ISSN 1440-771X

Department of Econometrics and Business Statistics
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Multivariate exponential smoothing for forecasting tourist arrivals to Australia and New Zealand

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February 2010

Working Paper 11/09

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22 February 2010

Abstract

In this paper we propose a new set of multivariate stochastic models that capture time varying seasonality within the vector innovations structural time series (VISTS) framework. These models encapsulate exponential smoothing methods in a multivariate setting. The models considered are the local level, local trend and damped trend VISTS models with an additive multivariate seasonal component. We evaluate their performances for forecasting international tourist arrivals from eleven source countries to Australia and New Zealand.

JEL classification: C32,C53

Keywords: Holt-Winters', stochastic seasonality, vector innovations state space models.

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1 Introduction

Forecasts from univariate exponential smoothing specifications have proven to be highly accurate in many areas of economics, including tourism (Athanasopoulos, Hyndman, Song & Wu 2008). However, univariate specifications are limited in that they are unable to capture important dynamic inter-relationships between variables of interest. In this paper we demonstrate that the relative forecast accuracy can be improved by extending standard univariate seasonal exponential smoothing models to capture inter-series dependencies. In particular, we extend the class of stochastic state space models presented by Hyndman, Koehler, Snyder & Grose (2002), thereby building on the Vector Innovations Structural Time Series (VISTS) framework outlined by de Silva, Hyndman & Snyder (2009), by explicitly modelling the seasonal component. This extension is crucial in the context of tourism data, as seasonality is one of the key characteristics of such data. Furthermore, the proposed multivariate framework is simple to implement, and is therefore a very useful and practical modelling tool.

Many papers have studied seasonal (monthly or quarterly) tourism demand from several countries of origin to various destination countries (see for example González & Moral 1995, Kulendran & King 1997, Kim & Moosa 2005, Song & Witt 2006, Wong et al. 2007, Athanasopoulos et al. 2008, amongst others). These papers employ multivariate specifications that include explanatory variables in multiple regression or vector autoregression frameworks. Although the forecasting results are conflicting at times, one can argue that these types of specifications do not improve the forecast accuracy over that of carefully specified pure time series alternatives.

In the aforementioned multivariate investigations, tourist flows are always modelled in isolation. Furthermore, the pure time series approaches considered are always univariate. Hence, these specifications do not allow for any interaction between the tourist flows from the various countries of origin. The one exception is du Preez & Witt (2003), who model monthly tourist arrivals from four countries of origin to the Seychelles, using multivariate state space models as specified by Akaike (1976). Their results show that in general univariate ARIMA models forecast monthly tourist flows more accurately than the univariate and multivariate state space models. However, it is important to note that the state space models fitted by du Preez & Witt (2003) are of an autoregressive nature, and are very different from the formulations we present in this paper.

In addition to specifying a new set of models, this study also differs from previous research in that we study eleven bivariate data sets of tourist arrivals from a common country of origin to Australia and New Zealand. We expect that tourism flows to Australia and New Zealand will be strongly associated, since the two countries are closely aligned both geographically and economically. We also believe that the forecast accuracy can be improved by capturing this association. To evaluate this hypothesis we perform a comprehensive forecast comparison in Section 4.

The structure of the paper is as follows. Section 2 provides a brief background on exponential smoothing specifications, and Section 3 presents an outline of the multivariate state space models we propose and the implementation procedures required to fit them. The results of the forecast evaluations are presented in Section 4, and we conclude the paper in Section 5.

2 Background

Exponential smoothing has been a popular forecast method for over half a century. It is commonly accepted that the method dates back to 1944, when R.G. Brown used it to model the trajectories of bombs fired at submarines in World War II (Gardner 2006). However, Brown's work did not appear in print until 1959 (Brown 1959). At this time, the method was also being used independently to model series containing seasonal components (Holt 1957, Winters 1960).

Interestingly, the multivariate form of this method has developed somewhat independently of the univariate contributions outlined above. The first multivariate specification appeared in 1966 (Jones 1966). Essentially, Jones proposed a multivariate equivalent to the simple exponential smoothing specification (Brown 1959) in the form of a state space model. The next installments were Enns et al. (1982) and Harvey (1986), both of which modified the stochastic properties of Jones' specification.

Other important multivariate contributions include Harvey (1989, chapter 8) and Harvey & Koopman (1997). The state space models outlined in these papers demonstrate the flexibility of the state space formulation.

The first paper to outline a multivariate exponential smoothing seasonal specification was that of Pfeffermann & Allon (1989). They considered two bivariate data sets, comprising *tourist arrivals by air* and *person-nights of tourists in tourist hotels*. They compared the multivariate exponential smoothing model with (vector) autoregressive integrated moving average models and univariate exponential smoothing models. Their results showed that the multivariate exponential smoothing model produced more accurate forecasts.

