. This is not the only by-product of aberrant Wnt signalling. Deregulation of the mechanism of Wnt signals in cancer leads to, or is a consequence of, internal dysregulation of proteins which are crucial for the normal function of other signals. The sequential breakdown of normal cellular behaviour allows the growth to be particularly malignant. This phenomenon in biology is known as signalling cross-talk. We will investigate the cross-talk mechanisms of Wnt. As there are a number of important cross-talks associated with Wnt, there is freedom to investigate that which is most interesting. Initially we will investigate these mechanisms using systems of ordinary differential equations but may then utilise spatial models (stochastic and deterministic) depending on the student’s research interests. For a comprehensive background on the mathematical approaches used in the literature see Ref [1].

References:
[1] B. Lloyd-Lewis, A. G. Fletcher, T. C. Dale and H. M. Byrne; "Toward a quantitative understanding of the Wnt/β-catenin pathway through simulation and experiment". Wiley Interdiscip Rev Syst Biol Med. 5(4):391-407 (2013).

Dr Justin Forlano

Poincare-Dulac Normal forms for nonlinear dispersive PDEs: (P)

You've likely seen footage on the internet of the infamous Tacoma narrows bridge collapse from 1940. This event dramatically highlighted the importance that resonances play in physical systems, and motivates our understanding of their mathematical underpinnings.

In the setting of (nonlinear) ordinary differential equations, the Poincare-Dulac theorem says that we can find a change of variables

which extracts the essential resonant parts of the nonlinearity modulo a better-behaved remainder term. The ideas behind this theorem have recently been extended to nonlinear dispersive partial differential equations (PDEs), and have proved to be a simple but highly effective technique in many different contexts.

This project will focus on understanding some of these developments for dispersive PDEs, with applications to the construction of low-regularity solutions without having to use heavy harmonic analysis.

Statistical properties of nonlinear dispersive PDEs: (P)

A central question in nonlinear PDEs is: how are initial data propagated by a nonlinear PDE?

In many physical situations, our initial data may not be perfectly known and we can model this issue by studying nonlinear PDE with random initial data. As our data is now random, it is very natural to ask what are the statistics (probability distribution) of the corresponding solutions at later times, which is an extremely active sub-area of dispersive PDEs. In this project, you will study this area and the latest developments. This mixes techniques from PDEs, harmonic analysis and probability theory.

Moreover, depending on your interest, this project could also include numerical simulations, with the aim to simulate the long-time behaviour of the statistics and form corresponding conjectures.

Singular stochastic PDEs: (P,S)

Many physical systems are influenced by some kind of noise, such as from thermal fluctuations These external forcings are random and well-modelled by the so-called space-time white noise. Unfortunately, the space-time white noise is so rough that our classical understanding of the PDE completely breaks down; products no longer make sense! Giving a meaning to these ill-posed problems has been a recent major breakthrough in stochastic PDEs over the last decade; indeed, a 2014 Fields medal was awarded to Martin Hairer who introduced the theory of regularity structures to solve such problems. Since then other techniques have been developed with particular note for this project being the theory of paracontrolled distributions by Gubinelli-Imkeller-Perkowski.

This project will serve as an introduction to this wide and highly active area of research concerning singular stochatstic parabolic, hyperbolic, and dispersive PDEs. Topics include: crash course in stochastic analysis, renormalisation, and the Da Prato-Debussche regime.

Prof Tim Garoni

Discrete models of critical phenomena (P, S)

Background: Many probabilistic models defined on graphs possess special "critical" values of their parameters, at which long-range order develops, and fractals emerge. Examples of such models include percolation, in which one studies the connectivity properties of random spanning subgraphs of some fixed graph, and the Ising/Potts models in which one studies random vertex colourings. The study of such "critical phenomena" has been a fertile area in mathematical physics for three quarters of a century. A wide variety of techniques are used to study these models, including probabilistic, combinatorial and algebraic methods.

Project Outline: The project would start with a review of the required basic background in mathematical physics (statistical mechanics); the references listed below being a good place to start. From there, a range of possible projects is available, and there would be scope for an investigation that may lead to new results.

References

[1] Slade, G "Probabilistic Models of Critical Phenomena" in The Princeton Companion to Mathematics, edited by T. Gowers, Princeton University Press (2008).
[2] Grimmett, G "Probability on Graphs" Cambridge University Press (2010), available online free (legally) at http://www.statslab.cam.ac.uk/~grg/books/pgs.html.
[3] Friedli and Velenik, "Statistical Mechanics of Lattice Systems" Cambridge University Press (2017), available online free (legally) at https://www.unige.ch/math/folks/velenik/smbook/.

Markov-chain Monte Carlo methods in statistical mechanics (P, S)

Background: Statistical mechanics began life as a branch of mathematical physics, but is now a central paradigm for studying all manner of complex systems, across fields as diverse as physics, chemistry, biology, economics and sociology. An important branch of statistical mechanics concerns discrete models, in which one studies random structures defined on graphs. These studies have significant overlap not only with probability theory, but also combinatorics and computer science. Since models in statistical mechanics are often mathematically intractable, Markov-chain Monte Carlo (MCMC) methods have become an indispensable computational tool in this field.

However, not all Markov chains are created equal, and while two Markov chains may have the same stationary distribution (and therefore approximate the same statistical mechanical model), their rates of convergence to stationarity (and therefore their practical efficiency) can be very different. While the classical theory of Markov chains considers the late-time asymptotics of fixed chains, the relevant asymptotics in statistical mechanics concerns the growth of "mixing times" as the size of the state space becomes large. Quantifying the size of such mixing times, and designing new Markov chains with reduced mixing times, are the central tasks in this field.

Project Outline: The project would start with a review of the required basic background in discrete statistical mechanics, and Markov-chain Monte Carlo; the reference listed below being a good place to start. From there, a range of possible projects is available, studying specific classes of Monte Carlo methods for specific classes of statistical-mechanical models. There would be scope for an investigation that may lead to new results.

References

[1] Levin, D and Peres, Y, "Markov Chains and Mixing Times" American Mathematical Society (2017), available online free (legally) at http://pages.uoregon.edu/dlevin/MARKOV/markovmixing.pdf.

Dr Ivan Guo

Topics in Martingale Optimal Transport (A, S, F)

The mathematical field of Optimal Transport, originally motivated by resource allocation problems in civil engineering, has been extensively developed in recent years with a wide range of applications, from various areas of pure mathematics to physics, cosmology, economics, finance, computer science and engineering. Martingale Optimal Transport (MOT) further extends the field to stochastic systems. Informally, given a pair of probability distributions, the primal MOT problem seeks a martingale which diffuses from the first onto the second, while optimising objective functions. The dual MOT problem involves optimising over the space of continuous functions.

There are many topics and problems to be explored in this area, for example:

- Convex analysis and dualities in various topological spaces
- Cyclically monotonic sets and the geometry of optimal transport maps
- Convex ordering of probability measures and Strassen's theorem
- "Peacocks" and Kellerer's theorem
- Shadow martingale couplings, left-curtain couplings and barcode couplings
- Semi martingale optimal transport in continuous time and duality with path-dependent PDEs
- Adapted Wasserstein distances, measuring similarity of stochastic processes
- Optimal transport with optimal stopping, optimal Skorokhod embedding
- Computational algorithms, from the Sinkhorn algorithm to deep learning
- Solving inverse problems via deep learning
- Applications in mathematical finance: robust finance, model calibration, portfolio optimisation
- Applications in generative modelling and AIs

This is suitable for those who are interested in further studies in areas such as probability, stochastic analysis, mathematical finance, optimisation or machine learning. Please contact Ivan Guo (ivan.guo@monash.edu) to discuss possible projects.

