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**Consumer Demand, Consumption, and Asset Pricing:
An Integrated Analsis**

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CONSUMER DEMAND, CONSUMPTION, AND ASSET PRICING:
AN INTEGRATED ANALYSIS

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Abstract

This paper integrates seemingly disjoint studies on consumer behavior in micro and macro analyses via the intertemporal two-stage budgeting procedure with durable goods and liquidity constraints. The model accounts for the influences of nondurables consumption, commodity prices, and durables stock on commodity demands as well as on risk aversion and asset returns. The demand functions for six nondurable goods are jointly estimated with the Euler equations for bonds, shares, and durables goods, with allowance for liquidity constraints. The integrated model proves useful with new findings for risk aversion and, particularly, an extended consumption CAPM with multiple goods and liquidity constraints.

Key Words: Intertemporal two-stage budgeting; Indirect utility function; Risk aversion; The stochastic discount factor; Consumption-based CAPM

JEL Numbers: D12, E21, G12

I. INTRODUCTION

Traditionally, studies on consumer behavior are conducted at two separate levels in micro and macro analyses. Micro analysis of consumer behavior is concerned with optimal allocation of the consumer's given expenditure to different goods by estimating demand systems conditional on the level of total expenditure (see Deaton and Meullbauer, 1980, for an extensive discussion). In macro analysis, in contrast, research on consumer behavior involves allocation of the consumer's wealth across periods by estimating the life-cycle or intertemporal pattern of consumption, usually by utilizing the Euler equation of the consumer's intertemporal optimization problem (Hall, 1978; see Deaton, 1992, for an excellent survey). Studies on asset pricing also employ this framework to analyze issues such as the equity premium and risk-free rate puzzles (see Cochrane, 2005, for a survey).

While the traditional approach based on separate research of consumer demand and consumption has greatly increased our understanding of consumer behavior, there are inherent shortcomings underlying this approach. Studies on consumer demand are essentially a static analysis and do not contribute to an understanding of the intertemporal behavior of consumers. In these studies, total expenditure is taken to be exogenous. However, to the extent that the consumer chooses the consumption levels to optimally allocate wealth across periods, consumption expenditure is endogenously determined in the consumer's intertemporal optimization problem. Studies on consumption and asset pricing, on the other hand, are highly aggregated and fail to provide disaggregate information about the components of consumption. These studies rest crucially on the tacit assumption that the consumer's within-period preferences are homothetic in order to define real consumption.[1] Homothetic preferences are, however, shown to be restrictive to characterize consumer behavior (see Deaton and Meullbauer, 1980). Moreover, durable goods are inadequately treated in both consumer demand and consumption or asset pricing studies. In consumer demand studies, they are usually handled like nondurable goods[2] without recognizing the inherent differences between the two classes of goods (see, e.g., Lewbel, 1991). In consumption and asset pricing studies, expenditure on nondurable goods is, by and large, used as a measure of consumption, implicitly assuming that durable goods are separable from nondurable goods (see Hall, 1978; Hansen and Singleton, 1983). However, the

validity of the separabilty assumption remains questionable. There are studies that incorporate both nondurable and durable goods using restrictive within-period utility functions such as CES (see Ogaki and Reinhart, 1998; Yogo, 2006), but, unlike nondurable goods, purchase of durable goods is much like a physical investment decision subject to costs of adjustment.[3]

In an attempt to redress the shortcomings of the traditional research on consumer behavior, this study develops and estimates a new integrated model for jointly analyzing consumer demand, consumption, and asset pricing under nonhomothetic within-period preferences by way of the intertemporal two-stage budgeting procedure (Blundell, Browning and Meagher, 1994; Kim, McLaren, and Wong, 2013) by allowing for durable goods and liquidity constraints. While the intertemporal two-stage budgeting procedure validates the traditional approach to consumer behavior under homothetic within-period preferences, it also provides a unifying framework to integrate three strands of studies on consumer demand, intertemporal consumption, and asset pricing under nonhomothetic within-period preferences. In this procedure, we first specify the familiar indirect utility function (see Deaton and Meullbauer, 1980) as a function of nondurables consumption and the prices of nondurable goods conditioned on the stock of durable goods, and obtain the demand or budget share equations for nondurable goods. Then by maximizing the expected lifetime indirect utility function with respect to bonds (risk-free asset), shares (risky assets), and durables stock, subject to the consumer's finance or budget constraint with costs of adjustment and allowing for liquidity constraints, we derive the Euler equations for bonds, shares, and durable goods. Optimal choice of bonds and shares and of durable goods determines the level for nondurables consumption and durables stock specified in the indirect utility function. Estimation consists of the system of the budget share equations for nondurable goods and the three Euler equations using the Generalized Method of Moments (GMM) with a careful implementation of the orthogonality conditions. These results suggest that while there is a separation of the consumption and asset decision from the consumer demand decision, these decisions are jointly – and inextricably – connected in the consumer's optimization problem; hence they cannot be studied in isolation of each other. This has not received deserved attention in previous studies on consumer behavior.[4]

The proposed integrated model coherently unifies apparently disjoint studies on consumer behavior in micro and macro analyses in a multicommodity framework, employing the

intertemporal two-stage budgeting procedure. The salient feature of the model is to take into account the influences of nondurables consumption, commodity (nondurables) prices, and durables stock on commodity demands as well as on risk aversion and asset returns, which enables us to appropriately address many relevant issues in these areas in a unifying framework with more reliable results. In studies on consumer demand, Marshallian price elasticities are typically used as a measure of the effect of changes in prices on commodity demands. In an intertemporal environment, however, the pertinent price elasticities are Frisch price elasticities conditioned on the marginal utility of consumption, which is not identified in traditional demand analysis; it is identified with estimation of the Euler equations in conjunction with the commodity demand functions. The Marshallian price elasticities identify effects of permanent changes in prices, while the Frisch price elasticities measure responses of commodity demands to temporary changes in prices.

Risk aversion is believed to determine the behaviour of asset prices. It is traditionally estimated with a tightly parameterized power utility function defined in terms of consumption (see, e.g., Hansen and Singleton, 1983). In this measure, consumption is the driving process for the marginal utility of consumption, and the degree of risk aversion is constant over time. In contrast, the integrated model allows the degree of risk aversion to vary with consumption as well as commodity prices and durable goods; hence it allows us to have a more reliable estimate of risk aversion than the traditional measure based on power utility.

The stochastic discount factor (SDF) plays an integral role in modern analysis of asset pricing (see Cochrane, 2005, for an extensive discussion). A core issue in asset pricing is to understand the underlying economic forces that determine the SDF and hence asset returns. In the well-known consumption-based capital asset pricing model (CCAPM) with power utility, consumption growth determines the SDF and is the fundamental factor driving asset returns. There have been attempts to improve the empirical performance of the CCAPM with relatively good success (see Cochrane, 2005, for a survey). Our integrated model accords a new perspective to this endeavour in a multicommodity framework with allowance for liquidity constraints. Using estimated parameters from the integrated model, the SDF, which measures the discounted growth of the marginal utility of nondurables consumption, is approximated by a time-varying linear function of nondurables consumption growth, discounted commodity

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(nondurables) price growth, and durables stock growth, suggesting that there are three risk factors that are fundamental to explaining equity returns. Identification of commodity price growth and durables stock growth as risk factors has not received due regard in previous studies, but evidence clearly attests to the relevance of these factors in explaining equity returns. In particular, recent experience suggests that higher energy prices and the housing crisis can influence the stock market and hence equity returns. Also, to the extent that spending on durable goods is affected by business cycle fluctuations, durables stock growth or the durables expenditure-stock ratio is expected to have a good explanatory power for the countercyclical variation in equity returns.

The linearized SDF equation gives rise to an extended, consumption-based *multifactor* pricing model with several risk factors – nondurables consumption growth, commodity price growth, and durables stock growth – where betas and risk prices vary over time with conditional variances and covariances. With low and relatively constant interest rates as well as small factor volatilities, the risk prices of the factors are a time-varying function of relative risk aversion (in the case of consumption growth), the liquidity constraint, and the marginal utility of consumption or income, producing time-varying equity returns. In traditional consumption-based models with power utility where consumption growth is the only risk factor, in contrast, the risk price is solely determined by risk aversion and consumption variance. Since disposable income measures the degree of liquidity constraints, this suggests that disposable income growth can have an important influence on equity returns. Altogether, commodity price growth, durables stock growth, and disposable income growth represent *business cycle* variables that account for the predictability of equity returns.[5] These variables are identified theoretically in this study, rather than being chosen empirically on the basis of goodness of fit, from the directives of the consumer's optimizing problem prescribed by the intertemporal two-stage budgeting procedure. The multifactor model allows us to estimate the beta and risk price of each risk factor that can be used to determine its contribution to explain equity returns. With the time-varying risk prices from many sources and additional state variables, the proposed multifactor model is expected to do a better job in explaining the magnitude of the equity premium than the traditional single-factor consumption-based models.

In the intertemporal two-stage budgeting procedure, the specification of a functional form for the indirect utility function is essential for empirical analysis. We employ a flexible specification of the indirect utility function that places minimal restrictions on the consumer's within-period preferences with respect to homotheticity and separability of durable goods from nondurable goods. This form also satisfies within-period and intertemporal regularity conditions. We jointly estimate the system of budget share equations for six nondurable goods and the three Euler equations for bonds, shares, and durable goods using GMM with quarterly U.S. data. We provide new results for estimating, in a unifying framework, demand elasticities, risk aversion, and the extended, *multifactor* consumption-based asset pricing model to re-examine the well-known asset pricing puzzles.

II AN INTEGRATED MODEL OF CONSUMER DEMAND, INTERTEMPORAL CONSUMPTION, AND ASSET PRICING, WITH DURABLE GOODS AND LIQUIDITY CONSTRAINTS

A. A Theoretical Framework: Intertemporal Two-Stage Budgeting

We deploy the intertemporal two-stage budgeting procedure (Kim, 1993; Blundell, Browning and Meghir 1994; Kim, McLaren, and Wong, 2013) as the frame of analysis by incorporating durable goods with allowance for liquidity constraints. In this procedure, the consumer chooses the level of nondurables consumption and durable goods by optimally allocating wealth across periods at the first stage. Then at the second stage each period's optimal allocation of consumption expenditure is distributed across nondurable goods conditioned on the stock of durable goods. The consumer, however, faces liquidity constraints because of limited opportunities to borrow against future labor income to finance current consumption expenditure. Thus there is a separation of the consumption and asset decision provided at the first stage and the consumer demand decision described at the second stage. Nevertheless, the intertemporal two-stage budgeting procedure allows for a joint link between these decisions in the consumer's optimization problem.

The consumer's second stage optimization problem is summarized by the within-period indirect utility function conditioned on the stock of durable goods, $v(C_t, \mathbf{p}_t, k_t)$, defined by

$$v(C_t, \mathbf{p}_t, k_t) = \max_{\mathbf{q}_t}\{u(\mathbf{q}_t, k_t) | \mathbf{p}_t \cdot \mathbf{q}_t \le C_t\}, \tag{1}$$

where C_t is consumption expenditure to be allocated among nondurable goods at period t, $\mathbf{p}_t$ is a price vector of nondurable goods whose elements are $p_{it}(i=1,...,N)$, the price of the ith nondurable good at period t, $\mathbf{q}_t$ is a quantity vector of nondurable goods whose elements are $q_{it}(i=1,...,N)$, the quantity of the ith nondurable good at period t, and k_t is the stock of durable goods at the beginning of period t. The indirect utility function (1) inherits the properties of the direct utility function $u(\mathbf{q}_t,k_t)$ under maximization of direct utility subject to the expenditure constraint with k_t taken as a conditioning variable, and possesses the following regularity conditions: it is continuous, increasing in C_t and k_t, decreasing in $\mathbf{p}_t$, homogeneous of degree zero in C_t and $\mathbf{p}_t$, and quasi-convex in $\mathbf{p}_t$. These conditions are well known in demand analysis in the absence of the conditioning variable k_t (see Deaton and Muellbauer, 1980). Applying Roy's identity to (1), we obtain the demand functions for nondurable goods:

$$q_{it}(C_t,\mathbf{p}_t,k_t) = -\frac{\partial v(C_t,\mathbf{p}_t,k_t)/\partial p_{it}}{\partial v(C_t,\mathbf{p}_t,k_t)/\partial C_t}, \; i=1,...,N. \tag{2}$$

At the first stage of the intertemporal two-stage budgeting procedure, the consumer enters a period holding bonds and shares purchased the period before with the given stock of durable goods. Bonds are taken to be a risk-free asset of duration one period, while shares or equities are risky assets. The consumer receives interest and dividends on those assets, may earn capital gains on shares, earns labor income,[6] and spends for nondurable goods and purchases of durable goods during the period. Savings are divided between increases in bonds and in the value of shares to be carried into the next period.

