ISSN 1440-771X

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February 2013

Working Paper 04/13

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Abstract

This paper builds a structural VARMA (SVARMA) model for investigating Canadian monetary policy. Despite the support for a VARMA model for monetary policy analysis, the traditional VAR and SVAR models have predominantly been used in the literature mainly due to difficulties associated with the identification and estimation of such a model. Using the scalar component model (SCM) proposed by Athanasopoulos and Vahid (2008a), this paper first identifies a VARMA model and then constructs a SVARMA model for Canadian monetary policy. We included the SVAR model in our study for a comparison purpose. Relative to this model, the impulse responses generated by the SVARMA model appear to be consistent with those predicted by various economic theoretical models, and solves the economic puzzles found commonly in the empirical literature on monetary policy. The successful construction and implementation of the SVARMA model for Canadian monetary policy analysis along with its promising impulse responses, indicate the suitability of this framework for small open economies.

Keywords: VARMA models, Identification, Impulse responses, Open economy, Transmission mechanism

JEL classification numbers: C32, E52, F41

1 Introduction

Over the past three decades, extensive investigations into modelling and analyzing monetary policies have led to the conclusion that differences in model specifications and parameter estimates across models can lead to widely different policy recommendations. In addition, the potential loss from basing monetary policy on an invalid model can be substantial. Since the seminal paper by Sims (1980), the use of vector autoregression (VAR) and structural VAR (SVAR) models has been prevalent in the empirical literature on monetary policy analysis. For an extensive review, see Leeper et al. (1996) and Christiano et al. (1999). Despite the sound theoretical and empirical justifications for the superiority of vector autoregressive moving average (VARMA) models over VAR-type models for policy modelling, the use of the former is still in its infancy; more on this later. The main reason for this is the lack of methodological advances in establishing uniquely identified VARMA models. Recently, however, Athanasopoulos and Vahid (2008a) proposed a complete methodology for identifying and estimating canonical VARMA models by extending the work of Tiao and Tsay (1989). They established necessary and sufficient conditions for exactly identifying a canonical VARMA model, so that all parameters can be efficiently identified and estimated simultaneously using full information maximum likelihood (FIML). Furthermore, Dufour and Pelletier (2002) illustrated that the VARMA representation is more appropriate for modeling monetary policy analysis than the VAR counterpart.

The main contribution of this paper is building a SVARMA model for Canadian monetary policy, and conducting an investigation into uncovering the underlying effects of the Bank of Canada's monetary policy in influencing the inflation rate and the output level in the economy, among others. These are accomplished by generating reliable dynamic impulse response functions. We apply the methodology of Athanasopoulos and Vahid (2008a) to examine the advantages of using VARMA models for the monetary policy framework of Canada - a small open economy. The aims of the paper are to: (i) adapt this new methodology to build a structural VARMA (SVARMA) framework for Canadian monetary policy analysis; (ii) compare the performance of the SVARMA framework with its SVAR counterpart in terms of impulse responses; and (iii) discover whether the new framework can resolve the economic puzzles commonly found in the empirical monetary literature of small open economies[1].

Monetary policy is widely implemented as a stabilization policy instrument for steering economies in the direction of achieving sustainable economic growth and price stability. The efficacy of monetary policy depends on the ability of policy makers to make an accurate assessment of the timing and effect of the policy on economic activities and prices. The Bank of Canada's policy instrument is the target rate it sets for the overnight interest rate, with the view to maintaining low and stable inflation. To achieve this objective, jointly with Canadian Government, the Bank adopted in 1991 an inflation targeting system that means to keep the annual inflation close to two per cent and within the range of one to three per cent. In such a low inflation environment, the spending, saving, investments and output are expected to increase which in turn lead to steadily increasing living standard in Canada; see, for example, Ragan (2007).

Although VARs provide useful tools for evaluating the effect of monetary policy shocks, there are ample warnings in the literature of their limitations on both theoretical and practical grounds. We shall discuss some of the justifications for the use of VARMA models over VARs provided in the recent literature. In studies of monetary policy, the dominant part of the analysis is based on the dynamics of impulse response functions of domestic variables to various monetary shocks; these impulse responses are derived using Wold's decomposition theorem. In a multivariate Wold representation, however, any covariance stationary time series can be transformed to an infinite order vector moving average (VMA(∞)) process of its innovations. A finite order VARMA model provide a

[1]The main economic puzzles are referred to as: (i) the liquidity puzzle (an unanticipated increase in money supply appears to be associated with increases rather than decreases in nominal interest rates); (ii) the price puzzle (an unanticipated tightening of monetary policy, which is identified with innovations in interest rates, is associated with an increase in the price level rather than a decrease); and (iii) the exchange rates puzzles (an unanticipated increase in interest rates is associated with a depreciation of the country's exchange rate relative to the US rather than an appreciation).

better approximation to the Wold representation than a finite order VAR, with the former producing more reliable impulse responses than the latter.

Several authors put forward several convincing arguments in support of VARMA processes over VARs for modelling macroeconomic variables[2]. In addition, economic and financial time series are invariably constructed data, involving for example seasonal adjustment, de-trending, temporal and contemporaneous aggregation. Such constructed time series would include moving average dynamics, even if their constituents were generated by pure autoregressive processes. Further, a subset of a system of variables that were generated by a vector autoregression would also follow a VARMA process[3].

To simplify the modelling and estimating a system of variables, applied researchers tend to approximate a VARMA process by a high-order VAR process. The use of VAR approximations requires models with extremely long lag lengths, much longer than those selected by typical information criteria such as the AIC or BIC, in order to describe a system adequately and to obtain reliable impulse responses[4]. However, in practice, the available sample sizes are inadequate to accommodate a sufficiently long lag structure, thus leading to poor approximations of the real business cycle models (see for example Chari et al., 2007). On the other hand, Dufour and Pelletier (2002) illustrate that the impulse-responses obtained from the more parsimonious VARMA representation are more precisely estimated than those from VAR counterparts, while Athanasopoulos and Vahid (2008b) show that VARMA models forecast macroeconomic variables more accurately than VARs. Then, via a simulation study, Athanasopoulos and Vahid (2008b) demonstrate that the superiority of the forecast comes from the presence of moving average components.

Despite the aforementioned justifications and recommendations to employ VARMA models rather than VARs, the use of the former is not prevalent in applied macroeconomics,

[2](See for example Zellner and Palm, 1974; Granger and Morris, 1976; Wallis, 1977; Maravall, 1993; Dufour and Pelletier, 2002; Lütkepohl, 2005; Fry and Pagan, 2005).

[3]Cooley and Dwyer (1998) claim that the basic real business cycle models follow VARMA processes. More recently, Fernández-Villaverde et al. (2005) demonstrated that linearized dynamic stochastic general equilibrium models in general imply a finite order VARMA structure.

[4]In a simulation study, Kapetanios et al. (2007) show that a sample size of 30,000 observations and a VAR of order 50 are required to sufficiently capture the dynamic effects of some of the economic shocks.

mainly due to difficulties in identifying a unique VARMA representation and its estimation. A search for an identified VARMA model is far more challenging than a simple VAR-type model specification, and the lack of enthusiasm for the use of VARMA models is due to these difficulties[5]. In this paper, we build a SVARMA model for Canadian monetary policy analysis in two stages: (i) we identify a VARMA model by implementing the methodology of Athanasopoulos and Vahid (2008a); and (ii) following Kim and Roubini (2000) and others, we impose a non-recursive structure on the contemporaneous matrix of SVARMA in order to identify the orthogonal policy and non-policy shocks. In light of the foregoing discussions on its suitability, we expect the SVARMA model to produce reliable dynamic impulse responses that are consistent with economic theoretical models and stylized facts, in comparison to the widely used SVAR model.