An important distinction between the state space models discussed so far and the specification adopted in this paper is the number of stochastic terms. We specify an innovations state space model, where a single source of error drives all state and measurement equations. In contrast, the contributions identified above specified a different source of error for each equation, and hence these models are often referred to as multiple source of error models. Arguably, multiple source of error specifications are more challenging to estimate (given that one must employ a Kalman Filter). Furthermore, it can be shown that the parameter range of multiple source of error specifications is smaller, making these models less general than single source of error specifications (see Harvey 1989, pp. 431–432, and de Silva et al. 2009, for further explanations).

Recently, Bermúdez et al. (2009) outlined a multivariate exponential smoothing approach in the form of an innovations state space model. By employing a Bayesian approach, they demonstrate how to calculate prediction intervals that take into account parameter uncertainty. They illustrate their method using hotel occupancy data relating to three provinces in Spain. There are three main differences between our formulation and the one proposed by Bermúdez et al. (2009). One, we expand the set of models to include the case where no trend or a damped trend is present; two, we present a model corresponding to a different parameter space;[1] and three, we constrain the contemporaneous correlations of the innovations to be zero.

3 Multivariate Exponential Smoothing Seasonal Models

The Holt-Winters' method is often chosen by practitioners and academics who want to forecast data with seasonal patterns. Perhaps the main reason for this is that the method has been shown to generate relatively accurate forecasts (Lim & McAleer 2001, Hyndman et al. 2002), and is also simple to implement. The latter feature is particularly attractive, given that seasonal ARIMA models can be complicated to identify and estimate in many business environments, because specialised statistical software is required.

Let the variable of interest at time t be denoted by y_t. A stochastic innovations state space model for the Holt-Winters' method can be written as:

$$\begin{aligned} y_t &= \ell_{t-1} + b_{t-1} + s_{t-p|t} + e_t, &(1)\\ \ell_t &= \ell_{t-1} + b_{t-1} + \alpha_\ell e_t, &(2)\\ b_t &= b_{t-1} + \alpha_b e_t, &(3)\\ s_{t|t} &= s_{t-p|t-1} + g_1 \alpha_s e_t, &(4)\\ s_{t-i|t} &= s_{t-i|t-1} + g_2 \alpha_s e_t, &(5) \end{aligned}$$

for $t = 1, 2, \ldots, T$, where ℓ_t and b_t denote the level and trend at time t respectively. The seasonal component for time $t - p$ conditional on information available at time t is denoted by $s_{t-p|t}$. The terms α_ℓ, α_b and α_s denote the smoothing coefficients. The smoothing coefficients are traditionally constrained to lie between zero and one (referred to earlier as the *usual* parameter region).

The terms g_1 and g_2 represent normalisation terms set to the values $\frac{p-1}{p}$ and $-\frac{1}{p}$ respectively. These terms ensure that the seasonal component adds to zero throughout the updating process and does not become contaminated by the level component (Hyndman, Akram & Archibald 2008).

In many situations it is desirable to generate forecasts for two or more variables simultaneously. Furthermore, these variables are often associated, and the forecast accuracy can often be improved by capturing this association. This is precisely the type of situation that occurs when forecasting tourism data. However, the standard Holt-Winters' specification has no capacity to model the inter-series association, and therefore the

[1] Bermúdez et al. (2009) constrain the smoothing parameters to be between zero and one (*usual* parameter region), whereas we impose *forecastable* parameter restrictions which are consistent with Hyndman, Akram & Archibald (2008). Importantly, Hyndman, Akram & Archibald (2008) show that the (univariate) parameter space subject to *forecastable* restrictions is larger than the parameter space subject to *usual* parameter restrictions.

following multivariate extension is proposed. The Vector Local Trend Seasonal model (VLTS) is specified as

$$
\begin{aligned}
\boldsymbol{y}_t &= \boldsymbol{\ell}_{t-1} + \boldsymbol{b}_{t-1} + \boldsymbol{s}_{t-p|t} + \boldsymbol{e}_t && (6)\\
\boldsymbol{\ell}_t &= \boldsymbol{\ell}_{t-1} + \boldsymbol{b}_{t-1} + \boldsymbol{A}_\ell \boldsymbol{e}_t && (7)\\
\boldsymbol{b}_t &= \boldsymbol{b}_{t-1} + \boldsymbol{A}_b \boldsymbol{e}_t && (8)\\
\boldsymbol{s}_{t|t} &= \boldsymbol{s}_{t-p|t-1} + \boldsymbol{G}_1 \boldsymbol{A}_s \boldsymbol{e}_t && (9)\\
\boldsymbol{s}_{t-i|t} &= \boldsymbol{s}_{t-i|t-1} + \boldsymbol{G}_2 \boldsymbol{A}_s \boldsymbol{e}_t. && (10)
\end{aligned}
$$

