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General Insurance Premiums When Tail Fatness Is Unknown: A Fat Premium Representation Theorem

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General insurance premiums when tail fatness is unknown;
A fat premium representation theorem

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Abstract

Fat-tailed distributions are used to model claims on general insurance contracts under which extremely large claims are a very real possibility. Since estimation of the tail-fatness parameter is notoriously difficult – it is one of the major outstanding statistical/actuarial problems – methods which do not require precise knowledge are valuable.
A characteristic feature of an important class of fat-tailed distributions, Pareto, is that ratios of expected values of large claims in the form

$$\{1+E[X^{(n)}]\}/\{1+E[X^{(n-k)}]\}$$

are independent of sample size. For suitably modelled uncertainty about the tail-fatness parameter, premiums to insurers with constant relative risk aversion can be represented in terms of these ratios.
Premiums increase with the insurers' risk-aversion and depend upon their perception of the fattest-tailed distribution generating claims.

Author keywords: Order statistics, constant relative risk-averse premiums, tail-fatness parameter, beta densities

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1. Introduction: a feature of fat-tailed risk

In an insurance context we define thin-tailed risk as that involving distributions for which moments of all orders exist.
Fat-tailed risk distributions have only finitely many integer moments.
One characteristic which uniquely differentiates fat-tailed risk from thin-tailed risk, and allows for discrimination between degrees of 'tail obesity' is the behaviour of order statistics. We investigate such behaviour for the Pareto distribution.

1.1 Expected value of Pareto order statistics

For Pareto X while the expected value of larger order statistics $X^{(n-k)}$ increases with n, ratios of the form $\{1+E[X^{(n)}]\}/\{1+E[X^{(n-k)}]\}$ *are independent of n* (and depend only on k and δ).
The actual form of the ratio is essential to our premium representation theorem; rather than relegate the proof to an Appendix we prove it now.

The density of the kth order statistic $X^{(k)}$ of a sample of size n from distribution F(x) and density f(x) is:

$$f_k(x) = \frac{n!}{(k-1)!(n-k)!} F(x)^{k-1}(1-F(x))^{n-k} f(x)$$

(see for instance, Kendall and Stuart, 1969, Vol. 1).

When Y (>0) is Pareto with $F(y) = 1-(1+y/\lambda)^{-\delta}$, i.e. we measure Y in units of the scale parameter λ, Y=λX then $F(x) = 1-(1+x)^{-\delta}$ (δ > 0). However it is necessary to have δ > 1 in order that $\mu = E[X] = (\delta-1)^{-1}$ exists and we assume this. When δ > 2 the variance of X exists and $O(n^{-1})$ convergence of the sample mean is ensured by the central limit theorem. Our main focus is thus mainly on values of δ in (1, 2].
This means that:

(i) for 1<δ≤2 the variance of X does not exist,

(ii) for 1 < δ ≤2 the variance of $X^{(n)}$ does not exist but all other order statistics have finite variance (this follows easily from substitution in $f_k(x)$).

Then:

$$E[1+X^{(k)}] = \int_0^\infty (1+x)\delta(1+x)^{-\delta-1}\{1-(x+1)^{-\delta}\}^{k-1}(1+x)^{-\delta(n-k)}\,dx$$

Substitution $u = (1+x)^{-\delta}$ leads directly to:

$$E[1+X^{(k)}] = \Gamma(n-k+1-1/\delta)\Gamma(n+1)/\{\Gamma n+1-1/\delta)\Gamma(n-k+1)\} \qquad (1)$$

Thus $E[1+X^{(n)}] = \frac{n}{(n-1/\delta)} \cdot \frac{n-1}{(n-1-1/\delta)} \cdot \;\cdot\;\cdot\; \frac{1}{(1-1/\delta)}$

$$\sim n^{1/\delta}\Gamma(1-1/\delta) \text{ for large } n, \text{ (See Feller, 1965, p.66)}$$

and

$$\approx n^{1/\delta}/(\delta-1) = \mu n^{1/\delta} \text{ if } \delta \text{ is close to 1.}$$

when we are dealing with large values when $\delta \in (1,2)$.