Prof. Zihua Guo

Fourier restriction estimates (P)

Fourier restriction estimate is one of the core topics in harmonic analysis. In 1960s, E. M. Stein observed for the first time the restriction phenomenon of the Fourier transform, and proposed the Fourier restriction conjecture. This conjecture in 3 and higher dimensions is still open and has been studied extensively. Besides revealing a fundamental property of the Fourier transform, Fourier restriction estimate has found tremendous applications in other fields, e.g. PDEs. This project will be devoted to study the classical Fourier restriction results and its applications in PDEs, providing a friendly introduction to the fascinating world of analysis and PDEs.

KdV equation: theory and numerical methods(P)

KdV equation is a fundamental equation for shallow water waves. This project will be devoted to study the KdV equation on the torus. It focuses on the low regularity well-posedness theory of the KdV equation, and study how to design the low-regularity numerical scheme using the theory obtained. This is an introductory project to the fascinating field: theory and numerics for dispersive PDE.

Fractal Uncertainty principle (P)

Fractal uncertainty principle states that no function can be localized in both position and frequency near a fractal set. It is a substantial extension of the classical uncertainty principle and has found wide applications in harmonic and semiclassical analysis. This project is to study the elegant theory and provide an overview of the recent studies.

3D axisymmetric Navier-Stokes equation (P)

The global existence of the large smooth solution for the 3D Navier-Stokes equation is a long-standing open problem. It is one of the millennium-problems. This project will focus on the axisymmetric case. This case was known to have some more structures so that there are rich results. Especially recently the criticality for this special case was revealed by Lei-Zhang basing on the work of Chen-Fang-Zhang. Their results showed it is only logarithmically far to finally solve the problem for this case. This project will be devoted to study the classical results, the rich structure as well as the recent mentioned works.

Some topics in nonlinear dispersive equations (P)

In the last 30 years, there were enormous developments in the fields of nonlinear dispersive equations, e.g. KdV, Schrodinger and wave equations. Many new tools were developed and new ideas from other fields have played important roles. This project will focus on one of the following topics: low regularity well-posedness problems, long-time behaviour, blow-up behaviour, and probabilistic well-posedness.

Dr Andy Hammerlindl

Topics in dynamical Systems (P)

Background: If we know exactly what happens to a system under tiny changes of time, can we use this information to determine the long-term behaviour of the system? This question is at the heart of dynamical systems research. The development of chaos theory shows that many systems which are highly predictable at small times are completely unpredictable over the long run. Nevertheless, many dynamical systems have invariant objects which are unchanged throughout time and aid greatly in understanding the overall qualitative behaviour of the system.

Project outline: This project will look at understanding the behaviour of certain families of dynamical systems based on invariant properties of the systems.

References:
[1] A. Katok, B. Hasselblatt, Introduction to the modern theory of dynamical system.
[2] D. Ruelle, Elements of Differentiable Dynamics and Bifurcation Theory

Topics in ergodic theory (P)

Imagine a billiard table, either rectangular or a more complicated shape with many straight or curved edges, but with no pockets for a ball to fall into. Then imagine a friction-less ball forever moving across the table and reflecting off of the walls. Does this ball visit all regions of the table? Does it favour some areas of the table over others? These questions, relating how one path through a space compares to the average behaviour over all points, define the notion of “ergodic theory.” This field uses measure theory to accurately describe the average behaviour of very chaotic dynamical systems. The study of billiards is just one example of its application.

Project outline: This project will begin with the study of invariant measures, ergodicity, mixing, and other related concepts. From there, further work could include the study of certain types of dynamical systems and determining when ergodicity and related properties hold.

References:
[1] P. Walters, An introduction to ergodic theory
[2] A. Katok, B. Hasselblatt, Introduction to the modern theory of dynamical system.
[3] D. Ruelle, Elements of Differentiable Dynamics and Bifurcation Theory

Numerical simulation of multiscale systems (P)

Multiscale or “slow-fast” systems are dynamical systems in which different parts of the system evolve at vastly different scales of time. As an example, cloud cover over a region of the Earth changes from minute to minute, but large-scale changes in climate can occur over decades or longer.

Numerical simulation of dynamical systems requires calculating the state of system from one instance of time to the next, and in a multiscale system there is no good choice of time step which ideally fits both the slow and fast dynamics.

This project will try to quantify the error in numerically simulating such systems, compare various techniques, and examine the effect of the floating-point precision of computer implementations of the algorithms.

References:
[1] E. Hairer, S. P. NÃ¸rsett, and G. Wanner. Solving ordinary differential equations I.
[2] P. Henrici. Discrete variable methods in ordinary differential equations.

Prof Kais Hamza

Financial mathematics (F)

(co-supervised with Prof Fima Klebaner)

Project Outline: There are many topics to choose from. Topics vary greatly in the degree of theoretical and practical work involved. Students can choose a topic with any mixture of the theory of stochastic calculus (used in the modelling of financial markets), and the practice of statistical analysis (applied to real data from the Australian market).

References
[1] Hull, J., “Options, futures and other derivative securities”, Prentice Hall, 1989.
[2] Klebaner, F.C., “Introduction to stochastic calculus with applications”, Imperial College Press, 1998.

Characterisations and probability distributions (S)

Project Outline: The normal and exponential distributions play essential roles in probability and statistics. This project aims at reviewing the main characterisation results and how these are used in various areas of probability and statistics.

References
[1] Galambos, János & Kotz, Samuel, “Characterizations of probability distributions: a unified approach with an emphasis on exponential and related models”, Springer-Verlag, 1978.
[2] Patel, J.K. & Read, C.B., “Handbook of the normal distribution”, Dekker, 1996.

Probability, a measure theory approach (S)

Project Outline: Probability (or stochastic) models are very widely used. A good understanding of the basic probability theory is an absolute necessity. This topic will cover the basic measure theory (measurability, integrability ….) as well as concepts specific to the area of probability theory (independence, conditioning, distributions…).

A/Prof Daniel Horsley

Topics in graph decompositions and combinatorial designs (P)

Background: Combinatorial design theory is an area of mathematics that studies arrangements of objects that are in some sense “balanced”. It has applications to the design of experiments, to coding and cryptography, to traffic grooming in networks, and to numerous other areas. Many problems in combinatorial design theory can be considered as problems concerning decomposing graphs into edge-disjoint subgraphs, and this way of viewing them has led to many important results and insights.

Objectives: This project will involve investigating one the many easily accessible problems in the area of graph decompositions and combinatorial designs. Possible problems could concern

- embedding Steiner triple systems;
- colouring graph designs;
- resolutions of Steiner triple systems;
- decompositions of graphs into cycles; or
- infinite designs.

Expectations: A reasonable degree of mathematical maturity.
Assumed Knowledge: None.
Reading: To be determined depending on the specific problem considered.

A/Prof Jonathan Keith

Integral-induced metrics on spaces of E-measurable (S)

Project outline: A *paving* is a family E of subsets of some set X, such that the empty set $\emptyset$ is an element of E. Greco defined the family $M(X,E)$ of E-measurable functions on a *paved set* (X,E), and an integral $f d\mu$ of an E-measurable function f with respect to a monotone set function on E (that is, a function $\mu : E \rightarrow [0,\infty]$ such that $\emptyset = 0$ and $A \leq \mu B$ for any A and B in E with $A \subseteq B$). Under certain conditions, such integrals induce a metric on $M(X,E)$. This project will explore the properties of such metrics, in particular, conditions under which they are complete.

