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Abstract

This paper tests for long memory in the volatility of the All Ordinaries Index and its Share Price Index (SPI) futures. This has important implications for those agents concerned with the long term volatility in these markets. We use daily data and a short span of high frequency data to estimate the fractional differencing parameter, examine the fit of the implied autocorrelation function, and calculate the modified R/S and KPSS test statistics. All procedures support the existence of long memory in volatility in both markets except the KPSS test on the index using daily data. We argue that this is due to the low power of the KPSS test.

1. Introduction

This paper tests for long memory in the volatility of the Australian All Ordinaries Index and its Share Price Index (SPI) futures. The presence of long memory in volatility has important implications for those agents concerned with volatility over the longer term. Long memory processes exhibit high degress of persistence and are associated with hypergeometric rates of decay in the autocorrelation function and impulse response coefficients. This is in contrast to the more popular ARMA class of processes which exhibit short memory and much faster exponential rates of decay. The presence of long memory in volatility therefore suggests that over longer time horizons, the popular GARCH class of processes will underestimate the extent of volatility persistence.

The presence of long memory in volatility has been documented across many financial time series (Taylor, 1986; Ding *et al*, 1993; Baillie *et al*, 1996; Bollerslev and Mikkelsen, 1996). Much of the research has focused on US equities and the Deutschemark-U.S.$. The findings of long memory in volatility across many financial time series, combined with the research supporting the cost of carry model between the All Ordinaries and its SPI futures (Twite, 1998; Brailsford and Hodgson, 1997), suggests that both markets are likely to exhibit long memory in volatility.

The existence of long memory and its source however are controversial. The high degrees of volatility persistence observed in financial markets may be explained by near long memory processes. Near long memory processes are those with autocorrelation functions that decay at a rate that is difficult to distinguish from long memory (Breidt and Hsu, 2002). These processes therefore exhibit high degrees of volatility persistence but do not exhibit hypergeometric rates of decay. This view refutes the use of long memory processes and supports the use of occasional break models.

In this paper we support the existence of long memory in volatility. We test for long memory using a long span of daily data and a short span of high frequency data. To test for long memory, we estimate the fractional differencing parameter via the spectral density, examine the fit of the implied autocorrelation function, and calculate the modified R/S and KPSS test statistics on squared returns. By using a variety of procedures along with data at different frequencies, robust conclusions regarding the existence of long memory in volatility should be obtained.

When using daily data, the results strongly support the existence of long memory in the SPI futures volatility. With respect to the index volatility, all procedures support the existence of long memory, except the KPSS test statistic which strongly rejects its presence. In contrast, the results using a short span of high frequency data strongly support the existence of long memory in the volatility in both markets. We argue that the inconsistent KPSS test results for the index at the daily frequency are explained by the tests low power. The high frequency data set overcomes this limitation by significantly increasing the sample size and hence the power of the test. These results highlight the importance of using a variety of procedures when testing for long memory, and illustrate the benefits of using short spans of high frequency data. The results also suggest that when longer term dependencies are important, short memory processes (like the GARCH class of processes) should not be used to model the volatility in these markets. Long memory processes like the Fractionally Integrated GARCH model (Baillie *et al*, 1996) are more likely to appropriately characterise the long term volatility dynamics.

Section 2 will review the literature providing justification for the use of a short span of high frequency data. The section will define long memory processes, outline the modified R/S and KPSS tests, overview the debate over the existence of long memory, and review the empirical evidence supporting long memory in financial market volatility. It will be shown that a short span of high frequency data increases the power of the tests for long memory, and decreases the likelihood of spuriously detecting long memory in volatility due to the presence of occasional breaks. Section 3 will discuss the data and methodology. Section 4 will present the results which are supportive of long memory in the volatility in both markets and highlight the benefits obtained from using a short span of high frequency data. Section 5 will conclude.

2. Literature Review

2.1 Long memory

Long memory processes have autocovariance functions (Ψ) that decay at the hypergeometric rate k^{2d-1} ($0<d<0.5$), where d is the fractional differencing parameter. The autocovariance function of a long memory processes is not absolutely summable [1]

$$\lim_{n\to\infty}\sum_{k=-n}^{n}\left|\Psi_k\right| = \infty \qquad (1)$$

This is in contrast to a short memory process which has an autocovariance function that is absolutely summable.

Granger (1980), Granger and Joyeux (1980) and Hosking (1981) were the first to apply long memory models to the field of econometrics. Each of these papers developed the fractional white noise and autoregressive fractionally integrated moving average (ARFIMA) models. See Baillie (1996) for details.

The extension of these models to the second moment has also produced a voluminous literature. See Baillie *et al* (1996) for the Fractionally Integrated GARCH (FIGARCH), Bollerslev and Mikkelsen (1996) for the Fractionally Integrated Exponential GARCH (FIEGARCH), Ding and Granger (1996) for the Long Memory ARCH (LM-ARCH), and Tse (1998) for the Fractionally Integrated Asymmetric Power ARCH (FIAPARCH) models. By imposing a hypergeometric rate of decay, these processes are able to capture long memory in volatility. This is in contrast to the GARCH class of processes which exhibit short memory and impose much faster exponential rates of decay.

2.2 Testing for long memory

There are a number of tests for long memory in a time series. We examine the rescaled range or R/S statistic of Hurst (1951), the modified R/S statistic of Lo (1991), and the KPSS test of Kwiatkowski *et al* (1992). Each of these statistics test an I(0) null against an I(d) alternative.

The rescaled range statistic of Hurst (1951) forms the basis of the modified R/S and KPSS statistics. Baillie (1996), Lo (1991) and Crato (1994) provide detailed reviews of this test statistic. The R/S statistic (Q_N), is the range of partial sums of deviations from the mean rescaled by its standard deviation

$$Q_N = \frac{1}{s_N}\left[\max_{1\le k\le N}\sum_{j=1}^{k}\left(X_j - \bar{X}_N\right) - \min_{1\le k\le N}\sum_{j=1}^{k}\left(X_j - \bar{X}_N\right)\right] \tag{2}$$

where N represents the sample size, $\bar{X}_N$ the sample mean of the variable X, and s_N the sample standard deviation.

