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A Categorical Time-varying Coefficient Approach**

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# Estimation of Technical Change and Price Elasticities: A Categorical Time-varying Coefficient Approach

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## Abstract

In this paper we outline a new procedure for estimating technical change and price elasticities. Specifically, we propose a categorical time-varying coefficient translog cost function, where each coefficient is expressed as a nonparametric function of a categorical time variable, thereby allowing each time period to have its own set of coefficients. Our application to U.S. electricity firms reveals that compared with the traditional time trend representation of technical change that has remained a cornerstone of the productivity literature, this model offers two major advantages: (1) it is capable of producing estimates of productivity growth that closely track those obtained using the Törnqvist approximation to the Divisia index; and (2) it can solve a well-known problem commonly referred to as “the problem of trending elasticities”, i.e., estimated price elasticities show little temporal variation even when in fact they do.

*JEL classification:* C33; D24.

*Keywords:* Semiparametric Method; Categorical Time-varying Coefficient Model; Technical Change and Productivity.

# 1 Introduction

Productivity and technical change have long been of interest to economists. Beginning with Tinbergen (1942), economists have used a time trend in economic functions (e.g., production functions or their dual representations such as cost functions) to represent the rate at which new technology is introduced into the production unit. Specifically, the time trend approach usually involves adding a linear term in the time trend, a quadratic term in the time trend, and/or interactions of the time trend with factor input prices or outputs to a *fixed* coefficient economic function. Given estimation of the economic function, technical change (thus productivity growth) can then be readily expressed in terms of the estimated coefficients of the economic function, e.g., by taking the derivative of the economic function with respect to the time trend. Due to its empirical tractability, this approach has enjoyed considerable popularity since its introduction, and in fact it remains the dominant econometric approach to productivity measurement.

Despite its popularity, the time trend approach has two major drawbacks. First, it “produces a smooth, slowly changing characterization of the pace of technical change” (Baltagi and Griffin, 1988). This pattern of technical change is not supported by the evidence from index number approaches to calculating rates of technical change. For example, Baltagi and Griffin (1988), using the Divisia productivity index, found that productivity growth in the U.S. electricity industry showed considerable variability across time periods. Feng and Serletis (2008), using the Fisher productivity index, found that productivity growth in the U.S. manufacturing industry varied substantially from year to year. In addition, the smooth, slowly changing pattern of technical change obtained using the time trend approach is also inconsistent with findings in the investment literature (Cooper *et al.*, 1999; Abel and Eberly, 1994) that suggest new technology adoptions occur in a “lumpy” fashion with discrete jumps.

Second, the time trend approach suffers “the problem of trending elasticities”, i.e., estimated price elasticities show little temporal variation even when in fact they do. Taking the standard time-trend normalized quadratic (NQ) functional form for example, the price elasticities produced by this functional form often exhibit little variation over time (see, for example, Feng and Serletis, 2008). This problem occurs mainly because the coefficients for the quadratic terms in input prices, on which price elasticities depend, are constant over time. Unfortunately, this problem is not confined to the NQ functional form.

Taking for example the standard time-trend translog cost function to be examined in this paper, the price elasticity of demand for input  $i$  with respect to input price  $j$  is calculated as  $\eta_{ij} = \beta_{ij}/s_i + s_j - \delta_{ij}$ , where  $\beta_{ij}$  is the coefficient for the quadratic term in log input prices,  $s_i$  ( $s_j$ ) is the cost share of input  $i$  ( $j$ ), and  $\delta_{ij} = 1$  if  $i = j$  and 0 otherwise. As can be seen,  $\delta_{ij}$  is a constant, and  $s_i$  and  $s_j$  are cost shares that do not vary much especially over a short period of time. If  $\beta_{ij}$  is also restricted to be constant over time, then price elasticities (i.e.,  $\eta_{ij}$ 's) are destined to show little temporal variation.

To overcome the first drawback, Baltagi and Griffin (1988) proposed an innovative procedure, which involves first replacing linear and quadratic terms in the time trend<sup>1</sup> in the standard translog cost function with a general index of technical change, and then estimating the index by use of a set of time-specific dummies and their interactions with input prices and output quantities. This procedure offers numerous advantages over the time trend approach, among which a major one is that it is capable of producing estimates of productivity growth that closely track the “observed” productivity growth represented by the Divisia productivity index. It is worth noting that while not discussed in their paper, the Baltagi and Griffin (1988) procedure still suffers the problem of trending elasticities, because the coefficients for the quadratic terms in log input prices in the translog cost function (i.e.,  $\beta_{ij}$  discussed above) remain constant over time. Since the seminal work of Baltagi and Griffin (1988), however, little progress has been made to overcome the two drawbacks. A possible reason is that the then existing econometric methods did not readily lend themselves to alternative methods of representing technical change.

The purpose of this paper is to propose a new procedure to simultaneously overcome the two drawbacks inherent in the standard time trend approach. Specifically, we propose a categorical time-varying coefficient translog cost function, whose primary feature is that each of its coefficients is expressed as a nonparametric function of a categorical time variable (which consists of  $T$  time points or  $T$  categories, where  $T$  is the total number of discrete time periods). The advantage of this feature is that it allows each time period to have its own set of coefficients and thus its own cost function. In other words, the new cost function has time-specific coefficients, thus enabling one to model production technology in a time-specific manner. To see this clearly, let  $t^c = 1, 2, \dots, T$  denote the categorical time variable,  $\mathbf{y}$  denote a vector of outputs, and  $\mathbf{w}$  denote a vector of input prices, then the categorical time-varying coefficient

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<sup>1</sup>Recent studies (see, for example, Example 5.4 of Chen, Gao and Li 2012) show that the time trending component can be chosen by a flexible form to improve model building and estimation.

cost function can be written as  $C^{t^c}(\mathbf{y}, \mathbf{w})$ , where the superscript  $t^c = 1, 2, \dots, T$  is used to indicate that the coefficients of the cost function differs across time periods<sup>2</sup>. In contrast, all the coefficients are restricted to be constant over time in standard time trend models. To distinguish the categorical time-varying coefficient translog cost function from the standard time trend translog cost function, we refer to the former as “the categorical time-varying coefficient model” and the latter as “the standard time trend model”.

The formulation of the categorical time-varying coefficient model is inspired by recent econometric advances in varying-coefficient models (Fan and Zhang, 1999; Fan and Zhang, 2008; Gao and Phillips, 2013), particularly in categorical varying coefficient models (Li et al., 2013). The main feature of varying-coefficient models is that their regression coefficients are not set to be constants but are allowed to evolve with certain characteristics (covariates). Because these models allow the exploration of dynamic features that may exist in the data set (Fan and Zhang 2008), they have received increasing attention in different areas of economics, such as monetary policy (Primiceri, 2005) and growth theory (Durlauf et al., 2001). However, to the best of our knowledge, this is the first study that uses a varying-coefficient model to model technical change and price elasticities.

The categorical time-varying coefficient model has two advantages. First, it is capable of producing estimates of productivity growth that closely track the Törnqvist discrete approximation to the Divisia productivity index (hereafter “the discrete Divisia index”). Index numbers (such as the discrete Divisia and Fisher indexes) are widely used as benchmarks to check the accuracy of productivity estimates obtained from econometric models (Baltagi and Griffin, 1988; Feng and Serletis, 2008). There are two reasons for this. First, these indexes are simple and transparent. As pointed out by Good, Nadiri and Sickles (1997), these indexes “embody less stringent assumptions than are required by econometric models” and thus “provide valuable checks on the results of those (econometric) models”. Hulten (2001) also recommended that researchers “exploit the relative simplicity and transparency of these indexes to serve as a benchmark for interpreting the more complicated results of the parametric (econometric) approach”. Second, these indexes, particularly the Divisia and Fisher indexes, satisfy many desirable statistical properties such as constant quantities, time reversal, and proportionality (Dean, Harper and

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<sup>2</sup>We also estimate a time-varying coefficient translog cost function, where the time variable is treated as a continuous variable. However, we find that this treatment results in less accurate estimates of technical change.

Sherwood, 1996). In this paper, we follow Baltagi and Griffin (1988) and use the discrete Divisia index as a benchmark to evaluate productivity estimates obtained from our model.

We apply the categorical time-varying coefficient model to a panel of 81 electricity firms in the U.S. over the period 1986–1998. We find that the productivity estimates obtained from the new model show considerable year-to-year variations. Particularly, we compare the productivity estimates obtained from the new model and those constructed from the discrete Divisia index and find that the former estimates closely track the latter ones, suggesting that the new model is capable of producing estimates of productivity growth with accuracy comparable to the discrete Divisia index. In addition, we find that our productivity estimates also closely track those obtained using the general index of technical change of Baltagi and Griffin (1988), further confirming the ability of the new model to closely track well-known productivity indexes. In contrast, we find that the standard time trend model yields only a smoothed version of the discrete Divisia index. Moreover, we find that the standard time trend model results in a misleading conclusion regarding the relative importance of technical change and scale effects.

Second, the categorical time-varying coefficient model is capable of producing price elasticities that show considerable year-to-year variations, as indicated by our empirical results. This is not surprising because all the coefficients of this new model, including  $\beta_{ij}$  on which all price elasticities are based, are allowed to vary from time period to time period. In contrast, we find that the standard time trend model generates price elasticities that show little temporal variations. Due to the importance and wide applications of price elasticities, this latter advantage should be of interest to economists in many fields, such as energy economics, public economics, international economics, and labor economics (Hochman *et al.*, 2010; Farrell and Walker, 1999).

The rest of the paper is organized as follows. Section 2 provides a brief summary of two approaches to estimating technical change — the econometric approach and the index number approach. Section 3 presents two competing econometric methods for estimating technical change and price elasticities: the standard time trend model and the categorical time-varying coefficient model. Section 4 discusses the estimation procedure for the categorical time-varying coefficient model. Section 5 deals with data issues. Section 6 compares the empirical results of the two models. Section 7 concludes the paper.

## 2 Overview of Index Number and Econometric Approaches to Measuring Technical Change

Generally, there are four approaches to measuring technical change: the growth accounting approach, the index number approach, the nonparametric frontier approach, and the econometric approach (see, for example, Hulten, 2001). In this section we focus on the two approaches that are related to this paper: the index number approach and the econometric approach.

### 2.1. Index number approach

In general, a total factor productivity growth (TFPG) index is defined as the growth in outputs not attributable to the growth in inputs. An advantage of index numbers is that they “embody less stringent assumptions than are required by econometric models” and thus “provide valuable checks on the results of those (econometric) models” (Good, Nadiri and Sickles, 1997). For example, Baltagi and Griffin (1988) used a Divisia productivity index as a benchmark to evaluate productivity estimates obtained from the general index of technical change. In this paper, we follow Baltagi and Griffin (1988) and use a Divisia productivity index to evaluate productivity estimates obtained from our categorical time-varying coefficient model. Therefore, in what follows we provide an overview of Divisia productivity indexes.

Solow (1957) was the first to propose a Divisia TFP index. Specifically, he began with an aggregate production function with a Hicksian neutral shift parameter and constant returns to scale. Assuming each input is paid the value of its marginal product, Solow (1957) showed that a Divisia TFP index based on this production function was calculated as output growth minus observed cost-share-weighted input growth. Later, Jorgenson and Griliches (1967) generalized this index to a multiple-output framework and showed that a Divisia TFP index based on a multiple output production function was calculated as the observed revenue-share-weighted output growth rate minus the observed cost-share-weighted input growth rate. Formally,

$$TFPG = \sum_{m=1}^M \tilde{s}_m \dot{y}_m - \sum_{n=1}^N s_n \dot{x}_n, \quad (1)$$

where  $y_m$  is output  $m$  ( $m = 1, 2, \dots, M$ );  $x_n$  ( $n = 1, 2, \dots, N$ ) is input  $n$ ; a dot over a variable indicates



the percentage growth of that variable (i.e.,  $\dot{y} = d \ln y / dt$ );  $\tilde{s}_m$  is the observed revenue share for output  $m$ ; and  $s_n$  is the observed cost share for input  $n$ .

While having enjoyed considerable popularity, these two indexes are restricted in the sense that they are obtained under perfect competition and constant returns to scale. Noting this problem, Denny *et al.* (1981) replaced the observed revenue shares with cost elasticity shares, resulting in an index that is valid in the presence of imperfect competition and increasing returns to scale. Specifically, Denny *et al.* (1981) assumed that the underlying production process of a cost-minimizing productive firm was represented by the following cost function

$$C(t) = \mathbf{w}'\mathbf{x} = C(\mathbf{y}, \mathbf{w}, t),$$

where  $C$  is total cost;  $\mathbf{w} = (w_1, \dots, w_N)'$  is an  $N \times 1$  vector of input prices;  $\mathbf{x} = (x_1, \dots, x_N)'$  is an  $N \times 1$  input vector;  $\mathbf{y} = (y_1, \dots, y_M)'$  is an  $M \times 1$  output vector; and  $t$  is a time trend. Denny *et al.* (1981) showed that the conceptually correct expression for TFPG for the cost-minimizing firm was

$$TFPG = - \left( \dot{C} - \sum_{m=1}^M \frac{\epsilon_m}{\epsilon} \dot{y}_m - \sum_{n=1}^N s_n \dot{w}_n \right) = \sum_{m=1}^M \frac{\epsilon_m}{\epsilon} \dot{y}_m - \sum_{n=1}^N s_n \dot{x}_n, \quad (2)$$

where  $\epsilon_m = \partial \ln C(\mathbf{y}, \mathbf{w}, t) / \partial \ln y_m$  is the elasticity of the cost function with respect to output  $m$ , and  $\epsilon = \sum_{j=1}^M \epsilon_j$  is the reciprocal of local returns to scale. The TFPG index defined in (2) has been widely used and discussed in the literature (see, for example, Jorgenson, 1991).

