ISSN 1440-771X

MONASH UNIVERSITY
AUSTRALIA

DEPARTMENT OF ECONOMETRICS AND BUSINESS STATISTICS

Does Beta React to Market Conditions? Estimates of Bull and Bear Betas Using a Nonlinear Market Model with an Endogenous Threshold Parameter

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Working Paper 9/2003

Does Beta React to Market Conditions? Estimates of Bull and Bear Betas using a Nonlinear Market Model with an Endogenous Threshold Parameter*

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ABSTRACT

We apply a logistic smooth transition market model (LSTM) to a sample of returns on Australian industry portfolios to investigate whether bull and bear market betas differ. Unlike other studies, our LSTM model allows for smooth transition between bull and bear states and allows the data to determine the threshold value. The estimated value of the smoothness parameter was very large for all industries implying that transition is abrupt. Therefore we estimated the threshold as a parameter along with the two betas in a dual beta market (DBM) framework using a sequential conditional least squares (SCLS) method. Using Lagrange Multiplier type tests of linearity, and the SCLS method our results indicate that for all but two industries the bull and bear betas are significantly different.

JEL Classification: G12, G14, C50, C51

Key Words: Logistic Smooth Transition Market Model (LSTM); Sequential Conditional Least Squares (SCLS); Linearity Tests; Bull/Bear Betas

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* This research was supported in part by a Monash Graduate School scholarship (MGS). We are grateful to Clive Granger, Timo Teräsvirta and Robert Brooks for their helpful suggestions. We would also like to thank the Financial Derivatives Centre for their support.

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1. INTRODUCTION

The simple linear market model has long been used, in tests of the Capital Asset Pricing Model (CAPM), as a benchmark for the performance of mutual funds, and for the measurement of abnormal returns in event studies. See Fama and French (1992), Sharpe (1966) and Fama et. al. (1969) for some examples. The stability of the beta coefficient in the market model over bull and bear market conditions is therefore of considerable interest since if beta does in fact differ with market conditions the single beta estimated over an entire period can result in erroneous conclusions in each case.[1] Direct evidence of the importance of the beta/market condition relationship issue is given by the fact that investment houses regularly publish separate alphas and betas over bull and bear markets, for a range of securities, to offer differing levels of upside potential and downside risk.

Many studies have investigated the relationship between beta risk and stock market conditions. These include studies of individual securities (Fabozzi and Francis (1977), Clinball et. al. (1993) and Kim and Zumwalt (1979)), mutual funds (Fabozzi and Francis (1979) and Kao et. al. (1998)), size based portfolios (Bhardwaj and Brooks (1993), Wiggins (1992) and Howton and Peterson (1998)), risk based portfolios (Spiceland and Trapnell (1983) and Wiggins (1992)) and past performance based portfolios (Wiggins (1992) and DeBondt and Thaler (1987)). While most of these studies have found evidence that beta varies with market conditions, this evidence is mixed and very weak. Furthermore most of these studies used the dual beta market (DBM) model and simple t- and F-testing method in conjunction with crude "up" and "down" market definitions of bull and bear markets to investigate this phenomenon.

[1] In particular with regard to tests of the CAPM, Jagannathan and Wang (1996), Kim and Zumwalt (1973) and Pettingill, Sundaram and Mathur (1995) each use a conditional CAPM to show that when beta is allowed to vary with market conditions, the importance of beta for explaining the cross-section of realized stock returns increases.

Contradicting the existing two-regime market models is the evidence of nonlinearities in stock prices and the evidence of asymmetric regime cycles found by various researchers. The nonlinear behavior of stock prices has been related to various behavioral dynamics of investors. Some prominent behavioral dynamics discussed in the recent papers are: Heterogenous objectives due to different risk profiles and different investment horizons by Peters (1994) and Guillaume et. al. (1995), Herd behavior by Lux (1995) and Heterogeneous beliefs on the market conditions by Brock and LeBaron (1998) and Brock and Hommes (1998).

There has been substantial divergence in the literature in the definition of bull and bear markets used in this context. Even with considerable refinement in the definition, almost all the existing definitions model the transition from bull to bear and vice versa as a discrete jump. Even the latest markov-switching model by Maher and McCurdy (2000) assumes the switch between regimes as abrupt. Such an assumption may contradict recent evidence of heterogeneous beliefs among investors. The transition is said to be abrupt when investors have homogeneous beliefs and they collectively switch from one market condition to another, as they share the same information. The homogeneous beliefs theory is hard to accept unless we believe in a strong form of efficient market theory.

The only study of beta nonstationarity over bull and bear markets, to our knowledge, that has used a continuously changing time varying parameter model is Chen (1982).

In this paper we investigate this phenomenon with three main aims in mind. First, like others we wish to determine whether bull and bear market betas differ. Second, unlike others, we allow for the possibility that transition between regimes is gradual in order to address the heterogeneous beliefs theory and third, unlike others we allow the data to determine an appropriate value of the threshold parameter. With these aims in mind we apply a logistic smooth transition market model (LSTM) to a sample of returns on Australian industry portfolios over the period 1979-2002[2]. While the threshold DBM model used in other studies implies a discrete jump between regimes, our new LSTM model replaces the indicator function with a logistic smooth function that allows for smooth and continuous transition between the two states. In stock markets with many participants, each switching at different times, due to heterogeneous beliefs and differing investment horizons, smooth transition between the states seems more appropriate. In addition the LSTM formulation allows for both the DBM and constant risk models as special cases. Furthermore, this formulation allows the data to choose an appropriate value for the threshold as a parameter of the model. Coutts et. al. (1997) also used a logistic smooth transition framework to model beta nonstationarity in the market model. Instead of a proxy for market conditions, as in our case, they use a polynomial trend as transition variable in an attempt to ascertain the timing of the changes in beta in response to major events.

