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Abstract

Many countries have implemented social programs providing long-term financial or in-kind entitlements. These programs often focus on specific age-groups and consequently their expenditure streams are subject to demographic change. Given the strains already existing on public budgets, long-term forecasts are an increasingly important instrument to monitor the budgetary consequences of social programs. The expected development of the labour force is a key input to these forecasts. We produce forecasts of age-specific labour market participation rates, combining a functional data approach with information on education, marital status and other exogenous variables.

Keywords: forecast, labour supply, age-profile, smoothing.

1 Introduction

Governments, national institutions, and international organisations use long-term economic forecasts as an instrument to assess the sustainability of existing government programs constituting long-term claims against future public budgets. Examples include public pension schemes, health care systems, state-subsidised education, and old-age care. Most of these programs focus their spending on specific age groups of the population and are therefore subject to future demographic change. Their budgetary consequences will become stronger if a country faces unfavourable changes in its population age structure. For example, most developed countries will undergo a substantial ageing process over the next 50 years causing expected expenditures on pensions, health care, and old age care to increase under current law. Because the expected expenditure stream of a program depends on the interaction between age-related claims and the future age structure of the population, most long-term forecasts are based on population forecasts with a more or less disaggregated age structure. Five-year cohorts are popular and may be appropriate if program usage does not strongly depend on specific cohorts within a five-year age bracket. If a specific eligibility age is important, projections based on 1-year cohorts will provide more accurate information on expected future expenditures.

Most long-term economic forecasts are regularly published at multi-year intervals (European Commission 2012, Productivity Commission 2013, Commonwealth of Australia 2015), while the US-Congressional Budget Office updates its long-term forecast every year to inform the US-Congress and the public about budgetary consequences of current laws and recently implemented policies (see Congressional Budget Office 2014). The forecast horizons for long-term economic forecasts typically stretch over a period of 25 or 50 years. These extended forecast horizons either reflect long transition periods until the effects of a government program become fully visible, or they run until major expected changes in the demographic structure will have passed through all age-cohorts. For many countries the forecast horizon will be longer than the sample which is available for statistical inference, which creates substantial uncertainty about the stability of the model, its parameters, and the size of sampling errors. For these reasons most forecasters follow a general forecasting guideline as summarised in Pindyck (2015) and rely on simple and transparent models that minimise the flexibility of the modeller and maximise the plausibility of the assumptions in the sense that a range of experts would be willing to accept them. This consensus-driven approach also takes into account the political sensitivity of the issues at stake and the need for an easy and credible way to communicate eventual follow-up policy reforms to the electorate. Long-term economic forecasts usually combine extrapolations of labour and capital inputs with an assumption on the rate of technical progress in a variation of the Solow-Swan growth model (Solow 1956, Swan 1956).

In this paper we will concentrate on long-term forecasts of labour market participation, which in combination with hours worked eventually forms labour input in the aggregate production function. So far, forecasters have used either judgemental trend extrapolations for age- and sex-specific population groups (Congressional Budget Office 2011) or a version of the dynamic cohort model (Productivity Commission 2005, European Commission 2012). Both approaches have their merits in terms of simplicity and transparency but they do not use the full information provided by the cross-sectional and the time dimensions of age-profiles for employment rates. Our alternative approach for long-term forecasting of participation rates builds on Hyndman & Ullah (2007) and combines elements from functional data analysis with nonparametric smoothing and robust statistics. This approach allows us to derive interesting curve features from discretely observed age-profiles of employment rates, and at the same time permits estimates of possible measurement error. We are also able to decompose the development of age-profiles over time into a few components closely associated with specific age groups. Our long-term projections are based on the persistence of age-profiles over time and incorporate additional explanatory variables related to long-term shifts in labour supply into the statistical model, thereby reducing the subjective input of the modeller.

The functional data approach has been successfully applied to long-term forecasting of fertility and mortality rates in demographic forecasting (Hyndman & Ullah 2007). Demographic data often stretch back to the 18th century and therefore offer enough observations for a thorough statistical evaluation of the forecast accuracy at the end of the sample. Collecting employment rates, however, especially at the 1-year cohort level, started only more recently; therefore any model evaluation will have to rely on plausibility and comparisons with alternative approaches rather than using measures of statistical accuracy. We will illustrate the quality of forecasts based on the functional data method by using employment rates from Austria and comparing our forecasts to predictions based on cohort-specific time series forecasts and on the dynamic cohort method which is often applied by institutional forecasters (e.g. European Commission 2012).

We proceed in Section 2 with a definition of employment rates and an illustration of age-profiles for employment rates from Austria, a typical example for a developed small open economy with a well established welfare system. In this section we also summarise the main explanations for the historical development of employment rates offered by labour economics. In Section 3 we outline the advantage of the functional data approach in dealing with age-profiles showing highly persistent characteristics, the estimation results and the resulting long-term forecasts. We close this section with robustness checks based on forecasts from cohort-specific time series models and from the dynamic cohort method. In the final section we summarise our results and conclude.

2 Stylised facts about age-profiles of employment rates

Long-term economic forecasts regularly use a standard and highly stylised aggregate production function embedded in a Solow-Swan growth model. Technical progress is labour augmenting and the production function combines capital and labour into the economy's market production of goods and services. The aggregate measure of labour input used for the production of goods and services is the total number of hours worked, H, in period t, which can be split into the number of individuals in work, L_t, and the average number of hours spent by an individual at market work, h_t, such that $H_t = L_t h_t$. This decomposition reflects two decisions at the individual level: first whether to work or not, and second how much to work. In this paper we will concentrate on the so-called extensive margin, and model aggregate employment L_t (i.e., the number of individuals employed) by taking demographic projections of the working age population as given and combining these with age- and sex-specific forecasts of employment rates. Let $L_{st}(x)$ denote the number of individuals observed working at mid-year (July) with s representing either men or women aged x in year t, and $P_{st}(x)$ be the annual average of the corresponding population size. Then the age- and sex-specific employment rate is the ratio

$$Y_{st}(x) = \frac{L_{st}(x)}{P_{st}(x)}, \tag{1}$$

where working age $x = 15, \ldots, 74$, and $s \in \{\text{men}, \text{women}\}$. To motivate the use of functional data methods for forecasting employment rates it is instructive to look at an example; in our case we use employment rates from Austria. For reasons of data availability, we are restricted to employees; i.e., to persons in active dependent employment at mid-year[1]. Figure 1 shows the employment rates for men and women in the years 1960, 1975, 1990 and 2013 and illustrates

[1] The data on active dependent employment in July are from the Federation of Austrian Social Insurance Institutions and cover all actively employed persons with a monthly labour income of more than € 387 as of 2013. The data on resident population are from Statistics Austria and based on annual averages.

the strong relationship between employment and age and its development over time. In 1960 the employment rate for men aged 15 was comparatively high at 62 percent and climbed up to a maximum of 86 percent at age 21. With further increasing age, fewer men engaged in dependent employment and at around age 55 the decline in employment rates started to accelerate with a steep drop around the statutory retirement for men of 65. With the introduction of a mandatory ninth school year in 1967 this picture transformed into a more pronounced inverted U-shape with a rather stable employment rate for prime-aged men (that is those aged 25–54 years) and low labour market activity for the younger and older cohorts. Panel b in Figure 1 shows that the employment rates for the youngest women also started at historically high values in 1960 and climbed up until the mid 1960s but they fell distinctly afterwards. Women at higher ages moved successively out of employment with pronounced drops occurring around the ages 22 and 60, the latter being the statutory retirement age of women. Similar to men, the age-profile of the employment rate for women became more inverted U-shaped over time and slowly approaches the level seen for men.

Country-specific features may result in more or less steep branches of the age-profile but between ages 30 and 55 employment rates across countries as different as France, the UK, and the USA are almost indistinguishable (Blundell et al. 2013). Different aggregate levels of employment result almost exclusively from the behaviour of the very young and the old, consequently the inverted U-shaped profile is typical for both sexes throughout industrial countries (Pencavel 1986, Killingsworth & Heckman 1986, Blundell & MaCurdy 1986). Time spent in formal education before entering the labour market reduces the labour supply of the young. The entry into the labour market after finishing school lifts employment rates and — facilitated by public and private retirement schemes — workers at higher ages retreat from the labour market. There appear to be three main stages in a work-life giving rise to (a) low employment rates among younger age groups; (b) a sudden increase in employment rates after finishing school; and (c) a drop in labour supply around the statutory retirement age. In some countries the age-profile for women also has a more or less pronounced dip during child-bearing and child-rearing years creating the M-shaped age-profile visible in the right hand panel of Figure 1.

Figure 1 already provides some insight into the development of age-profiles over time. In Figure 2 we show detailed time paths for the 16, 35, 50, and 65 year olds. Longer spells in formal education by a growing number of teenagers lowered employment rates for both sexes by some 20 percentage points. Consequently the left hand branch of the age-profile in Figure 1 moved downwards. This pattern matches international data (Blundell et al. 2013) but the movement in Austria was especially strong for 15 year olds because mandatory schooling was extended from eight to nine years in 1962. Women aged 35 to 55 more than doubled their labour supply between 1960 and 2013. Although Figure 1 also indicates an increase for prime-aged men, this development is mostly due to a change in the sampling of Austrian labour market statistics. Starting with 1980, civil servants were added to the sample and due to the higher average age of civil servants this caused a marked upward shift in the recorded employment of men above age 30. Also interesting are the years between 1980 and 1995 when men and women aged 55 and older left active employment and used the opportunities offered by early retirement schemes. This downward movement reversed in the second half of the 1990s in response to a series of pension reforms making early retirement less attractive and boosting particularly the labour supply of women.

The high persistence of age-specific employment rates in Figure 2 is useful for forecasting purposes. Although some business cycle variation is visible, especially for the 16 year olds, most of the development over time results from slow and long-term changes rather than unpredictable high frequency variation. Furthermore, Figure 1 clearly reveals the strikingly similar behaviour of neighbouring age groups which suggests imposing restrictions on the shape of the forecasted age-profile. Although some ripples appear for each of the age-groups depicted in Figure 1 they appear as random deviations from the stylised inverted U-shape. We conclude from these characteristics that the age-profile of employment rates follows a smooth function with some observational errors and that the variance of this error appears to be homoscedastic over age groups.