The stock of durable goods evolves according to

$$k_{s+1} = (1-\delta)k_s + q_s^k, s=t,t+1,...,t+T, \tag{3}$$

where q_s^k is the quantity of durables goods purchased at period s and δ is the depreciation rate assumed to be constant. There are costs of adjustment associated with changing the stock of durable goods, which is specified by $h(q_s^k,k_s)$ that is linear homogeneous in q_s^k and k_s, increasing in q_s^k but decreasing in k_s, and convex in q_s^k and k_s (see Hubbard, 1998). With this adjustment cost function,[7] the consumer's intertemporal finance or budget constraint is

$$B_{s+1} + p_s^e n_{s+1} = (1+r_s^b) B_s + (p_s^e + d_s) n_s + Y_s - C_s - p_s^k q_s^k - h(q_s^k, k_s), s = t, t+1, ..., t+T, \qquad (4)$$

where B_s is the value of bond holding at the beginning of period s carried over from the previous

period, n_s is the number of shares held at the beginning of period s, p_s^e is the price of an equity share

at period s, r_s^b is the interest rate on bonds at period s, d_s is dividends per share at period s, and Y_s is

labor income at period s. If the consumer faces a borrowing or liquidity constraint, debt cannot

exceed the total current value of traded bonds and shares. The liquidity constraint is specified by

$$B_{s+1} + p_s^e n_{s+1} \geq -L_s, s = t, t+1, ..., t+T, \qquad (5)$$

where L_s is the limit on net indebtedness at time s with $L_s \geq 0, s = t, t+1, ..., t+T$ (see Zeldes,

1989). If $L_s = 0,$ the consumer cannot borrow or incur debt at all, but he can save and earn

interest from his assets.

The direct and hence indirect utility function in (1) is ordinal; hence while the intratemporal

allocation of consumption across goods as captured by (2) is invariant to a monotonic

transformation of a utility function (1), intertemporal allocation of consumption is not. To

redress this problem, for the first-stage optimization problem of the intertemporal two-stage

budgeting procedure, we take the following lifetime or intertemporal utility function:

$$U = \sum_{s=t}^{t+T} \frac{(v(C_s, \mathbf{p}_s, k_s)^{1-\zeta} - 1)/(1-\zeta)}{(1+\rho)^{s-t}}, \qquad (6)$$

where ρ is the constant rate of the consumer's time preference and ζ is a Box-Cox parameter

that produces a transformation of the indirect utility function (1). The Box-Cox transformation

using parameter ζ allows the indirect utility function to be cardinal. It also allows for an

additional degree of flexibility in measuring the intertemporal properties of the indirect utility

function (1).

While the indirect utility function (1) has the usual regularity conditions with respect to $\mathbf{p}_t$,

the intertemporal utility function (6) has additional properties with respect to C_t and k_t : it is

continuous, increasing in C_t and k_t, but it is also concave in C_t and k_t. To examine these

properties, define the marginal utility of nondurables consumption C_s at time s (μ_s), $s = t, t+1,$

…, $t+T$, as

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$$\mu_s \equiv \frac{\partial U}{\partial C_s} = \frac{v(C_t,\mathbf{p}_t,k_t)^{-\zeta}}{(1+\rho)^{s-t}}\frac{\partial v(C_s,\mathbf{p}_s,k_s)}{\partial C_s} \text{ for all } s \geq t \tag{7}$$

so that

$$\frac{\partial \mu_s}{\partial C_s} \equiv \frac{\partial^2 U}{\partial C_s^2} = \frac{v(C_s,\mathbf{p}_s,k_s)^{-\zeta}}{(1+\rho)^{s-t}}\frac{\partial^2 v(C_s,\mathbf{p}_s,k_s)}{\partial C_s^2} - \zeta\frac{v(C_s,\mathbf{p}_s,k_s)^{-(\zeta+1)}}{(1+\rho)^{s-t}}\left(\frac{\partial v(C_s,\mathbf{p}_s,k_s)}{\partial C_s}\right)^2 \text{ for all } s \geq t. \tag{8}$$

From (7), the intertemporal utility function (6) is increasing in C_s, i.e., $\mu_t > 0$, if the within-period indirect utility function (1) is increasing in C_s, i.e., $\partial v(C_s,\mathbf{p}_s,k_s)/\partial C_s > 0$. From (8), the concavity of the intertemporal utility function (6) with respect to C_s implies that $\partial \mu_s / \partial C_s < 0$. For $\zeta > 0$, the concavity of the indirect utility function (1), guarantees the concavity of the intertemporal utility function (6). If $\zeta < 0$, however, even if the indirect utility function (1) is concave, the intertemporal utility function (6) can be convex so that $\partial \mu_s / \partial C_s > 0$. Similarly, the intertemporal utility function (6) is increasing in k_s if the within-period indirect utility function (1) is increasing in k_s. Also, the intertemporal utility function (6) is concave in k_s if the indirect utility function (1) is concave in k_s only with $\zeta > 0$. From this discussion, it is evident that the parameter ζ is essential in determining the marginal utility of consumption and durable goods and its curvature properties.

It is important to note that the indirect utility function (1) is, in general, nonhomothetic and places no restrictions on the structure of the consumer's within-period preferences. Intertemporal preferences described in (6) are, in general, nonhomothetic as well.[8] Suppose that there are no durable goods or consumption is taken to be expenditure on both durable and nondurable goods. If within-period preferences are homothetic, the indirect utility function is rank one (see Lewbel, 1991) and hence contains only one function of prices,[9] say $P(\mathbf{p}_t)$, and by homogeneity of degree zero, the function must be of the form $C_t / P(\mathbf{p}_t)$, which can be used to naturally define real consumption c_t as $C_t / P(\mathbf{p}_t)$ at each period (see Deaton and Muellbauer, 1980). If, in addition, we write the indirect utility function as $v(C_t,\mathbf{p}_t) = c_t$, then utility takes the form of a power or isoelastic function, $u(c_t) = (c_t^{1-\zeta} - 1)/(1-\zeta)$ or, more conveniently, $u(c_t) = c_t^{1-\zeta}/(1-\zeta)$ with $u'(c_t) = c_t^{-\zeta}$, giving us the familiar, conventional intertemporal utility function:

$$U = \sum_{s=t}^{t+T} \frac{1}{(1+\rho)^{s-t}} \frac{1}{1-\zeta} c_s^{1-\zeta}, \tag{9}$$

where ζ is known as the degree of relative risk aversion or its reciprocal as the intertemporal elasticity of substitution (see section III for a detailed discussion).[10] Moreover, with the existence of a price index under homothetic within-period preferences, we can re-express the nominal variables in the intertemporal budget constraint (4) in terms of real variables by dividing them by the price index. In the absence of durable goods with adjustment costs, this equation, together with the time-honored power utility function in (9), provides the traditional framework employed in macroeconomics and finance to address many issues in consumption and saving as well as asset pricing (Hall, 1978; Hansen and Singleton, 1983; Ljungqvist and Sargent, 2004).

Now, given the transformation of the indirect utility function in (6), the consumer's first stage optimization problem is

$$\max_{\{B_{s+1}, n_{s+1}, k_{s+1}\}_{s=t}^{t+T}} E_t \left[\sum_{s=t}^{t+T} \frac{(v(C_s, \mathbf{p}_s, k_s)^{1-\zeta} - 1)/(1-\zeta)}{(1+\rho)^{s-t}} \right], \tag{10}$$

where E_t denotes the expectation operator conditional on information available at time t, subject to the durables stock accumulation equation (3), the intertemporal budget constraint with adjustment costs (4), and the liquidity constraint (5), with the appropriate initial and terminal conditions on bonds, shares, and durables stock satisfied. With (7), the first-order conditions for the above problem at the initial point in time ($s = t$) are

$$B_{t+1}: \mu_t = E_t \left[(1+r_{t+1}^b)\mu_{t+1} \right] + \phi_t \tag{11}$$

$$n_{t+1}: p_t^e \mu_t = E_t \left[(d_{t+1} + p_{t+1}^e)\mu_{t+1} \right] + p_t^e \phi_t, \tag{12}$$

and

$$k_{t+1}: \left(p_t^k + \frac{\partial h(q_t^k, k_t)}{\partial q_t^k} \right)\mu_t = E_t \left[\begin{array}{l} \left\{ (1-\delta)\left(p_{t+1}^k + \frac{\partial h(q_{t+1}^k, k_{t+1})}{\partial q_{t+1}^k} \right) - \frac{\partial h(q_{t+1}^k, k_{t+1})}{\partial k_{t+1}} \right\}\mu_{t+1} \\ + \frac{v(C_{t+1}, \mathbf{p}_{t+1}, k_{t+1})^{-\zeta}}{1+\rho} \frac{\partial v(C_{t+1}, \mathbf{p}_{t+1}, k_{t+1})}{\partial k_{t+1}} \end{array} \right], \tag{13}$$

where ϕ_t is the Lagrange multiplier associated with the liquidity constraint (5) known at time t. ϕ_t will be positive when the liquidity constraint is binding and zero when it is not. When the

liquidity constraint is not binding, i.e., $\phi_t = 0$, equations (11) and (12) give the familiar Euler equations for bonds and shares. By optimally determining bonds, shares, and spending on durables over time, the consumer also chooses the level of nondurables consumption each period. From (13), it is of interest to note that purchase of durable goods is much like the decision to buy bonds and shares but with adjustment costs.

Equations (11) and (12) can be used to examine issues in asset pricing by expressing them in ratio form:

$$E_t \left[(1 + r_{t+1}^b) M_{t+1} \right] + \hat{\phi}_t = 1 \tag{14}$$

and

$$E_t \left[(1 + r_{t+1}^e) M_{t+1} \right] + \hat{\phi}_t = 1, \tag{15}$$

where $M_{t+1} \equiv \mu_{t+1} / \mu_t$, the intertemporal marginal rate of substitution, often referred to as the stochastic discount factor (SDF), $1 + r_t^e \equiv (d_{t+1} + p_{t+1}^e) / p_t^e$, the gross rate of return on equities, and $\hat{\phi}_t \equiv \phi_t / \mu_t$. As with ϕ_t, $\hat{\phi}_t$ will be positive when the liquidity constraint is binding and zero when it is not binding since $\mu_t > 0$. We can subtract (14) from (15) to obtain the central asset pricing equation (see Cochrane, 2005; Campbell, Lo, and MacKinley, 1997):

$$E_t \left[(r_{t+1}^e - r_{t+1}^b) M_{t+1} \right] = 0. \tag{16}$$

Manipulating the expectation in (16), we obtain an expression for the equity risk premium:

$$E_t r_{t+1}^e - r_{t+1}^b = -\text{Cov}_t(r_{t+1}^e, M_{t+1}) / E_t \left[M_{t+1} \right], \tag{17}$$

where Cov_t is covariance conditional on information available at time t. From this equation, we derive the Sharpe ratio as (Hansen and Jagannathan, 1991)

$$\frac{E_t[r_{t+1}^e] - r_{t+1}^b}{\sigma_t(r_{t+1}^e)} = -\rho_t(r_{t+1}^e, M_{t+1}) \frac{\sigma_t(M_{t+1})}{E_t[M_{t+1}]} \leq \frac{\sigma_t(M_{t+1})}{E_t[M_{t+1}]}, \tag{18}$$

where $\rho_t(r_{t+1}^e, M_{t+1})$ is the correlation coefficient between r_{t+1}^e and M_{t+1}, and $\sigma_t(M_{t+1})$ and $\sigma_t(r_{t+1}^e)$ are standard deviations of M_{t+1} and r_{t+1}^e. Since $\left| \rho_t(r_{t+1}^e, M_{t+1}) \right| < 1$, equation (18) shows that the Sharpe ratio for an equity places a lower bound on the volatility of the SDF given by its standard deviation divided by the conditional mean.

All of these results hold with nonhomothetic within-period preferences. Previous studies on consumption and asset pricing, however, in large part, rely crucially on homothetic preferences with no durable goods, typically employing a power utility function (see Hall, 1978; Hansen and Singleton, 1980). With power utility as in (9), a first-order approximation of the SDF gives the Hansen and Jagannathan's (1991) volatility bound as[11]

$$\frac{E_t[r_{t+1}^e] - r_{t+1}^b}{\sigma_t(r_{t+1}^e)} \leq \frac{\sigma_t[M_{t+1}]}{E_t[M_{t+1}]} = \zeta(1+\rho)\sigma_t(\Delta \ln c_{t+1}), \tag{19}$$

where Δ is the first difference operator, $\Delta \ln c_{t+1} = \ln c_{t+1} - \ln c_t$, real consumption growth between periods t and $t+1$, and $\sigma_t(\Delta \ln c_{t+1})$ is standard deviation of real consumption growth. From (19), we see that it is consumption growth, along with ρ and ζ, that plays a pivotal role in determining the equity premium in previous studies with power utility. Nonhomothetic preferences with durable goods, however, allow for the influences of consumption as well as commodity (nondurables) prices and durables stock on commodity demands as well as on risk aversion and asset returns, giving additional flexibility in explaining the issues in consumption and asset pricing. We will explore this issue further in the next section.

