MONASH University

MONASH BUSINESS SCHOOL

ISSN 1440-771X

Department of Econometrics and Business Statistics

http://business.monash.edu/econometrics-and-business-statistics/research/publications

# High-Frequency Jump Tests: Which Test Should We Use?

Worapree Maneesoonthorn, Gael M. Martin and Catherine S. Forbes

January 2020

Working Paper 03/20
(Revised working paper 17/18)

AACSB ACCREDITED EFMD EQUIS ACCREDITED ASSOCIATION OF MBAS AMBA ACCREDITED

# High-Frequency Jump Tests: Which Test Should We Use?*

Worapree Maneesoonthorn[†], Gael M. Martin[‡], Catherine S. Forbes[§]

December 12, 2019

**Abstract**

We conduct an extensive evaluation of price jump tests based on high-frequency financial data. After providing a concise review of multiple alternative tests, we document the size and power of all tests in a range of empirically relevant scenarios. Particular focus is given to the robustness of test performance to the presence of jumps in volatility and microstructure noise, and to the impact of sampling frequency. The paper concludes by providing guidelines for empirical researchers about which test to choose in any given setting.

*Keywords*: *Price jump tests; Nonparametric jump measures; Bivariate jump diffusion model; Volatility jumps; Microstructure noise; Sampling frequency*

*JEL Classifications: C12, C22, C58.*

## 1 Introduction

Extreme movements (or 'jumps') in asset prices play an important role in the tail behaviour of return distributions, with the perceived risk (and, hence, risk premium) associated with this extreme behaviour differing from that associated with small and regular movements (see, Bates, 1996, and Duffie *et al.*, 2000, for early illustrations of this point, and Todorov and Tauchen, 2011, Maneesoonthorn *et al.*, 2012, and Bandi and Renò, 2016, for more recent expositions). Indeed, the modelling of jumps, in both the price itself and its volatility, has been given particular attention in the option pricing literature, where the additional risk factors implied by random jumps have helped explain certain stylized patterns in option-implied volatility (Merton, 1976; Bates, 2000; Duffie *et al.*, Eraker, 2004; Todorov, 2010; Maneesoonthorn *et al.*; Bandi and Renò). Evidence of price jump clustering in spot returns - whereby price and/or volatility jumps occur in consecutive time periods - has also been uncovered, with various approaches having been adopted to model this dynamic behaviour, including the use of simultaneous price and volatility jumps over time (Chan

*This research has been supported by Australian Research Council Discovery Grants No. DP150101728 and DP170100729. We thank a co-editor, an associate editor and two anonymous referees for very helpful and constructive comments on earlier drafts of the paper. We are also grateful to John Maheu, Herman van Dijk, Maria Kalli and Jim Griffin, plus participants at the 11th Annual RCEA Bayesian Econometric Workshop (Melbourne, 2017), for very helpful comments on an earlier version of the paper.

[†]Melbourne Business School, The University of Melbourne, Australia.

[‡]Corresponding author. Department of Econometrics and Business Statistics, Monash University, Australia. Email: *gael.martin@monash.edu.*

[§]Department of Econometrics and Business Statistics, Monash University, Australia.

and Maheu, 2002; Eraker *et al.*, 2003; Maheu and McCurdy, 2004; Fulop *et al.*, 2014; Aït-Sahalia *et al.*, 2015; Bandi and Renò; Maneesoonthorn *et al.*, 2017).

Coincident with the trend towards more sophisticated models for asset prices, the use of high-frequency intraday data to construct nonparametric measures of asset price variation - including the jump component thereof - has become wide-spread. Multiple alternative methods are now available to practitioners, both for testing for jumps and for measuring price variation in the presence of jumps, with some empirical analyses exploiting such measures in addition to, or as a replacement of, measurements based on end-of-day prices. (See Koopman and Scharth, 2013, Christensen *et al.*, 2014, and Maneesoonthorn *et al.*, 2017, for recent examples, including references to earlier work.)

This short paper provides the results of an investigation into the relative accuracy of the many high-frequency price jump tests that are now on offer. Particular attention is given to the robustness of the tests to the presence of jumps in volatility, and to the effect of microstructure noise. The impact of sampling frequency on test performance is also documented. In addition to enabling key insights to be drawn, the current study also provides a template for future studies regarding jump tests that may be of interest.

We begin, in Section 2, by providing a very brief review of price jump tests that have been proposed to date. These methods are grouped into five categories: 1) those based on the difference between a measure of total (squared) variation and a jump-robust measure of integrated variation (Barndorff-Nielsen and Shephard, 2004, 2006; Huang and Tauchen, 2005; Corsi *et al.*, 2010; Andersen *et al.*, 2012); 2) those that exploit measures of higher-order variation (Aït-Sahalia and Jacod, 2009; Podolskij and Ziggel, 2010); 3) those based on returns, rather than measures of variation (Andersen *et al.*, 2007; Lee and Mykland, 2008); 4) those that exploit a variance swap (Jiang and Oomen, 2008); and 5) those that are expressly designed to mitigate the impact of microstructure noise (Aït-Sahalia *et al.*, 2012; Lee and Mykland, 2012). Section 3 makes note of the various tuning components that influence the price jump tests; with the performance of these tests documented in Section 4. Guidelines for practitioners are provided in Section 5.

# 2 Review of price jump tests

Defining $p_t = \ln(P_t)$ as the natural log of the asset price, $P_t$ at time $t > 0$, we begin by assuming the following jump diffusion process for $p_t$,

$$dp_t = \mu_t dt + \sqrt{V_t} dW_t^p + dJ_t^p, \tag{1}$$

where $W_t^p$ is the Brownian motion, and $dJ_t^p = Z_t^p dN_t^p$, with $Z_t^p$ denoting the random price jump size and $dN_t^p$ the increment of a discrete count process, with $P(dN_t^p = 1) = \delta^p dt$ and $P(dN_t^p = 0) = (1 - \delta^p)\, dt$.

The aim of a price jump test is to detect the presence of the discontinuous component, $dJ_t^p$, and to conclude whether or not $dN_t^p$ is non-zero over a particular period. The availability of high-frequency data has enabled measures of variation - incorporating both the continuous and discontinuous components of (1) - to be constructed over a specified interval of time, e.g. one day, with the statistical properties of such measures established using in-fill asymptotics. The relevant distributional results are then utilized in the construction of a price jump test, where the null hypothesis is usually that the asset price is continuous over the particular interval under investigation. All tests investigated in this paper entertain the same null hypothesis of a continuous price path, with the alternative hypothesis being that the price path contains jumps and, hence, is discontinuous.

This section reviews available tests based on the concepts of, respectively, squared variation (Section 2.1), higher-order power variation (Section 2.2), standardized daily returns (Section 2.3), and variance swaps (Section 2.4), as well as tests that are designed to be robust to microstructure noise (Section 2.5). The detail of eleven specific test statistics (and limiting distributions) are summarized in Tables 1 and 2. The role of certain tuning parameters, including those appearing in the test descriptions in Tables 1 and 2, is discussed in Section 3.