2.1 Smoothing age-profiles for employment rates

The advantage of combining the information from the cross-section and the time series dimensions can be seen in the surface created by stacking age-profiles period after period. The surface in Figure 3 reveals the smooth transition over time more strongly. For example, the trend towards longer formal and full-time education dragged the starting point of the left hand branch of the age-profile downwards and flattened the ascending part of the inverted U-shape first at a slow pace but more progressively after 1990. Furthermore, the increasing average fertility age caused a rightward shift of the peak moving it closer to age 30. Whereas in 1960 women rapidly withdrew from the labour market after age 20, creating a swift decline in the age-profile until age 35, by 2013 the trough in the M-shape almost disappeared. Finally, postponed retirement decisions after the year 2000 warped the descending branch of the women's age-profile outwards and made it comparatively steep at the statutory retirement age for women of 60. Above age 60 the employment rate is close to zero and almost flat with no discernible change between 1960 and 2013.

The upper panel of Figure 3 shows that the surface based on the original data already bends relatively smoothly over all ages and evolves gradually from year to year. The labour market outcome for neighbouring cohorts differs only slightly, changes from one year to the next are generally small, and business cycle variations seem to be limited to young age groups. To corroborate this visual impression we smooth the original data simultaneously along the age and the time dimension using the SMILE method suggested by Dokumentov & Hyndman (2013). The SMILE method was originally designed for mortality rates and can handle abrupt changes in the slope of age profiles; e.g., the swift switch-over from high infant mortality towards more regular levels. It is especially suitable for smoothing employment rates because they steeply ascend at age 16 and descend around ages 60 to 65. Moreover, employment rates of men experienced a considerable jump in 1980 (see Figure 2) due to a change in the sampling method. An interesting by-product of the SMILE method is a decomposition of the data into four components: the smooth surface reflecting the main effect depicted in the lower panel of Figure 3, period effects pointing towards a systematic deviation associated with a particular year in the sample, cohort effects showing special features of a cohort as it moves through time and grows older, and a residual effect capturing irregular deviations.

Period effects are negligible in our case, in Figure 4 we therefore concentrate on the cohort and residual effects. The heat map in the left hand panel visualises distinct cohort patterns running as diagonal blue and red stripes from the lower left hand to the upper right hand corner. The colour blue indicates below average labour market participation for a particular cohort while red shows above average employment ratios. Most of the cohort variation is small and bounded within ±0.5 percentage points, but the birth years 1938, 1940 and 1946 stand out with deviations of roughly ±1.5 percentage points over extended periods. These years of political upheaval and war time were characterised by postponing and catching up childbirth, e.g. 44 percent of all births in 1946 happened to be in the first half of the year, while in 1947 this share climbed up to 52 percent [2]. Shifts of births between the first and second half of the year twist the employment ratio which is based on a comparison of workers of age x sampled at mid-year in the numerator with the average population of age x in the corresponding year

[2] Source: Statistics Austria, Statistische Nachrichten 1947(8) p. 146 and 1948(3) p. 71.

in the denominator. Specifically for the cohort born in 1946 this creates a lower than average employment rate which is correctly identified by the cohort effect of the SMILE decomposition. Finally, the left hand panel in Figure 4 shows that the residual component is close to zero for the highest age groups from 60 through 74, but otherwise no systematic pattern emerges. The SMILE decomposition suggests that the wiggles along the age dimension in the original data can be interpreted as measurement error and we are able to use the advantages offered by functional data analysis (Ramsey & Silverman 2005).

2.2 Additional explanatory factors for long run labour supply

The smooth character of the age-profiles and their slow movement over time already provide useful restrictions and information for long-term forecasting. In this section we look for additional explanatory variables helping to forecast employment rates. We draw on the extensive research on labour supply as summarised in Pencavel (1986), Killingsworth & Heckman (1986), Blundell & MaCurdy (1986), and Meghir & Phillips (2010). This literature concentrates on identifying and measuring the elasticity of labour supply with respect to changes in real wages, alternative income sources, taxes and welfare benefits. The estimation of elasticities is based on a labour supply function derived from the first order conditions of either a within-period (static) or a multi-period (dynamic) utility maximisation problem subject to a budget constraint. The individual utility function describes the utility achieved from the consumption of goods and services and enjoying leisure. Within a general consumer demand model a trade-off between the benefits of higher labour income and the loss in utility associated with giving up leisure time arises. Individuals split their fixed endowment of time between hours worked in the market and hours spent on other activities. The typical labour supply function finally relates hourly labour supply to the real wage rate, a measure of non-labour income, and a set of observable personal characteristics from the utility function sometimes called "taste shifter" controls (Blundell & MaCurdy 1986). These personal characteristics account for the heterogeneity across individual labour supply found regularly in repeated cross section or panel data. Individual labour supply models can be extended to reflect household decisions with more than one potential earner or to allow for the accumulation of savings. Another extension is using a dynamic utility maximisation model with endogenous choice of human capital accumulation. This extension destroys the time separability of the inter-temporal decision rule and requires more complicated budgeting formulations.

The estimation of the wage elasticity is subject to simultaneity between the labour supply decision, the after tax wage rate, and the income from alternative sources. For example workers who value leisure less and work comparatively longer hours are likely to receive higher wages but due to progressive income taxation they also face higher marginal tax rates. In this case the hourly net of tax wage may become lower, causing (if not accounted for properly) a downward bias in the estimated wage elasticity. Moreover, different tastes for leisure create a selection bias and heterogeneity in the sample of workers, violating homogeneity assumptions. The heterogeneity across individuals' tastes can be controlled for by using additional explanatory variables in the regression, or by splitting the sample into homogeneous subgroups according to personal characteristics; alternatively such characteristics can be used in grouping estimators (see Blundell et al. 1998). A survey by Chetty et al. (2011) shows that the estimated values of Hicksian as well as the Frisch wage elasticities on the extensive margin are similar and low across studies with a mean of 0.25. Meghir & Phillips (2010) conclude that elasticities differ across groups. While men have a participation elasticity close to zero, the participation elasticity for single mothers and married women is typically quite high; that of lone mothers is among the highest of all demographic groups. Wernhart & Winter-Ebmer (2012) show, however, that never-married women in Austria have low wage elasticities of participation similar to those for men. Furthermore, since the end of the 1980s, married women in Austria show substantially falling wage elasticities of participation though they are still higher than those for men.

For the purpose of long-term forecasting, the low average sensitivity of participation decisions to changes in wages suggests that aggregate employment rates can be modelled separately from the wage development. If a long-term forecast is based on the assumption of a constant tax and benefit system, the response of the participation decision to changes in the tax and welfare system is irrelevant. Other individual and aggregate characteristics used in empirical labour economics, on the other hand, may be relevant for the purpose of long-term forecasting. Most common are personal characteristics like gender and age, both of which are already reflected by modeling gender specific age-profiles. Furthermore, variables like the education level, marital status and existence of children have been used to model heterogeneity. Bauernschuster & Schlotter (2015) suggest that child care facilities expand the labour supply of mothers with little children and Gruber & Wise (1999) show that the accessibility of early retirement schemes has strong effects on the employment decision of the elderly. Additional conditioning variables include information on race and ethnicity as well as regional variables like US-state unemployment rates or year dummies. In the following section we will use explanatory variables as suggested by labour economics and suitably adapt them to forecast age-profiles of Austrian employment rates.

2.3 Explanatory variables for Austrian employment rates

The descriptive analysis of the Austrian age-profiles shows several stylised facts for employment rates. While the employment decision of men is almost constant for prime-age groups, it declines substantially for younger ages and for workers close to the statutory retirement age. Women, on the other hand, show a secular increase in employment rates for all but the youngest ages. Employment rates of women advance at a steady pace and slowly catch up to the men's levels.

We will use aggregate explanatory variables corresponding closely to the list of personal characteristics suggested by micro-econometric evidence. For example, declining employment rates at younger ages are clearly associated with longer full-time attendance at school, and higher educational attainment in turn accelerates labour market participation (Pencavel 1986). In the following application we prefer using data on the educational attainment of the total working age population because the labour supply decision after completing formal full time education depends at all ages on opportunity costs and higher education gives access to higher paid jobs across all cohorts (Heckman et al. 2006). Consequently, individuals with higher educational attainment face higher opportunity costs — independent of their age — when staying out of the labour force. Our measure of educational attainment is a weighted average over individual cohorts (see the appendix A for details) and thus evolves only gradually over time because the entrance of new graduates and the retirement of less educated 66-year-olds affect the average only at the lower and upper margin, respectively. Over the last fifty years, more widespread higher education resulted in a rising share of individuals who completed more than the minimum mandatory schooling. Table 1 shows strikingly different starting values for men and women in 1960 but by 2013 the gap has narrowed substantially. Actually, in 2013 the difference between men and women aged 15–24 almost disappeared whereas the 60–65 years old still show a discrepancy of approximately 20 percentage points. This explains the disparity in average educational attainment of the year 2013. We assume that all future cohorts will have a common education level corresponding to the mean of women aged 15–24 years over the years 2009 through 2012. This will result in a closing of the gap between genders by the end of the forecasting period.

Aggregate business cycle fluctuations will move individuals in and out of employment due to variations in labour demand and cause more or less pro-cyclical fluctuations in employment rates. We capture these dynamics in the time dimension by using the aggregate unemployment rate, either measured as total, or as a gender-specific ratio of unemployed persons to persons being in active dependent employment. The total unemployment rate doubled between 1960 and 2013; most of the level shift occurred at the beginning of the 1980s in the aftermath of the second oil crisis; subsequently this upward movement eased. Though the difference between men and women reversed between the beginning and the end of our sample (see Table 1), both series show almost identical cyclical dynamics. We assume the unemployment rates of men and women will converge quickly towards a common steady state level, which we set according to Hofer et al. (2014). We fix the converge rate at 20 percent per year; i.e., the difference to the steady state value will decline by 20 percent per year.