B. An Empirical Model: Specification of the Indirect Utility and Adjustments Cost Functions and of the Liquidity Constraint

The foregoing discussion indicates that the indirect utility function (1) ties consumer demands, intertemporal consumption, and asset pricing together in a coherent way. For empirical analysis, the specification of an appropriate functional form for the indirect utility function is essential in order to obtain reasonable results. Admittedly, it is a challenge to find such a form for the indirect utility function that preserves the regularity conditions for within-period and intertemporal preferences, with allowance for durable goods. While existing functional forms are useful for demand analysis, their relevance may be limited for intertemporal analysis where the property of concavity in consumption is critical (see Deaton and Meullbauer, 1980). Kim, McLaren, and Wong (2013) find that a Modified PIGLOG (MPIGLOG) form (Cooper and McLaren, 1992) is useful in intertemporal analysis, but their analysis is limited to address many issues examined in this paper because it focuses on nondurables consumption with bonds, with no regard to durable goods and equities. We have adopted Kim, McLaren, and Wong's (2013)

MPIGLOG form in this study to characterize the indirect utility function (1) with allowance for durable goods. Our choice for this form is motivated by a number of considerations: the simplicity of the functional structure to capture nonhomothetic preferences and nonseparability of nondurable and durable goods, the ease of imposing and maintaining the within-period and intertemporal regularity conditions, and the fact that the number of parameters will not increase rapidly with the number of commodities under consideration.[12]

With MPIGLOG, the indirect utility function (1) is of the following form:

$$v(C_t, \mathbf{p}_t, k_t) = \ln\left[\frac{C_t k_t^{\omega}}{P_A(\mathbf{p}_t)}\right]\left(\frac{C_t^{\eta}}{P_B(\mathbf{p}_t)}\right), \tag{20}$$

where ω and η are parameters, and $P_A(\mathbf{p}_t)$ and $P_B(\mathbf{p}_t)$ are price indexes that are positive, increasing, homogeneous of degree one and η in $\mathbf{p}_t$ respectively, and concave in prices $\mathbf{p}_t$. The MPIGLOG indirect utility function (20) is a composite function of the product of two subutility functions, $v_A \equiv \ln[C_t k_t^{\omega} / P_A(\mathbf{p}_t)]$ and $v_B \equiv C_t^{\eta} / P_B(\mathbf{p}_t)$, each having strong regularity conditions of an indirect utility function. We assume that the price indexes take the forms:

$$P_A(\mathbf{p}_t) = \sum_j \gamma_j p_{jt} \text{ and } P_B(\mathbf{p}_t) = \prod_j p_{jt}^{\alpha_j} \text{ with } \sum_j \alpha_j = \eta.$$

The indirect utility function with MPIGLOG (20) characterizes nonhomothetic within-period preferences, with all variables being nonseparable with each other. MPIGLOG is a generalization of Deaton and Muellbauer's (1980) well-known AIDS (Almost Ideal Demand System) with allowance for durable goods and reduces to Muellbuer's (1975) seminal PIGLOG (Price Independent Generalized Loglinear) form if $\eta = \omega = 0$.

Given (20) together with associated price indexes, we obtain the following derivatives:

$$\frac{\partial v(C_t, \mathbf{p}_t, k_t)}{\partial C_t} = \frac{C_t^{\eta-1}}{P_B(\mathbf{p}_t)}\left(1 + \eta \ln\left[C_t k_t^{\omega} / P_A(\mathbf{p}_t)\right]\right), \tag{21}$$

$$\frac{\partial v(C_t, \mathbf{p}_t, k_t)}{\partial p_{it}} = -\frac{C_t^{\eta}}{P_B(\mathbf{p}_t)}\left(\frac{\gamma_{it}}{P_A(\mathbf{p}_t)} + \frac{\alpha_i}{p_{it}}\ln\left[C_t k_t^{\omega} / P_A(\mathbf{p}_t)\right]\right), i = 1,...,N, \tag{22}$$

$$\frac{\partial v(C_t, \mathbf{p}_t, k_t)}{\partial k_t} = \frac{\omega C_t^{\eta}}{k_t P_A(\mathbf{p}_t)}, \tag{23}$$

and

$$\frac{\partial^2 v(C_t, \mathbf{p}_t, k_t)}{\partial C_t^2} = \frac{C_t^{\eta-2}}{P_B(\mathbf{p}_t)} \left((2\eta-1) + \eta(\eta-1)\ln\left[C_t k_t^\omega / P_A(\mathbf{p}_t) \right] \right). \tag{24}$$

We can use (21) and (22) to obtain the demand functions for nondurable goods via Roy's identity (2). In budget share form, we have

$$s_{it}(C_t, \mathbf{p}_t, k_t) = -\frac{\partial v / \partial \ln p_{it}}{\partial v / \partial \ln C_t} = \frac{\left(\gamma_i p_{it} / P_A(\mathbf{p}_t) \right) + \alpha_i \ln\left[C_t k_t^\omega / P_A(\mathbf{p}_t) \right]}{1 + \eta \ln\left[C_t k_t^\omega / P_A(\mathbf{p}_t) \right]}, i = 1, \ldots N, \tag{25}$$

where $s_{it} \equiv p_i q_i / C_t \ (i = 1, \ldots, N)$ is the budget share of the ith nondurable good, with $\sum_{i=1}^N s_{it} = 1$. Moreover, given the Box-Cox transformation of the MIGLOG indirect utility function (20), equation (21) is used to evaluate the marginal utility of consumption (7) for $s = t$ and then, together with (23), to evaluate the Euler equations (11), (12), and (13).

The MPIGLOG indirect utility function (20) is regular with respect to the within-period properties over the region $C_t k_t^\omega > P_A(\mathbf{p}_t)$ for $0 \le \alpha_i, \eta \le 1, \omega \ge 0,$ and $\gamma_i \ge 0, i = 1, \ldots, N.$ Intertemporal properties of MPIGLOG with respect to the concavity of C_t can be examined from (24). If $\eta \ge 1,$ the MPIGLOG indirect utility function (20) is convex in C_t. If $0 < \eta \le 0.5,$ on the other hand, it is concave in C_t. If $0.5 < \eta < 1,$ the MPIGLOG indirect utility function is likely to be concave in C_t for certain combinations of data and parameters. Intertemporal optimization, however, requires the concavity of the intertemporal utility function (6). To this end, we can use the relevant expressions in (20), (21), and (24) to evaluate the concavity condition in (8). For $\zeta > 0$ and $0 < \eta < 0.5,$ with $C_t k_t^\omega > P_A(\mathbf{p}_t)$, the concavity of the indirect utility function (20) will preserve the concavity of the intertemporal utility function. However, if $\zeta > 0$ and $0 < \eta < 1,$ an additional sufficient condition is $\zeta > 1 + (0.5 / \eta).$ If η is in the range $0.5 < \eta < 1,$ then the Box-Cox transform has to be sufficiently concave to ensure overall concavity.[13]

In addition to the indirect utility function specified in (20) with MPIGLOG, we need a form for the adjustment cost function $h(q_t^k, k_t)$ to evaluate the Euler equation for durable goods (13). A convenient form that has desirable properties of the adjustment cost function is a quadratic function (see Hubbard, 1998):

$$h(q_t^k, k_t) = \frac{\varphi_0}{2} (\frac{q_t^k}{k_t} - \varphi_1)^2 k_t, \tag{26}$$

where φ_0 and φ_1 are parameters. From this equation, we obtain

$$\frac{\partial h(q_t^k, k_t)}{\partial q_t^k} = \varphi_0 (\frac{q_t^k}{k_t} - \varphi_1) \tag{27}$$

and

$$\frac{\partial h(q_t^k, k_t)}{\partial k_t} = \frac{\varphi_0}{2} (\varphi_1 - \frac{q_t^k}{k_t})(\varphi_1 + \frac{q_t^k}{k_t}). \tag{28}$$

Adjustment costs are increasing in q_t^k, i.e., $\partial h(q_t^k, k_t)/\partial q_t^k > 0$, if $q_t^k / k_t > \varphi_1$, and decreasing in k_t, i.e., $\partial h(q_t^k, k_t)/\partial k_t < 0$, if $q_t^k / k_t > \varphi_1$. Thus satisfying the properties of the adjustment cost function requires that $q_t^k / k_t > \varphi_1$.

Moreover, estimation of the Euler equations for bonds and shares in (11) and (12) requires the solution for the Lagrange multiplier for liquidity constraints ϕ_t. This variable is a non-differentiable function of many variables, which is difficult to derive analytically. However, when the consumer faces a liquidity constraint, his optimal consumption is constrained by current income and his ability to adjust current consumption is limited in response to a future increase in income. If the consumer is liquidity-constrained and his disposable income increases in the current period, the constraint will be relaxed and therefore ϕ_t will fall, suggesting a negative correlation between ϕ_t and disposable income (Zeldes, 1989). We therefore use disposable income as a measure of liquidity constraints and express ϕ_t as a function of this variable; that is, $\phi_t = \phi_1 \ln Y_t^d + \phi_2 Time$, where Y_t^d is disposable income at period t and *Time* is a time trend to account for some omitted variables, and ϕ_1 and ϕ_2 are parameters with the restriction that $\phi_1 < 0$ if the consumer is liquidity constrained.

III. DEMAND ELASTICITIES, RISK AVERSION, AND AN EXTENDED MULTIFACTOR CONSUMPTION-BASED CAPM WITH MULTIPLE GOODS AND LIQUIDITY CONSTRAINTS

The marginal utility of consumption (7), which is identified with estimation of the Euler equation for bonds (11) or shares (12), characterizes the consumer's intertemporal behavior. Intertemporal issues are usually investigated in studies on consumption or asset pricing but not in traditional studies on consumer demand because they are essentially a static analysis. In previous studies on consumption and asset pricing, however, the marginal utility of consumption depends on consumption only with no disaggregate information about its components [see (9)]. The integrated model allows for the effects of nondurables consumption, commodity (nondurables) prices, and durables stock on commodity demands as well as on risk aversion and asset pricing. Moreover, the intertemporal two-stage budgeting procedure allows us to examine intertemporal issues in consumer demand by estimating the Euler equations (11), (12), and (13) together with the commodity demand functions (2).

In demand analysis, the impact of prices and income on commodity demands is typically measured by Marshallian price and income elasticities. From (25), the Marshallian price elasticities ($G_{ijt}, i, j = 1, \ldots, N$) are given by

$$G_{ijt} \equiv \frac{\partial \ln q_{it}(C_t, \mathbf{p}_t, k_t)}{\partial \ln p_{jt}} = -\delta_{ij} + \frac{\left[\mathrm{EA}_{ijt} + \dfrac{\gamma_j p_{jt}}{P_A(\mathbf{p}_t)} (\eta s_{it} - \alpha_i) \right]}{\left\{ 1 + \eta \ln \left[C_t k_t^\omega / P_A(\mathbf{p}_t) \right] \right\} s_{it}}, i, j = 1, \ldots, N, \tag{29}$$

where δ_{ij} is the Kronecker delta and $\mathrm{EA}_{ijt} \equiv \dfrac{\partial^2 \ln P_A(\mathbf{p}_t)}{\partial \ln p_{it} \partial \ln p_{jt}} = (\gamma_j p_{jt}) \left[\dfrac{\delta_{ij}}{\gamma_i p_{it}} - \dfrac{1}{P_A(\mathbf{p}_t)} \right], i, j = 1,$

$\ldots, N$. Income or expenditure elasticities ($E_{it}, i, = 1, \ldots, N$) are expressed as

$$E_{it} \equiv \frac{\partial \ln q_{it}(C_t, \mathbf{p}_t, k_t)}{\partial \ln C_t} = 1 + \frac{\alpha_i - \eta s_{it}}{\left\{ 1 + \eta \ln \left[C_t k_t^\omega / P_A(\mathbf{p}_t) \right] \right\} s_{it}}, i = 1, \ldots, N. \tag{30}$$

The Marshallian price elasticities are conditioned on expenditure and can be considered long-run price elasticities with changes in prices interpreted as taking place permanently over time. In an intertemporal setting, however, the relevant price elasticities are Frisch price elasticities conditioned on the marginal utility of consumption (see Browning, Deaton and Irish 1985; Kim, 1993), which is identified with estimation of the Euler equation (11) or (12) in conjunction with

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the commodity demand functions (2). The Frisch price elasticities identify the responses of commodity demands to temporary changes in prices in the current period with the prices of future periods held constant; hence they are considered short run price elasticities (Kim, 1993). However, the Frisch price elasticities (F_{ijt}, $i, j = 1, \ldots, N$) are related to the Marshallian price and expenditure elasticities by the following relation (see Browning, Deaton and Irish 1985; McLaughlin 1995):

$$F_{ijt} \equiv \frac{\partial \ln q_{it}(\mu_t, \mathbf{p}_t, k_t)}{\partial \ln p_{jt}} = G_{ijt} + s_{jt} E_{it} E_{jt} \left[\frac{\partial \ln \mu_t}{\partial \ln C_t} \right]^{-1} + s_{jt} E_{it}, \quad i, j = 1, \ldots, N, \tag{31}$$

where $q_{it}(\mu_t, \mathbf{p}_t, k_t), i = 1 \ldots, N,$ is the Frisch demand function for nondurable goods and

$$\frac{\partial \ln \mu_t}{\partial \ln C_t} = \left[v(C_t, \mathbf{p}_t, k_t)^{-\zeta} \frac{\partial^2 v(C_t, \mathbf{p}_t, k_t)}{\partial C_t^2} - \zeta v(C_t, \mathbf{p}_t, k_t)^{-(\zeta+1)} \left(\frac{\partial v(C_t, \mathbf{p}_t, k_t)}{\partial C_t^n} \right)^2 \right] \frac{C_t}{\mu_t}, \tag{32}$$

which is derived from (8) for $s = t$. The concavity of the intertemporal utility function (6) implies that $\partial \ln \mu_t / \partial \ln C_t < 0$. Note that since Frisch demands reflect an intertemporal decision, intertemporal allocation of consumption also determines the Frisch price elasticities. This information is not provided by static, Marshallian demands (2) and requires estimation of the Euler equations.