The continuous-time Divisia TFPG indexes given in (1) and (2) must be approximated by reasonable discrete-time approximations as data do not come in continuous-time form. As is well known, the Törnqvist approximation to the Divisia index is “exact” if the production/cost function has the translog form. In other words, the Törnqvist index is not an approximation at all, but is actually exact under right conditions. In addition, because the translog production function is a second order approximation to other production/cost functions, the discrete-time Törnqvist index is a sensible choice even if the underlying true functional form is not a translog (see, for example, Hulten, 2001). The continuous-time Divisia TFPG index given in (1) can be approximated by the following discrete-time Törnqvist index

(Fuss, 1994):

$$TFPG = \sum_{m=1}^M \frac{1}{2} (\tilde{s}_{m,t} + \tilde{s}_{m,t-1}) \Delta \ln y_m - \sum_{n=1}^N \frac{1}{2} (s_{n,t} + s_{n,t-1}) \Delta \ln x_n. \quad (3)$$

With respect to the continuous-time Divisia TFPG index in (2), it can be approximated by the following discrete-time Törnqvist index (Fuss, 1994):

$$TFPG = \sum_{m=1}^M \frac{1}{2} \left( \frac{\epsilon_{m,t}}{\epsilon_t} + \frac{\epsilon_{m,t-1}}{\epsilon_{t-1}} \right) \Delta \ln y_m - \sum_{n=1}^N \frac{1}{2} (s_{n,t} + s_{n,t-1}) \Delta \ln x_n. \quad (4)$$

In this paper, we follow Baltagi and Griffin (1988) and use (3) as a benchmark to compare productivity estimates obtained using the categorical time-varying coefficient model with those obtained using the standard time trend model. This is because (3) does not require econometric specification and estimation of technology. In contrast, (4), while theoretically correct, involves the specification and estimation of a cost function to obtain estimates of elasticities of cost with respect to output. Consequently, TFPG estimates obtained using (4) may vary considerably depending on how the cost function is specified and estimated, making (4) less suitable as a benchmark.

## 2.2. Econometric approach

The econometric approach to productivity measurement involves estimating the parameters of an economic function — a production, cost, or profit function. Productivity growth can then be expressed in terms of the estimated parameters of the economic function. Compared with the index number approach, this approach has three advantages. First, it avoids the need to impose the marginal productivity conditions that are required by the Solow (1957) and Jorgenson and Griliches (1967) TFPG indexes. Second, it gives a full representation of the technology such that the estimated parameters can be used not only in the calculation of productivity but also in the calculation of substitution elasticities and scale parameters. Third, noncompetitive pricing behavior, nonconstant returns, and factor-augmenting technical change can be accommodated to help “explain” the sources of productivity (Hulten, 2001).

The dominant econometric approach is the standard time trend approach, which involves using a time trend in cost or production functions to represent the rate at which new technology is introduced

into the production unit. While this approach has enjoyed considerable popularity since its inception by Tinbergen (1942), no theoretical justification exists for the use of the time trend to proxy technical change. In fact, this approach produces a smooth, slowly changing characterization of the pace of technical change (Baltagi and Griffin, 1988), which is neither supported by the evidence from index number approaches to calculating rates of technical change, nor consistent with findings that suggest production technology proceeds in a “lumpy” fashion with discrete jumps.

Dissatisfaction with the standard time trend approach has led researchers to propose different techniques to overcome the problem associated with the standard time trend approach. For example, Baltagi and Griffin (1988) proposed an innovative procedure that involves the use of time-specific dummies and their interactions with input prices and output quantities. In this paper, we approach the problem from a different perspective — we approach the problem via the coefficients. More specifically, we allow the coefficients of our model (the categorical time-varying coefficient model) to potentially vary over time by expressing each coefficient as a nonparametric function of a categorical time variable. Such a flexible approach allows each time period to have its own set of coefficients and thus its own cost function, which in turn leads to two major advantages: (1) our model is capable of producing estimates of productivity growth that closely track those obtained using the Divisia productivity index, and (2) our model is capable of overcoming the well-known “trending elasticities problem”. In the following section we will explain the categorical time-varying coefficient model in more details.

### **3 Model Specifications**

In this section, we will discuss two competing econometric methods for estimating technical change and price elasticities: the standard time trend model and the categorical time-varying coefficient model. Because the former model serves as the benchmark model, we begin with this model.

#### *3.1. The standard time trend model*

For comparison purposes, we start by specifying the standard time trend model as follows:

$$\begin{aligned}
\ln C(\mathbf{y}, \mathbf{w}, t) &= \sum_{k=2}^K \tilde{\lambda}_k D_k + \alpha_0 + \sum_{i=1}^N \alpha_i \ln w_i + \sum_{j=1}^M \gamma_j \ln y_j + \tau t \\
&+ \frac{1}{2} \sum_{i=1}^N \sum_{n=1}^N \beta_{in} \ln w_i \ln w_n + \frac{1}{2} \sum_{j=1}^M \sum_{m=1}^M \gamma_{jm}^* \ln y_j \ln y_m + \frac{1}{2} \delta t^2 \\
&+ \sum_{i=1}^N \sum_{j=1}^M \psi_{ij} \ln w_i \ln y_j + \sum_{i=1}^N \phi_i t \ln w_i + \sum_{j=1}^M \varphi_j t \ln y_j,
\end{aligned} \tag{5}$$

where  $C$  is total cost;  $\mathbf{w} = (w_1, \dots, w_N)'$  is an  $N \times 1$  vector of variable input prices;  $\mathbf{y} = (y_1, \dots, y_M)'$  is an  $M \times 1$  output vector;  $t$  is a time trend;  $K$  is the number of firms;  $D_k$  ( $k = 2, \dots, K$ ) are firm-specific dummies; and  $\tilde{\lambda}_k$  ( $k = 2, \dots, K$ ) are the corresponding coefficients for the dummies. The usual symmetry restrictions require  $\beta_{in} = \beta_{ni}$  ( $i, n = 1, \dots, N$ ) and  $\gamma_{jm}^* = \gamma_{mj}^*$  ( $j, m = 1, \dots, M$ ). Moreover, homogeneity of degree one in input prices implies the following restrictions:

$$\sum_{i=1}^N \alpha_i = 1, \quad \sum_{i=1}^N \beta_{in} = \sum_{n=1}^N \beta_{ni} = \sum_{i=1}^N \psi_{ij} = \sum_{i=1}^N \phi_i = 0. \tag{6}$$

Although we could estimate (5) directly, efficiency gains can be realized by estimating (5) together with its cost share equations, which can be obtained by applying Shephard's lemma to the cost function (5):

$$s_i = \frac{w_i x_i}{C} = \alpha_i + \sum_{n=1}^N \beta_{in} \ln w_n + \sum_{j=1}^M \psi_{ij} \ln y_j + \phi_i t, \quad i = 1, \dots, N, \tag{7}$$

where  $s_i$  is the cost share for input  $i$ <sup>3</sup>. It is worth noting that the parameters  $\alpha_i$ ,  $\beta_{in}$ ,  $\psi_{ij}$ , and  $\phi_i$  are common across the system of equations.

Given the estimated parameters from equations (5) and (7), it is possible to compute technical change as follows:

$$TC = -\frac{\partial \ln C(\mathbf{y}, \mathbf{w}, t)}{\partial t} = -\left( \tau + \delta t + \sum_{i=1}^N \phi_i \ln w_i + \sum_{j=1}^M \varphi_j \ln y_j \right).$$

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<sup>3</sup>Because the shares in (7) sum to unity, the random disturbances corresponding to the share equations sum to zero, thus yielding a singular covariance matrix of errors. Barten (1969) has shown that full information maximum likelihood estimates of the parameters can be obtained by arbitrarily deleting any one equation. Alternatively, this problem can also be avoided by normalizing the cost and input prices by one of input prices such that only  $N - 1$  share equations are left (Griffith *et al.*, 2000).

Total factor productivity growth can then be computed as in Baltagi and Griffin (1988) and Fuss (1994)

$$TFPG = TC + \left(1 - \sum_{j=1}^M \epsilon_{cy_j}\right) \dot{y}, \quad (8)$$

where for  $j = 1, 2, \dots, M$ ,

$$\epsilon_{cy_j} = \frac{\partial \ln C(\mathbf{y}, \mathbf{w}, t)}{\partial \ln y_j} = \gamma_j + \sum_{m=1}^M \gamma_{jm}^* \ln y_m + \sum_{i=1}^N \psi_{ij} \ln w_i + \varphi_j t$$

is the cost elasticity of the  $j$ th output, and  $\dot{y} = \sum_{j=1}^M \left(\epsilon_{cy_j} / \sum_{j=1}^M \epsilon_{cy_j}\right) \dot{y}_j$  is the cost-elasticity-share weighted growth rate of outputs. According to (8), productivity growth can be decomposed into two components: technical change ( $TC$ ) and scale effects ( $\left(1 - \sum_{j=1}^M \epsilon_{cy_j}\right) \dot{y}$ ). The latter component is positive (negative) in the presence of increasing (decreasing) returns to scale.

### 3.2. The categorical time-varying coefficient model

Our categorical time-varying coefficient model involves specifying each coefficient of the standard translog cost function (without the usual time trend) as a function of a categorical time variable:

$$\begin{aligned} \ln C^{t^c}(\mathbf{y}, \mathbf{w}) &= \sum_{k=2}^K \lambda_k D_k + \alpha_0(t^c) + \sum_{i=1}^N \alpha_i(t^c) \ln w_i + \sum_{j=1}^M \gamma_j(t^c) \ln y_j \\ &+ \frac{1}{2} \sum_{i=1}^N \sum_{n=1}^N \beta_{in}(t^c) \ln w_i \ln w_n + \frac{1}{2} \sum_{j=1}^M \sum_{m=1}^M \gamma_{jm}^*(t^c) \ln y_j \ln y_m \\ &+ \sum_{i=1}^N \sum_{j=1}^M \psi_{ij}(t^c) \ln w_i \ln y_j, \end{aligned} \quad (9)$$

where  $t^c$  is the categorical time variable (which consists of  $T$  time points or  $T$  categories);  $C^{t^c}(\mathbf{y}, \mathbf{w})$  is the cost function for period  $t^c$ ;  $D_k$  ( $k = 2, \dots, K$ ) are firm-specific dummies; and  $\lambda_k$  are the corresponding parameters, which are assumed to be constant over time. Symmetry requires  $\beta_{in}(t^c) = \beta_{ni}(t^c)$  ( $i, n = 1, \dots, N$ ) and  $\gamma_{jm}^*(t^c) = \gamma_{mj}^*(t^c)$  ( $j, m = 1, \dots, M$ ). Linear homogeneity in  $\mathbf{w}$  implies

$$\sum_{i=1}^N \alpha_i(t^c) = 1, \quad \sum_{i=1}^N \beta_{in}(t^c) = \sum_{n=1}^N \beta_{ni}(t^c) = \sum_{i=1}^N \psi_{ij}(t^c) = \sum_{i=1}^N \phi_i(t^c) = 0. \quad (10)$$

Applying Shephard's lemma to the cost function (9) yields the following cost share equations:

$$s_i^{t^c} = \frac{w_i x_i}{C} = \alpha_i(t^c) + \sum_{n=1}^N \beta_{in}(t^c) \ln w_n + \sum_{j=1}^M \psi_{ij}(t^c) \ln y_j, \quad i = 1, \dots, N. \quad (11)$$

Note that the parameters  $\alpha_i(t^c)$ ,  $\beta_{in}(t^c)$ , and  $\psi_{ij}(t^c)$  are common across the cost system.

Given the estimated parameters from equations (9) and (11), it is possible to compute technical change. Noting that  $t^c$  is a categorical variable, technical change from period  $t - 1$  to  $t$  is computed as:

$$TC_{t-1,t} = -\frac{1}{2} \left\{ \left[ \ln C^{t^c}(\mathbf{y}^{t^c-1}, \mathbf{w}^{t^c-1}) - \ln C^{t^c-1}(\mathbf{y}^{t^c-1}, \mathbf{w}^{t^c-1}) \right] + \left[ \ln C^{t^c}(\mathbf{y}^{t^c}, \mathbf{w}^{t^c}) - \ln C^{t^c-1}(\mathbf{y}^{t^c}, \mathbf{w}^{t^c}) \right] \right\},$$

where  $\mathbf{y}^{t^c}(\mathbf{w}^{t^c})$  is the output (input price) vector for period  $t^c$ , and  $\mathbf{y}^{t^c-1}(\mathbf{w}^{t^c-1})$  is the output (input price) vector for the previous period. Given estimation of  $TC$ , the total factor productivity growth can be computed as

$$TFPG = TC + \left( 1 - \sum_{j=1}^M \epsilon_{cy_j}^{t^c} \right) \dot{y}, \quad (12)$$

where, for  $j = 1, 2, \dots, M$ , the cost elasticity of the  $j$ th output,  $\epsilon_{cy_j}^{t^c}$ , is

$$\epsilon_{cy_j}^{t^c} = \frac{\partial \ln C^{t^c}(\mathbf{y}, \mathbf{w})}{\partial \ln y_j} = \gamma(t^c) + \sum_{m=1}^M \gamma_{jm}^*(t^c) \ln y_m + \sum_{i=1}^N \psi_{ij}(t^c) \ln w_i \quad (13)$$

As in the case of the standard time trend model, (12) suggests that productivity growth can be decomposed into two components: technical change ( $TC$ ) and scale effects  $\left( \left( 1 - \sum_{j=1}^M \epsilon_{cy_j}^{t^c} \right) \dot{y} \right)$ .

## 4 Semiparametric Estimation

In this section we detail the semiparametric estimation procedure for the categorical time-varying coefficient model (i.e., (9) – (11)). In doing so, we draw on recent advances in semiparametric estimation for categorical varying coefficient models (Li *et al.*, 2013). For the standard time trend model ((5) – (7)), its estimation procedures have been widely documented in the traditional factor demand literature

(Barten, 1969; Christensen and Greene, 1976) and thus is not discussed in this paper.

