A Repayment Model of House Prices Oil Price Dynamics in a Real Business Cycle Model

Vipin Arora† and Pedro Gomis-Porqueras‡

Abstract:
We show the importance of endogenous oil prices and production in the real business cycle framework. Endogenising these variables improves the model’s predictions of business cycle statistics, oil related and non-oil related, relative to a situation where either is exogenous. This result is robust to the standard extensions (variable capacity utilisation and monopolistic competition) used in the literature. In particular, we first show that with either exogenous oil prices or production the standard real business cycle model and variants cannot match the oil-related and business cycle facts. In contrast, when both of these variables are endogenous, we can substantially improve the corresponding co-movements and slightly improve standard business cycle properties for consumption and investment.

Keywords: Oil price, two regions, variable capacity utilization
JEL Classification: E37, F47, Q43

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1 Introduction

There is a large body of work that explores the relationship between oil price changes and macroeconomic performance.\(^1\) In particular, there are many studies that use macroeconomic models to understand this relationship using a variant of the standard real business cycle model. This is because a dynamic general equilibrium framework is an ideal laboratory to analyze the links between oil and macroeconomic aggregates, as it allows for important feedback mechanisms between oil prices, oil production, and macroeconomic aggregates. As stressed by Barsky and Kilian (2004), allowing endogenous determination of this feedback within a model can help us improve our understanding of oil price dynamics. Yet macroeconomic models which have endogenous oil prices are rare, and those with both endogenous prices and production are even less common.

When endogenous oil prices or production are considered, the literature generally has not focused on their dynamics. Instead, it has studied the implications of their changes on macroeconomic aggregates. Typically these papers do not report the dynamics of oil and its co-movements with other macroeconomic observables. This makes it difficult to judge the reliability of some findings related to oil. Moreover, other papers have built on this framework to evaluate the impact of monetary policy. If the dynamics of the oil price are incorrect, the corresponding policy responses to changes in this price may be invalid. In order to provide policy prescriptions that take into account movements in oil prices, we first need to establish a framework that can account for the basic stylized facts of oil.\(^2\) This motivates our work.

Take for example the important contribution of Kim and Loungani (1992). They showed that oil price shocks could explain only a modest component of the variance of U.S. output growth in a standard real business cycle model. But their conclusions depend on the assumption that the oil price follows an exogenous process, and cannot respond to changes in macroeconomic conditions. Two other influential papers, one by Rotemberg and Woodford (1996) and one by Finn (2000), also take the price to be an exogenous process. Rotemberg and Woodford (1996) showed that with mark-up pricing the otherwise standard real business cycle framework can replicate some of the movements in U.S. GDP and wages seen after an oil price shock. Finn (2000) was able to obtain similar responses by using variable capacity utilisation in lieu of mark-up pricing. Neither of these papers reports the co-movements of the price or oil demand with macroeconomic aggregates.\(^3\)

Other papers use endogenous oil prices, but take production to be an exogenous process. In this case the price will adjust so that demand for oil matches the exogenous supply. Backus and Crucini (2000) argue that oil accounts for much of the variation in the terms of trade across a range of countries, and take the production of oil to be partially exogenous.\(^4\) They do not report the co-movements of the oil price or production with respect to macroeconomic aggregates. Bodenstein et al. (2011) use a similar model that highlights the importance of incomplete financial markets on consumption, the real interest rate, and trade balances. In their model each country has an oil endowment which follows a stochastic process. Oil price auto-correlations and variances are reported, but co-movements between oil and macroeconomic aggregates are not.\(^5\)

\(^1\)See Brown and Yucel (2002), Hamilton (2008), Jones et al. (2004), and Kilian (2008) for overviews.
\(^2\)This point is also made by Kilian and Lewis (2009).
\(^3\)See also Aguiar-Conraria and Wen (2007), Atkeson and Kehoe (1999), and Dhawan and Jeske (2008) for examples of other papers which take the oil price as an exogenous process.
\(^4\)OPEC supply is the exogenous component, while non-OPEC supply is determined in the model.
\(^5\)See also Hirakata and Sudo (2009) and Unalmis et al. (2009) for examples of other models which use an exogenous
Several other papers have analyzed monetary policies in an environment where oil is explicitly considered and taken to be exogenous. For example Blanchard and Gali (2007) use a New-Keynesian model to examine the reasons behind the relatively mild effects of recent oil price shocks on inflation and output. They find that luck, smaller oil shares in production, more flexible labour markets, and improvements in monetary policy all played a role. But their oil price is an exogenous process and no co-movements are reported. Leduc and Sill (2004) also use an exogenous oil price process in finding that monetary policy helps to exacerbate the negative impact of oil price rises. But no co-movements are reported in this case either.\footnote{See also Blanchard and Riggi (2009), Carlstrom and Fuerst (2006), and Kormilitsina (2011) for examples of similar papers. Both Campolmi (2008) and Leduc and Sill (2007) use an exogenous oil production process to study monetary policy.}

