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Parameterisation and Efficient MCMC Estimation of Non-Gaussian State Space Models

Chris M Strickland, Gael Martin and Catherine S Forbes

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Parameterisation and Efficient MCMC Estimation of Non-Gaussian State Space Models.

Chris M. Strickland, Gael M. Martin* and Catherine S. Forbes

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Abstract

The impact of parameterisation on the simulation efficiency of Bayesian Markov chain Monte Carlo (MCMC) algorithms for two non-Gaussian state space models is examined. Specifically, focus is given to particular forms of the stochastic conditional duration (SCD) model and the stochastic volatility (SV) model, with four alternative parameterisations of each model considered. A controlled experiment using simulated data reveals that relationships exist between the simulation efficiency of the MCMC sampler, the magnitudes of the population parameters and the particular parameterisation of the state space model. Results of an empirical analysis of two separate transaction data sets for the SCD model, as well as equity and exchange rate data sets for the SV model, are also reported. Both the simulation and empirical results reveal that substantial gains in simulation efficiency can be obtained from simple reparameterisations of both types of non-Gaussian state space models.

Keywords: Bayesian methodology, stochastic volatility, durations, non-centred in location, non-centred in scale, inefficiency factors.

JEL: C11; C22; G1.

*Corresponding Author: Department of Econometrics and Business Statistics, PO Box 11E, Monash University, Vic., 3800, Australia. Email: gael.martin@buseco.monash.edu.au. Phone: 61 3 9905 1189. Fax: 61 3 9905 5474.

1 Introduction

The state space form provides a convenient framework for building time series models for observed phenomena, whereby relatively simple model components are combined to explain potentially complex dependencies in observed data. Linear Gaussian state space models, in particular, have had a long history in both estimation and forecasting applications, with many traditional time series models, such as autoregressive integrated moving average (ARIMA) models, having a linear Gaussian state space representation. This representation is often exploited in computing algorithms, both Bayesian and classical, designed to infer parameter values and produce forecasts.

More recently, non-Gaussian state space models have enjoyed increasing popularity, partly due to developments in inferential simulation techniques. In particular, non-Gaussian state space models have been used to characterise the dynamic features of various financial time series, such as the time between transactions or the volatility of asset returns, with a range of Markov chain Monte Carlo (MCMC) methods having been employed to implement Bayesian analyses of such models. Concurrent with the increased application of MCMC sampling schemes to non-Gaussian state space models, has been the revelation that substantial improvements in the simulation efficiency of MCMC schemes, in a variety of contexts, can sometimes be obtained though simple model reparameterisation. Relevant work includes Gelfand, Sahu and Carlin (1995), Roberts and Sahu (1997), Pitt and Shephard (1999), Früwirth-Schnatter and Sögner (2003), Papaspiliopoulos, Roberts and Sköld (2003), Früwirth-Schnatter (2004), Roberts, Papaspiliopoulos and Dellaportas (2004) and Bos and Shephard (2006).

This paper contributes to the literature by examining the effect of particular types of reparameterisation in two specific non-Gaussian state space models. Firstly, a form of the stochastic conditional duration (SCD) model of Bauwens and Veredas (2004), based on the assumption of conditionally exponential data, is considered. Variants of such a model, under alternative distributional assumptions, have recently been applied to financial trade durations in Strickland, Forbes and Martin (SFM hereafter) (2006), with an MCMC algorithm developed for one particular parameterisation. In the present paper, in which the focus is on documenting computational performance under a range of scenarios, the use of the exponential distribution serves to minimize the number of parameters involved in the reparameterisations, such that the number of results to be produced and summarised is

manageable. The second model considered is the stochastic volatility (SV) model for financial returns, based on conditional normality; see for example Jacquier, Polson and Rossi (1994), Shephard and Pitt (1997), and Kim, Shephard and Chib (1998), amongst many others. Again, the assumption of conditional normality is chosen for expositional convenience.[1]

Comparisons are first conducted using artificially simulated data, based on multiple parameter settings, with the efficiency of the algorithms measured via inefficiency factors. The parameter settings are chosen to ensure that the simulated data resemble typical trade durations and financial returns data. Empirical evaluation of the alternative parameterisations of the SCD model is then conducted using trade durations data for two Australian listed companies: Broken Hill Proprietary Limited (BHP) and News Corporation (NCP), for the month of August 2001. These two data sets were analysed in SFM, using one particular parameterisation of an SCD model. The SV model is estimated using daily returns on the Morgan Stanley Capital Index (MSCI) between 1989 and 2002, and the pound/dollar daily exchange rate returns between 1981 and 1985. This exchange rate data was previously analysed in Harvey, Ruiz and Shephard (1994), Kim, Shephard and Chib (1998) and Durbin and Koopman (2001), using specific parameterisations and various numerical algorithms that are alternatives to those used in this paper.[2]

Four alternative parameterisations of both non-Gaussian state space models are considered. Using the nomenclature in the literature, the parameterisations are referred to as: 'centred', 'non-centred in location', 'non-centred in scale' and 'non-centred in both location and scale', with all parameterisations augmented to incorporate regressors in the state equation. In the centred parameterisation all of the parameters (persistence, scale and location) appear in the state equation, while the non-centred parameterisations are based on either the location or scale parameter, or both, appearing in the measurement equation. The simulation results reveal clear relationships between the simulation efficiency of the MCMC sampler, the chosen parameterisation and the magnitudes of the population parameters. Overall, both the experimental and empirical results indicate that parameterisations that place parameters in the measurement equation, as opposed to the state

[1] As is common in the literature, we use the term 'non-Gaussian' state space model to include any model in which the unconditional distribution of the observations is non-Gaussian. In the SV model the unconditional distribution is non-Gaussian due to a non-linearity in the measurement equation.

[2] The exchange rate data is publicly available at http://www.ssfpack.com/dkbook/.

equation, tend to perform better. Substantial improvements in efficiency are associated, in particular, with relocation of the scale parameter, either on its own, or in conjunction with the location parameter.

An outline of the paper is as follows. Section 2 defines the two non-Gaussian state space models that are considered, including the four alternative parameterisations of each model. Section 3 provides details of the algorithms used to estimate the different parameterisations. All algorithms are modifications of the hybrid Gibbs/Metropolis-Hastings (MH) MCMC sampling scheme proposed in SFM. Section 4 presents the criteria used to compare the performance of the algorithms and summarizes the results of the simulation experiment. Section 5 details the four empirical applications, two using the SCD model and two using the SV model. Section 6 concludes.

2 A Non-Gaussian State Space Framework

2.1 The Centred Parameterisation

Defining $\mathbf{y} = (y_1, y_2, \ldots, y_T)'$ as the $(T \times 1)$ observation vector and assuming conditional independence, the measurement equation of a non-Gaussian state space model may be represented by the following probability relation

$$p(\mathbf{y}|\boldsymbol{\alpha}) = \prod_{t=1}^{T} p(y_t|\alpha_t), \qquad t = 1, 2, ..., T, \tag{1}$$

where $p(y_t|\alpha_t)$ denotes the probability density function (pdf) for y_t, conditional on α_t. The $(t+1)^{th}$ element of the $(T \times 1)$ state vector $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_T)'$ is assumed to be generated by the following state equation,

$$\alpha_{t+1} = \delta_1 + \mathbf{W}_t'\boldsymbol{\delta}_2 + \phi\alpha_t + \sigma_\eta\eta_t, \qquad t = 1, 2, ..., T-1, \tag{2}$$

where δ_1 is a constant, $\boldsymbol{\delta}_2 = (\delta_2, \delta_3, \ldots, \delta_k)'$ is a $((k-1) \times 1)$ vector of coefficients and $\mathbf{W}_t'$ is the t^{th} row of the $(T \times (k-1))$ matrix of regressors, $\mathbf{W}$. It is further assumed that $\eta_t \sim N(0,1)$ is independent of $y_t|\alpha_t$ and that $|\phi| < 1$ and $\sigma_\eta^2 > 0$. Defining the parameter μ implicitly via $\delta_1 = \mu(1-\phi)$, with $\mu \in \mathbb{R}$, the full vector of unknown parameters is denoted by $\boldsymbol{\theta} = (\mu, \boldsymbol{\delta}_2', \phi, \sigma_\eta)'$. The assumed pdf for the initial state is given by

$$p(\alpha_1|\mathbf{W}_0, \boldsymbol{\theta}) \sim N\left(\mu + \frac{\mathbf{W}_0'\boldsymbol{\delta}_2}{1-\phi}, \frac{\sigma_\eta^2}{1-\phi^2}\right). \tag{3}$$

