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Long term hedging of the Australian All Ordinaries Index using a bivariate error correction FIGARCH model

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LONG TERM HEDGING OF THE AUSTRALIAN ALL ORDINARIES INDEX USING A BIVARIATE ERROR CORRECTION FIGARCH MODEL

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Abstract

This article compares the performance of bivariate error correction GARCH and FIGARCH models when estimating long term dynamic minimum variance hedge ratios (MVHRs) on the Australian All Ordinaries Index. The paper therefore introduces the bivariate error correction FIGARCH model into the hedging literature, which to date has only employed the GARCH class of processes. This is important for those interested in managing long term equity exposures, given that FIGARCH processes exhibit long memory, whilst the GARCH class of processes exhibit short memory. The naïve hedge ratio, the constant MVHR estimated via ordinary least squares (the OLS MVHR), the single period dynamic MVHR and the multi-period dynamic MVHR of Lee (1999) are considered. The results strongly support the estimation of dynamic MVHRs that allow for time varying correlations. Whilst long memory dependencies appear important, a multi-period dynamic MVHR that responds more rapidly to persistent changes in volatility dynamics requires development.

Keywords: long memory, bivariate FIGARCH, time varying correlations, multi period minimum variance hedge ratios.

JEL CLASSIFICATION: G0, G15

INTRODUCTION

This paper compares the performance of bivariate error correction GARCH and FIGARCH models when estimating long term dynamic minimum variance hedge ratios (MVHRs) on the Australian All Ordinaries Index. The paper therefore introduces the bivariate error correction FIGARCH model into the hedging literature, which to date has only employed the short memory GARCH class of processes. This paper is therefore of importance to those agents who are concerned with managing long term equity exposures.

The FIGARCH model of Baillie *et al* (1996) is one way of capturing long memory in volatility. Long memory is a characteristic commonly observed in financial market volatility and is associated with hyperbolically decaying autocorrelation functions and impulse response weights. This is in contrast to the GARCH (Bollerslev, 1986) class of processes which have short memory and exhibit much faster exponential rates of decay.

The paper examines the naïve hedge ratio, the constant MVHR estimated via ordinary least squares (the OLS MVHR), the single period dynamic MVHR and the multi-period dynamic MVHR of Lee (1999). If the volatility of the All Ordinaries, its SPI futures, and their covariance exhibit long memory, the bivariate error correction FIGARCH model should provide superior long term forecasts of volatility and the co-movements between assets. Long term dynamic MVHRs estimated using a bivariate error correction FIGARCH model should therefore outperform those estimated using a bivariate error correction GARCH model.

The contributions of this paper are as follows. First the bivariate error correction FIGARCH process is introduced into the hedging literature. This is significant given that the hedging literature to date has only considered the short memory GARCH class of processes. Second, the conventional single period dynamic MVHR is compared to the multi-period approach developed by Lee (1999). Third, the importance of allowing for time varying correlations when estimating dynamic MVHRs is examined.

The second section will provide a review of the literature. The section will briefly examine the long memory and FIGARCH literatures and develop the naïve, OLS and single period dynamic MVHRs. The section will then examine the previous research which considers the performance of these alternative approaches to MVHR estimation. The third section will outline the methodology and argue that when conducting a long term hedge, dynamic MVHRs should be estimated using a multi-period strategy with a bivariate error correction FIGARCH model. The fourth section will investigate this claim, comparing the risk reduction achieved using this procedure with the other methods of MVHR determination. The final section will conclude.

LITERATURE REVIEW

Long memory and FIGARCH

A process exhibits long memory if the autocovariance function is not absolutely summable and decays at the hypergeometric rate k^{2d-1} ($0<d<0.5$).[1] Subsequent to the pioneering work of Taylor (1986) a comprehensive literature has documented the presence of long memory in financial market volatility. Long memory has been documented across a range of equity indices; the S&P500 (Ding *et al*, 1993; Bollerslev and Mikkelsen, 1996; Ding and Granger, 1996; Granger and Ding, 1996; Andersen and Bollerslev, 1997b; Lobato and Savin, 1998; Liu, 2000), the NYSE (Ding *et al*, 1993), the Nikkei (Ding and Granger, 1996), the CRSP (Breidt *et al*, 1998), and the DAX (Ding *et al*, 1993). Long memory has also been documented in currency market volatility, including; the Deutschemark-U.S.$ (Dacorogna *et* *al*, 1993; Baillie *et al*, 1996; Ding and Granger, 1996; Andersen and Bollerslev, 1997a; 1997b; 1998) and the British pound-U.S.$ (Giriatis *et al*, 2001).

Baillie *et al* (1996) were the first to propose the FIGARCH(p,d,q) model as one way of modelling long memory in volatility. The FIGARCH (1,d,1) process can be expressed as;

$$\sigma^2_t = \frac{\omega}{1-\beta} + \left(1 - \frac{(1-\phi L)(1-L)^d}{1-\beta L}\right)\varepsilon^2_t \qquad (1)$$

where σ_t^2 is the conditional variance at time t, d is the fractional differencing parameter $0 < d < 1$ and

$$(1-L)^d = 1 - dL - \frac{d(1-d)}{2!}L^2 - \frac{d(1-d)(2-d)}{3!}L^3 - \ldots\ldots \qquad (2)$$

In contrast the GARCH class of processes are short memory processes, given that they have autocovariance functions that are geometrically bounded and are therefore absolutely summable.

Subsequent to the pioneering work of Teyssiere (1997), very little has been done on the estimation of multivariate FIGARCH processes. Teyssiere (1997) estimated bivariate FIGARCH processes between the Deutschemark-U.S.$ and the British pound-U.S.$ using daily data. Teyssiere (1998) subsequently extended this to a trivariate FIGARCH model between the U.S.$-Deutschemark, U.S.$-British pound and the U.S.$-Japanese yen, using data at the 30 minute frequency. Pafka and Matyas (2001) estimate trivariate FIGARCH processes between the same currencies using daily data. Brunetti and Gilbert (2000) estimate bivariate FIGARCH processes between the spot crude oil on the NYMEX and the IPE.

All of these papers find that the estimates of d tend to be very similar amongst assets of the same class. Teyssiere (1997, 1998) and Pafka and Matyas (2001) also support the use of models that allow for time varying correlations. This result is supported by the evidence against constant correlation between equity markets (Tse, 2000; Ramchand and Susmel, 1998) and the indices within those markets (Engle and Sheppard, 2001).

There has been no attempt to use the bivariate FIGARCH models for forecasting, let alone dynamic MVHR estimation, and it is this gap in the literature which motivates this paper.

The Hedging literature

The naïve hedge ratio sets the hedge ratio equal to one over the life of the hedge. Naïve hedging completely eliminates spot price risk and replaces it with basis risk. Risk reduction therefore only occurs if the variance of the basis is less than the variance of the spot.

Ederington (1979), Figlewski (1986) and Castelino (1992) overview the development of the OLS MVHR. The OLS MVHR adopts a portfolio approach to hedge ratio determination and addresses some of the shortcomings of the naïve strategy; allowing for partial hedging and the tradeoff between spot price risk and basis risk. It is assumed that the hedger has a spot position at time t and seeks to minimise the expected variability in the hedged return between time t and time $t+r$ (the hedge lifting date). The derived MVHR (Φ), can be expressed as

$$\Phi = \frac{\sigma_{sf}}{\sigma^2_{f}} \tag{3}$$

where σ_{sf} is the covariance between the spot and the futures and σ_f^2 is the futures variance. The OLS MVHR is equal to the slope coefficient of a regression of the spot on the futures.[2]

The OLS MVHR ignores conditional information and conditional heteroscedasticity. The single period dynamic MVHR therefore modifies the portfolio approach, minimising conditional rather than unconditional variances. Kroner and Sultan (1993) derive the dynamic single period MVHR. This can be expressed as

$$\Phi_t = \frac{\sigma_{sf,t+1}}{\sigma^2_{f,t+1}} \tag{4}$$

where Φ_t is the dynamic MVHR at time *t*, $\sigma_{sf,t+1}$ is the conditional covariance between the spot and the futures at time *t*+1, and $\sigma^2_{f,t+1}$ is the futures conditional variance at time *t+1*. Dynamic MVHR estimation over multiple periods therefore typically employs models from the bivariate GARCH family to make one period ahead forecasts of Φ_t for each period over the life of the hedge.

