ISSN 1440-771X

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Basis convergence and long memory in volatility when dynamic hedging with SPI futures

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Working Paper 06/04

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JEL CLASSIFICATION: G0, G15

Abstract

This paper examines the importance of basis convergence and long memory in volatility when estimating minimum variance hedge ratios (MVHRs) using SPI futures. The paper employs a bivariate FIGARCH model with a maturity effect to model the joint dynamics of the Australian All Ordinaries Index and the basis. This new approach allows for long memory in volatility, time varying correlations and the convergence between the All Ordinaries Index and its SPI futures over the life of the futures contract. The results illustrate the importance of these effects when modelling the joint dynamics and when estimating dynamic MVHRs.

Keywords: basis convergence, long memory, bivariates FIGARCH, dynamic minimum variance hedge ratios.

I. Introduction

The importance of managing risk exposure has seen a voluminous futures hedging literature develop over the last half a century. Working (1953a, 1953b, 1961) emphasised the importance of incorporating changes in the basis (the difference between the spot and the futures prices) into the hedging decision. Much of the subsequent literature however ignores the basis and its convergence to zero over the life of the contract, adopting the portfolio approach articulated by Ederington (1979). To estimate minimum variance hedge ratios (MVHRs), the portfolio approach requires estimates of the futures variance and the covariance between the spot and the futures markets. These estimates are typically obtained via a regression model between the spot and futures markets. This methodology however does not allow for basis convergence over the life of the contract.

Long memory in volatility has been documented across a range of equity indices; the S&P500 (Ding *et al*, 1993; Bollerslev and Mikkelsen, 1996; Ding and Granger, 1996) the NYSE (Ding *et al*, 1993), the Nikkei (Ding and Granger, 1996), the CRSP (Breidt *et al*, 1998), and the DAX (Ding *et al*, 1993). Despite these findings, the hedging literature fails to allow for long memory in volatility when estimating dynamic MVHRs.

This paper therefore seeks to examine the importance of basis convergence and long memory in volatility when conducting a minimum variance hedge using SPI futures on the Australian All Ordinaries Index. The paper supplements Dark (2003c) where bivariate error correction GARCH and FIGARCH models between the All Ordinaries

Index and its Share Price Index (SPI) futures are used to estimate dynamic MVHRs. Dark (2003c) documents the importance of allowing for long memory in volatility and time varying correlations. However the approach does not exploit the convergence between the All Ordinaries Index and its SPI futures over the life of the futures contract.

To allow for basis convergence we extend the procedure employed by Chen *et al* (1999). Rather than estimating a model between the equity index and its futures, Chen *et al* (1999) estimate a bivariate GARCH model with a maturity effect to describe the joint dynamics between the Nikkei 225 index and its basis. By including a maturity effect in the basis dynamics, the time to maturity is able to influence the behaviour of the basis and hence the estimated MVHRs.

Chen *et al* (1999) estimate a bivariate GARCH process that assumes constant correlation. In this paper we estimate bivariate GARCH and FIGARCH processes with and without maturity effects that allow for time varying correlations. We therefore compare the hedging performance of MVHRs estimated via the following; i) a bivariate GARCH process between the index and basis, with and without maturity effects; ii) a bivariate FIGARCH process between the index and basis, with and without maturity effects; and iii) a bivariate error correction FIGARCH process between the index and the futures.

The model results support the existence of long memory in the All Ordinaries Index and basis volatilities as well as their covariance. The model results are also consistent with Castelino and Franses (1982), who showed that the volatility of changes in the basis decreases as the futures contract approaches maturity. The importance of basis convergence and long memory in volatility is further illustrated when estimating MVHRs, given that the bivariate FIGARCH model with maturity effects generally provides superior hedging outcomes. Section II will review the relevant literature. This will be followed by the bivariate models in Section III. Section IV will present the data and the estimated models. Section V will examine the hedging outcomes over various horizons. Section VI will conclude.

II. Literature Review

The conventional approach to MVHR (Φ) determination seeks to minimise the variability in the expected hedged return. The MVHR is equal to

(1) $$\Phi = \frac{\sigma_{sf}}{\sigma_f^2}$$

where σ_{sf} is the covariance between the spot and the futures and σ_f^2 is the futures variance. A popular method of estimating the MVHR is via the slope coefficient from an ordinary least squares (OLS) regression between the spot and futures. See Ederington (1979) and Figlewski (1986) for further details.

This framework has been extended to allow for conditional heteroscedasticity (Kroner and Sultan, 1993) and cointegration between the spot and its futures (Ghosh, 1993; Lien, 1996). Dynamic MVHRs are commonly estimated by

(2) $$\Phi_t = \frac{\sigma_{sf,t+1}}{\sigma^2_{f,t+1}}$$

where Φ_t is the dynamic MVHR at time *t*, $\sigma_{sf,t+1}$ is the conditional covariance between the spot and the futures at time *t*+1, and $\sigma^2_{f,t+1}$ is the futures conditional variance at time *t*+1. Estimates of the dynamic MVHR are commonly made via a bivariate error correction GARCH model between the spot and the futures. See Cecchetti *et al* (1988), Baillie and Myers (1991), Sephton (1993), Park and Switzer (1995), Koutmos and Pericli (1998), Lien and Tse (1999) and Sim and Zurbreugg (2000).

