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Bivariate error correction FIGARCH and FIAPARCH models on the Australian All Ordinaries Index and its SPI futures.

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Bivariate error correction FIGARCH and FIAPARCH models on the Australian All Ordinaries Index and its SPI futures

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Abstract

In this paper we extend the univariate FIGARCH and FIAPARCH models to a bivariate framework. We estimate bivariate error correction FIGARCH and FIAPARCH models between the All Ordinaries Index and its SPI futures using constant correlation and diagonal parameterisations. We therefore employ a flexible estimation approach that captures the long run equilibrium relationship between the two markets, bi-directional return causality, long memory and asymmetries in volatility, and time varying correlations. The results strongly support the use of this approach. Strong bi-directional return causality exists with the index bearing the burden of adjustment to deviations from long run equilibrium. The results also illustrate the importance of allowing for long memory, asymmetries in volatility, and time varying correlations.

JEL classification: GO; C3; C51

Keywords: long memory, univariate and bivariate FIGARCH and FIAPARCH, asymmetries in volatility.

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1. Introduction

In this paper we extend the univariate Fractionally Integrated GARCH (FIGARCH) and Fractionally Integrated Asymmetric Power ARCH (FIAPARCH) models to a bivariate framework. The Fractionally Integrated GARCH (FIGARCH) model of Baillie *et al* (1996) exhibits long memory in volatility. This is in contrast to the GARCH (Bollerslev, 1986) class of processes which exhibit short memory. Both the GARCH and FIGARCH processes are unable to capture the widely documented asymmetries in equity market volatility (Engle and Ng, 1993; Bollerslev and Mikkelsen, 1996; Ostermark and Hoglund, 1997; Lien and Tse, 1998; Tse, 1999; Bhar, 2001; Koutmos and Tucker, 1996). This is addressed by Tse (1998) who develops the Fractionally Integrated Asymmetric Power ARCH (FIAPARCH) model, which allows for long memory and asymmetries in volatility.

This paper therefore supplements Dark (2003), where the presence of long memory in the volatility of the Australian All Ordinaries Index and its SPI futures is documented. Dark (2003) tests for long memory using a long span of daily data and a short span of high frequency data. Spectral density estimates of the fractional differencing parameter (d), the fit of the implied autocorrelation function, and the modified R/S (see Lo, 1991) and KPSS (see Kwiatkowski *et al*, 1992) test statistics, support the existence of long memory in the volatility in both markets.

Strong linkages between equity indices and their SPI futures are also well documented. The evidence generally indicates that violations in the cost of carry model are temporary (Buhler and Kempf, 1995; Brennan and Schwarz, 1990; Yadav and Pope, 1994; Klemkosky and Lee, 1991; Lim, 1992; Twite, 1998; Brailsford and Hodgson, 1997). This evidence supports the view that there is a long run equilibrium relationship between equity indices and their SPI futures. The evidence also supports the existence of uni-directional or bi-directional return causality (Kawaller *et al*, 1987; Stoll and Whaley, 1990; Koch, 1993, Frino *et al,* 2000; Turkington and Walsh, 1999).

The existence of a long run equilibrium relation, uni-directional or bi-directional return causality, and time varying volatility, has seen the bivariate error correction GARCH model become a very popular way of modelling the return and volatility dynamics between spot and futures markets (Koutmos and Tucker, 1996; Tse, 1999; Lee, 1994; Chatrah and Song, 1998; Lien and Luo, 1994; Lien and Tse, 1998; Koutmos and Pericli, 1998; Bhar, 2001). If the All Ordinaries Index and the SPI futures exhibit long memory and asymmetries in volatility, we would expect the bivariate error correction FIAPARCH model to outperform the bivariate error correction FIGARCH and GARCH models.

This paper seeks to make two contributions to this literature. The first is the estimation of bivariate error correction FIGARCH and FIAPARCH models between the All Ordinaries Index and the SPI futures. This is important given that univariate models have limited practical application. Financial analysts are typically interested in the co-movements across assets, particularly when considering asset allocation and hedging. This is significant given that multivariate FIGARCH processes are in their early stages of development, with there only being one publication in this area (Brunetti and Gilbert, 2000). It is also significant given that conventional models of the return and volatility dynamics between spot and futures markets impose short memory in volatility. By more appropriately capturing the long term volatility dynamics, the bivariate error correction FIGARCH and FIAPARCH models should be able to provide superior long term forecasts of volatility and the co-movements between assets. The second contribution is an investigation into the significance of allowing for time varying correlations. This is important given the popularity of the constant correlation assumption when estimating multivariate GARCH processes.

In Section 2 we will commence with a definition of long memory and present the univariate FIGARCH and FIAPARCH models. The section will then present the bivariate error correction GARCH, FIGARCH and FIAPARCH models. This will be followed by the empirical evidence supporting the estimation of univariate and multivariate FIGARCH processes. In Section 3 we detail the procedure used to estimate the bivariate error correction FIGARCH and FIAPARCH models. This is followed by the results in Section 4. Concluding remarks follow in Section 5.

2. Univariate and bivariate FIGARCH and FIAPARCH models

2.1 Definition of long memory, univariate FIGARCH and FIAPARCH

The most common definition of a long memory process is one where the autocovariance function decays at the hypergeometric rate k^{2d-1} ($0<d<0.5$).[1] Consequently, the autocovariance function (Ψ) of a long memory processes is not absolutely summable

[1] See Baillie (1996) and Davidson (2002) for other definitions of long memory.

$$\lim_{n\to\infty}\sum_{k=-n}^{n}\left|\Psi_k\right| = \infty \qquad (1)$$

In contrast a short memory process has an autocovariance function that is absolutely summable. Stationary and invertible ARMA processes exhibit this characteristic, given that they have autocovariances that are geometrically bounded (Baillie, 1996).

Baillie *et al* (1996) propose the FIGARCH(p,d,q) model as one way of modelling long memory in volatility.[2] They develop the FIGARCH(p,d,q) model by allowing the differencing parameter in the IGARCH(p,q) model to take non-integer values as follows

$$\phi(L)(1-L)^d \varepsilon_t^{\,2} = \omega + \left[1-\beta(L)\right] v_t \qquad (2)$$

where βL and ϕL represent lag polynomials of order p and q respectively, d is the fractional differencing parameter and

$$(1-L)^d = 1 - dL - \frac{d(1-d)}{2!}L^2 - \frac{d(1-d)(2-d)}{3!}L^3 - \ldots.. \text{ where } 0 < d < 1. \qquad (3)$$

Given that $v_t = \varepsilon^2{}_t - \sigma^2{}_t$, the FIGARCH(1,$d$,1) process can be expressed as an infinite order ARCH process

[2] Other long memory volatility models include; the long memory ARCH (LM-ARCH) of Ding and Granger (1996), the hyperbolic GARCH (HYGARCH) of Davidson (2002) and the FIEGARCH model of Bollerslev and Mikkelsen (1996).

$$\sigma_t^{\,2} = \frac{\omega}{1-\beta} + \left(1 - \frac{(1-\phi L)(1-L)^d}{1-\beta L}\right)\varepsilon_t^{\,2} \qquad (4)$$

The FIGARCH model imposes a rate of decay on the impulse response coefficients and the autocorrelation function that is eventually hypergeometric. This is in contrast to the GARCH class of models which exhibit short memory and impose much faster exponential rates of decay (see Baillie *et al*, 1996).