If the spot and futures prices are cointegrated and this is ignored, estimates of the MVHR may be downward biased (Ghosh, 1993; Lien, 1996). As a consequence, the bivariate error correction GARCH model has become a very popular method of estimating dynamic single period MVHRs.

Table I summarises the hedging effectiveness of various time invariant and dynamic MVHRs reported in the previous literature. The table only considers the *ex post* risk reduction achieved under each alternative. Here the data is divided into estimation and forecast periods, with the forecast period being used to determine the portfolio variance achieved.

(Insert Table I)

The table identifies a number of important issues. First, there is good support for the estimation of dynamic MVHRs using a bivariate error correction GARCH model. The earlier results (Cecchetti *et al*, 1988; Baillie and Myers, 1991; Sephton, 1993) rejected the time invariance in MVHRs, finding that bivariate GARCH models achieved greater risk reduction than the naïve and OLS MVHRs. Subsequent results further illustrated the benefits of also allowing for cointegration (Kroner and Sultan, 1993; Park and Switzer, 1995; Koutmos and Pericli, 1998; Lien and Tse, 1999; Sim and Zurbreugg, 2000). The research further demonstrates that the bivariate error correction GARCH hedge generally outperforms a variety of other approaches. This includes time invariant MVHRs estimated via; vector autoregressions (VAR), error correction models and fractionally integrated error correction (FIEC) models (Lien and Tse, 1999). It also includes time varying bivariate GARCH, VAR-GARCH and FIEC-GARCH (Koutmos and Pericli, 1998; Lien and Tse, 1999; Sim and Zurbreugg, 2000). These results also appear to be unaffected by the inclusion of transaction costs (Kroner and Sultan, 1993; Park and Switzer, 1995; Koutmos and Pericli, 1998). The results however are not universal with Myers (1991) showing that even though the bivariate GARCH models statistically outperform the time invariant models, they do not necessarily achieve superior risk reduction.

Second, much of the research estimates dynamic MVHRs via a bivariate GARCH model assuming constant correlation, yet fails to consider the effect of this assumption on hedging performance (Cecchetti *et al*, 1988; Sephton, 1993; Kroner and Sultan, 1993; Park and Switzer, 1995; Lien and Tse, 1998; 1999). This is despite the recognition that the constant correlation assumption imposes restrictions on the time path of the MVHRs (Lien and Tse, 1998).

Third, none of the approaches consider long memory in volatility. By employing the GARCH family of processes, the hedging literature imposes short memory on the volatility dynamics. If long memory in volatility is present, this suggests that superior long term hedging outcomes may be achieved if the models used for MVHR estimation capture the long memory dependencies. The development of a framework that is suitable under these circumstances is the subject of the next section.

METHODOLOGY

The previous section has presented evidence in support of the single period dynamic MVHR. At issue is whether this approach is consistent with the objective of minimising risk over the life of the hedge. By assuming time invariance in the joint distribution of the spot and futures, the OLS MVHR is able to treat the time from the commencement to the completion of the hedge as a single period. The MVHR is therefore derived under the assumption that the hedger seeks to minimise risk over the life of the hedge. By modifying this framework to allow for time varying moments, the dynamic single period MVHR only seeks to minimise the conditional variation in portfolio returns period by period. This single period dynamic MVHR may therefore be inconsistent with the objective of minimising the variability in portfolio returns over the life of the hedge, and fails to take into account any of the interperiod dependencies that may exist.

Lee (1999) addresses this by deriving a multi-period dynamic MVHR (MPMVHR), which seeks to minimise the conditional variability in portfolio returns over the life of the hedge. The approach allows for conditional information and conditional heteroscedasticity (Kroner and Sultan, 1993; Myers and Thompson, 1989; Chen *et al*, 1999) multiple assets (Gagnon *et al*, 1998; Giaccotto *et al,* 2001) and multiple periods (Howard and D'Antonio, 1991; Vukina and Anderson, 1993; Lien and Luo, 1994). The hedger is assumed to have a portfolio of assets with returns represented by the vector R_t, and portfolio weights represented by the vector x. Assuming a hedge over *r* periods, the end of period wealth (W_{t+r}) is therefore

$$W_{t+r} = W_t + \left(x'R_{t+1} + x'R_{t+2} + \ldots + x'R_{t+r}\right) \quad (5)$$

The hedger seeks to minimise the variability of wealth over the entire period of the hedge. The MPMVHR is therefore solved by minimising the variance of wealth over the life of the hedge conditional upon the information available (I_t) with respect to the portfolio weight vector x

$$Min_x\left[Var\left(W_{t+r} - W_t\right)/I_t\right] \quad (6)$$

Assuming no significant serial correlation in R_t, Lee (1999) shows that when confined to two assets, the solution to the above yields

$$x_t^* = \Phi_t = \frac{{}^*\sigma_{sf,t}}{{}^*\sigma^2_{f,t}} \qquad (7)$$

where $x_t = (1, x_t^*)$ represents the weight vector at time t and ${}^*\sigma_{sf,t}$ and ${}^*\sigma^2_{f,t}$ represent the appropriate element in the following 3x1 matrix $\left[vech(H_{t+1}) + vech(H_{t+2}) + ... + vech(H_{t+r})\right]$, where H_t represents the conditional covariance matrix at time t. Therefore the hedge ratio at time t, requires a forecast of the covariance matrix for each period over the life of the hedge.[3]

The approach captures the interperiod dependencies via the dynamic structure of the covariance matrix over the life of the hedge. Furthermore, the longer the hedge horizon, the less volatile the MPMVHR is likely to be. This is intentional, given that a hedging strategy that reflects short lived volatility fluctuations is unstable, costly and ineffective when hedging over the long term (Lee, 1999). The approach can therefore be expected to outperform the dynamic single period MVHR.

This paper compares the performance of the naïve, OLS and dynamic single and multi-period MVHRs. The OLS MVHR is estimated via the OLS regression of the spot return against the futures return. At issue is the most appropriate method of estimating dynamic MVHRs. Given that the cost of carry model imposes a long run equilibrium relation between the All Ordinaries Index and its SPI futures (Brailsford and Hodgson, 1997; Twite, 1998), the following error correction specification is estimated

$$R_{s,t} = a_1 + b_1 z_{t-1} + \sum_{i=1}^{k} c_{1,i} R_{s,t-i} + \sum_{i=1}^{k} d_{1,i} R_{f,t-i} + \varepsilon_{s,t}$$
$$R_{f,t} = a_2 + b_2 z_{t-1} + \sum_{i=1}^{k} c_{2,i} R_{s,t-i} + \sum_{i=1}^{k} d_{2,i} R_{f,t-i} + \varepsilon_{f,t} \quad (8)$$

with

$$\begin{bmatrix} \varepsilon_{s,t} \\ \varepsilon_{f,t} \end{bmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2{}_{s,t} & \sigma_{sf,t} \\ \sigma_{sf,t} & \sigma^2{}_{f,t} \end{pmatrix}\right] \quad (9)$$

where; $R_{s,t}$ and $R_{f,t}$ represents the returns in the spot and futures markets respectively (calculated as the difference in log of consecutive process multiplied by 100), z_{t-1} represents the error correction term, $\sigma^2_{s,t}$ and $\sigma^2_{f,t}$ represents the conditional variance in the spot and futures markets respectively, and $\sigma_{sf,t}$ represents their conditional covariance. A number of alternative conditional covariance specifications are estimated, including; [4] i) the constant correlation GARCH (1,1) model of Bollerslev (1990)

$$\begin{pmatrix} \sigma^2_{s,t} \\ \sigma_{sf,t} \\ \sigma^2_{f,t} \end{pmatrix} = \begin{pmatrix} \omega_1 + \alpha_{11}\varepsilon^2_{s,t-1} + \beta_{11}\sigma^2_{s,t-1} \\ \rho\sigma_{s,t}\sigma_{f,t} \\ \omega_3 + \alpha_{33}\varepsilon^2_{f,t-1} + \beta_{33}\sigma^2_{f,t-1} \end{pmatrix} \quad (10)$$