Similarly, from (1) $E[1+X^{(n-1)}] = \frac{n}{(n-1/\delta)} \cdot \frac{n-1}{(n-1-1/\delta)} \cdot \;\cdot\;\cdot\; \frac{2}{(2-1/\delta)}$

$$= (1-1/\delta)E[1+X^{(n)}]$$

And $E[1+X^{(n-2)}] = \frac{n}{(n-1/\delta)} \cdot \frac{n-1}{(n-1-1/\delta)} \cdot \;\cdot\;\cdot\; \frac{3}{(3-1/\delta)} \cdot \frac{1}{2}$

$$= (1-1/\delta)\{1-1/(2\delta)\}E[1+X^{(n)}]$$

More generally:

$$E[1+X^{(n-k)}] = (1-1/\delta)\{1-1/(2\delta)\}\cdots\{1-1/(k\delta)\}E[1+X^{(n)}] \qquad (2)$$

And

$$E[1+X^{(n)}]/E[1+X^{(n-k)}] = [(1-1/\delta)\{1-1/(2\delta)\}\cdots\{1-1/(k\delta)\}]^{-1} \qquad (3)$$

Some values of $E[1+X^{(n-k)}]$ for various values of n and δ are provided in Table 1 below for k = 0, 1, 2.

$\delta = 2.0$ ($\mu=1$)	n = 10	n = 100	n = 500
$E[1+X^{(n)}]$	5.68	17.75	39.64
$E[1+X^{(n-1)}]$	2.84	8.87	19.82
$E[1+X^{(n-2)}]$	2.13	6.66	14.87

$\delta = 1.5$ ($\mu=2$)	n = 10	n = 100	n = 500
$E[1+X^{(n)}]$	12.57	57.78	168.80
$E[1+X^{(n-1)}]$	4.19	19.26	56.27
$E[1+X^{(n-2)}]$	2.79	12.84	37.51

$\delta = 1.1$ ($\mu=10$)	n = 10	n = 100	n = 500
$E[1+X^{(n)}]$	85.57	691.5	2985.91
$E[1+X^{(n-1)}]$	7.78	62.86	271.45

$E[1+X^{(n-2)}]$	4.24	34.29	148.06

Table 1: Some expected values of (1+Pareto order statistics) for selected sample sizes and values of tail-fatness parameter. Numbers down columns are in constant ratio in each panel.

Thus the ratio

$$E[1+X^{(n)}]/E[1+X^{(n-1)}] = (1-1/\delta)^{-1} \rightarrow \infty \text{ as } \delta \rightarrow 1;$$

for the very fattest-tailed distributions for which the mean exists.

This is in contrast to thin-tailed risk where

$$E[1+X^{(n)}]/E[1+X^{(n-k)}] \rightarrow 1 \text{ for all } k \text{ as } n \rightarrow \infty$$

(heuristically, for all thin-tailed distributions $\delta = \infty$, and the result follows from (3)). We want to determine circumstances under which insurance premiums can be represented in terms of ratios like these.

2. Two classic measures of risk aversion

Noble laureate Kenneth Arrow (1971) defined two measures of risk aversion relating to a utility of wealth function U(x):

$R_A = -U''(x)/U'(x)$ is an investor's absolute risk aversion.

R_A should be decreasing with x (or at least not increasing)

$R_R = -xU''(x)/U'(x)$ is an investors' relative risk aversion

R_R should be increasing with x (or at least not decreasing)

Different functional forms of utility function are implied by holding these two measures constant.

2.1 Investors with constant absolute risk aversion

Investors with *constant* absolute risk aversion s have a utility function

$$U(x) = -\exp(-sx) \quad (s>0) \qquad (4)$$

Assets priced under the expected utility principle

$$E[U(X)] = U(P) \qquad (5)$$

with U(x) as in (4) lead to prices which depend on the Laplace tranform of the density of X.
Specifically,

$$P = -\ln M_X(-s)/s$$

where $M_X(-s)$ is the Laplace transform. These can be found in closed form for assets with both thin and fat-tailed payoff distributions (see for instance Bowers et. Al. 1986, Gay, 2003b).

Utility theory does not lend itself naturally to general insurance premium determination. The reason is that given Jensen's theorem/inequality*, utility functions are the wrong shape for premium pricing*. In the actuarial literature this difficulty is negotiated by using 'special versions' of the expected utility principle as follows:

For an insurer of wealth W the minimum acceptable premium P to insure against loss X is given by:

$$E[U(W+P-X)] = U(W) \qquad (6)$$

For customers with wealth W the rule is:

$$E[U(W-X)] = U(W-P) \qquad (7)$$

The equation need not involve W. There is the 'zero utility principle':

$$E[U(P-X)] = U(0) \qquad (8)$$

See for instance Rolski et al. (1998).