The forecast function for h periods into the future is:

$$
\hat{\boldsymbol{y}}_{T+h} = \boldsymbol{\ell}_T + (h-1)\boldsymbol{b}_T + \boldsymbol{s}_{T-p+h|T}, \tag{11}
$$

where the bold terms are N-vectors, and $\boldsymbol{A}_\ell$, $\boldsymbol{A}_b$ and $\boldsymbol{A}_s$ represent "persistence" matrices of N^2 smoothing parameters. The matrices $\boldsymbol{G}_1$ and $\boldsymbol{G}_2$ represent diagonal matrices with values of $\frac{p-1}{p}$ and $-\frac{1}{p}$ respectively.

The multivariate model above can also be modified such that the trend becomes a damped component. We specify the Vector Damped Local Trend Seasonal model (VDLTS) by introducing a new coefficient matrix $\boldsymbol{\Phi}$ into the trend equation:

$$
\boldsymbol{b}_t = \boldsymbol{\Phi} \boldsymbol{b}_{t-1} + \boldsymbol{A}_b \boldsymbol{e}_t, \tag{12}
$$

and as such, the forecast equation becomes

$$
\hat{\boldsymbol{y}}_{T+h} = \boldsymbol{\ell}_T + \boldsymbol{\Phi}^{h-1} \boldsymbol{b}_T + \boldsymbol{s}_{T-p+h|T}. \tag{13}
$$

The term $\boldsymbol{\Phi}$ represents an $N \times N$ diagonal matrix. The diagonal elements are constrained to have an absolute value of less than one. Thus, in this model the trend is modelled as a stationary process rather than as a random walk.

Another important variation of the multivariate Holt-Winters' specification is achieved by setting the local trend equal to zero. This produces the third model we introduce in this paper, the Vector Local Level Seasonal (VLLS) model.

3.1 Implementation

The implementation of the models is straightforward. The key is to recognise that these models are a special case of the Vector Innovations Structural Time Series framework outlined by de Silva et al. (2009):

$$\boldsymbol{y}_t = \boldsymbol{H}\boldsymbol{x}_{t-1} + \boldsymbol{e}_t \tag{14}$$

$$\boldsymbol{x}_t = \boldsymbol{F}\boldsymbol{x}_{t-1} + \boldsymbol{G}\boldsymbol{A}\boldsymbol{e}_t, \tag{15}$$

where $\boldsymbol{y}_t$ denotes an N-vector of observations at time t. The terms $\boldsymbol{H}$, $\boldsymbol{F}$ and $\boldsymbol{A}$ denote various coefficient matrices, $\boldsymbol{x}_t$ is a k-vector of states, and $\boldsymbol{e}_t$ is an N-vector of innovations. The innovations are assumed to follow a Gaussian distribution. It is also assumed that $\boldsymbol{\Sigma}$, the covariance matrix of $\boldsymbol{e}_t$, is diagonal.

Equations (14) and (15) are the *measurement* and *transition* equations, respectively. The measurement equation describes the vector of observations as a function of states (or unobservable components). Three unobservable components have been considered for each series, namely level, trend and seasonality. The way in which these components combine is determined by the structure matrix $\boldsymbol{H}$. The composition of this matrix is determined prior to fitting the model. In this paper it will comprise zeros and ones.

The evolution of the latent components is governed by the transition equation. There are two parts to this equation: the part that is determined prior to fitting the model, the composition of the transition matrix $\boldsymbol{F}$, and the part which is determined by the data, the components of matrix $\boldsymbol{A}$, which comprises the persistence matrices of equations 7 to 10, i.e., $\boldsymbol{A} = (\boldsymbol{A}_\ell' \boldsymbol{A}_b' \boldsymbol{A}_s' \ldots \boldsymbol{A}_s')'$.

We have designed the model to capture inter-series associations through the persistence matrices. Specifically, the off-diagonal elements of each persistence matrix capture the associations between various series at the latent component level. The covariance matrix $\boldsymbol{\Sigma}$ is constrained to be diagonal, and thus we do not allow for contemporaneous associations amongst the residuals. The structure and transition matrices $\boldsymbol{H}$ and $\boldsymbol{F}$ do not capture inter-series associations either, also by design.