Lo (1991) demonstrates that the R/S statistic is not robust if the series exhibits short memory or heteroscedasticity. Consequently Lo (1991) introduced the modified R/S statistic where the standard deviation (s_N) is replaced by the heteroscedastic and autocorrelation consistent (HAC) variance, $\sigma_X^2 = \operatorname{cov}\left(X_j, X_0\right)$, estimated by

$$s^2_{N,q} = \frac{1}{N}\sum_{j=1}^{N}\left(X_j - \bar{X}_N\right)^2 + 2\sum_{j=1}^{q}\omega_j(q)\gamma_j \tag{3}$$

where γ_j represents sample autocovariances and $\omega_j(q) = 1 - j/(q+1)$ represents the weights applied to the sample autocovariances at lag j to account for the possible short range dependence up to the qth order.[2] Lo (1991) shows that the statistic is consistent against a class of alternative stationary long range dependent processes. Giriatis *et al* (2001) extend these results to the second moment with $0<d<0.5$.

Kwiatkowski *et al* (1992) developed the KPSS statistic to test an I(0) null hypothesis against an I(1) alternative. This was subsequently extended by Lee and Schmidt (1996) to test for long memory in stationary fractionally integrated processes. The KPSS test statistic (T_N) standardises (via the HAC variance estimator $s^2_{N,q}$) the squared partial sums of deviations from the mean

$$T_N = \frac{1}{s^2_{N,q} N^2} \sum_{k=1}^{N} \left(\sum_{j=1}^{k} \left(X_j - \bar{X}_N \right) \right)^2 \qquad (4)$$

Lee and Amsler (1997) demonstrate the consistency of the statistic for the first moment, Giriatis *et al* (2001) extend this to the second moment when $0<d<0.5$. Further details can be found in Baillie (1996), Kirman and Teysierre (2000, 2001) and Giriatis *et al* (2000).

The size and power characteristics of these statistics when applied to the first moment have been examined by Lo (1991), Lee and Schmidt (1996) and Lee and Amsler (1997). These papers find that the tests suffer from low power and that their size is very sensitive to the choice of q in the HAC variance estimate. Unfortunately there there is a lack of statistical criteria that can be used to evaluate an appropriate lag length (Kirman and Teysierre, 2000).

Giriatis *et al* (2000, 2001) and Kirman and Teyssiere (2000) examine the size and power properties of these statistics when applied to the second moment. The KPSS statistic has good size properties which are not heavily influenced by the choice of q , whilst the size of the modified R/S test is heavily influenced by the choice of q (with significant size distortions occurring when q is small). Giriatis *et al* (2001) show that whilst power for both statistics increases with the sample size (due to the consistency of the tests), the power of the tests is even lower than the power when applied to the first moment.

In summary each of the statistics are consistent under long memory alternatives. When using finite samples, the low power of the tests may produce spurious conclusions in favour of short memory.

2.3 The long memory debate

The existence of long memory and its source are controversial. To date, explanations for long memory in volatility from the orthodox microeconomic literature have not been forthcoming (Goodhart and O'Hara, 1997). Long memory in volatility may arise from the aggregation of multiple volatility components caused by either heterogeneous information flows [3] (Andersen and Bollerslev, 1997a) or heterogeneous traders (Muller *et al*, 1997). These views are consistent with the use of fractional processes and extend the mixture of distributions hypothesis and the work of Granger (1980).[4] Long memory may also be a result of a heavy tailed regime switching process (Liu, 2000).[5]

There are however strong arguments against the existence of long memory. This view argues that the high degrees of persistence can be explained by near long memory processes. Breidt and Hsu (2002) define near long memory processes as those with autocorrelation functions that decay at a rate that is difficult to distinguish from long memory. These processes therefore exhibit high degrees of volatility persistence but do not exhibit hypergeometric rates of decay.

This view refutes the use of fractional processes and supports the use of occasional break models. Hyung and Franses (2001) and Granger and Hyung (1999) develop occasional break models that are ARMA (or short memory) processes within regimes. Kirman and Teysierre (2000, 2001) show that near long memory may arise from occasional breaks, where the market participants switch from being predominantly fundamentalist to chartist or vice versa.

Unfortunately the testing and estimation procedures are unable to differentiate between long memory and near long memory. If the data generating process (DGP) is an occasional break model, long memory testing and estimation procedures will spuriously detect long memory (Granger and Hyung, 1999; Breidt and Hsu, 2002). On the other hand, if the DGP is a long memory process, structural break tests will spuriously identify breaks (Granger and Hyung, 1999). Different break tests may also result in considerable differences in the detected break points (Granger and Hyung, 1999; Hyung and Franses, 2001).[6]

Andersen and Bollerslev (1997a; 1997b; 1998) argue that one way of resolving the controversy is by examining whether long run volatility dependencies can be uncovered from short spans of high frequency data. By using a short span of high frequency data, one minimises the chance that the data is subject to structural breaks. If long memory dependencies are found, it is therefore argued that long memory is an inherent characteristic of the DGP, not a result of occasional breaks. This approach is possible given that the value of *d* is unaffected by temporal aggregation (Andersen and Bollerslev, 1997a; Bollerslev and Wright, 2000).

In summary, there is very little consensus on the existence of long memory in volatility or its source. The debate cannot be resolved empirically given that current procedures are unable to differentiate between long memory and near long memory. In this paper we support the existence of long memory in volatility and the use of fractional processes. We also remain agnostic about the source of long memory.

The literature therefore provides two justifications for the use of a short span of high frequency data when testing for long memory. First, a short span of high frequency data facilitates the use of much larger data sets, thereby increasing the power of the modified R/S and KPSS tests. Second, a short span of high frequency data decreases the likelihood of spuriously detecting long memory due to the presence of occasional breaks.

2.4 Previous empirical evidence supporting long memory in volatility

A summary of the research documenting long memory in equity and currency market volatility is presented in Table 1. Much of the research has focused on US equities and the Deutschemark-U.S.$ and indicates that absolute returns, their power transformations or squared returns exhibit long memory.