Prof Fima Klebaner

Large deviations theory and applications (S)

(co-supervised with A/Prof Andrea Collevecchio)

The project focuses on the Large Deviations Theory and its application to find the most likely paths a stochastic system (a dynamical system with noise) can take. The project involves studies in probability, optimality and differential equations. Application can be in physics, biology or finance. This project will suit a strong mathematically minded student in probability.

References:
[1] Amir Dembo , Ofer Zeitouni, "Large deviations techniques and applications" Second ed., Springer-Verlag, New York, 1998
[2] M. I. Freidlin and A. D. Wentzell, "Random perturbations of dynamical systems". Second ed., Springer-Verlag, New York, 1998
[3] F. Klebaner and R. Lipster, "Asymptotic analysis of ruin in the constant elasticity of variance model theory" Probab. Appl. Vol. 55, No. 2, pp. 291–297.

Financial mathematics (S)

(co-supervised with Prof. Kais Hamza and Dr. Ivan Guo)

This project focuses on the so- called mimicking of stochastic processes, for example a fake Brownian motion, a process that is a martingale with Normal (0,t) marginals. There are two approaches to mimicking, one is purely probabilistic and one that involves PDEs and optimal transport. This project will touch on these approaches. This research is purely mathematical, however, it has wide applications in options research in finance.

References.
[1] K. Hamza and F. Klebaner. A family of non-Gaussian martingales with Gaussian marginals. J. Appl. Math. Stoch. Anal., pages Art. ID 92723, 19, 2007.
[2] J. Albin. A continuous non-Brownian motion martingale with Brownian motion marginal distributions. Statist. Probab. Lett., 78(6):682–686, 2008.
[3] H. Kellerer. Markov-Komposition und eine Anwendung auf Martingale. Math. Ann., 198:99–122, 1972.

Evolution of populations (S)

(co-supervised with Prof. Kais Hamza)

Often populations are modelled as Markov processes. This approach allows to compute their generators and to obtain an evolution equation. Further analysis is possible under suitable assumptions, for example when the carrying capacity is large. Such results include the fluid approximation and the diffusion approximation for the population size. This research is purely mathematical, however, it has wide applications in mathematical biology.

References
[1] Klebaner F. Population-dependent branching processes with a threshold. Stochastic Processes and Their Applications 46 (1993), 115-127.
[2] Jagers, P., Klebaner, F.C., 2000, Population-size-dependent and age-dependent branching processes, Stochastic Processes and their Applications, vol 87, pp. 235-254.
[3] Hamza K, Jagers, P. Klebaner FC. 2015, On the establishment, persistence, and inevitable extinction of populations. Journal of Mathematical Biology (2015)

Dr Ngan Le

Numerical methods for deterministic/stochastic micromagnetic modelling (A)

Micromagnetic modelling is a widely used tool, complimentary in many respects to experimental measurements. Several recent technological applications such as heat-assisted magnetic recording, thermally assisted magnetic random-access memories or spincaloritronics have shown the need to generalize the theory of micromagnetics to high temperatures.

A well-known model of magnetic material is strictly valid only at Curie temperatures. For high temperatures, a more thermodynamically consistent approach was introduced by Garanin who derived the Landau–Lifshitz–Bloch equation for ferromagnets. The equation essentially interpolates between the model at low temperatures and the Ginzburg-Landau theory of phase transitions. It is valid not only below but also above the Curie temperature.

Furthermore, mathematical theory of the magnetisation dynamics of ferromagnetic elements in presence of electric current is at an early stage. If the ferromagnetic element is small enough then the interaction between the electric current and the magnetisation results in the current-induced magnetisation switching and spin wave emission. It is expected that good understanding of those effects will allow us to develop new types of current-controlled magnetic memories and current controlled magnetic oscillators.

The first goal of the project is to compute approximate solutions of one of micromagnetic models introduced above, and provide order of convergence of these approximations. A follow up project aims to study numerical methods for their stochastic versions that describes noise-induced transitions between equilibrium states of the ferromagnet.

Numerical simulation/analysis for fractional partial differential equations (A)

Fractional derivatives provide an excellent instrument for the description of memory and hereditary properties of various materials and processes. Many complex systems in nature, such as the transport of chemical contaminants through water around rocks, the dynamics of viscoelastic materials as polymers, the atmospheric diffusion of pollution, cellular diffusion processes, have a macroscopic complex behaviour and their dynamics cannot be characterized by the classical integer-order differential models. Several experimental results revealed that the fractional order may vary in time or in space.

Many numerical studies of time-fractional differential equations assume more regularity on the solution than is actually possible. The project will investigate numerical methods for variable-order time fractional partial differential equations, in a setting that accounts for the significant fact that their solutions typically exhibit singularities.

Dr Greg Markowsky

Planar Brownian motion and complex analysis (P, S)

There is an intimate connection between planar Brownian motion and complex analysis, and the major theme of this project will be to tackle problems which lie in the intersection of these two topics. To be specific, a holomorphic map applied to a Brownian motion yields a new stochastic process which can be realized as a Brownian motion run at a variable speed. This connection can be used to motivate and provide proofs of many of the major results in complex analysis; conversely, this conformal invariance, as it is often called, can also be used to prove many interesting results on planar Brownian motion via complex analytic techniques. We will study these techniques in more detail and look for deeper connections.

Properties of entire functions and relationships with characteristic functions of random variables (P, S)

The characteristic function of a random variable is essentially a Fourier transform, and is generally an analytic function of a complex variable. In many cases it is an entire function, and properties of the random variable are reflected in complex analytic properties of this function. We will study this connection in great detail, and look at some recent papers which have made use of it.

Random power series (P, S)

A random power series is one in which the coefficients are random variables. In this project, we will study several recent papers on relationships between the complex analytic properties of such power series and the probabilistic properties of the coefficients.

A/Prof Daniel Mathews

Contact geometry in Physics (P)

Background: Contact geometry is a type of geometry which uses advanced concepts from differential geometry, but arises naturally all over mathematics and physics. In this project you can study some of these connections between mathematics and physics.

Project outline: This project would start by studying background differential geometry and contact geometry. Further topics could include
* studying how contact forms arise in thermodynamics
* study how contact geometry describes Huygens' wave principle in optics
* using contact geometry to solve differential equations
* recent developments claiming to relate contact geometry to superconductors,

Prerequisites: Undergraduate differential geometry, and some undergraduate physics.

References:
[1] Hansjorg Geiges, A Brief History of Geometry and Topology, Expo. Math. 19 (2001) 25-53.
[2] Hansjorg Geiges, An Introduction to Contact Topology
[3] V. I. Arnold, Contact geometry: the geometrical method of Gibbs's thermodynamics, Proceedings of the Gibbs Symposium (1990) 163--179.

Topics in Symplectic geometry (P)

Background: Classical Newtonian mechanics was reformulated in the 19th century by William Hamilton. The mathematical language of Hamiltonian mechanics is symplectic geometry. This beautiful subject encompasses much of classical physics, but has also enjoyed several breakthroughs in recent years, connections with string theory, and the development of powerful invariants.

Project outline: This project would begin with a study of symplectic vector spaces, which are the building blocks of symplectic geometry, and symplectic manifolds. From there, several directions are possible, depending on the interests of the student, such as:

- Reformulating various physical theories (mechanics, optics, electromagnetism) in terms of the mathematics of symplectic geometry
- Dynamical systems in symplectic geometry
- Holomorphic curves, Floer homology and Gromov-Witten theory

References:
[1] Stephanie Singer, Symmetry in Mechanics
[2] Dusa McDuff and Dietmar Salamon, Introduction to Symplectic Topology

Knots and Skein Theory (P)

Background: A knot is a loop in 3-dimensional space. One very interesting and important way to study them is to introduce a type of algebra on them, setting certain knots equal to combinations of other simpler knots by equations called skein relations. The resulting algebraic objects, called skein algebras and modules, are important mathematical objects.