In a stochastic intertemporal environment, risk aversion is typically estimated with a utility function defined in terms of consumption (see, e.g., Hansen and Singleton, 1983). For the integrated model, the coefficient of relative risk aversion (RRA) is defined by $RRA \equiv -\partial \ln \mu_t / \partial \ln C_t$ with $\partial \ln \mu_t / \partial \ln C_t$ given in (32). In the traditional measure based on the tightly parameterized power utility function in (9), $RRA = \zeta$. In this measure, consumption is the driving process for the marginal utility of consumption, and the degree of risk aversion is constant over time. In contrast, the integrated model allows for the effects of commodity prices and durable goods, and the degree of risk aversion varies over time with consumption as well as commodity prices and durable goods. It is interesting to note that the Frisch price elasticities (31) contain a term involving $\partial \ln \mu_t / \partial \ln C_t$. This means that the consumer's risk or intertemporal substitution behavior indirectly determines his intertemporal demand decision. Moreover, according to the consumption-based analysis of asset pricing, the risk aversion

coefficient ζ for power utility plays a central role in determining of equity returns, as illustrated in (19). Thus proper measurement of the coefficient of risk aversion is essential for a full understanding of intertemporal consumer behaviour and equity returns.

Identifying the fundamental factors governing the risk-return trade-off of an asset continues to be a central issue in finance or asset pricing. In the celebrated capital asset pricing model (CAPM), market or portfolio returns are the risk factor for an excess equity return over the risk-free asset. However, asset pricing models that use portfolio returns leave unanswered the question of what explains the return-based factors. Instead, the SDF model attempts to identify the underlying economic forces that determine asset returns.   Moreover, as noted by Cochrane (2005), various asset pricing models amount to alternative ways of applying the SDF as it appears in (16). In the traditional CCAPM with power utility, the SDF is a function of consumption growth (see footnote 11), so consumption growth is the fundamental risk factor driving asset returns. Despite its theoretical attraction, however, the empirical performance of the CCAPM has been disappointing, and there have been many attempts to improve its performance (see Cochrane, 2005, for a survey). These studies have displayed some success in explaining the observed equity returns, but they are, in large part, contingent on an analytical framework based on power utility with real consumption treated as the only consumption good. There are studies that incorporate durable goods (Yogo, 2005) and housing services (Piazzesia, Schneidera and Tuzelb, 2007) as additional consumption goods in the context of the traditional CCAPM.

We build on and expand previous studies on the CCAPM, extending to a multicommodity framework by accounting for the influences of nondurables consumption, commodity (nondurables) prices, and durables stock, with allowance for liquidity constraints on asset pricing. In general, the SDF is a nonlinear function of these variables in two adjacent periods. However, since the SDF measures the discounted growth of the marginal utility of nondurables consumption, it is reasonable to postulate that it can be expressed as some function of nondurables consumption growth, discounted commodity price growth, and durables stock growth. Thus, in the spirit of linear factor models but using the insights from the general model, we take a linear approximation of the SDF to obtain

$$M_{t+1} \approx b_{ot} + b_{ct}\Delta \ln C_{t+1} + \sum_{j=1}^{N} b_{jt}\Delta \ln \widehat{p}_{jt+1} + b_{kt}\Delta \ln k_{t+1}, \qquad (33)$$

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where $\Delta \ln \widehat{p}_{jt+1} = \ln(p_{jt+1} / p_{jt}(1+\rho))$, $j = 1, ..., N$, and b's are possibly time-varying parameters. The time-varying parameters are introduced to capture possible nonlinearity in the SDF. From (33), the SDF is a function of nondurables consumption growth $(\Delta \ln C_{t+1})$, discounted nondurables price growth $(\Delta \ln \widehat{p}_{jt+1}, j = 1, ..., N)$, and durables stock growth $(\Delta \ln k_{t+1})$, which is the ratio of (net) real durables expenditure to durables stock.[14] Since the marginal utility of consumption is decreasing in C_t and p_{jt}, $j = 1, ..., N$, but increasing in k_t, we expect that $b_{ct} < 0$ and $b_{jt} < 0$, $j = 1, ..., N$, but $b_{kt} > 0$. For homothetic within-period preferences, b_{ct} in absolute value is equal to the coefficient of relative risk aversion (i.e., $b_{ct} = \partial \ln \mu_t / \partial \ln C_t$), which is equal to ζ for power utility (see footnote 11). For nonhomothetic within-period preferences, the two coefficients will not be equal, but if (33) provides a good approximation to the exact, nonlinear SDF, they are expected to be close.

To obtain a tractable expression for excess equity returns in line with the traditional CCAPM, we assume that the three growth variables are orthogonal to one another (we will relax this assumption in empirical analysis in Section IV) and that these variables have a joint normal distribution with equity returns. Then we can use (33) and apply Stein's lemma (Cochrane, 2005; Liu, 2010)[15] to get

$$\text{Cov}_t(r_{t+1}^e, M_{t+1}) = b_{ct}\text{Cov}_t(r_{t+1}^e, \Delta \ln C_{t+1}^n) + \sum_{j=1}^{N} b_{jt}\text{Cov}_t(r_{t+1}^e, \Delta \ln \widehat{p}_{jt+1}) + b_{kt}\text{Cov}_t(r_{t+1}^e, \Delta \ln k_{t+1}). \quad (34)$$

With the expression for $E[M_t]$ obtained from (14) with the presence of a risk-free asset (bonds) and liquidity constraints, substituting (34) into (17) yields the following result:

$$\frac{E_t[r_{t+1}^e] - r_{t+1}^b}{1 + r_{t+1}^b} = \frac{-b_{ct}}{1 - \hat{\phi}_t}\text{Cov}_t(r_{t+1}^e, \Delta \ln C_{t+1}) - \sum_{j=1}^{N} \frac{b_{jt}}{1 - \hat{\phi}_t}\text{Cov}_t(r_{t+1}^e, \Delta \ln \widehat{p}_{jt+1}) - \frac{b_{kt}}{1 - \hat{\phi}_t}\text{Cov}_t(r_{t+1}^e, \Delta \ln k_{t+1}).$$
$$(35)$$

This equation has an equivalent representation in terms of multivariate betas $(\beta's)$ and prices of risk $(\lambda's)$:

$$E_t[r_{t+1}^e] = \lambda_{0t+1} + \beta_{ct+1}\lambda_{ct+1} + \sum_{j=1}^{N} \beta_{jt+1}\lambda_{jt+1} + \beta_{kt+1}\lambda_{kt+1}, \quad (36)$$

where

$$\lambda_{0t+1} \equiv r_{t+1}^b,$$

$$\beta_{ct+1} \equiv \mathrm{Cov}_t(r_{t+1}^e, \Delta \ln C_{t+1}) / \mathrm{Var}_t(\Delta \ln C_{t+1}),$$

$$\lambda_{ct+1} \equiv \frac{-b_{ct}(1+r_{t+1}^b)}{1-\hat{\phi}_t} \mathrm{Var}_t(\Delta \ln C_{t+1}),$$

$$\beta_{jt+1} \equiv \mathrm{Cov}_t(r_{t+1}^e, \Delta \ln \hat{p}_{jt+1}) / \mathrm{Var}_t(\Delta \ln \hat{p}_{jt+1}), \, j = 1, ..., N,$$

$$\lambda_{jt+1} \equiv \frac{-b_{jt}(1+r_{t+1}^b)}{1-\hat{\phi}_t} \mathrm{Var}_t(\Delta \ln \hat{p}_{jt+1}), \, j = 1, ..., N,$$

$$\beta_{kt+1} \equiv \mathrm{Cov}_t(r_{t+1}^e, \Delta \ln k_{t+1}) / \mathrm{Var}_t(\Delta \ln k_{t+1}),$$

$$\lambda_{kt+1} \equiv \frac{-b_{kt}(1+r_{t+1}^b)}{1-\hat{\phi}_t} \mathrm{Var}_t(\Delta \ln k_{t+1}).$$

Equation (36) has the standard structure of a multifactor or multibeta pricing model (see Cochrane, 2005; Lettau and Ludvigson, 2001) with several risk factors – nondurables consumption growth, commodity (nondurables) price growth, and durables stock growth – where the betas and risk prices are conditional on information at time t and vary over time, producing time-varying equity returns. The risk price of a factor depends on the associated parameter in the SDF equation (33), the risk-free bond rate, the factor's variance, and, recalling that $\hat{\phi}_t \equiv \phi_t / \mu_t$, the liquidity constraint and the marginal utility of consumption. In general, higher risk aversion (in the case of consumption growth) and a higher factor variance as well as risk-free rate raises the factor risk price by creating additional risk to investors, which requires additional return to hold stocks. The presence of liquidity constraints also raises the risk price, causing investors to demand a higher premium to hold stocks. The marginal utility of consumption has an opposite effect of the liquidity constraint. For an investor with low marginal valuation of consumption (i.e., a high income person because of diminishing marginal utility of income), the risk price is high relative to another investor with high marginal valuation of consumption (i.e., a low income person), requiring additional premium to bear the risk of consumption. It is worth noting that, with low and relatively constant interest rates as well as small factor variabilities, the factor risk prices vary over time due to the time-varying parameters of the SDF equation and the time-varying (adjusted) liquidity constraint, yielding time-varying equity returns. In the traditional CCAPM based on power utility, on the other hand, consumption growth is the only risk factor and its risk price is solely determined by risk aversion and consumption variance that are

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relatively constant over time (see Cochrane, 2005). To account for time variation in equity returns, the coefficient of consumption growth in the SDF equation (i.e., the RRA coefficient) is allowed to vary over time using some conditioning variables, and scaling factors are created (see Cochrane, 2005; Lettau and Ludvigson, 2001). Equation (36), however, discloses that this procedure may not be necessary because the time-varying (adjusted) liquidity constraint can also account for the time-varying risk prices and hence equity returns.