(Insert Table 1)

The research raises a number of important issues. First, a variety of procedures have found long memory in volatility across currency and equity markets. Nonetheless, findings of long memory have been sensitive to the test performed and the specified parameters within that test (Breidt *et al*, 1998). Second, those papers that employ short spans of high frequency data (Andersen and Bollerslev, 1997a; 1997b; 1998; Dacorogna *et al*, 1993) are able to uncover long memory dependencies. When operating in the time domain, these dependencies can only be uncovered once the effects of the intraday periodicity in volatility have been removed (see below). Third, whilst the implied autocorrelation functions from fractional processes provide a better fit than those from short memory processes, for higher order lags the implied autocorrelation function tends to be too high (Ding *et al*, 1993; Dacorogna *et al,* 1993; Ding and Granger, 1996). Fourth, most papers (particularly those that employ daily or lower frequencies) use samples that span long time periods. To avoid the spurious detection of long memory from structural breaks, some supplement their results with the same procedures over sub-samples (Lobato and Savin, 1998; Giriatis *et al*, 2001; Breidt *et al*, 1998). These papers however tend to perform no formal structural break testing, relying on the *ad hoc* creation of subsamples. The research therefore either ignores the possibility of structural breaks or seeks to address this by either employing short spans of high frequency data or through the *ad hoc* creation of sub-samples.[7]

3. Data and Methodology

This paper employs two data sets: (a) daily data commencing on January 4, 1988 and ending July 30, 1999. Data on the index was obtained from IRESS. The data for the futures was obtained from the Sydney Futures Exchange WWW site (http://www.sfe.com.au); (b) intraday data at the five minute frequency commencing on January 2, 1998 and ending on October 29, 1999. Intraday data on the index at the five minute frequency was obtained from IRESS. Tick by tick data on the futures was obtained from SIRCA.

We adopt the convention of creating continuously compounded returns as the difference between the log of consecutive prices multiplied by 100. Only those days were included where trading occurred in both markets. We use the nearby futures contract with rollover being performed 10 trading days prior to expiration.

There are a number of issues relevant in the construction of the intraday data series. First, there is a tradeoff between statistical considerations which suggest that high frequency data provides more reliable volatility estimates and the bias from microstructure effects that increase with data frequency (Andersen *et al*, 1999; Corsi *et al*, 2001). Five minute returns over higher frequencies are attractive in order to avoid some of these market microstructure effects.

Second, there is no consensus on how the five minute returns for the SPI futures series should be constructed from the tick by tick data. Methods include the use of linear interpolation or the first or last transaction in each time interval. We follow Fleming *et al* (1996) who generate five minute returns from tick by tick data, by establishing a grid that contains the *last* transaction in each interval.

Third, trading in equities occurs between 10am to 4.05pm with no lunch break. SPI futures trading is a combination of: a) floor trading from 9:50am to 12:30pm and 2pm to 4:10pm and; b) computerised trading on the Sydney Computerised Market (SYCOM) from 4.40pm to 6.00am. We only examine those time intervals where trading occurs in both markets. We follow Turkington and Walsh (1999), who construct five minute returns from 10.15am to 12.30pm and 2.05pm to 4pm. The opening fifteen minutes of trading plus the first five minutes after the lunchbreak are excluded to minimise the impact of any market opening effects. This procedure results in fifty five minute returns per day. Furthermore we exclude the post Christmas data given the thinness in trading over this period. We also only include those days where a full day of trading occurred in both markets.[8]

Fourth, whilst we use five minute returns to help minimise the market microstructure effects, the data is likely to exhibit intraday periodicity in volatility. Following Andersen and Bollerslev (1997a; 1997b; 1998) we treat the intraday periodicity as deterministic. By filtering out these deterministic effects, the remaining stochastic components of the process may be observed. If long memory in the short span of high frequency data can be uncovered from the deseasonalised series, long memory is likely to be an inherent characteristic of the data.

We employ the simple filter detailed in Andersen *et al* (2002). Here the intraday volatility pattern for each five minute return period is estimated by averaging the squared returns ($r_{i,t}^2$) across the (N) days in the various intraday intervals. This produces 50 intraday seasonal factors (s_i^2);

$$s_i^2 = \frac{1}{N}\sum_{i=1}^{N} r_{i,t}^2 \qquad i = 1,\ldots,50 \qquad (5)$$

Deseasonalised returns ($r_{i,t}^{s}$) are then calculated via the following transformation;

$$r_{i,t}^{s} = \frac{r_{i,t}}{s_i} \qquad (6)$$

To examine whether the All Ordinaries Index and its SPI futures exhibit long memory in volatility we perform two procedures. The first obtains spectral density estimates of *d* for squared returns using the procedure developed by Robinson (1994). The fit of the implied autocorrelation function is then used to assess the reasonableness of the estimate. Similar approaches can be found in Andersen and Bollerslev (1997a; 1998), Granger and Ding (1996), Ding and Granger (1996) and Breidt *et al* (1998). This approach however can be criticised given that it does not apply rigorous statistical testing procedures (Giriatis *et al*, 2001). The second procedure therefore employs the modified R/S and KPSS tests for long memory on the returns and squared returns.[9]

When using the daily data these procedures can be applied directly to the returns and squared returns. When using the high frequency data, the deseasonalised squared returns will be used to obtain a spectral density estimate and examine the fit of the implied autocorrelation function. The modified R/S statistic and KPSS test statistic will be applied to the raw and the deseasonalised squared returns. The results should be insensitive to the deseasonalisation, given that both statistics are robust against any short memory dependence.

4. Results

4.1 Daily data

In both markets, the squared returns measure of volatility displays two significant spikes on 16 October 1989 and 28 October 1997. In each case large negative returns were experienced in line with the anniversary of Black Monday. An additional large one off spike in volatility was observed in the futures on 11 January 1988. Given our desire to examine volatility under normal conditions the returns on these abnormal days were removed.[10] Figures 1 and 2 graph the squared returns for the corrected series. Both markets exhibit similar patterns of volatility clustering, with the futures displaying higher levels of volatility than the index.

(Insert Figures 1 and 2)

The autocorrelograms for the squared returns in Figures 3 and 4 exhibit a hypergeometric rate of decay similar to that observed in previous studies (see Ding *et al,* 1993; Dacorogna *et al*, 1993; Ding and Granger, 1996; Andersen and Bollerslev, 1997a; 1997b; 1998, Breidt *et al*, 1998). The spectral density estimate of d for the squared index returns $r_{s,t}^2$ is 0.2081, the estimate for the squared futures returns $r_{f,t}^2$ is 0.2565. These estimates also support the existence of long memory in volatility (given that $0 < d < 1$). Despite these estimates being low relative to that commonly observed in financial markets (usually between 0.3 to 0.4), the fit of the implied autocorrelation functions in Figures 3 and 4, suggest that the estimates of d are reasonable.[11]

(Insert Figures 3 and 4)

In the second stage we tested for long memory in returns and squared returns in the index and its futures using Lo's modified R/S and the KPSS test statistics. The results are presented in Tables 2 and 3 and indicate that returns in both markets exhibit short memory. This is consistent with Lo (1991), Crato (1994), Lobato and Savin (1998), Jacobsen (1996) and Bollerslev and Wright (2000).