The VISTS implementation procedure identified by de Silva et al. (2009) can easily be modified to incorporate the newly introduced seasonal state into the VISTS framework. Specifically, the following likelihood is maximised subject to an invertibility condition:

$$\log L(\boldsymbol{\theta}, \boldsymbol{x}_0) = -\frac{T}{2}\left(\log(2\pi) + \sum_{i=1}^{N} \log(\sigma_i^2)\right) - \frac{1}{2}\sum_{t=1}^{T}\sum_{i=1}^{N} e_{it}^2/\sigma_i^2,$$

where σ_i^2 is the ith diagonal element of $\boldsymbol{\Sigma}$, $\boldsymbol{\theta}$ is a vector of unknown smoothing parameters and $\boldsymbol{x}_0$ is a vector of the initial values for the states.

The invertibility condition required when a seasonal component is present is slightly different from that of de Silva et al. (2009), due to the introduction of the normalisation matrix $\boldsymbol{G}$. The modified invertibility condition is:

$$\lambda_s(\boldsymbol{D}) < 1, \quad \boldsymbol{D} = \boldsymbol{F} - \boldsymbol{G}\boldsymbol{A}\boldsymbol{H}, \tag{16}$$

where λ_s denotes the $N+1$ largest eigenvalue. The reason for this slight modification is that the first N eigenvalues equal one by default, owing to the normalised characteristic of the seasonal component.

Before the optimisation can take place, a set of initial values for the coefficients and components must be determined. Univariate estimates are employed for this purpose. Specifically, the seed values of the states are determined according to the heurisitc method outlined in Chapter 5 of Hyndman, Koehler, Ord & Snyder (2008), using an automated routine in R (R Development Core Team 2008). The coefficient matrices are set to be diagonal, with values corresponding to their optimised univariate estimates.

4 Forecasting application

4.1 Data

The data we consider are eleven bivariate data sets of monthly tourist arrivals to Australia and New Zealand. In Figure ?? we present time series plots for four of the bivariate data sets. The series have been standardised by their sample standard deviation, to enable them to be presented on the same graph. From the plots one can visually observe the associations between the series which multivariate models can potentially exploit and benefit from.

The data span the period from January 1980 to June 2007. We withhold the last 74 observations for forecast evaluation, which we perform using an "expanding window" approach. In particular, we first fit all models to the sample, withholding the last 74 observations, and generate $h = 1$- to 24-step-ahead forecasts. Note that this is the smallest sample which we fit all models to, with $n = 259$ observations. We then add the first observation of the withheld data to the estimation sample. All models are then re-estimated and a second set of $h = 1$- to 24-step-ahead forecasts is generated. We repeat this process 50 times. Hence, we generate 50 forecasts for each forecast horizon ($h = 1$- to 24-step-ahead forecasts). This means that we have 1,100 forecasts per forecast horizon in total (22 series and 50 forecasts per series), which we use for forecast evaluation.

4.2 Forecast error measures

In order to evaluate the forecasting performances of the models in a robust manner, we use three alternative forecast error measures: the Root Mean Square Forecast Error

$$\text{RMSFE}_h = \sqrt{\frac{1}{S \times I} \sum_{s=1}^{S} \sum_{i=1}^{I} (y_h^{s,i} - \hat{y}_h^{s,i})^2},$$