Household characteristics such as the presence of children and marital status also affect labour supply decision of individuals. We use two well-known aggregate demographic variables reflecting the number of children in a household and the age of women giving birth. At the beginning of the 1960s, the total fertility rate (i.e., the number of children per woman) started from high values associated with the end of the baby-boom. Around 1965, the fertility rate sharply declined levelling out at values around 1.6 at the end of the 1970s. After edging towards a minimum of 1.3 around the year 2000, the fertility rate started to move upwards; the population projection by Statistics Austria expects it to pick up moderately towards 1.55 over the forecast horizon. Another interesting development is related to a change in the shape of the female age-profile. The M-shaped age-profile of women slowly turned into an inverted U-shape while the trough moved rightwards during this transition. This peculiar change in shape is due to the increased average age of women giving birth. As measured by Statistics Austria, the average fertility age advanced by almost three years between 1960 and 2013. We assume that the average fertility age will remain constant over the remaining forecasting horizon.

Up to 1981, the marital status of couples was registered in Austria only at census dates, afterwards a survey forms the basis for published numbers, and from 2004 onwards the marital status is part of the regular labour force survey questionnaire. We compute the share of married couples in the total of families, where families are defined as adult couples living in the same household with or without children or single parents living in the same household with children. We fill the missing values for the years between population census and survey dates by linear interpolation. At the beginning of our sample, 83 percent of families were married. This share increased to 86 percent in 1971 and declined successively towards 72 percent. We assume that the share of married couples will remain at this value over the full forecasting horizon.

The availability of daycare facilities assists parents in reconciling market work with family life but it also increases the reservation wage depending on the extent of public subsidies to the costs of child care. We use the number of crèches, the number of kindergartens, and the share of children cared for in daycare facilities in the total number of children aged 3 to 6 years (see appendix A for details). Each of these variables measures slightly different aspects of daycare. A higher number of crèches, for example, allows parents with babies or toddlers to engage in market work activity while access to a kindergarten improves the flexibility of parents of pre-school aged children. Usually the access to day care facilities is easier in cities as compared to rural areas. Therefore a higher number of kindergartens also provides some indication for a broadened supply of child care on the country side. Finally, the share of children in day care is a measure for the intensity of use of existing facilities. Despite the fact that between 1960 and 2013 the number of children in the relevant age-group declined by almost one quarter, all three measures have seen a strong increase over time. In 2009, the share of children in day care shifted upwards by 4 percentage points from 2008. This level change was due to waiving the kindergarten attendance fee for the last year before children enter regular school. This fee was abolished in 2009 making day care for this group of children a public transfer in kind. For younger children the attendance fee still applies, but it is subject to a means tested subsidy depending on household income and family size. Thus the higher reservation wage associated with the costs of day care is attenuated for low income households.

The correlation among these explanatory variables is generally high. Table 2 indicates that even loosely related variables like the average education of men and the number of kindergartens are almost perfectly correlated, making statistical inference in a regression model difficult. Nevertheless, including highly correlated variables into the regressor matrix of a forecasting model may still improve forecasting accuracy.

The right hand branch of the age-profile is strongly affected by rules governing access to early retirement schemes. Figure 3 shows that between 1980 and 2000 the employment rate of women aged 50 to 60 dropped considerably; a similar pattern emerges for men in Figure 2. The retreat from the labour market was, on the one hand, a consequence of the maturing pension system with an increasing number of individuals fulfilling the requirements for early retirement, and on the other hand of introducing new opportunities for early retirement in the wake of the upward shift in unemployment after the year 1980 (Hofer & Koman 2006). Growing transfers from the federal budget to the public pension system caused a series of pension reforms aimed at increasing the effective retirement age. Starting with the year 2000 several reforms altered the retirement incentives of individuals close to the statutory retirement age, either immediately or stepwise within more or less long transition periods. The stepwise approach was often preferred by politics because some of the changes in the benefits' regulation were substantial and individuals close to the retirement age were considered to deserve protection against a radically different legal environment. Moreover, political parties had different views about the required extent of reforms and the speed of their implementation, resulting in compromise and a gradual approach, sometimes even reversing already implemented measures. Table 3 provides a survey of pension reforms between the years 2000 and 2012 and describes their most important features. The cautious approach is visible in Figure 2 where no sudden jumps after the year 2000 can be found, rather employment rates increased gradually in the relevant age groups while individuals above the statutory retirement age did not react at all. We capture this gradual effect of reforms by constructing a ramp dummy increasing by one unit every year after 2000 until 2013. We hold the value of the ramp dummy constant after 2013 because we do not assume further pension reforms to be enacted after 2013 and we do not have conclusive micro econometric evidence supporting the view that currently enacted laws will continue to drive up employment rates at constant pace.

Finally, Figure 2 shows for some age groups a jump in employment rates in 1980. After 1980 civil servants have been included in the sample due to a broader definition of employees. We model this one-off change by a step dummy variable switching from -1 to 0 in 1980.

3 Combining the functional data approach with dynamic regression

Let $y_{st}(x) = \log(Y_{st}(x)/(1 - Y_{st}(x)))$ denote the observed logit transformed employment rate for sex s at age x in year t. These data are smoothed to give

$$y_{st}(x_i) = f_{st}(x_i) + \sigma_{st}(x_i)\epsilon_{sti}$$

where $t = 1, 2, \ldots, T$, $s \in \{\text{men}, \text{women}\}$, i indexes single years of working age $\{x_{15}, x_{16}, \ldots, x_{65}\}$. ϵ_{sti} is a standard normal random error and $\sigma_{st}(x_i)$ allows the noise to vary over time and age. The observed data are not of a functional nature rather we assume there are underlying functional time series observed with error at discrete points in time. The smooth functions $f_{st}(x)$ are decomposed using a functional data model (Hyndman & Ullah 2007) given by

$$f_{st}(x) = \mu_s(x) + \sum_{k=1}^{K} \beta_{kst}\phi_{ks}(x) + e_{st}(x) \tag{2}$$

where $\mu_s(x)$ denotes the average (or median) of $y_{st}(x)$ across years, $\{\phi_{ks}(x)\}$ is a set of orthonormal basis functions obtained using (possibly robust) functional principal component analysis (Ramsey & Silverman 2005), and $\{\beta_{kst}\}$ are a set of time-varying coefficients that are contemporaneously uncorrelated with each other (these are the principal component scores). The final term $e_{st}(x)$ denotes normally distributed errors. Note that ϵ_{sti} denotes the observational errors of employment rates while e_{st} determines the model error.

The advantage of this model is that it allows the time and age dimensions to be entirely separated, with the coefficients $\{\beta_{kst}\}$ controlling how the basis functions $\{\phi_{ks}(x)\}$ affect the smooth functions $f_{st}(x)$ over time, and with the basis functions $\{\phi_{ks}(x)\}$ describing the labour supply decision of particular age groups if they strongly deviate from the average (or median) behaviour $\mu_s(x)$.

We assume that each of the time varying coefficients β_{kst} follows a dynamic regression model (e.g., Pankratz 1991):

$$\beta_{kst} = \gamma_{ks0} + \sum_{j=1}^{M} \gamma_{jks}\left(z_{jst} - z_{js}^{ss}\right) + \eta_{kst} \tag{3}$$

$$\text{where} \qquad \phi(L)\,\eta_{kst} = \theta(L)\,u_{kst}, \tag{4}$$

and L denotes the backshift operator $L\eta_t = \eta_{t-1}$. This is a linear regression model with serially correlated errors specified as an ARMA process. The explanatory variables discussed in the previous section are denoted here by z_{jst}. We subtract the steady values z_{js}^{ss} from all explanatory variables in order to ensure long-term convergence of the coefficients' forecasts to stable values.

Then conditioning on observed data $\mathcal{I} = \left\{y_{st}(x_i), z_{jst}; t = 1,\ldots,T; i = 15,\ldots,65\right\}$ and the set off basis functions Φ we combine the measurement equation for $y_{st}(x_i)$ with equation (2) to obtain h-step-ahead forecasts (Hyndman & Ullah 2007):

$$\mathrm{E}\left[\hat{y}_{s,T+h}(x)|\mathcal{I},\Phi\right] = \hat{\mu}_s(x) + \sum_{k=1}^{K} \tilde{\beta}_{ks,T+h}\hat{\phi}_{ks}(x) \tag{5}$$

where $\tilde{\beta}_{ks,T+h}$ denotes the h-step-ahead forecast of β_{ks} based on the estimated time series $\hat{\beta}_{sk1},\dots,\hat{\beta}_{skT}$. The functional data model also provides an estimate of the forecast variance. Because the functional principal component analysis gives approximately orthogonal basis functions we can approximate the forecast variance by the sum of component variances in equation (5):

$$\text{Var}\left[y_{s,T+h}(x)|\mathcal{I},\Phi\right] \approx \hat{\sigma}^2_{\mu_s}(x) + \sum_{k=1}^{K} \hat{\sigma}^2_{\beta_{ks}} \hat{\phi}^2_{ks}(x) + v(x) + \sigma^2_{s,T+h}(x) \tag{6}$$

where $\hat{\sigma}^2_{\beta_{ks}} = \text{Var}\left(\beta_{ks,T+h}|\beta_{ks1},\dots,\beta_{ksT}\right)$ is the variance of the time varying coefficients resulting from the dynamic regression model, $\hat{\sigma}^2_{\mu_s}(x)$ is the variance of the smooth estimate $\hat{\mu}_s$, $v(x)$ is the model error variance from equation (2), and $\sigma^2_{s,T+h}(x)$ is the measurement error variance, cf. Hyndman & Ullah (2007).

3.1 An application of the functional data model to Austrian employment rates

The Austrian labour market is similar to that of other industrial countries with mature pay-as-you-go pension systems and serves as an illustration for the application of the functional data method to long-term forecasting of employment rates. We first subtract the cohort effect resulting from the SMILE decomposition from observed employment rates $Y_{st}(x)$. In the second step we smooth the logit-transformed age-profiles $y_{st}(x)$ by penalised regression splines with 30 knots and estimate the curves $\hat{f}_{st}(x)$ for each year. Contrary to the SMILE decomposition we only smooth along the age dimension in this case in order to preserve variation across age-profiles over time. This improves the estimation results of dynamic regression models for the coefficients of the basis functions β_{kst}. The L_1-median of the estimated curves $\{\hat{f}_{s1}(x),\dots,\hat{f}_{sT}(x)\}$ gives a robust estimate of the main effect $\hat{\mu}_s(x)$ in equation (2). After subtracting $\hat{\mu}_s(x)$ from each smoothed curve we arrive at median-adjusted data $\hat{f}^*_{kst}(x) = \hat{f}_{kst}(x) - \hat{\mu}_s(x)$. Given the number of factors K, we obtain the basis functions $\phi_{ks}(x)$ by combining the weighted principal components method and the RAPCA algorithm according to the two-step procedure in Hyndman & Ullah (2007). We choose the number of basis functions $K = 3$ for both sexes. In the case of men, the first three basis functions explain 86.8, 6.0, and 4.5 percent of the variation leaving only 1.7 percent unexplained. The respective shares for women are 86.4, 11.4, and 1.0 percent, thus the remaining unexplained variation for women is only 0.7 percent. Choosing higher values for K does not change the forecast characteristics of the model because the effect of higher order basis functions are close to zero.