Equation (37) or (36) captures the gist of analysis of asset pricing in this study. It is an extended, *multifactor* CCAPM with multiple risk factors identified for equity returns, in accordance with the consumer's optimizing problem as prescribed by the intertemporal two-stage budgeting procedure. There are three fundamental risk factors, namely nondurables consumption growth, commodity (nondurables) price growth, and durables stock growth or the durables expenditure-stock ratio, that price equities. Also, equation (36) reveals that the presence of liquidity constraints is an important source of priced risks determining equity returns. While consumption growth is well recognized in previous studies, commodity price growth, durables stock growth, and liquidity constraints have not received due attention as the risk factors for equity returns (see Cochrane, 2005). Yet, these factors are clearly important in explaining equity returns. In particular, a recent housing crisis and resulting falling home prices has affected the stock market and hence stock returns. There is also considerable evidence that expected stock returns vary over business cycles (see Cochrane, 2005): the risk premium should be higher at the bottom of a business cycle when investors require a higher excess return to hold risky assets (stocks). The demand for durable goods is usually affected by business cycle fluctuations. Then durables stock growth or the durables expenditure-stock ratio should have a good predictive power in explaining the countercyclical variation in equity returns.[16] Moreover, studies often found that liquidity constraints can have an important influence on consumption (see Japelli and Pagano, 1989; Zeldes, 1989). Clearly then they will affect asset returns as well.[17]

IV. ESTIMATION AND EMPIRICAL ANALYSIS

A. Data and Estimation Methods

Empirical investigation was carried out using quarterly data for the United States spanning the

period 1959:1 to 2011:4. The required data are the prices and quantities of nondurable and durable goods in conjunction with the rates of return for bonds and equities. Data for the prices and quantities of nondurable and durable goods are obtained from Personal Consumption Expenditures in the National Income and Product Accounts (NIPA), published by the Bureau of Economic Analysis, U.S. Department of Commerce. There are three broad categories for consumption in Personal Consumption Expenditures: durables, nondurables, and services. We have regrouped nondurables and services into six broad categories: (1) food and beverages, (2) clothing and footwear,[18] (3) gasoline and other energy goods, (4) housing and utilities, (5) health care, and (6) others. We took the price indices for these six nondurables and services as the nondurables prices. Nondurables consumption is the sum of nominal expenditures on the six nondurables and services. Durable goods consist of items such as motor vehicles, furniture and appliances, and jewelry and watches. We utilized the price index for aggregate durable goods and took durables expenditure as the quantity index for durable goods. The durables stock series were constructed using past durables expenditure, which are available back to 1928.[19] Following Bernanke (1985), we used 5% per quarter for the rate of depreciation for the durables stock. Disposable income was also obtained from NIPA. Nondurables consumption, durables expenditure, and disposable income are expressed in per capita values by dividing them by population, and all data are seasonally adjusted, measured in 2000 dollars. For the bond interest rate, we use the three-month Treasury bill rate taken from Federal Reserve Board's H.15 Statistical Release. For the equity rate of return, we use the S&P500 Composite Index on stock prices and dividends, taken from Robert Shiller's website. Bond and equity returns are expressed in quarterly nominal rates.

Estimation consists of the demand function for six nondurable goods and services in (2) and the Euler equations for bonds, shares, and durable goods in (11), (12), and (13). We use the appropriate expressions derived from the empirical model in Section II.B to obtain these equations. For purposes of correcting for potential heteroscedasticity of error specification, however, it is convenient to work with a ratio form. To this end, we use the budget share equations (25) and the Euler equations for bonds and shares expressed in terms of the intertemporal marginal rate of substitution or the SDF in (14) and (15). We also express the Euler equation for durable goods (13) in a ratio form by dividing the right hand side by the left

hand expression. To implement the empirical analysis of the system of the nine equations, we assume that the budget share system (25) is stochastic due to errors of optimization.[20] Then the budget share system is jointly estimated with the three Euler equations using the Generalized Method of Moments (GMM) to deal with the possibility of endogeneity, measurement error, and nonnormality of errors, and allowing for the existence of cross-equation parameter restrictions in the nine equations. Joint estimation of the budget share system with the Euler equation for consumption or bonds was exploited by Kim, McLaren, and Wong (2013). They, however, considered one asset with no regard to durable goods and liquidity constraints. We adapt their estimation method to our analysis. There is, however, a potential econometric problem in using the budget share system in estimation. Nondurables consumption expenditure, the denominator of the budget shares, is an endogenous variable. To correct this problem, we use a commodity's expenditure by deflating it by disposable income as a dependent variable; that is, $p_{it}q_{it} / Y_t^d =$

$s_{it}(C_t, \mathbf{p}_t, k_t)C_t / Y_t^d, \ i = 1,...,6.$

B. Estimated Model

Table 1 reports estimation results for the empirical model based on joint estimation of the modified share and Euler equations. All estimation was carried out using the GMM procedure in TSP version 4.5, which is well-suited for estimation of systems with complex cross-equation constraints; it also allows for heteroscedasticity of an unknown form in the computation of the variance-covariance matrix. In the initial estimation, serial correlation of the errors was substantial for budget share equations, leading to the use of Moschini and Moro (1994)'s method for autoregressive errors. The model is highly nonlinear and, to ensure that the requisite within-period and intertemporal regularity conditions are satisfied (see the discussion in Section II.B, p. 13), we imposed some parameter restrictions in estimation by setting $\alpha_1 = 0$ in the MPIGLOG indirect utility function (20) and the Box-Cox parameter $\zeta = 1$ in (6). Setting $\zeta = 1$ amounts to producing a log form for the intertemporal utility function (6). The following comments are in order. First, the model satisfies the required regularity conditions for within-period and intertemporal preferences at every sample point. Second, the overall fit of the budget share system as indicated by the R^2 values is quite good. Third, the serial correlation properties of the

error terms as shown in the Durbin-Watson and Box-Pierce χ^2 statistics are no longer severely pathological, suggesting the appropriateness of the Moschini and Moro correction. Fourth, the χ^2 test for over-identifying restriction cannot reject the orthogonality conditions, implying that the restrictions producing the moment equations are valid.[21] This suggests that consumers in general obey the intertemporal two-stage budgeting rule in their optimization behaviour by separating within-period and intertemporal decisions. Fifth, while not all parameters are significant at conventional levels of significance,[22] it is worth noting that ϕ_1, though small in magnitude, is negative and significant, suggesting moderate evidence that disposable income plays a significant role for the presence of liquidity constraints. Evidence on liquidity constraints is mixed for U.S. households. Zeldes (1989) found that liquidity constraints can have important influences on consumption of a significant portion of the population.   Runkle (1991), on the other hand, found that there is no clear evidence for liquidity constraints for U.S. households.

C. Estimated Price and Expenditure Elasticities and the Risk Aversion Coefficient

Table 2 presents price and expenditure elasticities evaluated at the sample means of exogenous variables, using (29), (30), and (31). Looking at the sample mean budget shares, a substantial portion of expenditure on nondurables and services is spent on housing services (21%), followed by food (14%) and health care (13%). The estimated expenditure or income elasticities reveal that food is a necessity relative to other goods or services (see also Attanasio and Weber, 1995), while housing services appear to be a luxury. This implies that the budget share of food decreases with rising income; the budget share of housing services, on the other hand, tends to increase with rising income. The Marshallian price elasticities are long-run elasticities that identify effects of permanent changes in prices on commodity demands. The Frisch price elasticities are short-run elasticities that measure responses of commodity demands to temporary changes in prices. The estimated Frisch own-price elasticities are smaller in absolute value than the corresponding Marshallian elasticities in line with expectations. However, the estimated Marshallian price elasticities suggest that the demands for most goods and services are essentially unit-elastic [23] except for food, while the estimated Frisch elasticities reveal that they are inelastic.   By implication, we have

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RESULT 1. *The use of the traditional Marshallian, instead of Frisch, price elasticities as a measure of the effect of changes in prices on commodity demands is likely to lead to misleading results for analysis in most situations, especially in a dynamic, nonstationary environment.* This underscores the utility of the Frisch price elasticities in demand analysis (see Browning, Deaton and Irish 1985; Kim, 1993).

The integrated model allows us to obtain a more reliable estimate of risk aversion relative to those obtained in previous studies. The estimated RRA at the sample means of exogenous variables using (32) is 1.0209 and significant with the *t* value of 651.272. In the traditional measure based on tightly parameterized power utility, RRA is constant. In the RRA given in (32), RRA varies with consumption as well as commodity prices and durables stock.

RESULT 2. *The estimated RRA falls within the range of the usual a priori values considered reasonable for relative risk aversion* $(1 < RRA < 5)$ *(Cochrane, 2005). The low value of RRA indicates that an average American does not exhibit high risk aversion behavior; hence any attempt to explain the observed equity premium with an implausibly high level of risk aversion (Mehra and Prescott, 1985) may turn out to be unproductive.*

D. Analyzing the Asset Pricing Puzzles and the Risk Factors for Expected Returns

We want to investigate whether the integrated multifactor model provides new insights into the two well-known puzzles in asset pricing: the risk free rate and equity premium puzzles (see Kocherlakota, 1996, for a survey). These puzzles, in essence, concern examination of the co-movements of three variables: the return to short term risk free bonds, the excess return of stocks over risk free bonds, and the SDF (see (17) or (18)). In the integrated model, the SDF is estimated from the demand system together with the Euler equations and depends on nondurables consumption, commodity prices, and durables stock in two adjacent periods. In the traditional CCAPM with tightly parameterized power utility, on the other hand, it is estimated using only the Euler equations and depends only on real consumption growth (see footnote 11). As a consequence, we expect that our analysis provides new results in explaining the asset pricing puzzles.

The SDF measures the *gross* rate of the discounted marginal utility growth of nondurables consumption, i.e., $M_{t+1} \equiv \mu_{t+1} / \mu_t$, which is constructed using (7) together with (20) and (21) and

evaluated with the relevant parameter estimates in Table 1 and the sample values for nondurables consumption, nondurables prices, and durables stock. Plots of the constructed SDF, bond returns, and excess equity returns are depicted in Figures 1 and 2. The SDF and the excess equity return series appear stationary, while the bond return series show a stable pattern with little variation over time. Table 3 contains some summary statistics for the three variables. There are a few features of the table that give rise to the two puzzles. The average value of the SDF for the sample period, 1959-2011, is close to one. The average rate of return on bonds for the sample period is equal to 1.3% per quarter, while the excess equity return is equal to 1.24%, on average, per quarter for the sample period. The excess return has a higher variance than the SDF. While the covariance between the SDF and the excess return appears to be almost zero, the correlation between the two variables is -0.0567, which is much higher than the covariance.

Previous studies find that, with high risk aversion, the implied interest rate is too large to be consistent with the historically low government short-term bond rate. From (14), this puzzle involves whether the mean (gross) bond rate (1.013) is equal to the reciprocal of the mean SDF adjusted for the liquidity constraint (1.0089). The two values are very close to each other. This, together with the low value of risk aversion found in this study, suggests clearly that there is no risk-free rate puzzle from the data.

We can use (17) to evaluate the equity premium puzzle with Table 3. With the mean of the SDF close to one, this involves whether the excess return is equal to the absolute value of the covariance between the excess return and the SDF. Table 3 shows that the excess return (0.0124) is much higher than the covariance between this variable and the SDF (0.000033), suggesting the well-known equity premium puzzle. To further examine this issue, Table 4 presents the Sharpe ratio and Hansen-Jagannathan statistic in (18). For the sample period, the Sharpe ratio is 0.1716 and the Hansen-Jagannathan bound is 0.0101, which is far less than the Sharpe ratio. Thus the observed Sharpe ratio is too large to be explained by the Hansen-Jagannathan bound. Allowing for the correlation between the excess return and the SDF (0.0006) does not help but in fact makes it worse. Thus the equity premium still remains a puzzle even with the integrated model with nonhomothetic within-period preferences. A natural inquiry is to ascertain what causes this classical puzzle. From Table 3, we find that a low volatility of the SDF together with its low correlation with the excess equity return accounts for a low excess

return to be consistent with the high Sharpe ratio. In the traditional CCAPM, the SDF is determined by the coefficient of relative risk aversion and consumption growth (see footnote 11). Thus a low variance of consumption growth and low risk aversion are responsible for a low Hansen-Jagannathan bound, and, since consumption is smooth over time, a high value of relative risk aversion is suggested to resolve the puzzle (Mehra and Prescott, 1985). However, high relative risk aversion is not consistent with the data.

We want to re-examine the equity premium puzzle to see if we can provide additional insight into this unresolved issue. This requires us to empirically identify the relevant risk factors for determining excess returns by estimating the SDF equation (33) and then to evaluate the multibeta factor pricing model of the SDF (36). According to (33), the SDF, which measures the discounted growth of marginal utility of nondurables consumption, can be approximated by using the three growth variables: nondurable consumption growth, nondurables price growth, and durables stock growth. However, given the presence of a correlation that exists between the SDF and excess returns, an appropriate procedure is to jointly estimate the two equations as a system. Equation (36) shows that the same set of variables determining the SDF influence excess returns aside from the liquidity constraint. Thus we estimate the SDF and excess return equations jointly as a Seemingly Unrelated Regression by using the same set of explanatory variables. Disposable income is included to capture the effect of liquidity constraints on consumption and excess returns.[24] We treat the estimated values of the SDF as raw or generated data, as illustrated in Figure 1, and employ GMM to correct for possible endogeneity of the explanatory variables.