The specification of the state space model in (1), (2) and (3) is referred to here as the 'centred parameterisation'. Adopting the assumption of a conditional exponential distribution in the case of the SCD model, it follows that

$$p(y_t|\alpha_t) = \exp(-\alpha_t) \exp\left\{-y_t \exp\left(-\alpha_t\right)\right\}, \tag{4}$$

for all $t = 1, 2, ..., T$, with conditional mean $E\left[y_t|\alpha_t\right] = \exp\left(\alpha_t\right)$. The SV model assumes that the (potentially demeaned) y_t, conditional on α_t, has a normal distribution, with

$$p(y_t|\alpha_t) = \left(2\pi \exp\left(\alpha_t\right)\right)^{-1/2} \exp\left(-\frac{1}{2\exp\left(\alpha_t\right)} y_t^2\right), \tag{5}$$

for all $t = 1, 2, ..., T$, with conditional variance $Var\left(y_t|\alpha_t\right) = \exp\left(\alpha_t\right)$. Note that in this so-called 'centred' parameterisation none of the elements of $\boldsymbol{\theta}$ explicitly enter the measurement equation. This is in contrast to the reparameterisations presented in the following section.

2.2 Reparameterising the Non-Gaussian State Space Model

The centred parameterisation is the most commonly used parameterisation of the non-Gaussian state space model, at least within the SCD and SV literature. See, for example, Jacquier, Polson and Rossi (1994), Kim, Shephard and Chib (1998), Bauwens and Veredas (2004) and SFM. However, it is equally valid to modify the model specification by moving one or both of μ and σ_η into the measurement equation. Although, conditional on $\boldsymbol{\theta}$, the probability distribution of the data remains unchanged, reparameterisation may impact upon the simulation efficiency of the MCMC algorithms used to infer such model parameters from the observed data.

2.2.1 Non-centred in location

The first alternative parameterisation of the non-Gaussian state space model considered is based upon a location shift for the state variable. Define $\widetilde{\alpha}_t = \alpha_t - \mu$ and transform the model accordingly. The implied measurement equation is

$$p(\mathbf{y}|\boldsymbol{\alpha}, \mu) = \prod_{t=1}^{T} p(y_t|\widetilde{\alpha}_t, \mu), \quad t = 1, 2, ..., T, \tag{6}$$

with state equation given by

$$\widetilde{\alpha}_{t+1} = \mathbf{W}_t'\boldsymbol{\delta}_2 + \phi\widetilde{\alpha}_t + \sigma_\eta\eta_t. \tag{7}$$

From (3), the implied pdf of the initial state for the model that is non-centred in location is

$$p\left(\widetilde{\alpha}_1|\mathbf{W}_0, \phi, \sigma_\eta\right) \sim N\left(\frac{\mathbf{W}_0'\boldsymbol{\delta}_2}{1-\phi}, \frac{\sigma_\eta^2}{1-\phi^2}\right). \tag{8}$$

Under the assumption of a conditional exponential distribution for the SCD model, each component in (6) is given by

$$p(y_t|\widetilde{\alpha}_t, \mu) = \exp(-\left(\widetilde{\alpha}_t + \mu\right)) \exp\left\{-y_t \exp\left(-\left(\widetilde{\alpha}_t + \mu\right)\right)\right\}, \tag{9}$$

with the conditional mean now specified as $E\left[y_t|\widetilde{\alpha}_t, \mu\right] = \exp\left(\mu + \widetilde{\alpha}_t\right)$. For the SV model it follows that

$$p(y_t|\widetilde{\alpha}_t, \mu) = \left(\left(2\pi \exp\left(\widetilde{\alpha}_t + \mu\right)\right)^{-1/2} \exp\left(-\frac{1}{2\exp\left(\widetilde{\alpha}_t + \mu\right)} y_t^2\right)\right), \tag{10}$$

where the conditional variance is now given as $Var\left(y_t|\widetilde{\alpha}_t, \mu\right) = \exp\left(\mu + \widetilde{\alpha}_t\right)$. Adopting the terminology of Gelfand, Sahu and Carlin (1995), applied in the context of a random effects model, the model in (6) to (9) is referred to here as 'non-centred in location'.[3] From (9) and (10) it is clear that the location parameter, μ, now appears explicitly in the measurement equation.

2.2.2 Non-centred in scale

Next, a scale adjustment to the state variable from the centred parameterisation is considered. Defining the new state variable as $\alpha_t^* = \frac{\alpha_t}{\sigma_\eta}$ produces a parameterisation that is referred to as 'non-centred in scale'. In this case the measurement equation is

$$p(\mathbf{y}|\boldsymbol{\alpha}, \sigma_\eta) = \prod_{t=1}^{T} p(y_t|\alpha_t^*, \sigma_\eta), \quad t = 1, 2, ..., T \tag{11}$$

and the state equation,

$$\alpha_{t+1}^* = \delta_1^* + \mathbf{W}_t'\boldsymbol{\delta}_2^* + \phi\alpha_t^* + \eta_t, \tag{12}$$

[3] In the case where $\boldsymbol{\delta}_2 = \mathbf{0}$, this terminology is used to refer to the fact that the state variable, $\widetilde{\alpha}_{t+1}$, no longer has the unconditional expection μ, but rather has a zero unconditional mean.

where $\delta_1^* = \frac{\mu(1-\phi)}{\sigma_\eta}$ and $\boldsymbol{\delta}_2^* = \frac{\boldsymbol{\delta}_2}{\sigma_\eta}$. The implied pdf of the initial state is

$$p\left(\alpha_1^* | \mathbf{W}_0, \phi\right) \sim N\left(\frac{\delta_1^* + \mathbf{W}_0' \boldsymbol{\delta}_2^*}{(1-\phi)}, \frac{1}{1-\phi^2}\right). \tag{13}$$

Under the assumption of a conditional exponential distribution for the SCD model, each component in (11) is given by

$$p(y_t | \alpha_t^*, \sigma_\eta) = \exp(-\left(\sigma_\eta \alpha_t^*\right)) \exp\left\{-y_t \exp\left(-\left(\sigma_\eta \alpha_t^*\right)\right)\right\}, \tag{14}$$

with the conditional mean now given by $E\left[y_t | \alpha_t^*, \sigma_\eta\right] = \exp\left(\sigma_\eta \alpha_t^*\right)$. For the SV model it follows that

$$p(y_t | \alpha_t^*, \sigma_\eta) = \left(\left(2\pi \exp\left(\sigma_\eta \alpha_t^*\right)\right)^{-1/2} \exp\left(-\frac{1}{2\exp\left(\sigma_\eta \alpha_t^*\right)} y_t^2\right)\right), \tag{15}$$

where the conditional variance is now $Var\left(y_t | \alpha_t^*, \sigma_\eta\right) = \exp\left(\sigma_\eta \alpha_t^*\right)$. Under this parameterisation the scale parameter, σ_η, appears explicitly in the measurement equation only. It enters the state equation indirectly, via δ_1^* and δ_2^*.