## 2.1 Squared variation

The early literature on price jump testing exploits various measures of the squared variation of the asset price process. In the context of a continuous-time price process, as defined in (1), the object of interest is the difference between total quadratic variation over a discrete time period (typically a trading day), $\mathcal{QV}_{t-1,t} = \int_{t-1}^{t} V_s ds + \sum_{t-1<s\leq t}^{N_t^p} (Z_s^p)^2$ , and variation from the continuous component alone, quantified by the integrated variance, $\mathcal{V}_{t-1,t} = \int_{t-1}^{t} V_s ds$. By definition, the difference between these two quantities defines the contribution to price variation of the discontinuous jumps, $\mathcal{J}_{t-1,t}^2 = \sum_{t-1<s\leq t}^{N_t^p} (Z_s^p)^2$ , and price jump test statistics can thus be constructed from the difference between various *measures* of $\mathcal{QV}_{t-1,t}$ and $\mathcal{V}_{t-1,t}$. In Panel A of Table 1, we provide details of the test proposed by Barndorff-Nielsen and Shephard (2004, 2006) (also exploited by Huang and Tauchen, 2005), plus an alternative test proposed by Corsi *et al.* (2010) and two tests suggested by Andersen *et al.* (2012) (referenced hereafter as BNS, CPR and MINRV and MEDRV, respectively). In all cases, the test statistic is constructed using the difference between realized volatility, $RV_t = \sum_{i=1}^{M} r_{t_i}^2$, where $r_{t_i} = p_{t_i} - p_{t_{i-1}}$ denotes the $i^{th}$ of $M$ equally-spaced returns observed during day $t$, and a chosen measure of integrated variance. All tests are one-sided upper-tailed tests by construction.

## 2.2 Higher-order $\mathcal{P}$-power variation

A second class of price jump test exploits the behaviour of higher-order $\mathcal{P}$-power variation, and estimators thereof. Following Barndorff-Nielson and Shephard (2004), let an estimator of the $\mathcal{P}$-power variation of $p_t$ be defined as $\widehat{B}(\mathcal{P}, \Delta_M)_t = \sum_{i=1}^{M} |r_{t_i}|^{\mathcal{P}}$ , where $\Delta_M = 1/M$ denotes the common length of the time intervals between consecutive returns, and $\mathcal{P} > 0$. The limiting behaviour of this estimator, for different values of $\mathcal{P}$, sheds light on the different components of the variation in $p_t$. In the case of $\mathcal{P} = 2$, $\widehat{B}(\mathcal{P}, \Delta_M)_t \xrightarrow{p} \mathcal{QV}_{t-1,t}$ as $M \to \infty$, as is consistent with the distributional result that $RV_t \xrightarrow{p} Q\mathcal{V}_{t-1,t}$, as $M \to \infty$. For $0 < \mathcal{P} < 2$,

$$\frac{\Delta_M^{1-\mathcal{P}/2}}{m_{\mathcal{P}}} \widehat{B}\left(\mathcal{P}, \Delta_M\right)_t \xrightarrow{p} A\left(\mathcal{P}\right)_t \text{ as } M \to \infty, \tag{2}$$

where $A\left(\mathcal{P}\right)_t = \int_{t-1}^{t} \left|V_s^{1/2}\right|^{\mathcal{P}} ds$ denotes the $\mathcal{P}$-power *integrated* variation, $m_{\mathcal{P}} = E\left(|U|^{\mathcal{P}}\right) = \pi^{-1/2} 2^{\mathcal{P}/2} \Gamma\left(\frac{\mathcal{P}+1}{2}\right)$ and $U$ denotes a standard normal random variable. In contrast, for $\mathcal{P} > 2$, the increments from the jump component dominate, and the estimator converges in probability to the $\mathcal{P}$-power *jump* variation, $B\left(\mathcal{P}\right)_t = \sum_{t-1<s\leq t} |dJ_s|^{\mathcal{P}}$. If the jump component in (1) is not present and $p_t$ is continuous as a consequence, then the limiting result in (2) holds for any $\mathcal{P} > 0$.

These limiting results can be used in a variety of ways to detect jumps. Specifically, Aït-Sahalia and Jacod (2009) (ASJ, hereafter) compare $\widehat{B}\left(\mathcal{P}, \Delta_M\right)_t$ constructed over two different sampling intervals, while Podolskij and Ziggel (2010) (PZ, hereafter) rely on the limiting distribution of a modified version of $\widehat{B}\left(\mathcal{P}, \Delta_M\right)_t$. Both tests are one-sided, with the ASJ test being lower-tailed, while the PZ test is upper-tailed. These two approaches are outlined in Panel B of Table 1.

### 2.3 Standardized returns

Rather than construct price jump test statistics from various measures of variation, an alternative is to consider the behaviour of (appropriately standardized) returns themselves. In brief, based on the assumption of Brownian motion for the asset price, the return computed over a chosen interval length and scaled by the square root of a consistent estimator of the corresponding integrated variance, should be asymptotically standard normal if price jumps are absent. Two tests that exploit this property are proposed by Andersen *et al.* (2007) and Lee and Mykland (2008), referenced as ABD and LM, hereafter. ABD conduct multiple two-tailed tests on standardized returns observed over the trading day, while LM propose an upper-tailed test based on the maximum absolute standardized return. Details regarding the form of these two test statistics and their limiting distribution under the null hypothesis of no jumps are given in Panel A of Table 2.

### 2.4 Variance swaps

Variance swaps are instruments made up of financial assets and/or derivatives and are used as tools to hedge against volatility risk. The payoff of a variance swap can be replicated by taking a short position in the so-called "log contract" and a long position in the underlying asset, with the long position being continuously re-balanced (see Neuberger, 1994). The payoff of such a replicating strategy, computed as the accumulated difference between proportional returns and continuously compounded logarithmic returns, equates to half of the integrated variance when there is no price jump. When a jump is present, the replication error is completely determined by the realized jump, including the sign of such a jump. Jiang and Oomen (2008), JO hereafter, exploit this relation in their two-tailed test construction, with the test details given in Panel B of Table 2.

### 2.5 Treatment for microstructure noise

Finally, we investigate the performance of two tests that are specifically designed to be robust to the presence of microstructure noise. Lee and Mykland (2012), LM12 hereafter, provide an alternative to the LM test whereby the test statistic is computed from prices averaged over a small window. Aït-Sahalia *et al.* (2012) propose an alternative test to ASJ (denoted here by ASJL) that is based on locally smoothed prices. Similar to their corresponding predecessors, the LM12 test, based on extreme value theory, is an upper-tailed test, while the ASJL is a lower-tailed test. Details of these two alternative tests are provided in Panel C of Table 2.

## 3 Tuning parameter choice

As is clear from the outline of the test procedures in Section 2, all require a decision, of one form or another, to be made regarding the mechanism used to distinguish a continuous increment ($\sqrt{V_t}dW_t^p$) from a discontinuous increment ($dJ_t^p$) in (1). Such 'tuning' decisions will patently influence the outcome of any test, the values of the jump measures that are derived from preliminary application of the test and, hence, any inferential results based on those measures. In this section, we discuss the impact of alternative choices for the significance level and certain other tuning values.

### 3.1 Significance level

All tests are, of course, subject to the selection of a significance level, which determines the value beyond which the null hypothesis of 'no jump' is rejected. Although certain authors suggest the use of certain (typically small) significance levels (e.g. Tauchen and Zhou, 2011; ABD), there does not appear to be widespread consensus in the literature regarding this choice. It is important to recognize that use of a higher level of significance will automatically lead to the identification of a greater number of apparent 'jumps', including those having a relatively small magnitude, according to the usual trade-off between the size of a test and its power in a neighbourhood of the null hypothesis of no jump. Thus, if the desired focus is to detect (and subsequently measure) jumps with reasonably large magnitude only, then a small significance level should be selected. In Section 4 we conduct all size and power assessments based on two significance levels: 5% and 1%.

### 3.2 Threshold value

The higher-order $\mathcal{P}$-power variation-based PZ, ASJ and ASJL tests, along with the CPR test, also entail the choice of certain threshold values that determine the particular jump-free variations that are accumulated. That is, these threshold values determine whether an individual return belongs to the diffusive component or to the jump component in the calculation of $\mathcal{P}$-power variation. The thresholds for these tests are selected as a multiple ($c_\vartheta$) of the local volatility estimate (See Panel A, Table 1). For example, CPR suggest truncating returns at three times the local volatility measure. The choice of this multiplier is guided by the properties of a normal distribution, since if a jump is indeed absent, then the return (assumed to be normally distributed by approximation of the diffusive process) should cross the threshold only about 0.3% of the time. Naturally, a smaller multiplier will correspond to a larger proportion of returns being considered as part of the jump contribution. The PZ and ASJ tests prescribe truncations that involve the choice of the truncation root, $\varpi$ (see Panel B, Table 1). In both cases, a larger value of $\varpi$ corresponds to a smaller level of the actual truncation point, again implying that a larger number of returns are considered as part of the jump contribution. PZ recommend $\varpi = 0.4$, while ASJ recommend the use of $\varpi = 0.48$, recommendations that we adopt for our study of the performance of these tests, presented in Section 4.