We show the estimated main effect $\hat{\mu}_s(x)$, the three fitted basis functions $\hat{\phi}_{ks}(x)$, and the associated coefficients $\hat{\beta}_{kst}$ for the logit transformed data in Figures 5 and 6 for men and women, respectively. The main effect for men in Figure 5 replicates the inverted U-shape known from the original data with a rapid decline in employment rates starting after age 60. For an interpretation of the basis functions we have to keep in mind that they model deviations from the estimated main effect. The first basis function for men is slightly positive until age 55 and becomes strongly negative for higher ages reaching a local minimum around age 60. This particular shape suggests that the first basis function describes the retirement decision before the statutory retirement age for men of 65. The varying importance of the first basis function over time is shown by the development of coefficient 1. At the start of the sample, coefficient 1 was strongly negative, reversing the negative sign of the first basis function and implying an employment rate of men in the relevant age group above the main effect. This positive effect became smaller over time until 1983, when coefficient 1 is close to zero indicating that the first basis function created no divergence from the main effect around these years. Afterwards, coefficient 1 continues on its upward path and dampened employment rates with the maximum negative effect reached in year 2000. By the year 2013, the adverse effect of the first basis function on the employment rate of elderly men ceased.

The second basis function is less easy to interpret as it mixes changes in the labour market activity of two different age groups. The markedly negative values for the youngest age groups point at the progressive retreat of teenagers from the labour market. At the same time positive values during the mid working life indicate above average labour supply of men in these age groups. The development of coefficient 2 in Figure 5 corroborates this twofold interpretation. The most obvious characteristic of coefficient 2 is the sharp jump in 1980 capturing the inclusion of civil servants into the sampling of active dependently employed in this year. Nevertheless, the steady upward movement of coefficient 2 brings about a slowly decreasing employment rate for teenagers which accelerated temporarily in 1967, the year when a ninth school year has been added to minimum compulsory schooling years.

The third basis function shows negative spikes for young men and those aged 57 to 61, indicating the higher variability of labour supply in these age groups relative to prime age labour supply. Coefficient 3 implies that in 2013 this basis function creates a negligible deviation from the main effect but during the first half of the 1980s the third basis function exerted a dampening effect on the employment of men in these age groups while in the years around 2005 it increased male employment.

The womens' main effect in Figure 6 has a skewed M-shape with one peak at age 20 and the right hand branch of the M tilted downwards. This reflects more closely age-profiles of the female employment rate from the earlier years in our sample, see Figure 3. The flattening of the M-shape in the data is predominantly associated with the first basis function. This basis function has big positive values for women between age 25 and 55 with heightened values at higher ages; coefficient 1 shows a strong positive trend over the full sample flattening out slightly only in the years after 2000. Obviously, the first basis function reproduces the general increase in female labour market participation throughout our sample. We also observe a small jump of coefficient 1 in 1980 corresponding to the inclusion of civil servants into the sample. The second basis function captures two trends that emerged almost contemporaneously. The shrinking labour force participation of female teenagers started to gain pace around the year 1980 and coincides with a strengthened move into earlier retirement by women. Both movements are well mirrored in the development of the associated coefficient 2 in Figure 6 with high values until 1980, subsequently replaced by a downward trend reaching a minimum before the year 2000. Thereafter, the dampening effect of the second basis function on employment rates of pre-retirement age groups sharply reversed. This also implies that after the year 2000 the second basis function pushed teenage employment rates up, contradicting the trend to higher education. Strong negative values of the third basis function at ages 15 and 16 together with the development of coefficient 3 counteract this effect. The movement of coefficient 3 mirrors that of coefficient 2 and balances the dampening effect of the second basis function on teenagers' higher demand for full time education.

Forecasting Austrian employment rates

The time variation of the coefficients for all basis functions corresponds well with the Austrian labour market history. The coefficients, however, show peculiar turning points and — in a few cases — appear to be unstable suggesting caution when making model based forecasts. Pure time series models, for example, may generate unstable predictions or, in the case of exponential smoothing models, may require ad hoc assumptions on the factor controlling the dampening of the growth rate. Moreover, research in labour market economics suggests several potential explanatory factors related to long-term trends in labour market participation. We therefore choose dynamic regression models to forecast time varying coefficients β_{kst} and use the explanatory variables discussed in section 2.3. This extension improves plausibility as well as transparency of the forecasting process while still accounting for the dynamics of the adjustment to unexpected shocks η_{kst}.

The selection of explanatory variables for men and women, respectively, is based on the sum of individual Akaike Information Criteria (AIC) for each of the $K = 3$ dynamic regression models plus a correction term accounting for the bias towards large models. The identification of the ARMA part of each model relies on automated model selection criteria (Hyndman & Khandakar 2008, Hurvich & Tsai 1989).

Table 4 presents the final models and estimation results for men and women. The dynamic regression model for men includes average education, the male unemployment rate, the pension reform ramp dummy, and the step dummy for 1980 giving a minimum AICc of −124.8. The model search for women ends up at an AICc value of −207.4 and suggests the corresponding set of explanatory variables as used for men but adds the share of married couples, the number of kindergartens, and the average fertility age to the list of regressors. Because we do not have plausible forecasts for these variables we exclude them from the dynamic regression models presented in Table 4. To illustrate the value of explanatory variables for dynamic regression models it is illustrative to compare the above AICc values with those from pure time series based models supplemented only with the step dummy for the year 1980: −100.9 (men) and −153.2 (women).

The left hand part of Table 4 shows dynamic regression models for the time varying coefficients in Figure 5. Coefficient 1 transmits negative deviations from the main effect resulting from early retirement decisions of men into employment rates and consequently responds negatively to pensions reforms aimed at limiting access to early retirement schemes. The strongly positive effect from education on early retirement appears spurious and results from the steady increase in the average education level of men between the years 1960 to 2000 and its levelling off afterwards. Coefficient 2 grows over time and jumps in 1980, consequently the step dummy has a positive value. Higher average education levels and pension reforms also increase the relevance of the second basis function thus contributing to lower labour market participation of young cohorts as well as higher employment of prime aged men. Finally, Coefficient 3 also shows a spike in 1980 captured by the step dummy and responds negatively to variations in the business cycle as captured by the unemployment rate.

For women the lower panel of Figure 6 shows the time varying coefficients of the first three basis functions and the right hand panel of Table 4 presents the results from dynamic regressions. Coefficient 1 has a strong positive trend with a small jump in 1980 inducing a positive effect of the step dummy. The steady increase in education levels supports the secular upward trend in female employment but business cycle downturns dampen this effect temporarily. Longer periods of full time education increase Coefficient 2 and thereby lower employment within younger age groups. On the other hand, after the year 2000, Coefficient 2 starts to decline, which is associated with the negative effect of pension reforms on Coefficient 2. As a consequence the dampening effect of basis function 2 on employment rates quickly disappears. Coefficient 3 also has a lower turning point around the year 2000 and produces the biggest positive effect of basis function 3 in the last year of the sample when teenage employment reached its lowest in sample value. Coefficient 3 responds positively to pension reforms thus creating the corrective movement necessary to compensate for employment enhancing effects in younger age groups resulting from basis function 2.

The result of applying forecast equation (5) to Austrian employment rates is visualised as a rainbow plot in Figure 7. The grey lines represent historical values and the coloured lines are forecasts. The right hand panel shows almost stable forecasts for employment rates of men, while employment rates of women increase substantially over the forecast horizon. Stable employment rates for men are the result of rather flat forecasts for the three time varying coefficients $\tilde{\beta}_{k,T+h}$ as visible in the upper panel of Figure 8. This projection can be attributed to a stable average education level of men over the forecast horizon and it is due to the assumption that pension reforms will stop to produce further increases of employment rates during the period before the statutory employment age. i.e. the ramp dummy reflecting gradual pension reform activities is frozen at the year 2013 value. Though the explanatory variables for men do have some variation over time their development does not support a long-term level shift and all of the shaded 90 percent prediction intervals include the last value of the respective coefficient. On the other hand, the average education level for women will continue to grow until 2050 due to the ongoing substitution of old cohorts by young better educated cohorts. Only in the decade before 2050 the average education level of women will start to converge gradually to its new steady state level. The lower panel of Figure 8 shows that each of the time varying coefficients for women responds to this change permanently, specifically, prediction intervals of the first two coefficients do not include the last value from year 2013.

Forecast Evaluation and Robustness

In contrast to population forecasts, missing data prevents a rigorous statistical evaluation for long-term forecasts of employment rates. Statistical records usually do not provide the realizations necessary to compute prediction errors for very long horizons like 20 to 50 years.

Furthermore, comparing our forecasts to long-term forecasts from official sources suffers from the fact that official institutions incorporate expected future labour market effects of already implemented policy measures into their forecasts, e.g. the expected consequences of a prospective increase in the statutory retirement age. Our forecasts, on the other hand, do not account for such effects. We therefore check the robustness of forecasts from the functional data model in terms of their plausibility and by comparing them to predictions based on alternative methods.