In contrast to most other studies that have simply used the return on the market portfolio as transition variable, we use a rolling 12-month moving average of market returns to determine movement between bull and bear markets. This series is much smoother than the return on the market portfolio series itself. Therefore in this way, unlike others, we abstract from the small unsystematic and noisy movements to better capture long-run dependencies and drift in the data.

[2] We choose to analyse industry portfolios for two reasons. First, financial analysts recognize that firms within an industry have many common characteristics such as their sensitivity to the business cycle, degree of operating leverage, international tarriffs, raw material availability and technological development. As a result the existence of an industry risk is recognized. Second, given that changes of individual betas within a portfolio tend to be offsetting, one can be more confident of the response of a portfolio beta to changes in market conditions than in the case of a single security beta.

Our nonlinear least squares (NLS) estimates indicate that for all industries transition between bull and bear market states is not smooth and gradual but rather abrupt. This result fails to support the heterogeneous beliefs among investors theory by Brock and LeBaron (1998) and Brock and Hommes (1998). Further, the estimated threshold was negative for most industries and the bull and bear market betas were significantly different for all but two industries. Given that all prior research has arbitrarily imposed a nonnegative threshold value on the data, our finding that the threshold is in fact negative may be the reason for the unprecedented strength of our evidence of differential bull and bear market effects. Finally, we found that most industries spend the vast majority of their time in bull market states.

The plan of the paper is as follows. In section 2 we review the literature on definitions of bull and bear markets and describe the definition that will be used in this study. In section 3 we develop our model and describe the methodologies employed in the study. Section 4 discusses the data used and the results of our analysis, and section 5 finishes with some concluding remarks.

2. PHASES OF THE MARKET

The studies reviewed in section 1 either compared the market index to a critical threshold value to separate "up" from "down" market months, or defined markets as being either bull or bear using a trend based scheme. The "up" and "down" market scheme dichotomizes the market by comparing the market index to a critical threshold value. Wiggins (1993), for example, defined up (down) months as months when the market return was greater (less) than zero. Bhardwaj and Brooks (1993) used the median return on the market portfolio as the demarcating value with which to separate bull from bear months. Wiggins (1992) and Chen (1982) defined up (down) markets as months in which the market excess return was greater (less) than zero. Finally, Fabozzi and Francis

(1977,1979), in one of their three schemes, defined substantial up (down) months as months in which the return on the market portfolio was greater (less) than 1.5 times its standard deviation, thereby separating the market into periods when the market was substantially up or down or neither. Another, though very different, non-trend based way of defining the market is offered by Granger and Silvapulle (2002) who investigate the relative effectiveness of portfolio diversification over market phases. They separate the market into "bullish", "bearish" and "usual" using quantiles of the return distributions, and find that diversification is less effective in bear market states.

Several economists (e.g. Neftci (1984) and Skalin and Teräsvirta (2000)) have suggested that monthly observations on changes in economic time series are noisy and therefore do not reveal the cyclical nature of the data. Cognizant of this fact, several studies have used a trend based approach in their analysis of market conditions. Fabozzi and Francis (1977,1979), for example, used the dates published in Cohen, Zinbarg and Zeikel (1973,1987) to place most months when the market rose into the bull category and market fall months as well as market rise months that were surrounded by falling months into the bear market category. In a similar vein, Gooding and O'Malley (1977) defined two pairs of non-overlapping trend based bull and bear phases. They used daily price changes of the S&P425 Industrial Index to determine months in which major peaks and troughs occurred. Finally, Dukes, Bowlin and MacDonald (1987) used the S&P500 Index, to define bull (bear) markets as periods in which the index increased (decreased) by at least 20% from a trough (peak) to a peak (trough), to analyze the stability of the market model parameters.

More noteworthy are the recent studies by Pagan and Sossounov (2000) and Lunde and Timmermann (2001), who each developed sophisticated trend based definitions of bull and bear markets that focus on systematic movements in the market while ignoring the short-term noise effects. Both papers define bull and bear markets in terms of movements between peaks and troughs, and use pattern recognition dating algorithms to classify bull and bear markets. Both papers found that bull markets tend to last longer than bear markets.

We also use a trend based definition of bull and bear markets in our analysis. To capture the cyclical movement underlying the highly erratic, volatile and noisy nature of the stock market, we use the 12-month moving average of the logarithmic growth of the All Ordinaries Accumulation index to characterize the market[3]. In this way, like Pagan and Sossounov (2000) and Lunde and Timmermann (2001), we intend to capture sustained periods of growth or contraction that are normally associated with the concepts of bull and bear markets. As will be discussed in section 4, the estimated value of the threshold parameter is approximately –0.002 for most industries. A look at Figure 1 reveals that by using the erratic return on the market as transition variable most researchers have implicitly assumed that the market jumps in and out of market phases rapidly and with frequent regularity. Our use of the smoother 12-month moving average of this variable, however, implies a smooth and gradual transition in and out of market phases as can be seen by the way this transition variable hovers around the typical threshold value -0.002, in Figure 2. In support of our approach, as opposed to the simple up and down definitions discussed earlier, we note that Fama (1990) showed that the correlation between stock returns and real economic activity in the U.S.A. is much higher for annual than for monthly returns.