### 3.3 Value of $\mathcal{P}$

The $\mathcal{P}$-power variation-based tests are also subject to the choice of $\mathcal{P}$ itself, noting that, at least asymptotically, the jump contribution will dominate for values of $\mathcal{P} > 2$. However, with limited intra-day sampling available, a larger value of $\mathcal{P}$ will tend to accentuate jumps with relatively large magnitude and thereby diminish the role in the test outcome of relatively small jumps, for any given choice of significance level. In Section 4 we report results for the PZ test based on $\mathcal{P} = 2$ and $\mathcal{P} = 4$ (PZ2 and PZ4 respectively), to gauge the effect of this tuning parameter. We also report results for the ASJ and ASJL tests based on the author-recommended choice of $\mathcal{P} = 4$.

### 3.4 Sampling interval

The sampling interval over which intraday returns are computed also plays an important role in the performance of jump tests. A consensus seems to have developed in the literature that estimates of $Q\mathcal{V}_{t-1,t}$ and $\mathcal{V}_{t-1,t}$ are optimally computed over the five-minute interval, due to the mitigation of microstructure noise when sampling at that (relatively low) frequency (see Bandi and Russell, 2008). Whether such optimality carries over to the performance of the corresponding price jump test is still questionable. It is worth noting that the choice of the sampling interval will impact each of the ASJ and ASJL tests in two ways - first through the choice of $\Delta_M$, and secondly, via the tuning choice $k$, which determines the central location of the tests under the null hypothesis. We conduct both tests using $k = 2$, a choice that is recommended by the authors. We document test performance over four alternative sampling frequencies, in order to provide some insight into the influence of the choice of sampling frequency.

## 4 Assessment of test performance

### 4.1 Experimental design

We assess the finite sample size and power of each test in empirically relevant scenarios. In contrast to the earlier assessment of test performance by Dumitru and Urga (2012), our simulation exercise is used to shed particular light on the robustness of (an expanded set of) price jump tests to the presence of a discontinuous volatility process. This is something that has not, to our knowledge, been documented elsewhere, for any of the tests discussed, and which is relevant to recent empirical work, in which both (possibly dynamic) price and volatility jumps are modelled (see Fulop *et al.*, 2014 and Maneesoonthorn *et al.*, 2017, for examples).

We generate data from the process in (1), in conjunction with the following explicitly defined jump diffusion process for $V_t$,

$$dV_t = \kappa (V_t - \theta) + \sigma_v \sqrt{V_t} dW_t^v + dJ_t^v. \tag{3}$$

The Brownian increment $dW_t^v$ is assumed to be correlated with $dW_t^p$ (in (1)) with $corr(dW_t^v, dW_t^p) = \rho$, but is assumed to be uncorrelated with the increment in the volatility jump process, denoted by $dJ_t^v$. The data are generated using parameter values: $\mu_t = 0$, $\kappa = 5$, $\theta = 0.4^2$, $\sigma_v = 0.5$ and $\rho = -0.5$ (adhering to the theoretical restriction $2\kappa\theta \geqslant \sigma_v^2$), and with the diffusive variance process initialized at $\theta$.[1]

A very fine Euler discretization is employed to simulate high-frequency observations, with 21600 observations created per trading day, equivalent to generating price observations every one second over a six-hour trading period. The price jump test statistics are then constructed using four different sampling frequencies: five seconds, 30 seconds, one minute and five minutes. We then compute, over 1000 independent Monte Carlo replications, the proportion of times that a test detects a price jump, under several different scenarios. First, we set $dJ_t^p = 0$ for all $t$, and assess the size of the tests, both in the absence ($dJ_t^v = 0$) and in the presence ($dJ_t^v \neq 0$) of volatility jumps. When a volatility jump is present, only one jump occurs and its arrival time is random. The size of the volatility jump increment is assumed to be either 'moderate', $dJ_t^v = 3\theta$, or 'large', $dJ_t^v = 10\theta$. Secondly, we assess the power of the tests in the case where a single price jump arrives randomly over the day, and where the price jump size is either 'moderate', $dJ_t^p = 3\sqrt{\theta}$, or 'large', $dJ_t^p = 10\sqrt{\theta}$. Power is also assessed in both the absence and the presence of volatility jumps.[2]

Finally, we assess the size and power of all tests in the presence of microstructure noise, under both independent and identically distributed (*i.i.d.*) Gaussian and Student-t noise assumptions, as well as when the microstructure noise follows a Gaussian autocorrelated process (Aït-Sahalia and Mancini, 2008). Test performance in the absence and presence of microstructure noise is documented in Section 4.2 and Section 4.3 respectively.

[1]This DGP and its parameter settings are also used in the simulation exercise of Aït-Sahalia and Jacod (2009), and broadly reflect empirical results recorded in the literature. See Eraker *et al.* (2003) and Fulop *et al.* (2014) for examples.

[2]The size of the price jump is expressed as a proportion of the square root of the long run variance ($\theta$), to fit with the scale of $dp_t$. The size of the volatility jump, on the other hand, is expressed as a proportion of the long run variance itelf.

## 4.2 Test performance in the absence of microstructure noise

### 4.2.1 Empirical size

In Table 3, we report empirical size, based on nominal sizes of 5% and 1%, with the following summary relevant to the results for both significance levels.

Two of the four tests based on measures of squared variation, MINRV and MEDRV, have an empirical size that is closest to the nominal value, for all sampling frequencies, and in the presence of volatility jumps. In contrast, the CPR test is uniformly oversized, and the BNS test oversized in the presence of large volatility jumps, in particular.

The tests based on $\mathcal{P}$-power variation, standardized returns and variance swaps (PZ2, PZ4, ASJ, ABD, LM and JO) have reasonable size performance in the absence of volatility jumps, although ASJ and LM are quite undersized, in particular for the lower sampling frequencies (one and five minutes). In the presence of volatility jumps, the PZ and ABD tests are very (at times, grossly) over-sized, whilst the ASJ and LM tests remain somewhat under-sized[3]. Of this set, the JO test is the most robust to the presence of volatility jumps, and to the choice of sampling frequency; but is still not as accurately sized as MINRV and MEDRV.

In the absence of microstructure noise, the tests designed to accommodate such noise (LM12 and ASJL) perform well at the highest sampling frequency of five seconds, and under no volatility jump; but neither test is uniformly robust to either the sampling frequency or the presence of volatility jumps. It should be noted that these tests require a smoothing (averaging) process over sub-blocks throughout the day, so the effective sample size decreases rapidly as the sampling frequency is reduced.[4]

As a general rule, across all tests, the proportion of incorrect detections of a price jump increases with size of the volatility jump, highlighting the confounding influence of this feature of the DGP.

### 4.2.2 Empirical power

Table 4 reports the power of each test conducted at the 5% nominal size level.[5] As is to be expected, all tests exhibit greater power when the price jump size is larger. However, for any given price jump size, the level of power still varies, across test, sampling frequency and volatility jump size. It is interesting to note that for *all* tests, and for *all* designs, power is greatest when the test is conducted at the highest frequency.

Except for the LM12 and ASJL tests, power uniformly decreases as the volatility jump size gets larger, when the test is conducted at the highest frequency of five seconds. As the sampling

[3]Our detection of a decreasing test size at lower frequencies is in line with sizes reported in ASJ's Table 1, notwithstanding the fact that the lowest frequency they report corresponds to 30 seconds.