The FIAPARCH model of Tse (1998) modifies the FIGARCH process to allow for asymmetries [3]

$$\sigma_t^{\,\delta} = \frac{\omega}{1-\beta} + \left(1 - \frac{(1-\phi L)(1-L)^d}{1-\beta L}\right)\left(|\varepsilon_t| - \gamma\varepsilon_t\right)^{\delta} \qquad (5)$$

where $-1 < \gamma < 1$ and $\delta > 0$ .When $\gamma > 0$, negative shocks give rise to higher volatility than positive ones. The reverse applies if $\gamma < 0$. The FIAPARCH process therefore reduces to the FIGARCH process when $\gamma = 0$ and $\delta = 2$.

The statistical properties of the FIGARCH process, the source of long memory in volatility and even its existence are controversial. FIGARCH may not exhibit long memory (Giriatis *et al*, 2000a; 2000b) and may not be strictly stationary (Teyssiere, 1997; Kirman and Teyssiere, 2000). There is also controversy surrounding the interpretation of *d* (Davidson, 2002). Long memory may arise from; the aggregation

[3] The asymmetries can be explained by the leverage effect (Black, 1976). See Ding *et al* (1993) for the APARCH model and Engle and Ng (1993) for other approaches that may be used to capture an asymmetrical volatility response.

of multiple volatility components caused by heterogeneous information flows (Andersen and Bollerslev, 1997a), the aggregation of multiple volatility components caused by heterogeneous traders (Muller *et al*, 1997) or a heavy tailed regime switching process (Liu, 2000). Breidt and Hsu (2002), Hyung and Franses (2001), Granger and Hyung (1999) and Kirman and Teysierre (2001) do not support long memory in volatility arguing that volatility exhibits near long memory. They refute the use of fractional processes, supporting the use of occasional break models. In this paper we support the existence of long memory and the use of fractional processes, we also remain agnostic about the source of long memory.

2.2 Bivariate Error Correction GARCH, FIGARCH and FIAPARCH models

The motivations behind the development of multivariate ARCH models are varied. Univariate models have limited practical application given that financial analysts are generally interested in the co-movements across assets. This is particularly important when considering asset allocation and hedging. A multivariate approach may also provide efficiency gains.

The extension of the univariate GARCH process to the multivariate GARCH process has been well documented (see Bollerslev *et al,* 1992). Here we estimate the constant correlation and diagonal parameterisations and adopt the following error correction specification for the conditional mean

$$R_{s,t} = a_1 + b_1 z_{t-1} + \sum_{i=1}^{k} c_{1,i} R_{s,t-i} + \sum_{i=1}^{k} d_{1,i} R_{f,t-i} + \varepsilon_{s,t}$$
$$R_{f,t} = a_2 + b_2 z_{t-1} + \sum_{i=1}^{k} c_{2,i} R_{s,t-i} + \sum_{i=1}^{k} d_{2,i} R_{f,t-i} + \varepsilon_{f,t} \qquad (6)$$

with

$$\begin{bmatrix} \varepsilon_{s,t} \\ \varepsilon_{f,t} \end{bmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2{}_{s,t} & \sigma_{sf,t} \\ \sigma_{sf,t} & \sigma^2{}_{f,t} \end{pmatrix}\right] \qquad (7)$$

where; z_{t-1} represents the error correction term, and $R_{s,t}$ and $R_{f,t}$ represent the returns in the spot and futures markets respectively, calculated as the difference in the log of consecutive process multiplied by 100.

The constant correlation GARCH (1,1) model of Bollerslev (1990), expresses the conditional covariance matrix as

$$\begin{pmatrix} \sigma^2_{s,t} \\ \sigma_{sf,t} \\ \sigma^2_{f,t} \end{pmatrix} = \begin{pmatrix} \omega_1 + \alpha_{11}\varepsilon^2_{s,t-1} + \beta_{11}\sigma^2_{s,t-1} \\ \rho\sigma_{s,t}\sigma_{f,t} \\ \omega_3 + \alpha_{33}\varepsilon^2_{f,t-1} + \beta_{33}\sigma^2_{f,t-1} \end{pmatrix} \qquad (8)$$

whilst the diagonal GARCH (1,1) parameterisation of Bollerslev *et al* (1988), allows for time varying covariances and correlations.

$$\begin{pmatrix} \sigma_{s,t}^2 \\ \sigma_{sf,t} \\ \sigma_{f,t}^2 \end{pmatrix} = \begin{pmatrix} \omega_1 + \alpha_{11}\varepsilon_{s,t-1}^2 + \beta_{11}\sigma_{s,t-1}^2 \\ \omega_2 + \alpha_{22}\varepsilon_{s,t-1}\varepsilon_{f,t-1} + \beta_{22}\sigma_{sf,t-1} \\ \omega_3 + \alpha_{33}\varepsilon_{f,t-1}^2 + \beta_{33}\sigma_{f,t-1}^2 \end{pmatrix} \qquad (9)$$

The estimation of the bivariate FIGARCH(1,d,1) model was first performed by Teysierre (1997) using two alternative parameterisations of the conditional covariance. The first specification assumes constant correlation

$$\begin{pmatrix} \sigma_{s,t}^2 \\ \sigma_{sf,t} \\ \sigma_{f,t}^2 \end{pmatrix} = \begin{pmatrix} \dfrac{\omega_s}{1-\beta_s} + \left(1 - \dfrac{(1-\phi_s L)(1-L)^{d_s}}{1-\beta_s}\right)\varepsilon_{s,t}^2 \\ \rho\sigma_{s,t}\sigma_{f,t} \\ \dfrac{\omega_f}{1-\beta_f} + \left(1 - \dfrac{(1-\phi_f L)(1-L)^{d_f}}{1-\beta_f}\right)\varepsilon_{f,t}^2 \end{pmatrix} \qquad (10)$$

The second specification allows for time varying conditional covariances and correlations. It is therefore the FIGARCH(1,d,1) equivalent to the diagonal GARCH(1,1) specification.

$$\begin{pmatrix} \sigma_{s,t}^2 \\ \sigma_{sf,t} \\ \sigma_{f,t}^2 \end{pmatrix} = \begin{pmatrix} \dfrac{\omega_s}{1-\beta_s} + \left(1 - \dfrac{(1-\phi_s L)(1-L)^{d_s}}{1-\beta_s}\right)\varepsilon_{s,t}^2 \\ \dfrac{\omega_{sf}}{1-\beta_{sf}} + \left(1 - \dfrac{(1-\phi_{sf} L)(1-L)^{d_{sf}}}{1-\beta_{sf}}\right)\varepsilon_{s,t}\varepsilon_{f,t} \\ \dfrac{\omega_f}{1-\beta_f} + \left(1 - \dfrac{(1-\phi_f L)(1-L)^{d_f}}{1-\beta_f}\right)\varepsilon_{f,t}^2 \end{pmatrix} \qquad (11)$$

A straight forward extension of the constant correlation FIGARCH(1,d,1), is the constant correlation bivariate FIAPARCH(1,d,1)

$$\begin{pmatrix} \sigma_{s,t}^{\delta_s} \\ \sigma_{sf,t} \\ \sigma_{f,t}^{\delta_f} \end{pmatrix} = \begin{pmatrix} \frac{\omega_s}{1-\beta_s} + \left(1 - \frac{(1-\phi_s L)(1-L)^{d_s}}{1-\beta_s L}\right)\left(|\varepsilon_{s,t}| - \gamma_s \varepsilon_{s,t}\right)^{\delta_s} \\ \rho\sigma_{s,t}\sigma_{f,t} \\ \frac{\omega_f}{1-\beta_f} + \left(1 - \frac{(1-\phi_f L)(1-L)^{d_f}}{1-\beta_f L}\right)\left(|\varepsilon_{f,t}| - \gamma_f \varepsilon_{f,t}\right)^{\delta_f} \end{pmatrix} \tag{12}$$

Given ρ is positive, this constrains $\sigma_{sf,t}$ to be positive. The specification also indirectly captures asymmetries in the covariance via the $\sigma_{s,t}^{\delta_s}$ and $\sigma_{f,t}^{\delta_f}$ estimates.