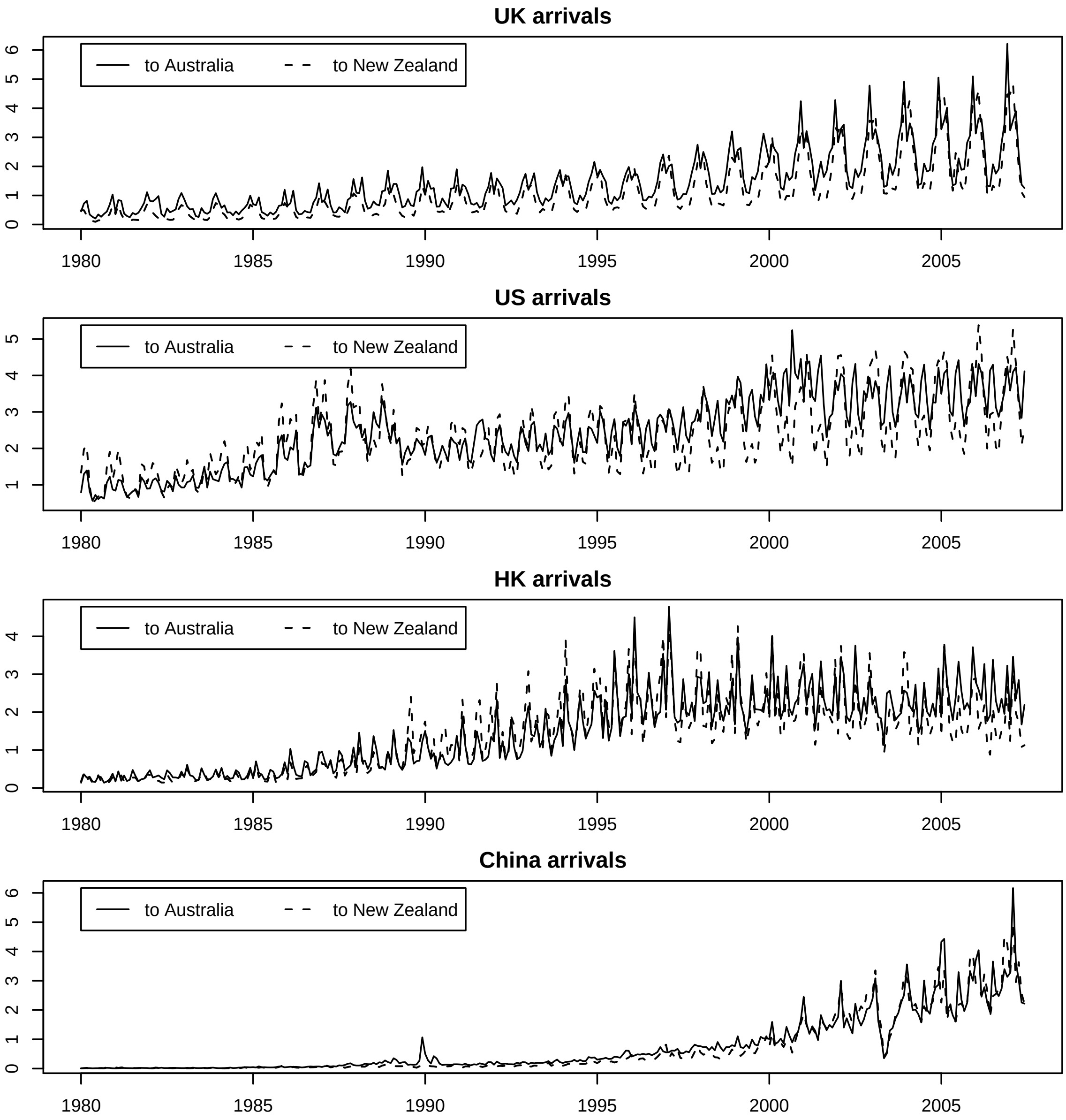


Figure 1: *Standardised tourist arrivals series, travelling from the UK, the US, Hong Kong and China to Australia and New Zealand.*

the Mean Absolute Percentage Error

$$\text{MAPE}_h = \frac{1}{S \times I} \sum_{s=1}^{S} \sum_{i=1}^{I} \left| \frac{y_h^{s,i} - \hat{y}_h^{s,i}}{y_h^{s,i}} \right|,$$

and the Mean Absolute Scaled Error

$$\text{MASE}_h = \frac{1}{S \times I} \sum_{s=1}^{S} \sum_{i=1}^{I} \left| \frac{y_h^{s,i} - \hat{y}_h^{s,i}}{\frac{1}{n^{s,i}-1} \sum_{t=2}^{n^{s,i}} |y_t^{s,i} - y_{t-1}^{s,i}|} \right|,$$

where S is the number of series (in our case 22), I is the number of iterations per forecast horizon (in our case 50), $\hat{y}_h^{s,i}$ is the h-step-ahead forecast for iteration i for series s, and $\frac{1}{n^{s,i}-1}\sum_{t=2}^{n^{s,i}}|y_t^{s,i} - y_{t-1}^{s,i}|$ is the average in-sample one-step-ahead forecast error from a random walk, for iteration i of series s, which has $n^{s,i}$ observations in the estimation sample.

Each measure is informative in its own way and may produce different outcomes. The RMSFE is scale dependent. Hence, it is sensitive to errors from series which are measured on a larger scale (and also to outliers). This can be informative, and is possibly an advantage when evaluating forecasts of tourist arrivals from several source countries to a particular destination. Tourism authorities and policy makers in the destination country are likely to consider the accuracy of forecasts for source countries that provide relatively more tourist arrivals to be of greater importance.

The MAPE is arguably the most popular measure in both forecasting practice and the academic literature. However, Hyndman & Koehler (2006) provide some warnings as to when this measure is unsuitable and Athanasopoulos et al. (2008) found significant practical evidence of cases in which the distribution of MAPE is highly positively skewed, due to some series being of very small scale (tourist arrivals close to zero). This has an adverse effect on the forecast error distribution, relative to that described above. Forecast errors for source countries with very small numbers of tourist arrivals are now very important. However, this is not a concern in this paper, as there are no observations in the holdout samples which are close to zero and which may therefore effect the distribution of the MAPE adversely.