[4]In both Lee and Mykland (2012) and Aït-Sahalia *et al.* (2012), test performance is assessed at extremely high frequency, with the *lowest* frequency recorded being three seconds for the LM12 test and five seconds for ASJL. Furthermore, Aït-Sahalia *et al.* only assess their test over three consecutive trading days, as opposed to a single trading day as is typically done in an empirically relevant context, and as we have done here.

[5]Findings at the 1% significance level were qualitatively similar and, hence, are not reported.

interval becomes longer however, a variety of patterns are observed, in particular when the price jump is only moderate.

Whilst the power of the ASJ test is always highest at the highest frequency (as tallies with the qualitative finding in Dumitru and Urga, 2012) it ranks lowest amongst all tests, overall, with a power that does not exceed 50% over all designs considered.[6] At the other end of the spectrum, the PZ2, PZ4 and ABD tests tend to have the highest power overall, in particular at the higher frequencies, and are the most robust to the size of the volatility jump.

The (relatively) well-sized MINRV and MEDRV tests also have high power in the presence or otherwise of volatility jumps, as long as the price jump is large and the sampling frequency is high (five seconds). This statement needs to be qualified somewhat for the lower sampling frequencies. In particular, when the volatility jump is also large a sampling interval beyond 30 seconds leads to quite a reduction in power for these two tests, as indeed is a feature for all tests.

The tests designed to cater for microstructure noise (LM12 and ASJL) have high power in its absence only when the sampling frequency is very high (five seconds), the price jump to be detected is large, and the (confounding) volatility jump is not.

### 4.3 Test performance in the presence of microstructure noise

The presence of microstructure noise is known to hamper the quality of measures constructed from high-frequency data, including any subsequent price jump tests conducted based on these measures (see Hansen and Lunde, 2006, and Bandi and Russell, 2008, amongst others). In Table 5, the empirical size and power of the tests conducted at a nominal level of 5%, is recorded, under the three assumptions of microstructure noise described in Section 4.1[7]. Power is assessed for the case where price jump size is large ($dJ_t^p = 10\sqrt{\theta}$) and volatility jumps are absent.

As a general observation, the presence of microstructure noise impacts negatively on the empirical size of the tests that are not expressly designed to cater for noise, and that impact varies according to the form of noise. For instance, the Student-t noise leads to the most inaccurate empirical sizes overall, including the least robustness of size to sampling frequency. Once again, the size of the MEDRV and MINRV tests tend to be the most robust to sampling frequency, although under the Student-t and (Gaussian) autocorrelated noise the tests are extremely undersized when computed using five second data.

Microstructure noise of all forms has arguably less impact on power than it does on size. Other than ASJ, all tests not designed to be robust to noise retain high power under Gaussian noise, in

[6] ASJ entertain two different null hypotheses: one where the price path is continuous under the null, and one where the null accommodates a discontinuous price path. In the latter case, the ability of their test to detect a price jump is measured by empirical size (only), and this is what they report. However, a key assumption underlying the distribution of the test statistic under this null hypothesis is that there are no common price and volatility jumps. We view this as a very restrictive assumption, particularly given the noted interest and empirical evidence found in support of this situation. Hence our use of the null hypothesis of a continuous process, and our documentation of empirical power under scenarios that allow for volatility jumps, including those that occur contemporaneously with a price jump.

[7] Results produced for the 1% nominal level are available but are not recorded here.

particular for the higher frequencies (five and 30 seconds), remembering that this assessment is now under large price jumps and zero volatility jumps.

Not surprisingly, the tests that are designed to accommodate microstructure noise are relatively robust to the type of noise assumed, at the highest frequency of five seconds, with the appropriate size and high power. However, the LM12 test is severely oversized, and the AJL test lacks power, when conducted at a lower frequency, mimicking their performance in the no microstructure noise case.

## 5 Guidelines for Practitioners

To conclude, we offer practitioners the following *guidelines*:

1. The ASJ is the least powerful test overall, over the variety of designs considered, as well as being consistently undersized. This confirms (albeit in slightly different scenarios) the findings of Dumitru and Urga (2012). Our finding holds both in the absence and presence of microstructure noise.

2. If microstructure noise alone is thought to be present, the two tests designed to cater for that feature - LM12 and ASJL - based on very high-frequency (five second) data, are the best choice. Importantly, these tests continue to perform well when microstructure is absent, but only when the sampling frequency remains very high, the price jump size is large and volatility jumps are absent.

3. If the data generating process is thought to feature volatility jumps, we advocate for the use of one of the two squared-variation tests, MINRV or MEDRV. To balance size and power performance, we also advise computation of the selected test statistic at a moderate frequency of 30 seconds.

While these guidelines flow from the settings covered in the experiments summarised in the paper, it is prudent to remind the reader that good (or poor) test performance under the available settings does not guarantee similarly good (or poor) performance under alternative simulation designs. Nevertheless, the approach used here, including the justifications used to determine the guidelines, may serve as a template for investigating the performance of price jump tests, and consequent price variation measures, under alternative DGPs and/or volatility jump size settings.

## References


[1] Aït-Sahalia, Y., Cacho-Diaz, J. and Laeven, R.J.A. 2015. Modeling Financial Contagion Using Mutually Exciting Jump Processes. *Journal of Financial Economics*, 109: 224-249.

[2] Aït-Sahalia, Y. and Jacod, J. 2009. Testing for Jumps in a Discretely Observed Process. *The Annals of Statistics*, 37: 184-222.

[3] Aït-Sahalia, Y., Jacod, J., and Li, J. 2012. Testing for jumps in noisy high frequency data. Journal of Econometrics, 168(2), 207-222.

[4] Aït-Sahalia, Y., and Mancini, L. 2008. Out of Sample Forecasts of Quadratic Variation. *Journal of Econometrics*, 147: 17-33.

[5] Andersen, T.G., Bollerslev, T. and Dobrev, D. 2007. No-Arbitrage Semi-Martingale Restrictions for Continuous-Time Volatility Models Subject to Leverage Effects, Jumps and IID Noise: Theory and Testable Distributional Implications. *Journal of Econometrics*, 138: 125-180.

[6] Andersen, T.G., Dobrev, D. and Schaumburg, E. 2012. Jump-Robust Volatility Estimation using Nearest Neighbor Truncation. *Journal of Econometrics*, 169: 75-93.

[7] Bandi, F.M. and Renò, R. 2016. Price and Volatility Co-Jumps. *Journal of Financial Economics*, 119: 107-146.

[8] Bandi, F.M. and Russell, J.R. 2008. Microstructure Noise, Realized Variance and Optimal Sampling. *The Review of Economic Studies*, 75: 339-369.

[9] Barndorff-Nielsen, O.E. and Shephard, N. 2004. Power and Bipower Variation with Stochastic Volatility and Jumps. *Journal of Financial Econometrics*, 2: 1-37.

[10] Barndorff-Nielsen, O.E. and Shephard, N. 2006. Econometrics of Testing for Jumps in Financial Economics Using Bipower Variation. *Journal of Financial Econometrics*, 4: 1-30.

[11] Bates, D.S. 1996. Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options. *Review of Financial Studies*, 9: 69-107.

[12] Bates, D.S. 2000. Post-87 Crash Fears in the S&P 500 Futures Option Market. *Journal of Econometrics*, 94: 181-238.

[13] Chan, W.H. and Maheu, J.M. 2002. Conditional Jump Dynamics in Stock Market Returns. *Journal of Business and Economic Statistics*, 20: 377-389.

[14] Christensen, K., Oomen, Roel, C.A. and Podolskij. Mark, 2014. Fact or Friction: Jumps at Ultra High Frequency. *Journal of Financial Economics*, 114, 576-599.

[15] Corsi, F., Pirino, D. and Renò, R. 2010. Threshold Bipower Variation and the Impact of Jumps on Volatility Forecasting. *Journal of Econometrics*, 159: 276-288.