Only a select number of studies endogenise both the oil price and production, but the dynamics of either variable usually go unreported. Nakov and Pescatori (2009) argue that oil played an important role in the Great Moderation, specifically in helping to reduce the volatility of both inflation and GDP growth in the US. Although oil price volatility is reported, there is no information on co-movements or oil production. Using the same benchmark model, Nakov and Pescatori (2010) find that the presence of a non-competitive oil producer results in sizeable costs for the US. In this case, no statistics on the price or production are provided. Finally, Balke et al. (2010) use a multi-region model with oil production and price both endogenous. They find that changes in the oil price are best understood as endogenous, and that oil price shocks have differing impacts depending on their source. No statistics on the price or production are reported here either.

This paper explicitly considers the dynamics of the oil price, oil production, and their macroeconomic determinants. We first take a standard real business cycle model with both the oil price and oil production exogenous, and compare its business cycle properties against data. Oil is an intermediate input in final goods production, and is not demanded directly by consumers. The driving process in the model is a productivity shock on final goods production. Keeping either the oil price or production exogenous, we then consider variable capacity utilisation as in Finn (2000), or market power as in Rotemberg and Woodford (1996). These two extensions are chosen because they are often used in conjunction with an exogenous price or production.\footnote{See for example Aguiar-Conraria and Wen (2007), Blanchard and Gali (2007), Blanchard and Riggi (2009), Campolmi (2008), Carlstrom and Fuerst (2006), Kormilitsina (2011), Leduc and Sill (2004), Leduc and Sill (2007), and Unalmis et al. (2009).}

We then extend the benchmark real business cycle model to include an oil exporting region so that both the oil price and production are endogenous. Oil is still an intermediate good into final production, but it must be imported from the other region. The model has a technology shock on final goods production (an oil demand shock), and a technology shock on oil production (an oil supply shock) which jointly drive the dynamics of the economic environment. We then re-evaluate the model while considering perfect competition, variable capacity utilisation, and market power in the oil consuming region. Our framework does not incorporate exhaustibility in oil.\footnote{In the short-run this is unlikely to be a major issue, but will change dynamics in the long-run (Mabro, 1998).} It also abstracts from storage of oil in inventories. We do so to focus on medium-term price dynamics, as storage has the largest impact in the short-run (Pindyck, 2001). Additionally, the only financial asset in this model is capital, which cannot be traded between regions.\footnote{This assumption simplifies the presentation and focuses the analysis on production and trade. See Arora (2011) for a model where the asset characteristics of oil are considered.}
The models with either the oil price or oil production exogenous do not account for observed co-movements well, and the results are similar irrespective of the extensions chosen. When the oil price is exogenous, the simulated correlation between the oil price and final goods output has the wrong sign. The relative volatility of the oil price with respect to final goods production only accounts for 22% of the data. Making production exogenous gives the correct sign between the oil price and final goods production, but it is double the actual value. The relative volatilities are even lower than when the price is exogenous, accounting for roughly 15% of the data. This result is robust to incorporating variable capacity utilisation or monopolistic competition. In contrast, endogenising the oil price and production improves the oil-related statistics. In particular, the different variants can account for roughly 70% of the correlation between the oil price and final goods production. However, the relative volatilities in each are too low, accounting for about 22% of the observed values. These findings are robust to a variety of different models, including the addition of non-contingent bonds, habit persistence in consumption, direct consumption of oil, variable capacity utilisation in the oil producing region, monopolistic competition in the oil producing region, and changing the calibrated capital to output ratio.