Lee (1999) is critical of this dynamic approach given that it does not seek to minimise the variability in portfolio returns over the life of the hedge, and fails to take into account any interperiod dependencies. When confined to two assets, and assuming a hedge over r periods, the multi period dynamic MVHR (MPMVHR) of Lee (1999) can be expressed as

(3) $$\Phi_t = \frac{\sigma_{sf,t+1} + ... + \sigma_{sf,t+r}}{\sigma^2_{f,t+1} + ... + \sigma^2_{f,t+r}}$$

The conventional approach to MVHR estimation (equations 1, 2 and 3) treats the futures as any other asset that is highly correlated with the spot. Castelino (1990a) is critical of this approach given its failure to allow for convergence. If the spot and futures converge over the life of the futures contract, basis risk decreases over the life of the hedge. If the spot and futures are equal on expiration, a hedge lifted on expiration of the futures contract is riskless and dictates a hedge ratio of one. By failing to impose basis convergence, the conventional approaches therefore ignore information that could be used when estimating MVHRs.[1] Castelino (1989, 1990a, 1990b, 1992) therefore develops a MVHR that reflects the time dimension to basis risk, by adjusting the hedge ratio away from unity as the hedge lifting date differs from the contract expiration date.[2]

Chen *et al* (1999) allow for convergence and conditional heteroscedasticity by modelling the basis and spot as a bivariate GARCH process with a maturity effect. By specifying the conditional mean and variance of the basis as a function of time to maturity, the maturity of the contract influences the behaviour of the basis. Chen *et al* (1999) re-express the dynamic MVHR (equation 2) as a function of the time to maturity as follows;

(4)
$$\Phi_t = \frac{\sigma_{bs,_{t+1}} + \sigma^2_{s_{t+1}}}{\sigma^2_{s_{t+1}} + \sigma^2_{b_{t+1}} + 2\sigma_{bs,_{t+1}}}$$

where $\sigma_{bs,_{t+1}}$ represents the covariance between the basis and the spot at time t+1, $\sigma^2_{s_{t+1}}$ represents the variance of the spot at time t+1, and $\sigma^2_{b_{t+1}}$ the variance of the basis at time t+1. Therefore by modelling the spot and the basis, rather than the spot and its futures, convergence can be explicitly allowed for.

Whilst the approach allows for the effects of convergence on the change in the basis and its volatility, it does not impose a MVHR of unity if the hedge is lifted on the expiration date. This seems appropriate given that there is unlikely to be perfect convergence between the All Ordinaries and its SPI futures, due to the large transaction costs associated with arbitrage.[3]

To estimate the MVHRs in equation 4, Chen *et al* (1999) employ a short memory GARCH process to model the volatility dynamics of the Nikkei 225 and its basis. The findings of Dark (2003a) however suggest that a long memory volatility process may be more appropriate when modelling the All Ordinaries Index. A wide variety of long memory in volatility models have been proposed, see Baillie *et al* (1996) for the Fractionally Integrated GARCH (FIGARCH), Bollerslev and Mikkelsen (1996) for the Fractionally Integrated Exponential GARCH (FIEGARCH), Ding and Granger (1996) for the Long Memory ARCH (LM-ARCH), and Tse (1998) for the Fractionally Integrated Asymmetric Power ARCH (FIAPARCH) models.

To examine the importance of long memory in volatility, we estimate bivariate GARCH and FIGARCH processes between the All Ordinaries Index and the basis. Subsequent to the pioneering work of Teyssiere (1997) very little research has considered the estimation of multivariate FIGARCH processes. See Teyssiere (1997, 1998), Pafka and Matyas (2002), Brunetti and Gilbert (2000) and Dark (2003b) for further details.

III. Model and estimation

To define the model, let S_t and F_t represent the index and its SPI futures at time *t,* and define the basis at time *t* as $B_t = F_t - S_t$. Following Chen *et al* (1999) we model the dynamics of the equity index return ($\frac{\Delta S_t}{S_{t-1}}$) and the normalised change in the basis $\frac{\Delta B_t}{S_{t-1}}$, where Δ represents the first difference. We model the All Ordinaries Index return as an AR(2) process and the normalised change in the basis as an ARMA(3,1) process with a maturity effect, where the variable m_t represents the number of days to maturity at time *t* divided by 100.

$$
\begin{aligned}
&\frac{\Delta S_t}{S_{t-1}} = a_1 + b_1 \frac{\Delta S_{t-1}}{S_{t-2}} + b_2 \frac{\Delta S_{t-2}}{S_{t-3}} + \varepsilon_{s,t} \\
&\frac{\Delta B_t}{S_{t-1}} = a_2 + \lambda_1 (m_t) + b_3 \frac{\Delta B_{t-1}}{S_{t-2}} + b_4 \frac{\Delta B_{t-2}}{S_{t-3}} + b_5 \frac{\Delta B_{t-3}}{S_{t-4}} + \varepsilon_{b,t} (m_t)^{\lambda_2} + b_6 \varepsilon_{b,t-1} (m_{t-1})^{\lambda_2}
\end{aligned} \tag{5}
$$

with

$$
\begin{pmatrix} \varepsilon_{s,t} \\ \varepsilon_{b,t} \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{s,t}^2 & \sigma_{sb,t} \\ \sigma_{sb,t} & \sigma_{b,t}^2 \end{pmatrix} \right] \tag{6}
$$

The conditional mean allows for basis convergence by allowing the intercept, $a_2 + \lambda_1 (m_t)$, to change as the contract approaches maturity. If λ_1>0 this suggests that the maturity effect decreases the constant term as the futures contract approaches maturity. Following Chen *et al* (1999), the volatility of the basis is a power function of maturity. The basis errors at time *t* ($\varepsilon_{b,t}$) and *t*-1 ($\varepsilon_{b,t-1}$), are therefore scaled by $(m_t)^{\lambda_2}$ and $(m_{t-1})^{\lambda_2}$ respectively. The power function means that the covariance and basis volatility estimates are a function of a GARCH/FIGARCH component and a maturity component (captured by $(m_t)^{\lambda_2}$). i.e $Cov_t\left[\frac{\Delta B_t}{S_{t-1}},\frac{\Delta S_t}{S_{t-1}}\right]=\sigma_{sb,t}m_t^{\lambda_2}$ and $Var_t\left[\frac{\Delta B_t}{S_{t-1}}\right]=\sigma_{b,t}^2 m_t^{2\lambda_2}$. If $\lambda_2 > 0$ then the basis volatility and the covariance approaches zero as the contract approaches maturity.[5]

