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A Semiparametric Approach to Value-at-Risk, Expected Shortfall and Optimum Asset Allocation in Stock-Bond Portfolios

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Abstract

This paper investigates stock-bond portfolios' tail risks such as value-at-risk (VaR) and expected shortfall (ES), and the way in which these measures have been affected by the global financial crisis. The semiparametric t-copula is found to be adequate for modelling stock-bond joint distributions of G7 countries and Australia. Empirical results show that weak (negative) dependence has increased for seven countries after the crisis, while it has decreased for Italy. However, both VaR and ES have increased for all eight countries. Before the crisis, the minimum portfolio VaR and ES were achieved at an interior solution only for the US, the UK, Australia, Canada and Italy. After the crisis, the corner solution was found for all eight countries. Evidence of "flight to quality" and "safety first" investor behaviour was found to be strong, after the global financial crisis. The semiparametric t-copula adequately forecasts the outer-sample VaR. These findings have implications for global financial regulators and the Basel Committee, whose central focus is currently on increasing the capital requirements as a consequence of the recent global financial crisis.

Keywords: Copula; Semiparametric method; Value-at-Risk; Investment decision

JEL Classifications: C14, C52, G11, F36, G15

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1 Introduction

Stocks and bonds are two of the most frequently considered asset classes in portfolio asset allocation strategies. Recently, several studies have found that the stock–bond correlation decreases and becomes negative during crises. This phenomenon has important implications for portfolio asset allocation decisions for investors, such as portfolio diversification, risk hedging, dynamic asset allocation and portfolio rebalancing. The recent global financial crisis (GFC) had a profound impact on many developed countries. In view of the expected strong negative correlation between stock and bond returns during the GFC, and the fact that value-at-risk (VaR) and the expected shortfall (ES) are popular measures of the downside market risk, one of the aims of this paper is to investigate the relationship between these two risk measures of stock–bond portfolios and the GFC.

The main contribution of this paper is the modelling of the joint distributions of stock–bond returns by employing semiparametric copulas, which fully capture the dependence structure and the distinct characteristics of bond and stock returns. The potential benefits of our investigation for applied researchers and investors come from the use of flexible copula models for estimating the portfolio VaRs and ES and finding the bond weights that minimise the stock–bond portfolio VaRs and ESs of Australia and the G7 countries. To the best of our knowledge, this line of research has not previously been pursued in the empirical finance literature. The reason for including the G7 countries is that the GFC has had catastrophic effects on these major economies. Australia is also included in this empirical investigation because it is widely known that the Australian financial market and its economy have been somewhat resilient to the GFC.

Moreover, this paper differs from previous studies in methodology in that a pattern recognition device known as the chi–plot[1] is used in this paper. This plot can reveal the significance of dependence that is closely associated with copulas. The chi–plot is explained briefly in Section 5 of this paper. The semiparametric copula models[2] are used to measure the inter-

[1] Introduced first by Fisher and Switzer (1985).

[2] Contributors to semiparametric methods include: Genest et al. (1995), Tsukahara (2005), Chen and

relationships of stock–bond markets, while previous studies have mostly used fully parametric copula models in a similar context; see, for example, Patton (2006). The semiparametric procedure, which is found to be robust to mis-specification of marginal distributions (Kim, Silvapulle and Silvapulle, 2007a), is employed for estimating the copula models which, in turn, are used for estimating risk measures such as Value-at-Risk and expected shortfall in our study. Following Genest, Rémillard and Beaudoin (2009), two rank-based goodness-of-fit testing procedures, known as "blanket tests", are employed to assess the suitability of some copulas and choose the one that fits the data well.

The existing literature on the stock–bond relationship is broadly in agreement as to the way in which the stock–bond dependence varies due to regime changes. Early studies used the simple linear correlation[3] to measure the degree of the stock–bond dependence and have established empirical evidence of varying correlation over long horizons; see, for example, Fleming, Kirby and Ostdiek (1998) and Connolly, Stivers and Sun (2005). These authors have found that, for a given asset allocation, the stock–bond portfolio diversification is also found to be changing. Furthermore, in line with the "flight to quality" phenomenon, their findings suggest that a high stock market uncertainty leads to a decoupling of the stock and bond prices. Gulko (2002), Scruggs and Glabadanidis (2003) and Cappiello, Engle and Sheppard (2006) have all contributed to this area and found "flight to quality" investor behaviour during market downturns.

The copula function has become very popular for modelling various dependence structures in multivariate data arising in areas such as economics, finance and insurance; see Cherubini, Mulinacci, Gobbi and Romagnoli (2011) for examples of applications of copulas in finance. One does not need to assume any parametric form for the marginals, as it has no role in the specification of copula.[4] The joint *cdf*, estimated via a copula approach, contains all

Fan (2006), Kim et al. (2007a,b) and Kim et al. (2008).

[3]See the studies by Ilmanen (2003), Jones and Wilson (2004) and Li (2004) on modelling the stock-bond dependence using various methodologies.

[4]Sklar (1959) proved that a multivariate joint cumulative density function (cdf) could be fully and uniquely characterized by its continuous marginal functions and a copula function. The Sklar theorem allows the marginals and the copula components to be estimated separately and independently of each other. See also Rosenblatt (1952) for some remarks on multivariate transformation.

the information about the marginals as well as all the information about the dependence structure that is captured by the copula. In addition, copula functions are able to capture different types of asymmetric dependence and the rich patterns of tail behaviour of joint distribution. Because of these properties, the copula models are found to be appropriate for capturing the underlying true dependence in financial time series. The semiparametric copula modelling of joint distributions would be very attractive due to its flexibility and robustness. In this approach, a nonparametric method produce is used to estimate the univariate margins, the shapes of which are largely unknown in practice, and then a parametric copula is fitted to the joint distribution. Thus, the estimates of tail risks of portfolios by semiparametric copula models are expected to be more reliable than their parametric counterparts.

Artzner, Delbaen, Eber and Heath (1997, 1999) showed that ES is a coherent risk measure, while VaR is not. Semiparametric copulas for computing the VaR and ES would help internationally active banks and other financial institutions to estimate reliable VaR and ES, and hence the right level of capital requirements which is currently the central focus of global regulators and the Basel Committee. Furthermore, the copula has become a popular and more advanced alternative to linear correlation in the area of financial risk management where extreme events need to be captured for accurate estimation of VaR and ES. See the survey paper by Embrechts, Lindskog and McNeil (2003) in regards to modelling extreme events.

Assuming independence between the stocks and bonds, Hyung and de Vries (2007) arrived at an interior solution, which tends to select the asset with the thinnest tail.[5] By adopting the semiparametric copula approach, we model the underlying dependence between the bond and stock returns which will, in turn, allow one to compute the VaR close to the true value and the bond weight that provides the minimum VaR and ES. Christoffersen's (1998) Conditional Coverage (CC) test is applied in order to evaluate the performance of the semiparametric copula in terms of quantile (the VaR) forecasting; Section 6.4 provides more details.

[5]See, for example, early contributors: Roy (1952), Arzac and Bawa (1977) and Jansen et al. (2000) for studies on portfolio selection, limited downside risk and safety first investors.

The rest of the paper is planned as follows: the next section describes the data and the results of some preliminary analysis. Section 3 describes the technical aspects of copula models. Section 4 presents the estimation methods for copula models, as well as the VaR and ES computations via the semiparametric copula approach. Section 5 describes the methodology of chi–plot, and Section 6 reports and analyses the empirical results. Finally, Section 7 concludes the paper.

2 Data and some preliminary analysis

The government bond index was collected for Australia and the G7 countries from the Thomson Reuters Datastream for the period 01/07/2003 to 17/08/2011. The stock price indices include: (1) Australian All Ordinaries Index [AUS]; (2) TSX Composite Index of Canada [CAD]; (3) CAC 40 Index of France [FRA]; (4) DAX 30 Performance Index of Germany [GER]; (5) FTSE MIB Index of Italy [ITL]; (6) NIKKEI 225 Index of Japan [JPN]; (7) FTSE 100 Index of the United Kingdom [UK]; and (8) Dow Jones Industrials Index of the United States [US]. The sample period includes the period of the global financial crisis (GFC) triggered by the sub-prime mortgage crisis in mid-2007. The starting date of our sample period was chosen to be mid-2003 to avoid confounding with the effect of the IT bubble burst in 2001. Since one of our aims is to examine the impact of the GFC on the measures such as stock–bond dependence, the VaR and ES of the stock–bond portfolio as well as on the optimum asset allocation, we split the sample period into two: the pre– and the post–GFC periods $01/07/2003 - 13/07/2007$, and $16/07/2007 - 17/08/2011$, respectively. The return, main variable of interest, is defined as $X = \log(P_t) - \log(P_{t-1})$, where P_t is the price index at time t.

2.1 Filtering the returns

Let X_{1t} and X_{2t} denote the bond returns and stock returns, respectively, at time t ($t = 1, \ldots, T$). For each country and sample period, the best fitting univariate GARCH type model of the following form was estimated:

$$X_{jt} = \gamma_{j0} + \sum_{k=1}^{m_j} \gamma_{jk} X_{j,t-j} + R_{jt}. \quad R_{jt} = h_{jt}\varepsilon_{jt}, \quad h_{jt}^2 = \alpha_{j0} + \sum_{k=1}^{m_j} \alpha_{jk}\varepsilon_{j,t-k}^2,$$

where $(\varepsilon_{1t}, \varepsilon_{2t})$ are assumed to be independent and identically distributed $(t = 1, \ldots, T)$. The *filtered returns* and *standardized residuals* were calculated as

$$\hat{R}_{jt} = X_{jt} - \hat{\gamma}_{j0} - \sum_{k=1}^{m_j} \hat{\gamma}_{jk} X_{j,t-j}, \quad \hat{\varepsilon}_{jt} = \hat{R}_{jt}/\hat{h}_{jt}, \quad (j = 1, 2; t = 1, \ldots, T).$$

Then, we estimated the copula $C(u_1, u_2)$ of $(\varepsilon_1, \varepsilon_2)$ by the semiparametric method outlined in Section 4.1.