2.2.3 Non-centred in both location and scale

By defining $\alpha_t^{**} = \frac{\alpha_t - \mu}{\sigma_\eta}$, both scale and location adjustments are made to the original centred parameterisation state variable. This parameterisation, referred to as 'non-centred in both location and scale', has measurement equation given by

$$p(\mathbf{y} | \boldsymbol{\alpha}, \mu, \sigma_\eta) = \prod_{t=1}^{T} p(y_t | \alpha_t^{**}, \mu, \sigma_\eta), \quad t = 1, 2, ..., T \tag{16}$$

and state equation given by

$$\alpha_{t+1}^{**} = \mathbf{W}_t' \boldsymbol{\delta}_2^* + \phi \alpha_t^{**} + \eta_t, \tag{17}$$

where $\boldsymbol{\delta}_2^*$ is as defined earlier. The implied pdf of the initial state is

$$p\left(\alpha_1^{**} | \mathbf{W}_0, \phi\right) \sim \left(\frac{\mathbf{W}_0' \boldsymbol{\delta}_2^*}{(1-\phi)}, \frac{1}{1-\phi^2}\right). \tag{18}$$

Under the assumption of a conditional exponential distribution for the SCD model, each component in (6) is given by

$$p(y_t|\alpha_t^{**},\mu,\sigma_\eta) = \exp(-(\sigma_\eta\alpha_t^{**}+\mu))\exp\left\{-y_t\exp\left(-(\sigma_\eta\alpha_t^{**}+\mu)\right)\right\}, \quad (19)$$

with the conditional mean specified as $E\left[y_t|\alpha_t^{**},\mu,\sigma_\eta\right] = \exp\left(\mu+\sigma_\eta\alpha_t^{**}\right)$. For the SV model each component in (6) is given by

$$p(y_t|\alpha_t^{**},\mu,\sigma_\eta) = \left(\left(2\pi\exp\left(\sigma_\eta\alpha_t^{**}+\mu\right)\right)^{-1/2}\exp\left(-\frac{1}{2\exp\left(\sigma_\eta\alpha_t^{**}+\mu\right)}y_t^2\right)\right), \quad (20)$$

with conditional variance, $Var\left(y_t|\alpha_t^{**},\mu,\sigma_\eta\right) = \exp\left(\mu+\sigma_\eta\alpha_t^{**}\right)$. Note that both the location and scale parameters enter the measurement equation, and neither explicitly enter the state equation.

3 Bayesian Estimation

The MCMC sampling scheme of SFM is used as the algorithm for the centred parameterisation, and is then modified to cater for the non-centred parameterisations. As highlighted in SFM, the algorithm is very flexible, being readily applicable beyond both the specific model specifications examined in that paper and the specifications examined in the current paper. In particular, it is more general than the distribution-specific algorithm outlined by Shephard (1994) and Carter and Kohn (1994), and implemented by Kim, Shephard and Chib (1998) for the SV model, whereby the non-Gaussian density in the (linearised) measurement equation is approximated by a mixture of normal densities. Shephard and Pitt (1997) also provide an alternative algorithm for non-Gaussian state space models, however this algorithm is arguably superseded by the scheme of SFM. The latter authors use the approach of Durbin and Koopman (2000, 2001) to produce a linear Gaussian approximation to the measurement equation, such that alterations to the algorithm required to accommodate different distributional assumptions (and, hence, data types) are relatively straightforward and transparent. In contrast to Durbin and Koopman, who use the approximation as a part of an importance sampling scheme, SFM use the approximating model to construct a candidate distribution in an MH step imbedded in an outer Gibbs chain. SFM argue that the MCMC approach is potentially more efficient than the importance sampling methodology, as the Gibbs sampler allows the high-dimensional latent vector

to be broken down into blocks of lower dimension. This has particular relevance to non-Gaussian financial data sets (such as durations and returns) that typically contain a large number of observations. Pitt (2000) also makes note of this drawback of the Durbin and Koopman approach.

3.1 The Joint Posterior

The joint posterior for the full set of unknowns in the non-Gaussian state space model is given by

$$p(\boldsymbol{\alpha}, \boldsymbol{\theta}|\mathbf{y}, \mathbf{W}) \propto p(\mathbf{y}|\boldsymbol{\alpha}, \boldsymbol{\theta}) \times p(\boldsymbol{\alpha}|\mathbf{W}, \boldsymbol{\theta}) \times p(\boldsymbol{\theta}), \tag{21}$$

where $p(\boldsymbol{\alpha}|\mathbf{W}, \boldsymbol{\theta})$ denotes the joint pdf of $\boldsymbol{\alpha}$ conditional on $\boldsymbol{\theta}$ and the observed $\mathbf{W}$, and $p(\boldsymbol{\theta})$ is the prior pdf for $\boldsymbol{\theta}$. The joint pdf $p(\mathbf{y}|\boldsymbol{\alpha}, \boldsymbol{\theta})$ is as defined in (1), (6), (11) or (16), depending on the chosen parameterisation. The joint density for the state vector is

$$p(\boldsymbol{\alpha}|\mathbf{W}, \boldsymbol{\theta}) = \left\{ \prod_{t=1}^{T-1} p(\alpha_{t+1}|\mathbf{W}_t, \alpha_t, \boldsymbol{\theta}) \right\} \times p(\alpha_1|\mathbf{W}_0, \boldsymbol{\theta}), \tag{22}$$

where $p(\alpha_{t+1}|\mathbf{W}_t, \alpha_t, \boldsymbol{\theta})$ is given by (2), (7), (12) or (17) and $p(\alpha_1|\mathbf{W}_0, \boldsymbol{\theta})$ is given by (3), (8), (13) or (18), once again according to the specified parameterisation.

The following subsections summarise the algorithm used for each of the parameterisations. Note that for all of the non-centred parameterisations, the MCMC algorithms use a combination of MH and multiple trial Metropolised (MTM) sub-steps; see Liu (2001)[4]. The resultant hybrid algorithms were found to be preferable, in terms of simulation efficiency, to the corresponding algorithms based only on MH sub-steps for the relevant parameters. Each of the algorithms outlined below requires only minor modifications to cater for different distributional assumptions in the measurement equation. As such, the ease with which the practitioner can modify the associated code, given alternative distributional assumptions, is maintained from SFM.

[4]The MTM algorithm is a generalisation of the MH algorithm in which multiple candidate draws are taken.

3.2 The Centred Parameterisation

Estimation of the model in Section 2.1 is essentially described in SFM (Section 3).[5] The steps of the Gibbs-based sampler are briefly summarized as follows:

1. Initialize $\boldsymbol{\alpha}$ and $\boldsymbol{\theta}$. Note that $\boldsymbol{\theta}$ needs to be initialised for the MH algorithm used in Step 2.

2. Sample $\boldsymbol{\theta}|\mathbf{y}, \mathbf{W}, \boldsymbol{\alpha}$.

3. Sample $\boldsymbol{\alpha}|\mathbf{y}, \mathbf{W}, \boldsymbol{\theta}$, where $\boldsymbol{\alpha}$ is broken up into blocks of size greater than one, as per SFM.