Project outline: This project would begin by studying knot theory in general. Then it will proceed to study skein relations and skein algebras. Further possibilities then include:
* calculations of skein algebras and modules in specific cases
* using skein theory techniques to calculate polynomial invariants of knots
* studying the AJ conjecture, a major open question relating polynomial invariants to geometry.

Prerequisites: Undergraduate abstract algebra and topology

References:
[1] Colin Adams, The Knot Book
[2] W. B. R. Lickorish, An Introduction to Knot Theory

[3] J. H. Przytycki, Fundamentals of Kauffman bracket skein modules

Contact category theory (P)

Background: Contact geometry is a type of geometry which arises from differential geometry, but it can also be described in an amazingly discrete and combinatorial way. This approach uses ideas from category theory. This project will study an object called the contact category, which arises in recent research in contact geometry.

Project outline: This project would start by studying background differential geometry and contact geometry, and a little category theory. Then it would study some known examples of contact categories, and attempt to make some further calculations.

Prerequisites: Undergraduate differential geometry and abstract algebra.

[1] Hansjorg Geiges, An Introduction to Contact Topology
[2] Mathews, Contact topology and holomorphic invariants via elementary combinatorics, https://arxiv.org/abs/1212.1759

Combinatorics of curves on surfaces (P)

Background: Take a surface, and mark some points on the boundary. Now ask how many ways there are to join those points up with curves? Depending on the types of curves allowed, some interesting answers can be found, connected to ideas from all over mathematics and physics.

Project outline: This project will begin by studying some background on surface topology, and then some existing results. It will then explore various extensions of these questions.

Prerequisites: Ability to read and write proofs, preferably undergraduate topology.

References:
[1] Do, Koyama, Mathews, Counting curves on surfaces https://arxiv.org/abs/1512.08853

Circle packings (P)

Background: Draw an arrangement of circles in the plane so that they are all externally tangent to each other. In what patterns can this be done? How much flexibility is there in these arrangements? There is a beautiful mathematical theory of such circle packings, which is related to complex analysis and important in 3-dimensional topology.

Project outline: This project will study the theory of circle packings. We may then proceed to compute some packings or try to prove results about them.

Prerequisites: At least one of undergraduate topology or complex analysis.

References:
[1] K. Stephenson, Introduction: The theory of discrete analytic functions

Advanced topics in Hyperbolic geometry (P)

Background: Hyperbolic geometry is a uniformly negatively curved geometry that is very important in low-dimensional topology, and has a beautiful mathematical structure. This project will explore some of the amazing properties of this geometry.

Project outline: After learning the background of hyperbolic geometry, several advanced topics are possible, such as the following.

* Studying Mostow rigidity: a deep result that says that, in certain circumstances, a hyperbolic geometry on a topological space is unique.
* Studying the volumes of polyhedra: The volume of hyperbolic objects can often be given exactly, but the algebra and analysis of it leads to some surprisingly sophisticated mathematics.

Prerequisites: Undergraduate differential geometry, preferably abstract algebra or topology.

References:
[1] Anderson, Hyperbolic Geometry
[2] Benedetti, Petronio, Lectures on Hyperbolic Geometry

Spinors and Clifford Algebras (P)

Background: Clifford algebras are wonderful mathematical objects that powerfully encode geometry within their algebraic structure. Included within their structure are objects called spinors, which arise in physics and have the strange property that turning 360 degrees does not return you to where you started, but turning 720 degrees does!

Project outline: The project will study Clifford algebras and their properties and applications.

Prerequisites: Undergraduate abstract algebra.

References:
[1] P. Lounesto, Clifford algebras and spinors, London Mathematical Society Lecture Note Series (1997) vol. 239.

Spinors in general relativity (P)

Background: General relativity is a fundamental physical theory which is described in the language of differential geometry. Roger Penrose and Wolfgang Rindler developed a program to rewrite this theory in terms of spinors.

Project outline: This project requires some knowledge of differential geometry and relativity. It will study spinors and Penrose and Rindler's work.

Prerequisites: Undergraduate differential geometry and relativity.

References:
[1] R. Penrose and W. Rindler, Spinors and Spacetime, vol. 1

Dr Stephen Muirhead

Variance of the number of percolation clusters(S)

Percolation is a model from statistical physics in which sites on a lattice are opened at random and the properties of the open clusters are studied. There are many variants, but the simplest is to open the sites independently with a fixed probability. It has been known since the 90's that, if you look in a large box, the number of open clusters satisfies a central limit theorem with variance growing in proportion to the volume of the box. However, the exact asymptotics of the variance is not well understood. The aim of the project would be to study the variance using some recently developed techniques. These should give at least three different methods to get an expression for the variance asymptotic, and the aim would be to compute these theoretically and then study them numerically.

Symmetric stable processes and Riesz potential theory [P,S]

The classical potential theory in physics studies the properties of an electric field when electric charges are distributed in space, interacting under the Newtonian kernel $|x|^{-1}$. Riesz potential theory is a generalisation of this to the kernels $|x|^{-\alpha}$, for general $\alpha \in (0,d)$ in d-dimensional space. Generally, this theory is more challenging, however in the regime $\alpha \in [d-2,d)$ there turns out to be a remarkable connection to a random process called the symmetric stable process. The aim of the project would be to use properties of the symmetric stable process to give insights into the Riesz potential theory. This would be mostly a theoretical project, but could also involve some numerics.

Dr Kihun Nam

Backward stochastic differential equations and stochastic optimization of marked point processes (S)

Backward stochastic differential equation (BSDE) is introduced by Bismut (1973) as a dual problem of stochastic control. Since then, BSDE has been one of the main tools to study stochastic optimization of a dynamical random system. One of the most frequently found dynamical random systems is the network such as SNS, or disease spreading. The dynamic random network can be seen as a multidimensional point process if one considers each edge as a point process (1 if connected, 0 if disconnected). The BSDE approach is desirable for controlling dynamic random network.

This project aims to develop a BSDE framework for a general optimizing problem of multidimensional point processes with a finite number of states. Our control will be the stochastic intensities on a restricted subset of point processes and our reward function is a functional on the path of point processes. The numerical methods for application will also be developed and implemented to a few toy examples.

Reference
[1] Confortola, Fulvia, Marco Fuhrman, and Jean Jacod. "Backward stochastic differential equation driven by a marked point process: an elementary approach with an application to optimal control. " The Annals of Applied Probability 26.3 (2016): 1743-1773.
[2] El Karoui, Nicole, Shige Peng, and Marie Claire Quenez. "Backward stochastic differential equations in finance. " Mathematical finance 7.1 (1997): 1-71.

FX estimation for non-trading hours (F)

Foreign exchange market is one of the biggest financial market with daily volume of 6.6 trillion dollars. The market opens 24 hour from Sunday 22:00 GMT to 22:00 GMT Friday. One cannot trade currencies from Friday 22:00 to Sunday 22:00 and FX rate is fixed. On the other hand, cryptocurrency is traded 24/7 around the globe with daily volume of 42 billion dollars. Using cryptocurrency, we would like to estimate the FX rate during non-trading hours, that is Friday 22:00 to Sunday 22:00. However, since the arbitrage trading is restricted by financial regulations in many countries, the price of cryptocurrency differs between countries. Therefore, one need properly normalize to obtain accurate FX rate from cryptocurrency market.
In this project, we will build stochastic models to generate FX rate from cryptocurrency prices and study whether the crypto-generated FX rate is indeed similar to the FX rate during trading hours.