2.2 Stock-bond dependence measures

The dependence measures of the Pearson correlation coefficient, Spearman's rank correlation and Kendall's tau between stock and bond returns are estimated for both the pre– and post–GFC periods for all eight countries, and the results are reported in Table 1.

(←-- Insert Table 1 Here --→)

The results in Table 1 show that the stock and bond returns were negatively related both before and after the GFC, for all eight countries. However, the negative relationship was rather weak before the GFC, but statistically significantly different from zero. These correlation estimates indicate that the negative relationship has increased, and has become significantly more negative during the GFC across all eight countries, except for Italy. This rise in negative associations is in line with the "flight to quality" phenomenon. For Italy, on the other hand, this negative association between stock and bond returns has actually become weak after the GFC. This is partly due to the exceptional turmoil in both the Italian stock and government bond markets since the outbreak of the Euro zone debt crisis. There is little perception of "safety" among investors looking to invest in Italian government bonds.

3 Methodologies

3.1 Copula

Let $\boldsymbol{X} = (x_1, \ldots, x_k)$ be a k-variate random variable. Let $G(x_1, \ldots, x_k)$ denote the cumulative distribution function of $\boldsymbol{X}$ and let $G_i(x_i)$ denote the marginal cumulative distribution function of X_i $(-\infty < x_i < \infty, i = 1, \ldots, k)$. Let $U_i = G_i(X_i)$ $(i = 1, \ldots, k)$, and $\boldsymbol{U} = (U_1, \ldots, U_k)^\top$. Then each U_i is uniformly distributed on $[0, 1]$ and $\boldsymbol{U}$ is distributed on the unit cube $\{(u_1, \ldots, u_k) : 0 \leq u_i \leq 1, i = 1, \ldots, k\}$. Let $C(u_1, \ldots, u_k)$ and $c(u_1, \ldots, u_k)$ respectively denote the cumulative distribution and the probability density functions of $\boldsymbol{U}$. The function C is called the *copula* of G or the copula of $\boldsymbol{X}$. It turns out that $G(x_1, \ldots, x_k) = C\{G_1(x_1), \ldots, G_k(x_k)\}$ and $g(x_1, \ldots, x_k) = c\{G_1(x_1), \ldots, G_k(x_k)\} \times g_1(x_1) \times \ldots \times g_k(x_k)$.

Conversely, if an arbitrary cumulative distribution function $C(u_1, \ldots, u_k)$ on the unit cube and univariate distributions $G_1, \ldots, G_k$ are given then $C\{G_1(x_1), \ldots, G_k(x_k)\}$ is a joint cumulative distribution function and its copula is C. Consequently, for specifying the joint distribution of $\boldsymbol{X}$, it suffices to specify the copula C and the marginal distributions. Similarly, for estimating the joint distribution of $\boldsymbol{X}$, it suffices to estimate the copula C and the marginal distributions separately. This separation of the joint distribution into its constituent parts $\{C, G_1, \ldots, G_k\}$ greatly simplifies the methodological tasks in finance.

The representation of the joint cumulative distribution function through a copula is particularly useful because it enables us to decompose any joint distribution into its copula and marginal distributions. The copula remains the same even if the marginal variables $X_1, \ldots, X_k$ are transformed to have different units. In this sense the copula captures the dependence between the marginal variables that is independent of the units of measurement of the marginal variables. From now on, we shall restrict ourselves to bivariate distributions and copulas, which is the focus of this paper.

Let $C(u_1, u_2)$ be a given bivariate copula and suppose that $C(u, u) \to \tau^L$ as $u \to 0$ for some $\tau^L \in (0, 1]$. Then, we say that τ^L is the *lower tail dependence* of C or of (U_1, U_2). It turns out that $\tau^L = \lim_{u \to 0} \mathrm{pr}(U_1 < u \mid U_2 < u) = \lim_{u \to 0} \mathrm{pr}(U_2 < u \mid U_1 < u)$. Therefore, lower tail dependence is a concept that is relevant for dependence between the extreme values in the lower tail of the bivariate distribution. The upper tail dependence τ^U is defined similarly, by replacing the lower corner probability $C(u,u)$ by the upper corner probability. A large number of different copulas have been proposed in the literature to capture different shapes of bivariate dependence. An extensive list of copulas may be found in Joe (1997).

The bivariate Gaussian copula.

Let $\Phi(x)$ denote the cumulative standard normal distribution function, and let $\Phi_\rho(x_1, x_2)$ denote the cumulative distribution function of the bivariate normal with marginal *cdf* s $\Phi(x_1)$ and $\Phi(x_2)$. Then, the Gaussian copula is defined as

$$C(u_1, u_2; \rho) = \Phi_\rho\{\Phi^{-1}(u_1), \Phi^{-1}(u_2)\} \quad (0 \leq u_1, u_2 \leq 1).$$

Consequently, the joint bivariate normal distribution $\Phi_\rho(x_1, x_2)$ can be expressed in terms of its copula as $C\{\Phi(x_1), \Phi(x_2); \rho\}$. Even if $\rho \neq 0$, the tail dependence measures τ^L and τ^U of $\Phi_\rho(x_1, x_2)$ are zero except when $\rho = 1$. Thus, while there may be positive correlation in the middle region of the distribution, there is no dependence in the tails. This is an intriguing feature of normal distribution, one that has supported the view that bivariate normal copula may not be suitable for some finance data even if the marginal distributions have very long tails. By contrast, for the bivariate t-copula, the tail dependence is nonzero, which has been found useful is the multivariate t-distribution and the corresponding t-copula. The other copula functions with various tail dependence properties have become popular in financial applications. In our study, we will apply: (i) two elliptic copulas, normal and t; (ii)two Archimedean copulas, Clayton with the lower tail dependence and Gumbel with the upper tail dependence); and (iii) The symmetrised Joe-Clayton copula with the both upper and lower tail dependence. Section 6.2 provides more details.

4 Estimation

For statistical inference about aspects of the relationship between the bond returns, X_{1t}, and the stock returns, X_{2t}, we need to model and estimate the joint distribution of (X_{1t}, X_{2t}). To this end, it has been demonstrated in the recent literature that using copulas would simplify the tasks for some empirical studies involving multivariate financial data; see Cherubini et al. (2011). In what follows, we shall provide a brief summary of the relevant parts of this large body of recent literature that we employed in this empirical study.

Let $\boldsymbol{X}_{j,t-1}$ denote $(X_{1,t-1}, \ldots, X_{1,t-a_j})$ for some a_j $(j = 1, 2)$. Suppose that

$$X_{jt} = g_j(\boldsymbol{X}_{j,t-1}, \boldsymbol{\beta}_j) + h_j(\boldsymbol{\varepsilon}_{j,t-1}, \boldsymbol{\alpha}_j)\varepsilon_{jt}, \tag{1}$$

where g_j and h_j are known functions and are defined in Section 2.1, $\{\boldsymbol{\varepsilon}_t, t = 1, \ldots, T\}$ are independent and identically distributed and $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \varepsilon_{2t})$. To estimate the joint distribution of (X_{1t}, X_{2t}), it suffices to estimate the unknown parameters of these two univariate time-series models separately,[6] and the joint distribution of $(\varepsilon_{1t}, \varepsilon_{2t})$, which we do by estimating the univariate marginal distributions and the copula, as indicated in the previous subsection.

Suppose that a consistent and asymptotically normal estimator $(\hat{\boldsymbol{\beta}}_j, \hat{\boldsymbol{\alpha}}_j)$ of $(\boldsymbol{\beta}_j, \boldsymbol{\alpha}_j)$ is available $(j = 1, 2)$. Typically, this would be a quasi-likelihood estimator. Let $\hat{\varepsilon}_{jt} = \{X_{jt} - g_j(\boldsymbol{X}_{j,t-1}, \hat{\boldsymbol{\beta}}_j)\}/h_j(\hat{\boldsymbol{\varepsilon}}_{j,t-1}, \hat{\boldsymbol{\alpha}}_j)$, the residual corresponding to ε_{jt} $(t = 1, \ldots, T)$. This is also called the *filtered* series because it filters out the systematic part. In this setting, it has been shown that $\{\hat{\boldsymbol{\varepsilon}}_t, t = 1, \ldots, T\}$ would be close to the true unobserved error terms $\{\boldsymbol{\varepsilon}_t, t = 1, \ldots, T\}$, and consequently the copula of $\boldsymbol{\varepsilon}_t$ can be estimated by substituting $\{\hat{\boldsymbol{\varepsilon}}_t, t = 1, \ldots, T\}$ for $\{\boldsymbol{\varepsilon}_t, t = 1, \ldots, T\}$ in the likelihood function. Finally, the marginal distributions of ε_{jt} is estimated by the empirical distributions of $\{\hat{\varepsilon}_{jt}, t = 1, \ldots, T\}$. In what follows, we discuss estimation of the copula of $\boldsymbol{\varepsilon}_t$.

Various diagnostics have been developed for inference involving copulas when the observations are independent and identically distributed. In our empirical study, we applied some of these to the residuals $\{\hat{\boldsymbol{\varepsilon}}_t, t = 1, \ldots, T\}$. While some of these may require additional theoretical work to provide a rigorous justification, one would expect that such results would hold, perhaps under some additional regularity conditions.

[6] Assuming the error distributions are unknown, the models in (1) are estimated by m.l.e. method. Gourieroux (1997) showed that the m.l.es of ARMA-GARCH parameters are consistent in this case.