[3] We also estimated our models using 6 and 18 month moving averages. The results were similar so to conserve space we do not report the details here. They are available from the authors upon request.

3. METHODOLOGY

3.1 THE LOGISTIC SMOOTH TRANSITION MARKET MODEL (LSTM)

An unconditional beta for any asset or portfolio can be estimated using the constant risk market model (CRM) regression:

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it} \qquad (1)$$

where R_{it} is the return on asset i for period t, R_{mt} is the return on the market index for period t, $\beta_i = \text{cov}(R_{it}, R_{mt}) / \sigma^2_{mt}$ and ε_{it} is the disturbance term which has zero mean and is assumed to be serially independent and homoscedastic. Under this specification α_i and β_i are constant with respect to time.

A dual beta market model (DBM) can be specified as:

$$R_{it} = \alpha_i + \beta_i R_{mt} + \beta_i^U \cdot D_t \cdot R_{mt} + \varepsilon_{it} \qquad (2)$$

where D_t is a dummy variable defining up and down markets by taking the value 1 if the return on the market portfolio, R_{mt} exceeds some critical value c and zero otherwise. Notice that in this specification the difference between the up and down market value of the slope coefficient is β_i^U

Now consider the logistic smooth transition regression (LSTR) model, henceforth called the logistic smooth transition market (LSTM) model, which has (1) and (2) as special limiting cases:

$$R_{it} = \alpha_i + \beta_i R_{mt} + \beta_i^U \cdot F(R_t^*) \cdot R_{mt} + \varepsilon_{it} \qquad (3)$$

with

$$F(R_t^*) = (1 + \exp[-\gamma(R_t^* - c)])^{-1}, \gamma > 0. \qquad (4)$$

The superscript U signifies an up market differential value of the parameter β, F is the logistic smooth transition function with transition variable R_t^* and critical threshold value c and $\varepsilon_{it} \sim niid(0, \sigma_i^2)$. Note that in our case R_t^* is the 12-month moving average of the return on the market index. Clearly, beta in the state dependent model (3) changes monotonically with the independent variable R_t^* as $F(R_t^*)$ in (4) is a smooth continuous increasing function of R_t^* and takes a value between 0 and 1, depending on the magnitude of $(R_t^* - c)$. When $R_t^* = c$ the value of the transition function is 0.5 and the current regime is half way between the two extreme upper and lower regimes. When $(R_t^* - c)$ is large and positive R_{it} is effectively generated by the linear model $R_{it} = \alpha_i + (\beta_i + \beta_i^U) R_{mt} + \varepsilon_{it}$, while when $(R_t^* - c)$ is large and negative R_{it} is virtually generated by $R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}$. Intermediate values of $(R_t^* - c)$ give a mixture of the two extreme regimes. Note that the DBM obtains as a special case since when γ approaches infinity in (4), $F(R_t^*)$ becomes an indicator function with $F(R_t^*) = 1$ for all values of R_t^* greater than c and $F(R_t^*) = 0$ otherwise. Also notice that the constant risk market model is a special case since as the smoothness parameter, γ, approaches zero, (3) becomes the constant risk market model (CRM). Since there is no theory with which to specify the value of c, we shall use nonlinear least squares to estimate c, along with the other four parameters.

Since the LSTM and DBM models are the same when γ approaches infinity, in cases where the γ estimate is very large a DBM will be estimated using a sequential conditional least squares (SCLS) technique that allows for consistent estimation of the threshold parameter c, along with the coefficient vector. This method involves estimating $\alpha_i, \beta_i,$ and β_i^U conditionally for each value of c as

$$(\hat{\alpha}_i, \hat{\beta}_i\ \hat{\beta}_i^U)' = (\sum_{t=1}^{n} x_t(c) x_t(c)')^{-1} (\sum_{t=1}^{n} x_t(c) y_t) \qquad (5)$$

where $x_t(c) = (1 \;\; R_{mt} \;\; R_{mt} I[R_t^* > c])'$ with $I[R_t^* > c] = 1$ if $R_t^* > c$ and zero otherwise and $y_t \equiv R_{it}$. A grid search over the potential values of c is then conducted to obtain that value of $\hat{c}$ which minimizes the sum of squared errors. In other words $\hat{c} = \arg\min_{c \in C} \hat{\sigma}^2(c)$ where C is the set of allowable threshold values. The final estimates of the parameters are: $\hat{\alpha}_i = \alpha_i(\hat{c}),\ \hat{\beta}_i(\hat{c})\ and\ \hat{\beta}_i^U(\hat{c})$. Note that under the assumption that the errors are normally distributed, the resulting estimates are equivalent to maximum likelihood estimates. Further, Chan (1993) demonstrated that the

estimator $\hat{c} = \arg\min_{c \in C} \hat{\sigma}^2(c)$ is consistent at the rate n even if this assumption does not hold.

3.2 TESTS OF LSTM AGAINST LINEARITY

As mentioned in section 3.1, when γ approaches zero (3) becomes the CRM, thus implying that the constant risk market model is nested in the LSTM model. Thus a natural first step in specifying the model is to test for linearity against the LSTM form. If the null of linearity cannot be rejected we shall conclude that the constant risk market model adequately represents the data generating process. On the other hand, if linearity is rejected we go on to estimate the highly nonlinear LSTM form using the nonlinear least squares (NLS) method.