We consider two alternative conditional covariance specifications. The first adopts the diagonal GARCH(1,1) specification of Bollerslev *et al* (1988) which allows for time varying correlations. Using the vech representation for the covariance matrix, the diagonal GARCH(1,1) model can be expressed as

(7)
$$\begin{pmatrix}\sigma_{s,t}^2\\ \sigma_{sb,t}\\ \sigma_{b,t}^2\end{pmatrix}=\begin{pmatrix}\omega_s+\alpha_s\varepsilon_{s,t-1}^2+\beta_s\sigma_{s,t-1}^2\\ \omega_{sb}+\alpha_{sb}\varepsilon_{s,t-1}\varepsilon_{b,t-1}+\beta_{sb}\sigma_{sb,t-1}\\ \omega_b+\alpha_b\varepsilon_{b,t-1}^2+\beta_b\sigma_{b,t-1}^2\end{pmatrix}$$

The second adopts the diagonal FIGARCH(1,*d*,1) specification of Teyssiere (1997)

(8)
$$\begin{pmatrix}\sigma_{s,t}^2\\ \sigma_{sb,t}\\ \sigma_{b,t}^2\end{pmatrix}=\begin{pmatrix}\frac{\omega_s}{1-\beta_s}+\left(1-\frac{(1-\phi_s L)(1-L)^{d_s}}{1-\beta_s L}\right)\varepsilon_{s,t}^2\\ \frac{\omega_{sb}}{1-\beta_{sb}}+\left(1-\frac{(1-\phi_{sb} L)(1-L)^{d_{sb}}}{1-\beta_{sb} L}\right)\varepsilon_{s,t}\varepsilon_{b,t}\\ \frac{\omega_b}{1-\beta_b}+\left(1-\frac{(1-\phi_b L)(1-L)^{d_b}}{1-\beta_b L}\right)\varepsilon_{b,t}^2\end{pmatrix}$$

We adopt the diagonal parameterisations, given that Dark (2003b,c) illustrates the importance of allowing for time varying correlations when modelling the dynamics between the All Ordinaries Index and its SPI futures. To examine the impact of convergence, we also estimate the diagonal GARCH and FIGARCH models where the maturity effects are disabled (so that $\lambda_1 = \lambda_2 = 0$).

We estimate the FIGARCH models via Quasi Maximum Likelihood methods. We use a truncation lag of 1000 observations, with the pre-sample values equal to the unconditional variance estimate. This procedure therefore approximates the fractional component as a long (but finite) autoregressive process. Numerical procedures are used to impose positive definiteness in the diagonal FIGARCH model.[6] Positive definiteness in the diagonal GARCH model is imposed via the conditions in Silberberg and Pafka (2003).

IV. Data and estimation results

The data set consists of daily data commencing on January 4, 1988 and ending October 22, 1999. The period from January 4, 1988 to July 30, 1999 is used for estimation, with the remainder of the sample being used for hedge ratio evaluation. Data on the All Ordinaries Index was obtained from IRESS, the futures was obtained from the Sydney Futures Exchange WWW site (http://www.sfe.com.au). Only those days were included where trading occurred in both markets. We use the nearby futures contract with rollover being performed 10 trading days prior to expiration.

Dark (2003a) establishes the presence of long memory in the volatility of the All Ordinaries Index. Our preliminary investigations therefore only consider the normalised change in the basis and the covariance between the All Ordinaries Index and the normalised change in the basis. The tests of Lo (1991) and Kwiatkowski *et al* (1992) in Table 1 demonstrate that the normalised change in the basis exhibits short memory, whilst the normalised squared change in the basis $\left(\frac{\Delta B_t}{S_{t-1}}\right)^2$ and the covariance $\left(\frac{\Delta S_t}{S_{t-1}}.\frac{\Delta B_t}{S_{t-1}}\right)$ exhibit long memory.

(Insert Table 1)

The presence of long memory in the volatilities and the covariance is further supported by the spectral density estimates of the fractional differencing parameter *(d)* in Table 2. These preliminary results suggest that the diagonal FIGARCH specification will outperform the diagonal GARCH specification.

(Insert Table 2)

The results in Tables 3 and 4 illustrate that the conditional mean estimates are insensitive to changes in the conditional variance specification. The estimates are also insensitive to the disabling of the maturity effects. The FIGARCH estimates of *d* are close to the spectral estimates, and the Box Pierce diagnostics and Nyblom (1989) tests of parameter stability are satisfactory.[7] The Jarque Bera test for normality suggests that the estimates are consistent but not efficient.

The results demonstrate that *both* long memory and maturity effects are important. Information criteria suggest that maturity effects are important and that the FIGARCH models outperform their GARCH counterparts. The results therefore clearly identify the diagonal FIGARCH model with maturity effects as being the best model. The FIGARCH model has the lowest information criteria, it is consistent with the preliminary findings that are supportive of long memory in the volatilities and the covariance, and the diagnostics in Table 4 are satisfactory.