4.1 A semiparametric method of inference on the copula model

Let F_1 and F_2 denote the marginal cumulative distribution functions of ε_{1t} and ε_{2t}, respectively, f_1 and f_2 denote their corresponding probability density functions, and $C(u_1, u_2; \boldsymbol{\theta})$ denote the copula of $(\varepsilon_{1t}, \varepsilon_{2t})$. Let $\hat{F}_j(x)$ denote the empirical distribution function $(T + 1)^{-1} \sum_{t=1}^{T} I(\hat{\varepsilon}_{jt} \leq x)$ of the residuals where I is the indicator function $(j = 1, 2)$.

If $\{\boldsymbol{\varepsilon}_t, t = 1, \ldots, T\}$ and (F_1, F_2) were known, the maximum likelihood estimator of the copula parameter $\boldsymbol{\theta}$ would be obtained by maximizing the kernel of the loglikelihood, $\sum_{t=1}^{T} \log c\{F_1(\varepsilon_{1t}), F_2(\varepsilon_{2t}); \boldsymbol{\theta}\}$. However, because these are unknown, we propose to estimate $\boldsymbol{\theta}$ by $\tilde{\boldsymbol{\theta}}$, the maximizer of $\sum_{t=1}^{T} \log c\{\hat{F}_1(\hat{\varepsilon}_{1t}), \hat{F}_2(\hat{\varepsilon}_{2t}); \boldsymbol{\theta}\}$ which is obtained by substituting estimates for (F_1, F_2) and $\boldsymbol{\varepsilon}_t$. It has been shown that $\tilde{\boldsymbol{\theta}}$ is a consistent and asymptotically normal estimator. This is a semiparametric method because it does not assume any parametric form for the distribution functions F_1 and F_2, but assumes parametric forms for the rest of the model. Since the primary purpose of this study is to use copula models to estimate VaR and ES, we would like to choose a parametric form for a copula.

4.2 Value-at-risk and expected shortfall of stock–bond portfolio

`Copulas method to value-at-risk estimation`

Let Y denote the value of an asset. The value-at-risk, denoted VaR_Y of Y, at confidence level α, is simply the α-quantile of Y defined by $\text{pr}\{Y \leq VaR\} = \alpha$. Although the VaR depends on α it is not shown explicitly for simplicity of notation. Thus, the probability that the value of the asset would fall below VaR_Y is α. While different values of α are of interest in different settings, the values often used are $0.01, 0.5$ and 0.1.

Now, consider the stock–bond portfolio of \$1 with \$$\beta$ and \$$(1-\beta)$ allocated to bond and stock, respectively. Let X_{1t}, X_{2t} and Z_t denote the rates of returns from bond, stock and the portfolio, respectively. Then $Z_t = \beta X_{1t} + (1-\beta)X_{2t}$. To estimate the VaR of Z_t, we would like to specify and estimate copulas for its components X_{1t} and X_{2t}.

$$
\begin{aligned}
0.05 &= \text{pr}\{\beta X_{1t} + (1-\beta)X_{2t} \le VaR \mid \mathcal{F}_{t-1}\} \\
&= \text{pr}\{\beta(g_{1t} + h_{1t}\varepsilon_{1t}) + (1-\beta)(g_{2t} + h_{2t}\varepsilon_{2t}) \le VaR \mid \mathcal{F}_{t-1}\} \\
&= \text{pr}\{A\varepsilon_{1t} + B\varepsilon_{2t} \le D \mid \mathcal{F}_{t-1}\}, \quad A = \beta h_{1t}, B = (1-\beta)h_{2t}, D = VaR - \beta g_{1t} - (1-\beta)g_{2t} \\
&= \int_{\varepsilon_2=-\infty}^{\infty} \int_{\varepsilon_1=-\infty}^{A^{-1}(D-B\varepsilon_2)} f(\varepsilon_1, \varepsilon_2) d\varepsilon_1 d\varepsilon_2 \\
&= \int_{\varepsilon_2=-\infty}^{\infty} \int_{\varepsilon_1=-\infty}^{A^{-1}(D-B\varepsilon_2)} c\{F_1(\varepsilon_1), F_2(\varepsilon_2)\} f_1(\varepsilon_1) f_2(\varepsilon_2) d\varepsilon_1 d\varepsilon_2 \\
&= \int_{\varepsilon_2=-\infty}^{\infty} \Big\{ \int_{x=0}^{F_1\{A^{-1}(D-B\varepsilon_2)\}} c\{x, F_2(\varepsilon_2)\} dx \Big\} f_2(\varepsilon_2) d\varepsilon_2 \\
&= \int_{y=0}^{1} \Big\{ \int_{x=0}^{F_1\{A^{-1}(D-BF_2^{-1}(\varepsilon_2))\}} c\{x, y\} dx \Big\} dy \\
&= \int_0^1 C_{1|2}[F_1\{A^{-1}(D - BF_2^{-1}(\varepsilon_2))\}, y] dy \\
&= \int_{-\infty}^{\infty} C_{1|2}[F_1\{A^{-1}(D - B\varepsilon_2)\}, F_2(\varepsilon_2)] f_2(\varepsilon_2) d\varepsilon_2 \\
&= E\big\{C_{1|2}[F_1\{A^{-1}(D - B\varepsilon_2)\}, F_2(\varepsilon_2)]\big\}
\end{aligned}
$$

where the expectation is taken with respect to the marginal distribution of ε_2.[7] Now, the VaR is estimated by solving

$$0.05 = E\big\{C_{1|2}[F_1\{A^{-1}(D - B\varepsilon_2)\}, F_2(\varepsilon_2)]\big\} \tag{2}$$

for VaR with all the unknown quantities replaced by sample estimates. To write down this equation, let $\hat{F}_j$ denote the empirical distribution of $\{\hat{\varepsilon}_{jt}\}$ $(j = 1, 2)$. Then the estimated equation is

$$0.05 = T^{-1} \sum_{t=1}^{T} \hat{C}_{1|2}[\hat{F}_1\{\hat{A}^{-1}(\hat{D} - \hat{B}\hat{\varepsilon}_{2t})\}, \hat{F}_2(\hat{\varepsilon}_{2t})]. \tag{3}$$

or equivalently,

$$0.05 = T^{-1} \sum_{t=1}^{T} \hat{C}_{2|1}[\hat{F}_2\{\hat{B}^{-1}(\hat{D} - \hat{A}\hat{\varepsilon}_{1t})\}, \hat{F}_1(\hat{\varepsilon}_{1t})]. \tag{4}$$

`Copulas method to estimating expected shortfall`

[7]The subscript t has been dropped for convenience.

By definition, VaR is a quantile measure and does not satisfy the axiom, subadditivity, whereas ES satisfies all four axioms; see Artzner et al. (1997, 1999) for the list of these four axioms and other details. Thus, ES is a coherent risk measure, while VaR is not.

As a specific high quantile measure, the VaR does not account for the extreme losses beyond the VaR. By contrast, the expected shortfall is defined as the expectation of losses conditional on them exceeding the VaR threshold loss. By definition, the estimate of ES is deemed to be a numerically higher than VaR. The ES for a portfolio with return Z_t can be expressed as follows:

$$ES = \int_{-\infty}^{VaR_Z} zf(z)dz,$$

where VaR_z is the value from equation (3). We apply a discrete approximation to ES of the following form:

$$ES \simeq \frac{1}{n}\sum_{i=1}^{n} VaR_{\alpha_i}. \tag{5}$$

4.3 Forecasting value-at-risk with semiparametric copulas

To evaluate the VaR forecasting performance of semiparametric copula models, the whole sample, $1, 2, \ldots, T$, is divided into an estimation sample, $1, 2, \ldots, K$ and an evaluation sample, $K+1, K+2, \ldots, T$.

Using the first K observations, the parameters β_j and α_j of model (1) are estimated by the Quasi-Maximum Likelihood approach, and the standardized residuals $\hat{\varepsilon}_{jt}$, are obtained. Thus, the one-step-ahead out-of-sample forecasts of conditional mean $\widehat{g}_{jt}$ and standard deviation $\widehat{h}_{jt}$ are generated for the remaining $n = T - K$ periods. These $\hat{\varepsilon}_{jt}, \widehat{g}_{jt}$ and $\widehat{h}_{jt}$, for $t = K+1, K+2, \ldots, T$, are then used to estimate the one-step-ahead stock-bond portfolio VaR using (3), providing $T - K$ out-of-sample VaR forecasts.

The Conditional Coverage (CC) test of Christoffersen (1998) is used to assess whether or not the empirical coverage of the VaR (quantile) forecasts by the semiparametric copula model is equivalent to the nominal level α. This CC test has been widely used in finance literature to evaluate the performance of return quantile forecasting models. In this empirical application, if the empirical coverage of its quantile forecasts is statistically equivalent to the nominal probability of return quantile α, then we can conclude that the semiparametric copula model performs well in VaR forecasting.

5 Chi–plot

The semiparametric copula method employed in this paper requires the user to specify a suitable parametric copula for the joint distribution of the error terms corresponding to the stock and bond return series. Because the true copula is not known in empirical studies, the best that can be done is to attempt to choose one that is as close to the true one as possible. To this end, various formal statistical tests with distribution theory and some diagnostic tests based on plots and diagrams have been proposed. In this regard, we found the *chi–plot* useful. To provide a brief description of this plot, let $\{(X_i, Y_i), i = 1, \ldots, n\}$ be a given bivariate sample. Let

$$\lambda_i = 4S_i \max\{(F_i - 1/2)^2, (G_i - 1/2)^2\} \quad \text{and} \quad \chi_i = (H_i - F_iG_i)\{F_i(1 - F_i)G_i(1 - G_i)\}^{-1/2},$$

where $H_i = \frac{1}{(n-1)}\sum_{i \neq j} I(X_j \leq X_i, Y_j \leq Y_i)$, $F_i = \frac{1}{(n-1)}\sum_{i \neq j} I(X_j \leq X_i)$, $G_i = \frac{1}{(n-1)}\sum_{i \neq j} I(Y_j \leq Y_i)$, $S_i = sign\{F_i - 1/2)(G_i - 1/2)$. Now consider the transformed set of points $\{(\lambda_i, \chi_i), i = 1, \ldots, n\}$. A scatter plot of these points is called a *chi–plot*. The clustering patterns of this plot reveals or accentuate some features in the dependence between the points in $\{(X_i, Y_i), i = 1, \ldots, n\}$ that may be difficult to see in a simple scatter plot of these observations. In fact, λ_i could be any function of (F_i, G_i), but in this study, we shall consider only the foregoing one.