Forecasts for men and women reflect the historical development very well. Figure 7 shows that the employment rates of men remain almost constant over the forecast horizon, whereas women continue to extent their labour market activity. This results in further convergence of employment rates for both sexes and corresponds well with international experience (Blundell et al. 2013). Predicted employment rates for the youngest cohorts continue to decrease for men and women alike, although at reduced pace. This implies a diminishing slope for the trend towards extended full time schooling and it reflects estimates of decreasing returns to education in Austria by Fersterer & Winter-Ebmer (2003). Around age 65, on the other hand, predictions for both sexes show only tiny differences to the last realisation from year 2013. Given the stable shape of employment rates in this age group in Figure 2 and the fact that we do not use additional assumptions on the future effects of pension reforms this prediction appears reasonable. Forecasts from the functional data model expect prime-age women to continue increasing their activity in the labour market by five to six percentage points, cf. Table 5, while men will reduce their employment status slightly below the average from the years 1980 through 2013. This movement is related to the converging educational attainment of both sexes, as older women with lower education drop out of the labour force and younger cohorts entering the labour market are better educated. This smooth development accelerates female labour market participation directly through higher opportunity costs of staying out of the labour force (Becker 1975) and indirectly by a higher probability of moving into a job after finishing education and delaying births (Bloemen & Kalwij 2001). Moreover, higher employment probabilities are also strengthened by additional political efforts to extent not only the number of child care facilities but also their operating hours (Bauernschuster & Schlotter 2015). Nevertheless, the functional data model predicts the M-shape of the age-profile to persist throughout the forecast horizon. The general increase in female employment rates carries over to age groups around 55, while for men at age 55 employment rates remain constant. Overall, the predictions from the functional data model remain in a plausible range and they are in accordance with theoretical and empirical labour market research.

We use several alternative forecasting approaches for a comparison. Specifically, we compare our forecasts for employment rates based on the functional data method (FDMc) to four alternative approaches: a functional data model with input data $Y_{st}(x)$ not adjusted for cohort effects (FDM), individual times series models for each logit transformed cohort $y_{st}(x)$ using only the 1980 step dummy for adjustments in official statistics as an explanatory variables (TSM). Age specific dynamic regression models (DRM) for $y_{st}(x)$ including the same set of explanatory variables as used for the prediction of time varying coefficients β_{kst}, cf. Table 4. To avoid outlying forecasts for a specific cohort of age x we smooth the predictions resulting from the time series and the dynamic regression models across ages. Finally, we use the dynamic cohort model (DCM) as applied by Productivity Commission (2005) and European Commission (2012) as an alternative applied by official institutions. Appendix B gives a short description of this approach.

Table 5 summarises point forecasts from a variety of models and, if available, 90 percent confidence intervals for the age groups 16, 35, 55, and 65 at the 1-, 30-, and 50-year forecast horizon from different models. The first line in each panel gives forecasts based on the functional data method with employment rates adjusted for cohort effects. The 90 percent prediction error intervals remain within three to four percentage points around the point forecast, even at long horizons. At very long forecast horizons, the adjustment by cohort effects does not create big differences in forecasts from functional data models. Jumps from the last observation in 2013 to the first forecasted value in year 2014, on the other hand, are distinctly smaller if we remove the cohort effects detected by the SMILE method from the original data.

The biggest deviations from functional data forecasts emerge in comparison to pure time series models (Hyndman & Khandakar 2008). These models project a further decline in employment rates for young women to 20.6 percent until 2063. For the cohort of 35 year old women a further increase in employment rates by more than 20 percentage points is predicted; the biggest change from 57.2 to 91.3 percent happens to be in the age group of 55 year old women. Pure time series models obviously reproduce and project the instability of age-specific employment rates as depicted in Figure 2 even if the jump in 1980 due to adjustments in official statistics is accounted for by a step dummy.

If we supplement the time series models by adding explanatory variables we avoid the instabilities associated with pure time series forecasts but dynamic regression models generate the most conservative set of forecasts. The deviations from the last observation in 2013 are small throughout all ages and forecast horizons presented in Table 5. This indicates that the additional information contained in explanatory variables does not exclusively determine the forecast patterns produced by functional data models, rather the combination of basis functions and time varying coefficients resulting from functional data analysis with dynamic regression models produces overall plausible forecasts.

The dynamic cohort model provides another check for predictions based on functional data models. We use 1-year cohorts in our example and take averages over the past ten years to compute age and sex specific entry and exit probabilities (see appendix B). For the youngest age groups the dynamic cohort model requires a no-change assumption and consequently employment rates at age 16 in Table 5 remain constant. For older age groups the dynamic cohort model picks up the recent cohort specific development and extrapolates it into the future. This gives slightly rising employment rates for men and women at age 55 and 65, but in the long run, DCM-forecasts show lower employment rates of prime-age men and women. Especially in the case of women this outcome appears implausible given the ongoing substitution of less educated cohorts by cohorts with higher educational attainment. Another caveat with respect to the dynamic cohort model is its sensibility to outsized changes in employment rates. The ten-year period used for the computation of average entry and exit probabilities includes the financial market crisis with a sharp drop in employment rates. This dampens average entry probabilities and exaggerates exit probabilities at the same time causing subdued predictions.

4 Conclusion

Long-term economic forecasts are a key input for monitoring the budgetary consequences of social programs in areas like education, health care, public old age pension systems, and old age care. These programs require forward looking political decisions which often have a serious impact on private households' intertemporal budget constraints; consequently they are politically highly sensitive. Pronounced individual exposure together with increasing prediction uncertainty at longer forecast horizons call for simple and transparent forecasting methods which at the same time provide minimal flexibility to the modeller. In this paper, we focus on the limited but central aim of forecasting long-term labour market developments. Currently, long-term forecasts of labour market participation rates are based on age-specific trend extrapolations or on the dynamic cohort method. We suggest an alternative statistical approach, combining functional data analysis with nonparametric smoothing and robust statistics. Our approach uses persistent properties of the combined cross-section and time series dimension for the prediction of participation rates, and integrates relevant explanatory variables as suggested by empirical and theoretical labour market research into the information set.

We use the functional data approach to derive interesting curve features from discretely observed age-profiles of employment rates and decompose the age-profile by functional principal components analysis into a few components offering a more informative interpretation with respect to specific age groups. This approach is particularly useful in raising forecast transparency, limiting the flexibility of the modeller, and integrating explanatory information on long-run labour market developments.

As an illustration, we apply the functional data method to Austrian employment rates, a typical example for developed small open economies with a well established welfare system. The predictions based on the functional data model for men and women deviate considerably from each other: whereas employment rates for men stay within a narrow range over the whole forecasting horizon, the functional data model predicts distinctly higher activity rates for women. This divergence reflects the increasing average education of women as younger cohorts with higher educational attainment successively replace older less educated cohorts dropping out of the labour force. Although the restrictions resulting from the historical curvature of the age-profile directly carry over into predictions of future age-profiles, we still need additional time varying information from explanatory variables to achieve plausible long-term developments of employment rates, specifically information on educational attainment turns out to be vital. Alternative approaches based only on information from the time series dimension or based on the dynamic cohort method do not provide plausible alternative scenarios.

Appendix A: Source and computation of explanatory variables

Statistics Austria provides information on educational attainment of men and women for the years 2009 through 2012 for 5-year cohorts from age 15 to 84. We use the share of individuals with successful completion of additional schooling after the statutory minimum number of 9 school years and interpolate the shares of the 5-year groups using cubic spline functions according to Forsythe et al. (1977). Given age-specific data at 1-year steps from the interpolation we construct an average educational attainment measure for the working age population by computing the weighted average for ages 15 through 65. Because data are only available for the years 2009 through 2012 we compute the age-specific shares for the period 1960 through 2008 recursively by shifting the shares backward in age and time; e.g., the educational attainment of the 15-years old in 2008 corresponds to the attainment of the 16-years old from 2009. When there is no more value available for the oldest cohorts, we take the value of the 84-years old from the year 2009 as a substitute:

$$\mathrm{educ}_t(x) = \begin{cases} \mathrm{educ}_{2009}(84), & \text{if } \mathrm{educ}_{t+1}(x+1) = \text{missing} \\ \mathrm{educ}_{t+1}(x+1), & \text{otherwise.} \end{cases}.$$

We proceed in a similar way to compute forecasts of the average educational attainment from 2013 onwards. We shift the shares forward in age and time; e.g., the educational attainment of the 65-years old in 2013 corresponds to the attainment of the 64-years old in 2012. In the first forecast year 2014, the value for the 15-year olds is missing and we substitute in the mean value of the 15–24 years old women from 2009 through 2012, $\mu_{\mathrm{educ}}(15-24)$, this allows the following recursive computation:

$$\mathrm{educ}_t(x) = \begin{cases} \mu_{\mathrm{educ}}(15-24), & \text{if } x = 15 \\ \mu_{\mathrm{educ}}(15-24), & \text{if } x > 15 \text{ and } \mathrm{educ}_{t-1}(x-1) = \text{missing} \\ \mathrm{educ}_{t-1}(x-1), & \text{otherwise} \end{cases} \cdot$$

This simple forecasting rule enables us to compute the weighted average educational attainment for all forecast years by using the number of persons of age x from the population forecast as weights. This weighted measure of educational attainment evolves only slowly over time because the average is only changed by the entrance of new graduates and the exit of the 66 year olds exceeding the maximum working age. According to this rule the educational attainment of men and women will converge to the same value $\mu_{\text{educ}}(15-24)$ after the 15-year old of the year 2012 will have become 65.

Back to 1972 the supply of day care is well documented by Statistics Austria. Before 1972 only the Federal Ministry of Education (2011) provides a number for the share of children aged between 3 to 6 years in day care by 1960 (23.5 percent). We replace missing values for this share in the years between 1961 and 1971 by linear interpolation and use the population statistics to compute the number of children in day care. In a second step, we compute the average number of children per kindergarten for the year 1971, and combine this average with our estimate for the number of children in day care to arrive at estimates for the number of kindergartens between 1960 and 1971. The number of créches has been almost constant during the 1970s. The minimum was 186 and the maximum 198 with ups and downs in between. Thus we assume that no change happened in the period before and we use the observation from 1971 to replace missing values for 1960 through 1970.