ii) the diagonal GARCH (1,1) parameterisation of Bollerslev *et al* (1988)

$$\begin{pmatrix} \sigma^2_{s,t} \\ \sigma_{sf,t} \\ \sigma^2_{f,t} \end{pmatrix} = \begin{pmatrix} \omega_1 + \alpha_{11}\varepsilon^2_{s,t-1} + \beta_{11}\sigma^2_{s,t-1} \\ \omega_2 + \alpha_{22}\varepsilon_{s,t-1}\varepsilon_{f,t-1} + \beta_{22}\sigma_{sf,t-1} \\ \omega_3 + \alpha_{33}\varepsilon^2_{f,t-1} + \beta_{33}\sigma^2_{f,t-1} \end{pmatrix} \quad (11)$$

iii) the constant correlation FIGARCH(1,*d*,1) of Teyssiere (1997)

$$\begin{pmatrix} \sigma_{s,t}^2 \\ \sigma_{sf,t} \\ \sigma_{f,t}^2 \end{pmatrix} = \begin{pmatrix} \frac{\omega_s}{1-\beta_s} + \left(1 - \frac{(1-\phi_s L)(1-L)^{d_s}}{1-\beta_s}\right)\varepsilon_{s,t}^2 \\ \rho\sigma_{s,t}\sigma_{f,t} \\ \frac{\omega_f}{1-\beta_f} + \left(1 - \frac{(1-\phi_f L)(1-L)^{d_f}}{1-\beta_f}\right)\varepsilon_{f,t}^2 \end{pmatrix} \quad (12)$$

and iv) the diagonal FIGARCH(1,d,1) of Teyssiere (1997)

$$\begin{pmatrix} \sigma_{s,t}^2 \\ \sigma_{sf,t} \\ \sigma_{f,t}^2 \end{pmatrix} = \begin{pmatrix} \frac{\omega_s}{1-\beta_s} + \left(1 - \frac{(1-\phi_s L)(1-L)^{d_s}}{1-\beta_s}\right)\varepsilon_{s,t}^2 \\ \frac{\omega_{sf}}{1-\beta_{sf}} + \left(1 - \frac{(1-\phi_{sf} L)(1-L)^{d_{sf}}}{1-\beta_{sf}}\right)\varepsilon_{s,t}\varepsilon_{f,t} \\ \frac{\omega_f}{1-\beta_f} + \left(1 - \frac{(1-\phi_f L)(1-L)^{d_f}}{1-\beta_f}\right)\varepsilon_{f,t}^2 \end{pmatrix} \quad (13)$$

The diagonal GARCH and FIGARCH models allow for time varying correlations and are therefore more flexible then their constant correlation counterparts.

Given that single period dynamic MVHR estimation only requires one period ahead forecasts, any long memory dependencies are unlikely to be important. When employing the dynamic single period MVHR, the bivariate error correction FIGARCH model is therefore unlikely to outperform the bivariate error correction GARCH model. The MPMVHR of Lee (1999) however requires forecasts for each period over the life of the hedge. As the horizon is extended, any long run volatility dynamics become more important. One would expect that if long memory is present, the longer the hedge horizon, the greater the benefits of using a bivariate error correction FIGARCH model over a bivariate error correction GARCH model.

If the individual long term forecasts of $\sigma^2_{f,t}$ and $\sigma_{sf,t}$ using the bivariate error correction FIGARCH model outperform the bivariate error correction GARCH model forecasts, the MPMVHR may still not provide superior risk reduction. This is because the MPMVHR represents a ratio of the forecasts. Therefore the long term bivariate error correction GARCH forecasts may understate the levels of volatility persistence in each series, yet the impact on the ratio of these forecasts (compared with the bivariate error correction FIGARCH forecasts) may be inconsequential.

To examine the importance of long memory in volatility, each dynamic MVHR will be estimated using the bivariate error correction GARCH and error correction FIGARCH models. To assess the importance of the constant correlation assumption, each of the dynamic MVHRs will be estimated using the constant correlation and diagonal parameterisations. To examine the effect of hedge horizon, hedges over 5, 20, 40 and 60 days will be considered. Table II summarises the MVHRs to be examined.[5]

(Insert Table II)

RESULTS

Data and Preliminary analysis

Daily data commencing on January 4, 1988 and ending October 22, 1999 is employed. The period from January 4, 1988 to July 30, 1999 is used for estimation, with the remainder of the sample being used for hedge ratio evaluation. Data on the index was obtained from IRESS, the futures was obtained from the Sydney Futures Exchange WWW site (http://www.sfe.com.au). Only those days were included where trading occurred in both markets. Given the desire to model volatility under normal conditions, observations on 16 October 1989 and 28 October 1997 are removed. Here large negative returns in both markets were experienced in line with the anniversary of Black Monday. The observations on 11 January 1988 are also removed, where an additional large one off spike in futures volatility was observed. The analysis uses the nearby futures contract with rollover being performed 10 trading days prior to expiration.[6]

Both series contain a unit root and are cointegrated in the presence of a structural break on October 11, 1993. This break was due to the SFE reducing the multiplier and the margin on the All Ordinaries SPI futures contract (see Bhar, 2001).[7] The autocorrelogram for squared returns, spectral density estimates using the procedure by Robinson (1994), and the modified R/S (Lo, 1991) and KPSS (Kwiatkowski, *et al* 1992) tests, support the existence of long memory in the volatility of the All Ordinaries, its SPI futures and their covariance.[8]

Model results

Tables XV to XX in the Appendix, present the results for each of the bivariate models. The diagonal models clearly outperform their constant correlation counterparts. The constant correlation models suffer from parameter instability (particularly in ρ) and have higher information criteria. Furthermore, despite the presence of long memory in volatility, the diagonal GARCH model has lower information criteria than the constant correlation FIGARCH model. Time varying correlations would therefore appear to be important.

The bivariate diagonal FIGARCH model has the lowest information criteria, is the only model with stable ARCH parameters, allows for long memory in volatility and rejects the constant correlation assumption. This model is therefore likely to provide superior estimates of the dynamic MPMVHRs. *A priori*, the relative performance of this model when estimating single period dynamic MVHRs is unclear, and depends on its ability to capture the short run dynamics.

MVHR estimation

This section considers the hedging performance of the naïve HR, the OLS MVHR, and the dynamic single period and multi-period MVHRs with and without transaction costs.

Tables III to VI summarise the *ex post* outcomes for each strategy excluding transaction costs. The tables detail the portfolio variance, the average hedge ratio, and the ranking of each MVHR according to the variance of the portfolio. This is followed by Figures 1 to 4 which display; the dynamic single period MVHRs, the dynamic MPMVHRs and a comparison between the dynamic single and multi-period MVHRs estimated using the diagonal FIGARCH model. An analysis of these results follows.

(Insert Tables III to VI)

(Insert Figures 1 to 4)

The results allow a number of observations to be made. First, hedging provides substantial risk reduction, with the unhedged position for all hedge horizons exhibiting the greatest variability in returns. Furthermore, all the dynamic MVHRs provide greater risk reduction than the time invariant naïve and OLS MVHRs.

Second, for all hedging horizons: a) the dynamic strategies produce MVHRs that exhibit an upward trend; b) the MVHRs estimated using the diagonal parameterisations are generally higher than their constant correlation counterparts; and c) all the diagonal models reduce portfolio risk more than their constant correlation counterparts. These results are due to an increase in the correlation between the returns in both markets over the periods of hedging, as revealed in Table VII.

(Insert Table VII)

By allowing for time varying correlations, the estimated covariances using the diagonal parameterisations have been able to capture the increase in the correlations over the hedge periods. The constant correlation models have only been able to partially reflect an increase in the covariance via changes in the index and SPI futures volatilities. The inability of the constant correlation models to capture the increase in the correlations, has resulted in an understatement of the conditional covariance forecasts and the MVHRs, resulting in inferior risk reduction.

Third, the MPMVHRs exhibit much less fluctuation than the single period MVHRs. This is consistent with the desire to decrease the susceptibility of the dynamic MVHRs to short term volatility fluctuations. Despite this, the evidence supporting the use of the MPMVHRs over the single period MVHRs is mixed and is summarised in Table VIII.