The extension of the FIAPARCH(1,d,1) model to a bivariate diagonal specification may be performed in two ways. The first allows for asymmetries but imposes the restriction of positive covariance estimates. It also restricts the asymmetries in the covariance to be a function of the asymmetries in the index and its futures. We will refer to this as the diagonal FIAPARCH(1,d,1) model – option 1

$$\begin{pmatrix} \sigma_{s,t}^{\delta_s} \\ \sigma_{sf,t} \\ \sigma_{f,t}^{\delta_f} \end{pmatrix} = \begin{pmatrix} \frac{\omega_s}{1-\beta_s} + \left(1 - \frac{(1-\phi_s L)(1-L)^{d_s}}{1-\beta_s L}\right)\left(|\varepsilon_{s,t}| - \gamma_s \varepsilon_{s,t}\right)^{\delta_s} \\ \frac{\omega_{sf}}{1-\beta_{sf}} + \left(1 - \frac{(1-\phi_{sf} L)(1-L)^{d_{sf}}}{1-\beta_{sf} L}\right)\left(|\varepsilon_{s,t}| - \gamma_s \varepsilon_{s,t}\right)\left(|\varepsilon_{f,t}| - \gamma_f \varepsilon_{f,t}\right) \\ \frac{\omega_f}{1-\beta_f} + \left(1 - \frac{(1-\phi_f L)(1-L)^{d_f}}{1-\beta_f L}\right)\left(|\varepsilon_{f,t}| - \gamma_f \varepsilon_{f,t}\right)^{\delta_f} \end{pmatrix} \tag{13}$$

The second model relaxes the restrictions in option 1. By introducing γ_{sf} and δ_{sf}, it allows for the asymmetries in the covariance to be modelled separately. This model also allows for positive and negative covariances.

$$\begin{pmatrix} \sigma_{s,t}^{\delta_s} \\ \sigma_{sf,t} \\ \sigma_{f,t}^{\delta_f} \end{pmatrix} = \begin{pmatrix} \frac{\omega_s}{1-\beta_s} + \left(1 - \frac{(1-\phi_s L)(1-L)^{d_s}}{1-\beta_s L}\right)\left(\left|\varepsilon_{s,t}\right| - \gamma_s \varepsilon_{s,t}\right)^{\delta_s} \\ \frac{\omega_{sf}}{1-\beta_{sf}} + \left(1 - \frac{(1-\phi_{sf} L)(1-L)^{d_{sf}}}{1-\beta_{sf} L}\right)\left\{\left(\varepsilon_{s,t}\varepsilon_{f,t}\right) - \gamma_{sf}\left|\varepsilon_{s,t}\varepsilon_{f,t}\right|\right\} \\ \frac{\omega_f}{1-\beta_f} + \left(1 - \frac{(1-\phi_f L)(1-L)^{d_f}}{1-\beta_f L}\right)\left(\left|\varepsilon_{f,t}\right| - \gamma_f \varepsilon_{f,t}\right)^{\delta_f} \end{pmatrix} \tag{14}$$

If the estimates of d in the diagonal FIGARCH models are found to be close, a third specification imposes the restriction of a common d. We refer to this as the common long range dependence model (CLRDM).

2.3 Previous evidence supporting long memory in equity market volatility

Long memory in volatility has been documented across a range of equity indices; the S&P500 (Ding *et al*, 1993; Bollerslev and Mikkelsen, 1996; Ding and Granger, 1996; Granger and Ding, 1996; Andersen and Bollerslev, 1997b; Lobato and Savin, 1998; Liu, 2000), the NYSE (Ding *et al*, 1993), the Nikkei (Ding and Granger, 1996), the CRSP (Breidt *et al*, 1998), the DAX (Ding *et al*, 1993) and the Australian All Ordinaries and its SPI futures (Dark, 2003). Long memory has also been documented in currency market volatility, including; the Deutschemark-U.S.$ (Dacorogna *et al*, 1993; Baillie *et al*, 1996; Ding and Granger, 1996; Andersen and Bollerslev, 1997a;

1997b; 1998) and the British pound-U.S.$ (Giriatis *et al*, 2001). See Dark (2003) for a more detailed review of this literature.

Subsequent to the pioneering work of Teyssiere (1997), very little has been done on the estimation of multivariate FIGARCH processes. The evidence which examines multivariate long memory processes is summarised in Table 1 and supports the development of bivariate FIGARCH models. No attempt has been made to estimate a bivariate FIGARCH or FIAPARCH model between a spot and futures market. Furthermore there has been no attempt to estimate a bivariate FIGARCH process with an error correction specification for the conditional mean. It is this gap in the literature which partially motivates this paper.

(Insert Table 1)

The evidence in Table 1 suggests that the assumption of constant correlation may be unreasonable. This is supported by the evidence against constant correlation between equity markets (Tse, 2000; Ramchand and Susmel, 1998) and the indices within those markets (Engle and Sheppard, 2001). Secondly, the research suggests that the CLRDM should be considered, given that estimates of d tend to be very similar amongst assets of the same class. The research above however fails to formally test for a common value of d, relying on information criteria to assess the most appropriate model. We will use this procedure along with the parameter stability test of Nyblom (1989), to determine the most appropriate models.

3. Estimation of the Bivariate Error Correction FIGARCH and FIAPARCH processes

3.1 Estimation

The QMLE procedure is used to maximize the log likelihood function derived by assuming conditional normality

$$L(\theta) = N\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{N}\left(\ln\left|H_t\right| + \varepsilon_t H_t^{-1}\varepsilon_t\right) \tag{15}$$

where $L(\theta)$ is the log of the likelihood function with respect to the parameter vector θ, H_t is the conditional covariance matrix and N is the sample size.

Estimation of the bivariate GARCH, FIGARCH and FIAPARCH processes requires inequality constrained optimisation procedures, given that the parameter estimates need to be constrained: i) to be within lower and upper bounds; ii) to impose non-negative conditional variance estimates and iii) to ensure positive definiteness of H_t. We employ the Feasible Sequential Quadratic Programming (SQPF) algorithm of Lawrence and Tits (2001) which allows one to place lower and upper bounds on the parameters and maximizes with respect to a matrix of inequality constraints.