Before proceeding to the semiparametric estimation procedure, we first impose the linear homogeneity restrictions in (10). This is done by normalizing the cost and input prices in (9) and (11) by one of the input prices (say,  $w_N$ )

$$\begin{aligned}
\ln \frac{C^{t^c}(\mathbf{y}, \mathbf{w})}{w_N} &= \sum_{k=2}^K \lambda_k D_k + \alpha_0(t^c) + \sum_{i=1}^{N-1} \alpha_i(t^c) \ln \frac{w_i}{w_N} + \sum_{j=1}^M \gamma_j(t^c) \ln y_j \\
&+ \frac{1}{2} \sum_{i=1}^{N-1} \sum_{n=1}^{N-1} \beta_{in}(t^c) \ln \frac{w_i}{w_N} \ln \frac{w_n}{w_N} + \frac{1}{2} \sum_{j=1}^M \sum_{m=1}^M \gamma_{jm}^*(t^c) \ln y_j \ln y_m \\
&+ \sum_{i=1}^{N-1} \sum_{j=1}^M \psi_{ij}(t^c) \ln \frac{w_i}{w_N} \ln y_j,
\end{aligned} \tag{14}$$

and

$$s_i = \alpha_i(t^c) + \sum_{n=1}^{N-1} \beta_{in}(t^c) \ln \frac{w_n}{w_N} + \sum_{j=1}^M \psi_{ij}(t^c) \ln y_j, \quad i = 1, \dots, N-1. \tag{15}$$

This normalization method has been widely used to impose linear homogeneity property on economic functions (Griffith *et al.*, 2000).

Equations (14) and (15) can then be combined to form a system of  $N$  equations that, upon appending idiosyncratic error terms, takes the common seemingly unrelated regression (SUR) form. In estimating this SUR system, we follow the spirit of Bai (2009) and use an iteration scheme, where each iteration involves two steps: 1) given the time-invariant coefficients for the firm-specific dummy variables (i.e.,  $\lambda_k$ ,  $k = 2, \dots, K$ ), we compute the time-varying coefficients for the non-dummy variables (i.e.,  $\alpha_0(t^c)$ ,  $\alpha_i(t^c)$ ,  $\gamma_j(t^c)$ ,  $\beta_{in}(t^c)$ ,  $\gamma_{jm}^*(t^c)$ , and  $\psi_{ij}(t^c)$ ) using the semiparametric estimation procedure developed by Li *et al.* (2013); and 2) given the time-varying coefficients, compute the time-invariant coefficients. As pointed out by Bai (2009), this iteration scheme is very robust and has an excellent convergence property. Considering that the second step is straightforward, we elaborate on the first step in what follows.

The SUR system in the first step is subject to many cross-equation restrictions implied by Shephard's lemma. Specifically, as can be seen from (14), the coefficients  $\alpha_i(t^c)$ ,  $\beta_{in}(t^c)$ , and  $\psi_{ij}(t^c)$  are common across the cost and share equations. To allow for such equality restrictions, we follow Wooldridge

(2010, p.188) and Cameron and Trivedi (2005, p.210) and redefine the regressors and coefficients given in (14) and (15) so that the SUR system in the first step can be estimated by least square method. Specifically, we first define the dependent variable vector and the disturbance vector. Let  $\mathbf{q}_l$  be an  $N \times 1$  vector representing the dependent variables associated with the  $l$ th observation with the first element being  $\left( \ln \frac{C_i^{t^c}(\mathbf{y}, \mathbf{w})}{w_{lN}} - \sum_{k=2}^K \hat{\lambda}_k D_{lk} \right)$  and the second to the last element being the  $N - 1$  shares (i.e.,  $s_{l,i}$ ,  $i = 1, \dots, N - 1$ ), and  $\mathbf{u}_l = (u_{l1}, \dots, u_{lN})'$  be an  $N \times 1$  disturbance vector, whose variance-covariance matrix is  $\Sigma = E(\mathbf{u}_l \mathbf{u}_l' | \mathbf{X}_l)$ . We then define regressors and coefficients equation by equation. For the normalized cost equation, let  $\mathbf{X}_{l1}$  be a  $1 \times \frac{N^2 + M^2 + 2MN + M + N}{2}$  vector representing all the non-dummy regressors in the normalized cost function (i.e., (14)):  $\boldsymbol{\beta}(t^c)$  be the corresponding coefficients, i.e., all coefficients for non-dummy variables in (14). The first equation of the  $N$  equation system can be written as

$$q_{l1} = \mathbf{X}_{l1} \boldsymbol{\beta}(t^c) + u_{l1}.$$

For the first normalized share equation, we still use  $\boldsymbol{\beta}(t^c)$  as our redefined coefficient vector. However, the regressor vector,  $\mathbf{X}_{l2}$ , is redefined in such a way that  $\mathbf{X}_{l2} \boldsymbol{\beta}(t^c)$  is equal to the right hand side of the first normalized share equation. Formally,  $\mathbf{X}_{l2} = \left( 0, 1, \mathbf{0}_{N+M-2}, \ln \frac{w_{l2}}{w_{lN}}, \dots, \ln \frac{w_{l,N-1}}{w_{lN}}, \mathbf{0}_{\frac{N^2 - 5N + 12}{2}}, \ln \frac{w_{l1}}{w_{lN}}, \mathbf{0}_{N-2 + \frac{M(M+1)}{2}}, \ln y_{l1}, \mathbf{0}_{N-2}, \dots, \ln y_{lM}, \mathbf{0}_{N-2} \right)$ , where  $\mathbf{0}_p$  is a  $1 \times p$  vector of zeros. Thus, the second equation of the  $N$ -equation system can be written as

$$q_{l2} = \mathbf{X}_{l2} \boldsymbol{\beta}(t^c) + u_{l2}.$$

The  $i^{th}$  ( $i = 2, 3, \dots, N - 1$ ) normalized share equation can be redefined in a similar manner. Stacking all the  $N$  equations associated with the  $l$ th ( $l = 1, \dots, KT$ ) observation in the data set yields:

$$\mathbf{q}_l = \mathbf{X}_l \boldsymbol{\beta}(t^c) + \mathbf{u}_l, \tag{16}$$



The entire system of equations associated with the  $KT$  observations can then be written as

$$\begin{bmatrix} \mathbf{q}_1 \\ \vdots \\ \mathbf{q}_{KT} \end{bmatrix} = \begin{bmatrix} \mathbf{X}_1 \\ \vdots \\ \mathbf{X}_{KT} \end{bmatrix} \boldsymbol{\beta}(t^c) + \begin{bmatrix} \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_{KT} \end{bmatrix}, \quad (17)$$

which can be written more compactly as

$$\mathbf{q} = \mathbf{X}\boldsymbol{\beta}(t^c) + \mathbf{u}, \quad (18)$$

where  $\mathbf{q}$  and  $\mathbf{u}$  are  $NKT \times 1$  vectors and  $\mathbf{X}$  is a  $NKT \times \frac{N^2+M^2+2MN+M+N}{2}$  matrix. The  $NKT \times 1$  disturbance vector  $\mathbf{u}$  has the following variance-covariance matrix:  $\boldsymbol{\Omega} = E(\mathbf{u}\mathbf{u}') = \mathbf{I}_{KT} \otimes \boldsymbol{\Sigma}$ , where  $\mathbf{I}_{KT}$  is an identity matrix of dimension  $KT$ .

The least-squares estimator of  $\boldsymbol{\beta}(t^c)$  in (18) is the solution to

$$0 = \mathbf{X}'\mathbf{L}(t^c)^{1/2} \boldsymbol{\Omega}^{-1} \mathbf{L}(t^c)^{1/2} [\mathbf{q} - \mathbf{X}\boldsymbol{\beta}(t^c)], \quad (19)$$

where  $\mathbf{L}(t^c)$  is a  $NKT \times NKT$  diagonal matrix with the  $i$ th ( $i = 1, 2, \dots, NKT$ ) diagonal element defined as in Li *et al.* (2013)<sup>4</sup>

$$l(t_i^c, t^c, \lambda) = \begin{cases} 1, & \text{when } t_i^c = t^c \\ \lambda, & \text{when } t_i^c \neq t^c \end{cases},$$

where  $\lambda$  is a smoothing parameter<sup>5</sup>. The range of  $\lambda$  is from 0 to 1.  $\lambda = 0$  leads to an indicator function, while  $\lambda = 1$  gives a uniform weight function.

<sup>4</sup>We also estimate the time-varying coefficient model (i.e., (9) – (11)), where the time variable ( $t^c$ ) is treated as an ordered discrete variable. However, we find that this treatment results in less accurate estimates of technical change. A possible reason is that in our particular case, the values (i.e., 1, 2, ..., T) of the time variable are used only as labels for cost functions of different time periods. Thus, the time variable should be treated as unordered rather than ordered.

<sup>5</sup>As in Li *et al.* (2013), we consider the case where  $K$  (the number of cross-sectional units) is large and  $T$  is small.

Solving for  $\beta(t^c)$  in (19) leads to the estimator

$$\widehat{\beta}(t^c) = \left[ \mathbf{X}' \mathbf{L}(t^c)^{1/2} \boldsymbol{\Omega}^{-1} \mathbf{L}(t^c)^{1/2} \mathbf{X} \right]^{-1} \mathbf{X}' \mathbf{L}(t^c)^{1/2} \boldsymbol{\Omega}^{-1} \mathbf{L}(t^c)^{1/2} \mathbf{q}, \quad (20)$$

where the error covariance matrix  $\boldsymbol{\Omega}$ , as in the case of the standard feasible generalized least squares (FGLS) method for SUR models (Wooldridge, 2010 p.176), can be estimated by using the consistent system estimator which ignores the information in the variance-covariance matrix (i.e., by setting  $\boldsymbol{\Omega} = I_{NKT}$ ). In this case, (20) reduces to

$$\widetilde{\beta}(t^c) = [\mathbf{X}' \mathbf{L}(t^c) \mathbf{X}]^{-1} \mathbf{X}' \mathbf{L}(t^c) \mathbf{q}. \quad (21)$$

Using (21), we can obtain the  $N \times 1$  vector of residuals associated with the  $l^{th}$  observation as  $\widetilde{\mathbf{u}}_l = \mathbf{q}_l - \mathbf{X}_l \widetilde{\beta}(t^c) = [\widetilde{u}_{l1}, \widetilde{u}_{l2}, \dots, \widetilde{u}_{lN}]'$ . The estimate of the variance covariance matrix is given by  $\widehat{\boldsymbol{\Sigma}} = \frac{1}{K^T} \sum_{l=1}^{K^T} \widetilde{\mathbf{u}}_l \widetilde{\mathbf{u}}_l'$ , and hence we can construct our estimator of  $\boldsymbol{\Omega}$ .

The choice of the smoothing parameter  $\lambda$  is crucial. When  $\lambda = 0$ , our estimator is equivalent to estimating  $T$  independent cost functions with one for each period, whereas when  $\lambda = 1$ ,  $\widehat{\beta}(t^c)$  becomes unrelated to  $t^c$ , implying that the coefficients are constant over time. When choosing  $\lambda$ , we follow Li *et al.* (2013) and minimize the following least squares cross-validation:

$$CV(\lambda) = \frac{1}{NKT} \sum_{j=1}^{NKT} \left[ g_j - \mathbf{z}_j \widehat{\beta}_{-j}(t_j^c) \right]^2,$$

where  $g_j$  is the  $j$ th row of  $\mathbf{q}$ ,  $\mathbf{z}_j$  is the  $j$ th row of  $\mathbf{X}$ , and the leave-one-out estimates of the time-varying coefficients is expressed as

$$\widehat{\beta}_{-j}(t_j^c) = \left( \mathbf{X}'_{-j} \mathbf{L}_{-j}(t_j^c)^{1/2} \boldsymbol{\Omega}_{-j}^{-1} \mathbf{L}_{-j}(t_j^c)^{1/2} \mathbf{X}_{-j} \right)^{-1} \mathbf{X}'_{-j} \mathbf{L}_{-j}(t_j^c)^{1/2} \boldsymbol{\Omega}_{-j}^{-1} \mathbf{L}_{-j}(t_j^c)^{1/2} \mathbf{q}_{-j}$$

and the notation  $-j$  implies that the  $j^{th}$  row is removed from  $\boldsymbol{\Omega}$ ,  $\mathbf{L}(t^c)$ ,  $\mathbf{X}$  and  $\mathbf{q}$ .

## 5 Data

Our data was provided by Rungsuriyawiboon and Stefanou (2007) and consisted of annual time-series data for 81 privately investor-owned electric utilities in the United States over the period 1986 – 1998. The choice of U.S. electric utilities is particularly relevant considering the numerous studies of productivity in this industry.

With regard to the specification of outputs and inputs, one output is specified (i.e.,  $M = 1$ ) and represented by net steam electric power generation in megawatt-hours, which is defined as the amount of power produced using fossil-fuel fired boilers to produce steam for turbine generators during a given period of time. On the input side, three inputs are specified (i.e.,  $N = 3$ ): the aggregate of labor and maintenance, fuels, and capital stocks. The aggregate price of labor and maintenance is a cost-share weighted price for labor and maintenance. The price of labor is a company-wide average wage rate. The price of maintenance and other supplies is a price index of electrical supplies from the Bureau of Labor Statistics. The weight is calculated from the labor cost share of nonfuel variable costs for those utilities with entirely steam power production. Quantities of labor and maintenance equal the aggregate costs of labor and maintenance divided by a cost-share weighted price for labor and maintenance. The price of fuel aggregate is a Törnqvist price index of fuels (i.e., coal, oil, gas). The fuel quantities are calculated by dividing the fuel expenses by the Törnqvist price of fuel aggregate. The values of capital stocks are calculated by the valuation of base and peak load capacity at replacement cost to estimate capital stocks in a base year and then updating it in the subsequent years based upon the value of additions and retirements to steam power plant. The price of capital is the yield of the firm's latest issue of long-term debt adjusted for appreciation and depreciation of the capital good using the Christensen and Jorgenson (1970) cost of capital formula.