(Insert Table VIII)

The poor performance of the MPMVHRs over the longer hedging horizons can also be explained by the rising correlations over the hedging periods. The longer the hedge horizon, the less responsive the MPMVHRs are to increases in correlations. Therefore, whilst the MPMVHR eliminates any of the transitory fluctuations in volatility, the MPMVHR suffers (relative to the single period MVHR) when the changes in volatility dynamics persist.

Fourth, the results generally support the estimation of dynamic MVHRs using the diagonal FIGARCH model. Table IX summarises the rankings of the dynamic MVHRs in Tables III to VI. Where the sum of the ranks are equal, the multi-period MVHR is favoured over the single period MVHR. The results illustrate that the diagonal FIGARCH model generally provides superior risk reduction, irrespective of hedge horizon or whether a single or multi-period MVHR is adopted. The diagonal FIGARCH model would therefore appear to be more appropriately capturing the short and long run volatility dynamics. [9]

(Insert Table IX)

These results assume that rebalancing of the portfolio occurs each period, ignoring the transaction costs associated with the buying and selling of futures contracts. The results are therefore unrealistic and overstate the benefits of the dynamic strategies. In order to incorporate transaction costs the approach of Kroner and Sultan (1993) is employed. A hedger will only rebalance if the expected utility from rebalancing exceeds the expected utility from not rebalancing. Under the assumption that the expected returns to the hedged portfolio are zero, rebalancing will occur if

$$-TC-\gamma\left(\sigma^{2}_{s,t+1}-2\Phi_{t}^{*}\sigma_{sf,t+1}+\Phi_{t}^{*2}\sigma^{2}_{f,t+1}\right)>-\gamma\left(\sigma^{2}_{s,t+1}-2\Phi_{t}\sigma_{sf,t+1}+\Phi_{t}^{2}\sigma^{2}_{f,t+1}\right) \quad (14)$$

where Φ_t^* represents the MVHR if rebalancing occurs, Φ_t represents the MVHR if rebalancing does not occur, TC represents the transaction costs associated with one round trip estimated at 0.10%,[10] γ is the risk aversion parameter, and the variance and covariance estimates represent one period ahead forecasts.

Given the high transaction costs, it is likely that rebalancing will occur only if estimated hedge ratios are volatile or if the degree of risk aversion is high. Furthermore, MVHRs are only utility maximising if futures are unbiased or if the hedger exhibits extreme risk aversion (Kahl, 1983). As a consequence, a number of alternative risk aversion measures ranging between $\gamma = 4$ to $\gamma = 100000$ are examined.

The results show that; i) as the degree of risk aversion increases, the number of rebalances increase and the portfolio variance approaches the variance achieved ignoring transaction costs. This is expected given that as the degree of risk aversion increases, the restrictions on rebalancing imposed by transaction costs diminish; ii) for low levels of risk aversion ($\gamma \leq 20$), the number of rebalances generally do not exceed 1, even over 60 day hedging horizons. Therefore hedgers with low degrees of risk aversion obtain little benefit from employing dynamic strategies; iii) when $\gamma = 100$, only a small number of rebalances occur, yet the portfolio variance is only slightly higher than the dynamic strategies that do not allow for transaction costs; and iv) for higher levels of risk aversion ($\gamma > 100$), further risk reduction is achieved, yet the benefit from the higher number of rebalances is small. The results obtained with a risk aversion parameter of 100 are therefore presented. Nonetheless the results and conclusions are quite insensitive to higher degrees of risk aversion. [11]

The results allowing for transaction costs of 0.1% and a risk aversion parameter of $\gamma = 100$ are presented in Tables X to XIII. For illustrative purposes, Figures 5 to 8 display the diagonal FIGARCH MVHRs with transaction costs (TC) against their respective diagonal FIGARCH MVHRs without transaction costs (NO TC). A discussion of these results follows.

(Insert Table X, XI, XII, XIII)

(Insert Figures 5 to 8)

The results are very similar to those excluding transaction costs. First, hedging provides substantial risk reduction. Second, all the dynamic strategies outperform the time invariant strategies. Third, the MVHRs estimated using the diagonal parameterisations outperformed their constant correlation counterparts. Fourth, the MPMVHRs rebalance less than the single period MVHRs. This is because the dynamic MPMVHRs without transaction costs exhibit much less fluctuation than the single period MVHRs without transaction costs. This effect is more pronounced, the longer the hedge horizon (given that the longer the hedge horizon the less volatile the MPMVHR). Fifth, the evidence supporting the use of the MPMVHRs over the single period MVHRs is mixed and can be explained by the rising correlations over the hedging period. Finally, the results in Table XIV generally support the estimation of dynamic MVHRs using a bivariate diagonal FIGARCH model.

(Insert Table XIV)

CONCLUSION

This paper has examined the performance of alternative methods of minimum variance hedge ratio estimation when hedging the Australian All Ordinaries Index. The paper considered the naïve approach, the OLS MVHR, the single period dynamic MVHR and the multi-period dynamic MVHR of Lee (1999). Dynamic MVHRs were estimated using bivariate error correction GARCH and FIGARCH models, where the constant correlation and diagonal parameterisations were employed.

The results provide support for the estimation of dynamic MVHRs, with all dynamic MVHRs providing greater risk reduction than the time invariant OLS and naïve hedge ratios. The results also strongly reject the estimation of dynamic MVHRs using models that assume constant correlation.

The single period MVHRs estimated via the diagonal FIGARCH model performed as well as the MPMVHRs. This suggests that either; i) long memory dependencies are unimportant when estimating dynamic MVHRs (due to the fact that the MVHR is a ratio of forecasts) or; ii) the MPMVHR is inappropriate. The persistently high correlation levels over the hedging horizons meant that all MPMVHRs suffered relative to their single period counterparts. This result points to the limitations in the MPMVHR, rather than any evidence against the use of models that allow for long memory in volatility. A dynamic MPMVHR that is able to more rapidly respond to persistent changes in volatility dynamics requires development.

For hedgers with high degrees of risk aversion ($\gamma \geq 100$), these conclusions are insensitive to the inclusion of transaction costs using the approach of Kroner and Sultan (1993). The same however cannot be said for hedgers with low degrees of risk aversion. The high transaction costs on the SPI futures contract mean that hedgers with low degrees of risk aversion will rarely rebalance their portfolios. As a consequence, the dynamic MVHR estimated for the first period often remained in place for most of the hedge. This significantly reduced the benefits of the dynamic strategies, with the differences between the dynamic strategies and the OLS MVHR being marginal. The results therefore suggest that only hedgers with high degrees of risk aversion should consider dynamic strategies when conducting a hedge on the All Ordinaries Index.

Table I The performance of alternative MVHRs

Reference	Data	MVHRs	Comments regarding risk reduction
Cecchetti *et al* (1988)	20 yr T-bond Monthly, 1/78-12/83	Biv-ARCH(3)	Time invariant MVHR inappropriate.
Baillie and Myers (1991)	Commodities Daily, 82-86 (periods vary)	OLS, Biv-GARCH	Time invariant MVHR inappropriate. GARCH outperforms OLS.
Myers (1991)	Wheat Weekly, 6/77-5/83	OLS, Biv-GARCH	MVHRs are time varying, however the GARCH model's performance is only marginally better. Suggested that once transaction costs are taken into account, the OLS strategy is probably the preferred strategy.
Sephton (1993)	Commodities Daily, 5/88-5/89	OLS, Biv-GARCH	Time invariant MVHR inappropriate. GARCH outperforms OLS.
Kroner & Sultan (1993)	Currencies vis-a vis USD Weekly, 2/85-2/90	Naïve,OLS, Biv-ECM, Biv-EC-GARCH	Biv-EC-GARCH provides best performance (with and without transaction costs).
Park and Switzer (1995)	S&P500, Toronto 35 Weekly, 6/88-12/91	Naïve, OLS, ECM, Biv-EC-GARCH	Biv-EC-GARCH hedge provides superior performance (with and without transaction costs).
Koutmos and Pericli (1998)	Commercial paper with T-bill futures, Weekly, 1/85- 3/96	OLS, Biv-GARCH, Biv-EC-GARCH	Biv-EC-GARCH provides best performance (with and without transaction costs). Both cointegration and time varying moment estimation improves hedging performance.
Lien and Tse (1999)	Nikkei 225 Daily, 1/89-8/97	OLS, VAR, ECM, FIECM Biv-VAR/EC/FIEC-GARCH	Including GARCH improves hedging performance. EC-GARCH is the dominant strategy. The use of the FIECM does not improve performance. OLS provides the worst performance.
Sim and Zurbruegg (2000)	KOSPI 200 Daily, 5/96-3/99	OLS, Biv-GARCH-X Biv-EC-GARCH-X	Biv-EC-GARCH-X provides best performance.