The tests using squared returns suggest that the SPI futures exhibit long memory in volatility. With respect to the volatility of the index, the same procedures provide conflicting results. Lo's R/S statistic supports long memory in index volatililty, while the KPSS test rejects long memory. The rejection of long memory by the KPSS test is therefore inconsistent with Lo's R/S test, the spectral density estimate and the autocorrrelation function, which all support long memory in index volatility. It is also inconsistent with the findings of long memory in the futures volatility.[12]

(Insert Tables 2 and 3)

If the acceptance of the short memory null for the index volatility is a function of the low power of the KPSS test, this may be addressed by increasing the sample size (given the consistency of the test statistic). The use of high frequency data is one way to address this issue and is the subject of the next section.

4.2 Intraday Data

In this section we present the results obtained using the intraday data set. We first examine the volatility of the index and its futures using squared five minute returns. An abnormally large index return from 10.15am to 10.20am on June 18, 1998 is observed. Given our desire to examine volatility under normal conditions this observation is removed from both markets.[13] Figures 5 and 6 graph the squared five minute returns for the corrected series. The futures exhibit greater volatility than the index, with most of the significant spikes in both markets occuring within the first hour of trading.

(Insert Figures 5 and 6)

Figures 7 and 8 display the autocorrelograms for the squared five minute returns. They display a distinct U shape across each day, where the U shapes appear to be slowly decreasing. These characteristics are very similar to those observed in the five minute squared return autocorrelograms on the Deutschemark-U.S $ (Andersen and Bollerslev, 1997a; 1997b; 1998) and the S&P500 (Andersen and Bollerslev, 1997b). The intraday periodicity however hides any of the low frequency dynamics in the process, highlighting the importance of deseasonalising the data.

(Insert Figures 7 and 8)

Figure 9 displays the estimated seasonals (s_i^2) for both markets. The intraday volatility of the index and its futures appear to exhibit the reverse J shape commonly identified in equities (Goodart and O'Hara, 1997; Andersen and Bollerslev, 1997b; Tse, 1999). The higher seasonals for the futures are expected given the higher levels of volatility in this market. Note that the lunch break (between observations 28 and 29) has a larger effect on the futures volatility than the index. This is a result of the data construction. Futures trading after the lunchbreak will respond to the information received over the previous 1.5 hour period. The lunchbreak for the index has been artificially created (given that trading occurs over this period) meaning that there is very little change in index volatility over this time.

(Insert Figure 9)

Figures 10 and 11 display the deseasonalised squared returns measures of volatility $\left(r_{i,t}^{s}\right)^{2}$. Both markets display similar patterns of volatility clustering.

(Insert Figures 10 and 11)

The spectral density estimate of *d* for the deseasonalised squared returns in both markets is 0.23, which is close to the estimates at the daily frequency. This is as expected given the insensitivity of *d* to the effects of temporal aggregation. The fit of the implied autocorrelation functions in Figures 12 and 13 suggests that these estimates are reasonable and support the existence of long memory in volatility in both markets.

(Insert Figures 12 and 13)

The results of the tests for long memory in returns, squared returns and deseasonalised returns on the index and its futures using the modified R/S and the KPSS test statistics can be seen in Tables 4 to 6. The finding of short memory in returns is consistent with the data at the daily frequency.

The evidence with respect to the volatilities strongly suggests that both markets exhibit long memory in volatility. This was detected in the raw and deseasonalised squared returns. The insensitivity of these results to the removal of the intraday periodicity in volatility is due to the HAC correction discussed above.

The strong findings of long memory in volatility in both markets using intraday data, support the view that the KPSS findings of short memory in daily index volatility, were due to the low power of the test. This finding highlights the importance of using a wide variety of procedures when testing for long memory in volatility. It also highlights the benefits obtained by using a short span of high frequency data.

(Insert Tables 4, 5 and 6)

5. Conclusion

In this paper we have conducted an investigation into the presence of long memory in the volatility of the Australian All Ordinaries Index and its SPI futures, using a long span of daily data and a short span of high frequency data. To test for long memory we obtained spectral density estimates of the fractional differencing parameter, examined the fit of the implied autocorrelation function, and calculated the modified R/S and KPSS tests for long memory. For both data frequencies, the results strongly support the presence of long memory in the volatility of the SPI futures. With respect to the index, the same procedures supported the existence of long memory in volatility, except for the KPSS test at the daily frequency, which supported short memory in volatility. This anomaly was explained by the low power of the KPSS test.

The results have therefore highlighted the importance of applying a number of alternative procedures when testing for long memory in volatility. They have also illustrated that there are very good reasons to employ short spans of high frequency data when testing for long memory. Not only does this procedure minimise the chance of spuriously detecting long memory (due to the presence of occasional breaks), it also enables a significant increase in the sample size, which is important given that the tests for long memory have low power.

These results suggest that when long term dependencies are important, short memory processes (like the GARCH class of processes) should not be used to model the volatility dynamics of the Australian All Ordinaries Index and its SPI futures. Under these circumstances, fractionally integrated processes (like the FIGARCH process) are likely to be more appropriate. These issues will be pursued in subsequent research.

Notes

[1] There are a number of other definitions of long memory in the literature. Refer Baillie (1996) and Davidson (2002).

[2] The modified R/S statistic collapses to the R/S statistic if $q = 0$.

[3] This view is supported by empirical studies which find that different information flows impart different volatility dynamics. See Andersen and Bollerslev (1998), Crain and Lee (1995), Ederington and Lee (1993), Jones *et al* (1998), Leng (1996).

[4] The mixture of distributions hypothesis was proposed by Clarke (1973) and subsequently developed by Epps and Epps (1976) and Tauchen and Pitts (1983).

[5] A heavy tailed regime duration implies that there may be a long lasting tranquil period, followed by a period of frequent regime switching. Liu (2000) shows that it is the heavy tailed regime duration that generates the long memory, given that the conventional Markov chain regime switching models exhibit short memory.

[6] See Kirman and Teysierre (2001), Granger and Hyung (1999), Hyung and Franses (2001) and Kim and Kon (1999) for alternative break point detection methods.