When these rules are applied to investors with constant absolute risk aversion, i.e. U(x) as in (4); all lead to:

$$\mathrm{Exp}(sP) = E[\exp(sX)] \qquad (9)$$

where the *pricing function* (the exponential) *is the right shape for premium determination.*
Acknowledging this, the simplest thing to do is admit a wider class of pricing functions of appropriate shape for premium determination into pricing repertoire. This case is argued in Gay (2003a).

Premiums determined under (9) are given by:

$$P = \ln M_X(s)/s$$

where $M_X(s)$ is the moment generating function of X. Rolski et. al refer to this as 'the exponential principle'.

The premium depends on all the moments of the distribution.

There is no constant absolute risk-averse price for any fat-tailed asset.

2.2 Investors with constant relative risk aversion

Investors with constant relative risk-aversion α ($\alpha > 0$) have

$$U(x) = x^{1-\alpha}/(1-\alpha) \qquad (10)$$

where we interpret $\alpha = 1$ to mean $U(x) = \ln(x)$.

Asset prices determined under (10) thus involve Mellin transforms (see for instance Oberhettinger, 1974). More precisely

$$P = E[X^{1-\alpha}]^{1/(1-\alpha)}$$

where in general α will need to be suitably restricted for different payoff distributions.

For instance if X is a fat-tailed payoff, $f(x) = \delta(1+x)^{-\delta-1}$ ($\delta > 1$)

$$P^{1-\alpha} = \Gamma(2-\alpha)\Gamma(\delta+\alpha-1)/\Gamma(\delta).$$

If the argument of the gamma function is restricted to be non-negative, ($0 < \alpha < 2$) all prices are in the range (δ^{-1}, $(\delta-1)^{-1}$).

2.3 Premiums which reflect constant relative risk aversion

We extend by analogy the pricing principle embodied in (5) to pricing functions $m_\theta(x)$, $x \geq 0$, via the rule

$$m_\theta(P) = E[m_\theta(X)] \qquad (11)$$

where θ is a suitable parameter set.

For constant relative risk aversion we choose $m(x) = x^{\alpha-1}$, ($\alpha \geq 2$, i.e. by analogy with the constant absolute risk aversion case, using the reciprocal of the utility function for α values which give the right premium pricing shape), then we obtain the pricing rule:

$$P^{\alpha-1} = E[X^{\alpha-1}] \qquad (12)$$

The simple device provides a coherent pricing framework. General insurance premiums for both thin-tailed and fat-tailed risk are available. The premium

$$M = E[X^{\alpha-1}]^{1/(\alpha-1)}$$

is classically an increasing function of α ($\alpha \geq 2$).

2.4 Premiums for fat-tailed risk: Pareto premiums

From (12) the Pareto premium P is determined by the rule:

$$P^{\alpha-1} = E[X^{\alpha-1}]$$

$$= \int_0^\infty \delta x^{\alpha-1}(1+x)^{-\delta-1}\,dx$$

$$= \delta B(\alpha, \delta-\alpha+1)$$

$$= \Gamma(\alpha)\Gamma(\delta-\alpha+1)/\Gamma(\delta)$$

If $\alpha = 2$, $P = (\delta-1)^{-1}$.

For Pareto, it is more convenient to price the claim (1+X) and use $m_\alpha(x) = x^{\alpha-1}$. This makes no difference to the asymptotics.

Then

$$P^{\alpha-1} = \delta/(\delta-\alpha+1)$$

$$P = \{1-\frac{\alpha-1}{\delta}\}^{1/(\alpha-1)} \qquad (13)$$

where again $\alpha = 2$ leads to $P = \delta/(\delta-1)^{-1}$ for (1+X) and risk-neutrality. It is also clear that we need $\alpha-1 < \delta$ since Pareto moments exist only up to order δ.

The diagram below depicts premiums based on constant relative risk-aversion for a fat-tailed Pareto risk ($\delta = 1.1$, i.e. $\mu = 10$).