In our empirical modeling of Canadian monetary policy, we use the same seven variables as Kim and Roubini (2000) who estimated a SVAR model for Canadian monetary policy, among others. Of the seven variables, the world oil price index and the Federal funds rates represent the foreign variables, while the industrial production index, consumer price index, monetary aggregate M1, short-term interest rate and exchange rates represent the domestic variables. Studies by Kim and Roubini (2000) and Brischetto and Voss (1999) demonstrated that these seven variables are sufficient to describe the monetary policy framework of small open economies. In fact, they provided evidence that these seven variables can capture the features of large dimensional and more complex open economy models, such as that investigated by Cushman and Zha (1997). In a quest for solving empirical puzzles (which are mentioned in the footnote 1) found largely in the VAR framework, Kim and Roubini (2000) developed SVARs for modelling Canadian and other non-US G7 economies. However, their results indicate that, in the case for Canada, though the price puzzle did not exist, contrary to the expectation, a monetary tightening induced a brief increase in output instead of a fall. In this paper, the SVARMA based empirical results for the extended period of study show that there do not exit any of the

[5](see for example Hannan and Deistler, 1988; Tiao and Tsay, 1989; Reinsel, 1997; Lütkepohl, 2005).

empirical puzzles and the monetary shock reduced inflation, output and money supply in the Canadian economy, whereas under the SVAR framework, in response to some shocks, the dynamic movements of economic variables were not what were predicted by theoretical models.

The paper is organized as follows: Section 2 discusses briefly the VARMA methodology proposed by Athanasopoulos and Vahid (2008a). Section 3 describes the variables used in the models and their time series properties. Section 4 explains the impulse response functions. Section 5 demonstrates in detail the identification of orthogonal shocks to monetary policy related variables and estimation of the SVARMA model for Canadian monetary policy, and reports and analyzes the empirical results. Section 6 concludes the paper.

2 Identification of a VARMA model

For identifying and estimating a VARMA model, we use the Athanasopoulos and Vahid (2008a) extension of the Tiao and Tsay (1989) scalar component model (SCM) methodology. The aim of identifying scalar components is to examine whether there are any simplifying embedded structures underlying a VARMA(p, q) process.

For a given K dimensional VARMA(p, q) process

$$\boldsymbol{y}_t = \boldsymbol{\Phi}_1 \boldsymbol{y}_{t-1} + \ldots + \boldsymbol{\Phi}_p \boldsymbol{y}_{t-p} + \boldsymbol{v}_t - \boldsymbol{\Theta}_1 \boldsymbol{v}_{t-1} - \ldots - \boldsymbol{\Theta}_q \boldsymbol{v}_{t-q}, \tag{1}$$

a non-zero linear combination $z_t = \boldsymbol{\alpha}' \boldsymbol{y}_t$ follows a SCM(p_1, q_1) if $\boldsymbol{\alpha}$ satisfies the following properties:

$$\begin{aligned} \boldsymbol{\alpha}'\boldsymbol{\Phi}_{p_1} &\neq \mathbf{0}' \text{ where } 0 \leq p_1 \leq p, \\ \boldsymbol{\alpha}'\boldsymbol{\Phi}_l &= \mathbf{0}' \text{ for } l = p_1 + 1, \ldots, p, \\ \boldsymbol{\alpha}'\boldsymbol{\Theta}_{q_1} &\neq \mathbf{0}' \text{ where } 0 \leq q_1 \leq q, \\ \boldsymbol{\alpha}'\boldsymbol{\Theta}_l &= \mathbf{0}' \text{ for } l = q_1 + 1, \ldots, q. \end{aligned}$$

The scalar random variable z_t depends only on lags 1 to p_1 of all variables and lags 1 to q_1 of all innovations in the system. The determination of embedded scalar component models is achieved through a series of canonical correlation tests.

Denote the estimated squared canonical correlations between $\boldsymbol{Y}_{m,t} \equiv \left(\boldsymbol{y}_t', \ldots, \boldsymbol{y}_{t-m}'\right)$ and $\boldsymbol{Y}_{h,t-1-j} \equiv \left(\boldsymbol{y}_{t-1-j}', \ldots, \boldsymbol{y}_{t-1-j-h}'\right)'$ by $\widehat{\lambda}_1 < \widehat{\lambda}_2 < \ldots < \widehat{\lambda}_K$. As suggested by Tiao and Tsay (1989), the test statistic for at least s SCM(p_i, q_i), i.e., s insignificant canonical correlations, against the alternative of less than s scalar components is

$$C(s) = -(n-h-j)\sum_{i=1}^{s} \ln\left\{1 - \frac{\widehat{\lambda}_i}{d_i}\right\} \overset{a}{\sim} \chi^2_{s\times\{(h-m)K+s\}} \tag{2}$$

where d_i is a correction factor that accounts for the fact that the canonical variates could be moving averages of order j, and is calculated as follows:

$$d_i = 1 + 2\sum_{v=1}^{j} \widehat{\rho}_v\left(\widehat{\boldsymbol{r}}_i'\boldsymbol{Y}_{m,t}\right)\widehat{\rho}_v\left(\widehat{\boldsymbol{g}}_i'\boldsymbol{Y}_{h,t-1-j}\right), \tag{3}$$

where $\widehat{\rho}_v(.)$ is the v^{th} order autocorrelation of its argument and $\widehat{\boldsymbol{r}}_i'\boldsymbol{Y}_{m,t}$ and $\widehat{\boldsymbol{g}}_i'\boldsymbol{Y}_{h,t-1-j}$ are the canonical variates corresponding to the i^{th} canonical correlation between $\boldsymbol{Y}_{m,t}$ and $\boldsymbol{Y}_{h,t-1-j}$. Let $\boldsymbol{\Gamma}(m,h,j) = E(\boldsymbol{Y}_{h,t-1-j}\boldsymbol{Y}_{m,t}')$. This is a sub-matrix of the Hankel matrix of the autocovariance matrices of $\boldsymbol{y}_t$. Note that zero canonical correlations imply and are implied by $\boldsymbol{\Gamma}(m,h,j)$ having a zero eigenvalue.

In what follows, we provide a brief description of the complete VARMA methodology based on scalar components. For further details, refer to Athanasopoulos and Vahid (2008a) and Tiao and Tsay (1989).

Stage I: Identifying the scalar components

First, by strategically choosing $\boldsymbol{Y}_{m,t}$ and $\boldsymbol{Y}_{h,t-1-j}$, we identify the overall tentative order of the VARMA(p,q) by searching for $s+K$ components of order SCM(p,q), given that we have found s SCM$(p-\kappa, q-\mu)$ for $\{\kappa,\mu\} = \{0,1\}$ or $\{1,0\}$ or $\{1,1\}$. The process of exploring the various possibilities of underlying simplifying structures in the form of SCMs is a hierarchical one. Hence, the identification process begins by searching for K SCMs of the most parsimonious possibility, i.e. SCM$(0,0)$ (which is a white noise process), by testing for the rank of $\boldsymbol{\Gamma}(0,0,0) = E(\boldsymbol{Y}_{0,t-1}\boldsymbol{Y}_{0,t}')$, where $\boldsymbol{Y}_{m,t} = \boldsymbol{Y}_{0,t}$ and

$\boldsymbol{Y}_{h,t-1-j} = \boldsymbol{Y}_{0,t-1}$. If we do not find K linearly independent white noise scalar processes, we set $m = h$, and by incrementing m and j we search for the next set of K linearly independent scalar components. First, we search for first order "moving average" components by testing for the rank of $\boldsymbol{\Gamma}(0,0,1) = E(\boldsymbol{Y}_{0,t-2}\boldsymbol{Y}_{0,t}^{'})$, and then we search for the first order "autoregressive" components by testing for the rank of $\boldsymbol{\Gamma}(1,1,0) = E(\boldsymbol{Y}_{1,t-1}\boldsymbol{Y}_{1,t}^{'})$, and then $\boldsymbol{\Gamma}(1,1,1) = E(\boldsymbol{Y}_{1,t-2}\boldsymbol{Y}_{1,t}^{'})$ for SCM(1,1), and so on.

Conditional on the overall tentative order (p,q), we then repeat the search process, but this time searching for individual components. So, starting again from the most parsimonious SCM(0,0), we sequentially search for K linearly independent vectors $(\boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_K)$ for $m = 0, \ldots, p$, $j = 0, \ldots, q$ and $h = m + (q - j)$. As for a tentative order of (p,q), each series is serially uncorrelated after lag q.