(Insert Tables 3 and 4)

The positive estimate of the maturity effect parameter λ_2, suggests that the volatility of the basis approaches zero as the contract approaches maturity. This result is consistent with Chen *et al* (1999) who estimate the volatility maturity effect parameter for the Nikkei 225 as 0.06. In contrast, the negative estimate of the parameter λ_1, suggests that the maturity effect may increase the constant term in the basis equation as the contract approaches maturity. This effect however appears weak given that the parameter is marginally significant for the GARCH model and insignificant for the FIGARCH model.[8]

V. Hedging outcomes

Section IV has illustrated that allowing for basis convergence and long memory in volatility is important when modelling the joint dynamics of the All Ordinaries Index and the basis. This section seeks to determine whether these effects are significant when estimating dynamic MVHRs.

When estimating MVHRs via the conventional approach, Dark (2003c) suggests that an error correction (EC) diagonal FIGARCH model between the All Ordinaries and its futures should be employed. We therefore seek to compare the hedging performance of this approach with the hedging performance achieved using the four models estimated above.[9]

The single period dynamic MVHR (SPMVHR) requires one period ahead forecasts of the variables in equation 4. We also consider Lee's (1999) MPMVHR, that is modified to allow for basis convergence as follows

(9)
$$\begin{aligned}
\Phi_t &= \frac{\sigma_{sf,t+1} + \dots + \sigma_{sf,t+r}}{\sigma^2_{f,t+1} + \dots + \sigma^2_{f,t+r}} \\
&= \frac{\sigma_{s_{t+1},(b_{t+1}+s_{t+1})} + \dots + \sigma_{s_{t+r},(b_{t+r}+s_{t+r})}}{\sigma^2_{b_{t+1}+s_{t+1}} + \dots + \sigma^2_{b_{t+r}+s_{t+r}}} \\
&= \frac{\left\{\sigma_{sb,_{t+1}} + \sigma^2_{s_{t+1}}\right\} + \dots + \left\{\sigma_{sb,_{t+r}} + \sigma^2_{s_{t+r}}\right\}}{\left\{\sigma^2_{s_{t+1}} + \sigma^2_{b_{t+1}} + 2\sigma_{sb,_{t+1}}\right\} + \dots + \left\{\sigma^2_{s_{t+r}} + \sigma^2_{b_{t+r}} + 2\sigma_{sb,_{t+r}}\right\}} \\
&= \frac{\left\{\sigma_{sb,_{t+1}} + \dots + \sigma_{sb,_{t+r}}\right\} + \left\{\sigma^2_{s_{t+1}} + \dots + \sigma^2_{s_{t+r}}\right\}}{\left\{\sigma^2_{s_{t+1}} + \dots + \sigma^2_{s_{t+r}}\right\} + \left\{\sigma^2_{b_{t+1}} + \dots + \sigma^2_{b_{t+r}}\right\} + 2\left\{\sigma_{sb,_{t+1}} + \dots + \sigma_{sb,_{t+r}}\right\}}
\end{aligned}$$

Table 5 summarises the various methods of MVHR estimation to be considered.

(Insert Table 5)

Tables 6 to 9 provide details of the hedging outcomes achieved over a five, twenty, forty and sixty day hedging horizon commencing 2/8/99. Each MVHR has been ranked according to the variance of the portfolio. Table 10 provides a summary of the ranks, and an overall ranking of each approach. The tables are also followed by Figures 1 to 5 which graph selected MVHRs.

(Insert Tables 6 to 10)

(Insert Figures 1 to 5)

The results illustrate that all MVHRs reduce risk relative to the unhedged position. They are also supportive of dynamic MVHR estimation, given that all dynamic MVHRs outperformed the time invariant naïve and OLS MVHRs.

The results support the estimation of dynamic MVHRs that allow for maturity effects. Tables 6 to 9 and Figures 1 to 2, suggest that MVHRs allowing for maturity effects generally exhibit greater variability in the MVHR, and provide better hedging outcomes. The FIGARCH model with maturity effects also appears to outperform the error correction FIGARCH model. Figure 3 shows that the major divergence between these MVHRs occurs between periods 35 to 42. This coincides with the rollover to the next futures contract and highlights the importance of directly modelling the basis dynamics.

Long memory in volatility also appears important given that all FIGARCH MVHRs outperformed their GARCH counterparts (e.g FIGARCH with maturity effects outperforms GARCH with maturity effects). Figure 4 demonstrates that the FIGARCH MVHRs are generally higher than their GARCH counterparts. This result is consistent with Dark (2003c) who demonstrates that over the hedging period, there was a significant rise in the correlation between the spot and futures markets. As a consequence those approaches to MVHR estimation that were able to capture this increase via higher MVHRs provided the best performance.

The results also demonstrate the poor performance of the MPMVHRs. Figure 5 demonstrates the inability of the MPMVHRs to capture the rise in correlations over the period of hedging. This result is again consistent with the results presented in Dark (2003c).

The results therefore indicate that a single period MVHR that allows for maturity effects and long memory in volatility is likely to provide superior risk reduction. This is confirmed by Table 10 which indicates that this approach generally outperforms the alternative methods of MVHR estimation.

VI. Conclusion

This paper has examined the importance of basis convergence and long memory in volatility when constructing a minimum variance hedge using SPI futures on the Australian All Ordinaries Index. The paper therefore supplements Dark (2003c) and Chen *et al* (1999), by estimating a bivariate diagonal FIGARCH model with a maturity effect to describe the joint dynamics between the All Ordinaries Index and its basis.

The results support the existence of long memory in the All Ordinaries Index and basis volatilities and demonstrate that the volatility of changes in the basis decreases as the futures contract approaches maturity. This modelling approach is in contrast to the conventional approach, which models the dynamics between spot and futures markets via short memory processes (typically the GARCH class of processes), ignoring the maturity effects. The paper has further demonstrated the benefits of this modelling approach when estimating dynamic MVHRs. The results show that a bivariate FIGARCH model between the All Ordinaries Index and the basis that allows for maturity effects generally provides superior hedging outcomes.