2 The Model

The simulations use different variants of an open economy real business cycle model, extended to include variable capacity utilisation and monopolistic competition. In the initial simulations, either the oil price or oil production are exogenous, whereas both are endogenous in the second set of simulations. In this section we present the most general case, which encompasses the other variations used in simulations. There is an oil producing and an oil consuming region, both with a representative consumer that owns the capital stock. The oil producing region has a final oil producer that exports oil to the consuming region, where it is used to produce intermediate goods. The intermediate goods producers are monopolistically competitive, and each can vary the rate at which they use capital in production. Each intermediate goods producer sells their output to a perfectly competitive final goods producer.

There are two shocks: the standard real business cycle technology shock on final production in the consuming region, and a technology shock on oil production. These processes induce a stochastic event, \( s_t \), in each period \( t \). There are finitely many possible events, and the history of events up to and including \( t \) is denoted by \( s^t = (s_o, s_1, ..., s_t) \). The initial realisation, \( s_o \), is known. The probability at period 0 of any history \( s^t \) is \( \chi(s^t) \). All equilibrium prices and allocations are a function of these histories, but the dependence will be suppressed throughout the paper for simplicity. Variables specific to the oil producing region are denoted with (*)

2.1 Consumers

Consumers in either region solve similar problems. In the oil consuming region they choose allocations of consumption (\( C_t \)), labour to supply (\( N_t \)), capital stock for the next period (\( K_{t+1} \)), and the capacity utilisation rate (\( T_t \)) to maximise expected utility:
\[
\max_{\{C_t, N_t, K_{t+1}, T_t\}} \mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \frac{\beta^t C_t^{1-\sigma_c}}{1-\sigma_c} + \frac{\xi_0 N_t^{1-\xi}}{1-\xi} \right\}
\]

(1)

each period this is subject to:

\[
C_t + I_t = w_t N_t + r_t T_t K_t + \pi_t
\]

(2)

and a capital accumulation equation:

\[
I_t = K_{t+1} - (1 - \delta_t) K_t
\]

(3)

where \(I_t\) is investment, \(w_t\) is the wage rate, \(r_t\) is the return to capital, \(\pi_t\) is firm profits, \(\beta\) is the discount factor, \(\delta_t\) the depreciation rate of capital, \(\sigma_c\) the coefficient of relative risk aversion (CRRA), \(\xi\) is a parameter which determines the labour supply elasticity, and \(\xi_0\) is a parameter which determines labour supply. Due to variable capacity utilisation, the depreciation rate is a function of the capital utilisation rate:

\[
\delta_t = \delta T_t^\eta
\]

(4)

with \(\delta\) a constant, and \(\eta\) \((1 < \eta)\) a utilisation parameter. This function is convex in the utilisation rate: an increase in utilisation raises depreciation, and successive increases raise depreciation by a larger and larger increment. Consumers in the oil producing region solve an identical problem, save they do not choose a capacity utilisation rate.

### 2.2 Firms

In the oil consuming region, there are a continuum of intermediate goods producing firms indexed by \(i \in [0, 1]\) that behave as imperfect competitors. The firms produce differentiated types of intermediate goods \([Y_t(i)]\) by choosing oil \([Q_t(i)]\), capital, and labour to maximise profits:

\[
\max_{Q_t(i), K_t(i), N_t(i)} p_{y,t}(i) Y_t(i) - p_{q,t} Q_t(i) - r_t T_t K_t(i) - w_t N_t(i)
\]

(5)

This is subject to a Cobb-Douglas production technology, with \(\psi\) \((0 < \psi < 1)\) the oil share in production:

\[
Y_t(i) = Z_t J_t(i)^\psi N_t(i)^{1-\psi}
\]

(6)

where \(p_{y,t}(i)\) is the price of a firm’s good, \(p_{q,t}\) is the price of oil, \(Z_t\) is an exogenous (aggregate) total factor productivity (TFP) shock, and \(J_t(i)\) is capital services. These capital services are a constant elasticity of substitution (CES) composite of the capital stock and oil:

\[
J_t(i) = [\gamma Q_t(i)^\tau + (1 - \gamma) [T_t(i) K_t(i)]^\tau]^{\frac{1}{\tau}}
\]

(7)
where $\gamma$ is an oil share parameter, and $\tau = \frac{(\sigma_{qk}-1)}{\sigma_{qk}}$, with $\sigma_{qk}$ the elasticity of substitution between oil and capital.\(^{10}\)