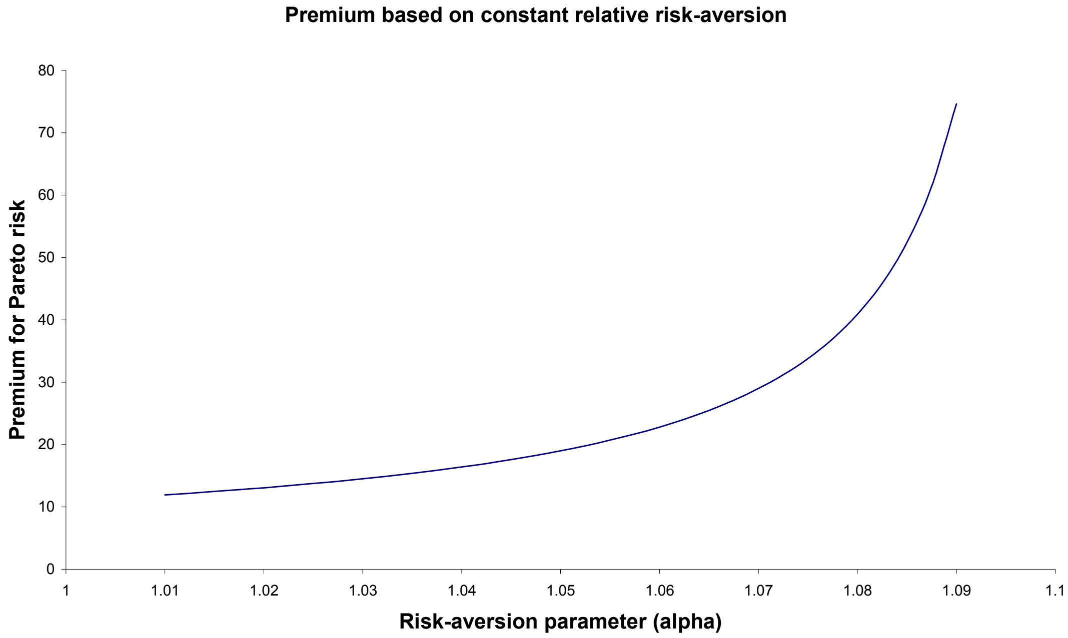

Figure 1: Cost of bearing risk incorporated in the insurance premium for a fat-tailed risk based on constant relative risk aversion ($\delta = 1.1$, $\mu = 10$).

Figure 2 below provides a comparison for insurance premiums involving fat-tailed (Pareto) risk and thin-tailed (negative exponential) risk based on constant relative risk aversion;

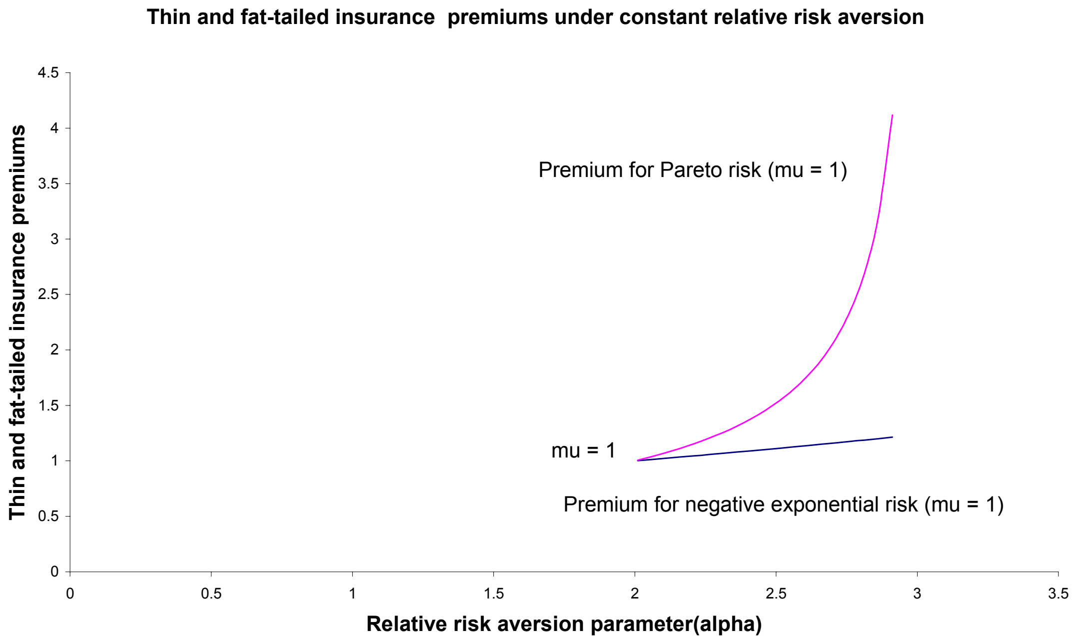

Figure 2: Thin and fat-tailed premiums under constant relative risk aversion compared; Pareto and negative exponential with the same mean ($\mu = 1$).