If X and Y are independent then the chi–plot would asymptotically show normal scatter with variance n^{-1} around the horizontal line $\chi = 0$. On the other hand, if X and Y are dependent then the pattern of the chi–plot would reflect some features of the dependence between X and Y values. To interpret the chi–plot, it is helpful to note that at each data point, χ_i is the correlation coefficient between the dichotomized values of X and Y. In this paper, the chi–plots for the two periods before and after the GFC are used to convey some changes in dependence between stock returns and bond returns.

The transformation from $\{(X_i, Y_i), i = 1, \ldots, n\}$ to $\{(\lambda_i, \chi_i), i = 1, \ldots, n\}$ is invariant under monotonic transformations. Therefore, diagnostics based on chi–plot are particularly suitable for indicating the potential copula that might fit the data, which is illustrated in the next section.

6 Empirical results

6.1 Chi–plots

The chi–plots for the US government bond index and stock index returns pair for both before and after the GFC periods are given in Figure 1.[8]

(←-- Insert Figure 1 Here --→)

Before the GFC, as indicated in Figure 1a), the overall dependence between the US stock and bond indices returns appears to be weak with most of the chi–plots falling inside the 95% confidence interval. The two separate chi–plots in Figure 1a) for the (lower) left tail returns and the (upper) right tail returns indicate a weak tail dependence, with the lower tail dependence being weaker than the upper tail dependence. However, from the shapes of these chi–plots, it is unclear as to what copula function would fit the data.

After the GFC, as shown in Figure 1b), the dependence between the US stock and bond indices returns has increased significantly with most of the chi–plots falling outside the 95% confidence interval. In addition, the two separate left and right tail returns plots in Figure 1b) indicate the presence of asymmetry in the tail dependence, with the upper tail dependence being marginally stronger than the lower tail dependence. These observations indicate that the symmetrized Joe-Clayton (SJC) copula function or the t-copula may be suitable for modelling this dependence. In the following section, various copula functions will be fitted to capture the dependence in the stock–bond returns series and goodness-of-fit tests will be used for choosing a suitable copula that fits the data well in each case.

[8] To save space, the plots for the other countries are not reported in this paper.

6.2 Semiparametric estimation of copulas

Employing the semiparametric copula modelling method discussed in Section 4.1, bivariate distributions of the stock–bond (filtered) returns pair for all eight countries before and after the GFC were estimated. We fit a number of bivariate copula functions such as normal, t, Clayton, Gumbul, and SJC. The parametric specifications of these copulas and their tail features are discussed in Kim et al (2008) and Patton (2006). Moreover, the adequacy of each fitted copulas model was tested using the goodness-of-fit tests, namely the "blanket tests". Genest, Rémillard and Beaudoin (2009) present a critical review of these tests, accompanied by a power comparison using a large scale simulation study. The two of these rank-based procedures, denoted as S_n and $S_n^{(K)}$, were found to be more powerful than their counterparts. Therefore, we use these two tests in this study for choosing the correct copula function; See Genest, Rémillard and Beaudoin (2009) for details of these two tests. Table 2 reports the p-values of the above two "blanket tests" as well as AIC only for the bivariate copula functions that best fit the stock–bond returns distributions for all eight countries and for the two (before and after the GFC) sample periods.[9]

(←-- Insert Table 2 Here --→)

We find that the Student-t copula is adequate for all the filtered stock–bond returns bivariate distributions considered in this study, both before and after the GFC. Before the GFC, the estimated copula model parameters are found to be significant at the 5% level, except for Australia and Canada. After the GFC, the dependence parameters of the student-t copula have increased for all countries, except for Italy. These results are somewhat consistent with the Kendall's tau estimates of some form of association (dependence) between the stock and bond returns, reported in section 2.2. The estimates of the tail index which measures the tail dependence are reported for all eight stock–bond returns distributions, and they are significant at the 5% level. More importantly, the tail dependence parameters

[9] A complete set of goodness-of-fit test results for all the fitted bivariate copulas are available from the authors upon request.

have increased after the GFC, except for the UK, indicating an overall increase in the tail dependence of the stock-bond return joint distributions.

In what follows, the estimated "best-fitting" copula models are used to quantify the VaR and ES of stock–bond portfolios with bond weights varying from 0 to 1.

6.3 Value at risk, expected shortfall and optimal bond weights in stock–bond portfolios

Using the methods outlined in Section 4.2, the downside tail risks VaR and ES of stock–bond portfolios (with bond weights β varying from 0 to 1) were computed at the probability levels $\alpha = 0.01, 0.05$ and 0.10. In doing this, we want to infer the optimal bond weight in each portfolio that yields the minimum VaR and ES across all eight countries. We further investigate whether or not the recent GFC has had any notable impact on the optimal asset allocations to these stock–bond portfolios and the corresponding minimum VaR and ES measures. The features of VaR and ES measures with respect to bond weights, the optimum asset mix in the -stock bond portfolios of all eight countries were found to be the same at all three probability levels 0.01, 0.05 and 0.10. Therefore, only the results corresponding to $\alpha = 0.05$ are reported in this paper.

`VaR estimates of stock-bond portfolios`

Using equation (3), the VaRs of stock–bond portfolios with the bond weight β varying from 0 to 1, were estimated with the best-fitting copula models established in the previous section for both the pre– and post–GFC periods, and the results are plotted in Figure 2.

(←-- Insert Figure 2 Here --→)

Figure 2 exhibits how the VaR measure changes as the bond weight β varies between 0 and 1. As can be seen, there are two curves for each country, with one representing the pre–GFC and the other the post–GFC period. Clearly, the VaR measures have (numerically) increased significantly after the crisis for all countries, with the post–GFC VaR curve lying below the pre–GFC one for all eight countries. Notably, for both periods, the VaR declines as β increases, reaching its minimum when $\beta = 0.9$ for four countries: Australia, Canada, Italy and the UK, and when $\beta = 0.8$ just for the US, before the GFC. For the other three countries (France, Germany and Japan), on the other hand, this minimum VaR is achieved when the bond weight is 100% - a corner solution. In the post–GFC period, the corner solution was found for all eight countries. The results are consistent with the downside risk portfolio selection criteria like the safety-first criterion,[10] which tends to select the asset with the thinnest tail. Ceteris paribus, the post-GFC VaRs would have been expected to decline as a result of the significant increase in the (negative) dependence in the stock-bond returns. Instead, they have increased for all countries due to unprecedented high volatilities and uncertainties in the stock and bond markets, following the GFC.

The gap between the pre– and post–GFC VaR curves is the largest at $\beta = 0$ (i.e. stock investment only) for all countries. This result suggests that the increase in VaR measures might be due to high levels of uncertainties in global stock markets after the GFC, leading to high portfolio risks. This is evident in the Euro zone nations, particularly Italy for which the VaR has notably increased after the GFC. The Euro zone debt crisis caused (and is still causing) chaos in both government bonds and stock markets. The crisis was instigated by high public debt levels in the Euro-zone nations during the GFC, due to low GDP growth rates and soaring budget deficits. Stock indices plunged as investors became concerned about the bleak future economic conditions in these countries. France and Germany were in better economic circumstances than the other countries in the region. Italy was among the top-five troubled countries in the region, and investors were concerned of the possibility that the country's rating would be downgraded.[11] The government bond index plunged, as did the country's stock index. For $\alpha = 0.05$, Italy has the highest reported VaR measures of at $\beta = 0$, indicating that its stock market risk is the highest among all eight countries after the GFC.

[10]Assuming independence between the stock and bonds, Hyung and de Vries (2007) arrived at an interior solution.

[11]Many Italian banks have been downgraded recently.

The slopes of both the pre- and post–GFC VaR curves are positive, since the (numerical) VaR increases as β increases. It's also clear that the slopes of the post–GFC curves are much steeper than those for the pre–GFC. Thus, the VaR has become more sensitive to the change in β after the GFC. These findings provide further evidence supporting the "flight to quality" phenomenon after the GFC, implying that greater risk reduction can be achieved by moving funds into bond investments after the GFC.

`Expected shortfall estimates of Stock-bond portfolios`

The ES is estimated using (5) for different bond weights for all eight countries. By definition, ES estimates are greater than the corresponding VaR counterparts. It is evident from Figure 3 that the ES curves' characteristics are very similar to those of the VaR curves in Figure 2, which are discussed in the previous paragraphs.

(←-- Insert Figure 3 Here --→)

This empirical investigation provides overwhelming evidence that portfolio VaR and ES have increased, after the GFC, and hence the required level of regulatory capital is higher than that during the tranquil period.

6.4 Out-of-sample evaluation of value-at-risk forecasting with Semiparametric copula models

To evaluate the performance of semiparametric copula models in forecasting VaRs in the out-of-sample, the data in both pre– and post–GFC subsamples is divided as follows:

Subsamples	Estimation Period	Evaluation Period
pre–GFC	01/07/2003 – 06/10/2006	09/10/2006 – 13/07/2007
post–GFC	16/07/2007 – 10/11/2010	11/11/2010 – 17/08/2011

We set the nominal probability level, $\alpha = 0.05$. Following the procedures described briefly in Section 4.3, one-step-ahead forecasts of stock–bond portfolio VaR were estimated for Australia and the G7-countries, for illustration, the results for both subsamples are plotted for US in Figure 4. As has been noted for the in-sample, the out-of-sample VaR forecasts are also minimized when β is close or equal to 1.