For cases when γ, the smoothness parameter, is very large, NLS estimates of γ can be very imprecise. When this happens, we estimate the virtually equivalent DBM using the sequential conditional least squares (SCLS) technique discussed above.

From (3) and (4) it can be seen that testing $H_0: \gamma = 0$ is a nonstandard testing problem since all the parameters of (3) are only identified under the alternative $H_1: \gamma \neq 0$. Following Luukkonen et. al. (1988) we replace $F(R_t^*)$ by either a first order or a third order Taylor series linear approximation in a version of (3), that allows the intercept to vary as well, and expand to form an auxiliary model with which to test the equivalent null hypothesis that both β_i^U are not zero or $\gamma \neq 0$ in equation (3). We describe the procedure for the case when a third order Taylor series approximation is used. When a first order Taylor series approximation is used the steps taken are similar.

When a third order Taylor series approximation is used the expanded and reparameterized equation is:

$$\begin{aligned} R_{it} = \phi_0 + \phi_1 R_{mt} + \phi_2 R_t^* + \phi_3 (R_t^*)^2 + \phi_4 (R_t^*)^3 + \phi_5 R_{mt} R_t^* \\ + \phi_6 R_{mt} (R_t^*)^2 + \phi_7 R_{mt} (R_t^*)^3 + u_{it} \end{aligned} \tag{6}$$

where in this reparameterized form the null hypothesis is: $H_0: \phi_j = 0\, (j = 2,...,7)$. The test is then carried out as follows:

(i) Regress R_{it} on $\{1, R_{mt}\}$, form the residuals $\hat{\varepsilon}_{it}\ (t = 1,...,T)$ and the residual sum of squares $SSE_0 = \sum \hat{\varepsilon}_{it}^2$.

(ii) Regress $\hat{\varepsilon}_{it}$ on $\{1, R_{mt}, R_t^*, (R_t^*)^2, (R_t^*)^3, R_{mt}R_t^*, R_{mt}(R_t^*)^2, R_{mt}(R_t^*)^3\}$, form the residuals $\hat{\eta}_{it}\ (t = 1,...,T)$ and $SSE_3 = \sum \hat{\eta}_{it}^2$

(iii) Compute the test statistic $S_3 = [(T-8)/6](SSE_0 - SSE_3)/SSE_3$

Under H_0, S_3 is approximately F distributed. When a first order Taylor series is used the test statistic is denoted S_1 and is derived similarly. In this case the test regressors are $\{1, R_{mt}, R_t^*, R_{mt}R_t^*\}$. An S_1^* test statistic with test regressors $\{1, R_{mt}, R_{mt}R_t^*, R_{mt}(R_t^*)^3\}$ will also be used since it has good power properties when the intercept is also time varying. Because S_1, S_1^* and S_3 can be regarded as Lagrange Multiplier type test statistics they can be expected to have reasonable power. Further, both Luukkonen et. al. (1988) and Petruccelli (1990) have shown that these tests are powerful in small samples when the true alternative is either the smooth transition regression or the abrupt regime switch form. Thus we can expect that in our case there will be reasonable power against the DBM as well. In this paper we will use the S_1, S_1^* and S_3 statistics since though S_3 is not as powerful as S_1 or S_1^* when the up market and down market intercept terms are the same it is generally more powerful if that assumption does not hold.

Another test of nonlinearity that will be used is Tsay's (1989) test. This procedure involves sorting the bivariate observations (R_{it}, R_{mt}) in ascending or descending order based on the ranked order of the corresponding threshold variable R_t^*. A sequence of OLS regressions is then conducted starting with the first b ranked bivariate observations. Then OLS is again performed for the first $b+1$ observations and so on until we come to the last ordered pair. The standardized one-step ahead predictive residuals $\hat{e}_t$ are then regressed on the corresponding (reordered) regressor R_{mt} :

$$\hat{e}_t = \omega_0 + \omega_1 R_{mt} + \varepsilon_t \qquad (7)$$

and the associated F-statistic $F(2, n-b-2) = ((\sum \hat{e}_t - \sum \hat{\varepsilon}_t^2)/2)/(\hat{\varepsilon}_t^2/(n-b-2))$ is calculated. The power of this test comes from the fact that the sequential OLS estimates are consistent estimates of the lower regime parameters as long as the last bivariate observation used in the regression does not belong to the upper regime and there are a sufficient number of observations to estimate the parameters of the lower regime. In this case the predictive residuals are orthogonal to the corresponding regressor R_{mt}. However, for the residuals corresponding to R_t^* greater than the unknown threshold value c the predictive residuals are biased because of the model change at this unknown change point.

4. RESULTS

The data used in this study is the adjusted price relatives information on the 24 Australian Stock Exchange industry classified groupings provided by the Securities Industry Research Centre of the Asia/Pacific (SIRCA). Observations are monthly, from December 1979 to December 2001 for 19 of the industries, giving 265 observations.[4] For the 3 industries Solid Fuels, Oil and Gas, and Entrepreneurial Investors, the observations end on October 1996, giving 203 observations. The Miscellaneous Services industry series ends on August 1997 giving 212 observations and the Tourism and Leisure industry series begins on August 1994, giving 144 observations. A continuously compounded percentage return series for each industry and the market index was calculated as the difference of the log of the prices. Some descriptive statistics for the returns data for each of the 24 industries and the market index are in Table 1. In keeping with other studies of financial time series all 24 return series are leptokurtotic and exhibit negative skewness. Jarque-Bera tests indicate that all 24 return series are not normal.