Estimation results are displayed in Table 5. R^2 for the SDF equation suggests strong evidence that the SDF can be approximated by the growth variables even for nonhomothetic within-period preferences. Consumption growth and durables stock growth have expected signs and are significant, while most of the price growth variables are insignificant. The coefficient for consumption growth in absolute value, 1.7694, is close to the mean value of the relative risk aversion coefficient, 1.0209. Thus the linearized SDF equation (33) provides a good approximation to estimation of the coefficient of relative risk aversion. There is, however, very weak evidence that the excess return can be forecasted by consumption growth and its related variables (see also Campbell, Lo, and MacKinley, 1997). The low R^2 is not unusual in equity

return regressions, and the value tends to be lower with quarterly data than annual or long horizon data (see Cochrane, 2005; Hansen and Singleton, 1983). Nevertheless, this equation provides useful information about the relevant risk factors affecting excess returns. As expected, consumption growth has a positive effect on excess returns and hence equity returns. However, all other variables have negative effects on excess returns. In particular, commodity price growth, though insignificant except for p_4 and p_6 is negatively related to excess or equity returns: equity returns tend to be lower with higher commodity prices but higher with lower commodity prices. This is in line with the negative relation between inflation and equity returns found in previous studies, but is in direct conflict with the view that the stock market is a perfect hedge against inflation. Lustig and Van Nieuwerburgh (2005) argue that a decrease in house prices reduces the collateral value of housing, which increases the exposure to risk and in turn the equity premium. This result is borne out by the negative relation between excess returns and housing service price. Durables expenditure or the service flow of durable goods displays a strong procyclical fluctuation (see Yogo, 2006), and durables stock growth, though not significant at the conventional levels of significance, also has a negative effect on equity returns. Disposable income growth has a negative sign. Since disposable income measures the effect of liquidity constraints, higher disposable income growth reduces the effect of liquidity constraints, while lower disposable income growth increases the effect of liquidity constraints. Thus liquidity-constrained households experiencing low income growth are likely to demand a higher equity premium to hold stocks.[25]

The variables identified in this study as the relevant risk factors – nondurables consumption growth, commodity (nondurables) price growth, and durables stock growth, and disposable income growth – can be considered *state* variables derived from the consumer's intertemporal optimizing problem. Cochrane (2012) holds that traditional asset pricing models fail to identify the relevant state variables and suggests business cycle or recession state variables to account for the predictability of equity returns. While we have indicated durables stock growth as a business cycle variable, Table 5 shows that commodity price growth and disposable income growth, to a certain extent, also capture the effects of a business cycle or recession on equity returns. These variables usually fluctuate with the business cycle and tend to be falling during recession. Equity returns tend to be low in inflationary times with high stock prices but high in recessionary times

with low stock prices because investors demand a higher premium to hold stocks. This is clearly evidenced with commodity price growth, durables stock growth, and disposable income growth in Table 7.

Encapsulating these results, we have

RESULT 3. *Commodity price growth, durables stock growth, and disposable income growth represent <u>business cycle</u> variables and can predict the countercyclical variation in excess returns.*

GDP or industrial production, investment, labor income, and inflation are often used as business cycle variables in previous studies (see Hodrick and Zhang, 2001; Cochrane, 2005). These variables are chosen based on intuitive analysis and empirical investigation. Instead, we have identified the relevant business cycle or recession state variables in this study, with theoretical underpinnings of the consumer's optimizing behavior as prescribed by the intertemporal two-stage budgeting procedure.

According to the multifactor pricing model (36), the predictable variation in equity returns can be driven by changes in betas as well as changes in risk prices of the factors. The beta of a factor measures the sensitivity of the equity return to exposure to the risk associated with the factor, and the factor risk price is the price of such risk exposure, i.e., the additional premium required for an investor to bear the risk of the factor. In general, the expected return is high if the equity has a high beta and/or a large risk price of a factor. Table 7 presents the estimated betas $(\beta's)$ and risk prices $(\lambda's)$ for the factors, which allow us to determine the contribution of each risk factor to expected excess returns. The multifactor pricing model developed in (36) is based on the assumption that the risk factors are orthogonal to each other, which is used to derive a tractable model. We relax this assumption to provide more reasonable estimates of betas and risk prices (see Cochrane, 2005 and Lettau and Ludvigson, 2001, for a derivation of the nonorthogonal case with no allowance for liquidity constraints). The betas are estimated as the regression coefficients of the risk factors in the excess return equation in Table 5, and the risk prices allow for sample covariances among the risk factors in Table 6. The signs and magnitudes of the estimated betas appear reasonable, but the values of the lambdas are close to zero, although all lambdas for the factors have positive signs in accordance with expectations. While there are several variables affecting the factor risk prices (see (36) in Section III), the virtual

nonexistence of the risk prices for excess returns is, in large part, due to very low variances of the risk factors (see Table 6).

Thus we have

RESULT 4. *With a general decline in volatility of the aggregate economy since the early 1980s (see Sills, 2005), aggregate time series variables are not noisy or volatile enough to have much effect on the risk prices for excess returns, suggesting that undiversifiable risk as measured by beta explains a change in expected stock market return better than the risk price.*[26]

For factor contribution, consumption growth accounts for most of the estimated excess return, while other risk factors have a negative contribution to excess returns. The estimated excess return (0.000221) is too small to explain the observed excess return (0.0124),[27] again suggesting the equity premium puzzle.

In response to the poor empirical performance of existing asset pricing models, Cochrane (2005) points out some desirable features that may be relevant for a new model to help explain the predictability of excess returns and hence the equity premium puzzle (pp. 465-467). According to him, recession state variables are the "natural source" to solving the empirical asset pricing puzzles. We have identified these variables – commodity price growth, durables stock growth, and disposable income growth – theoretically and have seen in Table 5 that they are useful predictors for excess returns. Table 7, however, reveals that they make negative contributions to explaining the equity premium. Their presence, in general, tends to lower, rather than raise, excess returns, negating the effect of consumption growth. This indicates that the resolution of the equity premium puzzle must be found from other sources – ones that complement the positive effect of consumption growth on excess returns. In addition to high risk aversion, Cochrane (2005) suggests "low labor income, high income uncertainty, liquidity, etc." as possible variables to help explain the equity premium puzzle. Interestingly, these variables describe a recession, which can be explained by the determinants of the consumption risk price in (36). The basic mechanism is time variation in risk aversion and liquidity constraints. Because risk aversion depends on consumption as well as commodity prices and durables stock that fluctuate with the business cycle, it can be high during a recession. Also, during a recession, consumers are likely to face a high degree of liquidity constraint because of low labor or disposable income. In essence, the variables that increase the consumption risk price will

contribute to a higher equity premium. Unfortunately, high risk aversion is not present in the data, and consumers, in general, appear to face a moderate level of liquidity constraints.

   Importantly, these results point to the following:

RESULT 5. *While it is possible to <u>rationalize</u> the presence of an equity premium, whether we will ever be able to <u>explain</u> the historical equity premium for the aggregate stock market using consumption-based variables with aggregate time series data is an open question. After all, the equity premium may still remain an unresolved puzzle!*

This appears to be the current state of research on the equity premium puzzle (see Kocherlakota, 1996 and Cochrane, 2005), which is clearly corroborated in this study with a more general, multifactor consumption-based model with multiple goods and liquidity constraints.

   Given that none of the existing models effectively is capable of explaining the historical equity premium, we want to see if our multifactor pricing model can do a better job in explaining the magnitude of the equity premium than previous consumption-based models. The estimated excess return from the well-known asset pricing formula (15) with the estimated SDF gives us the value of 0.000033 using Table 3. In traditional consumption-based models with tightly parameterized power utility, consumption growth is the only risk factor, so $\text{Cov}_t(r^e_{t+1}, M_{t+1}) = -\zeta \text{Cov}_t(r^e_{t+1}, \Delta \ln C_{t+1})$ (see footnote 11). Using the sample covariance between excess returns and consumption growth with the estimated RRA for ζ , we calculated the beta and lambda for consumption growth and then estimated the excess return, which is presented in Table 7.[28] The estimated excess return from the traditional CCAPM (0.000016) is lower than the one estimated with the SDF (17) (0.000033). Moreover, this value is much lower than the value estimated with the multifactor model (0.000221). Thus our model, especially the multifactor model, performs better in predicting the excess return than the traditional models. There are two possible reasons. The consumption beta for the multifactor model is higher than that of the traditional single-factor CCAPM. Also, the consumption risk price for the multifactor model is higher than that of the traditional single-factor CCAM. This is due to the presence of the risk-free bond rate and liquidity constraints in the multifactor model. Also, the inclusion of other risk variables – commodity price growth, durables stock growth, and disposable income growth – helps better explain the equity return for the multifactor model. Overall, the consumption contribution to the

excess return is higher with the multifactor model than the traditional model, thus making the multifactor model better able to explain the equity return.

In summary, we have

RESULT 6. *With the time-varying risk prices from many sources and additional state variables, the integrated multifactor model does a better job in explaining the magnitude of the equity premium than the traditional single-factor consumption-based models.*

V. CONCLUSION

Traditionally, studies on consumer demand and consumption or asset pricing are conducted independent of each other. This paper has examined the joint link that exists between these studies and provided an integrated model by way of the intertemporal two-stage budgeting procedure with allowance for durable goods and liquidity constraints. The model accounts for the influences of nondurables consumption, commodity prices, and durables stock on commodity demands as well as on risk aversion and asset returns. Results from U.S. data reveal that consumers in general follow the intertemporal two-stage budgeting procedure in their optimization behavior with the presence of liquidity constraints. Estimation of the demand elasticities show the relevance of the Frisch price elasticities in an intertemporal environment in contrast to the more common Marshallian price elasticities. The coefficient of relative risk aversion is found to be commensurate with the usual a priori values considered reasonable for risk aversion. An important finding for asset pricing is the identification of nondurable consumption growth, commodity price growth, and durables stock growth as the relevant risk factors for expected returns. We also find that the presence of liquidity constraints can have an important effect on equity returns. While we cannot explain the historical equity premium, we find that, with the time-varying risk prices from many sources and additional state variables, the proposed *multifactor* consumption-based model does a better job of predicting the magnitude of the excess return than the traditional single-factor models based on power utility.

Given that this analysis represents the first exploratory attempt to fully integrate the three areas of consumer behavior, these findings appear reasonable and promising. However, to draw firm conclusions, especially with respect to the asset pricing puzzles and to investigate other relevant issues left unaddressed due to space limitation, more empirical work is certainly in order

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with a possibly refined empirical model and better use of the data. Yet, our analysis indicates the relevance of the intertemporal two-stage budgeting procedure that can be exploited to improve traditional micro studies on consumer demand as well as macro studies on consumption and asset pricing.

FOOTNOTES

[1] Within-period preferences are homothetic if the marginal rate of substitution between two goods is homogeneous of degree zero (see Deaton and Muellbauer, 1980). This implies that income or expenditure elasticities are unity with the existence of a single price index. See Section II.A for a detailed discussion.

[2] Nondurable goods refer to nondurable goods and services. For convenience, we call them "nondurable goods." In empirical analysis, we consider both nondurable goods and services.

[3] There are studies in consumption that consider both nondurable and durable goods with adjustment costs (see, e.g., Bernanke, 1985). However, they are based on the implicit assumption that the consumer's within-period preferences are homothetic to define aggregate nondurable consumption.

[4] Blundell, Browning and Meghir (1994) provide an early attempt to apply the intertemporal two-stage budgeting procedure (see also Attanasio and Weber, 1995, for a related work). They estimate the AIDS (Almost Ideal Demand System) and then use the parameters from this demand system to estimate the intertemporal elasticity of substitution in the log-linear Euler equation for consumption. Kim, McLaren, and Wong (2013) show that the sequential estimation procedure adopted by Blundell, Browning and Meghir (1994) may be inefficient because the existence of cross-equation parameter restrictions requires that demand systems and the Euler equation be jointly estimated. Instead, they present an estimating framework for the budget share equations by incorporating the exact, nonlinear Euler equation for consumption using the Generalized Method of Moments (GMM). This paper benefits from Kim, McLaren, and Wong's (2013) work. However, while their work is useful in the applied demand analysis, it is limited to investigate many relevant issues in consumption and asset pricing. They include bonds as the only asset and do not consider equities, which are a more important asset. Also, they are concerned with nondurable goods only by assuming that durable goods are separable from nondurable goods, without allowing for liquidity constraints.

[5] Yogo (2006) includes durable goods and Piazzesia, Schneidera, and Tuzelb (2007) consider housing services as an additional risk factor in the context of the traditional CCAPM but do not consider commodity price growth and disposable income growth.

[6] We consider leisure or labor supply as fixed and treat labor income as exogenous to the consumer.

[7] Adjustment costs can be specified in the utility function (see, e.g., Bernanke, 1985). However, since they usually involve monetary payments to install and maintain durable goods, they are included in the budget constraint as an additional expenditure. This is in compliance with the investment literature that treats adjustment costs as an additional expenditure to the firm's costs of production (see Hubbard, 1998).

[8] Intertemporal preferences are homothetic if the marginal rate of substitution between consumption at any two periods is homogeneous of degree zero. This implies that the marginal propensity to consume (MPC) out of wealth is the same for all income or wealth levels so that the wealth elasticity of consumption is equal to unity.