## 6 Empirical Results

We estimate the standard time trend model and the categorical time-varying coefficient model separately for the electric utilities, and report the estimated parameters and their associated standard errors in Tables 1.1 – 1.2. For the categorical time-varying coefficient model, our estimated smoothing parameter ( $\lambda$ ) is

pretty close to zero (0.039), indicating that the categorical time variable,  $t^c$ , has a strong impact on the coefficients of the model.

With the estimated parameters, we check concavity in input prices by evaluating the Hessian matrix at each observation. However, concavity violations are observed for 149 of the 1,053 observations in the standard time trend model and for 340 of the 1,053 observations in the categorical time-varying coefficient model. A possible reason for the large number of violations in both models is that some electric utilities may not be cost minimizers throughout the sample period. Specifically, until the mid-1990s electric utilities in the U.S. typically operated as state-regulated monopolies under the jurisdiction of regulatory commissions in each state. Because of asymmetric information between these regulatory bodies and producers, effort-averse managers may engage in inefficient activities, resulting in possible deviations from cost-minimization (Fabrizio *et al.*, 2007). These distortions may be amplified because electric power rates were set by the asymmetrically informed regulatory bodies (Laffont and Tirole, 1993; Fabrizio *et al.*, 2007).

### *6.1. Comparison of the estimated coefficients of the two models*

It is of interest to compare the estimated coefficients of the two models. As can be seen from Tables 1.1 and 1.2, while the point estimates of the coefficients for the standard time trend model are constant over time (as expected), those for the categorical time-varying coefficient model vary considerably over time. Taking  $\alpha_1$  for example, its point estimate obtained from the former model is 0.370, whereas that obtained from the latter model varies markedly from 0.120 to 0.614. A possible reason for the coefficient variation in the latter model is that during the sample period the US electric industry underwent a profound restructuring and ceased to be operated by regulated monopolies, resulting in adjustments in the behavior and technology of producers in the sector (Fabrizio *et al.*, 2007).

We can also formally compare the estimated coefficients of the two models. To this end, we first

rewrite the standard time trend model as follows:

$$\begin{aligned} \ln C(\mathbf{y}, \mathbf{w}, t) = & \sum_{k=2}^K \tilde{\lambda}_k D_k + \left( \alpha_0 + \tau t + \frac{1}{2} \delta t^2 \right) + \sum_{i=1}^N (\alpha_i + \phi_i t) \ln w_i + \sum_{j=1}^M (\gamma_j + \phi_j t) \ln y_j \\ & + \frac{1}{2} \sum_{i=1}^N \sum_{n=1}^N \beta_{in} \ln w_i \ln w_n + \frac{1}{2} \sum_{j=1}^M \sum_{m=1}^M \gamma_{jm}^* \ln y_j \ln y_m + \sum_{i=1}^N \sum_{j=1}^M \psi_{ij} \ln w_i \ln y_j. \end{aligned} \quad (22)$$

If we treat  $(\alpha_0 + \tau t + \frac{1}{2} \delta t^2)$  in (22) as the coefficient for the constant term,  $(\alpha_i + \phi_i t)$  as the coefficient for  $\ln w_i$ , and  $(\gamma_j + \phi_j t)$  as the coefficient for  $\ln y_j$ , then a comparison of (22) with the categorical time-varying coefficient model in (9) reveals that the former model is a special case of the latter model at  $t = t^c = 1, 2, \dots, T$ . More specifically, let  $\theta_0(t^c)$  denote the vector of coefficients of (22) at period  $t^c$  and  $\theta(t^c)$  denote the vector of coefficients of the categorical time-varying coefficient model at the same period, then  $\theta(t^c)$  nests  $\theta_0(t^c)$  as a special parametric case. Thus, it would be of interest to test if  $\theta(t^c)$  is of the parametric form  $\theta_0(t^c)$  at  $t = t^c = 1, 2, \dots, T$ . If yes, we should therefore estimate the standard time trend model, because correctly specified parametric models are relatively more efficient than their semiparametric counterparts. Otherwise, we should estimate the semiparametric categorical time-varying coefficient model, because misspecified parametric models will lead to inconsistent results.

When conducting the test, we employ a parametric model specification testing procedure (see, for example, Chapter 3 of Gao, 2007). Specifically, we formulate the null hypothesis as follows:  $H_0 : \Pr(\theta(t^c) = \theta_0(t^c)) = 1$ , i.e., the probability of  $\theta(t^c)$  being equal to  $\theta_0(t^c)$  is one. The corresponding test statistic follows a standard normal distribution. In our particular case, the test statistic is 6.667 with an associated  $p$ -value of approximately zero; hence we reject the null that the general nonparametric coefficient function  $\theta(t^c)$  is of the parametric form  $\theta_0(t^c)$ .

## 6.2. Estimates of Total Factor Productivity

In this subsection, we compare the performance of these two models in terms of their ability to estimate total factor productivity growth. In doing so, we compute four industry aggregate TFPG indexes with the first one based on the standard time model (denoted by  $TFPG^{Time}$ ), the second one based on the categorical time-varying coefficient model (denoted by  $TFPG^{CTC}$ ), the third one based on the discrete Divisia TFPG index (denoted by  $TFPG^{Divisia}$ ), and the fourth one based on the general index

of technical change of Baltagi and Griffin (1988) (denoted by  $TFPG^{GI}$ )<sup>6</sup>. Specifically,  $TFPG^{Time}$  is obtained by first computing individual utility-level total factor productivity growth using (8) and then computing an industry aggregate index as an average of the 81 individual TFPG estimates.  $TFPG^{CTC}$ ,  $TFPG^{Divisia}$  and  $TFPG^{GI}$  are obtained in a similar way but by using (12), (3), and the general index of technical change of Baltagi and Griffin (1988) respectively. As discussed in the Introduction,  $TFPG^{Divisia}$  is used here as a benchmark for assessing  $TFPG^{Time}$  and  $TFPG^{CTC}$ , because the discrete Divisia index does not require direct estimation of the underlying technology, satisfies many desirable statistical properties, and also is widely used by major statistical agencies around the world (see, for example, Good, Nadiri and Sickles, 1997; Dean, Harper and Sherwood, 1996; Hulten, 2001). With regard to  $TFPG^{GI}$ , it is used as a second benchmark to further confirm the accuracy of  $TFPG^{CTC}$ . In addition, we also construct 95% confidence intervals for  $TFPG^{CTC}$  using 1,000 bootstrap replications.

Figure 1.1 plots  $TFPG^{Time}$ ,  $TFPG^{CTC}$ ,  $TFPG^{Divisia}$ , and  $TFPG^{GI}$  over the sample period. To avoid graphical clutter, the 95% bootstrap confidence intervals for  $TFPG^{CTC}$  are plotted in Figure 1.2. We first compare  $TFPG^{Time}$  and  $TFPG^{Divisia}$ . As can be seen from Figure 1.1,  $TFPG^{Time}$  shows much less variation than  $TFPG^{Divisia}$  and is roughly a smoothed version of  $TFPG^{Divisia}$ . To better observe this, we first examine the temporal pattern of the benchmark series,  $TFPG^{Divisia}$ . Roughly speaking, the series  $TFPG^{Divisia}$  can be divided into five segments: 1987 – 1988, 1988 – 1989, 1989 – 1993, 1993 – 1996, and 1996 – 1998, with the first, third and fifth segments representing three productivity slowdowns and the second and fourth segments representing two productivity resurgences. More specifically, in the first segment it decreases significantly from 8.37% in 1987 to 0.98% in 1988; in the second segment it rebounds from 0.98% in 1988 to 6.08% in 1989; in the third segment it declines substantially from 6.08% in 1989 to –3.14% in 1993; in the fourth segment it rebounds from –3.14% in 1993 to 3.68% in 1996; and in the last segment it drops sharply from 3.68% in 1996 to –3.78% in 1998. Turning now to  $TFPG^{Time}$ , we see that for each of the five  $TFPG^{Divisia}$  segments,  $TFPG^{Time}$  passes close by the mean of the segment. Considering the third segment as an example, the  $TFPG^{Time}$  series crosses this segment at a point where TFPG is approximately 0, which is close to the mean (0.0006) of

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<sup>6</sup>For details on the general technical change index model, see Baltagi and Griffin (1988).

this segment. This suggests that the series  $TFPG^{Time}$  can be regarded as being obtained by linking the means of the five segments. In this sense,  $TFPG^{Time}$  is roughly a smoothed version of  $TFPG^{Divisia}$ .

We now compare  $TFPG^{CTC}$  and  $TFPG^{Divisia}$ . From Figure 1.1, we note that  $TFPG^{CTC}$  closely tracks  $TFPG^{Divisia}$ , starting at 7.36%, dropping significantly to 2.09% in 1988, rebounding to 5.22% in 1989, dropping substantially to -1.42% in 1993, rebounding noticeably to 3.16% in 1996, and falling to -0.97% in 1998. Furthermore,  $TFPG^{CTC}$  suggests exactly the same technological segments as  $TFPG^{Divisia}$ , that is 1987 – 1988, 1988 – 1989, 1989 – 1993, 1993 – 1996, and 1996 – 1998, with the first, third and fifth segments representing three productivity slowdowns and the second and fourth segments representing two productivity resurgences.

In addition, we also compare  $TFPG^{CTC}$  and  $TFPG^{GI}$  (i.e., the one obtained using the general index of technical change of Baltagi and Griffin (1988)). As can be seen from Figure 1.1,  $TFPG^{CTC}$  also closely tracks  $TFPG^{GI}$ , further confirming the ability of the categorical time-varying coefficient model to closely track well-known productivity indexes.

To further compare  $TFPG^{CTC}$  and  $TFPG^{Time}$ , we calculate the mean squared error (MSE) for each of them. Formally,  $MSE = \frac{1}{T} \sum_{t=1}^T \left( T\widehat{FPG}_t - TFPG_t^{Divisia} \right)^2$ , where  $T\widehat{FPG}_t$  stands for an estimate of industry-level  $TFPG$  for period  $t$  (e.g.,  $TFPG_t^{CTC}$  or  $TFPG_t^{Time}$ ). Our results indicate that  $TFPG^{CTC}$  has an MSE of 0.016%, whereas  $TFPG^{Time}$  has a much higher MSE of 0.067%, further confirming that the categorical time-varying coefficient model tracks more closely with the discrete Divisia index than does the standard time trend model.

An interesting question to ask at this point is: what causes the lack of variations in  $TFPG^{Time}$ ? To answer this question, we decompose each of the two TFPG indexes,  $TFPG^{Time}$  and  $TFPG^{CTC}$ , into two components: technical change and scale effects. For notational clarity, let  $TC^{Time}$  ( $SC^{Time}$ ) denote technical change (scale effects) obtained using the standard time trend model, and let  $TC^{CTC}$  ( $SC^{CTC}$ ) denote technical change (scale effects) obtained using the categorical time-varying coefficient model. Figure 2.1 presents the estimates of  $SC^{Time}$  and  $SC^{CTC}$ . Looking at this figure, we see that  $SC^{Time}$  and  $SC^{CTC}$  closely track each other. This suggests that  $SC^{Time}$  and  $SC^{CTC}$  show a similar degree of variation, which in turn implies that scale effects cannot lead to the lack of variations in  $TFPG^{Time}$ .

Figure 2.2 presents the estimates of  $TC^{Time}$  and  $TC^{CTC}$ . We note that compared with  $TC^{CTC}$

that varies considerably from year to year,  $TC^{Time}$  declines in a linear fashion over the sample period, suggesting that it is the lack of variability in  $TC^{Time}$  that leads to the lack of variation in  $TFPG^{Time}$ . In fact,  $TC^{Time}$  is roughly a smoothed version of  $TC^{CTC}$ . To see this, we follow Feng and Serletis (2008) and obtain a smoothed  $TC^{CTC}$  series by regressing the raw  $TC^{CTC}$  series on firm dummies and a continuous time trend, calculating the fitted values, and aggregating across individual electric utilities. The smoothed  $TC^{CTC}$  is also plotted in Figure 2.2. As can be seen,  $TC^{Time}$  evolves in a similar pattern as the smoothed  $TC^{CTC}$ , confirming that  $TC^{Time}$  is roughly a smoothed version of  $TC^{CTC}$ .

It is also noted that one consequence of the lack of variation in  $TFPG^{Time}$  is that it does not have the right sign for 5 of the 12 time periods, namely 1990, 1991, 1992, 1993, and 1995. Specifically, looking at Figure 1.1, we see that  $TFPG^{Divisia}$  suggests that productivity growth is positive in 1990 and 1995 and negative in 1991, 1992, and 1993; however,  $TFPG^{Time}$  suggests exactly the opposite. In contrast,  $TFPG^{CTC}$  has the right sign for all the 12 time periods, as evidenced from Figure 1.1.

### 6.3. Decomposition of Total Factor Productivity into Technical Change and Scale Effects

As previously discussed, productivity growth can be decomposed into two components: technical change and scale effects. This decomposition raises an interesting question: What do the two competing models imply about the relative importance of technical change and scale effects? Table 2 presents the two components for each of the two models (i.e., the standard time trend model and the categorical time-varying coefficient model). For each model, firm-level estimates of technical change and scale effects are developed and then used to produce industry-level estimates of technical change and scale effects.