The Media industry offered the highest and the Miscellaneous Industrials industry the lowest mean return over this period. The standard deviation was highest for the Diversified Industrials industry and lowest for the Property Trust Industry. The constant risk market model beta estimate was highest for the Gold industry and lowest for the Property Trust industry.

[4] In tables 1-3 this corresponds to 253 returns after trimming off the first 11 observations when constructing the 12 month moving average of the return on the market.

As mentioned in section 3, in order to justify the estimation of the nonlinear DBM or LSTM market model formulations instead of the simpler constant risk model we must find evidence of nonlinearity in the data. In Table 2 we report the observed values of the Luukkonen and Tsay test statistics which are used for this purpose. Note that these statistics and their p-values are based on White's (1980) heteroscedasticity consistent standard error estimates. The Luukkonen and/or Tsay test statistics indicate nonlinearity at the 10% level for 16 industries: Gold, Other Metals, Oil and Gas, Diversified Resources, Developers and Contractors, Alcohol and Tobacco, Chemicals, Engineering, Transport, Insurance, Entrepreneurial Investors, Investment and Financial Services, Miscellaneous Services, Miscellaneous Industrials, Diversified Industrials, and Tourism and Leisure. To complement the Tsay tests, we plotted the sum of squared errors obtained from the recursively estimated models against the set of possible thresholds, and found that there was a very sharp and dramatic downward spike evident for each industry. Figure 3 illustrates this for the Building Materials (XBM) industry. The reason we chose to show this graph is that the null of linearity was not rejected for this industry and this graph is typical of all eight industries for which the null was not rejected. For the other 17 industries the downward spike was even more pronounced. Given this result and the fact that a Ramsey Reset test with the nonlinear terms as augmented variables indicated nonlinearity for all remaining eight industries and because several of the 8 industries for which linearity was not rejected the null was only a marginal non-rejection at the 10% level, we model all 24 industries as nonlinear.

We begin modelling the nonlinearity, assuming that transition between the two extreme regimes is gradual, using the LSTM form. The transition parameter, γ, in the estimated LSTM model is large, and imprecisely estimated for all 24 industries. The estimated values of this parameter ranged from a low of 118 to a high of 11,608. Therefore we do not report the results of our LSTM model estimations but instead choose to report the results of the optimal sequential conditional DBM estimations since the DBM representation is simpler and the parameters can be more accurately estimated using the associated closed form solution as opposed to the approximating search algorithm used to estimate the nonlinear LSTM form. Recall that the SCLS method is used to estimate the threshold parameter, c, consistently along with the other parameters in the DBM form. The results are reported in Table 3.

The large γ values indicate abrupt switch from one regime to another as the transition variable crosses the threshold. This may represent the fact that investors, with homogeneous beliefs, switch from one regime to the next instantaneously as a result of the symmetric information flow associated with an efficient market. The estimates of $\alpha_i, \beta_i, \beta_i^U$ and c are very close to those obtained for the LSTM estimations. A Wald test indicated that all but the Food and Household Goods and Building Materials industries had significantly different up and down market betas. In 14 of these 22 cases the down market beta was larger than the up market beta. This is an expected result as the literature suggests that risk is lower in up as compared to down markets. In 8 cases it was the other way around. Thus the 8 industries, Diversified Resources, Chemicals, Engineering, Paper and Packaging, Transport, Media, Insurance, and Miscellaneous Industrials, that had significantly greater bull than bear market betas, can offer upside potential with minimal downside risk. The two industries with the largest differential, Insurance and Miscellaneous Industrial, offer the greatest opportunity in this respect.

Interestingly, the estimated threshold parameter c was negative for 17 industries. This may be the reason that many previous studies failed to find evidence of differential bull and bear market effects. All of the studies to date that have not used trend based definitions of market phase have used arbitrary nonnegative values as demarcating thresholds to separate up from down markets. Our results imply that for most industries returns must be fairly poor before the market will react. Notice also the frequency with which the estimated value of the threshold parameter c is very close to -0.002. In the LSTM estimations for most of these cases the estimates are significantly less than zero. We used Zellner's (1962) multivariate Seemingly Unrelated Regression (SUR) to test whether the threshold value c is significantly different across related groups of industries. For the resources sector composed of Gold, Other Metals, Solid Fuels, Oil and Gas, and Diversified Resources together as a group the null of equal threshold values was rejected. However, for the investments sector composed of the Banks, Insurance, Entrepreneurial Investors, Investment and Financial Services, and Property Trusts industries we could not reject the null of one threshold value for the group. We also could not reject the null for the building sector composed of the Developers and Contractors and the Building Materials industries. The null hypothesis of equal thresholds was also not rejected for the group composed of Alcohol and Tobacco and the Food and Household Services industries. Finally, the group of industries Paper and Packaging, Retail, and Transport were also not found to have significantly different threshold values. Thus there exists some support for the idea that sectors of industries with similar characteristics may have their own unique threshold values.

Notice that for most industries the market is up more often than it is down as indicated by the large number of up market periods, T^U. This result concurs with Pagan and Sossounov (2000) and Lunde and Timmermann (2001) who both used trend based definitions of bull and bear markets to analyze market phase durations and amplitudes.

We performed some residual diagnostics and although heteroscedasticity was present for all industries, we found only mild evidence of serial correlation. The heteroscedasticity has been accounted for using White's (1980) heteroscedasticity consistent standard errors.