3. Premiums when the tail-fatness parameter is uncertain; the fat premium representation theorem

One of the major outstanding statistical problems in the area of fat-tailed risk is estimation of the tail-fatness parameter δ.
A recent publication outlining and improving the current state of affairs is Teugels and Vanroelen (2003).
The difficulties attaching to estimation of the tail-fatness parameter in the context of insurance risk are well understood. Large claims are rare observations. The large sample sizes needed to establish estimator precision especially when the tails are very fat, are invariably unavailable; it is in just this situation that the usual estimators are least efficient.

We now show that the tractable form of the constant relative risk-averse premium (13) provides an elegant and transparent premium when the tail-fatness parameter is subject to suitably modelled uncertainty.

We re-parametrize (13) so that

$$P = (1-\rho/\beta)^{-\beta} \qquad (14)$$

where $\rho = 1/\delta$, $\beta = 1/(\alpha-1)$. Because $1 \leq \alpha-1 < \delta$, it follows that $\rho < \beta \leq 1$.

In extreme value theory it is usual to put $\rho = 1/\delta$; so $0 < \rho < \infty$ for the tail-fatness index, but only distributions with $0 < \rho < 1$ have means and we impose this restriction.

In (14), β is a measure of 'risk tolerance'; (the largest value of β, $\beta = 1$ leads to $P = \mu = \delta/(\delta-1)^{-1}$ for claim $(1+X)$; the smallest, $\beta = \rho$ leads to $P = \infty$).

However *β loses this character* when it is used as the limit of integration in the expectation below. Its role as the upper limit of ρ (i.e. the fattest tail under consideration) means that its increase is certain to cause an increase in the insurer's premium.

Suppose ρ is unknown and $\beta_0 = 1/\delta_0$ is a putative or estimated value of ρ.
It ultimately will not matter if we want to adjust this value to a higher one – i.e. a fatter tail; this can be done by increasing β.

In the interests of prudence in premium setting, we want to be able to accord values near ρ_0 high probability density.
In this paper the uncertainty in the value of ρ near ρ_0 is expressed by use of beta densities restricted to $(0, \beta)$.

We have:

$$f_\rho(x) = \nu\beta^{-\nu}x^{\nu-1}, \quad (0 < x < \beta, \nu \geq 1, \rho < \rho_0 \leq \beta < 1)$$

The parameter ν now has the character of risk aversion. As ν increases the uncertainty attaching to ρ reflects the insurer's increasing conviction (or fear) of its value being the largest value ρ_0.
Figure 3 below indicates this.