We estimate the bivariate FIGARCH/FIAPARCH models by extending the procedure developed in Baillie *et al* (1996). The pre-sample values are set equal to the unconditional variance estimate with a truncation lag of 1000 observations used. We use numerical derivatives and impose non-negativity via the univariate sufficient but not necessary conditions in Bollerslev and Mikkelsen (1996).[4] Positive definiteness for the constant correlation model is obtained via the restriction $-1 < \rho < 1$. For the diagonal and CLRD models, an additional set of sufficient but not necessary conditions on the covariance parameters are imposed. Positive definiteness for the diagonal and CLRD models is imposed numerically given that analytical conditions have proven elusive. Different starting values are employed to ensure that the global maximum is found. Table 2 summarises the procedures employed.[5]

3.2 Model Estimator Properties

Baillie *et al* (1996) suggest that FIGARCH estimation using QMLE will produce consistent and asymptotically normal estimates. Caution is necessary given that the asymptotic properties of the estimates for the FIGARCH model are not well established (Davidson, 2002). Furthermore, the impact of violations in conditional normality are unknown. Assuming that the conditional mean and variance are correctly specified, GARCH estimates are consistent but asymptotically inefficient, with the degree of inefficiency increasing with the degree of departure from normality (Engle and Gonzalez-Rivera, 1991). We speculate that FIGARCH/FIAPARCH estimates behave similarly.

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[4] Sufficient but not necessary conditions for non-negativity have also been derived by Baillie *et al* (1996). The model results were insensitive to the conditions employed.
[5] Positive definiteness in the diagonal GARCH model is imposed using the constraints in Silberberg and Pafka (2003). The GARCH process is initialised by setting the unobservables equal to the unconditional variance.

To date our understanding of the QMLE procedure has come from Monte Carlo simulation. Baillie *et al* (1996) demonstrate the suitability of the QMLE procedure when estimating univariate FIGARCH processes with sample sizes of 1500 and 3000. Pafka and Matyas (2001) extend these results to the estimation of the multivariate CLRD model with sample sizes of 5000 and 10000. The QMLE procedures assuming conditional normality provide satisfactory estimates of the parameters, their standard errors and suitably adjust for conditional non-normality.

One however must exercise caution given that Monte Carlo simulations on models with more than one estimate of *d* (the constant correlation and diagonal specifications) have not been performed. There has also been no attempt to examine the QMLE procedure when estimating the bivariate FIAPARCH model. These issues are an important area for further research.

Lobato (1999) notes that one further drawback of the QMLE procedures is that they are inconsistent if the model is mis-specified. Any mis-specification of the short run dynamics will therefore result in inconsistent QMLE estimates. Spectral density estimates of *d* are unaffected by any of the higher frequency components which may effect estimates of *d* when estimating the FIGARCH and FIAPARCH processes using QMLE. The spectral density estimates can therefore be used to assess the reliability of the FIGARCH and FIAPARCH estimates of *d*.

4. Results

4.1 Data

We use daily data commencing on January 4, 1988 and ending July 30, 1999. Data on the index was obtained from IRESS, the futures was obtained from the Sydney Futures Exchange WWW site (http://www.sfe.com.au). We create continuously compounded returns (R_t) as the difference between the log of consecutive prices multiplied by 100. Only those days were included where trading occurred in both markets. We use the nearby futures contract with rollover being performed 10 trading days prior to expiration. We also remove three outliers on October, 16, 1989; October, 28, 1997 and January, 11, 1988.[6] The data set is therefore identical to the daily data set employed in Dark (2003).

4.2 Testing for Cointegration and Long Memory in the Covariance

In this section we test for unit roots, cointegration and long memory in the covariance between the two markets. This section therefore supplements Dark (2003), where the presence of long memory in the volatility of both markets is established.

On October 11, 1993 the SFE reduced the multiplier on the SPI futures contract from \$100 to \$25, reduced the margin from \$6,000 to \$1,500 and removed the exchange fees of \$1.17 per contract. Bhar (2001) documents a structural break on this date. We

[6] The removal of these observations had a minor impact on the results and does not effect the conclusions drawn.

therefore follow Bhar (2001) who makes allowance for the presence of the futures contract re-specification when testing for unit roots and cointegration. Zivot and Andrews (1992) use a modified augmented Dickey-Fuller (ADF) test, to test for the presence of a unit root when the series is subject to a structural break. Our results are consistent with Bhar (2001) who finds that both price series are I(1). Cointegration is examined using a modified Engle and Granger (1987) approach implemented in Gregory and Hansen (1996). Here the following regression is estimated

$$\ln S_t = \alpha + \beta t + \gamma D_t + \theta \ln F_t + \varepsilon_t \qquad (16)$$

where t represents a time trend and D_t represents a dummy variable equal to 0 for all observations up to and including October 11, 1993, otherwise equal to 1. We test ε_t for stationarity using the conventional ADF test. Our results are again consistent with Bhar (2001) who finds that the series are cointegrated. We therefore use the error from Equation 16 as the error correction term in Equation 6.[7]

To investigate the presence of long memory in the covariance, we consider the modified R/S test of Lo (1991), the KPSS test of Kwiatkowski *et al* (1992), the spectral density estimate of *d* and the fit of the implied autocorrelation function. The modified R/S and KPSS test statistics along with the spectral density estimates are presented in Tables 3 and 4 and indicate that the covariance exhibits long memory.[8]

(Insert Tables 3 and 4)

[7] Given that we multiplied the log differences by 100 to create returns, we also multiply the cointegrating residual by 100.
[8] The tests and spectral density estimates were performed using Long Memory Modelling version 2.

The estimates of *d* are low relative to that commonly observed in financial markets (usually between 0.3 to 0.4). Nonetheless the fit of the implied autocorrelation function is reasonable and supportive of long memory in the covariance.[9] These estimates are also quite close, suggesting that the CLRD model may be appropriate.

(Insert Figure 1)

In summary, the index and its futures are cointegrated with both volatility processes and their covariance appearing to exhibit long memory. These results support the estimation of the error correction FIGARCH and FIAPARCH models in the next section.

4.3 Bivariate Error Correction GARCH, FIGARCH and FIAPARCH models

This section discusses the results from the estimation of the bivariate error correction GARCH, FIGARCH and FIAPARCH models. Tables 5 to 8 present the best GARCH, FIGARCH and FIAPARCH models. All models are presented in the Appendix.

All models display non-normality in the standardized residuals along with parameter instability in the spot conditional mean equations. The conditional mean estimates are insensitive to the specification of the conditional variance. The index adjusts to deviations from long run equilibrium, with the sign, magnitude and significance of the error correction term being close to the result presented in Bhar (2001). In contrast to

[9] The fit of the implied autocorrelation function is performed using the asymptotic approximation for fractional white noise. See Baillie (1996) for details.

Bhar (2001), the futures do not adjust to deviations from long run equilibrium. Our result appears robust given its insensitivity to changes in the number of lags in the error correction model, and its consistency with Lien and Tse (1998, 1999). The short run return dynamics indicate that significant bi-directional causality exists. Each market responds negatively to its own lagged returns and positively to the lagged returns in the other market. This result is consistent with Chatrah and Song (1998), Sim and Zurbreugg (1999) and Lien and Luo (1994).