Another interesting finding is that although not reported we replaced the 12-month moving average switching variable with other commonly used leading and coincident indicators of economic conditions and repeated the analysis. In particular, given the evidence in Resnick and Shoesmith (2002) and Estrella and Mishkin (1998) who found that when compared to other financial variables, the yield spread between the 10-year T-Bill and the 3-month T-Bill, comes out on top in predicting economic recessions, we used this as our switching variable. We also conducted the analysis using the WESTPAC leading and coincident indicators, Seigel's (1998) suggestion that the business cycle is a key determinant of stock values. For all three of these switching variables the results were qualitatively similar to the results using the 12-month moving average reported in this paper.

5. CONCLUSION

Research on the relationship between beta and market phase offers, at best, only weak evidence that security and portfolio betas are influenced by the alternating forces of bull and bear markets. Most of these studies however, have used the simple threshold DBM model in conjunction with crude "up" and "down" market definitions that involve comparing the return on the market to an arbitrarily chosen nonnegative threshold value, to arrive at their conclusions.

In this paper we reinvestigated this phenomenon. Using a trend based definition of bull and bear markets we tested for differential bull and bear market effects. In addition we investigated the extent to which the transition between regimes was smooth or abrupt. In this way we addressed the hypothesis of heterogenous beliefs among investors. We also let the data determine an appropriate value for the threshold parameter c. To this end we estimated a logistic smooth transition market model which allows for smooth transition between the two extreme regimes while allowing for both the constant risk and DBM models as special cases.

Our LSTM estimates indicated that transition is indeed abrupt for all 24 industries investigated. Thus we can say that investors switch from one regime to the next instantly in response to movement of the transition variable around the threshold value. This we conclude may be attributed to homogeneous beliefs among investors due to information symmetry. Because the estimated value of the smoothness parameter in the LSTM model was very large for all industries we estimated a DBM using the sequential conditional least squares (SCLS) method for each industry. We found that the up market and down market betas were significantly different in 22 cases out of 24 with the down market beta larger than the up market beta for 14 industries and the up market beta greater than the down market beta for 8 industries. This is an expected result given the theory and evidence in the finance literature. The consistently estimated value of the threshold parameter, c, was negative for 17 of the 24 industries, thus indicating that for most industries returns must be fairly poor before the market will react. This contrasts sharply with the assumption of a nonnegative threshold value that has been imposed in prior research. Our finding that the threshold is in fact negative may be the reason for the unprecedented strength of our evidence of differential bull and bear market effects. Finally, consistent with Pagan and Sossounov (2000) and Lunde and Timmermann (2001), we found that for most industries, the stock market spends the vast majority of its time in bull market states.

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TABLE 1
Data Description, Summary Statistics and Constant Risk Beta For Monthly Returns On 24 Australian Industry Portfolios

ASX Industry	Sample Size	Mean	Std. Dev	Skewness	Kurtossis	Jarque Bera	Beta Estimate
Gold	253	0.0005	0.1205	-0.3974	7.6132	229.2	1.345
Other Metals	253	0.0031	0.0918	-1.9960	18.8683	2822.4	1.301
Solid Fuels	191	0.0028	0.0709	-1.1794	9.3428	362.5	0.824
Oil and Gas	191	0.0034	0.0857	-1.3325	10.4550	496.2	1.062
Diversified Resources	253	0.0100	0.1439	-2.7945	73.9801	53017.6	1.097
Developers and Contractors	253	0.0135	0.0700	-3.6030	36.7419	12549.2	1.039
Building Materials	253	0.0108	0.0585	-1.5066	12.4967	1046.4	0.846
Alcohol and Tobacco	253	0.0169	0.0563	-2.2205	20.9518	3605.1	0.708
Food and Household Goods	253	0.0118	0.0596	-1.3854	10.2356	632.8	0.718
Chemicals	253	0.0098	0.0649	-0.5882	8.3708	318.7	0.805
Engineering	253	0.0066	0.0615	-0.8202	6.8853	187.5	0.828
Paper and Packaging	253	0.0084	0.0565	-1.0373	8.0187	310.9	0.726
Retail	253	0.0131	0.0593	-2.0735	21.3646	3736.5	0.779
Transport	253	0.0117	0.0733	-2.2800	20.9636	3620.9	1.017
Media	253	0.0173	0.0937	-1.1227	8.0057	317.3	1.052
Banks	253	0.0162	0.0593	-0.9188	8.7302	381.7	0.774
Insurance	253	0.0137	0.0692	-1.6767	15.4683	1757.3	0.840
Entrepreneurial Investors	191	0.0087	0.0942	-3.9638	35.5699	8895.5	1.138
Investment and Financial Services	253	0.0096	0.0544	-3.6992	38.3985	13786.3	0.773
Property Trusts	253	0.0111	0.0359	-1.5816	15.6687	1797.4	0.423
Miscellaneous Services	201	0.0094	0.0524	-2.1323	17.3456	1866.5	0.662
Miscellaneous Industrials	253	0.0004	0.1137	-7.4317	84.8106	72884.0	1.106
Diversified Industrials	253	0.0130	0.0666	-2.5886	23.7545	4823.4	0.976
Tourism and Leisure	132	0.0121	0.0490	-1.0042	6.7339	90.6	0.893
Australian Market Index Return	253	0.0095	0.0576	-3.5783	35.9713	11999.8	-------

Note: The first eleven observations were trimmed to allow for construction of the 12 month moving average transition variable used in subsequent analysis. For all 24 industries the p-values of the beta estimates based on Whites (1980) Heteroscedasticity Consistent Standard Error estimates are zero.