[9] The well-known Hicks composite commodity theorem also justifies the existence of a price index. It says that that if the prices of goods within any group always move in the same proportion, then the demand for the group as a whole has the properties identical to that for a single commodity.

[10] We can see that the conventional intertemporal utility function of the form (9) is homothetic. Thus the power utility function is characterized by homothetic within-period as well as intertemporal preferences.

[11] For power utility, the SDF, which measures the growth of the marginal utility of consumption, is expressed as

$$M_{t+1} = \frac{1}{1+\rho}(c_{t+1}/c_t)^{-\zeta} \approx \frac{1}{1+\rho} - \zeta\Delta\ln c_{t+1}.$$

From this equation, we obtain $\mathrm{Var}_t(M_{t+1}) = \zeta^2\mathrm{Var}_t(\Delta\ln c_{t+1})$, implying that $\sigma_t(M_{t+1}) = \zeta\sigma_t(\Delta\ln c_{t+1})$. Assuming that $E_t[\Delta\ln c_{t+1}] = 0$, we get $E_t[M_{t+1}] = 1/(1+\rho)$. Substituting $E_t[M_{t+1}]$ and $\sigma_t(M_{t+1})$ into (18) gives the desired result (19).

[12] The MPIGLOG is a rank 2 demand system. Indeed, we tried more flexible rank 3 demand systems such as Banks, Blundell, and Lewbel's (1997) Quadratic Almost Ideal Demand System and McLaren and Wong's (2009) Composite Indirect Utility Function to represent the within-period indirect utility function. Results of initial estimation, however, revealed that these systems violated the required concavity condition (concave in C_t). This ill-behaved feature may cause serious problems when these estimated models are used in intertemporal analysis, particularly in situations where a model may be subjected to large shocks.

[13] These results are available from the authors upon request.

[14] Mathematically, a linear approximation in (33) using growth variables is justified only for homothetic within-period preferences. For nonhomothetic preferences, it requires levels and growth rates of variables. However, the inclusion of the level terms creates complications in analysis and is disregarded for analytical tractability. Yet, empirical analysis (see Table 7)

shows that (33) using growth variables provides a good approximation to the nonlinear stochastic discount factor.

[15] Normality is not necessary to apply Stain's lemma. See Soderlind (2009) for an application in the context of the traditional CCAPM.

[16] Notably, durables stock growth or the durables expenditure-stock ratio is a consumption counterpart of Cochrane's (1996, 2005) investment or investment-capital ratio that measures capital stock growth, which is a production variable. It is also related to Lettau and Ludvigson's (2001) (nondurables) consumption-wealth ratio. To the extent that the consumer's nondurables consumption is affected by the spending on durable goods and his wealth is related to the level of durables stock, the (nondurables) consumption-wealth ratio certainly captures the effect of durables stock growth or the durables expenditure-stock ratio. In empirical analysis, it may be easier to use the durables expenditure-stock ratio than the consumption-wealth ratio because of the well-known difficulty of a proper measurement of wealth, while the durables expenditure-stock ratio can be easily constructed from durables expenditure. The investment-capital ratio and the consumption-wealth ratio are found to be good explanatory variables in explaining equity returns (see Cochrane, 2005).

[17] He and Modest (1995) examine whether the presence of liquidity constraints and other market frictions can account for the apparent failure of comovements of consumption and asset returns implied by the Euler equations. They do so in the context of the traditional CCAPM without considering commodity prices and durable goods. They also do not investigate the effect of liquidity constraints on the risk prices.

[18] Following the NIPA classification and applied demand studies (see Lewbel, 1991; Blundell, Browning and Meghir, 1994), we treat clothing and footwear as nondurable goods. Some macro studies, on the other hand, exclude them from nondurables consumption (see Ogaki and Reinhart, 1998; Lettau and Ludvigson, 2001).

[19] Yogo (2006) employs the same data as ours for nondurables consumption and durable goods, and provides a good account of sample features of durables stock relative to nondurables consumption.

[20] Theoretically, stochastic error terms can be justified by allowing for random or stochastic disturbances or errors representing unobserved factors in the (deterministic) indirect utility function (20) (see Brown and Walker, 1989). The demand or budget share system (25) is then explicitly specified as a function of stochastic errors, but these errors are a function of consumption, prices, and durables stock, creating heteroscedasticity (Brown and Walker, 1989). The use of the GMM in this study accounts for heteroscedastic errors.

[21] We consider the following set of instruments: a constant, the time trend, and the time trend squared, private disposable income, and the first-order lags of the prices of durables and nondurables, interest rates on bonds, return rates on equities, private disposable income, stock and quantity of durable goods, and government expenditure.

[22] Some parameter estimates associated with the MPIGLOG indirect utility function (20) are insignificant at conventional levels of significance, which is not uncommon in demand estimation (see also Banks, Blundell, and Lewbel, 1997). However, the insignificance of these parameters does not necessarily mean that the associated variables are not economically important as can be seen from the estimated demand elasticities in Table 2.

[23] For British households, Banks, Blundell, and Lewbel (1997) also find similar values for most commodities.

[24] Campbell (1996) considers labor income growth as a proxy for the return to human capital and treats it as a risk factor for equity returns (see also Lettau and Ludvigson, 2001). To the extent that labor income growth is related to disposable income growth, our analysis suggests that it can be considered as reflecting the effect of liquidity constraints.

[25] Using the estimated SDF and excess returns equations in Table 5, we estimated the conditional Sharpe ratio and Hansen-Jagannathan statistics to see if this gives additional evidence for the equity premium puzzle. The results are not better than the unconditional ones in Table 4.

[26] This analysis appears to be the first attempt to identify and estimate the betas and risk prices for the aggregate stock market using consumption-based risk factors or state variables with time series aggregate or macro data. Previous studies use the well-known Fama-French 25 assets and obtain moderate values of risk prices for factors (see Hodrick and Zhang, 2001). Lettau and Ludvigson (2001) estimate the risk prices using the well-known Fama-MacBeth two-stage procedure with cross sectional data. They also found moderately high risk prices for several factors. It appears that there is much more variation in factor prices for cross sectional asset returns than for aggregate time series macro data.

[27] There has been a significant decline in the equity premium in the U.S. According to Sills (2005) and Lettau, Ludvigson, and Wachter (2008), a decline in volatility of most macroeconomic variables – aggregate consumption growth, in particular – is partly responsible for the decline in the equity premium. In our analysis, volatility comes from consumption growth, commodity price growth, durables stock growth, and disposable income growth.

[28] With $E_t[M_{t+1}] = 1/(1+\rho)$, $E_t[r_{t+1}^e] - r_{t+1}^b = -\text{Cov}_t(r_{t+1}^e, M_{t+1}) / E_t[M_{t+1}]$
$= \zeta(1+\rho)\text{Cov}_t(r_{t+1}^e, \Delta \ln C_{t+1})$. Expressing this in terms of beta and lambda, we get
$\beta_{ct+1} = \text{Cov}_t(r_{t+1}^e, \Delta \ln C_{t+1}) / \text{Var}_t(\Delta \ln C_{t+1})$ and $\lambda_{ct+1} = \zeta(1+\rho)\text{Var}_t(\Delta \ln C_{t+1})$.

REFERENCES

Attanasio, Orazio and Guglielmo Weber. 1995. "Is Consumption Growth Consistent with Intertemporal Optimization? Evidence from the Consumer Expenditure Survey. " *Journal of Political Economy* 103 (December): 1121-1157.

Banks, James, Richard Blundell and Arthur Lewbel. 1997. "Quadratic Engel Curves and Consumer Demand." *Review of Economics and Statistics* 79 (November): 527-539.

Bernanke, Ben. 1985. "Adjustment Costs, Durables, and Aggregate Consumption." *Journal of Monetary Economics* 15(March): 41-68.

Blundell, Richard, Martin Browning, and Costas Meghir. 1994. "Consumer Demand and the Life-Cycle Allocation of Household Expenditures. " *Review of Economic Studies* 61 (January): 57-80.

Brown, Bryan W. and Mary Beth Walker. 1989. "The Random Utility Hypothesis and Inference in Demand Systems." *Econometrica* 57 (July): 815–29.

Browning Martin, Angus Deaton, and Margaret Irish. 1985. "A Profitable Approach to Labor Supply and Commodity Demands over the Life-Cycle." *Econometrica* 55 (May): 503 -544

Campbell, John Y. 1996. "Understanding Risk and Return." *Journal of Political Economy* 104 (April): 298-345.

Campbell, John Y., Andrew W. Lo, and A. Craig MacKinley. 1997. *The Econometrics of Financial Markets.* Princeton University Press: Princeton, NJ.

Cochrane, John H. 1996. "A Cross-Sectional Test of an Investment-Based Asset Pricing Model." *Journal of Political Economy* 104 (June): 572-621.

Cochrane, John H. 2005. *Asset Pricing*. Princeton, NJ: Princeton University Press.

Cooper, Russel J and Keith R. McLaren. 1992. "An Empirically Oriented Demand System with Improved Regularity Properties." *Canadian Journal of Economics* 25 (August): 652–668.

Deaton, Angus. 1992. *Understanding Consumption*. Oxford: Oxford University Press.

Deaton, Angus and John Muellbauer. 1980. *Economics and Consumer Behavior*. Cambridge: Cambridge University Press.

Hall, Robert E. 1978. "Stochastic Implications of the Life Cycle - Permanent Income Hypothesis: Theory and Evidence. " *Journal of Political Economy* 86 (December): 971-987.

Hansen, Lars Peter and Ravi Jagannathan. 1991. "Implications of Security Market Data for

Models of Dynamic Economies." *Journal of Political Economy* 99 (April): 225-262.

Hansen, Lars P. and Kenneth J. Singleton. 1983. "Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns." *Journal of Political Economy* 91 (April): 249-265.

He, Hua and David M. Modest. 1995. "Market Frictions and Consumption-Based Asset Pricing." *Journal of Political Economy* 103 (February): 94-117.

Hodrick, Robert J. and Xiaoyan Zhang. 2001. "Evaluating the Specification Errors of Asset Pricing Models." *Journal of Financial Economics* 62 (November): 327-376.

Hubbard, R. Glenn. 1998. "Capital-Market Imperfections and Investment." *Journal of Economic Literature* 36 (March): 193-225.

Japelli, Julio and Marco Pagano. 1989. "Consumption and Capital Market Imperfections: An International Comparison." *American Economic Review* 79 (December): 1088-1105.

Kim, H.Youn. 1993. "Frisch Demand Functions and Intertemporal Substitution in Consumption, " *Journal of Money, Credit, and Banking* 25 (August): 445–454.

Kim, H. Youn, Keith R. McLaren, and K.K. Gary Wong. 2013. "Empirical Demand Systems Incorporating Intertemporal Consumption Dynamics." *Empirical Economics* 45 (August): 349-370.

Kocherlakota, Narayana. 1996. "The Equity Premium: It's Still a Puzzle." *Journal of Economic Literature* 35 (March): 42-71.

Lettau, Martin and Sydney Ludvigson. 2001. "Resurrecting the (C)CAPM: A Cross-Sectional Test When Risk Premia are Time-Varying. " *Journal of Political Economy* 109 (December): 1238-1287.

Lettau, Martin, Sydney C. Ludvigson, and Jessica A. Wachter. 2008. "The Declining Equity Premium: What Role Does Macroeconomic Risk Play?" *Review of Financial Studies* 21 (July): 1653-1687.

Lewbel, Arthur. 1991. "The Rank of Demand Systems: Theory and Non-parametric Estimation." *Econometrica* 59 (May): 711-730.

Liu, Jun S. 1994. "Siegel's Formula via Stein's Identities." *Statistics & Probability Letters* 21 (October): 247-251.

Ljungqvist, Lars and Thomas J. Sargent. 2004. *Recursive Macroeconomic Theory*, 2nd ed. Cambridge, MA: MIT Press.

Lustig, Hanno N. and Stijng Van Nieuwerburgh. 2005. "Housing Collateral, Consumption Insurance, and Risk Premia: An Empirical Perspective." *Journal of Finance* 60 (June): 1167-1219.

McLaren, Keith R. and K.K.Gary Wong. 2009. "Effective Global Regularity and Empirical Modelling of Direct, Inverse, and Mixed Demand Systems." *Canadian Journal of Economics* 42 (May): 749-770.

McLaughlin, Kenneth J. 1995. "Intertemporal Substitution and λ-constant Comparative Statistics. " *Journal of Monetary Economics* 35 (February): 193-213.

Mehra, R. and Edward C. Prescott. 1985. "The Equity Premium: A Puzzle." *Journal of Monetary Economics* 15 (March): 145-161.