Footnotes

[1] Castelino (1990a) also notes that the common practice of estimating the OLS MVHR via first differences in the spot and futures, implies that the basis is covariance stationary. This is invalid, given that basis convergence signifies a time varying mean and variance.

[2] Viswanath (1993) details an alternative approach that allows for basis convergence.

[3] Twite (1998) estimates the transaction costs associated with one round trip as 0.10% of the market value of the SPI futures contract. These costs are large relative to other markets. Kroner and Sultan (1993) employ 0.01% with currency futures, Koutmos and Pericli (1998) 0.0005% with T-bill futures.

[4] This does not necessarily mean that the change in the normalised basis decreases as the contract approaches maturity. This is because the conditional mean is also influenced by the other components of the ARMA(3,1) specification.

[5] This specification imposes the condition that at maturity the basis volatility is zero. The conditional mean specification however only allows for convergence towards zero. The approach is not entirely consistent with the specification in Chen *et al* (1999) where the constant term is also a power function of the time to maturity i.e . $a_2 m_t^{\lambda_1}$. The specification in Chen *et al* (1999) was not employed given that upon estimation this created convergence difficulties.

[6] Conditions for the positive definiteness of the bivariate FIGARCH process have remained elusive. Consequently the FIGARCH estimates may not produce positive definite covariance matrices for all observations.

[7] Nyblom (1989) parameter stability test results available on request.

[8] Furthermore, given that m_t is the number of days to expiration divided by 100, the size of this effect on the constant term is probably economically insignificant.

[9] The bivariate error correction FIGARCH model results used to estimate the dynamic MVHRs are not reported here and are available on request. The results are virtually identical to those presented in Dark (2003b,c) with slight differences due to the different methods used to create returns. Dark (2003b,c) creates continuously compounded returns by taking the log of the first differences. In this paper returns are calculated as $\frac{\Delta S_t}{S_{t-1}}$ and $\frac{\Delta F_t}{F_{t-1}}$.

Table 1 Testing for long memory

Test	Normalised change in the basis		Normalised squared change in the basis		Covariance	
	Statistic	Conclusion	Statistic	Conclusion	Statistic	Conclusion
Lo (1991)	0.56	SM	3.19	LM	2.00	LM
KPSS (1992)	0.01	SM	4.38	LM	1.23	LM

SM = short memory, LM = long memory
Significance level of 5% - Critical values – Lo = 1.747, KPSS = 0.463.

Table 2 Spectral estimates of d

	Squared index returns	Normalised squared change in the basis	Covariance
Spectral estimate of d	0.20	0.26	0.12

Spectral estimates obtained using procedure of Robinson (1994).

Table 3a Parameter estimates and *t* values for All Ordinaries and its basis – mean equations

	GARCH (maturity effect)	GARCH (no maturity effect)	FIGARCH (maturity effect)	FIGARCH (no maturity effect)
Index				
a_1	0.03 (2.08)	0.03 (2.20)	0.02 (1.65)	0.03 (1.76)
b_1	0.13 (6.88)	0.13 (6.87)	0.14 (7.13)	0.14 (7.13)
b_2	-0.05 (-2.46)	-0.05 (-2.45)	-0.04 (-2.39)	-0.04 (-2.25)
Basis				
a_2	0.006 (1.97)	0.001 (1.63)	0.005 (1.32)	0.001 (1.33)
λ_1	-0.01 (1.95)		-0.01 (1.39)	
b_3	0.55 (18.42)	0.55 (17.03)	0.53 (19.13)	0.54 (20.09)
b_4	0.11 (4.64)	0.11 (4.40)	0.12 (5.05)	0.12 (5.02)
b_5	0.08 (3.22)	0.07 (2.88)	0.08 (3.13)	0.07 (3.34)
b_6	-0.92 (40.57)	-0.92 (33.64)	-0.92 (43.60)	-0.92 (47.13)
λ_2	0.12 (2.22)		0.13 (2.31)	

Quasi maximum likelihood estimates of the mean equations are presented. The mean equations are as follows

$$\frac{\Delta S_t}{S_{t-1}} = a_1 + b_1 \frac{\Delta S_{t-1}}{S_{t-2}} + b_2 \frac{\Delta S_{t-2}}{S_{t-3}} + \varepsilon_{s,t}$$

$$\frac{\Delta B_t}{S_{t-1}} = a_2 + \lambda_1 (m_t) + b_3 \frac{\Delta B_{t-1}}{S_{t-2}} + b_4 \frac{\Delta B_{t-2}}{S_{t-3}} + b_5 \frac{\Delta B_{t-3}}{S_{t-4}} + \varepsilon_{b,t} (m_t)^{\lambda_2} + b_6 \varepsilon_{b,t-1} (m_{t-1})^{\lambda_2}$$

Table 3b Parameter estimates and *t* values for All Ordinaries and its basis – variance equations