Tables 9 to 11 in the Appendix present the results of the constant correlation and diagonal GARCH models. The diagonal specification is clearly superior to the constant correlation model. The instability in ρ coupled with the higher information criteria, suggests that the constant correlation assumption is inappropriate. There are however a number of issues surrounding the diagonal GARCH model. First, whilst the instability in the futures and covariance equations are reduced when moving to the diagonal specification, parameter instability is still significant. Second, the Engle and Ng (1993) diagnostic tests reveal significant sign and negative size bias. Third, the value of $\alpha + \beta$ in the GARCH models for both markets suggests near IGARCH behaviour. Baillie *et al* (1996) via Monte Carlo simulation demonstrate that if confined to the conventional GARCH and IGARCH paradigm, a FIGARCH process may easily be mistaken for an IGARCH process. These results suggest that the dynamics may be mis-specified, and support the estimation of more complicated covariance structures like the FIGARCH and FIAPARCH processes.

Tables 12 to 15 in the Appendix present the results of the bivariate FIGARCH estimation. The estimates of d are close to the spectral density estimates and are statistically significant for all models. Like the bivariate GARCH models, the constant correlation model is clearly the worst model. It has the highest information criteria and suffers from parameter instability, particularly in ρ. The CLRDM results appear satisfactory with the likelihood ratio test accepting this restriction at a 5% significance level ($\chi^2_{(3)} = 4.08$). The estimate of d however is unstable suggesting that the specification is overly restrictive. Furthermore, this restriction sometimes created convergence problems. On a number of occasions, different starting values provided local maxima that were clearly inferior to the results presented.

The best model within the FIGARCH class of processes is the diagonal FIGARCH model. This specification provides the best estimates of d (given their proximity to the spectral density estimates), has the lowest information criteria, is the only model with stable ARCH parameters and is insensitive to different starting values. The diagnostics in Tables 13 to 15 however indicate that all FIGARCH models suffer from sign and negative size bias, supporting the estimation of the FIAPARCH model.

Tables 16 to 19 present the bivariate FIAPARCH models. The estimates of d are similar in value and significant across the alternative specifications. They are also relatively close to the spectral density and FIGARCH estimates. The diagonal FIAPARCH model option 2 found γ_{sf} to be insignificant. This is consistent with the Engle and Ng (1993) diagnostics in the GARCH and FIGARCH equations, which found no evidence of asymmetries in the covariance.

Based on information criteria and evidence of parameter instability, the constant correlation model is again the worst performing model. The diagonal FIAPARCH models estimates are very similar. Option 1 is chosen as the best model given that it captures asymmetries better than the diagonal FIAPARCH Option 2 model. The performance of these models with respect to capturing asymmetries can be explained by the γ_s and γ_f parameters. As one moves from the most restricted model (the constant correlation model) to the least restricted model (option 2) the value of these parameters and their significance decreases (to a point where γ_s is insignificant). This result is inappropriate given that asymmetries are introduced as the values of γ_s and γ_f decreased.

Tables 5 to 8 summarise the best bivariate GARCH, FIGARCH and FIAPARCH models. The GARCH model is the worst model given that it has the highest information criteria, is inconsistent with the findings of long memory and suffers from parameter instability. The diagonal FIAPARCH model is chosen as the best model. Despite being more sensitive to starting values, the FIAPARCH model has the lowest information criteria and it captures asymmetries better than the FIGARCH model. Nonetheless the presence of asymmetries in the FIAPARCH model suggests that alternative methods of capturing asymmetries is an area for further research.

(Insert Tables 5 to 8)

5. Concluding Remarks

This paper has extended the univariate FIGARCH and FIAPARCH processes to a bivariate setting. We estimated bivariate error correction FIGARCH and FIAPARCH models between the Australian All Ordinaries Index and its SPI futures using constant correlation and diagonal parameterisations. The results strongly support the use of these models, with long memory being present in the spot, futures and their covariance. Our results support strong bi-directional causality between the two markets, with the index bearing the burden of adjustment to deviations from long run equilibrium. The results also support the existence of asymmetries in volatility and the use of models that allow for time varying correlations.

Table 1 Multivariate analysis of long memory in volatility

Reference	Data	Data frequency	Comments
Teysierre (1997)	DM/USD BP/USD	Daily	Bivariate FIGARCH estimation. CLRDM dominates diagonal model, which dominates constant correlation model.
Teysierre (1998)	USD/DM USD/GBP USD/JPY	30 minute returns	Trivariate FIGARCH estimation. Conditional variances and covariances have the same d. Diagonal FIGARCH dominates constant correlation FIGARCH.
Brunetti and Gilbert (2000)	Spot Crude oil on NYMEX and IPE	Daily	Bivariate FIGARCH estimation assuming constant correlation.
Pafka and Matyas (2001)	DM/USD BP/USD JPY/USD	Daily	Trivariate FIGARCH estimation. CLRDM dominates diagonal FIGARCH.

Table 2 Approach used to estimate bivariate FIGARCH/FIAPARCH models

Constant Correlation			Diagonal and CLRD		
Parameter restrictions	Non negativity	Positive definiteness	Parameter restrictions	Non negativity	Positive definiteness
Imposed via algorithm	Sufficient but not necessary conditions	Imposed via parameter restriction $-1<\rho<1$	Imposed via algorithm	Sufficient but not necessary conditions	Numerically

Table 3 Testing for long memory in covariance

Test	$R_{s,t}R_{f,t}$		$\left|R_{s,t}R_{f,t}\right|$	
	Statistic	Conclusion	Statistic	Conclusion
Lo's R/S	2.1145	Long memory	2.2082	Long memory
KPSS	0.5060	Long memory	0.6145	Long memory

Significance level of 5% - Critical values – R/S = 1.747, KPSS = 0.463.

Table 4 Spectral density estimates of d

	$R^2_{s,t}$	$R^2_{f,t}$	$R_{s,t}R_{f,t}$	$\lvert R_{s,t} \rvert$	$\lvert R_{f,t} \rvert$	$\lvert R_{s,t}R_{f,t} \rvert$
Spectral estimate	0.2081	0.2565	0.2152	0.2601	0.2544	0.2199

Spectral density estimate obtained using procedure of Robinson (1994)