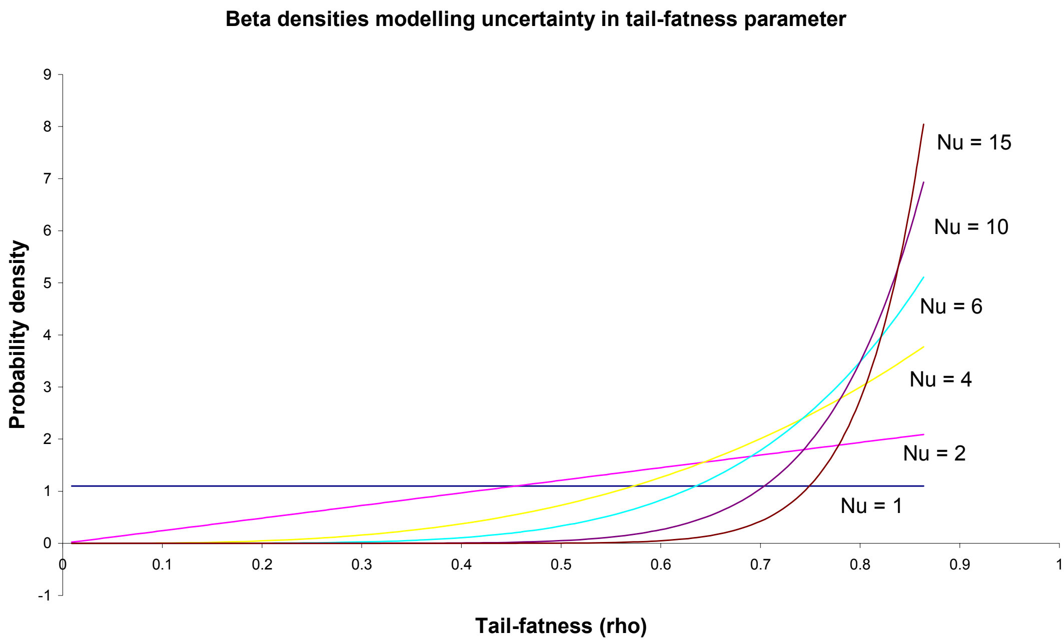

Figure 3: Beta densities with increasing ν reflecting higher conviction of ρ being close to ρ_0 (here *ρ_0 =1/1.1).*

Under these circumstances the expected value of the constant relative risk-averse premium is given by

$$I_\nu = E[P] = \int_0^\beta \nu\beta^{-\nu} x^{\nu-1}(1-\frac{x}{\beta})^{-\beta}\,dx$$

$$= \Gamma(\nu+1)\Gamma(1-\beta)/\Gamma(\nu-\beta+1) \qquad (15)$$

(see for instance, Gradshteyn and Ryzhik, 1965, p.284, 3.191(1))

In general ν need not be an integer, but it is very convenient if it is.

For suppose ν = k. Then (15) gives

$$E[P] = \frac{1\cdot 2\cdot 3\cdots k}{(1-\beta)(2-\beta)\cdots(k-\beta)} \qquad (16)$$

This expected value is a version of equation (3) relating expected values of large order statistics, a correspondence we now demonstrate.

For fixed values of ν minimum premiums occurs when $\beta = 1/\delta_0$ (the 'thinnest' fat tail).
Denote by $P_\nu(\beta)$ the premium determined as an expectation under (10), and P_ν the minimum premium when $\beta_0 = 1/\delta_0$.

If $\nu = 1$, $f_\rho(x) = 1/\beta = \delta_0$ (' the law of equal ignorance')

$$P_1 = (1-\beta_0)^{-1}$$

$$=(1-1/\delta_0)^{-1}$$

$$= E[1+X^{(n)}]/E[1+X^{(n-1)}]$$

i.e. $P = \mu_0 = \delta_0/(\delta_0-1)^{-1}$ the risk-neutral or globally minimum acceptable premium for claim (1+X) when δ_0 is the true tail-fatness.

As β increases towards 1, the premium increases according to

$$P_1(\beta) = 1/(1-\beta).$$

For $\nu = 2$, $f_\rho(x) = 2x/\beta$, the minimum acceptable premium when $\beta_0 = 1/\delta_0$ is given by

$$P_2 = 2/\{(1-\beta_0)(2-\beta_0)\}^{-1}$$

$$= (1-1/\delta_0)^{-1}\{1-1/(2\delta_0)\}^{-1}$$

$$= E[1+X^{(n)}]/E[1+X^{(n-2)}]$$

As β increases, the premium increases via

$$P_2(\beta) = \{(1-\beta)(1-\beta/2)\}^{-1}$$

and so on.

In general; minimum premiums for $\nu = k$ are given by

$$P_k = \{(1-1/\delta_0)(1-1/(2\delta_0)(1-1/(3\delta_0) \cdot \cdot \cdot (1-1/(k\delta_0))\}^{-1}$$

$$= E[1+X^{(n)}]/E[1+X^{(n-k)}]$$

and premiums for claim (1+X) and larger values of ρ_0 by increasing β in the formula

$$P_k(\beta) = \frac{1 \cdot 2 \cdot 3 \cdot \cdot \cdot k}{(1-\beta)(2-\beta) \cdot \cdot \cdot (k-\beta)}$$

Thus we have the remarkable result that *for premiums determined as expected values under constant relative risk aversion, the uncertainty attaching to the tail-fatness parameter (modelled by a restricted beta distribution) is reflected directly in a choice of ratios of expected values of the largest order statistics.*

The relation between minimum premiums and uncertainty in ρ is depicted by the end-points of the graphs in Figure 4 below. As β at each (integer) level increases, premiums increase. The putative value of ρ is $\rho_0 = 1/1.1$ (i.e. δ = 1.1).