Biv = bivariate, ECM = error correction model, VAR = Vector autoregression, FIECM = Fractionally Integrated error correction model, GARCH-X = GARCH model with lagged error correction term in the conditional variance (Lee, 1994)

Table II MVHRs to be examined over 5, 20, 40 and 60 day hedging horizons

Time invariant	Single period dynamic	Multi-period dynamic
Naïve	GARCH (const ρ)	GARCH (const ρ)
OLS	GARCH (diag)	GARCH (diag)
	FIGARCH (const ρ)	FIGARCH (const ρ)
	FIGARCH (diag)	FIGARCH (diag)

Table III MVHR estimation over 5 day horizon commencing 2/8/99

MVHR	Ranking	Variance	Avg MVHR
Unhedged	11	0.5571	
Naïve	10	0.0593	1
OLS	9	0.0527	0.6616
Single period			
GARCH (const ρ)	7	0.0479	0.6848
GARCH (diag)	3	0.0449	0.6982
FIGARCH (const ρ)	8	0.0494	0.6837
FIGARCH (diag)	2	0.0424	0.7095
Multi-period			
GARCH (const ρ)	6	0.0477	0.6857
GARCH (diag)	4	0.0449	0.6982
FIGARCH (const ρ)	5	0.0471	0.6897
FIGARCH (diag)	1	0.0418	0.7162

Rankings: 1-Best risk reduction, 11-Worst risk reduction
The portfolio consists of the underlying asset (the All Ordinaries Index) and SPI futures. The unhedged return is the return on the underlying asset over the 5 day period commencing 2/8/99.

Table IV MVHR estimation over 20 day horizon commencing 2/8/99

MVHR	Ranking	Variance	Avg MVHR
Unhedged	11	0.7439	
Naïve	10	0.0769	1
OLS	9	0.0713	0.6616
Single period			
GARCH (const ρ)	7	0.0570	0.7180
GARCH (diag)	5	0.0550	0.7285
FIGARCH (const ρ)	4	0.0546	0.7289
FIGARCH (diag)	2	0.0525	0.7454
Multi-period			
GARCH (const ρ)	8	0.0572	0.7169
GARCH (diag)	6	0.0558	0.7253
FIGARCH (const ρ)	3	0.0545	0.7287
FIGARCH (diag)	1	0.0519	0.7465

Rankings: 1-Best risk reduction, 11-Worst risk reduction
The portfolio consists of the underlying asset (the All Ordinaries Index) and SPI futures. The unhedged return is the return on the underlying asset over the 20 day period commencing 2/8/99.

Table V MVHR estimation over 40 day horizon commencing 2/8/99

MVHR	Ranking	Variance	Avg MVHR
Unhedged	11	0.7462	
Naïve	9	0.0964	1
OLS	10	0.1103	0.6616
Single period			
GARCH (const ρ)	7	0.0905	0.7372
GARCH (diag)	3	0.0877	0.7500
FIGARCH (const ρ)	4	0.0882	0.7464
FIGARCH (diag)	1	0.0854	0.7661
Multi-period			
GARCH (const ρ)	8	0.0923	0.7275
GARCH (diag)	6	0.0900	0.7382
FIGARCH (const ρ)	5	0.0897	0.7366
FIGARCH (diag)	2	0.0857	0.7580

Rankings: 1-Best risk reduction, 11-Worst risk reduction
The portfolio consists of the underlying asset (the All Ordinaries Index) and SPI futures. The unhedged return is the return on the underlying asset over the 40 day period commencing 2/8/99.

Table VI MVHR estimation over 60 day horizon commencing 2/8/99

MVHR	Ranking	Variance	Avg MVHR
Unhedged	11	0.7589	
Naïve	9	0.1053	1
OLS	10	0.1118	0.6616
Single period			
GARCH (const ρ)	5	0.0932	0.7419
GARCH (diag)	3	0.0903	0.7554
FIGARCH (const ρ)	4	0.0928	0.7498
FIGARCH (diag)	1	0.0884	0.7695
Multi-period			
GARCH (const ρ)	8	0.0957	0.7256
GARCH (diag)	6	0.0934	0.7358
FIGARCH (const ρ)	7	0.0938	0.7345
FIGARCH (diag)	2	0.0894	0.7558

Rankings: 1-Best risk reduction, 11-Worst risk reduction
The portfolio consists of the underlying asset (the All Ordinaries Index) and SPI futures. The unhedged return is the return on the underlying asset over the 60 day period commencing 2/8/99.

Table VII Correlation between index and futures returns

Period	Observations	Correlation
Estimation period	1 to 2928	0.8778
5 Day hedge	2929 to 2933	0.9695
20 day hedge	2929 to 2948	0.9697
40 day hedge	2929 to 2968	0.9466
60 day hedge	2929 to 2988	0.9455

Table VIII Dynamic single period versus multi-period MVHRs

Model	5 day	20 day	40 day	60 day
GARCH (const ρ)	Multi	Single	Single	Single
GARCH (diag)	Single	Single	Single	Single
FIGARCH (const ρ)	Multi	Multi	Single	Single
FIGARCH (diag)	Multi	Multi	Single	Single

Table entries represent the best MVHR. For example, for the GARCH (const ρ) model, when hedging over 5 days, the MPMVHR outperforms the single period MVHR.

Table IX Overall Ranking of dynamic MVHR estimation methods

	Hedges commencing 2/8/99				Summary	
	5 day	20 day	40 day	60 day	Total rank	Final rank
Single period						
GARCH (const ρ)	7	7	7	5	26	7
GARCH (diag)	3	5	3	3	14	3
FIGARCH (const ρ)	8	4	4	4	20	5
FIGARCH (diag)	2	2	1	1	6	2
Multi-period						
GARCH (const ρ)	6	8	8	8	30	8
GARCH (diag)	4	6	6	6	22	6
FIGARCH (const ρ)	5	3	5	7	20	4
FIGARCH (diag)	1	1	2	2	6	1

Rankings: 1-Best risk reduction, 8-Worst risk reduction

Table X MVHR estimation over 5 day horizon commencing 2/8/99, Risk aversion parameter 100

MVHR	Ranking	Variance	Avg MVHR	No of rebalances
Unhedged	11	0.5571		
Naïve	10	0.0593	1	
OLS	9	0.0527	0.6616	
Single period				
GARCH (const ρ)	7	0.0497	0.6745	0
GARCH (diag)	4	0.0466	0.6889	0
FIGARCH (const ρ)	8	0.0497	0.6808	1
FIGARCH (diag)	1	0.0423	0.7068	1
Multi-period				
GARCH (const ρ)	6	0.0492	0.6763	0
GARCH (diag)	3	0.0465	0.6891	0
FIGARCH (const ρ)	5	0.0484	0.6819	1
FIGARCH (diag)	2	0.0444	0.7051	1

Rankings: 1-Best risk reduction, 11-Worst risk reduction
The portfolio consists of the underlying asset (the All Ordinaries Index) and SPI futures. The unhedged return is the return on the underlying asset over the 5 day period commencing 2/8/99.