4. Repeat steps 2-3 until convergence has been achieved.

Depending on the form of $p(\boldsymbol{\theta})$, there may be no closed-form representation for $p(\boldsymbol{\theta}|\mathbf{y}, \mathbf{W}, \boldsymbol{\alpha})$. However, standard Bayesian linear regression theory provides a good candidate through which draws from $p(\boldsymbol{\theta}|\mathbf{y}, \mathbf{W}, \boldsymbol{\alpha})$ can be obtained indirectly using an MH algorithm. Sampling from $p(\boldsymbol{\alpha}|\mathbf{y}, \mathbf{W}, \boldsymbol{\theta})$ is also accomplished indirectly, as the non-Gaussian measurement equation implies that there is no closed form solution for the conditional posterior for $\boldsymbol{\alpha}$. This is the most complex component of the algorithm, with full details provided in SFM (Section 3.2.3). Crucially, this component of the algorithm is common to all four parameterisations considered in the current paper.

3.3 Non-Centred in Location

The steps required to implement the Gibbs-based sampler for the model in Section 2.2.1 can be summarized as follows:

1. Initialize $\widetilde{\boldsymbol{\alpha}} = (\widetilde{\alpha}_1, \widetilde{\alpha}_2, \ldots, \widetilde{\alpha}_T)'$ and $\boldsymbol{\theta}$. Note that $\boldsymbol{\delta}_2$, ϕ and σ_η need to be initialised, in addition to $\widetilde{\boldsymbol{\alpha}}$ and μ, for the MH algorithm used in Step 2.

2. Sample $\boldsymbol{\delta}_2, \phi, \sigma_\eta|\mathbf{y}, \mathbf{W}, \widetilde{\boldsymbol{\alpha}}, \mu$.

[5] Note that the modifications required for estimation of the SV model are not explicitly described in SFM. However the necessary modifications can be readily deduced from details provided in SFM and Durbin and Koopman (2001, Chp. 11).

3. Sample $\mu|\mathbf{y}, \mathbf{W}, \widetilde{\boldsymbol{\alpha}}, \boldsymbol{\delta}_2, \phi, \sigma_\eta$.

4. Sample $\widetilde{\boldsymbol{\alpha}}|\mathbf{y}, \mathbf{W}, \boldsymbol{\theta}$, where $\widetilde{\boldsymbol{\alpha}}$ is broken up into blocks of size greater than one. (Sample as per SFM, Section 3.2.3)

5. Repeat steps 2-4 until convergence has been achieved.

The linear Gaussian nature of the state equation once again implies that when sampling from $p\left(\boldsymbol{\delta}_2, \phi, \sigma_\eta|\mathbf{y}, \mathbf{W}, \widetilde{\boldsymbol{\alpha}}, \mu\right)$ a good candidate can be obtained using standard Bayesian linear regression theory, with draws from $p\left(\boldsymbol{\delta}_2, \phi, \sigma_\eta|\mathbf{y}, \mathbf{W}, \widetilde{\boldsymbol{\alpha}}, \mu\right)$ then obtained indirectly using an MH algorithm. Sampling from $p\left(\mu|\mathbf{y}, \mathbf{W}, \widetilde{\boldsymbol{\alpha}}, \boldsymbol{\delta}_2, \phi, \sigma_\eta\right)$ is conducted using the orientational bias Monte Carlo (OBMC) algorithm, which is a special case of the MTM algorithm; see Lui (2001) for details. For a general parameter ω (with $\omega = \mu$ in this case), let $p(\omega|.)$ denote the pdf corresponding to the full conditional posterior distribution from which the parameter ω is to be simulated. The steps of the OBMC algorithm to be inserted at the j^{th} iteration of the Gibbs chain are as follows:

1. Specify $\omega^{(j-1)}$ as an initial value for the algorithm.

2. Draw L candidates $\omega_l^* = \omega^{(j-1)} + \varepsilon_l$, $l = 1, 2, \ldots, L$, where the ε_l are independent draws from a common normal distribution with a mean of 0 and a variance of σ_ε^2.

3. Construct a probability mass function (pmf) by assigning to each ω_l^* a probability proportional to $p(\omega_l^*|.)$, $l = 1, 2, \ldots, L$, where $p(\omega|.)$ denotes the pdf corresponding to the relevant full conditional posterior distribution that is evaluated at the relevant argument.

4. Select ω^{**} randomly from this discrete distribution.

5. Draw $L-1$ reference points $r_l = \omega^{**} + \varepsilon_l$, $l = 1, 2, \ldots, L-1$, and set $r_L = \omega^{(j-1)}$.

6. Accept $\omega^{(j)} = \omega^{**}$ with probability equal to $\min\left(1, \frac{p(\omega_1^*|.)+\cdots+p(\omega_L^*|.)}{p(r_1|.)+\cdots+p(r_L|.)}\right)$

7. Otherwise accept $\omega^{(j)} = \omega^{(j-1)}$.

3.4 Non-Centred in Scale

Estimation of the non-Gaussian state space model for the non-centred in scale parameterisation, defined in (11), (12) and (13), is summarized by the following steps:

1. Initialize $\boldsymbol{\alpha}^* = (\alpha_1^*, \alpha_2^*, \ldots, \alpha_T^*)'$ and $\boldsymbol{\theta}$. Note that μ, $\boldsymbol{\delta}_2$, and ϕ need to be initialised, in addition to $\boldsymbol{\alpha}^*$ and σ_η, for the MH algorithm used in Step 2.

2. Sample $\mu, \boldsymbol{\delta}_2, \phi | \mathbf{y}, \mathbf{W}, \boldsymbol{\alpha}^*, \sigma_\eta$.

3. Sample $\sigma_\eta | \mathbf{y}, \mathbf{W}, \boldsymbol{\alpha}^*, \mu, \boldsymbol{\delta}_2, \phi$.

4. Sample $\boldsymbol{\alpha}^* | \mathbf{y}, \mathbf{W}, \boldsymbol{\theta}$, where $\boldsymbol{\alpha}^*$ is broken up into blocks of size greater than one. (Sample as per SFM, Section 3.2.3)

5. Repeat steps 2-4 until convergence has been achieved.

As equation (12) remains linear and Gaussian under this reparameterisation, standard Bayesian linear regression theory again provides an appropriate candidate for sampling from $p(\mu, \boldsymbol{\delta}_2, \phi | \mathbf{y}, \mathbf{W}, \boldsymbol{\alpha}^*, \sigma_\eta)$ using an MH algorithm. Sampling from $p(\sigma_\eta | \mathbf{y}, \mathbf{W}, \boldsymbol{\alpha}^*, \mu, \boldsymbol{\delta}_2, \phi)$ is conducted via the OBMC algorithm outlined in Section 3.3, with $\omega = \sigma_\eta$.

3.5 Non-Centred in Both Location and Scale

Estimation of the non-Gaussian state space model for the non-centred in location and scale parameterisation, defined in (16), (17) and (18) is summarised by the following steps:

1. Initialize $\boldsymbol{\alpha}^{**} = (\alpha_1^{**}, \alpha_2^{**}, \ldots, \alpha_T^{**})'$ and $\boldsymbol{\theta}$. Note that $\boldsymbol{\delta}_2$ and ϕ need to be initialised, in addition to $\boldsymbol{\alpha}^{**}$, μ and σ_η, for the MH algorithm used in Step 2.