Muellbauer, John. 1975. "Aggregation, Income Distribution, and Consumer Demand." *Review of Economic Studies* 42 (October): 525-543.

Ogaki, Masao and Carman M. Reinhart. 1998. "Measuring Intertemporal Substitution: The Role of Durable Goods." *Journal of Political Economy* 106 (October): 1078-1098.

Piazzesia, Monika, Martin Schneidera, and Selale Tuzelb. 2007. "Housing, Consumption, and Asset Pricing." *Journal of Financial Economics* 83 (March): 531-569.

Runkle, David E. 1991. "Liquidity Constraints and the Permanent-Income Hypothesis." *Journal of Monetary Economics* 27 (February): 73-98.

Sills, Keith. 2005. "Macroeconomic Volatility and the Equity Premium." Working Paper No. 06-1 (December 28), Federal Reserve Bank of Philadelphia.

Soderlind, Paul. 2009. "An Extended Stein's Lemma for Asset Pricing." *Applied Financial Economics Letters* 16 (10): 1005-1008.

Yogo, Motohiro. 2006. "A Consumption-Based Explanation of Expected Stock Returns." *Journal of Finance* 61 (April): 539-580.

Zeldes, Stephen P. 1989. "Consumption and Liquidity Constraints: An Empirical Analysis." *Journal of Political Economy* 97 (April): 305-346.

Table 1: Joint Estimation Results: Budget Share System and Euler Equations
(Standard Errors in Parentheses)

Parameter	Estimate	Parameter	Estimate
α_1	0.000	γ_5	1.1043
			(0.1916)
α_2	0.0399	γ_6	0.5669
	(0.0152)		(0.4330)
α_3	0.0019	η	0.6488
	(0.0069)		(0.1479)
α_4	0.0470	ω	0.0519
	(0.0350)		(0.0893)
α_5	0.0456	ϕ_1	-0.0011
	(0.0274)		(0.0001)
α_6	0.5144	ϕ_2	0.0000
	(0.1048)		(0.0000)
γ_1	1.8086	φ_1	0.0007
	(0.2386)		(0.0000)
γ_2	0.0617	φ_2	8.3691
	(0.0400)		(0.2078)
γ_3	0.3191	ρ	0.0080
	(0.0460)		(0.0008)
γ_4	2.2457	ζ	1.0000
	(0.3281)		

Test for the over-identifying restrictions: χ^2 statistic (df = 111) = 92.343, p value = 0.901		Log-likelihood value =7597.530

	R^2	D-W Statistics	Box-Pierce χ^2 Statistics ($\chi^2_{1\%,6}$ = 16.810)
q_1: Food	0.999	1.792	6.530
q_2: Clothing	0.998	1.998	7.100
q_3: Energy	0.997	2.242	7.710
q_4: Housing Services	0.968	1.861	4.010
q_5: Health Care	0.999	1.164	55.900
q_6: Others	0.996	1.766	6.890

Table 2: Estimated Price and Expenditure Elasticities
(Standard Errors in Parentheses)

Commodity	Mean Budget Share	Expenditure Elasticity	Own Price Elasticity	
			Marshallian	Frisch
Food	0.1409	0.8963 (0.0034)	-0.6960 (0.0581)	-0.4550 (0.0199)
Clothing	0.0654	1.0405 (0.0226)	-1.0005 (0.0001)	-0.8610 (0.0337)
Energy	0.0446	1.1130 (0.0477)	-1.0067 (0.0039)	-0.9016 (0.0225)
Housing Services	0.2101	1.2077 (0.0156)	-1.0754 (0.0075)	-0.5085 (0.0118)
Health Care	0.1299	1.1411 (0.0543)	-1.0183 (0.0029)	-0.7044 (0.0859)
Others	0.4090	1.1348 (0.0196)	-1.0125 (0.0019)	-0.0130 (0.0395)

Table 3: Summary Statistics for the Stochastic Discount Factor and Asset Returns

Mean	
M	0.9913
r^b	0.0130
$R^e = r^e - r^b$	0.0124

Variance-Covariance Matrix		
	M	R^e
M	0.000078	-0.000033
R^e	-0.000033	0.005223

Table 4: Estimated Sharpe Ratio and Hansen-Jagannathan Statistic

	Value
$\dfrac{\bar{R}^e}{s(R^e)}$	0.1716
$\dfrac{s(M)}{\bar{M}}$	0.0101
$\dfrac{-\mathrm{Corr}\left(R^e,\, M\right)s(M)}{\bar{M}}$	0.0006

Note: $\bar{R}^e$ and $\bar{M}$ are the sample mean of R^e and M.

$s(R^e)$ and $s(M)$ are the sample standard deviations of R^e and M.

$\mathrm{Corr}\left(R^e,\, M\right)$ is the sample correlation coefficient of R^e and M.

Table 5: Joint Estimation of the Stochastic Discount Factor and Excess Equity Return Equations (Standard Errors in Parentheses)

Explanatory Variable	M	R^e
Constant	1.0040 (0.0010)	0.0686 (0.3780)
$\Delta \ln C$	-1.7694 (0.0967)	9.5972 (3.5308)
$\Delta \ln \hat{p}_1$	0.1321 (0.0155)	-0.0293 (0.6141)
$\Delta \ln \hat{p}_2$	0.0138 (0.0177)	-0.5663 (0.8008)
$\Delta \ln \hat{p}_3$	0.0361 (0.0037)	-0.1247 (0.1557)
$\Delta \ln \hat{p}_4$	0.3171 (0.0037)	-1.8166 (1.0452)
$\Delta \ln \hat{p}_5$	0.1329 (0.0436)	-1.3775 (1.5158)
$\Delta \ln \hat{p}_6$	0.1979 (0.0374)	-6.7431 (1.6675)
$\Delta \ln k$	0.3980 (0.1079)	-5.8508 (3.8468)
$\Delta \ln Y^d$	0.0035 (0.0175)	-1.7948 (0.7386)
R^2	0.9572	0.0309
DW Statistics	0.8117	1.9960

Note: C = nondurables consumption, p_1 = price of food and beverages, p_2 = price of clothing and footwear, p_3 = price of gasoline and other energy goods, p_4 = price of housing services, p_5 = price of health care, p_6 = price of other goods, k = durables stock, and Y^d = disposable income.

Table 6: Sample Variance-Covariance Matrix of Different Variables

	R_t^e	$\Delta \ln C$	$\Delta \ln \hat{p}_1$	$\Delta \ln \hat{p}_2$	$\Delta \ln \hat{p}_3$	$\Delta \ln \hat{p}_4$	$\Delta \ln \hat{p}_5$	$\Delta \ln \hat{p}_6$	$\Delta \ln k$	$\Delta \ln Y^d$
R_t^e	0.005177	0.000016	-0.000064	0.000000	0.000429	-0.000051	-0.000068	-0.000062	0.000026	0.000001
$\Delta \ln C$	0.000016	0.000055	0.000033	0.000025	0.000209	0.000027	0.000031	0.000026	0.000006	0.000046
$\Delta \ln \hat{p}_1$	-0.000064	0.000033	0.000100	0.000023	0.000102	0.000025	0.000028	0.000027	0.000004	0.000030
$\Delta \ln \hat{p}_2$	0.000000	0.000025	0.000023	0.000063	0.000080	0.000012	0.000027	0.000019	-0.000003	0.000020
$\Delta \ln \hat{p}_3$	0.000429	0.000209	0.000102	0.000080	0.004077	0.000053	0.000033	0.000050	0.000024	0.000118
$\Delta \ln \hat{p}_4$	-0.000051	0.000027	0.000025	0.000012	0.000053	0.000041	0.000034	0.000025	-0.000006	0.000027
$\Delta \ln \hat{p}_5$	-0.000068	0.000031	0.000028	0.000027	0.000033	0.000034	0.000059	0.000031	-0.000011	0.000028
$\Delta \ln \hat{p}_6$	-0.000062	0.000026	0.000027	0.000019	0.000050	0.000025	0.000031	0.000032	-0.000006	0.000023
$\Delta \ln k$	0.000026	0.000006	0.000004	-0.000003	0.000024	-0.000006	-0.000011	-0.000006	0.000023	0.000006
$\Delta \ln Y^d$	0.000001	0.000046	0.000030	0.000020	0.000118	0.000027	0.000028	0.000023	0.000006	0.000100

Note: C = nondurables consumption, p_1 = price of food and beverages, p_2 = price of clothing and footwear, p_3 = price of gasoline and other energy goods, p_4 = price of housing services, p_5 = price of health care, p_6 = price of other goods, k = durables stock, and Y^d = disposable income.

Table 7: Estimated Betas and Risk Prices and Factor Contribution to Excess Returns

Risk Factor	β	λ	$\beta \times \lambda$
$\Delta \ln C$	9.597180	0.000066	0.000635
$\Delta \ln \widehat{p}_1$	-0.029348	0.000023	-0.000001
$\Delta \ln \widehat{p}_2$	-0.566266	0.000028	-0.000016
$\Delta \ln \widehat{p}_3$	-0.124683	0.000171	-0.000021
$\Delta \ln \widehat{p}_4$	-1.816590	0.000023	-0.000041
$\Delta \ln \widehat{p}_5$	-1.377490	0.000030	-0.000041
$\Delta \ln \widehat{p}_6$	-6.743110	0.000025	-0.000167
$\Delta \ln k$	-5.850780	0.000005	-0.000028
$\Delta \ln Y^d$	-1.794750	0.000054	-0.000098
Estimated R^e	N/A	N/A	0.000221
Observed R^e	N/A	N/A	0.012400
Comparison with Other Models			
Estimated R^e (SDF based on (15))	N/A	N/A	0.000033
Estimated R^e (Traditional CCAPM)	0.290909	0.000057	0.000016

Note: C = nondurables consumption, p_1 = price of food and beverages, p_2 = price of clothing and footwear, p_3 = price of gasoline and other energy goods, p_4 = price of housing services, p_5 = price of health care, p_6 = price of other goods, k = durables stock, and Y^d = disposable income.
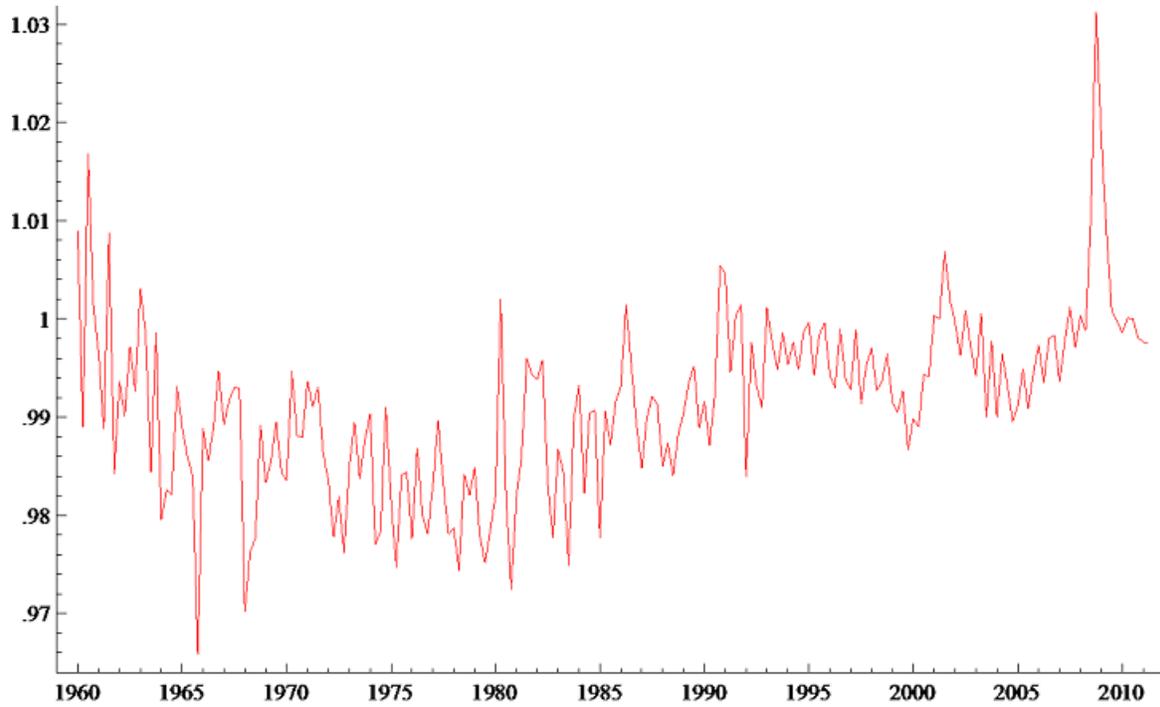
Figure 1: The Constructed Stochastic Discount Factor (M_t)

Figure 2: Bond and Excess Equity Returns

Bond Return r^b

Excess Equity Return R^e