The MASE was proposed by Hyndman & Koehler (2006) in order to overcome the limitations of other forecast measures, whether they are scale dependent, relative or percentage error measures. A MASE value of less than one indicates that the average forecast error is smaller than the average one-step-ahead in-sample error from a random walk.

4.3 Alternative forecasting methods

We evaluate the forecasting performance of the proposed class of multivariate models against the performances of two univariate approaches which form natural benchmarks.

Seasonal Naïve (SN)

The first is a naïve approach for seasonal data (SN), where all forecasts are equal to the most recent observation from the corresponding season. Formally

$$\hat{y}_{n+h|n} = y_{n-12+h_{12}},$$

where $h_{12} = [(h-1) \mod 12] + 1$.

Univariate exponential smoothing (ETS)

The second univariate approach is a fully automated algorithm for univariate exponential smoothing (ETS) methods which was initiated by Hyndman et al. (2002), and was first introduced to the tourism literature by Athanasopoulos et al. (2009). The algorithm provides a means of forecasting time series data by selecting from an extensive range of innovations state space models, which have been shown to generate optimal forecasts for all exponential smoothing methods (including non-linear methods). The statistical foundations on which the algorithm is built allow for maximum likelihood estimation, point and interval forecasting, and procedures for model selection. This algorithm has proven very successful for forecasting, and in particular it was found to forecast monthly tourism data extremely well in the recent extensive forecasting competition performed by Athanasopoulos et al. (2008). We apply the algorithm by minimising the AIC across all additive and multiplicative models. Further details of the algorithm and all possible models are provided by Hyndman, Koehler, Ord & Snyder (2008) and Hyndman & Khandakar (2008).

4.4 Results

The results of the forecasting exercise by horizon are presented in Figures 2 to 4. Despite using three distinct measures of forecast accuracy, the relative performances of the models are fairly similar. In particular, the charts indicate that the multivariate models we introduce in this paper generate reasonably accurate forecasts relative to the univariate alternatives.

Importantly, the vector local level seasonal (VLLS) model is the best performing model across all measures and forecast horizons. According to the RMSFE (Figure 2) and the MAPE (Figure 3), the VLLS model clearly produces the most accurate forecasts. However, the findings are less definitive according to the MASE. The VLLS model clearly produces the most accurate forecasts in the second year of the forecast horizons. The results are not very different, either between the models or relative to SNaïve, for the first year.

Table 1 presents the results for each of the three forecast accuracy measures. In addition to providing the forecast measures at horizons one and twelve, it also presents the average accuracy measures over selected horizons. The models are listed according to their "*Av. rank*", which represents an average rank across all forecast horizons. A smaller value indicates that the model is more accurate.

The values in Table 1 confirm previous observations, namely that the multivariate models, and in particular the VLLS model, perform well. This is typified by the second and third columns, which indicate that the VLLS model produces the most accurate forecasts at the first and twelfth horizons on average. The accuracy of the VLLS model is further demonstrated by the noticeably lower average rank scores for the RMSFE and MAPE measures (note that the difference between the SNaïve and VLLS models according to the average rank of the MASE is marginal).

Another interesting observation is that the VDLTS model is consistently the second best multivariate model. Furthermore, it seems to be at least as accurate as the ETS alternative. One possible explanation for this is that