The test results from first identifying the overall tentative order and then the individual SCMs are tabulated in what are referred to as *Criterion* and *Root* tables. Reading from the Criterion table allows us to identify the overall tentative order of the model, while reading from the Root table allows us to identify the individual orders of the scalar components. Since an $SCM(m,j)$ nests all scalar components of order $(\leq m, \leq j)$, for every one $SCM(p_1 < p, q_1 < q)$ there will be $s = \min\{m - p_1 + 1, j - q_1 + 1\}$ zero canonical correlations at position $(m \geq p_1, j \geq q_1)$. Therefore, for every increment above s, a new $SCM(m,j)$ is found. We demonstrate the reading of these tables in Section (5). For a complete exposition of how to read from these tables and recognize the patterns of zeros, as well as for further details on the sequence of testing, see Athanasopoulos and Vahid (2008a).

Suppose that we have identified K linearly independent scalar components characterized by the transformation matrix $\boldsymbol{A}_0 = (\boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_K)'$. If we rotate the system in (1) by $\boldsymbol{A}_0$, we obtain

$$\boldsymbol{A}_0\boldsymbol{y}_t = \boldsymbol{A}_1\boldsymbol{y}_{t-1} + \ldots + \boldsymbol{A}_p\boldsymbol{y}_{t-p} + \boldsymbol{\eta}_t - \boldsymbol{M}_1\boldsymbol{\eta}_{t-1} - \ldots - \boldsymbol{M}_q\boldsymbol{\eta}_{t-q}, \tag{4}$$

where $\boldsymbol{A}_i = \boldsymbol{A}_0\boldsymbol{\Phi}_i$, $\boldsymbol{\eta}_t = \boldsymbol{A}_0\boldsymbol{\upsilon}_t$ and $\boldsymbol{M}_i = \boldsymbol{A}_0\boldsymbol{\Theta}_i\boldsymbol{A}_0^{-1}$. This rotated model incorporates whole rows of zero restrictions in the AR and MA parameter matrices on the RHS, as each row represents one identified SCM(p_i, q_i). However, we should note that obtaining the orders of SCMs does not necessarily lead to a uniquely identified system. For example, if two scalar components were identified such that $z_{r,t} = SCM\,(p_r, q_r)$ and $z_{s,t} = SCM\,(p_s, q_s)$, where $p_r > p_s$ and $q_r > q_s$, the system will not be identified. To obtain an identified system, we need to set $\min\{p_r - p_s, q_r - q_s\}$, i.e. set either the autoregressive or moving average parameters to be zero. This process is known as the "general rule of elimination", and in order to identify a canonical VARMA model as defined by Athanasopoulos and Vahid (2008a), we set the moving average parameters to zero.

Stage II: Imposing identification restrictions on matrix $\boldsymbol{A}_0$

Athanasopoulos and Vahid (2008a) recognized that some of the parameters in $\boldsymbol{A}_0$ are redundant and can be eliminated. This stage mainly outlines this process, and a brief description of the rules of placing restrictions on the redundant parameters is as follows:

1. Given that each row of the transformation matrix $\boldsymbol{A}_0$ can be multiplied by a constant without changing the structure of the model, one parameter in each row can be normalized to one. However, there is a danger of normalizing the wrong parameter, i.e. a zero parameter might be normalized to one. To overcome this problem, we add tests of predictability using subsets of variables. Starting from the SCM with the smallest order (the SCM with minimum $p+q$), exclude one variable, say the K^{th} variable, and test whether a SCM of the same order can be found using the $K-1$ variables alone. If the test is rejected, the coefficient of the K^{th} variable is then normalised to one, and the corresponding coefficients in all other SCMs that nest this one are set to zero. If the test concludes that the SCM can be formed using the first $K-1$ variables only, the coefficient of the K^{th} variable in this SCM is zero, and should not be normalised to one. It is worth noting that if the order of this SCM is uniquely minimal, then this extra zero restriction adds to the restrictions discovered

before. Continue testing by leaving out variables $K-1$ and testing whether the SCM could be formed from the first $K-2$ variables only, and so on.

2. Any linear combination of a SCM(p_1, q_1) and a SCM(p_2, q_2) is a SCM$(\max\{p_1, p_2\}, \max\{q_1, q_2\})$. The row of matrix $\boldsymbol{A}_0$ corresponding to the SCM(p_1, q_1) is not identified if there are two embedded scalar components with weakly nested orders, i.e., $p_1 \geq p_2$ and $q_1 \geq q_2$. In this case arbitrary multiples of SCM(p_2, q_2) can be added to the SCM(p_1, q_1) without changing the structure. To achieve identification, if the parameter in the i^{th} column of the row of $\boldsymbol{A}_0$ corresponding to the SCM(p_2, q_2) is normalized to one, the parameter in the same position in the row of $\boldsymbol{A}_0$ corresponding to SCM(p_1, q_1) should be restricted to zero. A detailed explanation on this issue, together with an example, can be found in Athanasopoulos and Vahid (2008a).

Stage III: Estimating the uniquely identified system

Finally, in the third stage, the identified model is estimated using FIML. As in Hannan and Rissanen (1982), a long VAR was used to obtain initial values of the parameters.

3 Variables and their time series properties

As was mentioned in the introduction, in this study, we use the same seven variables as Kim and Roubini (2000) and Brischetto and Voss (1999) for modelling Canadian monetary policy; these variables are listed in Table 1. The variables OPI and R_U represent the foreign block. The OPI is included to account for inflation expectations, mainly to capture the non-policy induced changes in inflationary pressure to which the central bank may react when setting monetary policy. Hence, it is essential to include OPI in the monetary model to account for forward-looking monetary policy (see Brischetto and Voss, 1999). It is also common in the monetary literature of small open economies to use the US Federal fund rates as a proxy for foreign monetary policy (see for example Cushman and Zha, 1997; Kim and Roubini, 2000; Dungey and Pagan, 2000). Since Canada is an open economy and has relatively open capital markets, it is also reasonable to assume that domestic interest rates are related to US interest rates. The remaining five variables are the standard set of variables used in the monetary literature to represent open economy monetary business cycle models (see for example Sims, 1992; Cushman and Zha, 1997; Kim and Roubini, 2000; Christiano et al., 1999). YP and CPI are taken as the target variables of monetary policy, known as non-policy variables while $M1$ and R_C represent policy variables and ER is the information market variable.

Table 1: Variables included in the Canadian Monetary Policy Models

Variable	**Variables**	**Abbrevation**
Foreign		
Oil Price	World Oil Price Index, Logs	OPI
US Interest Rate	Federal Funds Rate, Percent	R_U
Canada		
Output	Industrial Production (SA), Logs	YP
Price	Consumer Price Index (SA), Logs	CPI
Money	Monetary Aggregate M1 (SA), Logs	$M1$
Interest Rate	Overnight Inter-Bank Rate, Percent	R_C
Exchange Rate	Exchange Rate (USA/CAN), Logs	ER

Sources: International Financial Statistics

The data are collected from the International Financial Statistics (IFS), covering January 1974 to December 2007, excluding the global financial crisis. The variables are seasonally adjusted and in logarithms, except for interest rates, which are expressed in percentages. The results of unit root tests - Augmented Dickey Fuller and Philips-Perron - of all variables over the whole sample show that the variables are I(1) and I(0) in first-differences. In addition, Johansen's co-integration test also provides evidence of long run relationships among the seven variables. Given that the variables in levels are non-stationary and cointegrated, VAR or VARMA models with variables in first differences lead to loss of information in the long run relationships. Since the objective of this study is to assess the interrelationships among the variables, we concur with Ramaswamy and Sloke (1997) that VAR and VARMA with the variables in levels remain appropriate measures to correctly identify the effects of monetary shocks.[6]

4 Impulse response functions

Impulse response functions are estimated to assess the persistence and dynamic effects of various macroeconomic shocks on policy and non-policy related variables. It is also apparent that economically interpretable shocks are obtained to assess these responses and these issues are briefly discussed below.