Table 5 Summary of best GARCH, FIGARCH and FIAPARCH models

	Diagonal GARCH(1,1) Equation 9		Diagonal FIGARCH(1,d,1) Equation 11		Diag FIAPARCH(1,d,1) Option 1 – Equation 13	
	Coeff't	Nyblom	Coeff't	Nyblom	Coeff't	Nyblom
Mean						
Index						
a_1	0.04 (2.52)	0.34	0.03 (2.33)	0.34	0.02 (2.09)	0.31
b_1	-0.07 (-8.75)	2.01**	-0.06 (-8.26)	1.78**	-0.06 (-8.43)	1.75**
$c_{1,1}$	-0.17 (-8.81)	0.96**	-0.17 (-8.74)	1.25**	-0.18 (-8.61)	1.27**
$d_{1,1}$	0.22 (14.03)	0.71*	0.23 (14.37)	0.91**	0.23 (14.10)	0.81**
Futures						
a_2	0.04 (2.23)	0.28	0.04 (1.95)	0.29	0.02 (1.80)	0.25
$c_{2,2}$	0.13 (5.31)	0.22	0.14 (5.75)	0.18	0.13 (5.75)	0.23
$d_{2,2}$	-0.12 (-6.10)	0.10	-0.13 (-6.51)	0.07	-0.13 (-6.59)	0.11
$d_{2,3}$	-0.02 (-2.56)	0.15	-0.02 (-2.69)	0.14	-0.03 (-3.46)	0.16
Variance						
Index						
ω_s	0.02 (1.18)	0.14	0.08 (1.95)	0.08	0.09 (1.98)	0.17
d_s			0.19 (4.43)	0.09	0.19 (4.01)	0.06
ϕ_s #	0.04 (2.51)	0.08	0.45 (3.30)	0.20	0.38 (2.72)	0.08
β_s	0.93 (23.05)	0.13	0.59 (3.87)	0.30	0.52 (3.31)	0.24
γ_s					0.25 (2.39)	0.59*
δ_s					1.92 (17.11)	0.15
Futures						
ω_f	0.03 (1.12)	0.43	0.10 (2.31)	0.13	0.09 (2.21)	0.22
d_f			0.22 (6.21)	0.12	0.23 (5.35)	0.19
ϕ_f #	0.04 (2.34)	0.44	0.48 (4.67)	0.03	0.42 (3.83)	0.04
β_f	0.93 (24.17)	0.46	0.64 (5.57)	0.08	0.59 (4.67)	0.07
γ_f					0.24 (2.92)	0.19
δ_f					2.00 (23.03)	0.37
Covariance						
ω_{sf}	0.02 (1.07)	0.27	0.08 (2.31)	0.10	0.08 (2.11)	0.21
d_{sf}			0.20 (5.35)	0.05	0.20 (4.92)	0.07
ϕ_{sf} #	0.04 (2.44)	0.17	0.46 (4.49)	0.10	0.41 (3.72)	0.07
β_{sf}	0.93 (22.16)	0.24	0.62 (5.37)	0.22	0.56 (4.45)	0.20
LL function	-5278.01		-5251.01		-5234.65	
AIC	3.6168		3.6004		3.5920	
Schwarz	3.6515		3.6413		3.6410	
Shibata	3.6167		3.6003		3.5918	
Hann Quinn	3.6293		3.6151		3.6096	

Mean specification – equation 6.

QMLE t statistics are in parentheses. # = when reading the GARCH estimates ϕ is replaced by α.

Nyblom statistics-* = significant at 5% (critical value 0.47), ** = significant at 1% (critical value 0.75).

Table 6 Diagnostics - Diagonal GARCH

Test	Index	Futures	Covariance
Q(10)	0.82	0.83	0.57
Q(15)	0.37	0.31	0.84
Q(20)	0.41	0.22	0.97
Q2(10)	0.08	0.56	1.00
Q2(15)	0.28	0.74	1.00
Q2(20)	0.57	0.93	1.00
Sign bias	0.14	0.01	0.88
Negative size bias	0.00	0.00	0.72
Positive size bias	0.69	0.18	0.93
Joint test	0.04	0.01	0.99
Skewness	<0.001	<0.001	
Excess kurtosis	<0.001	<0.001	
Jarque-Bera	<0.001	<0.001	

Entries represent p values, Q(10) = Box Pierce statistic on ε_t / σ_t for 10 lags, Q2(10) is the statistic for $\varepsilon^2{}_t / \sigma^2{}_t$

Table 7 Diagnostics – Diagonal FIGARCH

Test	Index	Futures	Covariance
Q(10)	0.83	0.80	0.60
Q(15)	0.38	0.27	0.73
Q(20)	0.40	0.18	0.93
Q2(10)	0.16	0.70	1.00
Q2(15)	0.37	0.58	1.00
Q2(20)	0.69	0.81	1.00
Sign bias	0.03	0.02	0.25
Negative size bias	0.01	0.04	0.95
Positive size bias	0.71	0.05	0.69
Joint test	0.04	0.04	0.69
Skewness	<0.001	<0.001	<0.001
Excess kurtosis	<0.001	<0.001	<0.001
Jarque-Bera	<0.001	<0.001	<0.001

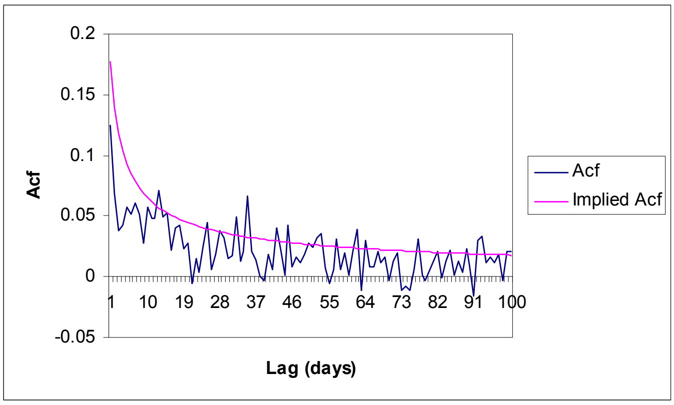
Entries represent p values, Q(10) = Box Pierce statistic on ε_t / σ_t for 10 lags, Q2(10) is the statistic for $\varepsilon^2_t / \sigma^2_t$

Table 8 Diagnostics – Diagonal FIAPARCH Option 1

Test	Index	Futures	Covariance
Q(10)	0.84	0.88	0.65
Q(15)	0.35	0.31	0.79
Q(20)	0.37	0.22	0.95
Q2(10)	0.20	0.72	1.00
Q2(15)	0.45	0.64	1.00
Q2(20)	0.76	0.84	1.00
Sign bias	0.08	0.06	0.23
Negative size bias	0.05	0.23	0.99
Positive size bias	0.73	0.25	0.68
Joint test	0.12	0.31	0.66
Skewness	<0.001	<0.001	<0.001
Excess kurtosis	<0.001	<0.001	<0.001
Jarque-Bera	<0.001	<0.001	<0.001

Entries represent p values, Q(10) = Box Pierce statistic on ε_t / σ_t for 10 lags, Q2(10) is the statistic for $\varepsilon^2_t / \sigma^2_t$

Figure 1 Fit of implied autocorrelation function, covariance



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Appendix – Estimation Results

Table 9 Bivariate GARCH

	Constant Correlation Equation 8		Diagonal Equation 9	
	Coefficient	Nyblom	Coefficient	Nyblom
Mean				
Index				
a_1	0.04 (2.51)	0.27	0.04 (2.52)	0.34
b_1	-0.07 (-9.26)	1.45**	-0.07 (-8.75)	2.01**
$c_{1,1}$	-0.17 (-8.31)	0.87**	-0.17 (-8.81)	0.96**
$d_{1,1}$	0.23 (13.66)	0.55*	0.22 (14.03)	0.71*
Futures				
a_2	0.05 (2.49)	0.21	0.04 (2.23)	0.28
$c_{2,2}$	0.11 (4.45)	0.27	0.13 (5.31)	0.22
$d_{2,2}$	-0.11 (-5.51)	0.10	-0.12 (-6.10)	0.10
$d_{2,3}$	-0.02 (-2.20)	0.11	-0.02 (-2.56)	0.15
Variance				
Index				
ω_s	0.03 (2.12)	0.09	0.02 (1.18)	0.14
α_s	0.05 (3.38)	0.13	0.04 (2.51)	0.08
β_s	0.91 (29.72)	0.10	0.93 (23.05)	0.13
Futures				
ω_f	0.03 (2.29)	1.59**	0.03 (1.12)	0.43
α_f	0.04 (3.62)	2.36**	0.04 (2.34)	0.44
β_f	0.94 (47.32)	2.23**	0.93 (24.17)	0.46
Covariance				
ω_{sf}			0.02 (1.07)	0.27
α_{sf}			0.04 (2.44)	0.17
β_{sf}			0.93 (22.16)	0.24
ρ	0.90 (188.93)	2.50**		
LL function	-5302.86		-5278.01	
AIC	3.6324		3.6168	
Schwarz	3.6631		3.6515	
Shibata	3.6324		3.6167	
Hann Quinn	3.6435		3.6293	

Mean specification – equation 6
QMLE t statistics are in parentheses. Nyblom statistics - * = significant at 5% (critical value 0.47)
** = significant at 1% (critical value 0.75).