Table XI MVHR estimation over 20 day horizon commencing 2/8/99, Risk aversion parameter 100

MVHR	Ranking	Variance	Avg MVHR	No of rebalances
Unhedged	11	0.7439		
Naïve	10	0.0769	1	
OLS	9	0.0713	0.6616	
Single period				
GARCH (const ρ)	8	0.0598	0.7140	2
GARCH (diag)	5	0.0569	0.7204	2
FIGARCH (const ρ)	4	0.0539	0.7333	2
FIGARCH (diag)	1	0.0529	0.7380	4
Multi-period				
GARCH (const ρ)	7	0.0589	0.7121	2
GARCH (diag)	6	0.0573	0.7172	2
FIGARCH (const ρ)	3	0.0539	0.7272	3
FIGARCH (diag)	2	0.0534	0.7367	2

Rankings: 1-Best risk reduction, 11-Worst risk reduction
The portfolio consists of the underlying asset (the All Ordinaries Index) and SPI futures. The unhedged return is the return on the underlying asset over the 20 day period commencing 2/8/99.

Table XII MVHR estimation over 40 day horizon commencing 2/8/99, Risk aversion parameter 100

MVHR	Ranking	Variance	Avg MVHR	No of rebalances
Unhedged	11	0.7462		
Naïve	9	0.0964	1	
OLS	10	0.1103	0.6616	
Single period				
GARCH (const ρ)	6	0.0907	0.7401	4
GARCH (diag)	4	0.0885	0.7536	3
FIGARCH (const ρ)	3	0.0877	0.7486	10
FIGARCH (diag)	1	0.0861	0.7595	9
Multi-period				
GARCH (const ρ)	8	0.0922	0.7328	3
GARCH (diag)	7	0.0913	0.7361	3
FIGARCH (const ρ)	5	0.0907	0.7362	4
FIGARCH (diag)	2	0.0869	0.7531	5

Rankings: 1-Best risk reduction, 11-Worst risk reduction
The portfolio consists of the underlying asset (the All Ordinaries Index) and SPI futures. The unhedged return is the return on the underlying asset over the 40 day period commencing 2/8/99.

Table XIII MVHR estimation over 60 day horizon commencing 2/8/99, Risk aversion parameter 100

MVHR	Ranking	Variance	Avg MVHR	No of rebalances
Unhedged	11	0.7589		
Naïve	9	0.1053	1	
OLS	10	0.1118	0.6616	
Single period				
GARCH (const ρ)	4	0.0922	0.7455	6
GARCH (diag)	3	0.0907	0.7599	4
FIGARCH (const ρ)	5	0.0926	0.7510	16
FIGARCH (diag)	1	0.0891	0.7669	15
Multi-period				
GARCH (const ρ)	8	0.0951	0.7266	2
GARCH (diag)	6	0.0930	0.7387	4
FIGARCH (const ρ)	7	0.0942	0.7367	6
FIGARCH (diag)	2	0.0906	0.7531	4

Rankings: 1-Best risk reduction, 11-Worst risk reduction
The portfolio consists of the underlying asset (the All Ordinaries Index) and SPI futures. The unhedged return is the return on the underlying asset over the 60 day period commencing 2/8/99.

Table XIV Overall Ranking of MVHR estimation methods with transaction costs

	Hedges commencing 2/8/99				Summary	
	5 day	20 day	40 day	60 day	Total rank	Final rank
Single period						
GARCH (const ρ)	7	8	6	4	25	7
GARCH (diag)	4	5	4	3	16	3
FIGARCH (const ρ)	8	4	3	5	20	5
FIGARCH (diag)	1	1	1	1	4	1
Multi-period						
GARCH (const ρ)	6	7	8	8	31	8
GARCH (diag)	3	6	7	6	22	6
FIGARCH (const ρ)	5	3	5	7	20	4
FIGARCH (diag)	2	2	2	2	8	2

Figure 1 Single and Multi-period MVHRs over 5 day horizons

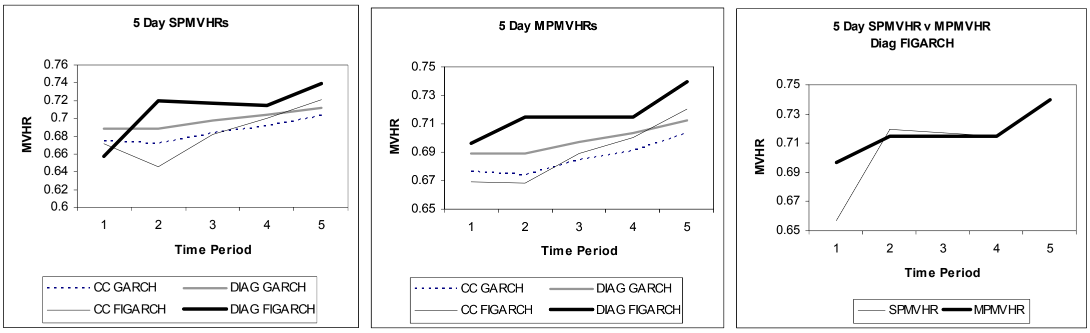


Figure 2 Single and Multi-period MVHRs over 20 day horizons

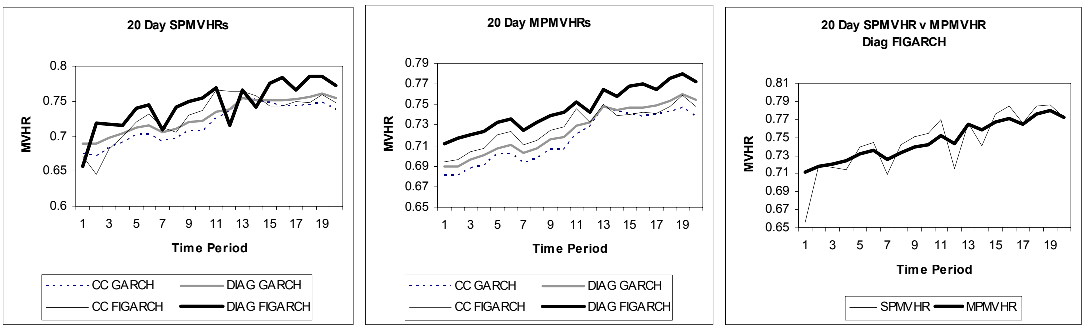


Figure 3 Single and Multi-period MVHRs over 40 day horizons

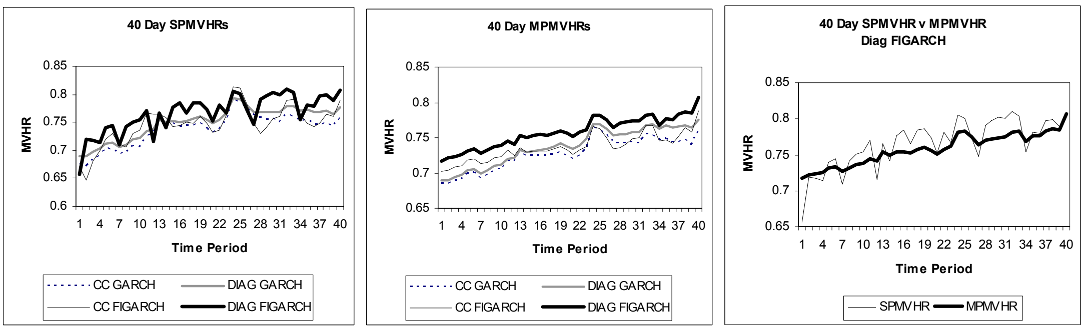


Figure 4 Single and Multi-period MVHRs over 60 day horizons

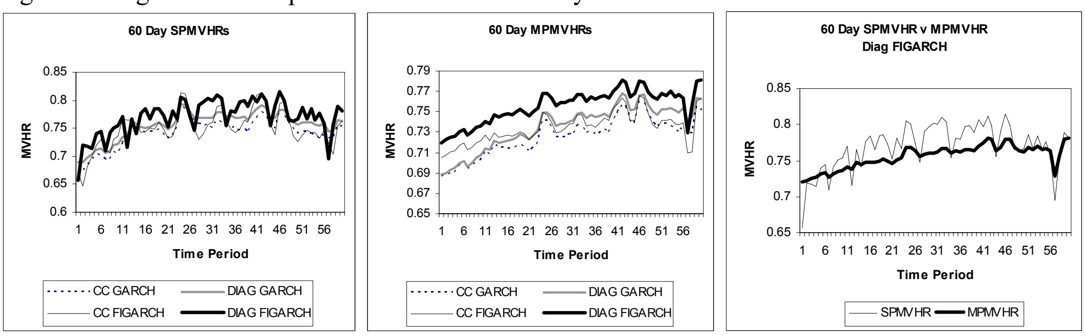


Figure 5 5 day single and multi-period Diagonal FIGARCH MVHRs with and without transaction costs