On-chain analysis of Bitcoin trading (F)

In contrast to conventional financial asset, cryptocurrency’s transaction record is public data on blockchain. Therefore, one can analyse the on-chain data to extract the information about cryptocurrency trading. In this project, using Bitcoin on-chain data, we will study whether large deposit/withdraw from exchange is related to the price change or not. After the study, we will investigate how these large deposits are liquidated in the exchange.

Prof Todd Oliynyk

Isoperimetric surfaces in geometry (P)

Background: Isoperimetric surfaces are surfaces that minimize area subject to a volume constraint. They arise naturally in calculus of variation and provide a natural tool to attack some basic geometric problems.

Possible project topics under this area include:

- Review Hubert Bray's approach to Volume Comparison using isoperimetric surfaces techniques.
- Investigate potential applications in the study of manifolds with boundary.
- Review Gerhard Huisken's approach to the definition of quasi-local mass via isoperimetric surfaces in general relativity.

Newtonian limit and Post-Newtonian expansions (P)

Background: The Newtonian limit is the study of solutions to Einstein gravity coupled to matter in the limit that v/c -> 0, where v is a characteristic velocity scale associated to the gravitating matter and c is the speed of light. In this limit, one expects that solutions of general relativity approach solutions of Newtonian gravity in some sense. Starting from a fully relativistic solution with a well-defined Newtonian limit, one can try and expand the solution in powers of v/c. The resulting expansion is known as the Post-Newtonian expansion. This produces a sequence of equations beginning with the Newtonian gravitational one. These equations can be solved to yield an approximation to the fully relativistic solution to a certain order in v/c for v/c sufficiently small.

Possible project topics under this area include:

- (numerical) In general, there will be a critical time Tc after which the Newtonian solution or more generally the Post-Newtonian expansions will no longer be a valid approximation to the fully relativistic solution. The aim of this project would be to solve the spherically symmetric Einstein equations coupled to a perfect fluid and try to identify the critical time Tc for specific classes of initial data.
- (analytical) In the physics literature, Post-Newtonian expansions are computed using formal expansions without any rigorous justification. Recently, I have, using PDE techniques, established the validity of the Post-Newtonian expansions in the so-called near zone. The goal of this project would be to try and understand the relationship between the formal expansions used in the physics literature and the rigorous expansions I obtained using PDE techniques.

Renormalization group flow (P)

Background: The Renormalization Group (RG) flow arise from demanding cut-off independence of classical field theory quantization. For the special case of nonlinear sigma models, the RG equations correspond to geometrical flow equations for a Riemannian metric on a manifold. In general, the RG equations are extremely complicated. However, they do depend a small parameter and are often studied by expanding in the parameter and truncating at a certain (loop) order.

Possible project topics under this area include:

- (analytical) Ricci flow appears as the approximation to RG flow at the one-loop level for the non-linear sigma model. The aim of this project would be to carefully understand this relationship between RG and Ricci flow.
- (analytical) Entropies are important quantities for understanding the behaviour of RG flow. The goal of this project would be to interpret the Perelman's entropy for Ricci flow in terms of an entropy for the full RG flow.
- (numerical) In spherical symmetry, solve numerically both the first and second order RG equations. The goal of this project would be to identify regions in space-time where the first and second order RG equations are qualitatively the same and also where they differ significantly.
- (analytical) The aim of this project would be to extend existing work on spherically symmetric Ricci flow to prove either global existence or singularity formulation of solutions to the second order RG equations.

Prescribed mean curvature surfaces (P)

Background: Soap films and soap bubbles are examples of surfaces of constant mean curvature. The mathematical equations describing them are non-linear, second order, elliptic partial differential equations. Other examples of prescribed mean curvature are capillary surfaces and also maximal hypersurfaces in relativity. There is extensive literature, covering different mathematical approaches for proving existence, regularity and obtaining information on the shapes of such surfaces.

Possible projects:

- Isoperimetric property of the sphere
- Delaunay surfaces (classification of axially symmetric constant mean curvature surfaces)
- Existence of solutions (either by classical partial differential equations methods, or measure theoretical ones)
- Gradient estimates (classical partial differential methods)
- Construction of minimal surfaces - Weierstrass representation
- Construction of minimal surfaces - geometric heat flow methods
- Construction of maximal hypersurfaces in spacetime – apriori estimates: general relativity.

Discretely self-similar (DSS) singularities (P)

Background: Motivated by the pioneering analysis of the spherically symmetric Einstein equations with massless scalar field matter by Demitri Christodoulou, around 1990 Choptuik discovered numerically that spacetimes at the borderline of forming a black hole, instead developed a naked singularity with a new and totally unexpected structure. One critical issue is to discover whether this DSS singularity is a true (non-removable) singularity or not. Numerical evidence suggests that it is removable, but this evidence is not conclusive and may be numerical artefact.

Possible projects:

- (numerical) solve the resulting 1+1 Einstein-massless scalar field PDE, using Choptuik's adaptive mesh refinement;
- (numerical) solve the 1+1 hyperbolic PDE using Stewart's (1996) double null formulation;
- (numerical) Find the underlying DSS solution to high accuracy, using Gundlach's Fourier expansion;
- (analytic) Prove the existence of the DSS solution using Gundlach's technique;
- (analytic) Review Christodoulou's proof of global existence for the Einstein-massless scalar field equations, and the proof that some initial data can evolve to create a black hole;
- (analytic or numerical) Study similar questions for the case of massive scalar field, with or without electromagnetic charge, or for the Einstein-Yang-Mills equations.

Spacetime energy (P)

The proofs of the Positive Mass Theorem by Schoen and Yau (1979) and Witten (1981) formed a watershed in the application of mathematical techniques to the Einstein Equations. The two proofs are quite different and their inter-relationship is still far from understood. The Schoen-Yau proof uses quasi-linear elliptic PDE, minimal surface theory and differential geometry; the Witten proof uses spinors and existence of solutions to the elliptic Dirac equation.

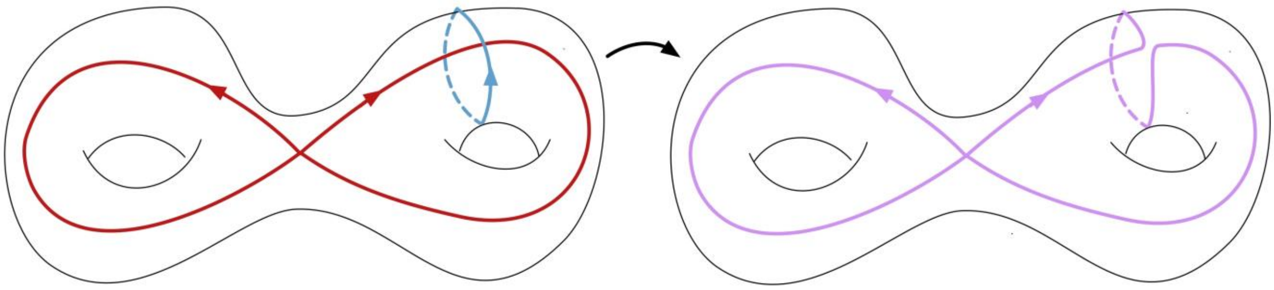
Possible projects:

- Review the Schoen-Yau proof of the PET and the related Positive Mass Theorem, which incorporates the linear momentum of the spacetime;
- Study the use of special foliations (polar coordinates) in partial proofs of the PMT based on monotone functionals, in particular the relation between the Hawking mass and the Geroch inverse mean curvature flow;
- Review properties of spinors and the Witten proof of the PMT.
- (Numerical) Solutions of the initial 3D spatial geometry may be constructed by solving a parabolic equation. This project will construct such solutions numerically to study their geometry.