[16] Duffie, D., Pan J. and Singleton, K. 2000. Transform Analysis and Asset Pricing for Affine Jump-Diffusions. *Econometrica*, 68: 1343-1376.

[17] Dumitru, A.M. and Urga, G. 2012. Identifying Jumps in Financial Assets: A Comparison Between Nonparametric Jump Tests. *Journal of Business and Economic Statistics*, 30: 242-255.

[18] Eraker, B., Johannes, M. and Polson, N. 2003. The Impact of Jumps in Volatility and Returns. *The Journal of Finance*, LVIII: 1269-1300.

[19] Eraker, B. 2004. Do Stock Prices and Volatility Jump? Reconciling Evidence from Spot and Option Prices. *The Journal of Finance*, LIX: 1367-1403.

[20] Fulop, A., Li, J. and Yu, J. 2014. Self-Exciting Jumps, Learning, and Asset Pricing Implications. *Review of Financial Studies*, 28: 876-912.

[21] Hansen, P.R. and Lunde, A. 2006. Realized Variance and Microstructure Noise. *Journal of Business and Economic Statistics*, 24: 127-161.

[22] Huang, X. and Tauchen, G. 2005. The Relative Contribution of Jumps to Total Price Variance. *Journal of Financial Econometrics*, 3: 456-499.

[23] Jiang, G.J. and Oomen, R.C. 2008. Testing for Jumps When Asset Prices are Observed with Noise: A 'Swap Variance' Approach. *Journal of Econometrics*, 144: 352-370.

[24] Koopman, S.J. and Scharth, M. 2013. The Analysis of Stochastic Volatility in the Presence of Daily Realized Measures. *Journal of Financial Econometrics*, 11: 76-115.

[25] Lee, S.S. and Mykland, P.A. 2008. Jumps in Financial Markets: A New Nonparametric Test and Jump Dynamics. *Review of Financial Studies*, 21: 2535-2563.

[26] Lee, S.S. and Mykland, P.A. 2012. Jumps in Equilibrium Prices and Market Microstructure Noise. *Journal of Econometrics*, 168: s396-406.

[27] Maheu, J.M. and McCurdy, T.H. 2004. News Arrival, Jump Dynamics, and Volatility Components for Individual Stock Returns. *The Journal of Finance*, LIX: 755-793.

[28] Maneesoonthorn, W., Forbes, C.S. and Martin, G.M. 2017. Inference on Self-Exciting Jumps Using High-Frequency Measures. *Journal of Applied Econometrics*, 32: 504-532.

[29] Maneesoonthorn, W., Martin, G.M., Forbes, C.S. and Grose, S. 2012. Probabilistic Forecasts of Volatility and its Risk Premia. *Journal of Econometrics*, 171: 217-236.

[30] Merton, R.C. 1976. Option Pricing When Underlying Stock Returns Are Discontinuous. *Journal of Financial Economics*, 3: 125-144.

[31] Neuberger, A. 1994. The Log Contract. *The Journal of Portfolio Management*, 20: 74-80.

[32] Podolskij, M. and Ziggel, D. 2010. New Tests for Jumps in Semimartingale Models. *Statistical Infererence for Stochastic Processes*, 13: 15-41.

[33] Podolskij, M., and Vetter, M. 2009. Estimation of Volatility Functionals in the Simultaneous Presence of Microstructure Noise and Jumps. *Bernoulli*, 15: 634-658.

[34] Tauchen G. and Zhou, H. 2011. Realized Jumps on Financial Markets and Predicting Credit Spreads. *Journal of Econometrics*, 160: 102-118.

[35] Todorov, V. 2010. Variance Risk-Premium Dynamics: The Role of Jumps. *Review of Financial Studies*, 23: 345-383.

[36] Todorov, V. and Tauchen, G. 2011. Volatility Jumps. *Journal of Business & Economic Statistics*, 29: 356-371.

Table 1: The jump tests outlined in Section 2.1 (squared variation) and Section 2.2 (higher-order power variation). All test statistics reported have a $N(0,1)$ limiting distribution under the null hypothesis of no jump.

**Test** **Test Statistic**

**Panel A: Tests based on squared variation**

**BNS**

$$T_{BNS,t} = \frac{1-\frac{BV_t}{RV_t}}{\sqrt{\left(\left(\frac{\pi}{2}\right)^2+\pi-5\right)M^{-1}\max\left(1,\frac{TP_t}{BV_t^2}\right)}}, \text{ where } BV_t = \frac{\pi}{2}\left(\frac{M}{M-1}\right)\sum_{i=2}^{M}\left|r_{t_i}\right|\left|r_{t_{i-1}}\right|,$$

$$TP_t = \mu_{4/3}^{-3}\left(\frac{M^2}{M-2}\right)\sum_{i=3}^{M}\mid r_{t_{i-2}}\mid^{4/3}\mid r_{t_{i-1}}\mid^{4/3}\mid r_{t_i}\mid^{4/3}, \text{ and } \mu_{4/3} = 2^{2/3}\Gamma(7/6)\Gamma(1/2)^{-1}$$

**CPR**

$$T_{CPR,t} = \frac{1-\frac{CTBV_t}{RV_t}}{\sqrt{\left(\left(\frac{\pi}{2}\right)^2+\pi-5\right)M^{-1}\max\left(1,\frac{CTriPV_t}{CTBV_t^2}\right)}}, \text{ where } CTBV_t = \frac{\pi}{2}\left(\frac{M}{M-1}\right)\sum_{i=2}^{M}\tau_{1,t_i}\tau_{1,t_{i-1}},$$

$$CTriPV_t = \mu_{4/3}^{-3}\left(\frac{M^2}{M-2}\right)\sum_{i=3}^{M}\tau_{4/3,t_i}\tau_{4/3,t_{i-1}}\tau_{4/3,t_{i-2}}, \quad \tau_{1,t_i} = \begin{cases} |r_{t_i}| & \text{for } r_{t_i}^2 < \vartheta_{t_i} \\ 1.094\sqrt{\vartheta_{t_i}} & \text{for } r_{t_i}^2 > \vartheta_{t_i} \end{cases},$$

$$\tau_{4/3,t_i} = \begin{cases} |r_{t_i}|^{4/3} & \text{for } r_{t_i}^2 < \vartheta_{t_i} \\ 1.129\vartheta_{t_i}^{2/3} & \text{for } r_{t_i}^2 > \vartheta_{t_i} \end{cases}, \text{ and } \vartheta_{t_i} = c_\vartheta^2\widehat{V}_{t_i}, \text{where } \widehat{V}_{t_i} \text{ denotes a local variance estimator}$$

**MINRV**

$$T_{MinRV,t} = \frac{1-\frac{MinRV_t}{RV_t}}{\sqrt{1.81M^{-1}\max\left(1,\frac{MinRQ_t}{MinRV_t^2}\right)}}, \text{ where } MinRV_t = \frac{\pi}{\pi-2}\left(\frac{M}{M-1}\right)\sum_{i=2}^{M}\min\left(\left|r_{t_i}\right|,\left|r_{t_{i-1}}\right|\right)^2$$

and $MinRQ_t = \frac{\pi}{3\pi-8}\left(\frac{M^2}{M-1}\right)\sum_{i=2}^{M}\min\left(\left|r_{t_i}\right|,\left|r_{t_{i-1}}\right|\right)^4$

**MEDRV**

$$T_{MedRV,t} = \frac{1-\frac{MedRV_t}{RV_t}}{\sqrt{0.96M^{-1}\max\left(1,\frac{MedRQ_t}{MedRV_t^2}\right)}}, \text{ where } MedRV_t = \frac{\pi}{\pi+6-4\sqrt{3}}\left(\frac{M}{M-2}\right)\sum_{i=3}^{M} med\left(\left|r_{t_i}\right|,\left|r_{t_{i-1}}\right|,\left|r_{t_{i-2}}\right|\right)^2$$

and $MedRQ_t = \frac{3\pi}{9\pi+72-52\sqrt{3}}\left(\frac{M^2}{M-2}\right)\sum_{i=3}^{M} med\left(\left|r_{t_i}\right|,\left|r_{t_{i-1}}\right|,\left|r_{t_{i-2}}\right|\right)^4$