Table 10 Diagnostics – Constant Correlation GARCH

Test	Index	Futures	Covariance
Q(10)	0.86	0.87	0.70
Q(15)	0.42	0.33	0.92
Q(20)	0.46	0.24	0.99
Q2(10)	0.27	0.61	1.00
Q2(15)	0.63	0.81	1.00
Q2(20)	0.87	0.95	1.00
Sign bias	0.02	0.01	0.26
Negative size bias	0.01	0.00	0.98
Positive size bias	0.59	0.11	0.71
Joint test	0.04	0.01	0.71
Skewness	<0.001	<0.001	<0.001
Excess kurtosis	<0.001	<0.001	<0.001
Jarque-Bera	<0.001	<0.001	<0.001

Entries represent p values, Q(10) = Box Pierce statistic on ε_t / σ_t for 10 lags, Q2(10) is the statistic for $\varepsilon^2_t / \sigma^2_t$

Table 11 Diagnostics – Diagonal GARCH

Test	Index	Futures	Covariance
Q(10)	0.82	0.83	0.57
Q(15)	0.37	0.31	0.84
Q(20)	0.41	0.22	0.97
Q2(10)	0.08	0.56	1.00
Q2(15)	0.28	0.74	1.00
Q2(20)	0.57	0.93	1.00
Sign bias	0.14	0.01	0.88
Negative size bias	0.00	0.00	0.72
Positive size bias	0.69	0.18	0.93
Joint test	0.04	0.01	0.99
Skewness	<0.001	<0.001	<0.001
Excess kurtosis	<0.001	<0.001	<0.001
Jarque-Bera	<0.001	<0.001	<0.001

Entries represent p values, Q(10) = Box Pierce statistic on ε_t / σ_t for 10 lags, Q2(10) is the statistic for $\varepsilon^2_t / \sigma^2_t$

Table 12 Bivariate FIGARCH

Coefficient	Constant Correlation Equation 10		Diagonal Equation 11		CLRD	
	Coeff't	Nyblom	Coeff't	Nyblom	Coeff't	Nyblom
Mean						
Index						
a_1	0.03 (2.42)	0.29	0.03 (2.33)	0.34	0.03 (2.33)	0.32
b_1	-0.07 (-9.41)	1.28**	-0.06 (-8.26)	1.78**	-0.06 (-8.29)	1.91**
$c_{1,1}$	-0.17 (-8.15)	0.84**	-0.17 (-8.74)	1.25**	-0.17 (-8.75)	1.27**
$d_{1,1}$	0.23 (13.64)	0.51*	0.23 (14.37)	0.91**	0.23 (14.32)	0.94**
Futures						
a_2	0.04 (2.41)	0.22	0.04 (1.95)	0.29	0.04 (2.00)	0.28
$c_{2,2}$	0.11 (4.31)	0.24	0.14 (5.75)	0.18	0.13 (5.61)	0.18
$d_{2,2}$	-0.11 (-5.25)	0.08	-0.13 (-6.51)	0.07	-0.13 (-6.29)	0.08
$d_{2,3}$	-0.02 (-2.06)	0.12	-0.02 (-2.69)	0.14	-0.02 (-2.67)	0.14
Variance						
Index						
ω_s	0.19 (5.71)	0.09	0.08 (1.95)	0.08	0.06 (2.90)	0.12
d_s	0.16 (7.57)	0.08	0.19 (4.43)	0.09	0.22 (6.88)	0.52*
ϕ_s			0.45 (3.30)	0.20	0.46 (5.50)	0.24
β_s	0.09 (3.57)	0.12	0.59 (3.87)	0.30	0.63 (7.17)	0.38
Futures						
ω_f	0.25 (5.91)	1.07**	0.10 (2.31)	0.13	0.10 (2.79)	0.20
d_f	0.19 (9.14)	1.58**	0.22 (6.21)	0.12		
ϕ_f			0.48 (4.67)	0.03	0.48 (6.04)	0.03
β_f	0.13 (4.23)	0.15	0.64 (5.57)	0.08	0.63 (7.13)	0.10
Covariance						
ω_{sf}			0.08 (2.31)	0.10	0.07 (2.95)	0.16
d_{sf}			0.20 (5.35)	0.05		
ϕ_{sf}			0.46 (4.49)	0.10	0.47 (6.41)	0.12
β_{sf}			0.62 (5.37)	0.22	0.63 (7.99)	0.28
ρ	0.90 (196.72)	2.43**				
LL function	-5293.46		-5251.01		-5253.11	
AIC	3.6260		3.6004		3.6005	
Schwarz	3.6566		3.6413		3.6373	
Shibata	3.6259		3.6003		3.6004	
Hann Quinn	3.6370		3.6151		3.6137	

Mean specification – equation 6, QMLE t statistics are in parentheses. Nyblom statistics - * = significant at 5% (critical value 0.47), ** = significant at 1% (critical value 0.75).

Table 13 Diagnostics – Constant Correlation FIGARCH

Test	Index	Futures	Covariance
Q(10)	0.85	0.85	0.88
Q(15)	0.38	0.34	0.91
Q(20)	0.42	0.24	0.98
Q2(10)	0.71	0.85	1.00
Q2(15)	0.86	0.69	1.00
Q2(20)	0.97	0.85	1.00
Sign bias	0.03	0.01	0.24
Negative size bias	0.07	0.03	0.98
Positive size bias	0.34	0.04	0.51
Joint test	0.15	0.03	0.64
Skewness	<0.001	<0.001	<0.001
Excess kurtosis	<0.001	<0.001	<0.001
Jarque-Bera	<0.001	<0.001	<0.001

Entries represent p values, Q(10) = Box Pierce statistic on ε_t / σ_t for 10 lags, $Q_2(10)$ is the statistic for $\varepsilon^2_t / \sigma^2_t$

Table 14 Diagnostics – Diagonal FIGARCH

Test	Index	Futures	Covariance
Q(10)	0.83	0.80	0.60
Q(15)	0.38	0.27	0.73
Q(20)	0.40	0.18	0.93
Q2(10)	0.16	0.70	1.00
Q2(15)	0.37	0.58	1.00
Q2(20)	0.69	0.81	1.00
Sign bias	0.03	0.02	0.25
Negative size bias	0.01	0.04	0.95
Positive size bias	0.71	0.05	0.69
Joint test	0.04	0.04	0.69
Skewness	<0.001	<0.001	<0.001
Excess kurtosis	<0.001	<0.001	<0.001
Jarque-Bera	<0.001	<0.001	<0.001

Entries represent p values, Q(10) = Box Pierce statistic on ε_t / σ_t for 10 lags, Q2(10) is the statistic for $\varepsilon^2_t / \sigma^2_t$