The effects of monetary policy shocks are analysed from impulse response functions, which are derived from pure moving average representations of models. For a VARMA(p, q) process

$$\mathbf{A}(L)\mathbf{y}_t = \mathbf{M}(L)\boldsymbol{\upsilon}_t \tag{5}$$

the impulse responses can be obtained from

$$\mathbf{y}_t = \boldsymbol{\Xi}(L)\boldsymbol{\upsilon}_t = \boldsymbol{\upsilon}_t + \sum_{i=1}^{\infty} \boldsymbol{\Xi}_i \boldsymbol{\upsilon}_{t-i}, \tag{6}$$

where $\boldsymbol{\Xi}_i = \mathbf{M}_i + \sum_{j=1}^{i} \mathbf{A}_j \boldsymbol{\Xi}_{i-j}$, $\boldsymbol{\Xi}_0 = \mathbf{I}_k$ and $\boldsymbol{\upsilon}_t$ is a white noise process with $E(\boldsymbol{\upsilon}_t) = 0$ and $E(\boldsymbol{\upsilon}_t \boldsymbol{\upsilon}_t') = \boldsymbol{\Sigma}_{\boldsymbol{\upsilon}}$. Similarly, we obtain the impulse responses from orthogonal shocks for a reduced form VAR(p) model

$$\boldsymbol{\Phi}(L)\mathbf{y}_t = \mathbf{e}_t \tag{7}$$

with a pure VMA representation $\mathbf{y}_t = \boldsymbol{\Phi}^*(L)\mathbf{e}_t = \mathbf{e}_t + \sum_{i=1}^{\infty} \boldsymbol{\Phi}_i^* \mathbf{e}_{t-i}$ where $\boldsymbol{\Phi}_i^* = \sum_{j=1}^{i} \boldsymbol{\Phi}_j \boldsymbol{\Phi}_{i-j}^*$ and $\boldsymbol{\Phi}_0^* = \mathbf{I}_K$.

In order to directly attribute the responses of variables to economically interpretable shocks, we need to transform the exogenous shocks in equation (6) to a new set of orthogonal shocks. A traditional and convenient method is to use the Choleski decomposition,

[6] The choice between a VAR in levels (unrestricted VAR) and a VECM (restricted VAR) depends on the economic interpretation attached to impulse response functions from the two specifications (see Ramaswamy and Sloke, 1997, for details). The impulse response functions generated from VECM models tend to imply that the impact of monetary shocks is permanent, while the unrestricted VAR/VARMA allows the data series to decide whether the effects of the monetary shocks are permanent or temporary. It is also common in the monetary literature to estimate the unrestricted VAR model in levels (see for example Sims, 1992; Cushman and Zha, 1997; Bernanke and Mihov, 1998; Kim and Roubini, 2000).

as first applied by Sims (1980). A major criticism of the Choleski decomposition approach is that the assumed Wold ordering of the variables is considered atheoretical. In contrast, SVARMA and SVAR models use economic theory to identify the contemporaneous relationships between variables (see for example Bernanke, 1986; Sims, 1986; Blanchard and Watson, 1986). The relationship between the reduced form VARMA disturbances ($\boldsymbol{\upsilon}_t$) and the orthogonal shocks $\mathbf{v}_t$ is

$$\mathbf{B}_0\boldsymbol{\upsilon}_t = \mathbf{v}_t, \tag{8}$$

where $\mathbf{B}_0$ is an invertible square matrix, $E(\mathbf{v}_t) = 0$, $E(\mathbf{v}_t\mathbf{v}_t') = \boldsymbol{\Sigma}_{\mathbf{v}}$ and $\boldsymbol{\Sigma}_{\mathbf{v}}$ is a diagonal matrix. $\mathbf{B}_0$ is normalized across the main diagonal, so that each equation in the system has a designated dependent variable. The innovations of the structural model are related to the reduced form innovations by $\boldsymbol{\Sigma}_{\boldsymbol{\upsilon}} = \mathbf{B}_0^{-1}\boldsymbol{\Sigma}_{\mathbf{v}}(\mathbf{B}_0^{-1})'$. The impulse responses from the SVARMA are obtained from

$$\mathbf{y}_t = \mathbf{B}_0^{-1}\mathbf{v}_t + \sum_{i=1}^{\infty} \boldsymbol{\Xi}_i\mathbf{B}_0^{-1}\mathbf{v}_{t-i}. \tag{9}$$

while the the impulse responses from the SVAR are obtained as follows

$$\mathbf{y}_t = \mathbf{B}_0^{-1}\boldsymbol{\epsilon}_t + \sum_{i=1}^{\infty} \boldsymbol{\Phi}_i^*\mathbf{B}_0^{-1}\boldsymbol{\epsilon}_{t-i}, \tag{10}$$

where $\boldsymbol{\epsilon}_t = \mathbf{B}_0\mathbf{e}_t$. For both the SVARMA and SVAR models, a non-recursive identification structure on the contemporaneous matrix $\mathbf{B}_0$ is imposed. Kim and Roubini's identification structure is as follows

$$\mathbf{B}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ b_{21}^0 & 1 & 0 & 0 & 0 & 0 & 0 \\ b_{31}^0 & 0 & 1 & 0 & 0 & 0 & 0 \\ b_{41}^0 & 0 & b_{43}^0 & 1 & 0 & 0 & 0 \\ 0 & 0 & b_{53}^0 & b_{54}^0 & 1 & b_{56}^0 & 0 \\ b_{61}^0 & 0 & 0 & 0 & b_{65}^0 & 1 & b_{67}^0 \\ b_{71}^0 & b_{72}^0 & b_{73}^0 & b_{74}^0 & b_{75}^0 & b_{76}^0 & 1 \end{bmatrix} \tag{11}$$

with the variables ordered as in Table 1. The above contemporaneous structure is used to estimate the orthogonal shocks for Canada where the sizes of these shocks actually represent the one standard deviation of the corresponding orthogonal errors obtained from the SVARMA and SVAR models.

The two foreign variables are identified recursively, with the assumption that the OPI is contemporaneously exogenous to all other variables in the model, while the R_U is assumed to be contemporaneously affected by OPI. YP is influenced contemporaneously only by OPI, while CPI is affected by both OPI and YP. $M1$ equation which represents the demand for real money balances is contemporaneously dependent on YP, CPI and R_C. The domestic monetary policy equation is assumed to be the reaction function of Bank of Canada, which sets the interest rate after observing the current OPI, $M1$ and ER and assume that the central bank does not react immediately to the unexpected R_U shock. Kim and Roubini (2000)'s justification for this assumption is that within a month the Bank of Canada is more concerned about the impact of global and foreign exchange rate shocks on the economy than the impact of foreign monetary shock. Finally, ER is seen as an information market variable that reacts quickly to all relevant economic disturbances and hence is contemporaneously affected by all the variables in the SVARMA/SVAR systems.

Apart from the restrictions imposed on the contemporaneous structure, no restrictions are imposed on the lag structures of the SVAR model. On the other hand, due to the identification issues discussed in Section (2), further restrictions are imposed for the SVARMA model in order to identify a unique VARMA process.

5 Empirical results

In this section, we apply the VARMA methodology outlined in Section (2) to the Canadian monetary model of seven variables. The impulse responses generated from the identified SVARMA and SVAR models are then used to assess the effects of various monetary shocks.

5.1 Identifying SVARMA and SVAR models

In Stage 1, we identify the overall order of the VARMA process and the orders of embedded SCMs in the data for Canada. In Panel A of Table 2 we report the results of all canonical correlations test statistics, divided by their χ^2 critical values and this table is known as the "Criterion Table". If the entry in the $(m, j)^{th}$ cell is less than one, this shows that there are seven SCMs of order (m, j) or lower in this system.