**Panel B: Tests based on $\mathcal{P}$-power variation**

**PZ**

$$T_{PZ,t} = \frac{M^{\frac{\mathcal{P}-1}{2}}\sum_{i=1}^{M}\left|r_{t_i}\right|^{\mathcal{P}}\left(1-\eta_i\mathbf{1}_{\{\left|r_{t_i}\right|<\vartheta(\Delta_M)^{\varpi}\}}\right)}{\sqrt{Var(\eta_i)M^{\frac{\mathcal{P}}{2}-1}\sum_{i=1}^{M}\left|r_{t_i}\right|^{\mathcal{P}}\mathbf{1}_{\{\left|r_{t_i}\right|<\vartheta(\Delta_M)^{\varpi}\}}}} \text{ where } \eta_i \text{ is a symmetric IID random variable}$$

with $E(\eta_i) = 1$, $Var(\eta_i) < \infty$ and $E\left(|\eta_i|^{2+d}\right) > 0$ for some $d > 0$; and $\mathcal{P} \geq 2$.

**ASJ**

Under $H_0$ of continuous price path

$$T_{ASJ,t} = \left(\widehat{\Sigma}_{M,t}^{c}\right)^{-1/2}\left(\widehat{S}(\mathcal{P},k,\Delta_M)_t - k^{\frac{\mathcal{P}}{2}-1}\right), \text{ where } \widehat{S}(\mathcal{P},k,\Delta_M)_t = \frac{\widehat{B}(\mathcal{P},k\Delta_M)_t}{\widehat{B}(\mathcal{P},\Delta_M)_t}, \ \widehat{\Sigma}_{M,t}^{c} = \frac{\Delta_M M(\mathcal{P},k)\widehat{A}(2\mathcal{P},\Delta_M)_t}{\widehat{A}(\mathcal{P},\Delta_M)_t^2}$$

and $\widehat{A}(\mathcal{P},\Delta_M)_t = \frac{\Delta_M^{1-\mathcal{P}/2}}{m_{\mathcal{P}}}\sum_{i=1}^{M}\left|r_{t_i}\right|^{\mathcal{P}}\mathbf{1}_{\{\left|r_{t_i}\right|<\vartheta\Delta_M^{\varpi}\}}$, for $k \geq 2$ and $\mathcal{P} > 2$.

Here, $M(\mathcal{P},k)$ and $m_{\mathcal{P}}$ are constants defined by expectations of absolute power of standard normal variables.

Table 2: The jump tests outlined in Section 2.3 (standardized daily returns), Section 2.4 (variance swaps) and Section 2.5 (microstructure noise). The limiting distribution for each test statistic under the null hypothesis of no price jump given in column 3 below.

| Test | Test Statistics | Limiting Dist. |
|---|---|---|
| **Panel A: Tests based on standardized returns** | | |
| ABD | $T_{ABD,t_i} = \frac{r_{t_i}}{\sqrt{M^{-1}BV_t}}$. The significance level needs to be adjusted for multiple testing. | N(0,1) |
| LM | $T_{LM,t}=\frac{\left(\max\left(\widetilde{T}_{LM,t_i}\right)-C_M\right)}{S_M}$ where $\widetilde{T}_{LM,t_i} = \frac{\lvert r_{t_i}\rvert}{\sqrt{\hat{V}_{t_i}}}$,<br>$C_M = \frac{(2\log M)^{1/2}}{0.8} - \frac{\log\pi+\log(\log M)}{1.6(2\log\pi)^{1/2}}$,<br>$S_M = \frac{1}{0.6(2\log\pi)^{1/2}}$ and $\hat{V}_{t_i}$ denotes the local variance estimate | Gumbel |
| **Panel B: Test based on variance swap** | | |
| JO | $T_{JO,t} = \frac{BV_t}{M^{-1}\sqrt{\widehat{\Omega}_{SwV}}}\left(1-\frac{RV_t}{SwV_t}\right)$, where $SwV_t = 2\sum_{i=1}^{M}\left(R_{t_i}-r_{t_i}\right)$<br>with $R_{t_i}$ = arithmatic returns<br>and $\widehat{\Omega}_{SwV} = 3.05\frac{M^3}{M-3}\sum_{i=1}^{M}\prod_{k=0}^{3}\left\lvert r_{t_{i-k}}\right\rvert^{3/2}$ | N(0,1) |
| **Panel C: Tests that account for microstructure noise** | | |
| LM12 | $T_{LM12} = \max_{t_j\in G_n^k}\frac{\lvert\chi(t_j)\rvert-A_n}{B_n}$ where $A_n = \left(2\log\lfloor\frac{n}{kM}\rfloor\right)^{1/2} - \frac{\log\pi+\log(\log\lfloor\frac{n}{kM}\rfloor)}{2\left(2\log\lfloor\frac{n}{kM}\rfloor\right)^{1/2}}$,<br>$B_n = \left(2\log\lfloor\frac{n}{kM}\rfloor\right)^{1/2}$, $\chi(t_j) = \sqrt{\frac{M}{V_n}}\left(\widehat{p}_{t_j+kM}-\widehat{p}_{t_j}\right)$,<br>and $\widehat{p}_{t_j}$ is the average log price over a block size $M$,<br>with the average price computed using every $k^{th}$ observations.<br>$V_n$ is computed using Podolskij & Vetter (2009) | Gumbel |
| ASJL | Test statistic as in ASJ, but with the power variation computed<br>from smoothed log prices. The estimator of the asymptotic variance<br>of the test statistic is modified accordingly.<br>Code to conduct the test is available at https://sites.duke.edu/jiali/research/. | N(0,1) |

Table 3: The empirical size of price jump tests constructed at the five second, 30 second, one minute and five minute sampling frequencies. The tests are conducted under three DGPs. DGP1 assumes volatility jumps are absent (labelled as '$dJ_t^v = 0$'); DGP2 assumes that a volatility jump is present and is of moderate size (labelled as '$dJ_t^v = 3\theta$'); DGP3 assumes a large volatility jump is present (labelled as '$dJ_t^v = 10\theta$'). Nominal sizes of 5% and 1% are shown in columns 3-6 and 7-10, respectively. Microstructure noise is absent.