Appendix B: The Dynamic Cohort Method

The dynamic cohort method is based on the rates of entry and exit in the labour market as observed in the last year of the sample, T, and assumes that future lifetime employment profiles will be parallel to the historic development (Scherer 2002, Carone 2005). Let $Y_{s,t}(x)$ be the age, x, and sex, s, specific employment rate in year t, then the probability of a person exiting the employment status before period t, ex_s, is given by the average exit probability over the last ten years in the sample:

$$ex_s(x-1) = \frac{1}{10}\sum_{t=T-9}^{T}\left[1 - \frac{Y_{s,t}(x)}{Y_{s,t-1}(x-1)}\right].$$

The probability of entering employment is

$$en_s(x-1) = \frac{1}{10}\sum_{t=T-9}^{T}\left[1 - \frac{\bar{Y} - Y_{s,t}(x)}{\bar{Y} - Y_{s,t-1}(x-1)}\right],$$

where $\bar{Y}$ is an upper limit on employment rates which we assume at 99 percent for men and women alike. Forecasts based on the dynamic cohort method assume that these entry and exit probabilities are constant over the forecast horizon and predictions of employment rates follow from:

$$Y_{s,t}(x+1) = \begin{cases} Y_{s,t}(x)\left(1 - ex_s(x)\right), & \text{if } ex_s > 0 \\ \bar{Y}\dot{e}n_s(x) + Y_{s,t}(x)\left(1 - en_s(x)\right), & \text{if } en_s(x) > 0 \\ Y_{s,t}(x), & \text{otherwise.} \end{cases}$$

We keep the employment rate of young cohorts constant, i.e. those aged 15-19, to avoid a further decline in employment rates due to increased full time education in this age group. The recursive structure of the prediction formulas would automatically translate such a decline into a negative trend for the employment rates of prime-age persons.

Tables and Figures

Table 1: *Description and steady state values of explanatory variables for long run labour supply.*

		1960	2013	Steady state
Avg. education	Share of persons with more than mandatory schooling			
	Males	0.65	0.84	0.84
	Females	0.35	0.76	0.84
Unemployment	Total unemployment rate	3.49	7.62	6.98
	Male unemployment rate	2.51	8.19	6.98
	Female unemployment rate	5.18	6.96	6.98
Fertility	Total fertility rate (kids per woman)	2.85	1.44	1.55
Fertility age	Average fertility age	27.53	30.32	30.32
Marriage	Share of married couples in families	0.83	0.72	0.72
Crèches	Number of crèches (in 1000)	0.19	1.45	1.45
Kindergarten	Number of Kindergartens (in 1000)	1.59	4.69	4.69
Daycare	Share of 3–6 year old kids in daycare	0.24	0.66	0.66
Ramp dummy 2000–2013	Incentive effect of pension reforms 2000–2013	−14	0	0
Step dummy 1980	Inclusion of civil servants into labour market statistic	−1	0	0

Source: Austria Labour Market Service, Statistics Austria, Hofer et al. (2014), own computations.

Table 2: *Correlation among explanatory variables.*

	Educ male	Educ female	Unempl total	Unempl male	Unempl female	Fertility	Fertility age	Marriage	Crèches	Kinder	Daycare
Educ male	1.00										
Educ female	0.99	1.00									
Unempl total	0.89	0.90	1.00								
Unempl male	0.92	0.93	0.99	1.00							
Unempl female	0.76	0.77	0.96	0.92	1.00						
Fertility	−0.89	−0.84	−0.67	−0.72	−0.50	1.00					
Fertility age	0.71	0.78	0.77	0.78	0.73	−0.34	1.00				
Marriage	−0.86	−0.91	−0.83	−0.86	−0.72	0.59	−0.94	1.00			
Crèches	0.78	0.85	0.73	0.76	0.62	−0.50	0.95	−0.96	1.00		
Kinder	0.99	0.97	0.87	0.90	0.73	−0.92	0.63	−0.81	0.71	1.00	
Daycare	0.99	0.98	0.88	0.92	0.74	−0.89	0.69	−0.85	0.76	0.99	1.00

Source: Austria Labour Market Service, Statistics Austria, own computations.

Table 3: *Pension reforms in Austria between the years 2000 and 2012*

Year	Reform measures
2000	Early retirement due to reduced capacity to work abolished, stepwise increase of minimum age for early retirement by 2 months until year 2002, deductions for early retirement increased to 3 accrual points per year of entry before statutory age, survivor pensions fully means tested.
2003	Early retirement due to long-term unemployment abolished, continued stepwise increase of minimum age for early retirement until year 2004, lowering of new pension benefits by stepwise deterioration of the benefit calculation formula until the year 2028, deductions for early retirement increased to 4 accrual points per year of entry before statutory age, stepwise convergence of civil servants' pension system to private sector rules until 2028.
2004	Reformed early retirement scheme introduced, stepwise convergence of all public pension systems towards a harmonized contribution based pension account until 2050, further deterioration of the benefit calculation formula effective between 2028 and 2033, dynamic adjustment of existing pensions switched from wage to CPI based indexation.
2012	Entry requirements for early retirement stepwise tightened until 2017, possibility to enter disability pension tightened, deductions for early retirement increased to 4.2 accrual points per year of entry before statutory age.

Source: Miscellaneous Austrian federal law gazettes.

Table 4: *Dynamic Regression Models for time varying coefficients in functional data method*

	Men			Women		
	Coeff. 1	Coeff. 2	Coeff. 3	Coeff. 1	Coeff. 2	Coeff. 3
Avg. Education	50.96	6.96	−1.99	24.15	−12.65	−4.37
	(4.13)	(2.03)	(4.34)	(1.36)	(1.26)	(1.72)
Unempl. rate	0.04	−0.05	−0.21	−0.12	−0.04	−0.16
	(0.05)	(0.03)	(0.04)	(0.04)	(0.04)	(0.05)
Pens.ref.dummy	−0.37	0.08	0.03	0.02	0.32	0.22
	(0.04)	(0.02)	(0.03)	(0.04)	(0.03)	(0.04)
Step dummy 1980	−0.13	1.54	0.88	0.54	−0.16	0.32
	(0.21)	(0.10)	(0.14)	(0.10)	(0.12)	(0.15)
AR(1)	0.90	0.91	1.33	1.15	0.93	0.93
	(0.06)	(0.06)	(0.12)	(0.13)	(0.05)	(0.05)
AR(2)	–	–	−0.39	0.03	–	–
	–	–	(0.13)	(0.20)	–	–
AR(3)	–	–	–	−0.27	–	–
	–	–	–	(0.14)	–	–
constant	−0.86	1.81	–	7.05	–	1.17
	(0.44)	(0.22)	–	(0.33)	–	(0.42)
St.Error of est.	0.20	0.09	0.14	0.10	0.12	0.14
Box-Ljung test	0.13	0.18	0.21	0.75	0.30	0.14
Shap.Wilk test	0.40	0.01	0.35	0.24	0.24	0.04

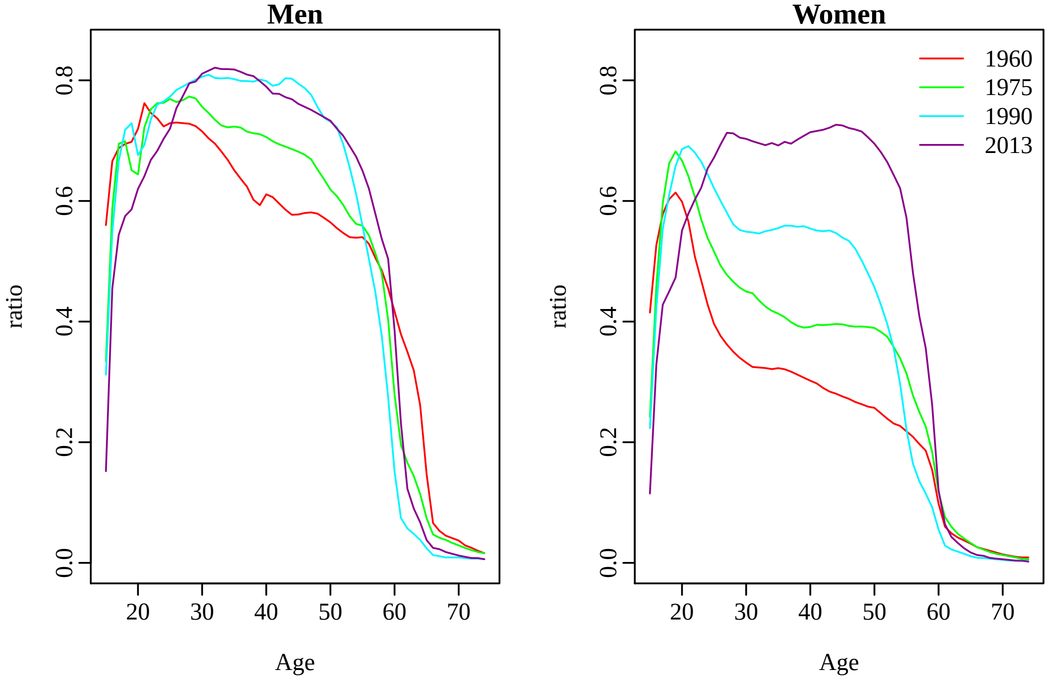
Notes: Values in brackets below coefficients are standard errors. P-value of Box-Ljung test of no autocorrelation at the first lag not modelled by the ARIMA model. P-value of Shapiro-Wilk test (SW test) of null hypothesis that residuals are normally distributed.

Table 5: *Forecast comparison from various models for specific cohorts and forecast horizons*

		Men				Women			
Age	Model	2013	2014	2043	2063	2013	2014	2043	2063
16	FDMc	45.5	43.2	43.8	43.8	32.9	31.6	27.2	27.0
			(38.3,48.3)	(38.4,49.3)	(38.4,49.4)		(27.6,35.8)	(23.3,31.4)	(23.1,31.2)
	FDM	45.5	41.9	42.4	42.5	32.9	29.3	25.7	25.5
			(37.2,46.7)	(37.2,47.8)	(37.2,47.9)		(25.8,33.0)	(22.1,29.7)	(22.0,29.5)
	TSM	45.5	42.5	35.9	32.2	32.9	31.3	23.2	20.6
	DRM	45.5	43.1	46.3	46.4	32.9	32.6	32.9	32.9
	DCM	45.5	45.5	45.5	45.5	32.9	32.9	32.9	32.9
35	FDMc	81.8	79.1	78.5	78.5	69.2	69.2	74.8	75.1
			(76.5,81.4)	(75.6,81.1)	(75.6,81.1)		(66.7,71.7)	(72.1,77.3)	(72.4,77.6)
	FDM	81.8	78.8	78.8	78.8	69.2	68.7	74.4	74.7
			(75.6,81.6)	(75.3,81.9)	(75.4,81.9)		(66.1,71.1)	(71.7,76.9)	(72.0,77.2)
	TSM	81.8	81.4	81.4	81.4	69.2	69.9	85.7	92.0
	DRM	81.8	81.2	80.1	80.1	69.2	67.5	67.2	67.3
	DCM	81.8	81.6	81.3	81.3	69.2	69.7	65.0	65.0
55	FDMc	64.4	63.8	64.5	64.5	57.2	58.9	61.9	62.1
			(60.9,66.6)	(61.3,67.5)	(61.3,67.5)		(56.0,61.7)	(57.6,66.0)	(57.8,66.2)
	FDM	64.4	63.7	63.6	63.6	57.2	59.4	62.3	62.5
			(60.4,66.9)	(60.0,67.1)	(59.9,67.1)		(56.5,62.2)	(58.0,66.4)	(58.2,66.6)
	TSM	64.4	64.4	62.0	60.8	57.2	57.4	81.3	91.3
	DRM	64.4	63.9	65.8	65.8	57.2	55.7	56.4	56.5
	DCM	64.4	65.8	66.7	67.0	57.2	57.9	61.0	59.8
65	FDMc	3.6	3.9	4.7	4.7	1.8	1.7	1.2	1.2
			(3.4,4.6)	(3.8,5.8)	(3.9,5.8)		(1.4,2.0)	(1.0,1.6)	(1.0,1.5)
	FDM	3.6	3.8	3.9	3.9	1.8	1.6	1.2	1.2
			(3.2,4.4)	(3.1,4.9)	(3.1,4.9)		(1.3,2.0)	(0.9,1.5)	(0.9,1.5)
	TSM	3.6	4.1	17.3	39.2	1.8	1.9	7.3	14.7
	DRM	3.6	3.9	4.9	4.9	1.8	2.0	2.0	2.0
	DCM	3.6	4.4	5.1	5.0	1.8	1.9	3.4	3.2