From Panel A in Table 2, we infer that the overall order of the system is VARMA$(2, 1)$. Conditional on this overall order, the canonical correlation tests are employed to identify the individual orders of embedded SCMs. The number of insignificant canonical correlations identified are tabulated in Panel B of Table 2, which is referred to as the "Root Table". The root table shows the test results from identifying the individual orders of the SCMs conditional on the overall order being VARMA(2,1). We first identify two SCM(1,0)s and one SCM(0,1). As it is possible for an SCM(0,1) to be observationally equivalent to an SCM(1,0), which leads to an identification problem (see Tiao and Tsay, 1989, page 161), we proceed with only the autoregressive components. Next, we find five SCM(2,0)s and five SCM(1,1)s. However, every SCM(m, j) nests all scalar components of order $(\leq m, \leq j)$. For each individual SCM$(p_1 < p, q_1 < q)$, there will be $\xi = \min\{m - p_1 + 1, j - q_1 + 1\}$ zero canonical correlations at position $(m \geq p_1, j \geq q_1)$. Therefore, a new SCM(m, j) is found for every increment above ξ, and hence only three of the five SCMs found are new. We proceed with three SCM(1,1)s. Finally, we also find 2 new SCMs of order (2,1). The identified VARMA$(2, 1)$ consists of two SCM$(1, 0)$s, three SCM$(1, 1)$s and two SCM$(2, 1)$s.

Table 2: Stage I of the identification process of a VARMA model for the Canadian Monetary System

PANEL A: Criterion Table

m	j = 0	1	2	3	4
0	164.98[a]	13.28	7.05	4.69	3.50
1	3.67	1.50	1.32	0.86	0.99
2	1.46	**0.75**	0.86	0.88	0.96
3	1.29	0.84	0.85	0.90	1.02
4	0.90	0.97	0.94	0.91	0.98

PANEL B: Root Table

m	j = 0	1	2	3	4
0	0	1	1	1	1
1	**2**	**5**	7	8	8
2	**5**	**9**	12	14	15
3	6	12	16	19	20
4	7	13	19	23	26

[a] *The statistics are normalized by the corresponding 5% χ^2 critical values*

Implementing Stage II of the identification process described in Section (2), has led to additional zero restrictions on the matrix containing the contemporaneous relationships between the variables, and the canonical SCM representation of the identified VARMA$(2, 1)$

of the Canadian monetary model is given below,

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \alpha_{23} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \alpha_{56} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \boldsymbol{y}_t = \boldsymbol{c} + \begin{bmatrix} \psi_{11}^{(1)} & \psi_{12}^{(1)} & \psi_{13}^{(1)} & \psi_{14}^{(1)} & \psi_{15}^{(1)} & \psi_{16}^{(1)} & \psi_{17}^{(1)} \\ \psi_{21}^{(1)} & \psi_{22}^{(1)} & \psi_{23}^{(1)} & \psi_{24}^{(1)} & \psi_{25}^{(1)} & \psi_{26}^{(1)} & \psi_{27}^{(1)} \\ \psi_{31}^{(1)} & \psi_{32}^{(1)} & \psi_{33}^{(1)} & \psi_{34}^{(1)} & \psi_{35}^{(1)} & \psi_{36}^{(1)} & \psi_{37}^{(1)} \\ \psi_{41}^{(1)} & \psi_{42}^{(1)} & \psi_{43}^{(1)} & \psi_{44}^{(1)} & \psi_{45}^{(1)} & \psi_{46}^{(1)} & \psi_{47}^{(1)} \\ \psi_{51}^{(1)} & \psi_{52}^{(1)} & \psi_{53}^{(1)} & \psi_{54}^{(1)} & \psi_{55}^{(1)} & \psi_{56}^{(1)} & \psi_{57}^{(1)} \\ \psi_{61}^{(1)} & \psi_{62}^{(1)} & \psi_{63}^{(1)} & \psi_{64}^{(1)} & \psi_{65}^{(1)} & \psi_{66}^{(1)} & \psi_{67}^{(1)} \\ \psi_{71}^{(1)} & \psi_{72}^{(1)} & \psi_{73}^{(1)} & \psi_{74}^{(1)} & \psi_{75}^{(1)} & \psi_{76}^{(1)} & \psi_{77}^{(1)} \end{bmatrix} \boldsymbol{y}_{t-1}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \psi_{31}^{(2)} & \psi_{32}^{(2)} & \psi_{33}^{(2)} & \psi_{34}^{(2)} & \psi_{35}^{(2)} & \psi_{36}^{(2)} & \psi_{37}^{(2)} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \psi_{71}^{(2)} & \psi_{72}^{(2)} & \psi_{73}^{(2)} & \psi_{74}^{(2)} & \psi_{75}^{(2)} & \psi_{76}^{(2)} & \psi_{77}^{(2)} \end{bmatrix} \boldsymbol{y}_{t-2}$$

$$+\boldsymbol{u}_t - \begin{bmatrix} \mu_{11}^{(1)} & \mu_{12}^{(1)} & \mu_{13}^{(1)} & \mu_{14}^{(1)} & \mu_{15}^{(1)} & \mu_{16}^{(1)} & \mu_{17}^{(1)} \\ \mu_{21}^{(1)} & \mu_{22}^{(1)} & \mu_{23}^{(1)} & \mu_{24}^{(1)} & \mu_{25}^{(1)} & \mu_{26}^{(1)} & \mu_{27}^{(1)} \\ \mu_{31}^{(1)} & \mu_{32}^{(1)} & \mu_{33}^{(1)} & 0 & 0 & \mu_{36}^{(1)} & \mu_{37}^{(1)} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mu_{61}^{(1)} & \mu_{62}^{(1)} & \mu_{63}^{(1)} & \mu_{64}^{(1)} & \mu_{65}^{(1)} & \mu_{66}^{(1)} & \mu_{67}^{(1)} \\ \mu_{71}^{(1)} & \mu_{72}^{(1)} & \mu_{73}^{(1)} & 0 & 0 & \mu_{76}^{(1)} & \mu_{77}^{(1)} \end{bmatrix} \boldsymbol{u}_{t-1}.$$

where $\boldsymbol{y}_t = (OPI_t, R_{U,t}, YP_t, CPI_t, M1_t, R_{C,t}, ER_t)'$. Among the variables, CPI_t and $M1_t$ are found to be loading as SCM$(1, 0)$, while OPI_t, $R_{U,t}$ and $R_{C,t}$ were loading as SCM$(1, 1)$ and YP_t and ER_t were loading on as SCM$(2, 1)$. We also ensured that the individual tests described in Section (2) do not contradict the normalization of the diagonal parameters of the contemporaneous matrix to one.

For the SVAR model, we found that twelve lags are necessary to capture all of the dynamics in the data. To obtain the orthogonal shocks, we use the the non-recursive identification structure of Kim and Roubini (2000) described in Section (4). As in Kim and Roubini (2000), five additional restrictions were imposed on Canada and the over identifying restrictions were not rejected at the 1% significance level, thus suggesting that the identified model specifications are appropriate.[7]

5.2 Responses of policy and non-policy variables to various shocks

The dynamic impulse response functions of domestic variables to various independent shocks are generated from SVARMA and SVAR models and revealed in Figures 1 to 7. The behavior of these responses over a period of 48 months is analyzed and discussed in this section. The sizes of the shocks are measured by one-standard deviation of the orthogonal errors of the respective models and are presented in Table 3 below. The sizes of the orthogonal shocks in the SVARMA and SVAR models appear to be somewhat similar.