Looking at Panel A, Table 2, we see that the categorical time-varying coefficient model suggests that on average scale effects is dominant over technical change. Specifically, this model suggests that on average scale effects accounts for 58.0% (i.e.,  $0.0102/(0.0074 + 0.0102)$ ) of the productivity growth over the sample period, whereas technical change accounts for 42.0% (i.e.,  $0.0074/(0.0074 + 0.0102)$ ). This finding is consistent with the long tradition of individual plant studies emphasizing the importance of scale economies and their potential for stimulating productivity growth (see, e.g., Christensen and Greene 1976; Atkinson and Halvorsen, 1984). We also note that while on average scale effects contributes positively to productivity growth over the whole sample period, it contributes negatively to productivity growth over the subperiod 1990 – 1995, implying that these electric utilities exhibit de-



creasing returns to scale during this subperiod. A possible explanation for the decreasing returns to scale over this subperiod is due to deviations from cost-minimization by the electric utilities caused by price regulation by state governments (Fabrizio *et al.*, 2007).

Looking at Panel B, Table 2, we see that the standard time trend model suggests that on average technical change plays a more important role than scale effects. Specifically, this model suggests that on average technical change accounts for 72.1% (i.e.,  $0.0106/(0.0106 + 0.0041)$ ) of the total productivity growth, compared to 27.9% (i.e.,  $0.0041/(0.0106 + 0.0041)$ ) accounted for by scale effects, thus implying that technical change is the dominant component. To examine why technical change plays a more important role in the case of the standard time trend model, we plot in Figure 3 the estimates of technical change implied by this model together with those of scale effects implied by the same model. As can be seen from this figure, the dominance of technical change over scale effects occurs because the former component is positive throughout the sample period, whereas the latter one is negative for half of the sample period (i.e., 1990 – 1993, 1995, and 1998). However, as previously discussed, the positiveness of technical change in the standard time trend model is mainly the result of the model's inability to accurately capture the shift of the cost function over time.

In summary, the use of the standard time trend model not only results in mis-measured productivity growth (as discussed in the previous subsection), but also results in a misleading conclusion regarding the relative importance of technical change and scale effects.

#### 6.4. Estimates of Price Elasticities

As discussed in the Introduction, the standard normalized quadratic (NQ) functional form suffers “the problem of trending elasticities”, that is, the price elasticities produced by this functional form often exhibit little variation over time. This problem raises an intriguing question: Do the two competing translog models in the present paper suffer the same problem? Figures 4.1 – 4.9 present the estimates of own price elasticity of labor ( $\eta_{11}$ ), elasticity of demand for labor with respect to the price of fuel ( $\eta_{12}$ ), elasticity of demand for labor with respect to the price of capital ( $\eta_{13}$ ), elasticity of demand for fuel with respect to the price of labor ( $\eta_{21}$ ), own price elasticity of fuel ( $\eta_{22}$ ), elasticity of demand for fuel with respect to the price of capital ( $\eta_{23}$ ), elasticity of demand for capital with respect to the price of labor ( $\eta_{31}$ ), elasticity of demand for capital with respect to the price of fuel ( $\eta_{32}$ ), and own price elasticity of

capital ( $\eta_{33}$ ). For each model, firm-level estimates of price elasticity of demand for input  $i$  with respect to input price  $n$  are calculated as

$$\eta_{in} = \beta_{in}/s_i + s_n - \delta_{in}, \quad (23)$$

where  $\beta_{in}$  is the coefficient for  $\ln w_i \ln w_n$ ,  $s_i$  ( $s_n$ ) is the cost share of input  $i$  ( $n$ ), and  $\delta_{in} = 1$  if  $i = n$  and 0 otherwise. These firm-level estimates are then used to produce industry-level estimates of price elasticities. In each figure, the solid line shows the estimates obtained from the categorical time-varying coefficient model and the dotted lines show the associated 95% bootstrap confidence intervals. The dashed line shows the estimates obtained from the standard time trend model.

Looking at the industry-level estimates of the own elasticity of labor demand implied by the standard time trend model (i.e.,  $\eta_{11}^{Time}$ ) in Figure 4.1, we see that it varies within an very narrow range between  $-0.7510$  and  $-0.7398$ . This result is not surprising. As can be seen from (23),  $\delta_{in}$  is a constant, while  $s_i$  and  $s_n$  are cost shares that do not vary much in practice, especially over a short period of time. If the coefficient,  $\beta_{in}$ , is also restricted to be a constant over time as in the standard time trend translog cost function, then  $\eta_{in}$  is doomed to show little variation. Thus, as with the standard normalized quadratic functional form, the standard translog functional form suffers a similar problem (i.e., lacking variation in price elasticities), due largely to the time-invariant nature of  $\beta_{in}$ .

Turning to the industry-level estimates of the own elasticity of labor demand implied by the categorical time-varying coefficient model (i.e.,  $\eta_{11}^{CTC}$ ) in Figure 4.1, we see that it shows considerably more year-to-year variation. Specifically, it starts at  $-0.8550$  in 1986, rises considerably to  $-0.7612$  in 1987, remains at roughly the same value in 1988, drops appreciably to  $-0.8128$  in 1989, rebounds to  $-0.7463$  in 1990, falls notably to  $-0.8032$  in 1991, rises substantially to  $-0.6042$  in 1993, falls considerably to  $-0.6955$  in 1994, rebounds to  $-0.6249$  in 1995, and falls significantly to  $-1.0217$  in 1998. The finding that  $\eta_{11}^{CTC}$  shows much more variation is not surprising, because  $\beta_{11}$  is no longer a constant in the categorical time-varying coefficient model. Instead, it varies from time period to time period, thus allowing  $\eta_{11}$  to vary from one period to another even when  $\delta_{11}$  is a constant and  $s_1$  does not vary much. More specifically,  $\beta_{11}$  is a nonparametric function of the categorical time variable ( $t^c$ ) and thus is very flexible with respect to time. This flexible treatment allows  $\beta_{11}$  to vary considerably over time, which in turn allows price elasticities (i.e.,  $\eta_{11}$ ) to vary considerable over time as in our case (see (23)).

It is worth noting that in Figure 4.1 the elasticity estimates obtained from the standard time trend model (i.e.,  $\eta_{11}^{Time}$ ) fall into the 95% bootstrap confidence intervals for those obtained from the categorical time-varying coefficient model (i.e.,  $\eta_{11}^{CTC}$ ). There are two possible reasons why this happens. First, as can be seen from the figure, the dashed line representing  $\eta_{11}^{Time}$  is roughly a smoothed version of the solid line representing  $\eta_{11}^{CTC}$ , making it very likely that the former line falls into the 95% confidence intervals associated with the latter line. Second, the categorical time-varying coefficient model allows each period to have its own set of coefficients, which in turn allows price elasticities to vary considerably from year to year. Accordingly, this model requires large number of observations for each period in order to obtain tight elasticity confidence intervals. In our particular case, however, we have only 81 observations (firms) for each period, which may not be large enough to produce tight elasticity confidence intervals.

We turn now to Figures 4.2 – 4.9. These figures, though not explained here due to space limitations, further confirm that the standard time trend model produces price elasticities that vary within very narrow ranges, whereas the categorical time-varying coefficient model is capable of producing elasticities that vary considerably from year to year. In sum, the time-varying nature of the categorical time-varying coefficient model has another advantage in addition to the model’s flexibility to model productivity growth and technical change: the model is free of the problem of trending elasticities inherent in the standard time trend translog model. Due to the importance and wide applications of price elasticities, this latter advantage should be of interest to applied economists in many fields.

## 7 Conclusion

The econometric approach to productivity measurement literature has long been dominated by the time trend approach. Despite its popularity, this approach has two major drawbacks. First, it produces a smooth, slowly changing characterization of the pace of technical change. This pattern of technical change is neither supported by the evidence from index number approaches to calculating rates of technical change, nor consistent with findings in the investment literature that suggest technologies are introduced in a “lumpy” fashion with discrete jumps. Second, it suffers the problem of trending

elasticities.

To overcome the two drawbacks associated with the standard time trend approach, we propose in the present paper a categorical time-varying coefficient translog cost function. The main feature of this model is that each of its coefficients is expressed as a nonparametric function of a categorical time variable (which consists of  $T$  time points or  $T$  categories, where  $T$  is the total number of discrete time periods), thus allowing each time period to have its own set of coefficients and cost function. In this sense, the time-varying feature of the new cost function relaxes the restrictive implicit assumption underlying the standard time trend models that all sample years have to share the set of coefficients, thus making the new cost function a more general representation of production technology. Our technique requires panel data on firms within the same industry to allow the coefficients to differ across time periods.

We apply the categorical time-varying coefficient model to a sample of 81 electric utilities in the United States over the period 1986 – 1998. We find that the categorical time-varying coefficient model is capable of producing estimates of productivity growth that closely track those obtained using the Divisia productivity index. In contrast, the standard time trend model produces estimates of technical change that is only a smoothed version of those implied by the Divisia productivity index. We also find that the use of the standard time trend model not only results in mis-measured productivity growth, but also results in a misleading conclusion regarding the relative importance of technical change and scale effects.

We also find that the categorical time-varying coefficient model free of the problem of trending elasticities. Specifically, we find the price elasticities produced by the model shows considerable year-to-year variations, whereas those produced by the standard time trend model vary within very narrow ranges. Considering the importance and wide applications of price elasticities, this latter advantage should be of interest to applied economists in many fields.

## References

Abel, Andrew B. and Eberly, Janice C. 1994. "A Unified Model of Investment under Uncertainty." *The American Economic Review* 84: 1369–1384.

- Atkinson, Scott E. and Halvorsen, Robert. 1984. "Parametric Efficiency Tests, Economies of Scale, and Input Demand in U.S. Electric Power Generation." *International Economic Review*, 25: 647-62.
- Bai, Jushan. 2009. "Panel Data Models with Interactive Fixed Effects." *Econometrica*, 77(4): 1229-1279.
- Baltagi, Badi H. and Griffin, James M. 1988. "A General Index of Technical Change." *Journal of Political Economy*, 96: 20-41.
- Barten, Anton P. 1969. "Maximum Likelihood Estimation of a Complete System of Demand Equations." *European Economic Review*, 1: 7-73.
- Cameron, A. Colin and Trivedi, Pravin K. 2005. *Microeconometrics: Method and Applications*. Cambridge University Press, U.K.
- Chen, Jia, Gao, Jiti and Li, Degui. 2012. "Semiparametric trending panel data models with cross-sectional dependence". *Journal of Econometrics*, 171: 71-85.
- Christensen, Laurits R. and Green, William H. 1976. "Economies of Scale in U.S. Electric Power Generation." *Journal of Political Economy*, 84(4): 655-76.
- Christensen, Laurits R. and Jorgenson, Dale W. 1970. "US Real Product and Real Factor Input, 1929-1967." *Review of Income and Wealth*, 16: 19-50.
- Cooper, Russell, Haltiwanger, John and Power, Laura. 1999. "Machine Replacement and the Business Cycle: Lumps and Bumps." *The American Economic Review*, 89(4): 921-946.
- Dean, Edwin R., Harper, Michael J. and Sherwood, Mark S. 1996. "Productivity Measurement with Changing-Weight Indices of Outputs and Inputs," *Industry Productivity: International Comparison and Measurement Issues*, Paris: The Organization for Economic Co-Operation and Development, p. 183-215. Paris: Organization for Economic Cooperation and Development.
- Denny, Michael, Fuss, Melvyn and Waverman, Leonard. 1981. "The Measurement and Interpretation of Total Factor Productivity in Regulated Industries with an Application to Canadian Telecommunication." In *Productivity Measurement in Regulated Industries*, edited by Thomas G. Cowing and Rodney E. Stevenson. New York: Academic Press.
- Durlauf, Steven N., Kourtellos, Andros and Minkin, Artur. 2001. "The local Solow growth model." *European Economic Review* 45 (4): 928-940.
- Fabrizio, Kira R., Rose, Nancy L. and Wolfram, Katherine D. 2007. "Do Markets Reduce Costs? Assessing the Impact of Regulatory Restructuring on US Electricity Generating Efficiency." *The American Economic Review*, 97: 1250-1277.

- Fan, Jianqing and Zhang, Wenyang. 1999. "Statistical estimation in varying coefficient models." *Annals of Statistics* 27 (5): 1491-1518.
- Fan, Jianqing and Zhang, Wenyang. 2008. "Statistical Methods with Varying Coefficient Models." *Statistics and its Interface* 1 (1): 179-195.
- Farrell, Lisa and Walker, Ian. 1999. "The Welfare Effects of Lotto: Evidence from the UK." *Journal of Public Economics*, 72: 99–120.
- Feng, Guohua and Serletis, Apostolos. 2008. "Productivity Trends in U.S. Manufacturing: Evidence from the NQ and AIM Cost Functions." *Journal of Econometrics*, 142(1): 281–311.
- Fuss, Melvyn A. 1994. "Productivity Growth in Canadian Telecommunications." *Canadian Journal of Economics*, 27: 371-392.
- Gao, Jiti. 2007. *Nonlinear Time Series: Semiparametric and Nonparametric Methods*. Chapman & Hall/CRC.
- Gao, Jiti and Phillips, Peter C.B. 2013. "Functional Coefficient Nonstationary Regression with Non- and Semi-Parametric Cointegration." Available at <https://ideas.repec.org/p/msh/ebswps/2013-16.html>.
- Good, David H., Nadiri, M. Ishaq and Sickles, Robin. 1997. "Index Number and Factor Demand Approaches to the Estimation of Productivity," In: M. Pesaran and P. Schmidt (eds.) *Handbook of Applied Econometrics, Vol II: Microeconometrics*, Basil Blackwell.
- Griffiths, William E., O'Donnell, Christopher J. and Cruz, Agustina Tan. 2000. "Imposing Regularity Conditions on a System of Cost and Cost-Share Equations: A Bayesian Approach." *Australian Journal of Agricultural and Resource Economics*, 44: 107-127.
- Hochman, Gal, Rajagopal, Deepak and Zilberman, David. 2010. "Are Biofuels the Culprit? OPEC, Food, and Fuel." *The American Economic Review*, 100(2): 183-187.
- Hulten, Charles. 2001. "Total Factor Productivity: A Short Biography." in *New Developments in Productivity Analysis*, Charles R. Hulten, Edwin R. Dean, and Michael J. Harper (eds.), *Studies in Income and Wealth*, 63, (Chicago: The University of Chicago Press for the National Bureau of Economic Research, Chicago): 1-47.
- Jorgenson, Dale W. 1991. "Productivity and Economic Growth." in *Fifty Years of Economic Measurement*, Ernst R. Berndt and Jack E. Triplett (eds.) *Studies in Income and Wealth*, 63, (Chicago: The University of Chicago Press for the National Bureau of Economic Research, Chicago): 19-118.
- Jorgenson, Dale W. and Griliches, Zvi. 1967. "The Explanation of Productivity Change." *The Review of Economic Studies*,

34: 249-83.