2. Sample $\boldsymbol{\delta}_2, \phi | \mathbf{y}, \mathbf{W}, \boldsymbol{\alpha}^{**}, \mu, \sigma_\eta$.

3. Sample $\mu | \mathbf{y}, \mathbf{W}, \boldsymbol{\alpha}^{**}, \phi, \sigma_\eta, \boldsymbol{\delta}_2$.

4. Sample $\sigma_\eta | \mathbf{y}, \mathbf{W}, \boldsymbol{\alpha}^{**}, \phi, \mu, \boldsymbol{\delta}_2$.

5. Sample $\boldsymbol{\alpha}^{**}|\mathbf{y}, \mathbf{W}, \boldsymbol{\theta}$, where $\boldsymbol{\alpha}^{**}$ is broken up into blocks of size greater than one. (Sample as per SFM, Section 3.2.3)

6. Repeat steps 2-4 until convergence has been achieved.

As equation (17) is still linear and Gaussian, a candidate for $p(\boldsymbol{\delta}_2, \phi|\mathbf{y}, \boldsymbol{\alpha}^{**}, \mu, \sigma_\eta)$ can be obtained using Bayesian linear regression theory, and a standard MH algorithm applied. Sampling μ and σ from their full conditional posterior distributions is achieved using the OBMC algorithm in Section 3.3, with $\omega = \mu$ and $\omega = \sigma_\eta$ respectively.[6]

4 Simulation Experiment

A simulation experiment is used to explore the efficiency of the MCMC algorithms under the four different parameterisations of each of the SCD and SV non-Gaussian state space models. To simplify the experiment, $\boldsymbol{\delta}_2$ is set equal to a vector of zeros. The parameters ϕ and σ_η are assigned a range of empirically plausible values for each of the specified models, which leads to a total of 18 simulated data sets, each of which has $T = 5000$ observations. The values of ϕ that are considered for both the SCD and SV models are $\{0.8, 0.9, 0.95\}$. The values of σ_η under consideration are $\{0.1, 0.2, 0.3\}$ and $\{0.2, 0.3, 0.4\}$ for the SCD and the SV model, respectively. Specific references for the range of parameter values considered for the SCD model are Bauwens and Veredas (2004) and SFM, and for the SV model, Jacquier, Polson and Rossi (1994), Kim, Shephard and Chib (1998), Liesenfeld and Richard (2003) and Bos and Shephard (2006). The values also accord with certain of the empirical estimates produced in Section 5.

For the SCD model, the value of δ_1 is specified as

$$\delta_1 = -\frac{\sigma_\eta^2(1-\phi)}{2(1-\phi^2)}, \tag{23}$$

[6] An interesting alternative blocking scheme is implemented in Früwirth-Schnatter (2004) in the context of the non-centred in location and scale parameterisation of the linear Gaussian state space model. The algorithm is subtly different from that presented here in that it is based on conditioning on the state disturbance vector ($\boldsymbol{\eta}$), rather than on the transformed state vector (α^{**}), in steps 2, 3 and 4 above. This blocking scheme was implemented for the non-Gaussian models considered here, but with some convergence problems occurring in certain of the empirical applications. Hence, we decided not to document the performance of this scheme.

for given values of ϕ and σ_η, where this setting implies that the unconditional mean of the durations is equal to 1. This is consistent with empirical applications of the SCD model, as the latter is typically applied to a transaction data set that has been de-seasonalised assuming a multiplicative intraday pattern (producing observations that average to about 1); see, for example, Bauwens and Veredas (2004) and SFM. The expression in (23) implies a range of values for μ between -0.14 and -1.53.

For the SV model δ_1 is also set conditionally on the specified values for σ_η and ϕ, such that

$$\delta_1 = (1-\phi)\left[\ln(0.4) - \frac{\sigma_\eta^2}{2(1-\phi^2)}\right], \tag{24}$$

where equation (24) implies an unconditional variance in the SV model of 0.4. This value is comparable to that implied by typical empirical estimates of the SV parameters for daily exchange rate returns data and some stock index returns data, including the two data sets analysed in Section 5.2. Specifically, it corresponds to an expected annualised volatility of approximately 10% for continuously compounded returns.

Given the nature of the experiments it is natural to assume diffuse priors. Specifically, for δ_1 we assume a uniform prior over $\mathbb{R}$ and for ϕ we assume a uniform distribution over the $(-1,1)$ interval. An inverted-gamma distribution is assumed for σ_η, such that $p(\sigma_\eta) \sim IG\left(\frac{\sigma_r}{2}, \frac{S_\sigma}{2}\right)$ (see Zellner, 1996), with the hyper-parameters σ_r and S_σ set to 1.0001 and 0.01 respectively, implying very diffuse prior information on σ_η.

4.1 Simulation Efficiency Comparison

Under each parameterisation the so called inefficiency factor (IF) is used to benchmark simulation efficiency. The IF features prominently in the literature as a measure for comparing the performance of various alternative algorithms; see for example Chib and Greenberg (1996), Shephard and Pitt (1997) and Kim, Shephard and Chib (1998). The IF is calculated using the following formula

$$IF = 1 + 2\frac{B}{B-1}\sum_{i=1}^{B} K_{QS}\left(\frac{i}{B}\right)\widehat{\rho}_i, \tag{25}$$

where $\widehat{\rho}_i$ is the estimate of the correlation at lag i of the MCMC iterates, K_{QS} is the Quadratic Spectral (QS) kernel and B is the bandwidth.[7] The QS kernel is defined as

$$K_{QS}(x) = \frac{25}{12\pi^2 x^2}\left(\frac{\sin(6\pi x/5)}{6\pi x/5} - \cos(6\pi x/5)\right). \tag{26}$$

The bandwidth B is selected automatically following Andrews (1991). Implementation of this procedure is summarised in SFM (Appendix B).

Further understanding of the IF comes through its relationship with the (estimated) Monte Carlo standard error (MCSE) for the posterior mean of any given parameter. That is,

$$MCSE^2 = \frac{\sigma^2_{MCMC}}{M} IF, \tag{27}$$

where σ^2_{MCMC} is the sample variance of the MCMC iterates and M denotes the number of iterations. From (27) it is apparent that the IF represents the ratio of the simulation variance (square of the estimated MCSE) to the variance that would arise from a hypothetical sample of M independent draws. For example, a value of 100 for IF implies that, for a given number of iterations M, correlation in the MCMC iterates produces an estimated MCSE for the posterior mean that is ten times greater than that which would be associated with an independent sample. The aim of reparameterisation is to reduce the dependence in the MCMC draws and, hence, the IF, to the greatest extent possible.

In the simulation experiment, the MCMC sampler is run for 10000 iterations with a burn-in period of 2000 iterations. IFs are calculated for each sequence of simulated draws of ϕ, σ_η, and δ_1, for each of the four parameterisations and for data simulated under each of the nine different parameter settings (for the SCD and SV models respectively). We say that parameterisation A is preferred to parameterisation B, for a particular parameter set $\boldsymbol{\theta} = (\phi, \sigma_\eta, \delta_1)$, if the largest IF for the elements of $\boldsymbol{\theta}$ under parameterisation A is less than the largest IF for the elements of $\boldsymbol{\theta}$ under parameterisation B.[8] Parameterisation A is then deemed to be superior to parameterisation B

[7] Note that Andrews (1991) finds the QS kernel to be superior, in terms of an asymptotic truncated mean squared error criterion, to other kernels.

[8] The rational in only comparing the largest IF for each relevant parameter setting is that the largest IF measures the minimum level of accuracy across the parameter set.

across all parameter settings if A is preferred to B in the majority of parameter settings considered for the model. Further, parameterisations will be ranked from 1 to 4, corresponding to most efficient to least efficient, if there is a clear ordering of pairwise comparisons between the four parameterisations.