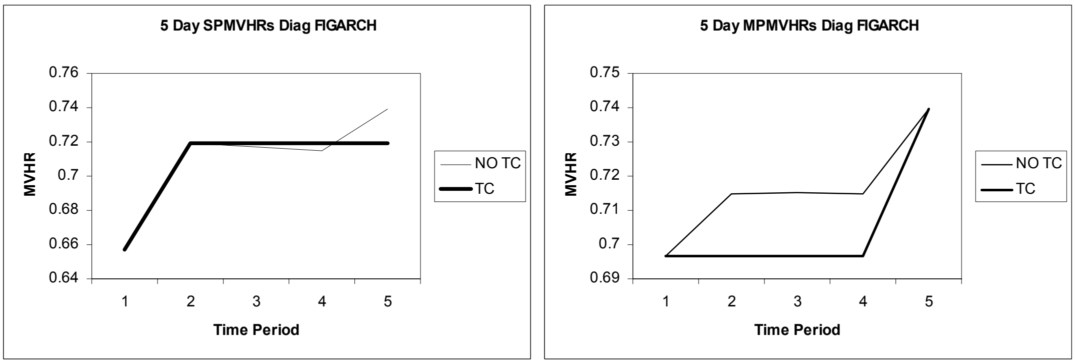


Figure 6 20 day single and multi-period Diagonal FIGARCH MVHRs with and without transaction costs

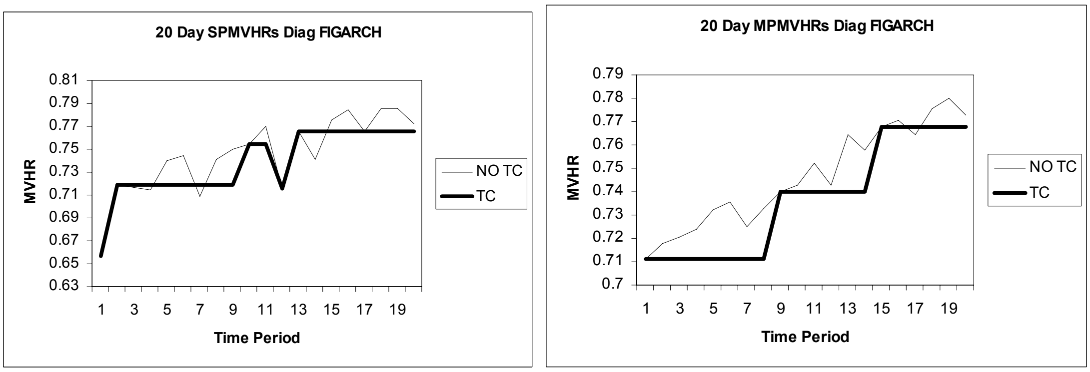


Figure 7 40 day single and multi-period Diagonal FIGARCH MVHRs with and without transaction costs

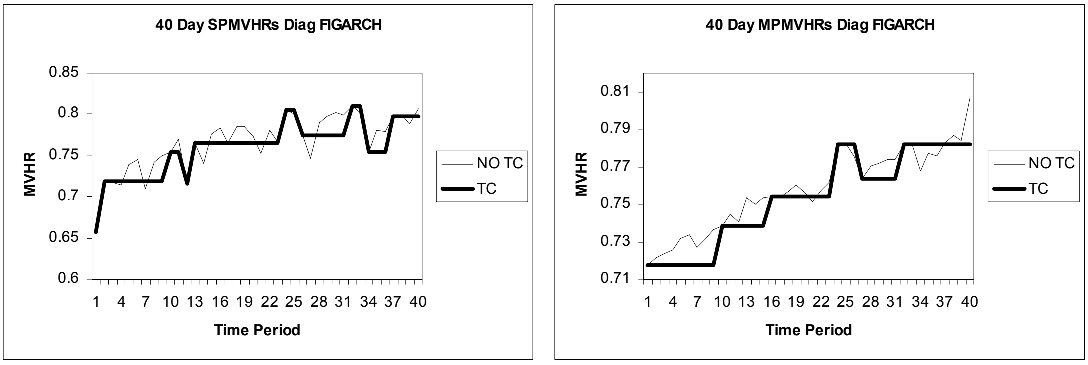
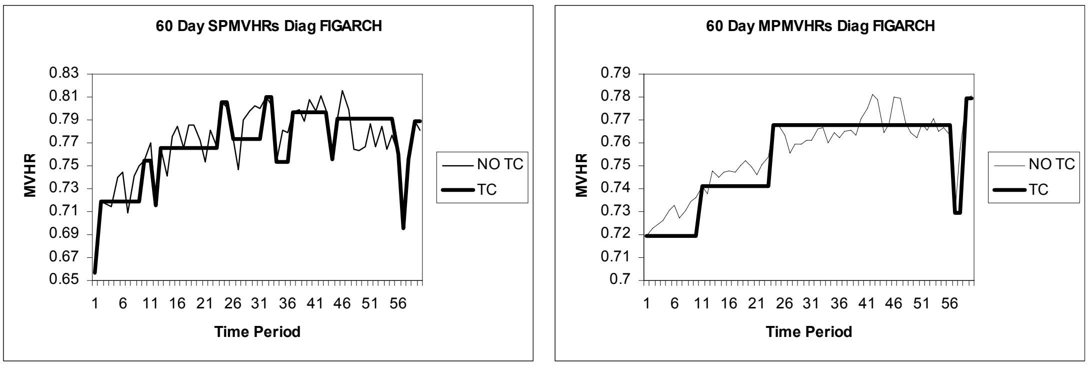


Figure 8 60 day single and multi-period Diagonal FIGARCH MVHRs with and without transaction costs



Appendix – Estimation Results

Table XV Bivariate GARCH estimation

	Constant Correlation		Diagonal	
	Coefficient	Nyblom	Coefficient	Nyblom
Mean				
a_1	0.04 (2.51)	0.27	0.04 (2.52)	0.34
b_1	-0.07 (-9.26)	1.45**	-0.07 (-8.75)	2.01**
$c_{1,1}$	-0.17 (-8.31)	0.87**	-0.17 (-8.81)	0.96**
$d_{1,1}$	0.23 (13.66)	0.55*	0.22 (14.03)	0.71*
a_2	0.05 (2.49)	0.21	0.04 (2.23)	0.28
$c_{2,2}$	0.11 (4.45)	0.27	0.13 (5.31)	0.22
$d_{2,2}$	-0.11 (-5.51)	0.10	-0.12 (-6.10)	0.10
$d_{2,3}$	-0.02 (-2.20)	0.11	-0.02 (-2.56)	0.15
Variance				
ω_s	0.03 (2.12)	0.09	0.02 (1.18)	0.14
α_s	0.05 (3.38)	0.13	0.04 (2.51)	0.08
β_s	0.91 (29.72)	0.10	0.93 (23.05)	0.13
ω_f	0.03 (2.29)	1.59**	0.03 (1.12)	0.43
α_f	0.04 (3.62)	2.36**	0.04 (2.34)	0.44
β_f	0.94 (47.32)	2.23**	0.93 (24.17)	0.46
ω_{sf}			0.02 (1.07)	0.27
α_{sf}			0.04 (2.44)	0.17
β_{sf}			0.93 (22.16)	0.24
ρ	0.90 (188.93)	2.50**		
LL function	-5302.86		-5278.01	
AIC	3.6324		3.6168	
Schwarz	3.6631		3.6515	
Shibata	3.6324		3.6167	
Hann Quinn	3.6435		3.6293	

QMLE t statistics are in parentheses. Nyblom statistics - * = significant at 5% (critical value 0.47), ** = significant at 1% (critical value 0.75) – see Nyblom (1989).