(←-- Insert Figure 4 Here --→)

Christoffersen's (1998) Conditional Coverage (CC) test is applied in order to evaluate the performance of the semiparametric copula in terms of quantile (the VaR) forecasting. The size of the forecast evaluation period is $n = 200$, which is sufficiently large for the reliable statistical inference. For $\alpha = 0.05$, the p-values of the CC test are reported in Table 3.

(←-- Insert Table 3 Here --→)

Table 3 shows that, for the pre–GFC period, the p-values are greater than the nominal α for all stock–bond portfolios with the bond weight varying from 0 to 1. The VaR forecasts generated by semiparametric copula model have the empirical coverage rate that are statistically the same as the nominal probability of the return quantile at $\alpha = 0.05$, indicating that that the null hypothesis that the semiparametric copula models that were established for the stock-bond returns distributions are adequate for out-of-sample VaR forecasting. On the contrary, for the post–GFC period, the results show that the p-values are less than the α for various bond weights and for all countries, except Italy. In particular, the p-values of the CC test for Japan are less than 0.05 for all bond weights except for $\beta = 1$. The out-of-sample quantile forecast performance of copula models in the post–GFC period appears to be inadequate. This lack of forecast performance may be partly due to high levels of uncertainties in the stock and bond markets, causing very high volatilities in these markets in the post–GFC period.

Figure 4 plots the out-of-sample VaR forecasts for the US before and after the GFC, for bond weights range between 0 and 1. To illustrate the forecasting performance further, the out-of-sample VaR forecasts for the US stock–bond portfolio are plotted in Figure 5 along with the observed empirical returns, for both the pre– and post–GFC periods. The results from Table 3 and Figure 5 are indicative of what can be expected for the other countries considered in this study. Note that for illustration purpose, only results for $\beta = 0, 0.5$ and 1 are shown in Figure 5.[12]

[12] A set of comprehensive results can be obtained from the authors upon request.

(←-- Insert Figure 5 Here --→)

The left panel of Figure 5 illustrates that for the pre–GFC period, the number of observed US stock–bond portfolio returns that are less than the VaR forecasts lies between 8 and 11 (out of 200 forecasts) for different bond weights, indicating the satisfactory coverage rate of VaR forecasts at $\alpha = 0.05$. This finding is consistent with the large p-values reported for the CC test in Table 3, reinforcing the good performance of forecastability of semiparametric copula models in the pre–GFC period.

On the other hand, the right panel of Figure 5 illustrates that the coverage rates produced by the VaR forecasts are not satisfactory overall for the post–GFC period. In particular, the number of observed stock–bond portfolio returns that are less than the forecasted quantile is very few (between 1 and 3 out of 200 forecasts), implying the over-estimation of the potential loss for the US stock–bond portfolio by the copula models, which is consistent with the small p-values (< 0.01) of the CC test in Table 3. However, in the context of capital requirements for fund managers, the over–estimation of VaRs is perceived better by the regulators, than the under-estimation. It is considered good business practice to set aside more capitals to buffer against potential losses from investment portfolios.[13]

7 Conclusion

This paper builds semiparametric copula models for the bivariate joint distribution of government bond index returns and stock index returns for Australia and the G7 countries. These copula models are then used to estimate the value-at-risk (VaR) and expected shortfall (ES) measures of stock–bond portfolios, with varying weights on bond investments. We then use these results to infer the optimum bond weights that yield the minimum stock–bond portfolio VaR and ES for these countries. We are also interested in finding out the ways in which the stock–bond dependence structures, VaR and ES measures, and the optimum bond weights have changed as a consequence of the recent GFC. The semiparametric Student's t

[13]Matlab code was used to carry out all the computing required for the empirical work presented in this paper.

copula was found to be the best fit for all bond and stock pairs, both before and after the GFC. Using the semiparametric copula models, the VaR and ES of stock–bond portfolios with bond weights varying from 0 to 1 were estimated.

The stock-bond dependence was found to be negative but weak in the pre-GFC period, and this negative dependence has become very strong in the post-GFC period. As a consequence, ceteris paribus, the VaR and ES are expected to have declined in the latter period. Contrary to these expectations, these down-side tail risk measures have increased in the post-GFC period due to increased volatilities of stock and bond markets during the financial turmoil. The minimum portfolio VaR and ES were achieved at the interior solution only for five countries, except for France, Germany and Japan for which a corner solution was found. After the GFC, a corner solution was found for all eight countries. That is, VaR and ES were minimum at the 100% bond weight. In addition, the results of Christoffersen's (1998) conditional coverage test used for evaluating the out-of-sample performance of semiparametric copula models for VaR forecasting indicate the satisfactory performance of copula models in the pre–GFC period. On the other hand, the these models were found to over-predict the VaRs in the post–GFC period. The empirical results of this paper suggest that both VaR and ES can notably increase after financial crises, and hence the required level of regulatory capital can be much higher than that one would expect. These findings are useful to global financial regulators and the Basel Committee whose central focus is currently on increasing capital requirements as a consequence of the recent GFC.

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Table 1: Estimates of stock–bond linear correlation, rank correlation and Kendall's tau for Australia and the G7 countries before and after the GFC.

	Sample	Linear correlation		Rank correlation		Kendall's τ	
	period	Estimates	P-value	Estimates	P-value	Estimates	P-value
AUD	Pre-GFC	-0.0580	0.0596*	-0.0553	0.0726*	-0.0378	0.0668*
	Post-GFC	-0.4525	0.0000	-0.4436	0.0000	-0.3139	0.0000
CAD	Pre-GFC	-0.0423	0.1704*	-0.0585	0.0577*	-0.0391	0.0576*
	Post-GFC	-0.3461	0.0000	-0.3249	0.0000	-0.2283	0.0000
FRA	Pre-GFC	-0.2081	0.0000	-0.2060	0.0000	-0.1426	0.0000
	Post-GFC	-0.4790	0.0000	-0.4600	0.0000	-0.3254	0.0000
GER	Pre-GFC	-0.1924	0.0000	-0.1934	0.0000	-0.1335	0.0000
	Post-GFC	-0.4835	0.0000	-0.4729	0.0000	-0.3353	0.0000
ITL	Pre-GFC	-0.1434	0.0000	-0.1437	0.0000	-0.0979	0.0000
	Post-GFC	-0.0130	0.6713*	-0.0694	0.0233	-0.0477	0.0198
JPN	Pre-GFC	-0.4098	0.0000	-0.3943	0.0000	-0.2776	0.0000
	Post-GFC	-0.4096	0.0000	-0.4721	0.0000	-0.3361	0.0000
UK	Pre-GFC	-0.1746	0.0000	-0.1703	0.0000	-0.1148	0.0000
	Post-GFC	-0.3901	0.0000	-0.3937	0.0000	-0.2747	0.0000
US	Pre-GFC	-0.0615	0.0458	-0.0508	0.0995*	-0.0340	0.0979*
	Post-GFC	-0.4195	0.0000	-0.4308	0.0000	-0.3062	0.0000

Note: The p-values were computed for the two-tail t-test at the 5% level. * indicates insignificance at the 5% level.

Table 2: Estimates of the copula model and the dependence parameters of bivariate stock–bond joint distributions for Australia and the G7 countries

	Country	Dependence Parameter	Degree of freedom Parameter	Tail index	AIC	S_n test p-value	$S_n^{(K)}$ test p-value
	AUD	0.0601*	14.9384	0.0017	-6.9300	0.8155	0.7915
	CAD	0.0535*	14.3441	0.0020	-7.5059	0.4000	0.0914
	FRA	0.2053	6.8834	0.0526	-58.2865	0.6353	0.7250
Pre-	GBR	0.1903	6.2201	0.0611	-52.4479	0.8872	0.7169
GFC	ITL	0.1383	10.0063	0.0148	-26.5596	0.6711	0.5866
	JPN	0.4166	7.2212	0.1020	-207.7538	0.7186	0.4296
	UK	0.1791	12.9304	0.0077	-37.8701	0.4854	0.6660
	US	0.0491	6.3817	0.0345	-20.7416	0.7501	0.4443
	AUD	0.4802	13.1256	0.0427	-277.7368	0.7184	0.2243
	CAD	0.3462	6.3541	0.0987	-149.8895	0.7687	0.8256
	FRA	0.5037	9.1116	0.0973	-318.8751	0.6980	0.7190
Post-	GBR	0.5236	7.5940	0.1372	-353.3797	0.8982	0.1325
GFC	ITL	0.1067	6.9648	0.0351	-28.5855	0.3397	0.4905
	JPN	0.5035	6.6038	0.1537	-316.1302	0.6988	0.8368
	UK	0.4224	31.1561	0.0010	-205.9733	0.8462	0.9666
	US	0.4820	6.2886	0.1528	-295.2217	0.9686	0.0811