Dr Tali Pinsky

To The Goldman Bracket (P)

The Goldman bracket is an operation defined over the set of homotopy classes of curves on a two-dimensional surface. The bracket is a simple resolution at intersection points of the curves, as in the figure below. In this project we'll see how this operation gives rise to a Lie algebra structure, and we'll investigate the specific algebras arising in the case of the torus and of the once punctured torus.



Knots in the Lorenz system (P)

The Lorenz flow is probably the most famous chaotic flow, defined by a simple set of ordinary differential equations but demonstrating unpredictable behaviour. The Lorenz butterfly shaped strange attractor can be studied from many different perspectives, but one of them is the unique set of knots that appear as the periodic orbits contained in the attractor. These have been well studied. Out of the first 1.7 million knots, when organized by crossing numbers, only 20 are Lorenz knots, and they have special properties (for example they are all fibered and prime).

It turns out that one can consider different parameters in the Lorenz equations, and then the shape of the attractor changes. Presumably this means that we can get different sets of knots for different parameters in the equations. In this project we will use a bit of numerics to draw the attractors at other parameters, and then use some basic knot theory to try and establish some of the knot properties or even better: Find a non-Lorenz knot in the Lorenz system.

Prerequisites: Some knowledge of topology such as MTH3130 is a must. Some knowledge of Matlab or Python/Julia would be useful but is not required.

Dr Tomasz Popiel

Topics in finite group theory and related areas of discrete mathematics (P)

Group theory is the mathematics of symmetry. Groups can be extremely interesting in their own right, and they also provide a way of understanding various physical/geometric/combinatorial objects. I can offer a range of projects on finite groups and related topics. I am particularly interested in

* finite simple groups, including the famous “Monster” group;
* finite permutation groups;
* group actions on various geometric structures, particularly in the realm of finite algebraic geometry.

I also enjoy problems that have a computational/programming component, which can be tackled using the computer algebra system GAP (gap-system.org).

I would be happy to talk to any interested students and see whether we can come up with a specific project that you would enjoy.

Prerequisites: Algebra 1 is essential, as is a willingness to do some background reading and write clear proofs/explanations. Algebra 2 would be very useful, as would a basic familiarity with programming/coding. A general interest in related topics such as number theory and networks (graphs) would also be good, but it is not necessary to have taken the relevant units.

Prof Jessica Purcell

Alternating shuffle braids (P)

Background: Many braids have an associated hyperbolic volume. I want to find braids with the largest possible volume per crossing number. A simpler sub-problem is to analyse a type of braid called an alternating shuffle braid, which seems to have very large relative volume, and to find bounds on its volume depending on its form. For classes of these braids, a simple pattern seems to be occurring in the geometry. An example is shown in the image below, which is a computer printout of the geometry of one alternating shuffle braid.

Project: Which alternating shuffle braids have largest volume per crossing number? A first step is to analyse geometric structures of alternating shuffle braids, and to prove inductively that tetrahedra and octahedral appear in predicted patterns in the braids. The next step is to find estimates on the shapes of the tetrahedra and octahedra, and use them to bound volumes from above and below.

References:

[1] This project would generalise work in the paper: Champanerkar, Kofman, Purcell, "Volume bounds for weaving knots", Alg. Geom. Topol., Vol. 16 (2016), No. 6, 3301-3323. Also available at https://arxiv.org/abs/1506.04139

More information on geometry and knots can be found in a book I wrote on Hyperbolic Knot Theory:
https://bookstore.ams.org/cdn-1598077516917/gsm-209
Or ArXiv version: https://arxiv.org/abs/2002.12652

Knot invariants: Classical, Geometric, and/or Quantum (P)

Background: In the late 19th century, knots were studied by their diagrams and classified by things like their minimal number of crossings. By the middle of the 20th century, they were often studied by considering their complement: the space obtained by removing the knot from the 3-sphere. In the 1980s and 1990s, several new invariants were found, including geometric, algebraic and quantum invariants. On the geometric side, it was discovered that knot complements often admit a unique hyperbolic metric, and so geometric quantities like volume become knot invariants. On a more algebraic side, new polynomial invariants such as the Jones polynomial were discovered. Even more recently, knot invariants have been discovered arising from quantum topology and homology theories. Many of these modern invariants are difficult to compute for particular knots. It is also often unknown how various invariants relate to each other.

Project: Possible projects include computing knot invariants for new classes of knots, investigating relationships between invariants, such as quantum and geometric.

Prerequisites: Ability to read and write proofs, interest in geometry and topology.

References:

[1] Adams, The Knot Book
[2] Purcell, Hyperbolic Knot Theory. https://bookstore.ams.org/cdn-1598077516917/gsm-209
Or ArXiv version: https://arxiv.org/abs/2002.12652

Note: Other projects on the geometry, topology, and algebra of knots and links are also available under the supervision of Jessica Purcell.

Dr Janosch Rieger

Topics in numerical analysis and optimisation (A and P)

The fields of numerical analysis and nonlinear optimisation study the interplay of the properties of numerical algorithms and the often geometrical properties of the problems they intend to solve. I am particularly interested in algorithms for computing sets and shapes, and in iterative algorithms for solving nonlinear optimisation problems.

In this project, several papers from this context will be reviewed with the intention to improve existing results.

The project requires a strong background in real analysis and linear algebra, as well as the ability to read, understand and write mathematical proofs.

Please contact supervisor by email if you are interested.

Dr Sergio Rojas

Numerical Methods for PDEs: From Finite Elements to Physics-Informed Neural Networks (A)

I have a wide range of topics in the development and mathematical analysis of numerical methods for solving partial differential equations (PDEs). Areas of focus include advanced Finite Element Methods (both conforming and non-conforming) and Neural Network-based approaches such as Physics-Informed Neural Networks (PINNs). A central theme across these topics is the use of Residual Minimisation techniques in discrete dual norms, enabling robust and accurate solutions to complex PDEs. Students will have the opportunity to explore state-of-the-art numerical methods, contribute to ongoing research, and tailor the topic to their interests and background.

If interested, please contact Dr Sergio Rojas at sergio.rojas@monash.edu.

A/Prof Ricardo Ruiz Baier

Analysis of displacement-traction boundary conditions for rotation-based formulations in elasticity (P)

New variational formulations for elasticity equations involving displacement, rotations and pressure have been recently proposed, where the analysis of the continuous problem and the discrete finite element and finite volume element schemes was however restricted to pure displacement boundary conditions. Considering mixed boundary conditions (e.g. displacement-traction) modifies substantially the definition of the weak formulation on which the discretisation methods are based, and in particular makes it difficult to apply classical techniques for saddle-point problems in the analysis, due to the structure and regularity of the problem. The first goal would be to study the well-posedness of the problem under the new set of boundary conditions and to extend the numerical methods accordingly. Next, we intend to use the new formulation in a coupled elasticity-diffusion model for limb morphogenesis and carry out a thorough mathematical and numerical analysis.

The proposed tasks involve PDE analysis (solvability of saddle-point problems, regularity of solutions on the boundary), fundamental topics of numerical analysis (stability and convergence of approximate solutions), and eventually an application in the simulation of the interaction between pattern formation and domain growth.