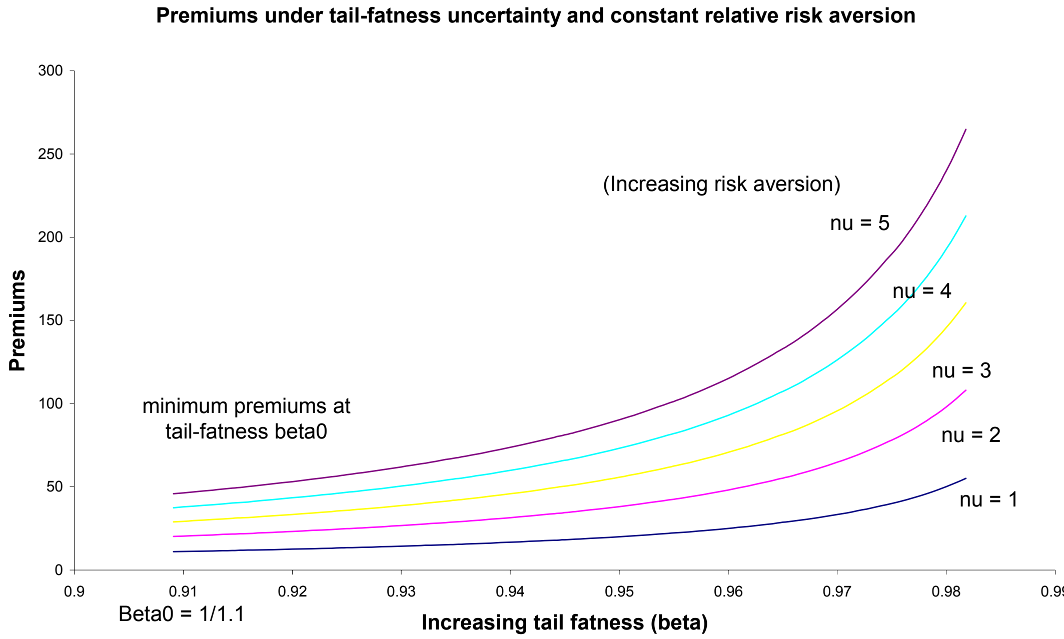

Figure 4; Fat-tailed insurance premiums under constant relative risk aversion when the tail-fatness parameter value is uncertain (in the vicinity of δ_0 = 1.1)

<u>When ν is not an integer</u>

Because I_ν is a continuous function of ν, premiums 'deform continuously' between the integer values. The non-integer curves determined from numerical evaluation of

$$P_\nu(\beta) = \int_0^\beta \nu\beta^{-\nu}x^{\nu-1}(1-\frac{x}{\beta})^{-\beta}\,dx$$

$$= \Gamma(\nu+1)\Gamma(1-\beta)/\Gamma(\nu-\beta+1)$$

'fill in the spaces' along curves between the integer value curves (and between ratios of the sort:

$$E[1+X^{(n)}]/E[1+X^{(n-k)}]$$

The results in the present framework are summarised as follows:

4.1 The fat premium representation theorem

When the claim distribution is Pareto with tail-fatness parameter ρ in the range $0 < \rho \le \beta < 1$, uncertainty in ρ being modelled by restricted beta densities, minimum premiums for insurers with constant relative risk aversion for claim (1+X) are determined as expected values at the value $\rho = 1/\delta$ and are ratios of expected values of the order statistics according to the following scheme:

When uncertainty is modelled by $f_\rho(x) = kx^{k-1}/\beta^k$ on $(0, \beta)$,

$$P_k(\beta) = (1+E[X^{(n)}])/(1+E[X^{(n-k)}]$$

$$= [(1-1/\delta_0)\{1-1/(2\delta_0)\}\cdots(1-1/(k\delta_0))]^{-1}$$

$$= \frac{1\cdot 2\cdot 3\cdots k}{(1-\beta)(2-\beta)\cdots(k-\beta)}$$

In general the value of the minimum premium for a given value of risk-aversion ν is given by

$$P_\nu(\beta) = \Gamma(\nu+1)\Gamma(1-\beta)/\Gamma(\nu-\beta+1)$$

$$= \nu B(1-\beta, \nu)$$

For $k-1<\nu<k$, P_ν falls between

$$P_{k-1} = (1+E[X^{(n)}])/(1+E[X^{(n-k+1)}]$$

and

$$P_k = (1+E[X^{(n)}])/(1+E[X^{(n-k)}]$$

5. Remarks

1. The representation of premium in terms of expected values of the largest claims is interesting. The *raison d'etre* of fat-tailed distributions in insurance is for large claim modelling. Large claims represent greatest risk for insurers. It is pleasing to be able to frame premium structure in terms of ratios of expected values of order statistics which are independent of sample size.

2. Uncertainty about the value of the tail-fatness parameter ($\rho = 1/\delta$) is expressed in very simple terms via the family of beta densities.

3. The insurer has considerable flexibility in management of risk. It must:

(i) choose an estimated or putative value $\beta = 1/\delta_0$ for the fattest tailed risk it is prepared to entertain; this choice will depends on the insurer's level of relative risk-aversion

(ii) decide upon an appropriate value of ν for the beta distribution

$$f_\rho(x) = \nu\beta^{-\nu}x^{\nu-1}, \quad (0 < x < \beta, \nu \geq 1, \rho < \rho_0 \leq \beta<1)$$

As ν increases, premiums increase, so the value of ν can also reflect risk-aversion, in the sense it reflects the degree of belief that fat tails (i.e. high ρ values) dominate the claims generation process.

These two features, although artefacts of the model, reflect practical issues facing insurers.

4. Since premiums are determined as expected values (equation (15)), it is sensible to ask 'when do the actual premiums have variances?' The variance will exist when

$$E[P^2] = \int_0^\beta \nu\beta^{-\nu}x^{\nu-1}(1-x/\beta)^{-2\beta}\,dx$$

exists.

It is easy to show that β must be less than ½ i.e. $\delta > 2$. That is, only when the variance exists for the original claims distribution (the result follows from the fact that we need equation (16) to make sense when β is replaced by 2β).

5. Summary and conclusion

Fat-tailed distributions are used to model insurance business with potential large claims. It is possible to structure premiums in terms of expected values of these claims.

For Pareto claims, when the tail-fatness parameter is unknown but its uncertainty suitably modelled, insurance premiums for investors with constant relative risk-aversion determined as expected values can be represented in terms of ratios of expected values of the largest order statistics.

Premiums for more risk-averse insurers increase with their conviction as to the true value of the tail-fatness parameter being near the extreme end of the range, and with the actual range value δ_0.

The premiums form a continuum, with values between the ratios expressive of non-integer beta distribution models for tail-fatness uncertainty.

The premium structure articulates practical decisions and some of the fears which face an insurer setting premiums

References

Arrow, K. 1971, The theory of risk aversion, in *Essays on the Theory of Risk Bearing*, by Kenneth J. Arrow, Markham, Chicago, 90-120

Bowers, N. Gerber, H. Hickman, J. Jones, D, Nesbitt, C. 1986 *Actuarial Mathematics*, Society of Actuaries, USA

Feller, W. 1965, An Introduction to Probability Theory and Its Applications, 3rd Edn. J. Wiley.

Gay, R. (2004) Pricing risk when distributions are fat-tailed, to appear in J.Appl. Prob. **41a**

Gay, R. 2003b, Fat-tailed premium prices based on constant risk-aversion (submitted to ASTIN Bulletin, preprint available).

Gradsthetyn, I.S. and Ryzhik, I.M. 1965, *Tables of Integrals, Series and Products*, Academic Press

Kendall, M.G. and Stuart, A. 1969, The Advanced Theory of Statistics, Vol. 1, Griffin

Oberhettinger, F.L. 1974, Tables of Mellin Transforms, Springer-Verlag, N.Y.

Rolski, T. Schmidli, H. Schmidt, V. and Teugels, J. 1998, *Stochastic Processes for Insurance and Finance,* J. Wiley

Teugels, J. and Vanroelen, G. 2003 Box-Cox transformations and heavy tailed distributions. to appear in *"Festschrift for C.C. Heyde*", Applied Probability Trust