[7] Granger and Hyung (1999) seek to address these limitations. They remove the effects of identified breaks on the S&P500 and show that some sub-periods exhibit long memory in volatility whilst others do not. It is therefore concluded that a model

incorporating both structural breaks and the possibility of a fractional unit root may be optimal. This avenue is not pursued here and is an area for further research.

[8] The procedures were also applied to a series constructed as follows; a) rollover on expiration, b) inclusion of the post Christmas period and c) inclusion of returns from 10am to 10.15am and from 2pm to 2.05pm. This had very little effect on the results.

[9] The spectral density estimates and the modified R/S and KPSS test statistics are estimated using Davidson's Long Memory Modelling version 2.

[10] The inclusion of these observations do not effect the conclusions drawn.

[11] We employ the asymptotic approximation for the implied autocorrelation function for fractional white noise. See Baillie (1996) for details.

[12] The presence of first order serial correlation in index returns meant that the tests were also performed on AR(1) filtered index returns. The filtering of returns had very little effect on the test statistics and have not been reported.

[13] The results are insensitive to the removal of this observation.

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Table 1 Evidence supporting long memory in volatility

Reference	Market	Data	Series	Method	Results
Taylor (1986)	40 financial time series	Daily	$\lvert r_t \rvert$, r_t^2	ACF	ACF decays slowly and exhibits long memory
Ding *et al* (1993)	S&P500 NYSE DAX	Daily 1/28 – 8/91	$\lvert r_t \rvert$, r_t^2	ACF, GARCH, APARCH	ACF significant for over 10 years and exhibits long memory. Implied ACF from GARCH inappropriate.
Dacorogna *et al* (1993)	DM-USD	20 minute 3/86 – 3/90	$\lvert r_t \rvert$	ACF	ACF follows hyperbolic rate of decay. For higher lags implied acf too high.
Baillie *et al* (1996)	DM-USD	Daily 3/79 – 12/92	r_t^2	FIGARCH, GARCH & IGARCH	FIGARCH(1,d,0) best model.
Bollerslev & Mikkelsen (1996)	S&P500	Daily 1/53 – 12/90	r_t^2	GARCH, IGARCH, FIGARCH, IEGARCH, FIEGARCH	AR(3)-FIEGARCH(1,*d*,1) best model.
Ding & Granger (1996)	S&P500 Nikkei DM-USD	Daily S&P 1/28-8/91	$\lvert r_t \rvert$, r_t^2	ACF LM-ARCH.	ACFs decay hyperbolically. S&P500 best modeled with MA(1)-LM-ARCH. From lag 500-2500 implied ACF is too high.
Granger & Ding (1996)	S&P500	Daily 1/28 – 8/90	$\lvert r_t \rvert$, r_t^2	ACF, FIARCH Estimate of d via time and spectral domain	Estimate of d via time domain more appropriate than spectral domain. Implied ACF from FIARCH better fit than GARCH. Implied ACF not appropriate over the first 20 lags.
Andersen & Bollerslev (1997a)	DM-USD	5 minute 10/92 – 9/93	$\lvert r_t \rvert$	Spectral density function, ACF, Time and spectral estimates of d	Spectrum and ACF generally consistent with long memory. Autocorrelogram only considered for lags up to 10 days. Estimate of d supportive of long memory.
Andersen & Bollerslev (1997b)	DM-USD S&P500	5 minute 10/92 – 9/93	$\lvert r_t \rvert$	ACF	ACF decays at a hyperbolic rate
Lobato & Savin (1998)	S&P500	Daily 7/62-12/94	r_t, $\lvert r_t \rvert$, r_t^2	Test for long memory	r_t short memory, $\lvert r_t \rvert$ & r_t^2 long memory.Dividing sample into 2 sub-samples does not change the result.

Andersen & Bollerslev (1998)	DM-USD	5 minute 10/92 –/93	$\|r_t\|$	Time domain estimation of d, ACF	Estimate of d supports long memory. Implied ACF closely follows ACF. Only examines lags up to 10 days.
Breidt *et al* (1998)	Value & equally weighted CRSP	Daily 7/62 – 7/89	r_t^2	ACF, Spectral estimate of d, R/S statistic, LMSV	Value weighted supports long memory. Mixed results for the equally weighted. LMSV better fit than GARCH and EGARCH.
Liu (2000)	S&P500	Daily 4/28 – 12/95	r_t^2	FIEGARCH/ LMSV & regime switching models.	FIEGARCH and LMSV with spline errors preferred. All LM models have similar ACFs and forecasts.
Giraitis et al (2001)	£ / USD	Daily, 4000 obs to 1/97	r_t^2	Modified R/S KPSS, V/S	Sample divided into 4 blocks of 1000 observations. LM in 3 out of 4 blocks.

$|r_t|$ represents absolute returns, r_t^2 represents squared returns.

ACF = the autocorrelation function
LM = Long Memory
SM = Short Memory
GARCH = Generalised Autoregressive Conditional Heteroscedasticity (Bollerslev, 1986)
APARCH = Asymmetric Power ARCH (Ding *et al*, 1993)
EGARCH = Exponential GARCH (Nelson, 1991)
IGARCH = Integrated GARCH (Engle and Bollerslev, 1986)
FIGARCH = Fractionally Integrated GARCH (Ballie *et al*, 1996)
FIEGARCH = Fractionally Integrated Exponential GARCH (Bollerslev and Mikkelsen, 1996)
LM-ARCH = Long Memory ARCH (Ding and Granger, 1996)
LMSV = Long Memory Stochastic Volatility (Breidt *et al*, 1998)

Table 2 Testing for long memory in daily returns

Test	Index		Futures	
	Statistic	Conclusion	Statistic	Conclusion
Lo's R/S	1.0089	Short memory	0.9738	Short memory
KPSS	0.0329	Short memory	0.0373	Short memory

Significance level of 5% - Critical values – R/S = 1.747, KPSS = 0.463.

Table 3 Testing for long memory in daily squared returns

Test	Index		Futures	
	Statistic	Conclusion	Statistic	Conclusion
Lo's R/S	1.8837	Long memory	2.5461	Long memory
KPSS	0.2018	Short memory	2.0493	Long memory

Significance level of 5% - Critical values – R/S = 1.747, KPSS = 0.463.

Table 4 Testing for long memory in five minute returns

Test	Index		Futures	
	Statistic	Conclusion	Statistic	Conclusion
Lo's R/S	1.0800	Short memory	1.2243	Short memory
KPSS	0.1190	Short memory	0.0920	Short memory

Significance level of 5% - Critical values – R/S = 1.747, KPSS = 0.463.