The main directions of the project can be discussed and modified accordingly to the skills or preferences of the student.

Computational models for the electromechanics of heart and torso (A)

In this project we will advance a model for the coupling of the bidomain equations describing the propagation of electric potential through the heart and interfacing the torso; together with the interaction with the large deformations exhibited by the active contraction of the myocardial muscle in contact with the surrounding structure, represented by an interface finite elasticity problem. Such a model has the potential to contribute in the understanding of the electromechanical properties of both heart and torso, and phenomena of this kind do occur in many medical procedures, including the very common conduction of electrocardiograms.

The problem involves setting of adequate transmission conditions between the heart and the surrounding tissue and assessing the effects of the electromechanical coupling in the outcome of ECGs, implementation of finite element schemes using appropriate open source libraries.

The main directions of the project can be discussed and modified accordingly to the skills or preferences of the student.

Dr Anja Slim

Topics in geological fluid dynamics (A)

Geological fluid dynamics encompasses a broad range of natural flows from mantle dynamics to volcanic ash clouds. Many of these flows occur at relatively low Reynolds numbers and/or have extreme aspect ratios which allow the governing Navier-Stokes equations to be reduced to simpler forms more amenable to analytic solutions. There are many projects that are possible, depending on your strengths and interests. Please contact Anja (anja.slim@monash.edu) to discuss possibilities.

Prerequisites
MTH3360 Fluid dynamics. Or MTH3011 Partial differential equations and an interest in geological modelling

A/Prof Tianhai Tian

Statistical inference of genetic regulatory networks (S)

Background: Investigating the dynamics of genetic regulatory networks through high throughput experimental data, such as microarray gene expression profiles, is a very important research topic in biological sciences. Although a variety of inference methods have been designed over the last ten years, it is still a challenge problem in bioinformatics and computational biology. One of the major hindrances in building detailed mathematical models for genetic regulation is the large number of unknown model parameters.

Project outline: This project commences with the analysis of gene expression data such as microarray data and RNA-seq data. Then you will implement different modelling approaches to reconstruct the regulatory network. The p53 gene network will be used as the test problem. There are two possible projects under this topic:

1. You may study the top-down approach uses probabilistic graphical models to predict the network structure of genetic regulatory networks. You may consider different graphic models such as Gaussian graphic model or Bayesian graphic model.
2. You may study the bottom-up approach using differential equation models to investigate the detailed genetic regulations based on either a fully connected regulatory network or a gene network obtained by the top-down approach.

Reference
[1] Wang J, Wu Q, Hu XT, Tian T. An integrated approach to infer dynamic protein-gene interactions - A case study of the human P53 protein. Methods. 2016 Nov 1;110:3-13.
[2] Wang J, Tian T. Quantitative model for inferring dynamic regulation of the tumor suppressor gene p53. BMC Bioinformatics. 2010 Jan 19;11:36.

Stochastic simulation of biochemical reaction systems (S)

Background: There is a growing body of evidence which suggests the dynamics of biological systems in the cell, especially genetic regulation, is stochastic. One of the major reasons is the small molecular numbers of proteins in the cell such as transcriptional factors and message RNA. Biochemical reaction systems are typically studied using the stochastic simulation algorithm (SSA). Recent progress in computational biology has proposed mathematical models for large-scale complex biological systems. There is a strong need to develop efficient and effective numerical methods for simulating the dynamics of stochastic biological systems.

Project outline: This project commences with the implementation of the SSA for simulating genetic regulatory networks. Then you will implement more efficient methods such as the tau-leap methods and multi-scale simulation methods. There are two possible projects under this topic:

1. You may study the dynamic property of a specific biological network in genetic regulation or cell signalling transduction by using stochastic simulation methods. An interesting question of this project is the function of noise in maintaining bistability property of gene networks.
2. You may study effective simulation techniques, including the implementation of stochastic simulation on high performance computers, to simulating biochemical reaction systems.

References
[1] Gillespie D.T., “Stochastic simulation of chemical kinetics”, Annual Review of Physical Chemistry, 58, 35-55, 2007.
[2] Tian, T. and Burrage, K., “Binomial leap methods for simulating stochastic chemical kinetics”, Journal of Chemical Physics 121, 10356-10364, 2004.
[3] Burrage, K., Tian, T. and Burrage, P.M., “A multi-scaled approach for simulating chemical reaction systems”, Progress in Biophysics and Molecular Biology, 85, 217-234, 2004.

Dr Theodore Vo

Detection of rate-induced tipping events (A)

Rapid environmental changes can cause ecosystems to collapse suddenly. For example, sudden heat waves can cause bleaching of coral reefs, or forests vanish in explosive wildfires. These "tipping events" happen because nature cannot adapt to the speed of climate change. Detecting these critical rates is an urgent problem of key importance to conservation goals in ecosystems. Further compounding the issue is that most natural systems evolve over multiple timescales, which can result in tipping cascades, wherein the current tipping event triggers another one, which can cause irreversible damage to a system. In this project, we will develop geometric methods to detect these tipping events and apply our methods to ecosystems, such as multitrophic food web models.

Pattern formation in the life sciences (A)

Alan Turing is well-known for deciphering the German Enigma code during World War II and laying the foundations of computer science as a new discipline. But toward the end of his short life, he made a lesser-known yet groundbreaking contribution. He showed that a homogeneous system of chemical substances that react with each other and diffuse in space can self-organise into spatially periodic distributions. His work received limited attention until 4 decades later, when the behaviour was experimentally observed, first in chemical systems, then in biological systems, and more recently, in ecological systems. In this project, we will explore the surprising morphologies that arise in arid and semiarid regions, such as parallel vegetation bands observed in northwestern Australia.

Other projects in pattern formation are available, including applications to chemical oscillators, and neuronal communication.

Topics in neuroscience (A)

Binocular rivalry is a visual phenomenon where different images are presented to each eye at the same time, and instead of blending/averaging the images, the brain alternates your perception of which image it actually sees. In this project, we will study mathematical models of binocular rivalry and explore what these perceptual switches tell us about consciousness.

Other projects in neuroscience are available. Please discuss with supervisor.

Topics in cardiology (A)

Early afterdepolarisations (EADs) are diseased rhythms that are seen in heart muscle cells and are correlated with potentially fatal arrhythmias. In network of heart muscle cells, it has been observed that if one cell has a diseased rhythm, then it can pass on its diseased state to nearby cells. In this project, we will study small networks of heart muscle cells and explore the resilience of the network to the propagation of disease states, what drug interventions need to be induced to recover from the disease states, and the underlying mechanisms that might trigger these disease states to begin with.

Dr Thuan Vu Vu Ho

Projects in differential geometry, fluid mechanics & internal wave modelling (A)

Background: Classical mechanics has provided us with the structure of Galilean space-time and axioms in mechanics, and Newton's equation to describe the motion of a point particle in a 3-dimensional Euclidean space. Lagrangian mechanics considers this motion in a generalized coordinate system (with or without constraints), in which motion is described in a finite (or infinite) dimensional manifold. To this end, we need mathematical notions from differential geometry, Lie group theory, calculus of variation, etc. In this setting, the equation of motion is described by the Euler-Lagrange equation. Instead of taking the Lagrangian mechanics point of view, the Hamiltonian approach to studying dynamics can provide some benefits. By deriving the Hamiltonian structure of a dynamical system, we can then study the associated conserved quantities, as well as the stability of the system and related integrability issues. This Hamiltonian approach can be applied to the study of internal waves in stratified fluids, previously proposed by T.B.Benjamin.