Laffont, Jean-Jacques and Tirole, Jean. 1993. *A Theory of Incentives in Procurement and Regulation*. Cambridge MA: MIT Press.

Li, Qi, Ouyang, Desheng and Racine, Jeffrey S. 2013. "Categorical Semi-parametric Varying Coefficient Models." *Journal of Applied Econometrics*, 28(40): 551-579.

Primiceri, Giorgio E. 2005. "Time varying structural vector autoregressions and monetary policy." *The Review of Economic Studies* 72 (3): 821-852.

Rungsuriyawiboon, Supawat and Stefanou, Spiro. 2007. "Stochastic Estimation of Efficiency and Deregulation in The US Electricity Industry Using Dynamic Efficiency Model." *Journal of Business and Economic Statistics*, 25: 226-238.

Solow, Robert M. 1957. "Technical Change and The Aggregate Production Function." *The Review of Economics and Statistics*, 39: 312-20.

Tinbergen, Jan. 1942. "Zur Theorie Der Langfristigen Wirtschaftsentwicklung." *Weltwirtschaftliches Archiv*, 55(1): 511-49.

Wooldridge, Jeffrey. M. 2010. *Econometric Analysis of Cross Section and Panel Data*, 2nd ed. The MIT Press.

TABLE 1.1  
PARAMETER ESTIMATES FOR THE CATEGORICAL  
TIME-VARYING COEFFICIENT MODEL FOR YEARS 1986-1998

Year → Parameter ↓	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998
$\alpha_0$	11.512 (3.370)	11.806 (2.810)	11.409 (2.674)	9.661 (2.161)	11.721 (2.937)	10.267 (2.171)	10.487 (2.256)	11.354 (2.212)	11.252 (1.994)	10.930 (1.996)	10.723 (2.114)	10.147 (2.329)	11.643 (2.397)
$\alpha_1$	0.600 (0.112)	0.430 (0.101)	0.504 (0.100)	0.472 (0.084)	0.120 (0.224)	0.405 (0.109)	0.326 (0.090)	0.279 (0.111)	0.369 (0.113)	0.356 (0.101)	0.426 (0.115)	0.521 (0.079)	0.614 (0.135)
$\alpha_2$	0.204 (0.186)	-0.212 (0.215)	0.313 (0.200)	0.151 (0.166)	-0.403 (0.525)	0.107 (0.128)	-0.019 (0.105)	-0.167 (0.106)	-0.149 (0.129)	-0.230 (0.121)	-0.107 (0.146)	-0.090 (0.160)	-0.251 (0.141)
$\alpha_3$	0.196 (0.204)	0.782 (0.235)	0.183 (0.202)	0.377 (0.202)	1.283 (0.730)	0.488 (0.184)	0.694 (0.140)	0.888 (0.139)	0.781 (0.163)	0.874 (0.150)	0.681 (0.185)	0.569 (0.180)	0.638 (0.186)
$\gamma_1$	-0.499 (0.428)	-0.410 (0.329)	-0.465 (0.334)	-0.225 (0.281)	-0.255 (0.302)	-0.266 (0.266)	-0.255 (0.280)	-0.330 (0.274)	-0.352 (0.253)	-0.295 (0.248)	-0.307 (0.265)	-0.268 (0.296)	-0.465 (0.301)
$\beta_{11}$	-0.001 (0.016)	0.013 (0.014)	0.013 (0.014)	0.004 (0.012)	0.013 (0.012)	0.005 (0.014)	0.022 (0.017)	0.039 (0.016)	0.022 (0.016)	0.035 (0.018)	0.018 (0.016)	-0.002 (0.016)	-0.029 (0.024)
$\beta_{12}$	-0.092 (0.014)	-0.086 (0.020)	-0.111 (0.017)	-0.078 (0.017)	-0.086 (0.019)	-0.055 (0.015)	-0.041 (0.019)	-0.047 (0.020)	-0.050 (0.021)	-0.065 (0.017)	-0.065 (0.018)	-0.063 (0.023)	-0.049 (0.018)
$\beta_{13}$	0.093 (0.021)	0.073 (0.021)	0.098 (0.020)	0.074 (0.018)	0.072 (0.024)	0.050 (0.020)	0.019 (0.020)	0.008 (0.026)	0.028 (0.026)	0.031 (0.025)	0.047 (0.025)	0.065 (0.021)	0.078 (0.022)
$\beta_{22}$	0.213 (0.039)	0.271 (0.040)	0.204 (0.046)	0.212 (0.042)	0.127 (0.059)	0.183 (0.045)	0.168 (0.040)	0.179 (0.043)	0.206 (0.048)	0.251 (0.044)	0.192 (0.051)	0.223 (0.042)	0.281 (0.030)
$\beta_{23}$	-0.121 (0.033)	-0.185 (0.034)	-0.093 (0.040)	-0.134 (0.040)	-0.042 (0.065)	-0.128 (0.043)	-0.127 (0.030)	-0.132 (0.031)	-0.155 (0.036)	-0.185 (0.038)	-0.127 (0.046)	-0.160 (0.042)	-0.232 (0.030)
$\beta_{33}$	0.028 (0.035)	0.112 (0.040)	-0.005 (0.039)	0.060 (0.045)	-0.030 (0.078)	0.078 (0.048)	0.108 (0.030)	0.124 (0.032)	0.128 (0.033)	0.155 (0.037)	0.080 (0.046)	0.096 (0.048)	0.154 (0.032)
$\gamma_{11}$	0.067 (0.027)	0.057 (0.020)	0.063 (0.021)	0.049 (0.018)	0.033 (0.021)	0.051 (0.017)	0.050 (0.018)	0.054 (0.017)	0.057 (0.016)	0.053 (0.016)	0.053 (0.017)	0.051 (0.019)	0.064 (0.019)
$\psi_{11}$	-0.012 (0.004)	-0.006 (0.004)	-0.007 (0.003)	-0.008 (0.003)	0.014 (0.016)	-0.007 (0.004)	-0.010 (0.004)	-0.011 (0.005)	-0.011 (0.004)	-0.012 (0.003)	-0.011 (0.004)	-0.011 (0.004)	-0.010 (0.004)
$\psi_{12}$	0.015 (0.010)	0.029 (0.012)	0.016 (0.009)	0.015 (0.006)	0.067 (0.042)	0.015 (0.005)	0.020 (0.005)	0.030 (0.006)	0.026 (0.006)	0.028 (0.006)	0.030 (0.007)	0.023 (0.007)	0.020 (0.006)
$\psi_{13}$	-0.004 (0.010)	-0.023 (0.012)	-0.008 (0.009)	-0.007 (0.006)	-0.081 (0.058)	-0.008 (0.006)	-0.011 (0.006)	-0.019 (0.005)	-0.014 (0.006)	-0.016 (0.006)	-0.018 (0.007)	-0.012 (0.007)	-0.010 (0.007)

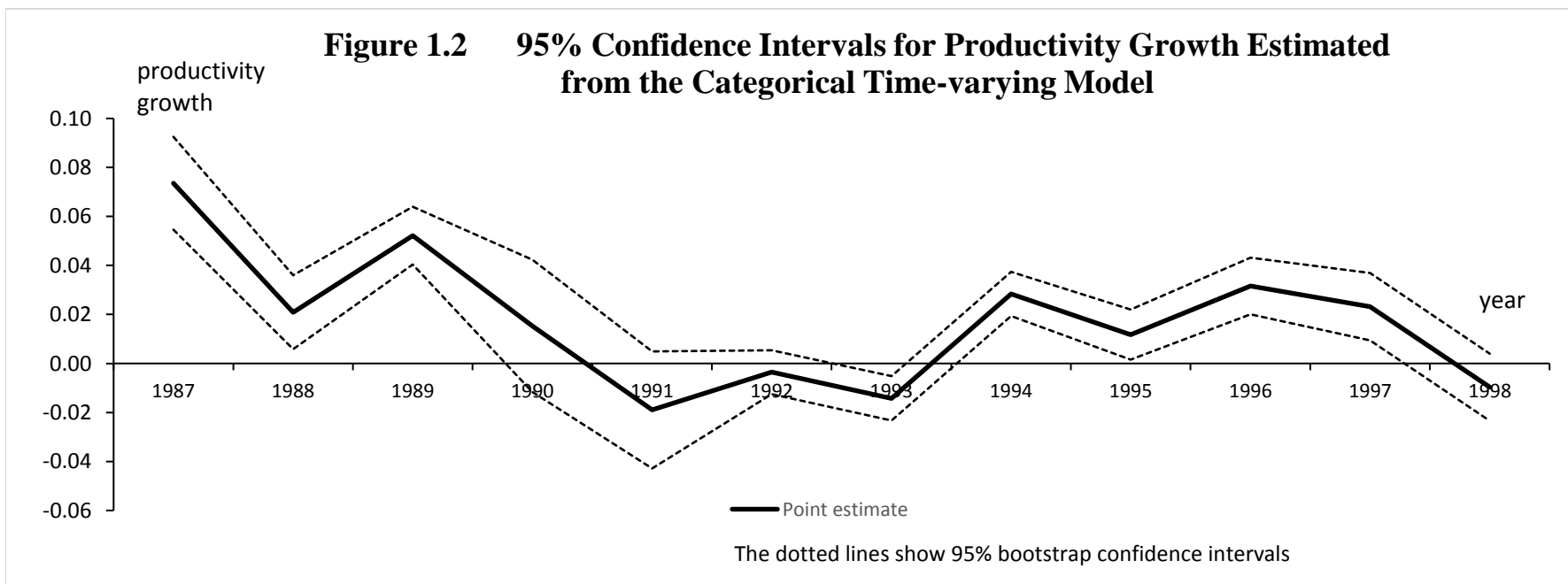
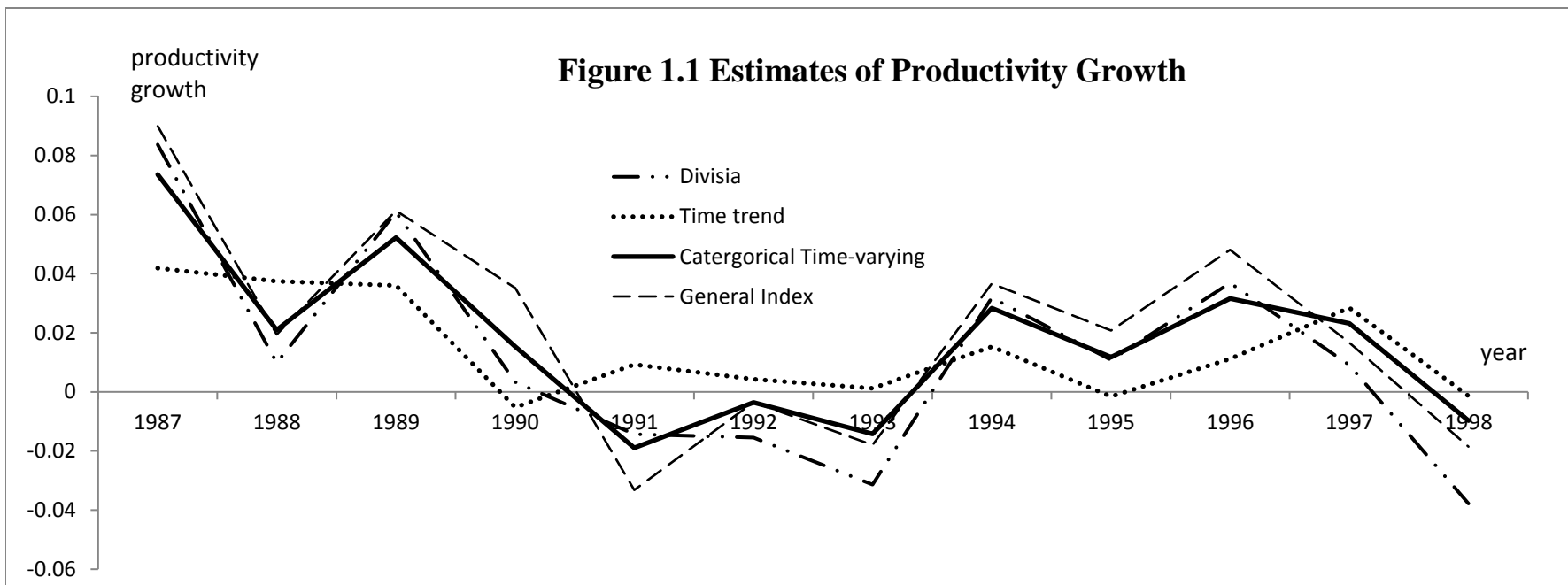