Table 5 Testing for long memory in five minute squared returns

Test	Index		Futures	
	Statistic	Conclusion	Statistic	Conclusion
Lo's R/S	3.5644	Long memory	3.9192	Long memory
KPSS	1.4494	Long memory	3.9009	Long memory

Significance level of 5% - Critical values – R/S = 1.747, KPSS = 0.463.

Table 6 Testing for long memory in deseasonalised five minute squared returns

Test	Index		Futures	
	Statistic	Conclusion	Statistic	Conclusion
Lo's R/S	3.4821	Long memory	3.8006	Long memory
KPSS	1.3661	Long memory	3.8652	Long memory

Significance level of 5% - Critical values – R/S = 1.747, KPSS = 0.463.

Figure 1 Daily squared index returns

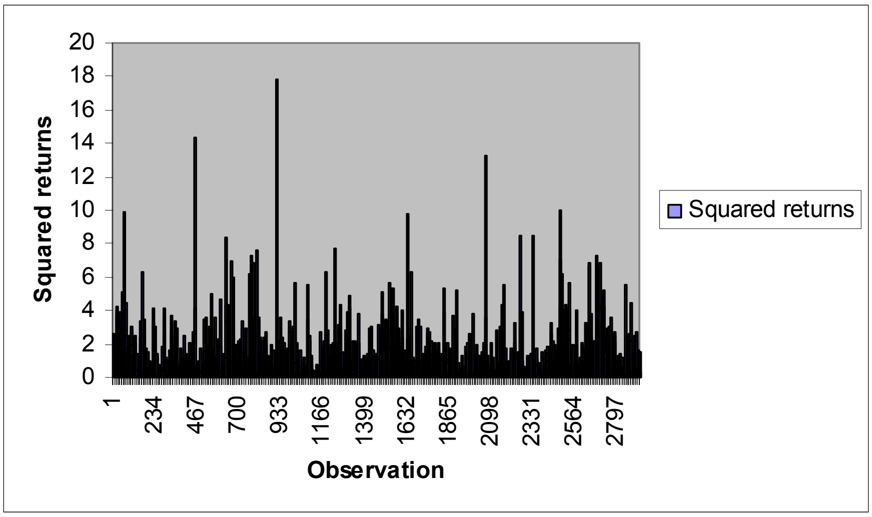

Figure 2 Daily squared futures returns

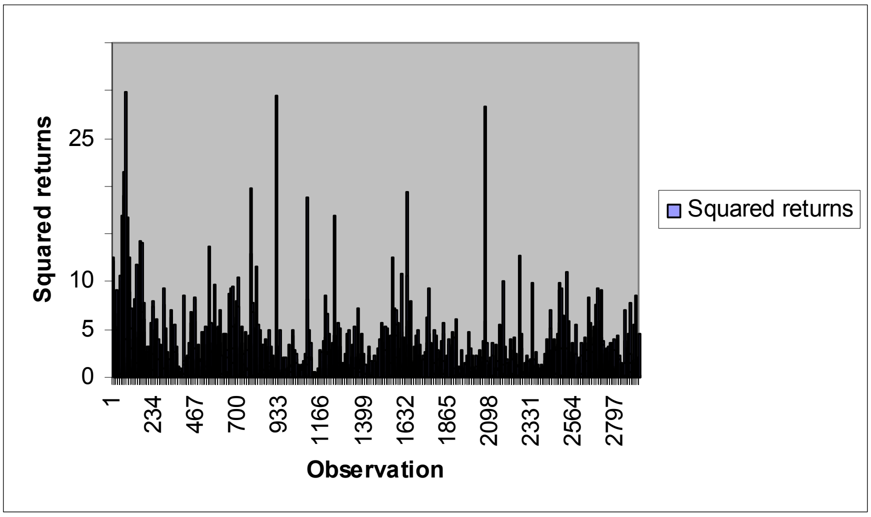

Figure 3 Fit of implied Acf, Daily squared index returns

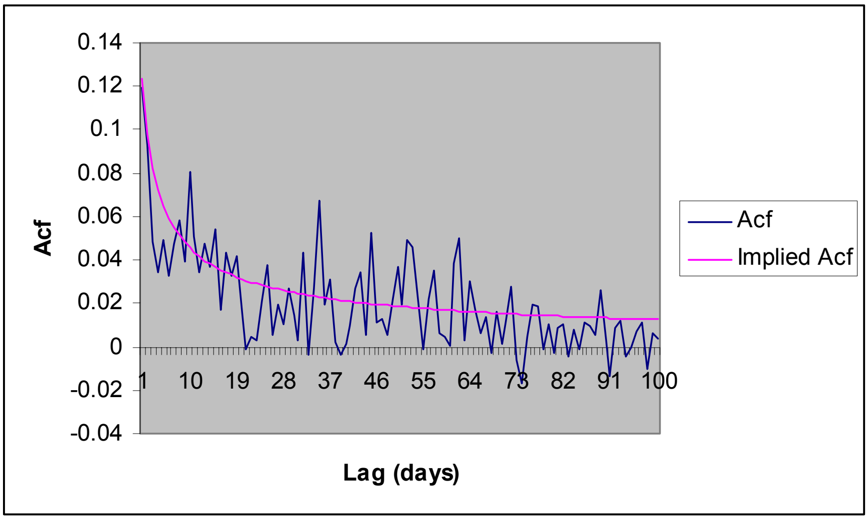

Figure 4 Fit of implied Acf, Daily squared futures returns

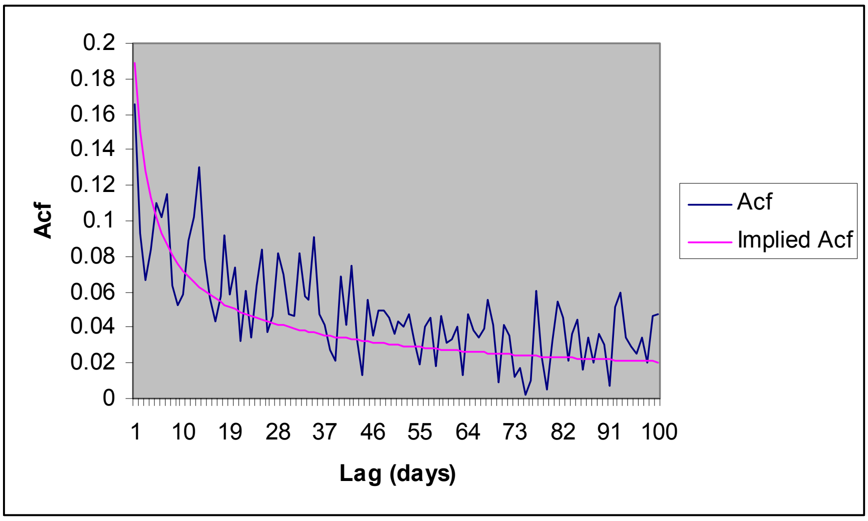

Figure 5 Five minute squared index returns

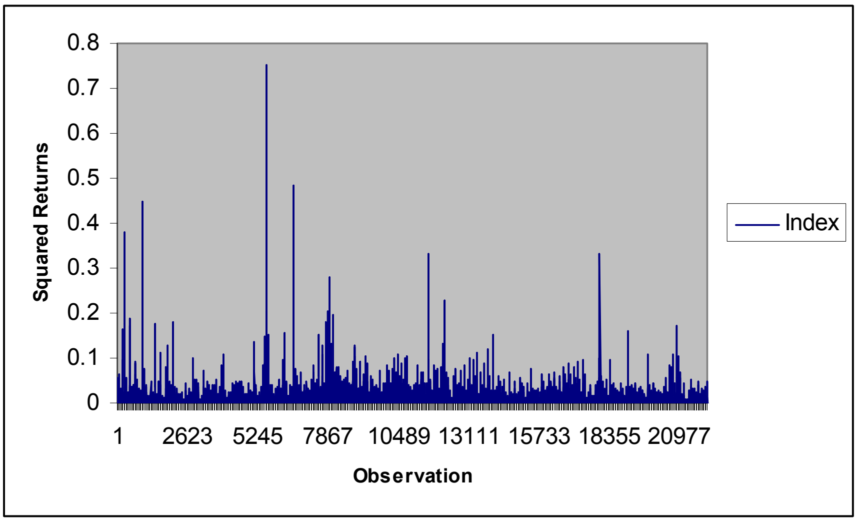

Figure 6 Five minute squared futures returns

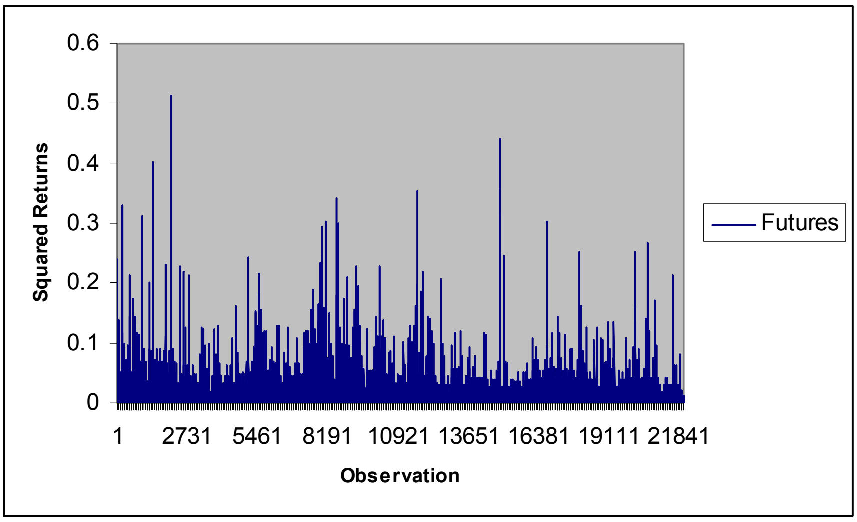

Figure 7 Acf, squared index returns