If you have interests in differential geometry, fluid mechanics, internal wave modelling, feel free to discuss possible projects with me, thuanvu.vuho@monash.edu

Possible project topics:

1. Lagrangian and Hamiltonian dynamics: a differential geometric approach
2. Internal wave models: asymptotic approach

References:

[1] Darryl D. Holm, Tanya Schmah, and Cristina Stoica, *Geometric Mechanics and Symmetry. From Finite to Infinite Dimensions*

[2] Camassa, R., Falqui, G., Ortenzi, G. *et al. Hamiltonian Aspects of Three-Layer Stratified Fluids.* https://doi.org/10.1007/s00332-021-09726-0

[3] Camassa, R., Falqui, G., Ortenzi, G. *et al. Simple two-layer dispersive models in the Hamiltonian reduction formalism.* DOI: 10.1088/1361-6544/ace3a0

Prof Ian Wanless

A permutation game (P)

Let n be a positive integer and let S denote the set of all permutations of the numbers 1, 2, 3, ..., n. So if n=3 then S={123,132,213,231,312,321}. Consider the following game played with these permutations:

The game has three stages, and each person sees the other person's choices:
1) I choose a subset T of S.
2) You choose a permutation p in S.
3) I choose a permutation q in T.

The score is then calculated as the number of positions in which p and q agree. For example, if p=132 and q=231 then the score is 1 since p and q agree in the second position.
My aim is to score as highly as possible and your aim is to keep the score as low as possible.

Question: Suppose I want to guarantee a score of at least s. How small can T be?

This question has been answered only in very simply cases, such as s=1 or s=n-1. Even solving the s=2 case might answer some important problems in discrete mathematics. However, there have been several interesting papers written on this problem and its connections to other areas of mathematics. These papers would be the subject of the reading project.

Latin squares (P)

Latin squares are a two-dimensional analogue of permutations. A Latin square of order n is an n by n matrix in which each of n symbols occurs once in each row and in each column. These days you'll find Latin squares on the puzzle page of every major newspaper as well as in the schedule for sports tournaments and statistical experiments. In pure mathematics, Latin squares are fundamental objects. Every finite group is defined by its Cayley table ("multiplication" table), which is a Latin square. Similarly, in finite geometry projective planes are defined by certain sets of Latin squares.

Project Outline: There are a range of possible projects goals available, exploring the structure and uses of Latin squares. All would be combinatorial in nature but depending on your interests they could also have an algebraic, algorithmic, probabilistic, geometric or enumerative flavour. Drop in to discuss the options if you are interested.

References:
[1] Laywine, C.F. and Mullen, G.L. “Discrete mathematics using Latin squares”, Wiley, New York, 1998.
[2] Recorded Lectures (especially lectures 1,3,5,6,7) at: http://qtss.amsi.org.au/SummerSchool2004gra.html also the latter part of the mathematical/historical/comedy piece: https://www.youtube.com/watch?v=gONv139Jd2Y

Graph factorisations (P)

Background: Suppose n points are placed in general position (no 4 are co-planar). Between each pair of points, colour the line joining those points subject to the conditions:
(i) Only n-1 colours are available.
(ii) No two lines meeting at a point may have the same colour.
(iii) For every choice of two points and two colours it must be possible to travel between the chosen points along lines of the chosen colours.

In graph theory terminology you have found a perfect 1-factorisation of the complete graph Kn.

For a project in this area you would study what is known (and what is not known) about perfect factorisations. This could include some group theory if you want to study their symmetries, or some computational work, looking for perfect factorisations (but it doesn’t have to involve either).

References:
[1] Wallis, W.D., “One-factorizations”, Math. Appl. 390, Kluwer Academic, Dordrecht, 1997.

Prof David Wood

Topics in structural graph theory (P)

Abstract: Graph theory is the mathematics of networks. Structural graph theory looks at topics like graph colouring, planarity, graphs on surfaces, graph minors, and tree width. In this project, several papers on a specific area of research will be reviewed with the goal of improving on the existing results. Come and talk to me about your specific interests.

Skills required**:** A love of mathematics, and the ability to read, understand and write mathematical proofs.

Units Required: MTH3170 Network Mathematics (preferably MTH3175) and the honours Combinatorics or Advanced Graph Theory unit.

Prof Nick Wormald

Random graphs (P)

Background: A graph or network is a set of points called 'vertices' and links between the nodes called 'edges'. Random graphs involve some random choices in determining the edges. Random graph theory was first used last century to show the existence of graphs with special hard-to-construct properties. Since then, it has been used to answer many questions arising in computer science, as well as generating questions of intrinsic interest. These questions usually ask for the properties of a large random graph. Sometimes the problem is to make a random graph in some weird way so that it is likely to have some desirable property, such as with graphs called random "lifts".

Project outline: There are many possible projects. Here is one. The game of "cops and robbers" is played on a graph. The cops are moved by one player and the robbers by another. First the cops are placed on the graph, then the robber. Then each cop can move to an adjacent vertex, then the robber can move similarly, and so on. For a given graph, how many cops are required to ensure being able to catch the robber eventually? An unsolved problem (Meyniel's conjecture) is to show that some constant times the square root of the number of vertices is always sufficient. The best-known bounds are far from this. We can ask whether random graphs satisfy the conjecture.

Reference
[1] Janson, Luczak and Rucinski, "Random Graphs".

Modelling the World Wide Web (P)

A modern topic at the interface of mathematics with computer science and physics is task of making simple theoretical models of "naturally" occurring network structures. Even with simple-sounding rules for their evolution, randomly evolving networks can have some surprising properties. One of the well-known examples of these regards the degrees of the nodes, i.e. the numbers of links they have to other nodes. These often follow a somewhat mysterious "power law". Another property was popularised under "six degrees of separation": there is usually a "short" path between any two nodes. The aim of the project is to look at these questions from the pure mathematical side, investigating the rigorous proofs of such properties. Such questions are solved for some kinds of networks but unsolved for many others.

Skills required: Some familiarity with probability is highly desirable.

Counting using complex numbers (P)

Background: If you want to count something, like the ways to insert balls into boxes under certain restrictions, or the ways to walk from A to B on some kind of grid in a given number of steps, then power series can help. The number of things is a coefficient in the power series.

Sometimes the coefficient is horribly complicated and a much simpler approximate answer exists. Important methods of obtaining such answers use complex analysis, including complex integration.

The project: After learning about the methods of counting, there are a number of problems that can be addressed. Suppose you are allowed to walk to the right along the integer line taking steps of a restricted set of lengths. In how many ways can you reach distance n? What if you are restricted so that you must "touch base" on each multiple of k along the way? The same techniques can answer not just counting questions, but questions about random walks, where each step length has a certain probability.

Prerequisite: Knowledge of complex analysis and power series is required.

References
[1] H. Wilf, "Generating functionology"
[2] P Flajolet and R Sedgewick, "Analytic Combinatorics"

How fast do rumours spread? (P)

Background: In the beginning, somebody starts a rumour by telling a friend. Each day, everybody who knows the rumour calls one of their friends at random and tells them the rumour. How long do we expect it to take before everybody knows the rumour? This simple question has implications for distributed computing.

Project outline: The answer to the rumour question depends on the structure of the network of friends. We can ask which network structures of n friends are likely to take longest for the rumour to spread. We can ask if the rules of spreading are changed, how does it affect the spread time. In considering these questions you will learn about some simple probability processes, some graph theory, and possibly something about random graphs.

Skills required: some familiarity with probability is highly desirable.

▪