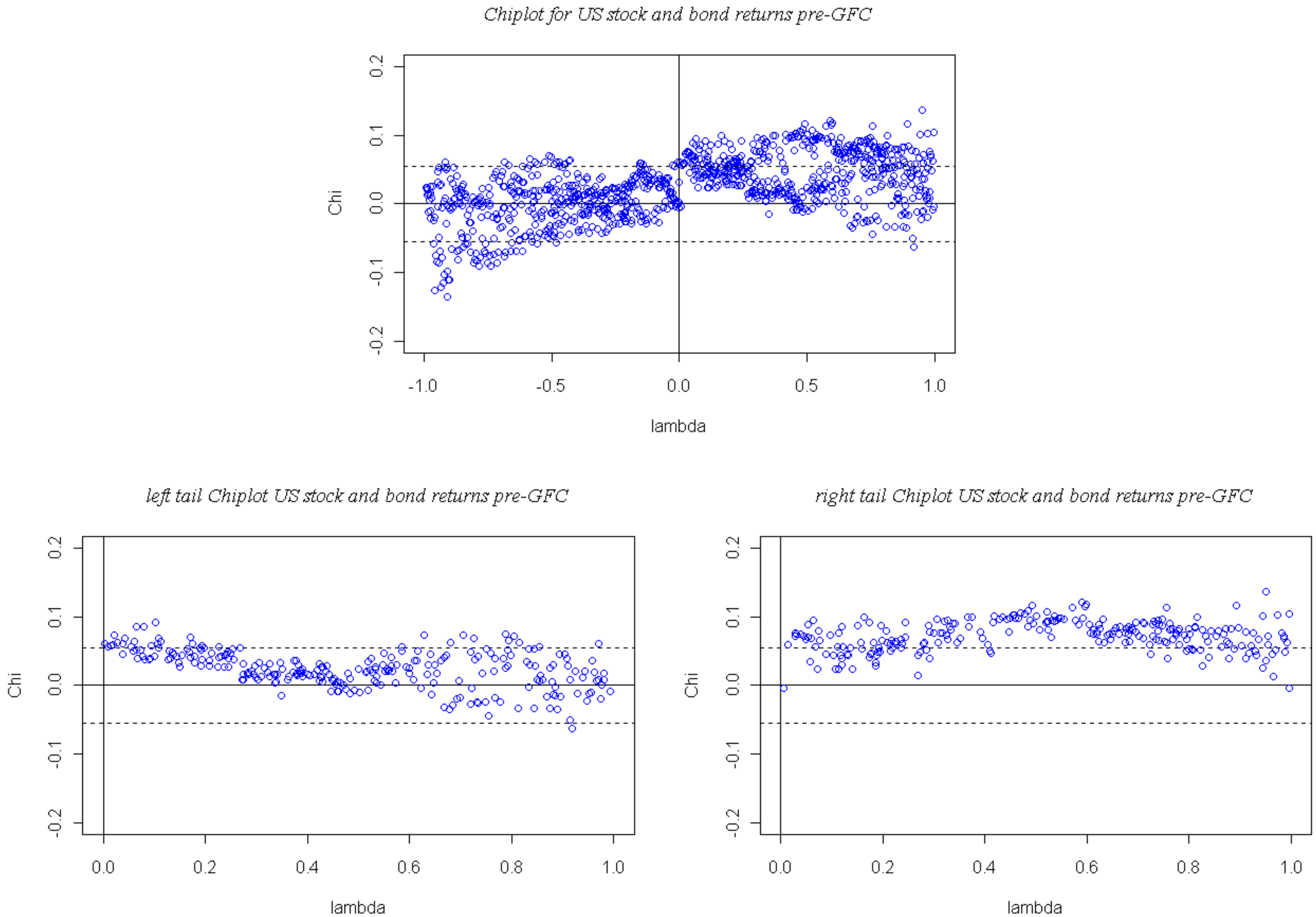
Notes: The best fitted copula for all countries is the Student-t copula. The above table reports the p-values of the the two goodness-of-fit (GoF) tests, S_n and $S_n^{(K)}$ from Genest et al. (2009), as well as the AIC model selection criterion for selected Student-t copula. The tail index measure of the corresponding student-t copula is also reported here. * indicates insignificance at the 5% nominal level. A complete set of results is available upon request.

Table 3: Conditional Coverage Test for $\alpha = 0.05$

		Conditional Coverage test p-values for varying bond weights										
	Country	$\beta = 0.0$	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.3$	$\beta = 0.4$	$\beta = 0.5$	$\beta = 0.6$	$\beta = 0.7$	$\beta = 0.8$	$\beta = 0.9$	$\beta = 1.0$
	AUD	0.5890	0.5890	0.5890	0.5890	0.4991	0.6183	0.5709	0.1776	0.5958	0.6184	0.7226
	CAD	0.8042	0.8042	0.8042	0.8042	0.8042	0.8042	0.8420	0.8042	0.3124	0.4750	0.4750
	FRA	0.8420	0.8420	0.8042	0.4750	0.2827	0.2827	0.2827	0.3124	0.3124	0.3224	0.4747
Pre-	GBR	0.3124	0.4574	0.4574	0.3124	0.3124	0.2827	0.2827	0.2827	0.1776	0.3224	0.4747
GFC	ITL	0.5890	0.5709	0.5709	0.6682	0.6682	0.4750	0.3124	0.3124	0.4574	0.3124	0.5958
	JPN	0.5709	0.5709	0.5709	0.5709	0.6183	0.6183	0.6183	0.5709	0.3124	0.0829	0.4688
	UK	0.4750	0.4750	0.2827	0.1366	0.1366	0.1366	0.2468	0.1823	0.1823	0.3224	0.1823
	US	0.6682	0.5709	0.5709	0.6183	0.6183	0.6183	0.6183	0.4574	0.6183	0.3795	0.4991
	AUD	0.4547	0.5709	0.3124	0.3124	0.1776	0.1776	0.0070*	0.0070*	0.0284*	0.0284*	0.1823
	CAD	0.5890	0.5890	0.6183	0.6183	0.5709	0.4574	0.0514	0.0070*	0.0140*	0.1366	0.5709
	FRA	0.4991	0.5890	0.5709	0.5709	0.4574	0.1776	0.0812	0.0812	0.0284*	0.0284*	0.4574
Post-	GBR	0.4054	0.4054	0.5300	0.6218	0.5958	0.1823	0.1823	0.0070*	0.0010*	0.0070*	0.1776
GFC	ITL	0.2897	0.6382	0.6382	0.3143	0.4709	0.4709	0.4709	0.4709	0.3166	0.4688	0.7534
	JPN	0.0407*	0.0407*	0.0407*	0.0407*	0.0407*	0.0101*	0.0101*	0.0101*	0.0025*	0.0002*	0.4574
	UK	0.0667	0.0667	0.0349*	0.0013*	0.0514	0.0514	0.0284*	0.0010*	0.0000*	0.0010*	0.6183
	US	0.6183	0.4574	0.3124	0.0812	0.0284*	0.0284*	0.0070*	0.0000*	0.0070*	0.0025*	0.8420

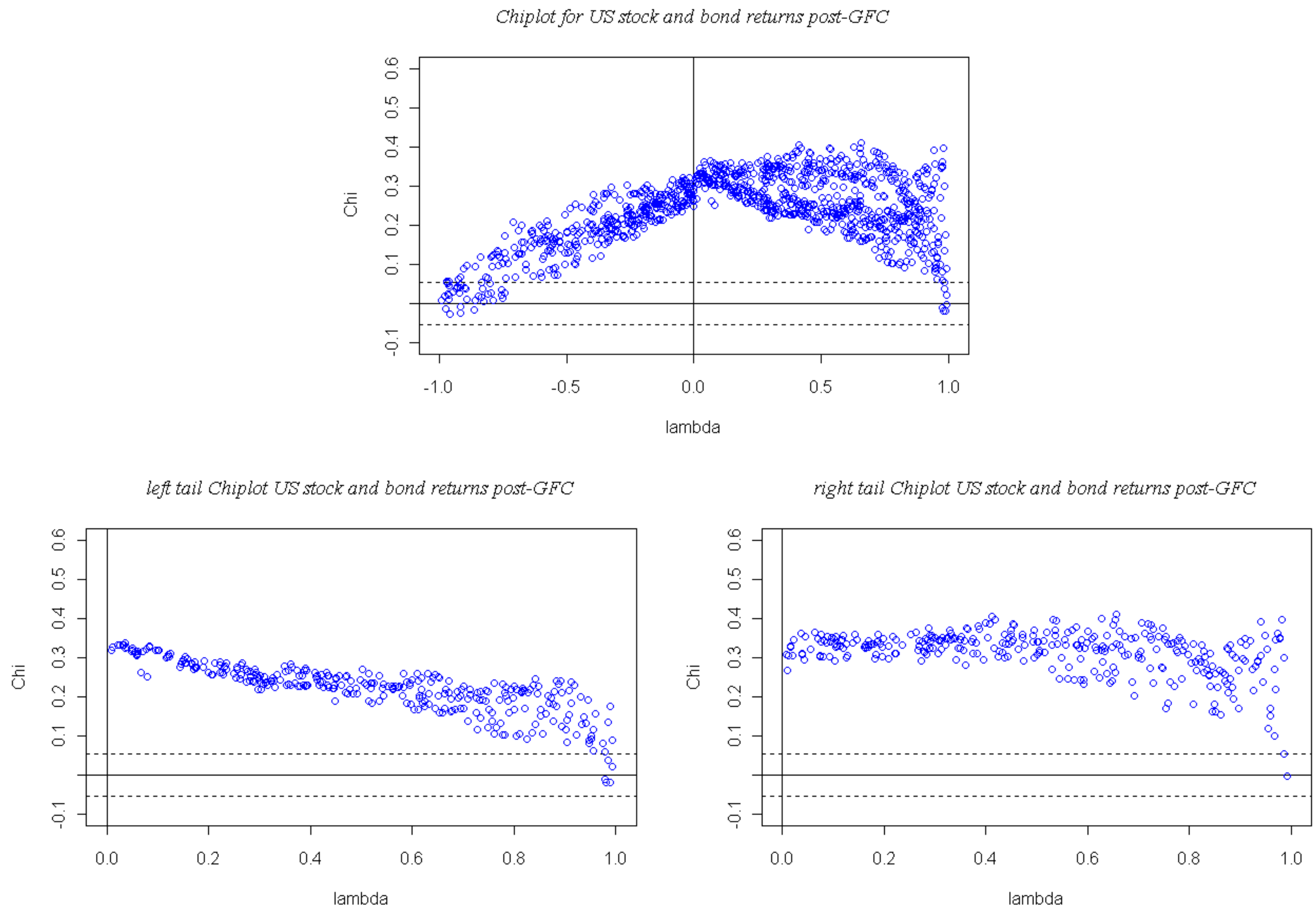
* indicates the p-values for the Conditional Coverage test are less than the significance level of 5%.