Figure 2: *RMSFE*

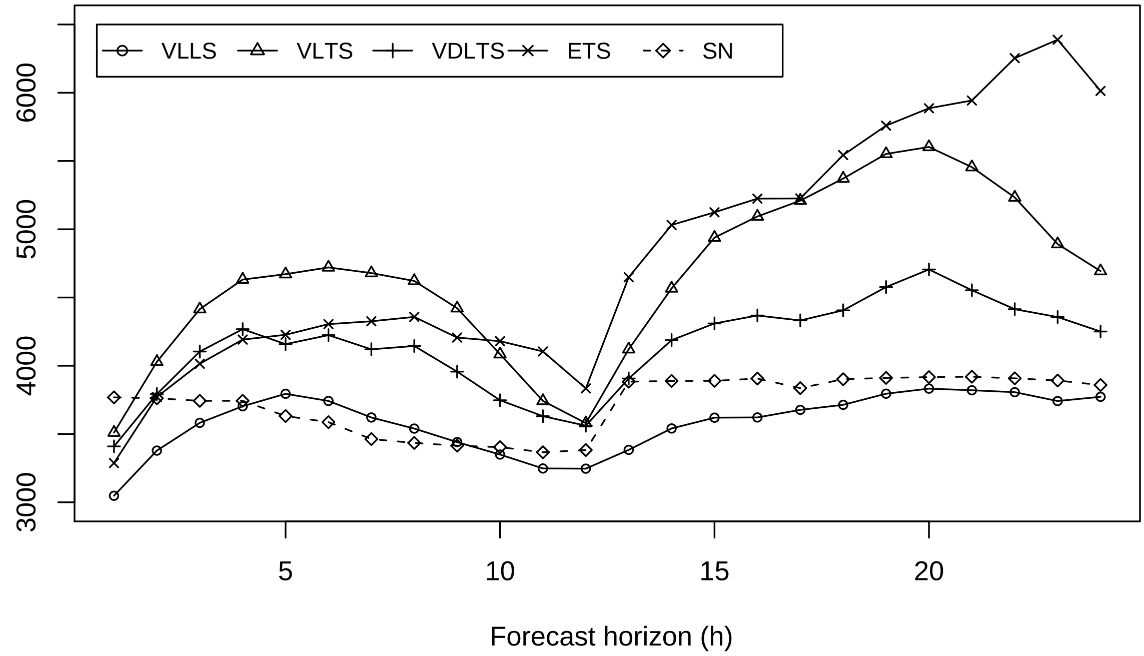
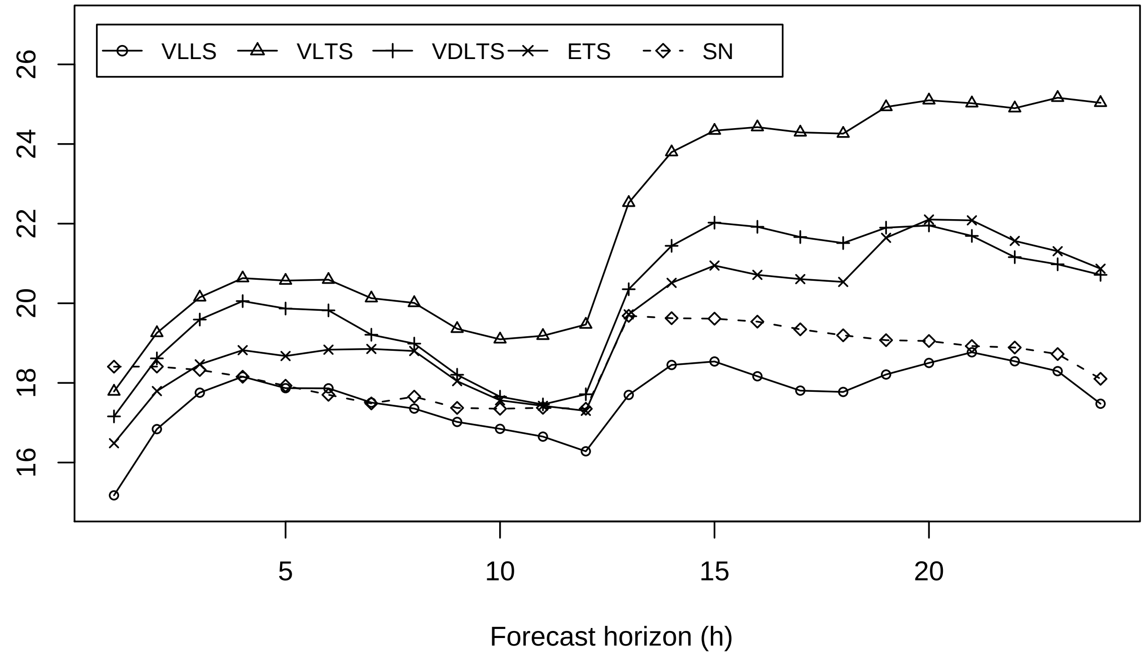