TABLE 1.2  
 PARAMETER ESTIMATES FOR  
 THE STANDARD TIME TREND MODEL

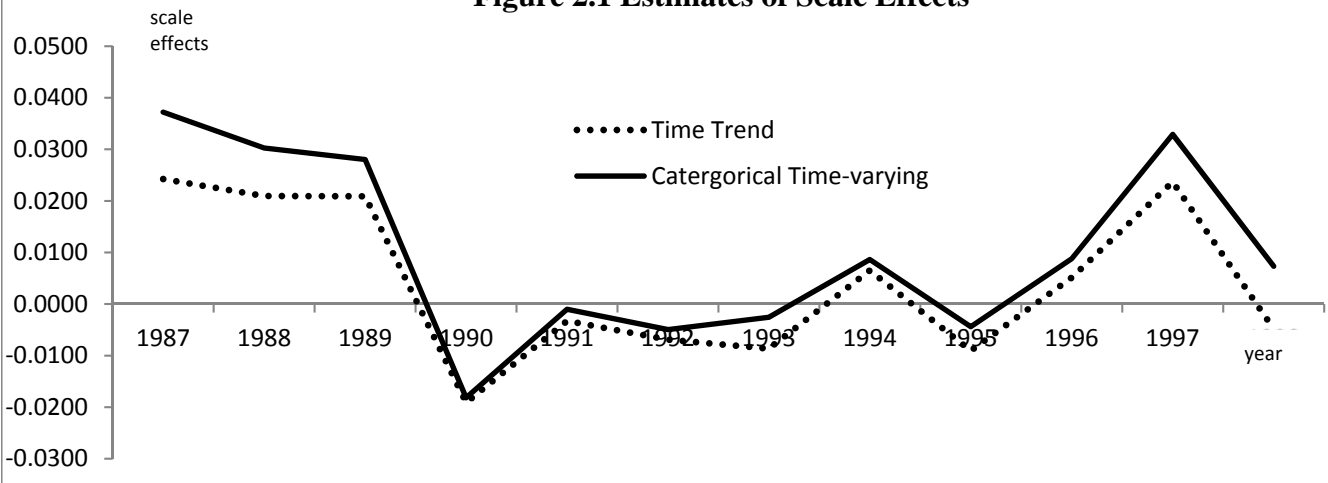
Parameter	Estimate	Standard Error
$\alpha_0$	12.040	1.881
$\alpha_1$	0.370	0.069
$\alpha_2$	-0.140	0.122
$\alpha_3$	0.769	0.159
$\gamma_1$	-0.458	0.238
$\tau$	-0.044	0.001
$\beta_{11}$	0.014	0.010
$\beta_{12}$	-0.053	0.012
$\beta_{13}$	0.039	0.014
$\beta_{22}$	0.204	0.026
$\beta_{23}$	-0.151	0.028
$\beta_{33}$	0.112	0.035
$\gamma_{11}$	0.063	0.015
$\delta$	0.001	0.000
$\psi_{11}$	-0.008	0.003
$\psi_{21}$	0.026	0.005
$\psi_{31}$	-0.018	0.007
$\phi_1$	0.001	0.001
$\phi_2$	0.001	0.002
$\phi_3$	-0.002	0.002
$\varphi_1$	0.001	0.001

TABLE 2  
TECHNICAL CHANGE AND SCALE EFFECTS

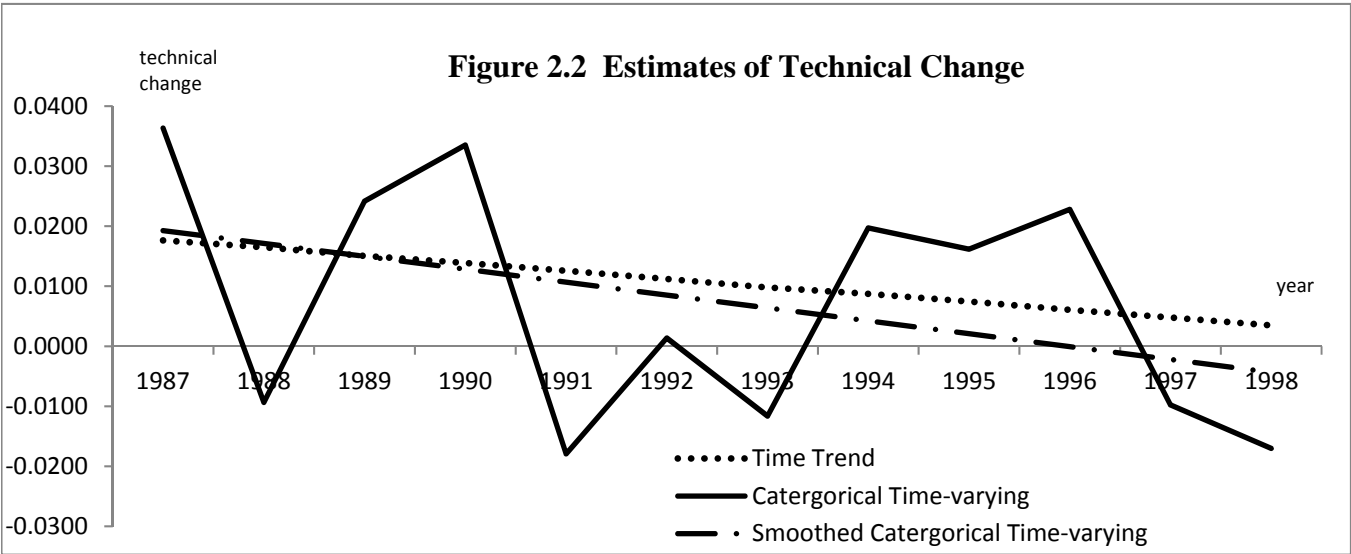
Year	Panel A: Categorical Time-Varying Coefficient Model				Panel B: Standard Time Trend Model			
	Technical Change		Scale Effects		Technical Change		Scale Effects	
	Estimate	Standard Error	Estimate	Standard Error	Estimate	Standard Error	Estimate	Standard Error
1987	0.0363	0.0095	0.0372	0.0016	0.0176	0.0025	0.0242	0.0014
1988	-0.0094	0.0079	0.0303	0.0016	0.0165	0.0020	0.0210	0.0012
1989	0.0242	0.0059	0.0280	0.0013	0.0151	0.0016	0.0209	0.0011
1990	0.0335	0.0145	-0.0182	0.0011	0.0139	0.0012	-0.0191	0.0010
1991	-0.0179	0.0122	-0.0010	0.0010	0.0126	0.0009	-0.0033	0.0002
1992	0.0014	0.0046	-0.0049	0.0000	0.0112	0.0008	-0.0069	0.0004
1993	-0.0116	0.0046	-0.0026	0.0002	0.0098	0.0010	-0.0086	0.0013
1994	0.0197	0.0047	0.0086	0.0009	0.0087	0.0013	0.0065	0.0003
1995	0.0162	0.0052	-0.0044	0.0003	0.0075	0.0017	-0.0091	0.0006
1996	0.0228	0.0059	0.0088	0.0003	0.0061	0.0022	0.0051	0.0007
1997	-0.0098	0.0070	0.0329	0.0006	0.0048	0.0026	0.0237	0.0012
1998	-0.0170	0.0070	0.0073	0.0013	0.0035	0.0030	-0.0048	0.0004
Average	0.0074	0.0013	0.0102	0.0005	0.0106	0.0009	0.0041	0.0003



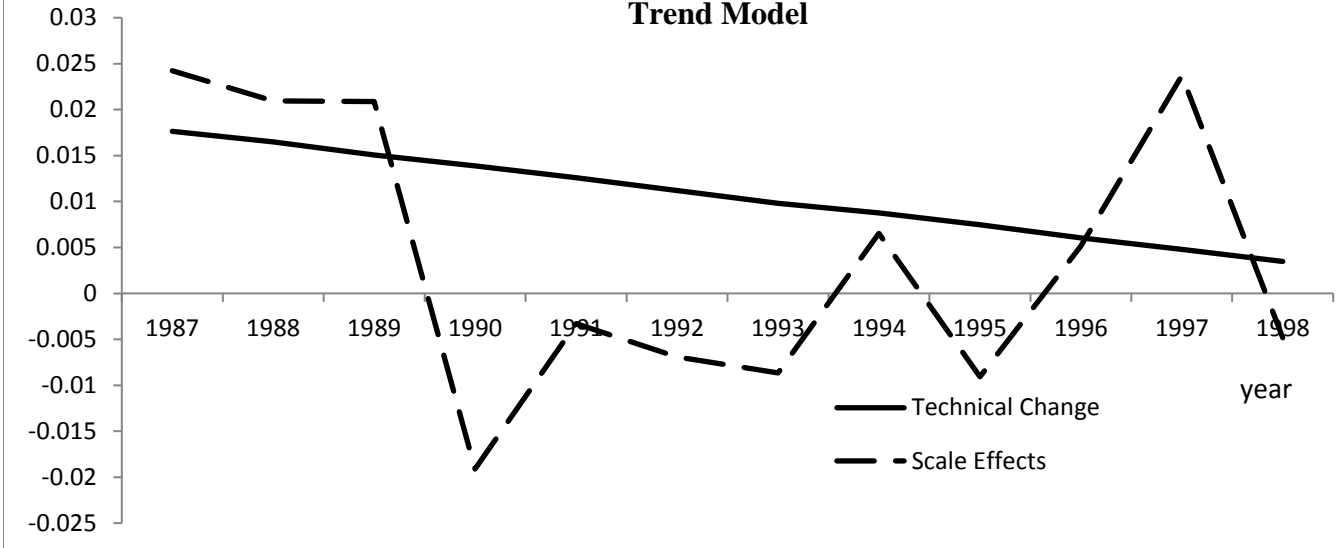
**Figure 2.1 Estimates of Scale Effects**



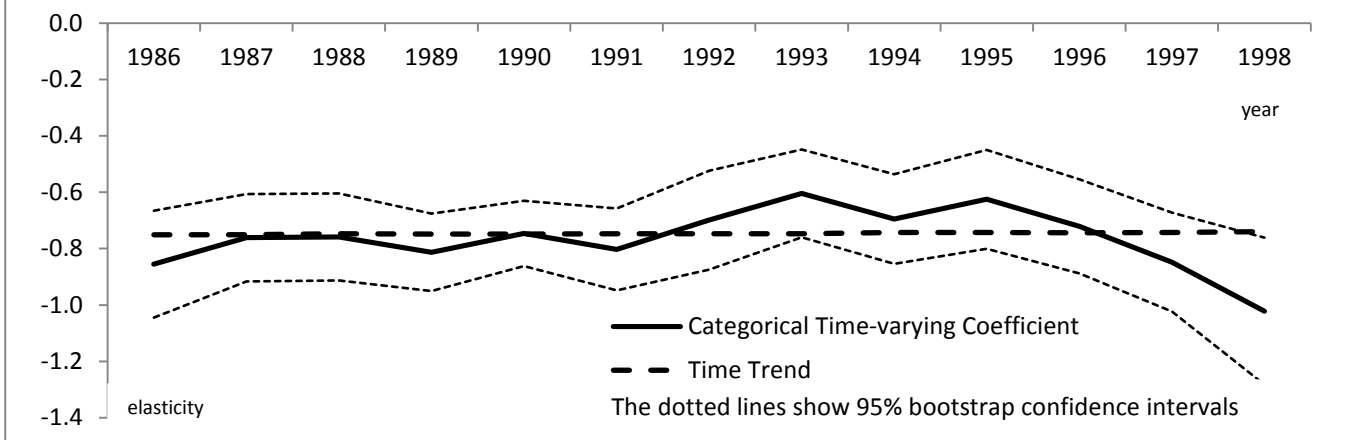
**Figure 2.2 Estimates of Technical Change**



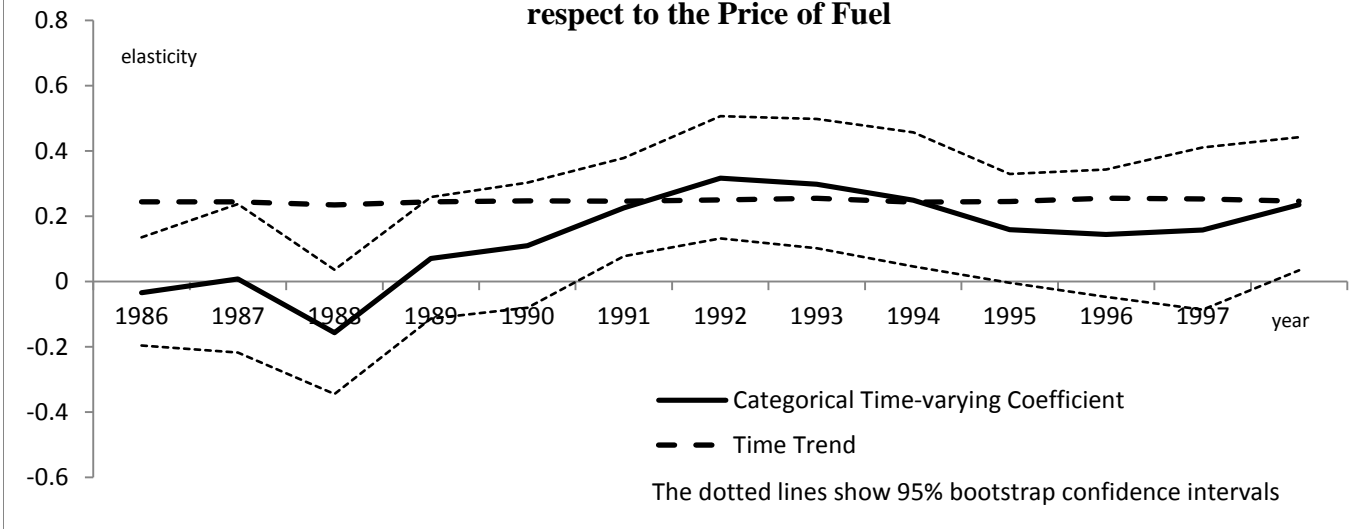
**Figure 3 Technical Change and Scale Effects for the Standard Time Trend Model**