4.1.1 The centred parameterisation

Table 1 reports the IFs for the MCMC sampler for the centred (C) parameterisation of both the SCD and SV models. It is apparent that there are large differences in the magnitude of the IFs for different parameter settings. Further, there are systematic patterns in the IFs that are dependent upon the magnitudes of the parameters σ_η and ϕ. For both models, as the true value of σ_η increases there is a corresponding increase in the simulation efficiency (i.e. a reduction in the IFs) of the marginal posterior mean estimates of all three parameters, ϕ, σ_η and δ_1, irrespective of the true value of ϕ. An increase in efficiency associated with all parameters is also apparent as the degree of persistence in the state variable, measured by ϕ, increases, for any given true value of σ_η.[9] For all nine parameter settings, for both models, the maximum IF is associated with the parameter σ_η, indicating that dependence in the MCMC draws is the most marked for this particular parameter.[10]

4.1.2 Non-centred in location

Table 2 reports the IFs for the MCMC sampler for the non-centred in location (NCL) parameterisation for both the SCD and SV models. Based on the benchmarking criterion defined in Section 4.1, the NCL parameterisation is superior to the C parameterisation for both models. Improvements in efficiency are seen in seven of the nine parameter sets considered for the SCD model, and the NCL parameterisation is uniformly superior (i.e. across all nine parameter settings) to the C parameterision for the SV model. Importantly, taking the maximum IF for each parameter setting (recorded in bold in Table 2) and comparing it to the corresponding maximum for the C parameterisation (recorded in bold in Table 1), we find reductions of as

[9] See Früwirth-Schnatter (2004), Gelfand, Sahu and Carlin (1995) and Pitt and Shephard (1999), amongst others, for related results pertaining to the impact on efficiency of changes in the degree of heterogeneity and persistence in the state variable.

[10] See also Kim, Shephard and Chib (1998).

much as 52% for the SCD model and 60% for the SV model, where these reductions are denoted by the symbol $\Delta\%$ in Table 2. In addition, the few IFs that have increased with the NCL parameterisation are still quite small in magnitude.

As is the case with the C parameterisation, the simulation efficiency of the estimates of both ϕ and σ_η increases as the true values of ϕ and σ_η increase. Interestingly, however, the systematic relationships between the efficiency of the estimates of δ_1 and the true values of ϕ and σ_η no longer hold. The highest IF values are still associated with σ_η.

4.1.3 Non-centred in scale

Table 3 reports the IFs for the MCMC sampler for the non-centred in scale (NCS) parameterisation of the SCD and SV models. For eight of the nine cases of the SCD model, the NCS parameterisation is superior to both the C and NCL parameterisations. Furthermore, in many cases, the gains in efficiency are substantial, with the maximum IF being up to 81% lower than the corresponding figure for the C parameterisation, and 60% lower than the corresponding figure for the NCL parameterisation.[11] Thus, for the SCD model, there is a clear ranking of the three alternative parameterisations: 1) NCS; 2) NCL; 3) C.

For the SV model the results are somewhat different, in that the NCL parameterisation is preferred to the NCS for the SV model in all nine cases. Given that the NCS parameterisation is also superior to the C parameterisation in five of the nine cases, the three parameterisations can be ranked as: 1) NCL; 2) NCS; 3) C.

As with the C parameterisation, for both models there remains, in most cases, a positive relationship between the simulation efficiency of the estimates of ϕ and δ_1 and the magnitudes of the true values of ϕ and σ_η. For the relocated scale parameter σ_η, however, this relationship no longer holds. Also in contrast with both the C and NCL parameterisations, the IF values for σ_η are not uniformly the largest for all parameter settings. For the SCD model in particular, the IFs for σ_η are markedly reduced as a result of the relocation of the scale parameter to the measurement equation.

[11] The 81% figure is recorded in the row labelled as '($\Delta\%$)' in Table 3. The 60% reduction is deduced from the maximum IF figures recorded in Tables 2 and 3.

4.1.4 Non-centred in both location and scale

Table 4 records the IFs of the MCMC sampler for both the SCD and SV models, for the parameterisation that is non-centred in both location and scale (NCLS). For the SCD model, further simulation efficiency, over and above that produced by the NCS parameterisation, is produced by relocating both μ and σ_η into the measurement equation. Specifically, the dual relocation produces improvement in eight of the nine parameter settings, with up to a 70% reduction in the maximum IF, compared with the NCS parameterisation.[12] Further, in the one case where there is no improvement, the IF is not a great deal larger than the IFs for other parameter settings. Overall then, for the SCD model, the ranking of the four alternative parameterisations is: 1) NCSL; 2) NCS; 3) NCL; 4) C.

Interestingly, the dual relocation also produces substantial efficiency gains for the SV model, compared with the relocation of μ only, with the NCLS parameterisation being superior to the NCL parameterisation in all cases but one. Overall then, the appropriate ranking for the SV model is: NCSL; 2) NCL; 3) NCS; 4) C.

For both models, only the positive relationship between the simulation efficiency of the estimator of ϕ and the magnitude of the true values of ϕ and σ_η remains. The IFs of the parameters δ_1 and σ_η no longer have a clear relationship with the true values of ϕ and σ_η. Interestingly, the IFs for σ_η are, in general, much more in line with those of the remaining parameters, for both models.

4.1.5 Summary of simulation efficiency results

The experiments clearly illustrate that substantial gains in simulation efficiency can be achieved through simple reparameterisation of the state space models. For the empirically important regions of the parameter space for both the SCD and the SV models, the main efficiency gains are to be had by relocating both the location and scale parameters from the state equation into the measurement equation, with there being substantial reductions in the IF values in virtually all cases. The dual relocation also produces much more uniform IFs for the different parameters, for virtually all parameter settings.

[12] This figure can be deduced from the maximum IF figures reported in both Table 4 and Table 3.

Most notably, the dual relocation improves the efficiency associated with the estimation of σ_η, bringing the IFs for that parameter in line with those for the other parameters in most cases. Finally, relocation of either the location or scale parameter or both into the measurement equation tends to diminish (or eliminate) the systematic relationships that hold between the IFs and the magnitudes of the population parameters in the centred formulation.

5 Empirical Evaluation

In this section four empirical data sets are examined, with a view to ascertaining the extent to which the results in the previous section hold when the data is not artificially generated. Regressors are also reintroduced into the analysis in the case of the SCD model.

For the empirical analysis, priors reflecting those used in the literature are specified. A uniform prior over $\mathbb{R}$ and $\mathbb{R}^k$ is assumed for δ_1 and $\boldsymbol{\delta}_2$ respectively. However, in contrast to the uniform prior specified for ϕ in the simulation exercise, the prior for ϕ is now derived from a beta distribution that has been stretched over the $(-1, 1)$ interval, with density,

$$p(\phi) \propto \left\{\frac{1+\phi}{2}\right\}^{(\phi_1-1)} \left\{\frac{1-\phi}{2}\right\}^{(\phi_2-1)},$$

and with hyper-parameters ϕ_1, $\phi_2 > 0.5$. The hyperparameters ϕ_1 and ϕ_2 are set to 15 and 1.5 respectively, implying a prior mean of 0.82 and variance of 0.02 for ϕ. As before, an inverted-gamma distribution is assumed for the prior on σ_η, such that $p(\sigma_\eta) \sim IG\left(\frac{\sigma_r}{2}, \frac{S_\sigma}{2}\right)$, but with $\sigma_r = 3$ and $S_\sigma = 0.03$, implying a prior mean of 0.14 and prior variance of 0.01 for σ_η. The overall prior specification is similar to that used in several papers in the high frequency financial data literature, including Kim, Shephard and Chib (1998) and SFM.