	GARCH (maturity effect) Equation 7	GARCH (no maturity effect) Equation 7	FIGARCH (maturity effect) Equation 8	FIGARCH (no maturity effect) Equation 8
Index				
ω_s	0.04 (2.14)	0.04 (2.61)	0.21 (4.61)	0.22 (4.58)
d_s			0.15 (5.40)	0.15 (5.31)
α_s / ϕ_s [a]	0.05 (3.29)	0.05 (3.56)		
β_s	0.89 (23.44)	0.90 (28.61)	0.07 (1.71)	0.06 (1.63)
Covariance				
ω_{sb}	0.007 (0.63)	0.005 (1.09)	0.02 (2.19)	0.02 (2.07)
d_{sb}			0.17 (5.21)	0.16 (4.61)
α_{sb} / ϕ_{sb} [a]	0.04 (2.29)	0.04 (3.53)	0.30 (2.10)	0.36 (2.26)
β_{sb}	0.92 (9.94)	0.92 (19.76)	0.47 (3.22)	0.52 (3.18)
Basis				
ω_b	0.006 (1.25)	0.003 (1.65)	0.03 (1.55)	0.01 (1.55)
d_b			0.24 (5.11)	0.24 (4.98)
α_b / ϕ_b [a]	0.04 (1.97)	0.03 (2.56)	0.42 (1.87)	0.58 (3.83)
β_b	0.94 (26.92)	0.95 (46.40)	0.56 (2.48)	0.71 (4.69)
Log Likelihood	-5298.04	-5314.16	-5274.71	-5291.55
AIC	3.6356	3.6452	3.6210	3.6311
Schwarz	3.6744	3.6800	3.6639	3.6700
Shibata	3.6355	3.6452	3.6209	3.6311
Hann Quinn	3.6496	3.6578	3.6365	3.6451

[a] α is the parameter for the GARCH estimates, ϕ is the parameter for the FIGARCH estimates

Table 4 Diagnostics

Test	GARCH (maturity effect)		FIGARCH (maturity effect)	
	Index	Basis	Index	Basis
Q(10)	0.76	0.10	0.74	0.06
Q(15)	0.34	0.21	0.31	0.16
Q(20)	0.48	0.39	0.46	0.31
Q2(10)	0.12	0.04	0.33	0.32
Q2(15)	0.43	0.17	0.68	0.70
Q2(20)	0.71	0.46	0.89	0.85
Sign bias	0.06	0.09	0.06	0.63
Negative size bias	0.02	<0.005	0.18	0.15
Positive size bias	0.93	0.74	0.52	0.81
Joint test	0.09	<0.005	0.27	0.47
Jarque Bera	<0.005	<0.005	<0.005	<0.005

Entries represent p values, Q(10) = Box Pierce statistic on ε_t / σ_t for 10 lags, Q2(10) is the statistic for $\varepsilon^2{}_t / \sigma^2{}_t$
The diagnostics for the models without maturity effects are very similar and have not been reported.

Table 5 Alternative methods of MVHR estimation

Time invariant	Single period	Multi period
Naïve (Φ=1)	GARCH – with and without maturity effect (equation 4)	GARCH – with and without maturity effect (equation 9)
OLS (equation 1)	FIGARCH – with and without maturity effect (equation 4)	FIGARCH – with and without maturity effect (equation 9)
	EC FIGARCH – spot and futures (equation 2)	FIGARCH – spot and futures (equation 3)

Table 6 MVHR estimation over 5 day horizon commencing 2/8/99

MVHR	Ranking	Portfolio Variance	Average MVHR	Variance of MVHR
Unhedged	13	0.5582	0	0
Naïve	12	0.0597	1	0
OLS	11	0.0528	0.6616	0
Single period				
GARCH (no mat)	9	0.0457	0.6951	0.000157
GARCH (mat)	10	0.0458	0.6947	0.000244
FIGARCH (no mat)	2	0.0410	0.7155	0.000792
FIGARCH (mat)	3	0.0421	0.7108	0.000448
FIGARCH (spot & futures)	6	0.0430	0.7075	0.000617
Multi-period				
GARCH (no mat)	8	0.0455	0.6961	0.000133
GARCH (mat)	7	0.0454	0.6964	0.000199
FIGARCH (no mat)	1	0.0409	0.7192	0.000204
FIGARCH (mat)	4	0.0422	0.7122	0.000254
FIGARCH (spot & futures)	5	0.0425	0.7126	0.000226

Rankings: 1-Best risk reduction, 13-Worst risk reduction

The portfolio consists of the underlying asset (the All Ordinaries Index) and SPI futures. The unhedged return is the return on the underlying asset over the 5 day period commencing 2/8/99

Table 7 MVHR estimation over 20 day horizon commencing 2/8/99

MVHR	Ranking	Portfolio Variance	Average MVHR	Variance of MVHR
Unhedged	13	0.7467	0	0
Naïve	12	0.0773	1	0
OLS	11	0.0710	0.6616	0
Single period				
GARCH (no mat)	10	0.0548	0.7276	0.000726
GARCH (mat)	8	0.0529	0.7346	0.000880
FIGARCH (no mat)	3	0.0512	0.7427	0.000850
FIGARCH (mat)	2	0.0504	0.7490	0.001091
FIGARCH (spot & futures)	6	0.0518	0.7474	0.001045
Multi-period				
GARCH (no mat)	9	0.0547	0.7277	0.000547
GARCH (mat)	7	0.0521	0.7381	0.000569
FIGARCH (no mat)	5	0.0515	0.7398	0.000327
FIGARCH (mat)	1	0.0498	0.7507	0.000464
FIGARCH (spot & futures)	4	0.0514	0.7484	0.000493

Rankings: 1-Best risk reduction, 13-Worst risk reduction
The portfolio consists of the underlying asset (the All Ordinaries Index) and SPI futures. The unhedged return is the return on the underlying asset over the 20 day period commencing 2/8/99

Table 8 MVHR estimation over 40 day horizon commencing 2/8/99

MVHR	Ranking	Portfolio Variance	Average MVHR	Variance of MVHR
Unhedged	13	0.7474	0	0
Naïve	11	0.0969	1	0
OLS	12	0.1102	0.6616	0
Single period				
GARCH (no mat)	8	0.0882	0.7487	0.000921
GARCH (mat)	6	0.0867	0.7544	0.001364
FIGARCH (no mat)	4	0.0859	0.7561	0.000879
FIGARCH (mat)	1	0.0847	0.7634	0.001541
FIGARCH (spot & futures)	2	0.0852	0.7677	0.001061
Multi-period				
GARCH (no mat)	10	0.0892	0.7420	0.000565
GARCH (mat)	9	0.0886	0.7409	0.000370
FIGARCH (no mat)	7	0.0874	0.7484	0.000275
FIGARCH (mat)	5	0.0866	0.7491	0.000324
FIGARCH (spot & futures)	3	0.0856	0.7589	0.000457

Rankings: 1-Best risk reduction, 13-Worst risk reduction
The portfolio consists of the underlying asset (the All Ordinaries Index) and SPI futures. The unhedged return is the return on the underlying asset over the 40 day period commencing 2/8/99.