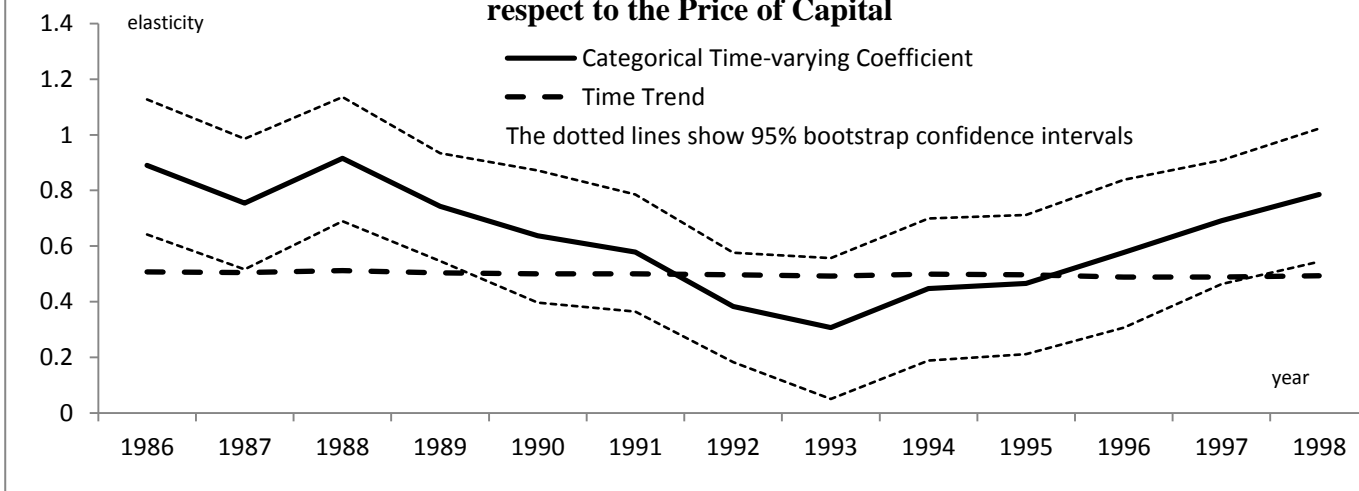
**Figure 4.1 Estimates of Own Elasticity of Labor**



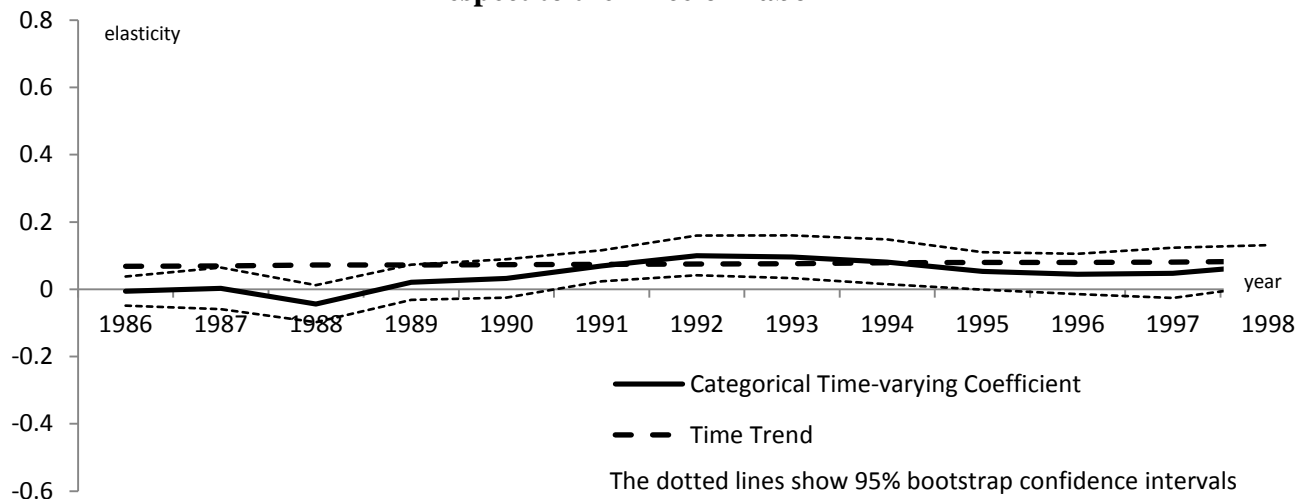
**Figure 4.2 Estimates of Elasticity of Demand for Labor with respect to the Price of Fuel**



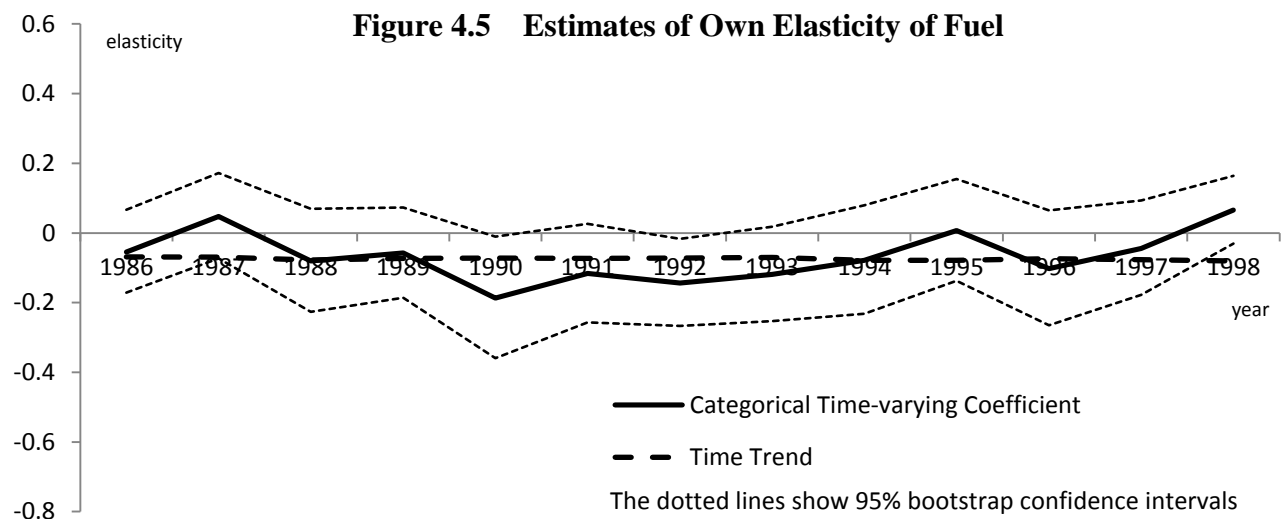
**Figure 4.3 Estimates of Elasticity of Demand for Labor with respect to the Price of Capital**



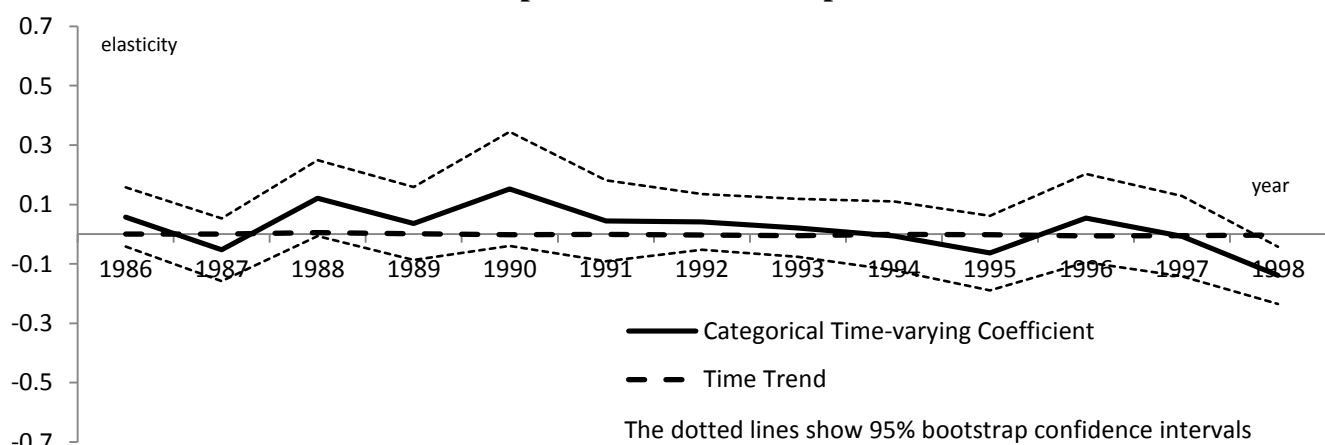
**Figure 4.4 Estimates of Elasticity of Demand for Fuel with respect to the Price of Labor**



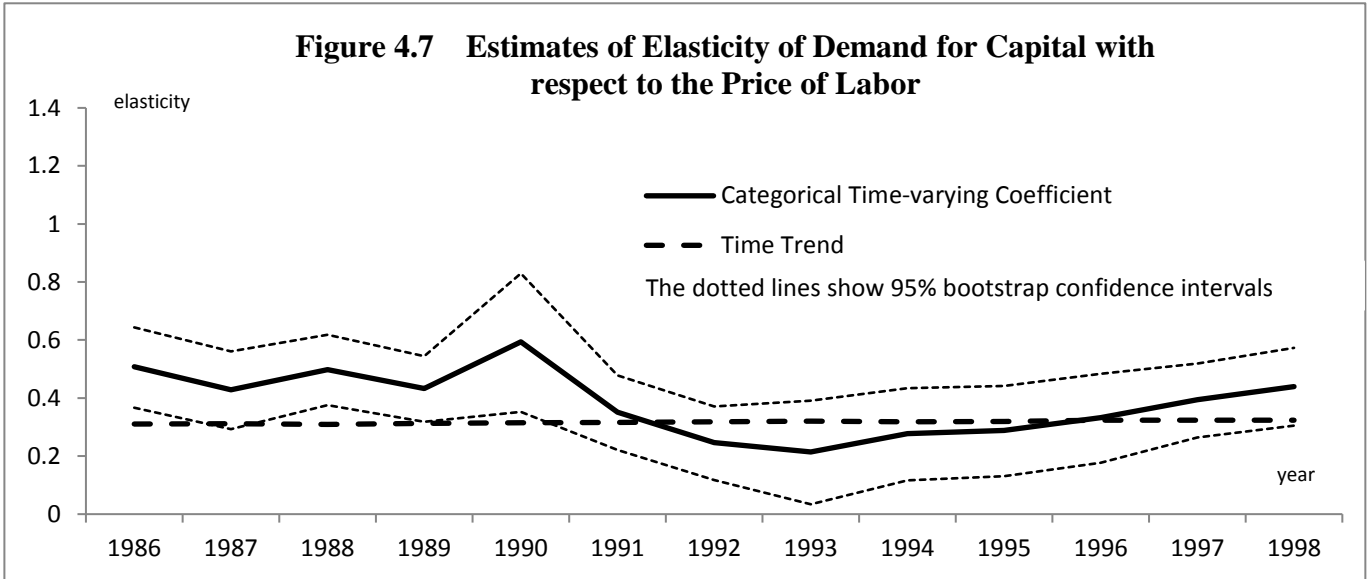
**Figure 4.5 Estimates of Own Elasticity of Fuel**



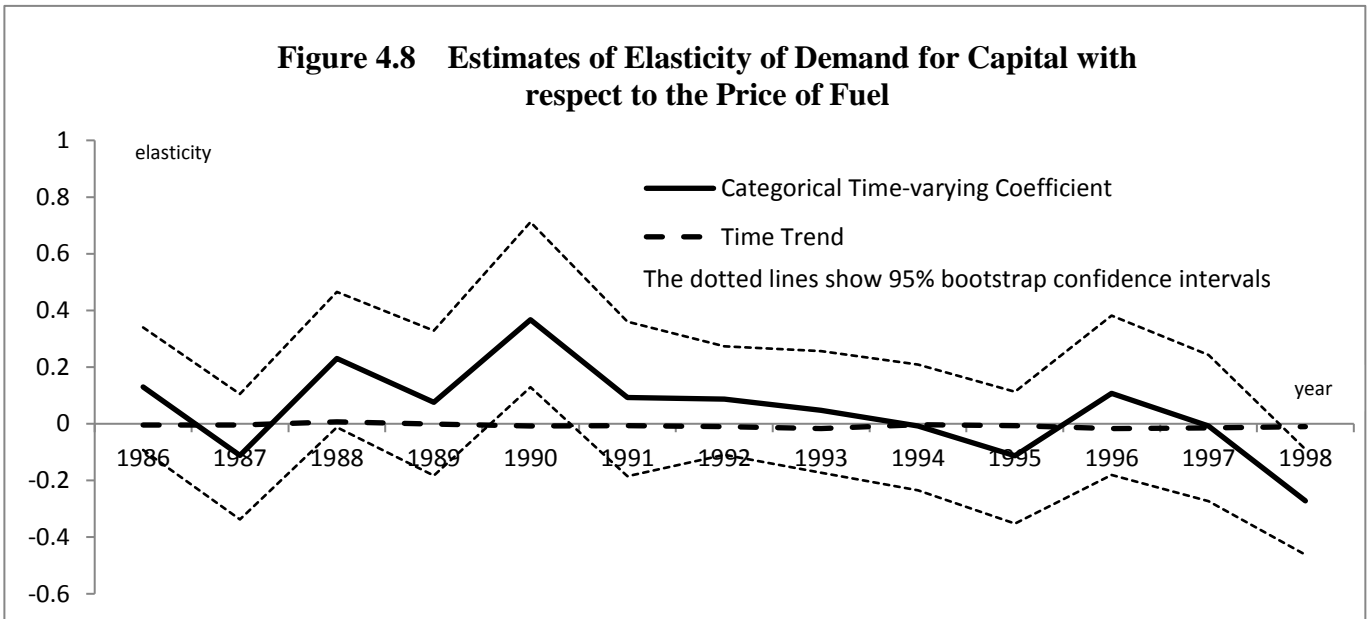
**Figure 4.6 Estimates of Elasticity of Demand for Fuel with respect to the Price of Capital**



**Figure 4.7 Estimates of Elasticity of Demand for Capital with respect to the Price of Labor**



**Figure 4.8 Estimates of Elasticity of Demand for Capital with respect to the Price of Fuel**



**Figure 4.9 Estimates of Own Elasticity of Capital**