Table 3: Magnitude of One Standard Deviation Shocks from the SVAR and SVARMA Models

Model	OPI	R_U	YP	CPI	$M1$	R_C	ER
SVAR	0.132	0.143	0.041	0.008	0.014	0.771	8.752
SVARMA	0.139	0.153	0.040	0.008	0.017	0.746	9.167

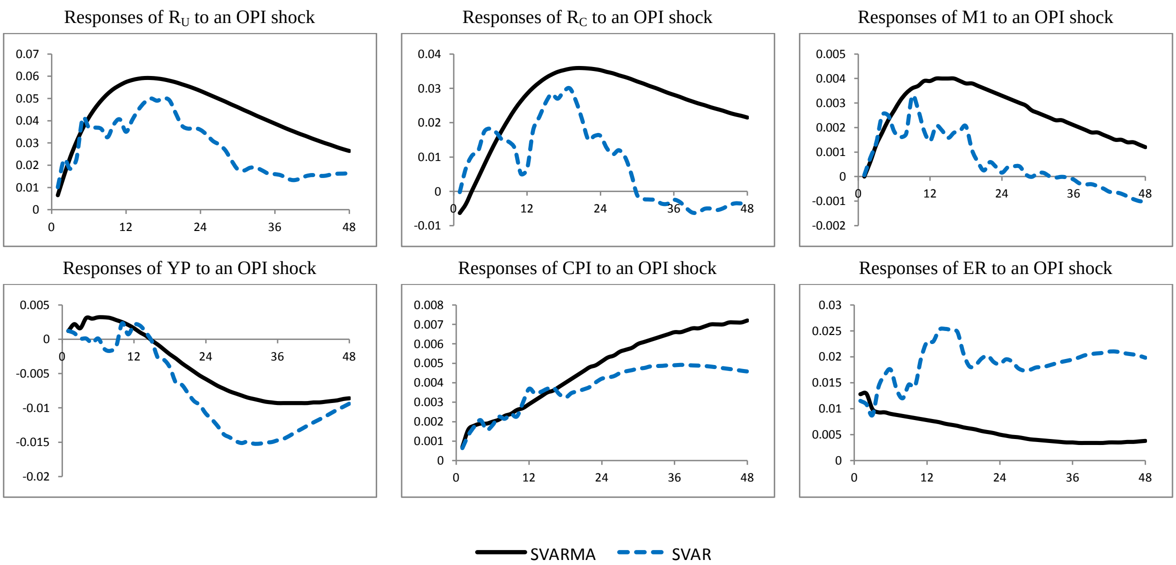
5.2.1 Impulse responses to an oil price shock

The responses of Canadian variables to an oil price shock are shown in Figure 1. A positive OPI shock is expected to induce inflationary pressure on the economy. As expected, both the US and Canadian monetary policies responded to higher oil prices, resulting in increasing policy rates, R_U and R_C. Both of these rates continue to rise for about 18 months and peaked around 6% and 3.5% respectively. Since OPI is based on the world market price, the rise in oil price leads to an inflationary pressure in the economy. To combat the rise in CPI and the demand for money, $M1$, the Bank of Canada has responded by increasing the R_C. As observed in Figure 1, the rise in R_C is higher than the rise in CPI, leading to a positive rise in real interest rate. Consequently, these changes lead to an immediate appreciation of the Canadian dollar, ER, followed by a fall and then reaching a new lower steady state after 2 years.

[7]The contemporaneous matrix $\mathbf{B}_0$ requires $((7^2 - 7)/2 = 21)$ restrictions for exact identification while in (9) there are 26 restrictions imposed, leading to over identification.

Considering that Canada is a net oil exporter, a positive movement in OPI led to a positive movement in YP within the first year, followed by a negative response. This outcome is not surprising as Canada, unlike many oil producers, does not heavily subsidize fuel. Further, both a rise in R_C and an appreciation of ER reduce the size of the positive impact of oil shock on Canadian output. After a year, output falls as a result of falling in consumption, investment and non-oil exports. It is evident that these expected movements of all six variables: R_U, R_C, $M1$, CPI, and ER, to a positive oil price shock are well captured by impulse responses generated by the SVARMA model.

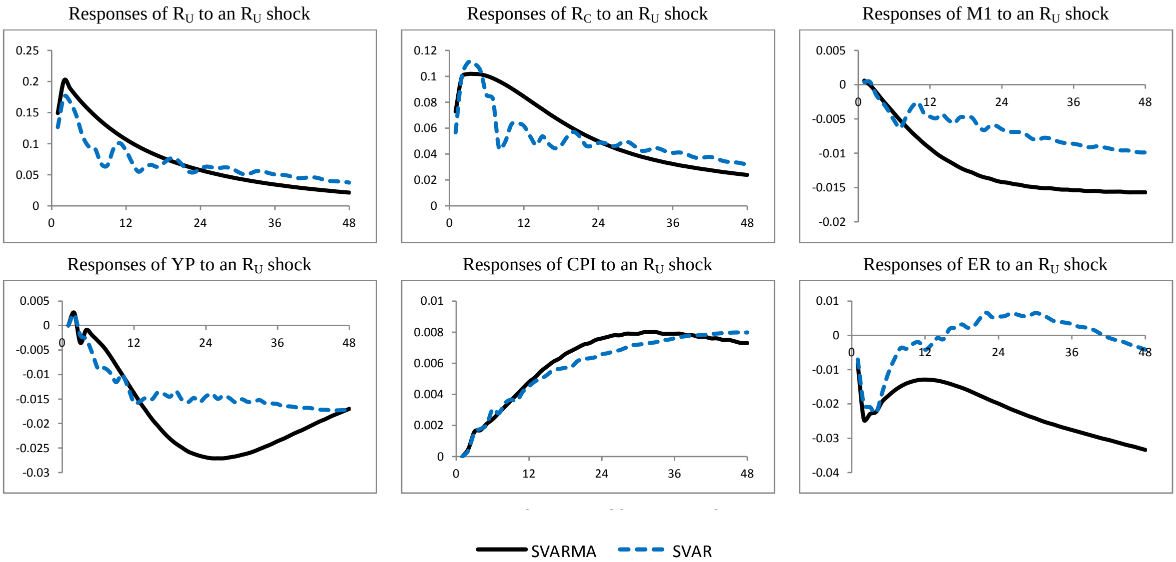
Figure 1: Impulse responses of variables to an oil price shock



5.2.2 Impulse responses to an US monetary policy shock

A positive R_U shock, which is defined as the unanticipated monetary contraction, leads to a rise in US interest rate, resulting in excess demand for US currency in the foreign exchange market. The US dollar appreciates, while the Canadian dollar depreciates. To lean against the exchange rate depreciation, the Bank of Canada increases its policy rate R_C and consequently $M1$ and YP fall. As observed in Figure 2, the rise in both R_U and R_C peaked around 3 months and then declined and reached a steady state after 3 years, while $M1$ and YP continue to fall for 2 years. Throughout these periods the R_U is higher than R_C, suggesting that the ER should be depreciating, which is clearly demonstrated by SVARMA responses. The depreciating Canadian dollar could also explain the continuous rise in CPI in response to US monetary shock.