5.1 SCD Model

5.1.1 Data

The alternative parameterisations of the SCD model are estimated using trade durations data for two Australian listed companies: Broken Hill Proprietary Limited (BHP) and News Corporation (NCP), for the month of

August 2001. Following Engle and Russell (1998) and SFM, only distinct trades between 10:20 a.m. and 4:00 p.m. are used, leaving $T = 27746$ observations for BHP and $T = 13832$ observations for NCP. The intraday pattern is modelled using a cubic smoothing spline, $g(y_t)$, that is estimated using the 'fields' package in 'R'. The adjusted durations are calculated as

$$\widehat{y_t} = \frac{y_t}{g(y_t)}. \tag{28}$$

Following SFM, trading volume is included as a regressor, with coefficient denoted by δ_2, in the state equation. In addition, and similar to Zhang, Russell and Tsay (2001), an additional regressor is defined, with coefficient denoted by δ_3, and value equal to the number of distinct trades occurring simultaneously. For both regressors, the intraday pattern is removed assuming the same type of relationship as used to adjust the duration series in (28).

5.1.2 Empirical Results

Table 5 reports estimates of the marginal posterior mean of each parameter of the SCD model, and associated IF, for each data set: BHP (Panel A) and NCP (Panel B). The algorithm for each parameterisation of the SCD model is run for 100000 iterations with a burn-in period of 20000 iterations. The (average) time taken (in seconds) for 1000 iterations is reported at the bottom of each panel. For each data set, IFs are reported for the C, NCL, NCS and NCLS parameterisations.

For each data set, the algorithm for the NCS parameterisation performs best followed closely by that for NCLS. The superior performance of the NCS parameterisation compared with the C and NCL parameterisations, certainly mimics the result in the simulation exercise, as does the overall similarity of the NCS and NCLS results. However, the fact that the dual relocation NCSL parameterisation does not improve efficiency is somewhat surprising, given the rankings obtained from the simulation experiment.[13]

[13]It is worth reiterating at this point that the simulation experiments do not incorporate regressors in the state equation.

5.2 SV model

5.2.1 Data

Estimation of the SV model is illustrated using two sets of returns data. The first data set comprises observations on the pound/dollar daily exchange rates from 1 October 1981 to 28 June 1985. This data has been used by Harvey, Ruiz and Shephard (1994), Kim, Shephard and Chib (1998) and Durbin and Koopman (2001) to illustrate their alternative estimation methodologies for the SV model. The second data set comprises observations on the Morgan Stanley Capital Index (MSCI) from 29 December 1989 to 31 May 2002. For the exchange rate series $T = 945$, while $T = 3240$ for the MSCI series.

5.2.2 Empirical Results

Table 6 reports estimates of the marginal posterior mean of each parameter of the SV model, and associated IF, for each set of data. Panel A refers to the pound/dollar exchange rate data whilst Panel B contains the output for the MSCI data set. As with the SCD model, the algorithm for each parameterisation of the SV model is run for 100000 iterations with a burn-in period of 20000 iterations. The (average) time taken (in seconds) for 1000 iterations is reported at the bottom of each panel. Results are reported for all four parameterisations for each data set.

As in the simulation study, the NCLS parameterisation clearly produces the most efficient algorithms for estimating the SV model, in both empirical settings. However, for both of these data sets, the NCS parameterisation outperforms the NCL parameterisation. Although this seems contrary to the conclusions of the simulation study, it is nevertheless consistent with the specific simulation results for the region of the parameter space that corresponds most closely to the empirical estimates.

6 Conclusion

Previous studies have documented the fact that substantial gains in the simulation efficiency of MCMC algorithms may be obtained through the use of simple reparameterisation. This paper has contributed to this literature by focussing on the impact of reparameterisation in the context of two non-Gaussian state space models that feature in empirical finance, namely the

SCD and SV models. A further contribution of the paper is the presentation of the modifications to the general algorithm of SFM required to accommodate the alternative parameterisations of the non-Gaussian state space specification.

Simulated and empirical data are used to explore the impact of parameterisation on simulation efficiency. The parameter settings used in the simulation experiment are representative of parameter estimates appearing in both the existing empirical literature and in the empirical analysis conducted herein. Four different parameterisations are examined in relation to each of the SCD and SV models, with the impact of reparameterisation measured using inefficiency factors.

For all parameterisations, the simulation results indicate that systematic patterns exist between the efficiency of simulation estimators and the true value of certain of the parameters, with these patterns being most marked for the centred parameterisation. Most importantly, the experimental results reveal that for the two models considered, and for the empirically relevant parameter ranges explored, gains in simulation efficiency are produced by moving either the location or scale parameter from the state equation to the measurement equation. For the SCD model in particular, relocation of the scale parameter produces a marked increase in efficiency. For both models, however, dual relocation of the location and scale parameters produces the most substantial gains in the majority of parameter settings, and only minimal efficiency loss in the remaining cases. The empirical results tend to mimic the simulation results, except for the fact that relocation of μ alone does not produce efficiency gains, overall, relative to the centred parameterisation. For the SCD model in particular, this result which may well be linked to the fact that regressors remain in the state equation, so that repositioning μ alone produces only a partial 'noncentred in location' parameterisation. Certainly, the main conclusion to be drawn from both the simulation and empirical results is that the dual relocation parameterisation (NCLS) is the safest choice for practitioners applying these particular non-Gaussian models to typical data sets.

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are produced using C++ run on a Pentium 4, 3.0GHz computer. The C++ code also makes use of the Template Numerical Toolkit which can be found at http://math.nist.gov/tnt/index.html. Software for the random number generator used can be found at http://www.agner.org /random/randomc.htm.

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Inefficiency Factors (IF) for the Centred (C) Parameterisation

Panel A : SCD Model					Panel B: SV Model				
True σ_η :		*0.1*	*0.2*	*0.3*	*True* σ_η :		*0.2*	*0.3*	*0.4*
True ϕ		IF			*True* ϕ		IF		
0.95	ϕ	39	11	6	*0.95*	ϕ	26	14	8
	σ_η	**221**	**84**	**44**		σ_η	**167**	**100**	**66**
	δ_1	8	3	2		δ_1	22	10	6
0.9	ϕ	115	25	13	*0.9*	ϕ	103	36	20
	σ_η	**248**	**126**	**66**		σ_η	**294**	**157**	**99**
	δ_1	17	6	4		δ_1	95	31	17
0.8	ϕ	417	115	50	*0.8*	ϕ	936	144	66
	σ_η	**988**	**285**	**159**		σ_η	**981**	**307**	**175**
	δ_1	59	19	12		δ_1	920	135	60
Time (seconds)	27				**Time** (seconds)	29			

Table 1: C parameterisation simulation results for the SCD model (Panel A) and the SV model (Panel B). The top row of each panel contains the true values (in italics) of the scale parameter, while the first column contains the true values (in italics) of the persistence parameter. The second column in each panel contains the parameters to which IFs (reported in the subsequent columns) refer, with the maximum IF for each parameter setting indicated in bold. The average time (across parameter settings) to obtain 1000 iterations is reported in the bottom row of each panel.

Inefficiency Factors (IF) for the Non-Centred in Location (NCL) Parameterisation

Panel A: SCD Model					Panel B: SV Model				
True σ_η :		*0.1*	*0.2*	*0.3*	*True* σ_η :		*0.2*	*0.3*	*0.4*
True ϕ		IF			*True* ϕ		IF		
0.95	ϕ	31	12	7	*0.95*	ϕ	22	13	9
	σ_η	**140**	**76**	**51**		σ_η	**113**	**83**	**63**
	δ_1	30	35	30		δ_1	24	17	13
	$(\Delta\%)^{(a)}$	*-37*	*-10*	*16*		$(\Delta\%)$	*-32*	*-17*	*–5*
0.9	ϕ	81	28	16	*0.9*	ϕ	66	33	18
	σ_η	**211**	**112**	**77**		σ_η	**179**	**115**	**85**
	δ_1	23	31	37		δ_1	66	34	20
	$(\Delta\%)$	*-15*	*-11*	*17*		$(\Delta\%)$	*-39*	*-27*	*-14*
0.8	ϕ	253	83	45	*0.8*	ϕ	321	101	51
	σ_η	**478**	**168**	**127**		σ_η	**390**	**168**	**113**
	δ_1	20	26	35		δ_1	323	97	50
	$(\Delta\%)$	*-52*	*-41*	*-20*		$(\Delta\%)$	*-60*	*-45*	*-35*
Time (seconds)	26				**Time** (seconds)	28			

(a) $\Delta\%$ denotes the percentage change in the maximum IF for a particular parameter setting in the NCL parameterisation, relative to the C parameterisation.