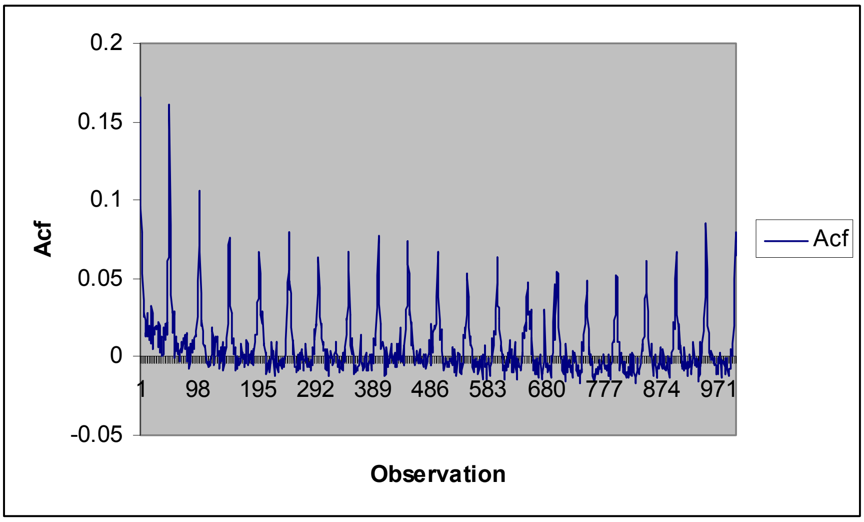

Figure 8 Acf, squared futures returns

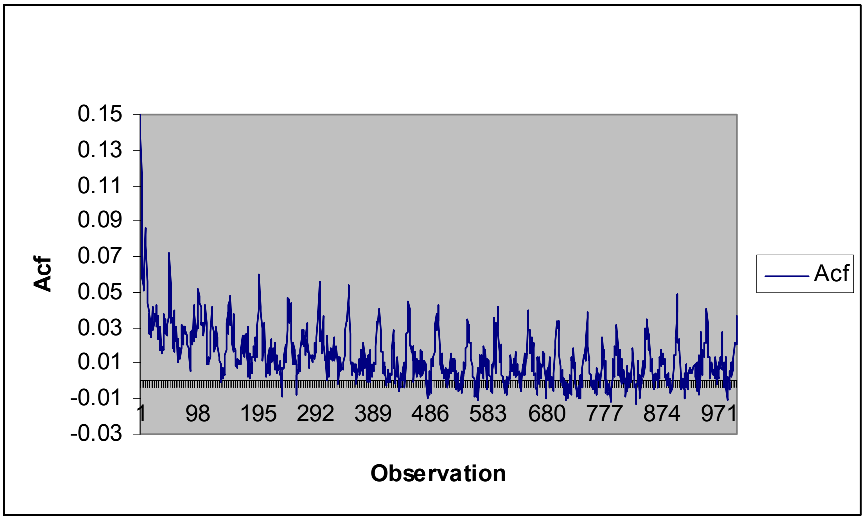

Figure 9 Seasonals

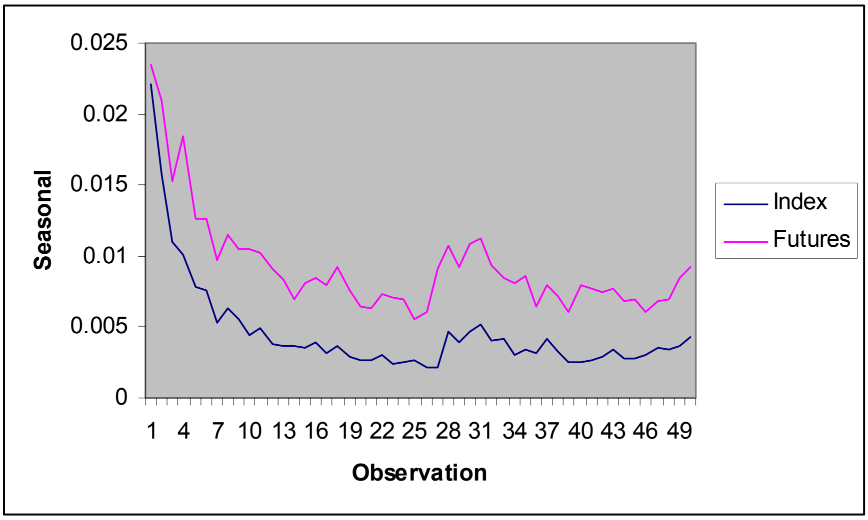

Figure 10 Five minute deseasonalised squared index returns

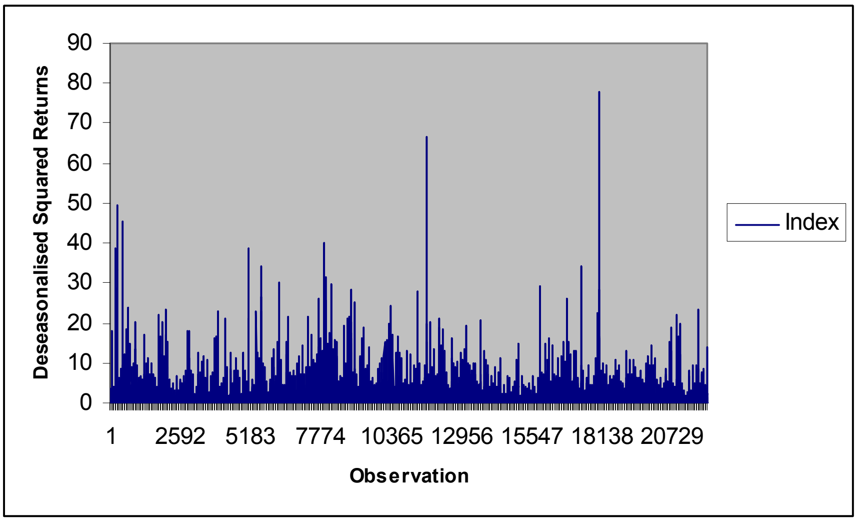

Figure 11 Five minute deseasonalised squared futures returns

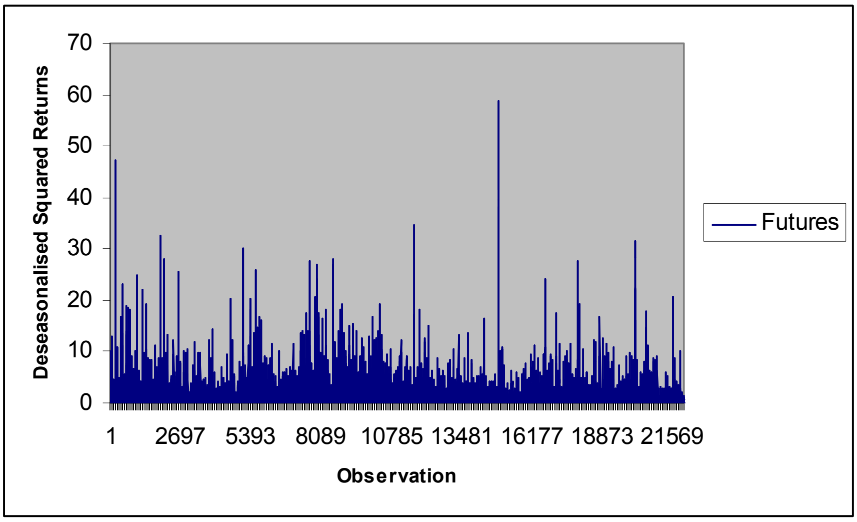

Figure 12 Fit of Implied Acf, deseasonalised squared index returns

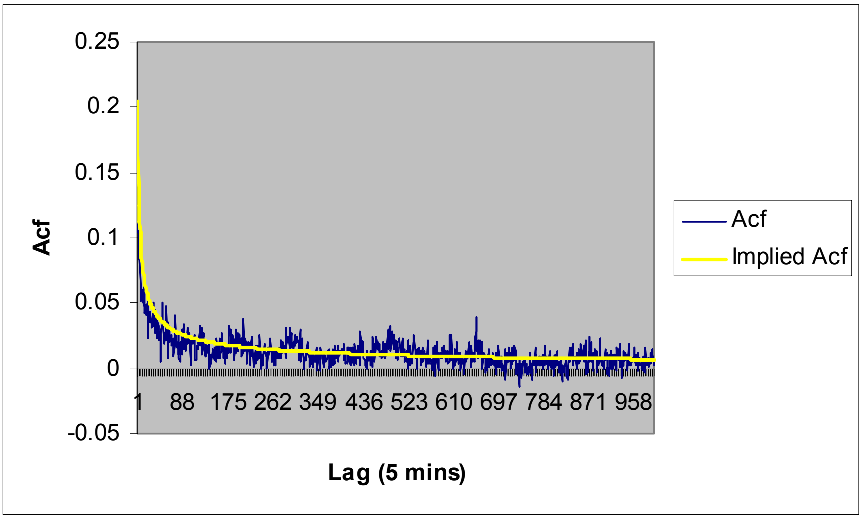

Figure 13 Fit of Implied Acf, deseasonalised squared futures returns