Table 9 MVHR estimation over 60 day horizon commencing 2/8/99

MVHR	Ranking	Portfolio Variance	Average MVHR	Variance of MVHR
Unhedged	13	0.7587	0	0
Naïve	11	0.1055	1	0
OLS	12	0.1120	0.6616	0
Single period				
GARCH (no mat)	7	0.0905	0.7535	0.000707
GARCH (mat)	4	0.0894	0.7535	0.000981
FIGARCH (no mat)	3	0.0881	0.7616	0.000711
FIGARCH (mat)	1	0.0875	0.7626	0.001102
FIGARCH (spot & futures)	2	0.0885	0.7711	0.000906
Multi-period				
GARCH (no mat)	9	0.0919	0.7412	0.000364
GARCH (mat)	10	0.0933	0.7304	0.000301
FIGARCH (no mat)	6	0.0895	0.7511	0.000226
FIGARCH (mat)	8	0.0907	0.7413	0.000184
FIGARCH (spot & futures)	5	0.0895	0.7565	0.000258

Rankings: 1-Best risk reduction, 13-Worst risk reduction
The portfolio consists of the underlying asset (the All Ordinaries Index) and SPI futures. The unhedged return is the return on the underlying asset over the 20 day period commencing 2/8/99.

Table 10 Summary of rankings for hedges commencing 2/8/99

MVHR	5 day rank	20 day rank	40 day rank	60 day rank	Total	Overall rank
Unhedged	13	13	13	13	52	13
Naïve	12	12	11	11	46	12
OLS	11	11	12	12	46	11
Single period						
GARCH (no mat)	9	10	8	7	34	9
GARCH (mat)	10	8	6	4	28	7
FIGARCH (no mat)	2	3	4	3	12	2
FIGARCH (mat)	3	2	1	1	7	1
FIGARCH (spot & futures)	6	6	2	2	16	3
Multi-period						
GARCH (no mat)	8	9	10	9	36	10
GARCH (mat)	7	7	9	10	33	8
FIGARCH (no mat)	1	5	7	6	19	6
FIGARCH (mat)	4	1	5	8	18	5
FIGARCH (spot & futures)	5	4	3	5	17	4

Rankings are based on the rankings in Tables 6 to 9.

Figure 1 60 day bivariate FIGARCH SPMVHRs with and without maturity effects

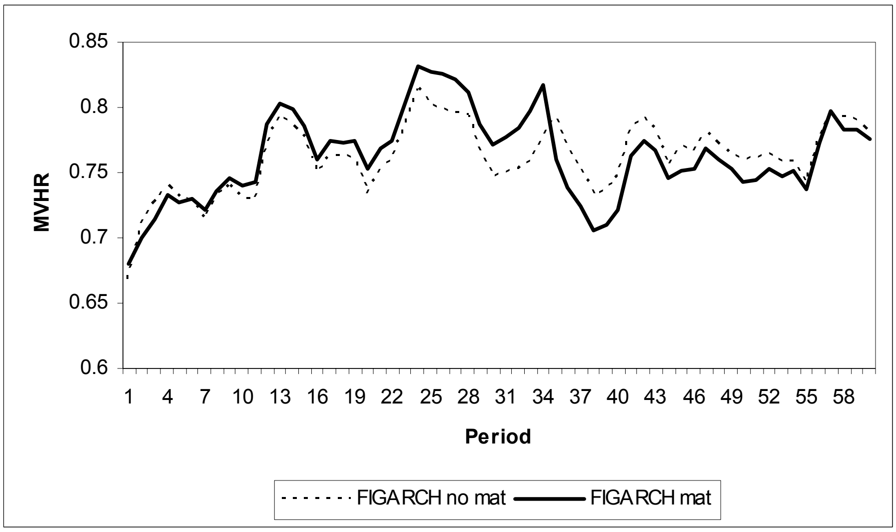


Figure 2 60 day bivariate GARCH SPMVHRs with and without maturity effects

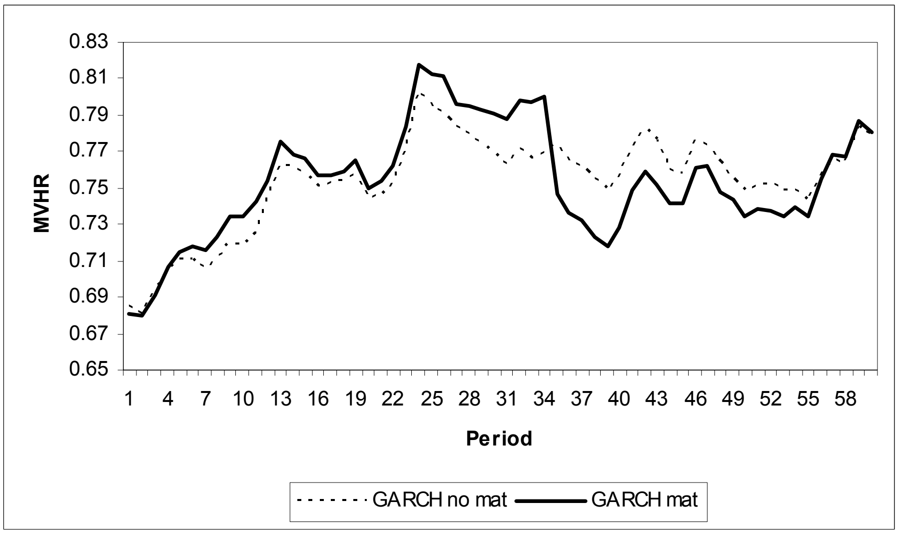


Figure 3 60 day bivariate FIGARCH SPMVHRs with maturity effects versus bivariate error correction FIGARCH SPMVHRs