Table XVI Diagnostics – Constant Correlation GARCH (1,1)

Test	Index	Futures	Covariance
Q(10)	0.86	0.87	0.70
Q(15)	0.42	0.33	0.92
Q(20)	0.46	0.24	0.99
Q2(10)	0.27	0.61	1.00
Q2(15)	0.63	0.81	1.00
Q2(20)	0.87	0.95	1.00
Sign bias	0.02	0.01	0.26
Negative size bias	0.01	0.00	0.98
Positive size bias	0.59	0.11	0.71
Joint test	0.04	0.01	0.71
Skewness	<0.001	<0.001	
Excess kurtosis	<0.001	<0.001	
Jarque-Bera	<0.001	<0.001	

Entries represent p values, Q(10) = Box Pierce statistic on ε_t / σ_t for 10 lags, Q2(10) is the statistic for $\varepsilon^2_t / \sigma^2_t$

Table XVII Diagnostics – Diagonal GARCH (1,1)

Test	Index	Futures	Covariance
Q(10)	0.82	0.83	0.57
Q(15)	0.37	0.31	0.84
Q(20)	0.41	0.22	0.97
Q2(10)	0.08	0.56	1.00
Q2(15)	0.28	0.74	1.00
Q2(20)	0.57	0.93	1.00
Sign bias	0.14	0.01	0.88
Negative size bias	0.00	0.00	0.72
Positive size bias	0.69	0.18	0.93
Joint test	0.04	0.01	0.99
Skewness	<0.001	<0.001	
Excess kurtosis	<0.001	<0.001	
Jarque-Bera	<0.001	<0.001	

Entries represent p values, Q(10) = Box Pierce statistic on ε_t / σ_t for 10 lags, Q2(10) is the statistic for $\varepsilon^2_t / \sigma^2_t$

Table XVIII Bivariate FIGARCH Estimation

Coefficient	Constant Correlation		Diagonal	
	Coeff't	Nyblom	Coeff't	Nyblom
Mean				
a_1	0.03 (2.42)	0.29	0.03 (2.33)	0.34
b_1	-0.07 (-9.41)	1.28**	-0.06 (-8.26)	1.78**
$c_{1,1}$	-0.17 (-8.15)	0.84**	-0.17 (-8.74)	1.25**
$d_{1,1}$	0.23 (13.64)	0.51*	0.23 (14.37)	0.91**
a_2	0.04 (2.41)	0.22	0.04 (1.95)	0.29
$c_{2,2}$	0.11 (4.31)	0.24	0.14 (5.75)	0.18
$d_{2,2}$	-0.11 (-5.25)	0.08	-0.13 (-6.51)	0.07
$d_{2,3}$	-0.02 (-2.06)	0.12	-0.02 (-2.69)	0.14
Variance				
ω_s	0.19 (5.71)	0.09	0.08 (1.95)	0.08
d_s	0.16 (7.57)	0.08	0.19 (4.43)	0.09
ϕ_s			0.45 (3.30)	0.20
β_s	0.09 (3.57)	0.12	0.59 (3.87)	0.30
ω_f	0.25 (5.91)	1.07**	0.10 (2.31)	0.13
d_f	0.19 (9.14)	1.58**	0.22 (6.21)	0.12
ϕ_f			0.48 (4.67)	0.03
β_f	0.13 (4.23)	0.15	0.64 (5.57)	0.08
ω_{sf}			0.08 (2.31)	0.10
d_{sf}			0.20 (5.35)	0.05
ϕ_{sf}			0.46 (4.49)	0.10
β_{sf}			0.62 (5.37)	0.22
ρ	0.90 (196.72)	2.43**		
LL function	-5293.46		-5251.01	
AIC	3.6260		3.6004	
Schwarz	3.6566		3.6413	
Shibata	3.6259		3.6003	
Hann Quinn	3.6370		3.6151	

QMLE t statistics are in parentheses. Nyblom statistics - * = significant at 5% (critical value 0.47), ** = significant at 1% (critical value 0.75) – see Nyblom (1989).

Table XIX Diagnostics – Constant Correlation FIGARCH

Test	Index	Futures	Covariance
Q(10)	0.85	0.85	0.88
Q(15)	0.38	0.34	0.91
Q(20)	0.42	0.24	0.98
Q2(10)	0.71	0.85	1.00
Q2(15)	0.86	0.69	1.00
Q2(20)	0.97	0.85	1.00
Sign bias	0.03	0.01	0.24
Negative size bias	0.07	0.03	0.98
Positive size bias	0.34	0.04	0.51
Joint test	0.15	0.03	0.64
Skewness	<0.001	<0.001	
Excess kurtosis	<0.001	<0.001	
Jarque-Bera	<0.001	<0.001	

Entries represent p values, Q(10) = Box Pierce statistic on ε_t / σ_t for 10 lags, Q2(10) is the statistic for $\varepsilon^2{}_t / \sigma^2{}_t$

Table XX Diagnostics – Diagonal FIGARCH

Test	Index	Futures	Covariance
Q(10)	0.83	0.80	0.60
Q(15)	0.38	0.27	0.73
Q(20)	0.40	0.18	0.93
Q2(10)	0.16	0.70	1.00
Q2(15)	0.37	0.58	1.00
Q2(20)	0.67	0.81	1.00
Sign bias	0.03	0.02	0.25
Negative size bias	0.01	0.04	0.95
Positive size bias	0.71	0.05	0.69
Joint test	0.04	0.04	0.69
Skewness	<0.001	<0.001	
Excess kurtosis	<0.001	<0.001	
Jarque-Bera	<0.001	<0.001	

Entries represent p values, Q(10) = Box Pierce statistic on ε_t / σ_t for 10 lags, Q2(10) is the statistic for $\varepsilon^2_t / \sigma^2_t$

Footnotes

[1] There are also a number of other definitions of long memory, refer Baillie (1996) and Davidson (2002).

[2] Myers and Thompson (1989) and Castelino (1990) note that there is no consensus on whether estimation should be in levels, first differences or returns.

[3] To illustrate, the MVHR between time *t* and *t*+1 is based on $^{*}\sigma_{sf,t}$ and $^{*}\sigma^{2}{}_{f,t}$, obtained from the appropriate element in $\left[vech\left(H_{t+1}\right)+vech\left(H_{t+2}\right)+...+vech\left(H_{t+r}\right)\right]$. At time *t*+1, in light of the additional information that is available, the hedger revises the forecasts and uses $\left[vech\left(H_{t+2}\right)+vech\left(H_{t+3}\right)+...+vech\left(H_{t+r}\right)\right]$ to construct the MVHR between *t*+1 to *t*+2. This procedure continues over the life of the hedge.

[4] All models are estimated using Quasi Maximum Likelihood methods. For the FIGARCH process, the pre-sample values are set equal to the unconditional variance estimate with a truncation lag of 1000 observations used. Numerical procedures are used to impose positive definiteness in the diagonal model. Forecasts of the conditional covariance elements are made recursively and independently of the conditional mean.

[5] Note that the time invariant ECM MVHR has not been considered. Lien (1996) shows that if $b_2 = 0$ in equation 8, then failure to employ the ECM will not bias the MVHR estimates.

The estimation results in the Appendix support this restriction. Consequently, the naïve and OLS MVHRs are the only time invariant approaches considered.

[6] The removal of these observations had a minor impact on the results and do not effect the conclusions drawn.

[7] The modified Augmented Dickey Fuller test of Zivot and Andrews (1992) is used to test for unit roots. Cointegration is examined using the modified Engle and Granger (1987) approach in Gregory and Hansen (1996).

[8] Details are available from the author upon request.

[9] The significant decrease in the 60 day single and multi-period MVHRs that is evident near the completion of the hedge (October, 19, 1999), is due to a large negative return in the futures on October 18, 1999. This observation also coincided with the anniversary of Black Monday. It had the effect of increasing the forecasts of the futures conditional variance, which consequently decreased the single and multi-period MVHRs. The observation was not removed in the initial stages given that it was not abnormally large.

[10] A round trip involves the buying and selling of one futures contract. Following Twite (1998) transaction costs of 0.10% are employed. These costs are large in comparison to other markets; Kroner and Sultan (1993) employ 0.01% on a currency futures hedge, Koutmos and Pericli (1998) use 0.0005% on a T-bill futures hedge.

[11] More detailed results are available from the author upon request.

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