Figure 1 a): Chi-plots for US Government bond index returns and Dow-Jones Industrial stock index returns before GFC.



Note: The dotted lines are the 95% confidence bands for the chi-plots.

Figure 1 b): Chi-plots for US Government bond index returns and Dow-Jones Industrial stock index returns after GFC.



Note: The dotted lines are the 95% confidence bands for the chi-plots.

Figure 2: Five per cent Value-at-Risk estimates of stock–bond portfolios with varying bond weights for Australia and the G7 countries before and after the global financial crisis.

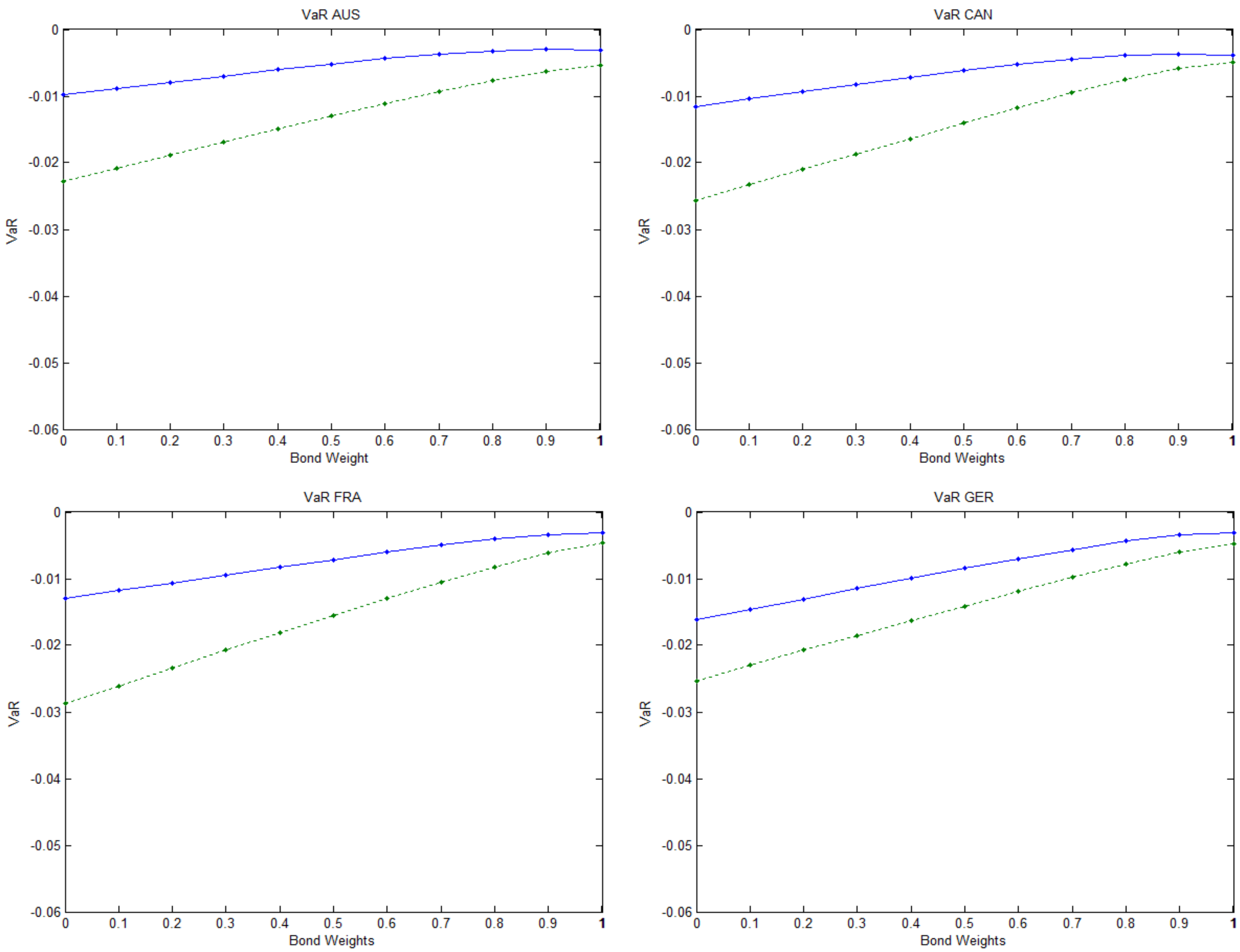
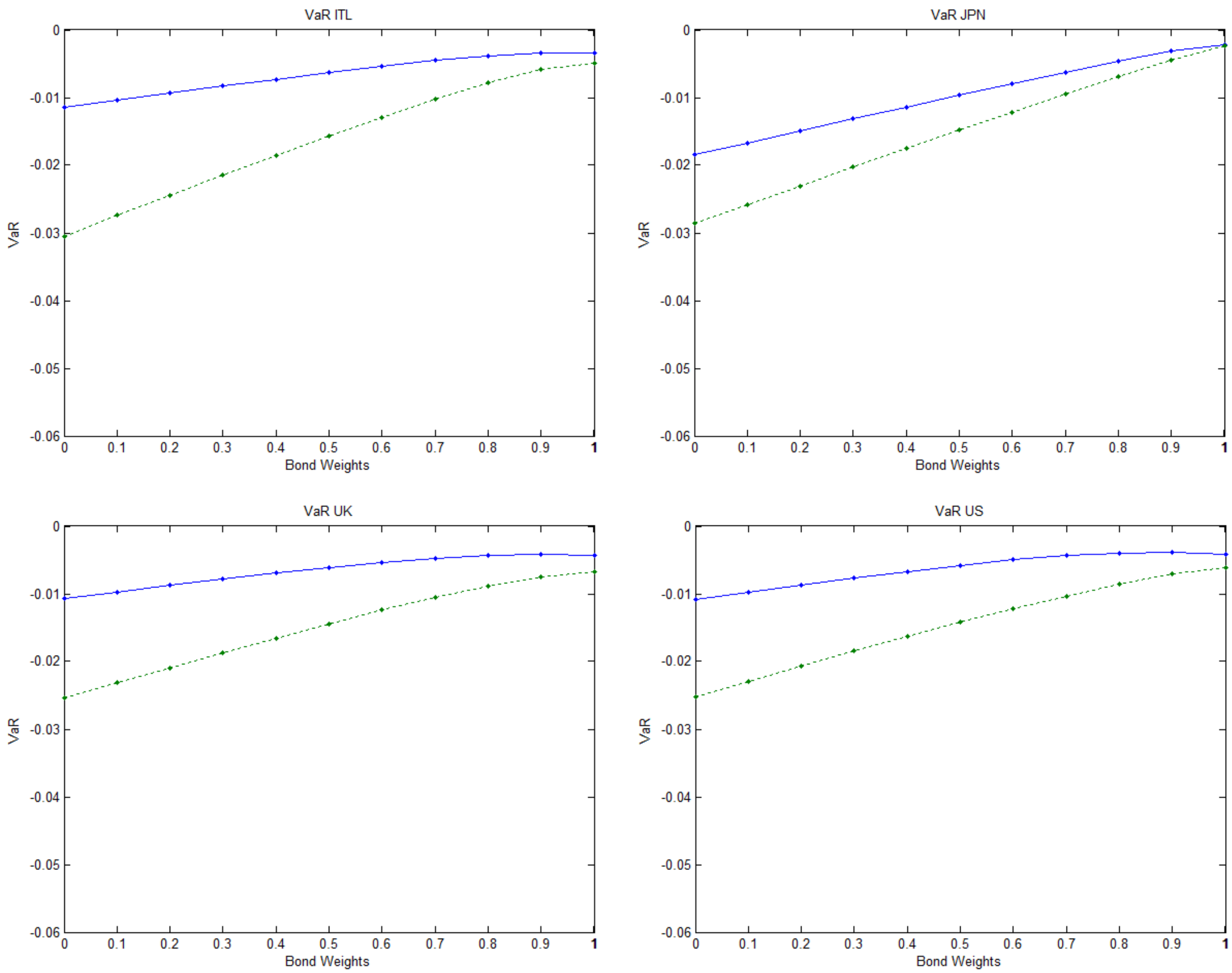


Figure 2 (continued): Five per cent Value-at-Risk estimates of stock–bond portfolios with varying bond weights for Australia and the G7 countries before and after the global financial crisis.



Note: The solid curves represent the stock-bond portfolio VaRs at the 5% level before the GFC, and the dashed curves represent the VaRs after the GFC.

Figure 3: Five per cent Expected Shortfall estimates of stock–bond portfolios with varying bond weights for Australia and G7 countries before and after the global financial crisis.