Table 2: NCL parameterisation simulation results for the SCD model (Panel A) and the SV model (Panel B). The top row of each panel contains the true values (in italics) of the scale parameter, while the first column contains the true values (in italics) of the persistence parameter. The second column in each panel contains the parameters to which IFs (reported in the subsequent columns) refer, with the maximum IF for each parameter setting indicated in bold. In addition, the percentage change in maximum IF for each parameter setting, relative to the C parameterisation, is given (in italics) in the row immediately below. The average time (across parameter settings) to obtain 1000 iterations is reported in the bottom row of each panel.

Inefficiency Factors (IF) for the Non-Centred in Scale (NCS) Parameterisation

Panel A: SCD Model					Panel B: SV Model				
True σ_η :		*0.1*	*0.2*	*0.3*	*True* σ_η :		*0.2*	*0.3*	*0.4*
True ϕ		IF			*True* ϕ		IF		
0.95	ϕ	25	11	7	*0.95*	ϕ	27	11	7
	σ_η	**45**	**48**	**72**		σ_η	**203**	**141**	**147**
	δ_1	6	3	3		δ_1	22	9	6
	$(\Delta\%)^{(a)}$	*-80*	*-43*	*64*		$(\Delta\%)$	*22*	*41*	*123*
0.9	ϕ	**70**	25	15	*0.9*	ϕ	98	33	17
	σ_η	**70**	**44**	**40**		σ_η	**374**	**148**	**91**
	δ_1	13	6	4		δ_1	87	27	14
	$(\Delta\%)$	*-72*	*-65*	*-39*		$(\Delta\%)$	*27*	*-6*	*-8*
0.8	ϕ	**189**	**72**	31	*0.8*	ϕ	**805**	148	55
	σ_η	108	70	**41**		σ_η	450	**288**	**140**
	δ_1	33	16	9		δ_1	709	130	47
	$(\Delta\%)$	*-81*	*-75*	*-74*		$(\Delta\%)$	*-18*	*-6*	*-20*
Time (seconds)	28				**Time** (seconds)	31			

(a) $\Delta\%$ denotes the percentage change in the maximum IF for a particular parameter setting in the NCS parameterisation, relative to the C parameterisation.

Table 3: NCS parameterisation simulation results for the SCD model (Panel A) and the SV model (Panel B). The top row of each panel contains the true values (in italics) of the scale parameter, while the first column contains the true values (in italics) of the persistence parameter. The second column in each panel contains the parameters to which IFs (reported in the subsequent columns) refer, with the maximum IF for each parameter setting indicated in bold. In addition, the percentage change in maximum IF for each parameter setting, relative to the C parameterisation, is given (in italics) in the row immediately below. The average time (across parameter settings) to obtain 1000 iterations is reported in the bottom row of each panel.

Inefficiency Factors (IF) of the Non-Centred in both Location and Scale (NCLS) Parameterisation

Panel A: SCD Model					Panel B: SV Model				
True σ_η :		*0.1*	*0.2*	*0.3*	*True* σ_η :		*0.2*	*0.3*	*0.4*
True ϕ		IF			*True* ϕ		IF		
0.95	ϕ	11	11	8	*0.95*	ϕ	17	12	8
	σ_η	**46**	**42**	**56**		σ_η	**36**	**42**	**43**
	δ_1	27	29	25		δ_1	18	15	10
	$(\Delta\%)^{(a)}$	*-79*	*-50*	*27*		$(\Delta\%)$	*-78*	*-58*	*-35*
0.9	ϕ	22	23	13	*0.9*	ϕ	59	25	18
	σ_η	**36**	**38**	**38**		σ_η	**60**	**39**	**34**
	δ_1	21	21	25		δ_1	58	26	18
	$(\Delta\%)$	*-85*	*-70*	*-42*		$(\Delta\%)$	*-80*	*-75*	*-66*
0.8	ϕ	**56**	**54**	32	*0.8*	ϕ	**449**	**73**	**39**
	σ_η	52	50	**39**		σ_η	60	46	35
	δ_1	23	20	23		δ_1	395	70	38
	$(\Delta\%)$	*-94*	*-81*	*-75*		$(\Delta\%)$	*-54*	*-76*	*-78*
Time (seconds)		36			**Time** (seconds)		39		

(a) $\Delta\%$ denotes the percentage change in the maximum IF for a particular parameter setting in the NCLS parameterisation, relative to the C parameterisation.

Table 4: NCLS parameterisation simulation results for the SCD model (Panel A) and the SV model (Panel B). The top row of each panel contains the true values (in italics) of the scale parameter, while the first column contains the true values (in italics) of the persistence parameter. The second column in each panel contains the parameters to which IFs (reported in the subsequent columns) refer, with the maximum IF for each parameter setting indicated in bold. In addition, the percentage change in maximum IF for each parameter setting, relative to the C parameterisation, is given (in italics) in the row immediately below. The average time (across parameter settings) to obtain 1000 iterations is reported in the bottom row of each panel.

Empirical Application of the SCD Model

Panel A: BHP

Parameter	Mean	IF			
		C	NCL	NCS	NCLS
ϕ	0.88	91	110	63	86
σ_η	0.18	**259**	**352**	**90**	**143**
δ_1	0.02	11	48	10	34
δ_2	0.00	27	26	25	25
δ_3	-0.03	62	74	34	43
Time (secs)		173	145	162	197

Panel B: NCP

Parameter	Mean	IF			
		C	NCL	NCS	NCLS
ϕ	0.57	82	83	71	75
σ_η	0.66	**138**	**140**	**81**	**87**
δ_1	-0.05	17	85	18	74
δ_2	-0.01	11	19	11	17
δ_3	-0.04	13	19	12	17
Time (secs)		86	78	90	106

Table 5: Panel A contains the SCD results for BHP, whilst Panel B contains the results for NCP. For each panel, column 1 reports the relevant parameters, column 2 reports estimated marginal posterior means, and columns 3 - 6 report IFs for the four alternative parameterisations. The maximum IF under each parameterisation is indicated in bold. The average time to obtain 1000 iterations, for each parameterisation, is reported in the bottom row of each panel.

Empirical Application of the SV Model

Panel A: Pound/Dollar

		IF			
Parameter	Mean	C	NCL	NCS	NCLS
ϕ	0.98	27	31	26	26
σ_η	0.16	**213**	**207**	**147**	**63**
δ_1	-0.02	20	40	19	35
Time (seconds)	5	5	5	6	7

Panel B: MSCI

		IF			
Parameter	Mean	C	NCL	NCS	NCLS
ϕ	0.97	28	33	31	30
σ_η	0.21	**216**	**230**	**169**	**95**
δ_1	-0.03	17	43	19	37
Time (seconds)		18	18	20	25

Table 6: Panel A contains the results for the Pound/Dollar exchange rate data, whilst Panel B corresponds to the MSCI data. For each panel, column 1 reports the relevant parameters, column 2 reports estimated marginal posterior means, and columns 3 - 6 report IFs for the four alternative parameterisations. The maximum IF under each parameterisation is indicated in bold. The average time to obtain 1000 iterations, for each parameterisation, is reported in the bottom row of each panel.