Figure 2: Impulse responses of variables to an US Monetary shock

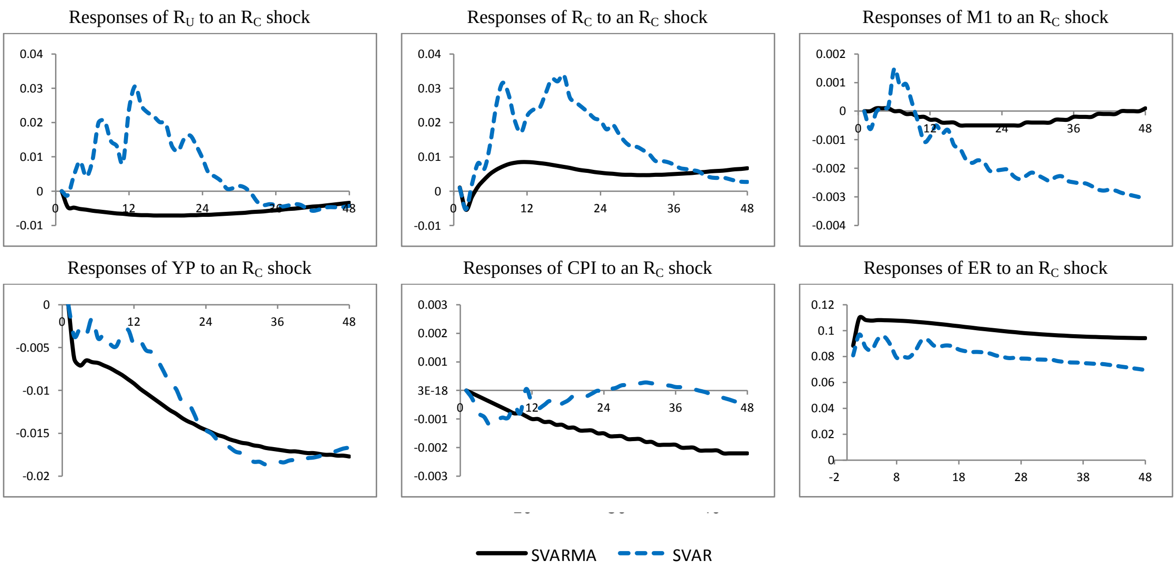


5.2.3 Impulse responses to a domestic monetary policy shock

Canada is a small open economy, and hence its monetary policy changes are not expected to affect the US interest rate - a proxy for the world interest rate. The SVARMA response function is flat near zero line, indicating a positive R_C shock has no significant impact on R_U. On the contrary, the SVAR responses are positive and large. According to Kim and Roubini (2000), if the identified monetary shock is indeed orthogonal, in the sense that it is not a systematic response to any shock, a tighter monetary policy stance would lead to a rise in R_C and a fall in $M1$, and subsequently these results will be reversed due to persistent deflationary pressures in the economy. It is worth noting that, as observed in Figure 3, the SVARMA responses were consistent with this expectation: the R_C continues to rise for nearly next 12 months and then falls, and reaching subsequently to a new lower steady state, while $M1$ continues to decline for the next 18 months, followed by a rise thereafter.

Although a sudden fall in SVAR CPI response to a contractionary monetary policy shock is observed, it is more pronounced and persistent only in the corresponding SVARMA response, where it declines smoothly, becoming persistent over the entire 48 months horizon. A rise in R_C followed by a fall in CPI, leads to both a rise in real interest rate and an appreciation of the nominal exchange rate. That is, a positive interest differential in favour of Canadian financial assets is associated with a persistent appreciation of the Canadian dollar. This result is consistent with those of Eichenbaum and Evans (1995) and ?. In response to a positive R_C shock, YP declines, indicating that Canadian money is non-neutral in the short run. As indicated by both STAR and SVARMA models, the negative persistent YP responses to an R_C shock may be due to an increase in the real cost of borrowing and the appreciation of the currency. Overall, the absence of price and liquidity puzzles highlights the adequacy of the SVARMA model in identifying an appropriate monetary policy shock and producing impulse responses, which are consistent with economic theoretical model predictions.

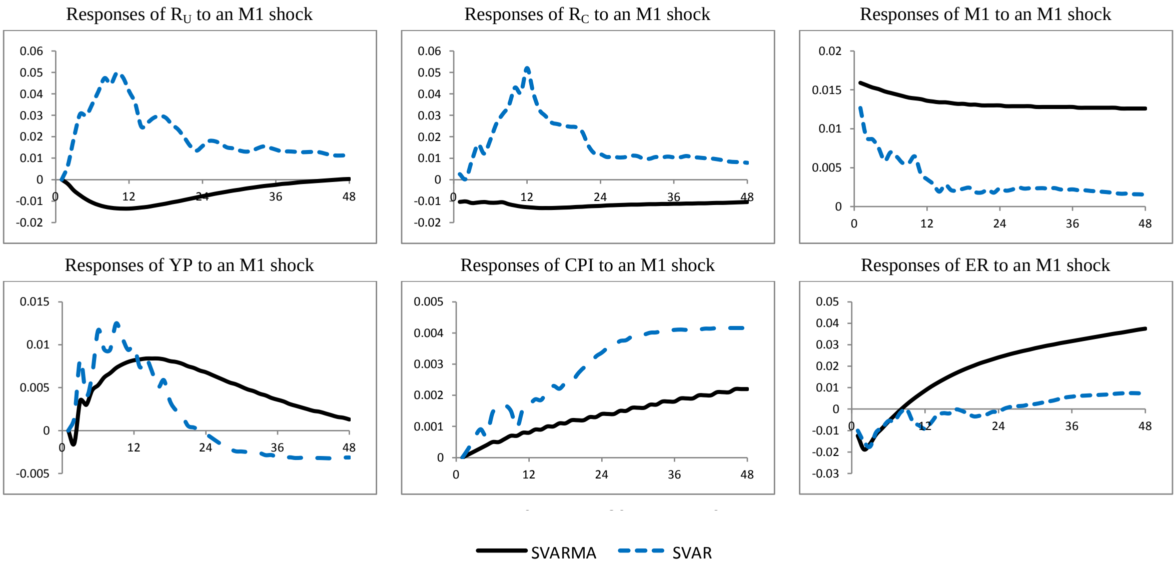
Figure 3: Impulse responses of variables to a domestic monetary shock



5.2.4 Impulse responses to a money shock

According to Christiano et al. (1999), a positive $M1$ shock is expected to trigger an easing of monetary policy, followed by a fall in interest rate - a process known as the liquidity effect. In addition, a rise in monetary aggregates and output, a slow positive response of price level and a deprecation of the currency are also expected responses to a positive $M1$ shock. As observed in Figure 4, this liquidity effect is clearly captured only by the SVARMA model responses. By contrast, the SVAR response is positive and notably large, creating a liquidity puzzle. As expected, the positive SVARMA CPI response is also slow and less prominent compared to that of SVAR CPI response.

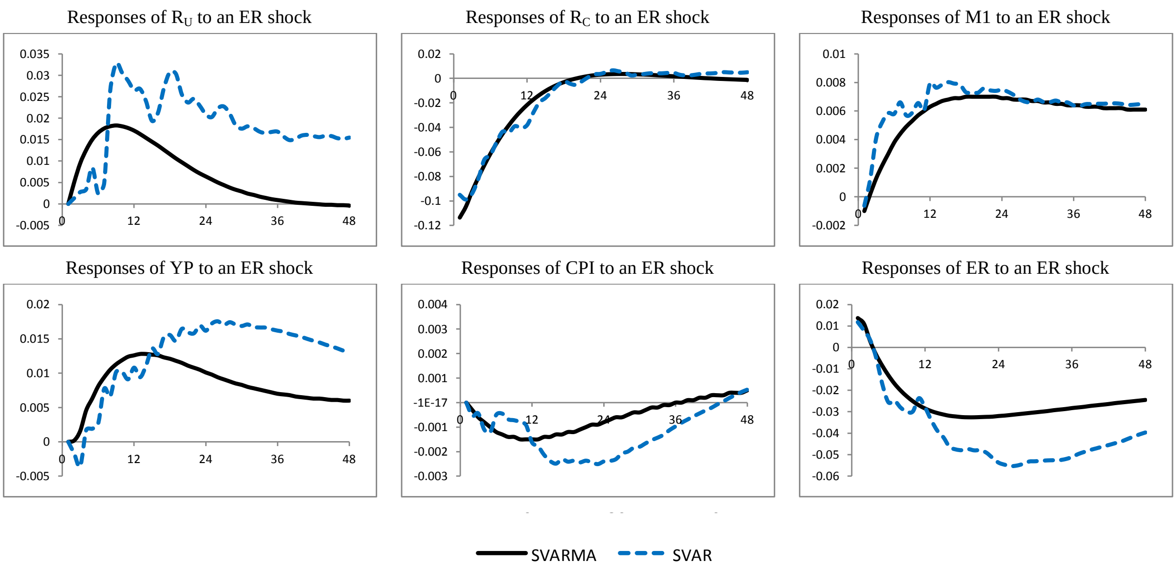
Figure 4: Impulse responses of variables to a money shock



5.2.5 Impulse responses to an exchange rate shock

R_C is expected to decline to a positive ER shock, where an unanticipated appreciation of currency should prompt policy makers to lean against currency appreciation. Referring to Figure 5, as anticipated, a fall in SVARMA/SVAR R_C responses is immediate, followed by a rising $M1$ lasting for about 18 months, before returning to a steady state. The currency appreciation is expected to have two opposing effects on YP. On the one hand, it decreases the net exports as they become more expensive than the imports. On the other hand, it reduces the cost of production through lower prices of imported intermediate goods. These combined effects transpiring through the demand and supply channels would determine the net influence of an ER shock on YP. It is noted that in both SVAR/SVARMA YP responses increase for first few years and then become persistent. The reason for these models capturing the expansionary output maybe due to the fact that both the monetary policy and the exchange rate shocks were allowed to contemporaneously interact in both SVAR/SVARMA models. As a result, a decline in interest rate followed by a currency appreciation might have led to an expansionary effect on output. CPI is expected to respond negatively to an ER shock, due to lower import prices and production costs. This outcome is observed in both the SVAR and SVARMA responses.