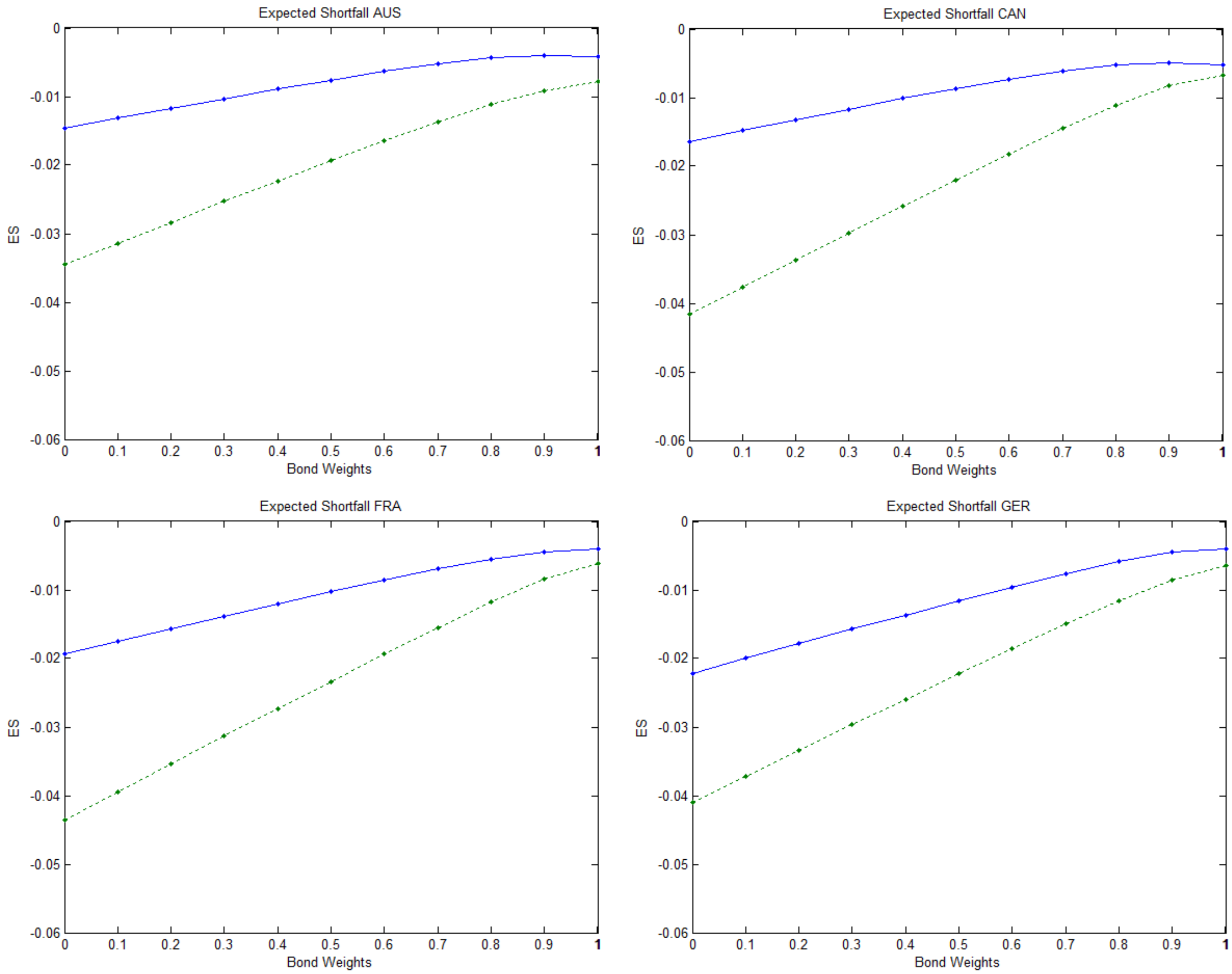
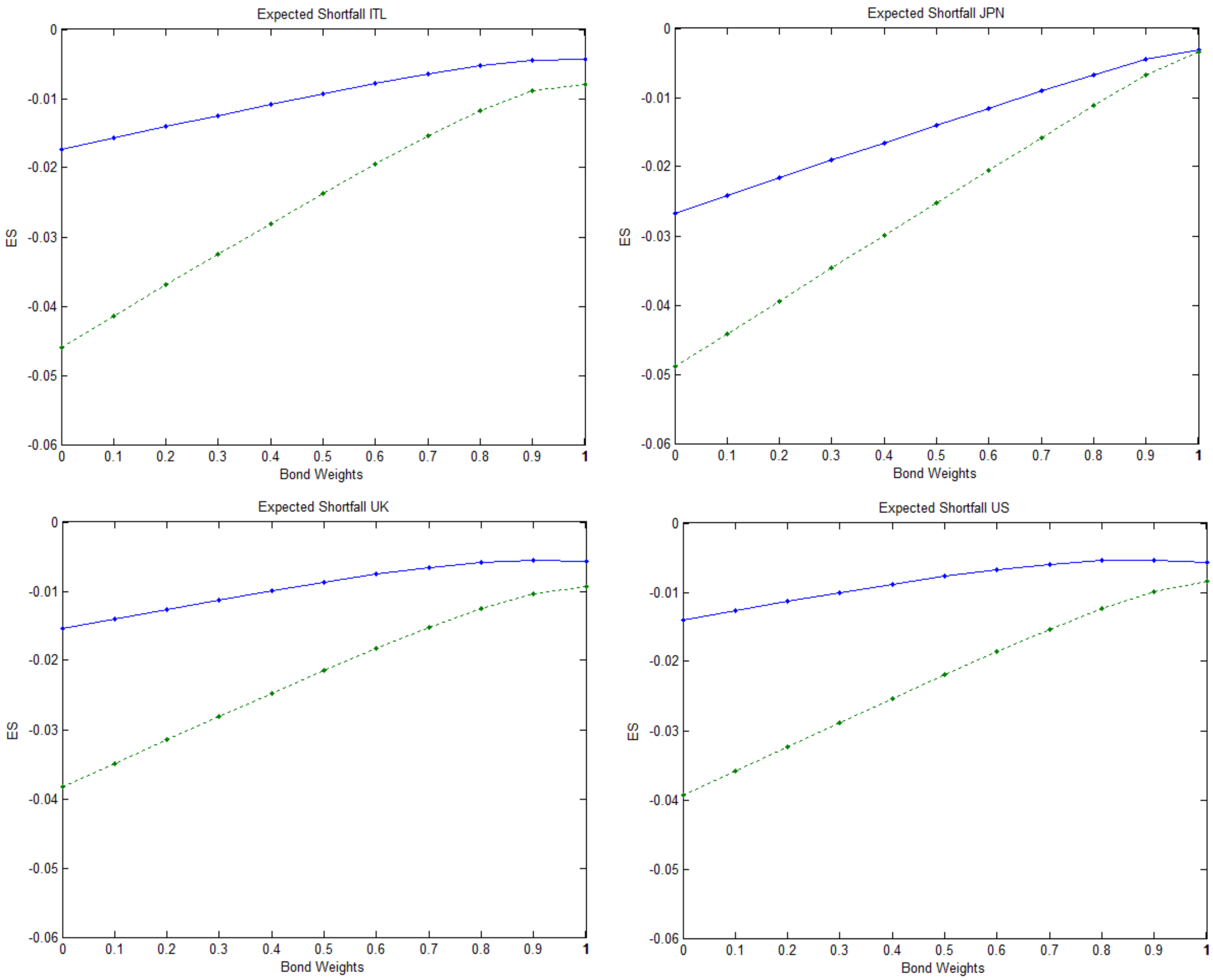


Figure 3 (continued): Five per cent Expected Shortfall estimates of stock–bond portfolios with varying bond weights for Australia and G7 countries before and after the global financial crisis.



Note: The solid curves represent the stock-bond portfolio ESs at the 5% level before the GFC, and the dashed curves represent the ESs after the GFC.

Figure 4: Five per cent out-of-sample Value-at-Risk forecasts of stock–bond portfolios with varying bond weights for US before and after the global financial crisis.

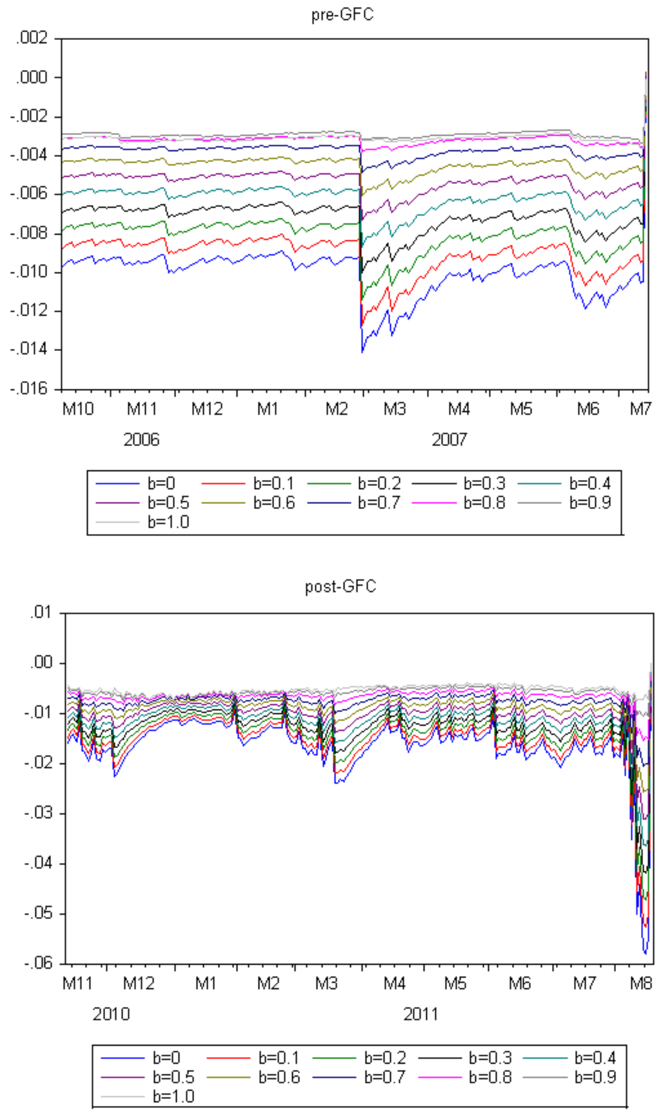


Figure 5: Five per cent out-of-sample Value-at-Risk forecasts against realized returns of US stock–bond portfolio with varying bond weights: pre–GFC and post–GFC periods