| | | Nominal Size = 5% | | | | Nominal Size = 1% | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 5 sec | 30sec | 1min | 5min | 5 sec | 30sec | 1min | 5min |
| | BNS | 0.051 | 0.052 | 0.042 | 0.055 | 0.008 | 0.009 | 0.010 | 0.020 |
| | CPR | 0.086 | 0.133 | 0.165 | 0.464 | 0.016 | 0.045 | 0.059 | 0.241 |
| | MINRV | 0.051 | 0.046 | 0.048 | 0.044 | 0.006 | 0.005 | 0.005 | 0.010 |
| | MEDRV | 0.054 | 0.041 | 0.057 | 0.060 | 0.010 | 0.008 | 0.005 | 0.014 |
| DGP1 | PZ2 | 0.047 | 0.057 | 0.070 | 0.092 | 0.009 | 0.018 | 0.028 | 0.051 |
| no VJ | PZ4 | 0.046 | 0.051 | 0.058 | 0.078 | 0.014 | 0.015 | 0.026 | 0.046 |
| ($dJ_t^v = 0$) | ASJ | 0.018 | 0.011 | 0.004 | 0.000 | 0.002 | 0.000 | 0.000 | 0.000 |
| | ABD | 0.086 | 0.058 | 0.053 | 0.063 | 0.020 | 0.012 | 0.011 | 0.020 |
| | LM | 0.032 | 0.009 | 0.007 | 0.005 | 0.003 | 0.001 | 0.002 | 0.000 |
| | JO | 0.054 | 0.049 | 0.068 | 0.079 | 0.007 | 0.011 | 0.016 | 0.025 |
| | LM12 | 0.037 | 0.124 | 0.209 | 0.521 | 0.011 | 0.055 | 0.103 | 0.386 |
| | ASJL | 0.055 | 0.057 | 0.011 | 0.093 | 0.018 | 0.009 | 0.002 | 0.091 |
| | BNS | 0.078 | 0.078 | 0.066 | 0.066 | 0.021 | 0.030 | 0.018 | 0.022 |
| | CPR | 0.104 | 0.127 | 0.155 | 0.275 | 0.033 | 0.045 | 0.048 | 0.105 |
| | MINRV | 0.050 | 0.057 | 0.049 | 0.049 | 0.014 | 0.021 | 0.007 | 0.010 |
| | MEDRV | 0.054 | 0.056 | 0.044 | 0.061 | 0.011 | 0.017 | 0.010 | 0.021 |
| DGP2 | PZ2 | 0.216 | 0.268 | 0.271 | 0.179 | 0.185 | 0.230 | 0.237 | 0.152 |
| moderate VJ | PZ4 | 0.214 | 0.260 | 0.268 | 0.180 | 0.189 | 0.229 | 0.233 | 0.150 |
| ($dJ_t^v = 3\theta$) | ASJ | 0.022 | 0.004 | 0.004 | 0.000 | 0.001 | 0.000 | 0.000 | 0.000 |
| | ABD | 0.827 | 0.532 | 0.409 | 0.194 | 0.568 | 0.259 | 0.212 | 0.083 |
| | LM | 0.023 | 0.009 | 0.009 | 0.011 | 0.003 | 0.000 | 0.000 | 0.001 |
| | JO | 0.053 | 0.068 | 0.078 | 0.110 | 0.011 | 0.017 | 0.021 | 0.045 |
| | LM12 | 0.205 | 0.287 | 0.372 | 0.579 | 0.079 | 0.156 | 0.224 | 0.438 |
| | ASJL | 0.113 | 0.088 | 0.035 | 0.118 | 0.041 | 0.030 | 0.007 | 0.111 |
| | BNS | 0.121 | 0.119 | 0.115 | 0.101 | 0.040 | 0.060 | 0.045 | 0.048 |
| | CPR | 0.140 | 0.153 | 0.157 | 0.192 | 0.053 | 0.079 | 0.065 | 0.093 |
| | MINRV | 0.048 | 0.072 | 0.054 | 0.068 | 0.016 | 0.023 | 0.012 | 0.023 |
| | MEDRV | 0.056 | 0.063 | 0.050 | 0.084 | 0.012 | 0.018 | 0.017 | 0.029 |
| DGP3 | PZ2 | 0.800 | 0.723 | 0.636 | 0.421 | 0.794 | 0.713 | 0.624 | 0.400 |
| large VJ | PZ4 | 0.802 | 0.718 | 0.635 | 0.424 | 0.794 | 0.711 | 0.619 | 0.399 |
| ($dJ_t^v = 10\theta$) | ASJ | 0.018 | 0.002 | 0.004 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | ABD | 0.996 | 0.922 | 0.818 | 0.477 | 0.975 | 0.765 | 0.585 | 0.272 |
| | LM | 0.031 | 0.033 | 0.030 | 0.042 | 0.005 | 0.008 | 0.006 | 0.016 |
| | JO | 0.059 | 0.070 | 0.087 | 0.140 | 0.016 | 0.018 | 0.031 | 0.066 |
| | LM12 | 0.544 | 0.535 | 0.595 | 0.703 | 0.324 | 0.379 | 0.433 | 0.581 |
| | ASJL | 0.220 | 0.172 | 0.083 | 0.165 | 0.130 | 0.077 | 0.024 | 0.155 |

Table 4: The empirical power of price jump tests constructed at the five second, 30 second, one minute and five minute sampling frequencies. The tests are conducted under two scenarios for the price jump size: moderate (left panel, with $dJ_t^p = 3\sqrt{\theta}$) and large (right panel, with $dJ_t^p = 10\sqrt{\theta}$), and under three DGPs for the volatility jump process. DGP1 assumes volatility jumps are absent (labelled as '$dJ_t^v = 0$'); DGP2 assumes that a volatility jump is present and is of moderate size (labelled as '$dJ_t^v = 3\theta$'); DGP3 assumes a large volatility jump is present (labelled as '$dJ_t^v = 10\theta$'). All tests are conducted using a nominal size of 5%. Microstructure noise is absent.

| | | Moderate Price Jump ($dJ_t^p = 3\sqrt{\theta}$) | | | | Large Price Jump ($dJ_t^p = 10\sqrt{\theta}$) | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 5 sec | 30sec | 1min | 5min | 5 sec | 30sec | 1min | 5min |
| | BNS | 0.793 | 0.201 | 0.120 | 0.068 | 1.000 | 1.000 | 0.998 | 0.573 |
| | CPR | 0.881 | 0.384 | 0.311 | 0.479 | 1.000 | 1.000 | 1.000 | 0.862 |
| | MINRV | 0.540 | 0.149 | 0.101 | 0.050 | 1.000 | 1.000 | 0.992 | 0.490 |
| | MEDRV | 0.776 | 0.219 | 0.124 | 0.064 | 1.000 | 1.000 | 1.000 | 0.705 |
| DGP1 | PZ2 | 1.000 | 0.813 | 0.375 | 0.098 | 1.000 | 1.000 | 1.000 | 0.961 |
| no VJ | PZ4 | 1.000 | 0.811 | 0.370 | 0.091 | 1.000 | 1.000 | 1.000 | 0.960 |
| ($dJ_t^v = 0$) | ASJ | 0.492 | 0.191 | 0.050 | 0.000 | 0.492 | 0.487 | 0.484 | 0.017 |
| | ABD | 1.000 | 0.907 | 0.486 | 0.079 | 1.000 | 1.000 | 1.000 | 0.968 |
| | LM | 0.646 | 0.073 | 0.019 | 0.002 | 1.000 | 0.936 | 0.730 | 0.108 |
| | JO | 0.999 | 0.334 | 0.178 | 0.104 | 1.000 | 1.000 | 1.000 | 0.818 |
| | LM12 | 0.626 | 0.244 | 0.245 | 0.508 | 1.000 | 0.960 | 0.931 | 0.675 |
| | AJL | 0.130 | 0.059 | 0.014 | 0.083 | 0.998 | 0.813 | 0.511 | 0.112 |
| | BNS | 0.474 | 0.158 | 0.099 | 0.087 | 1.000 | 0.995 | 0.928 | 0.363 |
| | CPR | 0.579 | 0.254 | 0.190 | 0.294 | 1.000 | 0.999 | 0.965 | 0.605 |
| | MINRV | 0.284 | 0.111 | 0.066 | 0.061 | 1.000 | 0.976 | 0.826 | 0.277 |
| | MEDRV | 0.418 | 0.130 | 0.077 | 0.064 | 1.000 | 0.997 | 0.941 | 0.415 |
| DGP2 | PZ2 | 1.000 | 0.598 | 0.338 | 0.204 | 1.000 | 1.000 | 1.000 | 0.806 |
| moderate VJ | PZ4 | 1.000 | 0.591 | 0.335 | 0.189 | 1.000 | 1.000 | 1.000 | 0.800 |
| ($dJ_t^v = 3\theta$) | ASJ | 0.475 | 0.027 | 0.006 | 0.000 | 0.492 | 0.471 | 0.389 | 0.003 |
| | ABD | 1.000 | 0.808 | 0.515 | 0.214 | 1.000 | 1.000 | 1.000 | 0.841 |
| | LM | 0.457 | 0.052 | 0.013 | 0.006 | 0.995 | 0.785 | 0.547 | 0.073 |
| | JO | 0.818 | 0.138 | 0.118 | 0.121 | 1.000 | 1.000 | 0.989 | 0.524 |
| | LM12 | 0.316 | 0.302 | 0.369 | 0.566 | 0.998 | 0.859 | 0.801 | 0.639 |
| | AJL | 0.152 | 0.092 | 0.033 | 0.120 | 0.980 | 0.539 | 0.259 | 0.122 |
| | BNS | 0.283 | 0.160 | 0.117 | 0.119 | 1.000 | 0.881 | 0.666 | 0.237 |
| | CPR | 0.322 | 0.211 | 0.165 | 0.214 | 1.000 | 0.905 | 0.745 | 0.368 |
| | MINRV | 0.137 | 0.088 | 0.051 | 0.080 | 0.996 | 0.589 | 0.371 | 0.162 |
| | MEDRV | 0.173 | 0.098 | 0.064 | 0.081 | 1.000 | 0.787 | 0.530 | 0.198 |
| DGP3 | PZ2 | 0.995 | 0.739 | 0.653 | 0.416 | 1.000 | 1.000 | 0.997 | 0.609 |
| large VJ | PZ4 | 0.995 | 0.742 | 0.652 | 0.415 | 1.000 | 1.000 | 0.997 | 0.607 |
| ($dJ_t^v = 10\theta$) | ASJ | 0.118 | 0.002 | 0.000 | 0.000 | 0.492 | 0.318 | 0.101 | 0.000 |
| | ABD | 1.000 | 0.940 | 0.823 | 0.465 | 1.000 | 1.000 | 0.999 | 0.664 |
| | LM | 0.376 | 0.085 | 0.041 | 0.042 | 0.940 | 0.625 | 0.449 | 0.094 |
| | JO | 0.180 | 0.087 | 0.096 | 0.157 | 1.000 | 0.923 | 0.640 | 0.236 |
| | LM12 | 0.541 | 0.496 | 0.588 | 0.696 | 0.971 | 0.742 | 0.718 | 0.682 |
| | ASJL | 0.240 | 0.160 | 0.079 | 0.164 | 0.779 | 0.278 | 0.128 | 0.160 |