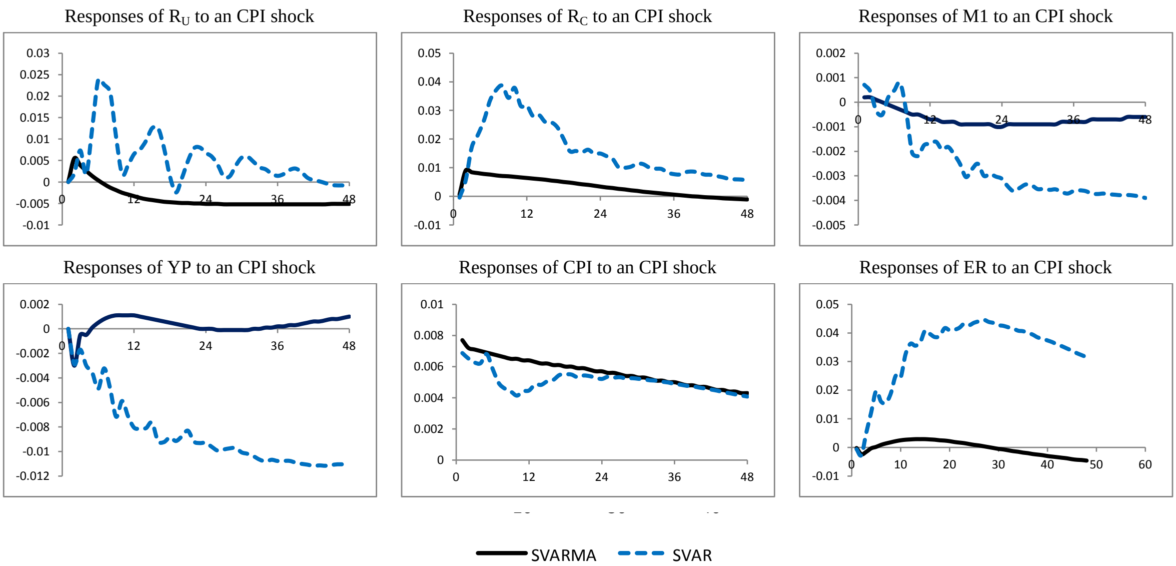
Figure 5: Impulse responses of variables to an exchange rate shock



5.2.6 Impulse responses to a price shock

A positive shock to CPI can be regarded as an unanticipated inflationary pressure on the economy. As a consequence,R_C is expected to rise while $M1$ is expected to fall. Figure 6 shows the SVARMA R_C response to a CPI shock is positive as expected but less pronounced, lasting for only about two years, while the corresponding SVAR response increases sharply and then declines gradually. A comparison of both models' responses shows that the SVAR model tends to overstate the dynamic response of monetary policy to an inflationary shock. Although an unanticipated inflationary pressure resulting in depreciation of ER is expected, a rise in R_C can offset this negative effect. This outcome is captured neatly only by the SVARMA model, while this response is positive and large in the SVAR model.

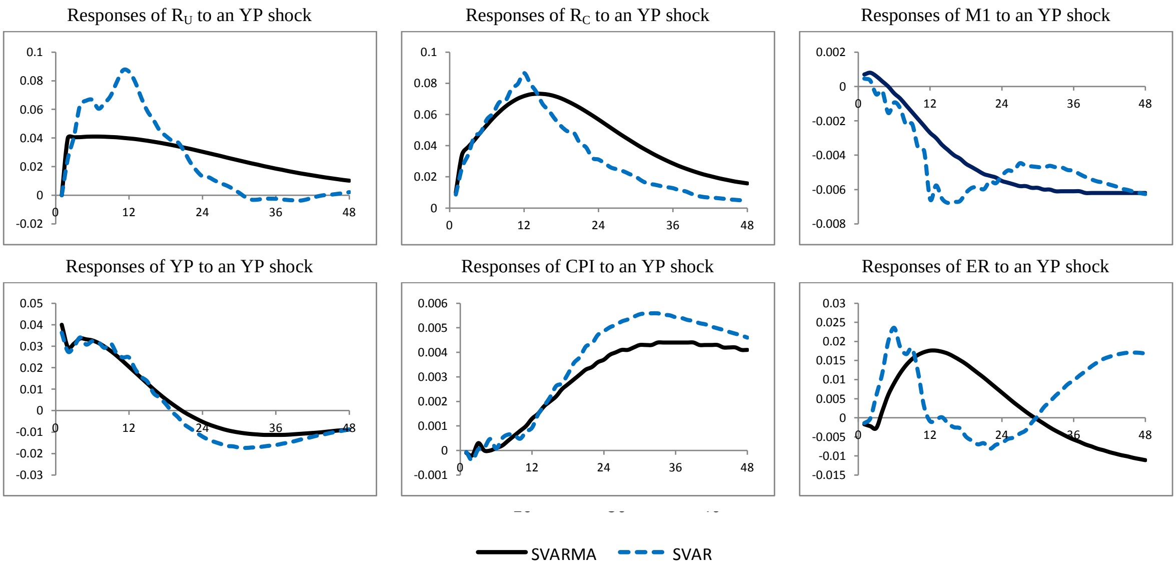
Figure 6: Impulse responses of variables to a price shock



5.2.7 Impulse responses to an output shock

The R_C response increases and $M1$ response declines to a positive YP shock. These outcomes are consistent with the contractionary policy measure usually undertaken by Central Banks against expanding economies. The expansion in YP also induces inflationary pressure in the economy causing an increase in CPI. Figure 7 shows that all the above expected directions of responses are observed in both SVAR/SVARMA models. However, a larger than expected increase in YP leads to an increase in R_C, resulting in ER appreciation. Such movements of responses are clearly evident in SVARMA model, while the SVAR responses appear to be unstable.

Figure 7: Impulse responses of variables to an output shock



6 Conclusion

This paper builds a structural VARMA (SVARMA) model for investigating Canadian monetary policy in two stages. Firstly, using the scalar component model (SCM) developed by Athanasopoulos and Vahid (2008a), this paper identifies a VARMA model. Secondly, imposing a non-recursive identifying structure on its contemporaneous matrix, it establishes the identification conditions for a SVARMA model for Canadian monetary policy. Although the VARMA model has long been known as the preferred model for monetary policy analysis, the traditional VAR and SVAR models have been widely used, mostly due to difficulties associated with the identification and estimation of such a model. The SVAR model is included in this study for a comparison purpose. To our knowledge, this is the first paper to successfully construct a canonical SVARMA model for Canadian monetary policy analysis. All computations are carried out using Gauss code, which will be available from authors on request.

The results of our investigation are very promising. The impulse response functions generated by the new SVARMA framework appear to have resolved the anomalies commonly found in the empirical monetary literature on small open economies. These anomalies include liquidity, price, output and exchange rate puzzles. The presence of moving average components combined with the simultaneous interaction between the monetary policy and exchange rate shocks in the SVARMA model appeared to have resolved these economic anomalies. By contrast, the impulse responses generated by SVAR reveals the existence of the liquidity puzzle. In addition, the SVAR exchange rate response to oil price, foreign monetary policy, domestic prices level and output shocks are not consistent with those predicted by economic theoretical models and stylized facts. We recommend the SVARMA methodology be adapted for analyzing the monetary policy of other open economies, because the potential gain from basing monetary policy on an adequate model is immense as shown in this investigation.

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