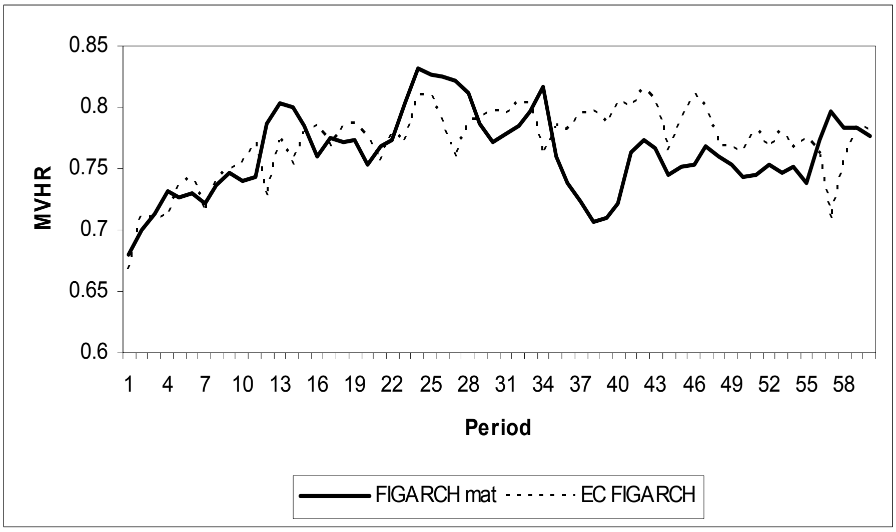


Figure 4 60 day bivariate GARCH and FIGARCH SPMVHRs with maturity effects

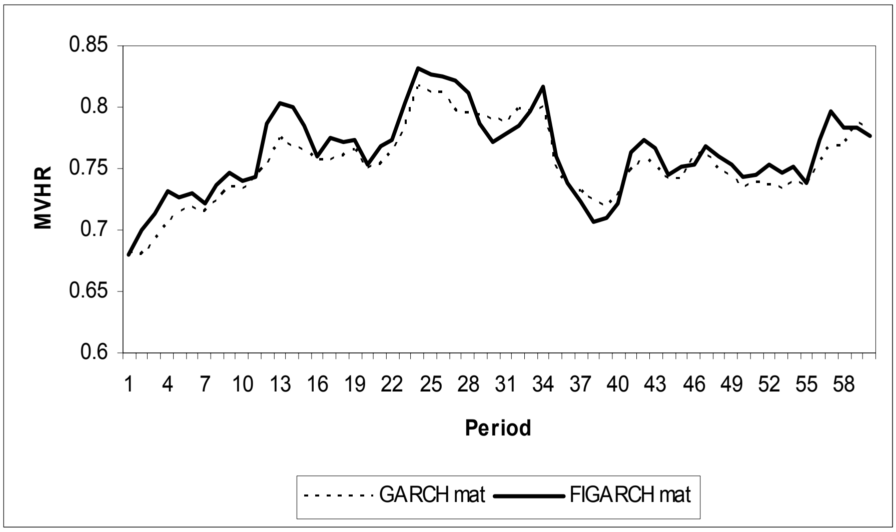
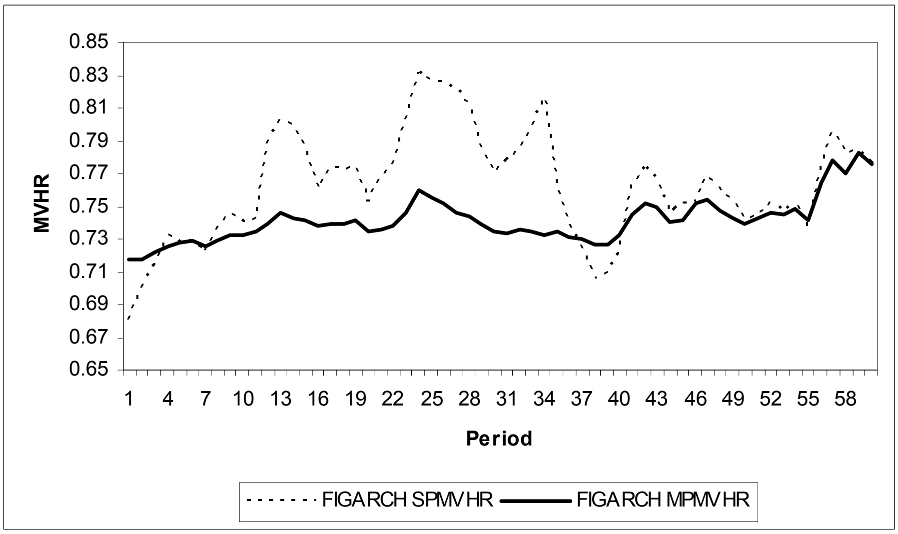


Figure 5 60 day bivariate FIGARCH SPMVHRs and MPMVHRs



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