TABLE 2
Linearity Test Statistics

ASX Industry	S_3	S_1^*	S_1	$TSAY$	$TSAY^*$
Gold (XGO)	3.773 (0.001)	1.358 (0.256)	0.242 (0.785)	0.728 (0.484)	0.175 (0.840)
Other Metals (XOM)	1.878 (0.085)	1.855 (0.138)	0.982 (0.376)	1.082 (0.341)	0.690 (0.503)
Solid Fuels (XSF)	1.172 (0.323)	0.858 (0.434)	1.282 (0.280)	0.892 (0.412)	1.794 (0.169)
Oil and Gas (XOG)	5.175 (0.000)	2.753 (0.044)	0.515 (0.598)	2.140 (0.121)	0.195 (0.823)
Diversified Resources (XDR)	4.408 (0.003)	3.579 (0.015)	0.482 (0.618)	0.838 (0.434)	0.771 (0.464)
Developers and Contractors (XDC)	1.043 (0.398)	0.902 (0.441)	0.844 (0.431)	0.506 (0.603)	2.438 (0.090)
Building Materials (XBM)	1.247 (0.283)	0.814 (0.487)	0.632 (0.532)	0.372 (0.689)	0.307 (0.736)
Alcohol and Tobacco (XAT)	2.173 (0.046)	1.105 (0.348)	0.048 (0.953)	0.282 (0.754)	0.659 (0.519)
Food and Household Goods (XFH)	1.092 (0.368)	1.112 (0.345)	1.609 (0.202)	1.925 (0.148)	0.416 (0.660)
Chemicals (XCE)	2.865 (0.010)	5.455 (0.001)	1.319 (0.252)	0.217 (0.805)	2.202 (0.113)
Engineering (XEG)	1.402 (0.214)	2.379 (0.070)	2.675 (0.071)	2.325 (0.100)	3.101 (0.047)
Paper and Packaging (XPP)	1.463 (0.192)	1.988 (0.116)	0.502 (0.606)	0.533 (0.587)	0.405 (0.667)
Retail (XRE)	0.686 (0.661)	0.487 (0.692)	0.467 (0.628)	1.360 (0.259)	0.710 (0.493)
Transport (XTP)	3.724 (0.002)	7.764 (0.000)	1.377 (0.254)	1.531 (0.219)	1.470 (0.232)
Media (XME)	0.967 (0.448)	1.563 (0.199)	1.436 (0.240)	1.825 (0.163)	1.114 (0.330)
Banks (XBF)	0.498 (0.810)	0.217 (0.805)	0.145 (0.933)	0.859 (0.425)	0.200 (0.819)
Insurance (XIN)	15.843 (0.000)	3.497 (0.016)	4.447 (0.013)	4.955 (0.008)	3.275 (0.040)
Entrepreneurial Investors (XEI)	1.700 (0.123)	1.855 (0.139)	2.720 (0.069)	2.410 (0.093)	4.326 (0.015)
Investment and Financial Services (XIF)	1.709 (0.119)	0.925 (0.429)	1.009 (0.366)	0.088 (0.916)	6.102 (0.003)
Property Trusts (XPT)	0.788 (0.580)	0.063 (0.979)	0.091 (0.913)	0.049 (0.952)	0.400 (0.671)
Miscellaneous Services (XMS)	4.080 (0.001)	1.054 (0.370)	0.415 (0.661)	1.060 (0.349)	0.448 (0.639)
Miscellaneous Industrials (XMI)	3.206 (0.005)	2.262 (0.082)	0.345 (0.709)	1.512 (0.216)	0.140 (0.869)
Diversified Industrials (XDI)	2.784 (0.012)	0.260 (0.854)	0.349 (0.705)	0.364 (0.695)	1.247 (0.289)
Tourism and Leisure (XTU)	3.135 (0.007)	0.319 (0.812)	0.378 (0.686)	0.425 (0.655)	0.572 (0.566)

Note: S_1, S_1^* and S_3 are respectively the Luukkonen first order, augmented first order and third order F-versions of the Lagrange Multiplier type tests of nonlinearity. $TSAY$ and $TSAY^*$ are the Tsay F-statistics for the data sorted in ascending and descending order respectively. P-values are in parentheses next to the calculated values of the statistics. The code names, given by SIRCA, are in parentheses next to the unabbreviated descriptions of the industries.

TABLE 3

Parameter Estimates for threshold models corresponding to threshold value giving minimum sum of squared errors