Table 15 Diagnostics – CLRDM

Test	Index	Futures	Covariance
Q(10)	0.84	0.80	0.57
Q(15)	0.39	0.26	0.74
Q(20)	0.42	0.18	0.93
Q2(10)	0.16	0.70	1.00
Q2(15)	0.41	0.58	1.00
Q2(20)	0.72	0.80	1.00
Sign bias	0.03	0.01	0.25
Negative size bias	0.01	0.03	0.95
Positive size bias	0.62	0.05	0.66
Joint test	0.06	0.04	0.69
Skewness	<0.001	<0.001	<0.001
Excess kurtosis	<0.001	<0.001	<0.001
Jarque-Bera	<0.001	<0.001	<0.001

Entries represent p values, Q(10) = Box Pierce statistic on ε_t / σ_t for 10 lags, Q2(10) is the statistic for $\varepsilon^2_t / \sigma^2_t$

Table 16 Bivariate FIAPARCH

	Constant Correlation Equation 12		Diagonal-Option1 Equation 13		Diagonal-Option2 Equation 14	
	Coeff't	Nyblom	Coeff't	Nyblom	Coeff't	Nyblom
Mean						
Index						
a_1	0.02 (1.23)	0.25	0.02 (2.09)	0.31	0.03 (2.28)	0.32
b_1	-0.07 (-9.50)	1.59**	-0.06 (-8.43)	1.75**	-0.06 (-8.23)	1.67**
$c_{1,1}$	-0.17 (-8.20)	0.95**	-0.18 (-8.61)	1.27**	-0.17 (-8.74)	1.15**
$d_{1,1}$	0.23 (13.76)	0.55*	0.23 (14.10)	0.81**	0.23 (14.19)	0.83**
Futures						
a_2	0.02 (1.05)	0.60*	0.02 (1.80)	0.25	0.03 (1.76)	0.31
$c_{2,2}$	0.11 (4.61)	0.28	0.13 (5.75)	0.23	0.14 (5.81)	0.17
$d_{2,2}$	-0.12 (-5.70)	0.10	-0.13 (-6.59)	0.11	-0.13 (-6.63)	0.07
$d_{2,3}$	-0.02 (-2.09)	0.12	-0.03 (-3.46)	0.16	-0.02 (-2.60)	0.13
Variance						
Index						
ω_s	0.15 (2.29)	0.07	0.09 (1.98)	0.17	0.11 (1.09)	0.13
d_s	0.20 (5.23)	0.08	0.19 (4.01)	0.06	0.19 (3.71)	0.05
ϕ_s	0.25 (1.68)	0.04	0.38 (2.72)	0.08	0.38 (1.31)	0.12
β_s	0.38 (2.37)	0.07	0.52 (3.31)	0.24	0.52 (1.63)	0.28
γ_s	0.26 (2.90)	0.08	0.25 (2.39)	0.59*		
δ_s	1.62 (5.70)	0.08	1.92 (17.11)	0.15	1.81 (8.74)	0.14
Futures						
ω_f	0.11 (2.80)	0.95**	0.09 (2.21)	0.22	0.12 (1.37)	0.20
d_f	0.26 (5.45)	1.22**	0.23 (5.35)	0.19	0.21 (3.30)	0.22
ϕ_f	0.36 (3.90)	0.23	0.42 (3.83)	0.04	0.43 (2.12)	0.04
β_f	0.58 (5.29)	0.06	0.59 (4.67)	0.07	0.58 (2.36)	0.07
γ_f	0.34 (3.40)	0.67*	0.24 (2.92)	0.19	0.03 (1.55)	0.08
δ_f	1.72 (8.04)	1.78**	2.00 (23.03)	0.37	2.00 (14.32)	0.32
Covariance						
ω_{sf}			0.08 (2.11)	0.21	0.10 (1.26)	0.17
d_{sf}			0.20 (4.92)	0.07	0.19 (3.47)	0.07
ϕ_{sf}			0.41 (3.72)	0.07	0.41 (1.73)	0.05
β_{sf}			0.56 (4.45)	0.20	0.54 (2.02)	0.18
δ_{sf}						
ρ	0.90 (192.02)	2.15**				
LL function	-5270.49		-5234.65		-5245.99	
AIC	3.6144		3.5920		3.5990	
Schwarz	3.6573		3.6410		3.6460	
Shibata	3.6143		3.5918		3.5989	
Hann Quinn	3.6299		3.6096		3.6160	

Mean specification – equation 6
QMLE t statistics are in parentheses. Nyblom statistics - * = significant at 5% (critical value 0.47)
** = significant at 1% (critical value 0.75).

Table 17 Diagnostics – Constant Correlation FIAPARCH

Test	Index	Futures	Covariance
Q(10)	0.84	0.88	0.77
Q(15)	0.35	0.31	0.86
Q(20)	0.38	0.22	0.97
Q2(10)	0.51	0.68	1.00
Q2(15)	0.74	0.59	1.00
Q2(20)	0.94	0.81	1.00
Sign bias	0.09	0.05	0.25
Negative size bias	0.24	0.22	0.98
Positive size bias	0.96	0.33	0.59
Joint test	0.27	0.28	0.67
Skewness	<0.001	<0.001	<0.001
Excess kurtosis	<0.001	<0.001	<0.001
Jarque-Bera	<0.001	<0.001	<0.001

Entries represent p values, Q(10) = Box Pierce statistic on ε_t / σ_t for 10 lags, Q2(10) is the statistic for $\varepsilon^2_t / \sigma^2_t$

Table 18 Diagnostics – Diagonal FIAPARCH Option 1

Test	Index	Futures	Covariance
Q(10)	0.84	0.88	0.65
Q(15)	0.35	0.31	0.79
Q(20)	0.37	0.22	0.95
Q2(10)	0.20	0.72	1.00
Q2(15)	0.45	0.64	1.00
Q2(20)	0.76	0.84	1.00
Sign bias	0.08	0.06	0.23
Negative size bias	0.05	0.23	0.99
Positive size bias	0.73	0.25	0.68
Joint test	0.12	0.31	0.66
Skewness	<0.001	<0.001	<0.001
Excess kurtosis	<0.001	<0.001	<0.001
Jarque-Bera	<0.001	<0.001	<0.001

Entries represent p values, Q(10) = Box Pierce statistic on ε_t / σ_t for 10 lags, Q2(10) is the statistic for $\varepsilon^2_t / \sigma^2_t$

Table 19 Diagnostics – Diagonal FIAPARCH Option 2

Test	Index	Futures	Covariance
Q(10)	0.84	0.81	0.70
Q(15)	0.38	0.28	0.79
Q(20)	0.40	0.19	0.95
Q2(10)	0.18	0.81	1.00
Q2(15)	0.40	0.65	1.00
Q2(20)	0.72	0.84	1.00
Sign bias	0.03	0.02	0.24
Negative size bias	0.01	0.04	0.95
Positive size bias	0.67	0.07	0.70
Joint test	0.04	0.06	0.69
Skewness	<0.001	<0.001	<0.001
Excess kurtosis	<0.001	<0.001	<0.001
Jarque-Bera	<0.001	<0.001	<0.001

Entries represent p values, Q(10) = Box Pierce statistic on ε_t / σ_t for 10 lags, Q2(10) is the statistic for $\varepsilon^2{}_t / \sigma^2{}_t$