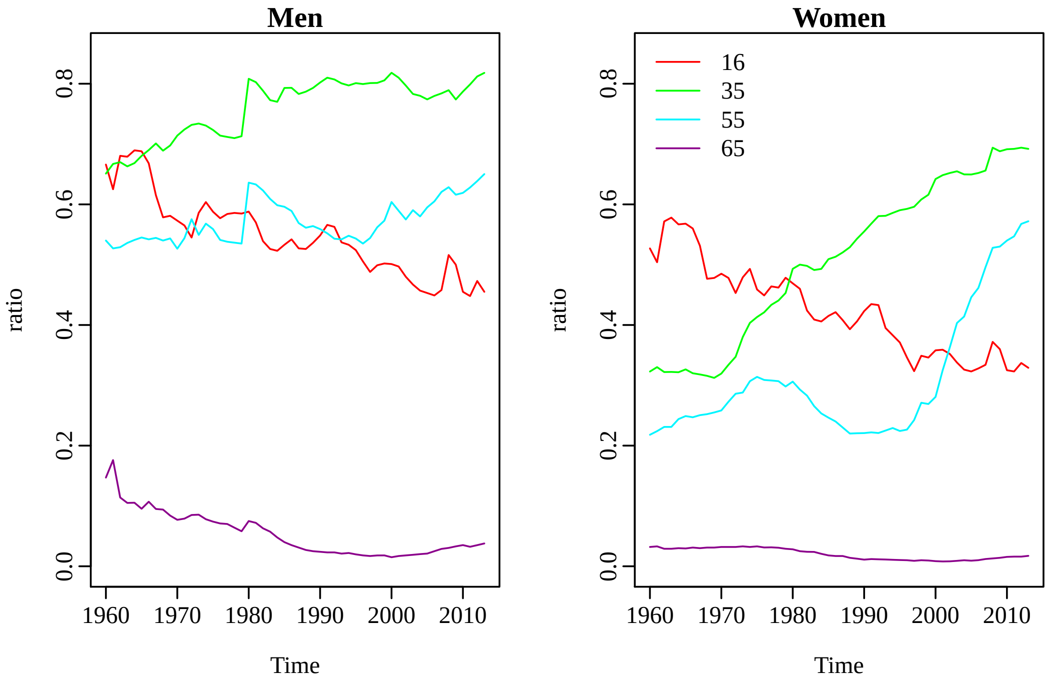
Notes: FDMc: Functional Data Model based on data corrected for cohort effects according to SMILE decomposition; FDM: Functional Data Model without correction for cohort effects; TSM: smoothed Time Series Model using only the step dummy 1980 as an explanatory variable; DRM: Dynamic Regression Model including all explanatory variables corresponding to FDM; DCM: Dynamic Cohort Model. Values in brackets below coefficients are 90 percent prediction intervals.

Figure 1: *Age-profiles of employment rates in Austria by sex, 1960–2013*



Notes: Ratio of employees at each age to respective population group.

Figure 2: *Development of employment rates in Austria by sex and age groups 15, 35, 55, and 65*



Notes: Ratio of employees at each age to respective population group.

Figure 3: *Age-profiles of employment rates in Austria for women, 1960–2013. Original data and the data smoothed with SMILE method.*

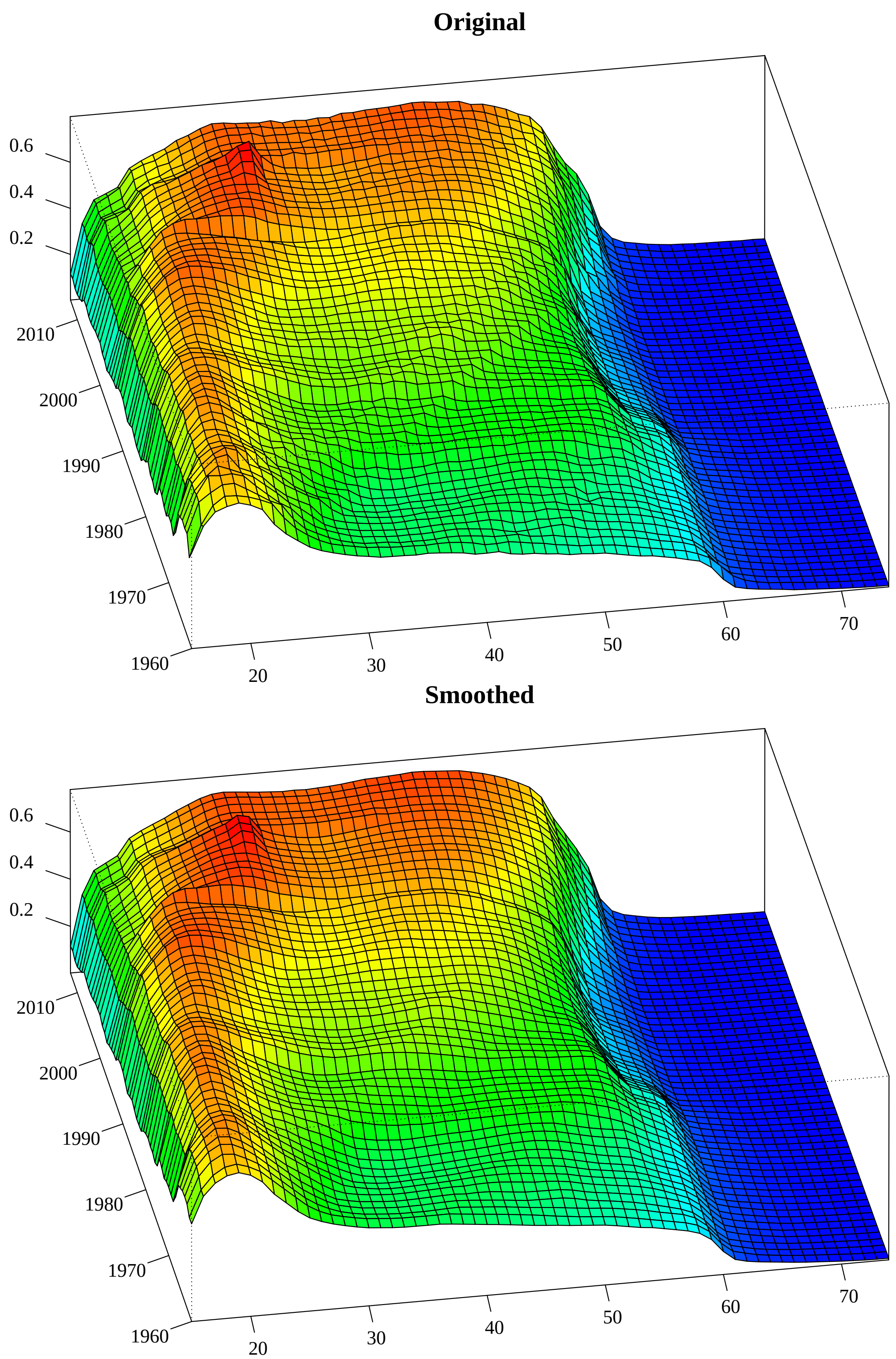
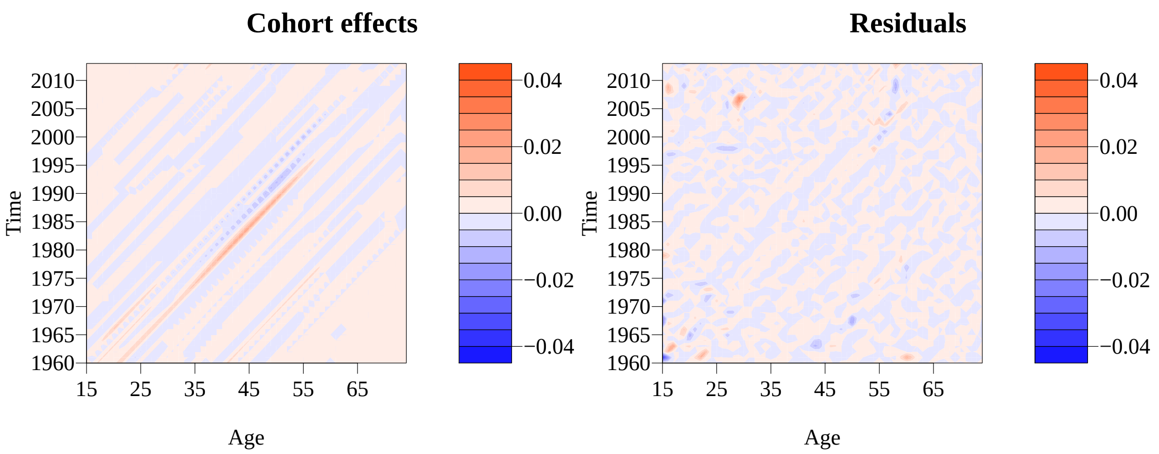
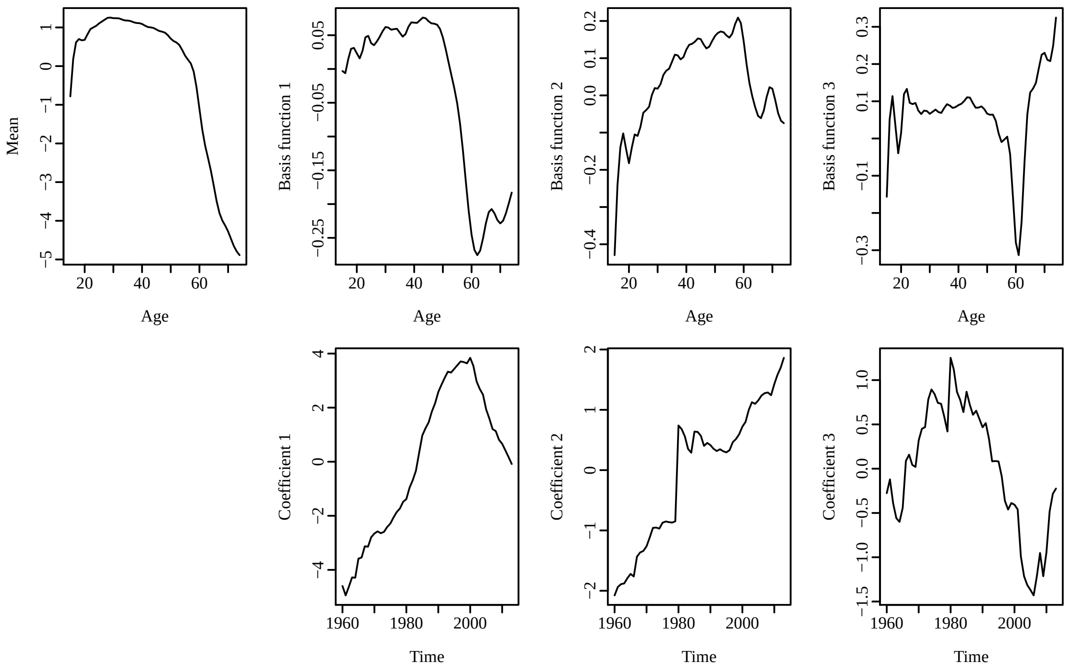