ASX Industry	α	β^L	β^U	$\beta^L - \beta^U$ Wald	c	T^L	T^U
Gold	-0.010 (-1.655)	2.761 (6.196)	1.270 (16.090)	12.673 (0.001)	-0.01487	15	238
Other Metals	-0.007 (-2.226)	1.382 (27.405)	1.033 (6.811)	4.816 (0.029))	0.01508	181	72
Solid Fuels	-0.003 (-0.857)	0.890 (12.482)	0.583 (5.130)	5.328 (0.022)	0.01798	128	63
Oil and Gas	-0.005 (-1.356)	1.758 (5.130)	1.000 (14.595)	4.743 (0.031)	-0.01222	22	169
Diversified Resources	-0.003 (-1.231)	0.962 (20.042)	1.216 (19.813)	10.958 (0.001)	-0.00285	41	212
Developers and Contractors	0.006 (2.296)	1.175 (24.917)	0.905 (15.406)	12.631 (0.001)	-0.002335	42	211
Building Materials	0.002 (0.742)	0.771 (30.581)	0.922 (17.550)	6.430 (0.12)	-0.002098	43	210
Alcohol and Tobacco	0.012 (4.849)	0.799 (18.483)	0.613 (9.846)	6.253 (0.013)	-0.000947	47	206
Food and Household Goods	0.004 (1.279)	0.644 (15.396)	0.796 (9.535)	2.550 (0.112)	-0.0008054	48	205
Chemicals	0.001 (0.238)	0.763 (16.46)	1.241 (8.993)	10.769 (0.001)	0.03086	231	22
Engineering	-0.004 (-1.454)	0.665 (13.624)	0.999 (16.499)	17.860 (0.000)	-0.000686	49	204
Paper and Packaging	-0.000 (-0.069)	0.613 (18.100)	0.844 (11.530)	7.861 (0.006)	-0.001901	45	208
Retail	0.008 (3.028)	0.912 (22.480)	0.640 (9.884)	12.135 (0.001)	-0.001745	46	207
Transport	0.002 (0.553)	0.752 (5.00)	1.056 (27.041)	3.960 (0.048)	-0.00405	38	215
Media	0.005 (1.087)	0.973 (14.364)	1.379 (8.148)	5.219 (0.023)	0.01844	191	62
Banks	0.009 (3.835)	1.045 (8.290)	0.734 (17.338)	5.440 (0.021)	-0.00405	38	215
Insurance	0.005 (1.484)	0.102 (1.039)	0.891 (16.951)	49.285 (0.000)	-0.01244	21	232
Entrepreneurial Investors	0.002 (0.591)	1.406 (13.219)	0.822 (9.184)	17.756 (0.000)	-0.00285	40	151
Investment and Financial Services	0.004 (2.015)	0.912 (16.474)	0.638 (11.503)	12.137 (0.001)	-0.002961	39	214
Property Trusts	0.008 (4.445)	0.469 (16.507)	0.377 (9.121)	3.220 (0.074)	-0.002098	43	210
Miscellaneous Services	0.004 (1.612)	0.691 (19.865)	0.453 (4.008)	4.068 (0.045)	0.03007	172	29
Miscellaneous Industrials	-0.011 (-1.651)	0.383 (1.982)	1.164 (10.607)	10.910 (0.001)	-0.01222	22	231
Diversified Industrials	0.005 (2.301)	1.037 (31.506)	0.834 (13.483)	8.531 (0.004)	0.00962	129	128
Tourism and Leisure	0.005 (1.617)	1.068 (6.380)	0.743 (7.420)	3.083 (0.082)	0.007876	54	78

Note: t-statistics are in parentheses beneath the parameter estimates. A Wald test of the restriction $\beta^L = \beta^U$ is in column 4. T^L and T^U represent the number of observations in the lower and upper regimes respectively.

Figure 1

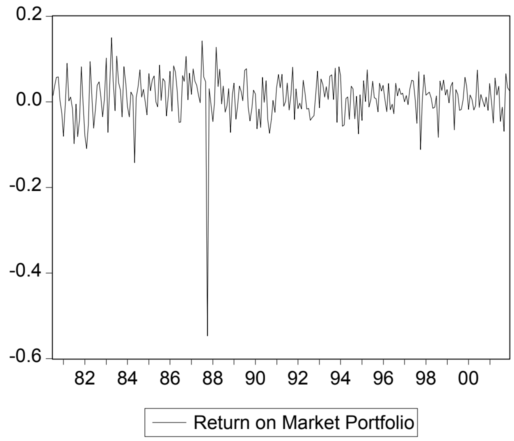


Figure 2

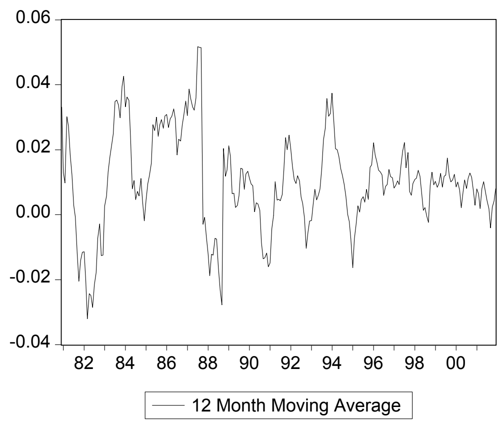
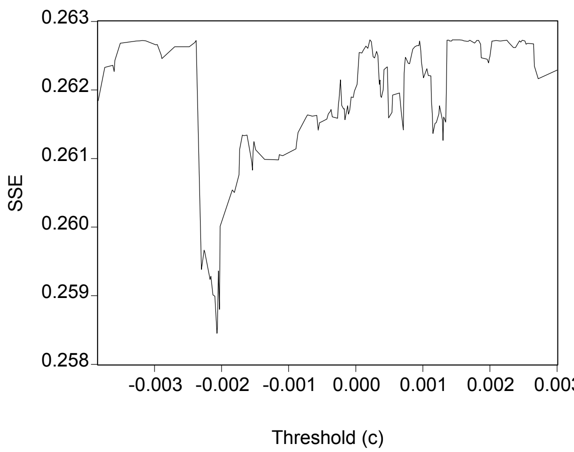


Figure 3



Graph of DBM sum of squared errors against corresponding threshold values for Building Materials Industry