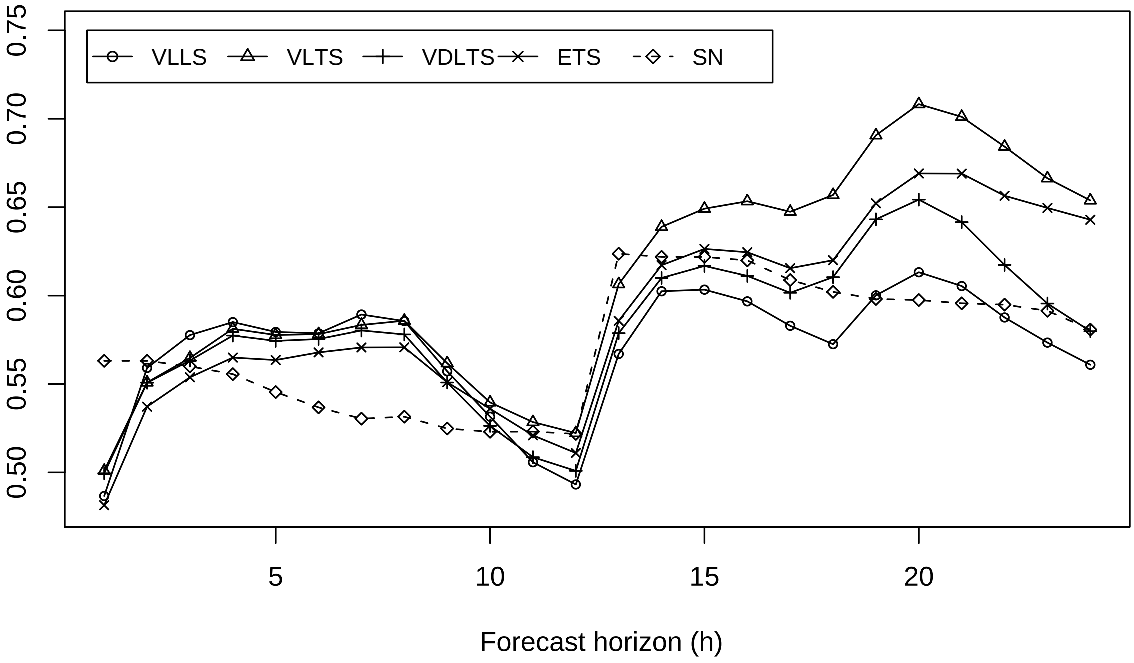
Figure 3: *MAPE*



the VLLS and VDLTS models are both unit root processes, unlike the VLTS model, which is integrated of order two[2] (see Roberts 1982, de Silva 2008, for more details). The VLTS model appears to be the least accurate of all models considered. This suggests that the tourism data considered here are modelled better by processes of

[2]All multivariate models are seasonally integrated of order one.

Figure 4: *MASE*



an integration order of one rather than two. This explanation is supported by the plots in Figure ??, which indicate that a forecast function with a deterministic trend (see equation 11) may not be the most appropriate.

5 Conclusion and directions for future research

In this paper, a multivariate extension of the Hyndman et al. (2002) exponential smoothing models was presented, and its forecasting performance was evaluated. The specification extends the work of de Silva et al. (2009), in showing how seasonality can be incorporated into three different VISTS formulations.

The forecasting results demonstrated that these extensions can produce relatively accurate forecasts. In particular, in Section 4, the forecast accuracy of the three new specifications was evaluated in a large scale forecasting competition against univariate alternatives. Two of the proposed specifications (the Vector Local Level and Vector Damped Local Trend Seasonal models) were demonstrated to produce very accurate forecasts on a consistent basis.

Finally, the VISTS framework is easy to implement. Given its encouraging forecasting performance, we are aiming to develop this framework into a general class of multivariate models. We would envisage this development as lead to fully automated multivariate algorithms along the same lines as current univariate ones (see for example Hyndman & Khandakar 2008).

Table 1: *Forecast results for some selected forecast horizons.*

	Forecast horizon		*Average measures*					*Av. rank*
	1	12	1–3	1–12	1–24	13–24	19–24	1–24
	RMSFE							
VLLS	3047.74	3246.41	3336.12	3474.54	3584.19	3693.84	3794.89	1.21
SN	3768.53	3382.67	3757.75	3558.42	3725.53	3892.64	3901.03	1.92
VDLTS	3409.32	3561.22	3768.84	3926.56	4145.29	4364.02	4476.25	3.13
ETS	3287.34	3834.14	3691.28	4067.21	4826.98	5586.76	6040.54	4.42
VLTS	3510.42	3579.56	3984.75	4258.82	4659.76	5060.69	5237.88	4.33
	MAPE							
VLLS	15.18	16.28	16.59	17.11	17.65	18.19	18.30	1.13
SN	18.41	17.35	18.38	17.79	18.47	19.15	18.79	2.08
ETS	16.49	17.30	17.58	18.09	19.57	21.05	21.60	3.08
VDLTS	17.16	17.71	18.46	18.69	20.07	21.44	21.40	3.75
VLTS	17.78	19.47	19.06	19.69	22.08	24.48	25.02	4.96
	MASE							
SN	0.56	0.52	0.56	0.54	0.57	0.60	0.59	2.38
VLLS	0.49	0.49	0.54	0.55	0.57	0.59	0.59	2.46
VDLTS	0.50	0.50	0.54	0.55	0.58	0.61	0.62	2.54
ETS	0.48	0.51	0.52	0.54	0.59	0.64	0.66	3.00
VLTS	0.50	0.52	0.54	0.56	0.61	0.66	0.68	4.63

6 Acknowledgments

We thank Andrew Maurer and Tourism Research Australia for their continual support and for providing data and explanations.

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