Figure 4: *Cohort effect and residuals after applying SMILE method to employment rates in Austria for women, 1960–2013.*



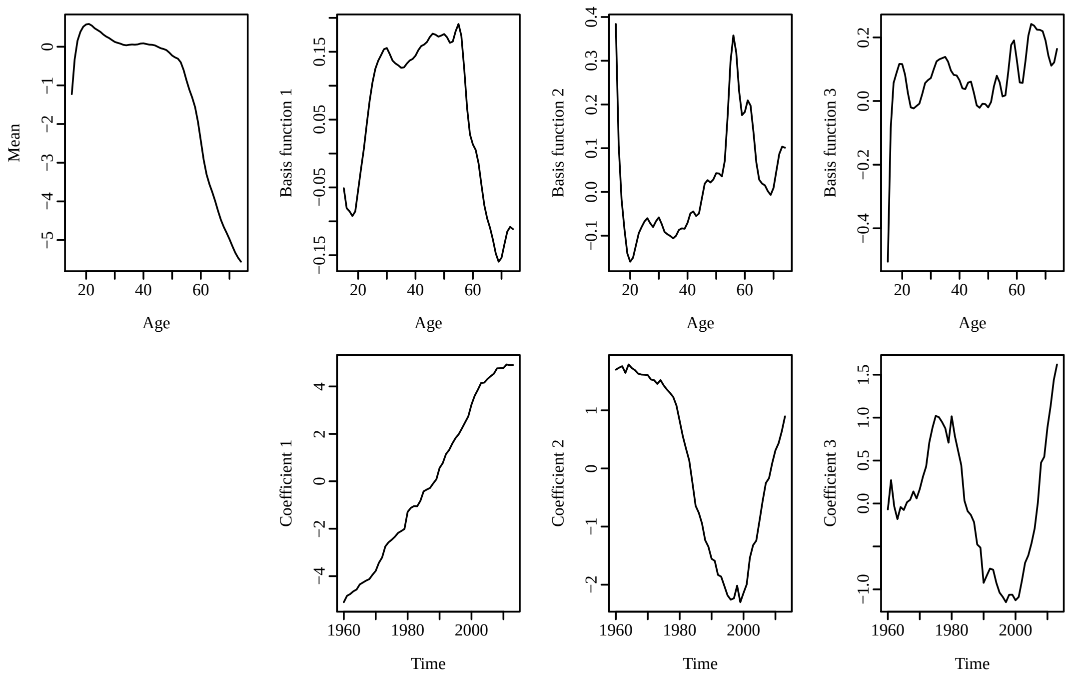
Notes: Heat map with blue colours indicating below average values for a particular cohort while red colours show above average values.

Figure 5: *Main effect, basis functions and associated coefficients for mens' employment rate*



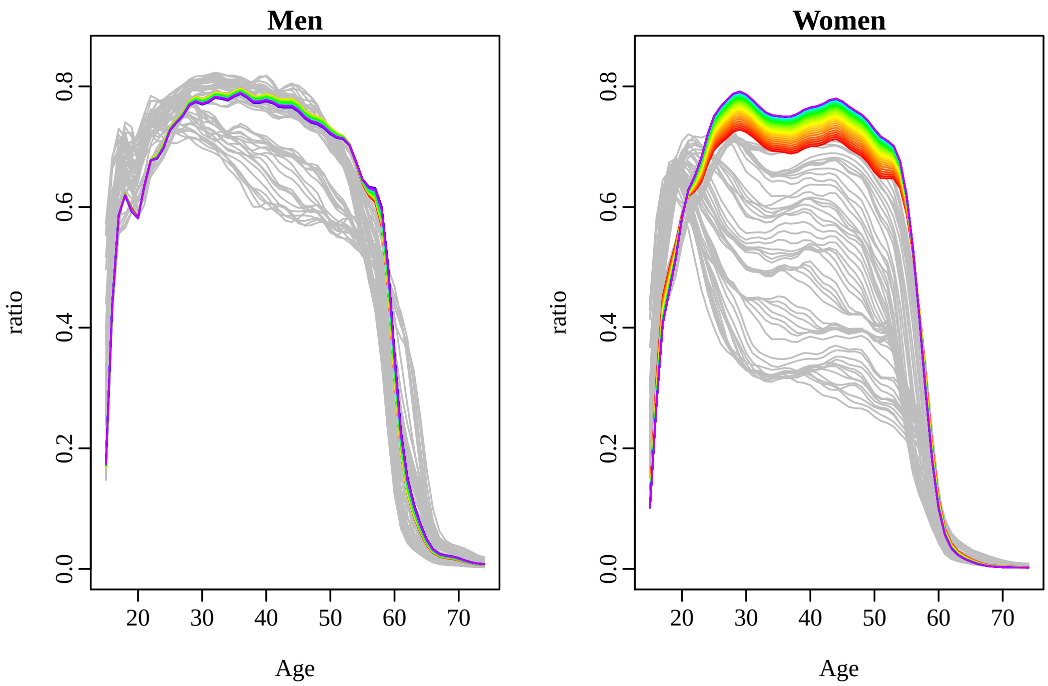
Notes: The decomposition is based on order $K = 3$.

Figure 6: *Main effect, basis functions and associated coefficients for womens' employment rate*



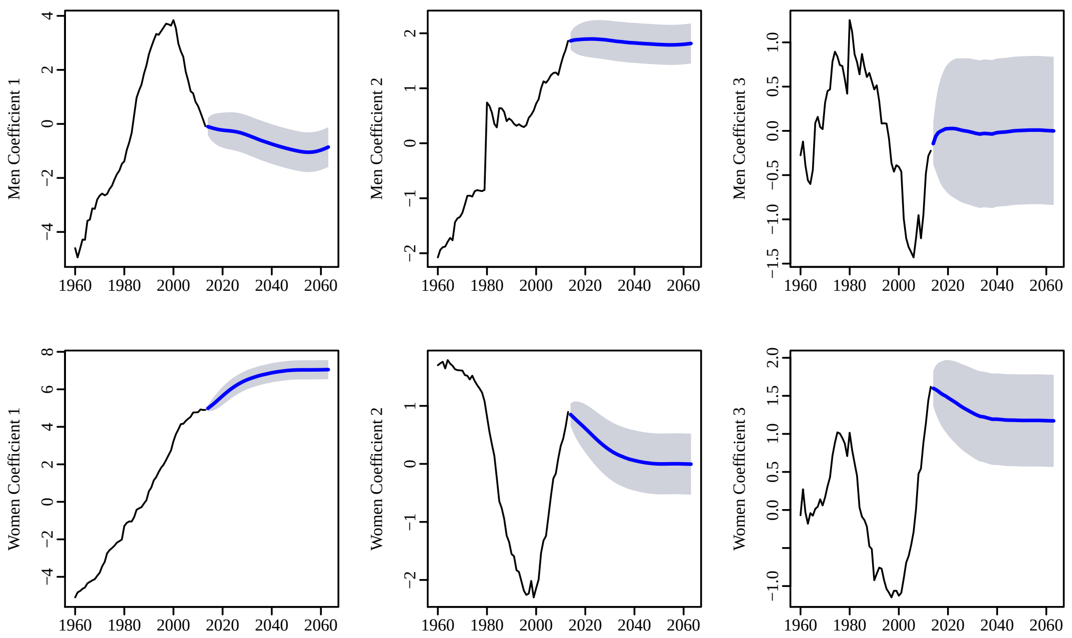
Notes: The decomposition is based on order $K = 3$.

Figure 7: *Long-term forecast of employment rates in Austria by sex, 2014–2062*



Notes: Ratio of employees at each age to respective population group. In sample values in grey and forecasts coloured in "rainbow order" starting at red (2014) and moving forward in time through: yellow, green, cyan, blue to magenta (2062).

Figure 8: *Long-term forecasts of time varying coefficients based on dynamic regression models, 2014–2062*



Notes: In sample values coloured black and forecasts blue; grey areas around forecast values are 90 percent prediction intervals.

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