Table 5: The empirical size and power of price jump tests constructed at the five second, 30 second, one minute and five minute sampling frequencies, in the presence of microstructure noise. The tests are conducted under DGP1, but with three different types of microstructure noise: Gaussian noise; Student-t noise and autocorrelated noise. Each test is conducted using a nominal size of 5%, with the power assessment conducted using a large price jump size $dJ_t^p = 10\sqrt{\theta}$. Volatility jumps are absent.

| | | Size | | | | Power | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 5 sec | 30sec | 1min | 5min | 5 sec | 30sec | 1min | 5min |
| | BNS | 0.000 | 0.040 | 0.048 | 0.060 | 0.999 | 1.000 | 0.997 | 0.533 |
| | CPR | 0.000 | 0.109 | 0.181 | 0.443 | 0.999 | 1.000 | 0.999 | 0.847 |
| | MINRV | 0.054 | 0.056 | 0.064 | 0.068 | 0.869 | 0.999 | 0.975 | 0.440 |
| | MEDRV | 0.054 | 0.056 | 0.064 | 0.068 | 1.000 | 1.000 | 0.996 | 0.649 |
| DGP1 | PZ2 | 0.059 | 0.057 | 0.059 | 0.072 | 1.000 | 1.000 | 1.000 | 0.950 |
| Gaussian noise | PZ4 | 0.054 | 0.056 | 0.064 | 0.068 | 1.000 | 1.000 | 1.000 | 0.950 |
| | ASJ | 0.019 | 0.013 | 0.002 | 0.000 | 0.529 | 0.501 | 0.517 | 0.020 |
| | ABD | 0.062 | 0.082 | 0.074 | 0.052 | 1.000 | 1.000 | 1.000 | 0.967 |
| | LM | 0.025 | 0.006 | 0.007 | 0.001 | 0.990 | 0.875 | 0.683 | 0.118 |
| | JO | 0.006 | 0.039 | 0.042 | 0.072 | 1.000 | 1.000 | 1.000 | 0.809 |
| | LM12 | 0.043 | 0.155 | 0.216 | 0.511 | 1.000 | 0.965 | 0.921 | 0.671 |
| | AJL | 0.053 | 0.046 | 0.014 | 0.082 | 0.999 | 0.818 | 0.507 | 0.105 |
| | BNS | 0.117 | 0.050 | 0.046 | 0.065 | 0.999 | 0.999 | 0.991 | 0.548 |
| | CPR | 0.410 | 0.192 | 0.210 | 0.488 | 0.999 | 1.000 | 0.997 | 0.846 |
| | MINRV | 0.000 | 0.022 | 0.031 | 0.044 | 0.353 | 0.910 | 0.926 | 0.434 |
| | MEDRV | 0.000 | 0.036 | 0.044 | 0.054 | 0.610 | 0.952 | 0.964 | 0.639 |
| DGP1 | PZ2 | 0.997 | 0.374 | 0.172 | 0.088 | 1.000 | 1.000 | 1.000 | 0.935 |
| Student-t noise | PZ4 | 0.997 | 0.370 | 0.183 | 0.079 | 1.000 | 1.000 | 1.000 | 0.935 |
| | ASJ | 0.000 | 0.006 | 0.005 | 0.000 | 0.157 | 0.394 | 0.460 | 0.016 |
| | ABD | 1.000 | 0.463 | 0.198 | 0.070 | 1.000 | 1.000 | 1.000 | 0.948 |
| | LM | 0.722 | 0.063 | 0.018 | 0.002 | 0.996 | 0.893 | 0.680 | 0.114 |
| | JO | 0.208 | 0.074 | 0.056 | 0.084 | 0.997 | 0.996 | 0.997 | 0.798 |
| | LM12 | 0.051 | 0.148 | 0.221 | 0.512 | 1.000 | 0.955 | 0.925 | 0.722 |
| | AJL | 0.063 | 0.054 | 0.018 | 0.101 | 0.998 | 0.798 | 0.488 | 0.110 |
| | BNS | 0.000 | 0.050 | 0.063 | 0.058 | 1.000 | 1.000 | 0.994 | 0.560 |
| | CPR | 0.000 | 0.133 | 0.189 | 0.468 | 1.000 | 1.000 | 0.998 | 0.827 |
| | MINRV | 0.000 | 0.047 | 0.044 | 0.041 | 1.000 | 1.000 | 0.985 | 0.459 |
| | MEDRV | 0.000 | 0.056 | 0.048 | 0.051 | 1.000 | 1.000 | 1.000 | 0.619 |
| DGP1 | PZ2 | 0.062 | 0.062 | 0.068 | 0.082 | 1.000 | 1.000 | 1.000 | 0.939 |
| Gaussian | PZ4 | 0.053 | 0.058 | 0.063 | 0.077 | 1.000 | 1.000 | 1.000 | 0.937 |
| Autocorrelated noise | ASJ | 0.023 | 0.015 | 0.004 | 0.000 | 0.517 | 0.486 | 0.475 | 0.014 |
| | ABD | 0.039 | 0.047 | 0.056 | 0.060 | 1.000 | 1.000 | 1.000 | 0.956 |
| | LM | 0.020 | 0.009 | 0.007 | 0.000 | 0.999 | 0.894 | 0.710 | 0.115 |
| | JO | 0.013 | 0.040 | 0.048 | 0.086 | 1.000 | 1.000 | 1.000 | 0.797 |
| | LM12 | 0.030 | 0.164 | 0.216 | 0.536 | 1.000 | 0.958 | 0.925 | 0.736 |
| | ASJL | 0.070 | 0.058 | 0.015 | 0.095 | 0.997 | 0.822 | 0.488 | 0.118 |