The total factor productivity shock evolves according to:

$$\ln Z_t = \rho \ln Z_{t-1} + \epsilon_t$$

where $\rho$ is the persistence. The innovation $\epsilon_t \sim i.i.d N(0,\sigma^2_v)$, and $\sigma_v$ is the respective standard deviation. The final goods producing firm behaves competitively and chooses inputs of these different intermediate types each period to maximise profits:

$$\max \int Y_t(i)di - \int p_{yt}(i)Y_t(i)di$$

subject to:

$$Y_t = \left[ \int Y_t(i)^\theta di \right]^\frac{1}{\theta}$$

In the oil producing region, the representative firm chooses capital and labour to maximise profits:

$$\max_{K_t^*,N_t^*} p_{q,t}Q_t - r^*_t K_t^* - w^*_t N_t^*$$

subject to:

$$Q_t = Z_t^* K_t^{*\alpha} N_t^{*(1-\alpha)}$$

where $\alpha (0 < \alpha < 1)$ is the capital share in production. $Z_t^*$ follows the same process as $Z_t$, but may have different a persistence and shock volatility.

### 2.3 Optimality and Equilibrium

The consumer problems in both regions yield the following optimality conditions:

$$\frac{1}{C_t} = \mathbb{E}_t \left\{ \frac{\beta}{C_{t+1}} \left[ \frac{\beta}{C_{t+1}^*} (r_{t+1} T_{t+1} + (1-\delta_t)) \right] \right\}$$

$$\frac{1}{C_t^*} = \mathbb{E}_t \left\{ \frac{\beta^*}{C_{t+1}^*} \left[ \frac{\beta^*}{C_{t+1}^*} (r^*_{t+1} T^*_{t+1} + (1-\delta^*)) \right] \right\}$$

$$w_t = \frac{\xi_0 C_t}{(1-N_t)^\xi}$$

---

\(^{10}\)The assumption taken here is that oil and capital are complements, and this is standard in the current framework as outlined in Kim and Loungani (1992). Apostolakis (1990) provides some evidence in support of this assumption.
The first two state that any reduction of consumption today (in marginal utility terms) which is used for investment must be equal to the discounted gain from the return on that investment (in marginal utility terms). The third and fourth equations show that at an optimum the wage equals the marginal rate of substitution between labour and consumption. The fifth equates the benefit from utilising an additional unit of capital (its rate of return) to the cost (a higher depreciation rate).

In equilibrium (defined below), the intermediate goods producers demand the same oil, capital, and labour; charge the same price; and produce the same level. This implies that $Y_t(i)=Y_t$, $Q_t(i)=Q_t$, $K_t(i)=K_t$, and $N_t(i)=N_t$, yielding three optimality conditions:

$$r_t = \eta \delta T_t^{\eta-1}$$ (17)

$$w_t = \frac{\xi^*_0 C_t^*}{(1 - N_t^*)^\xi^*}$$ (16)

These equate the factor payments to value of their marginal products with one difference. Due to monopolistic competition, there is a markup ($\frac{1}{\theta}$) associated with the price of final goods. This is reflected in the optimality conditions by changing the marginal products of all factors so they are below what they would be in the perfectly competitive case ($\theta=1$). Optimality conditions for the oil producer are standard, factor payments are equated to the values of their marginal products:

$$r_t^* = \alpha p_{q,t} Q_t$$ (21)

$$w_t^* = \frac{(1 - \alpha) p_{q,t} Q_t}{N_t^*}$$ (22)

The model uses a symmetric equilibrium, implying that all intermediate goods producers make identical decisions. In particular, the equilibrium for this economy is a process of prices $\{p_{y,t}, p_{q,t}, r_t, r_t^*, w_t, w_t^*\}_{t=0}^{\infty}$, a process of allocations $\{C_t, C_t^*, N_t, N_t^*, I_t, I_t^*, K_t, K_t^*, J_t, Q_t, Y_t\}_{t=0}^{\infty}$, a process of utilisation rates $\{T_t\}_{t=0}^{\infty}$, and exogenous technology processes $\{Z_t, Z_t^*\}_{t=0}^{\infty}$ such that (i) taking prices as given, consumers solve (1); (ii) taking all prices save their own as given, each intermediate goods producer solves (5); (iii) taking all prices as given, the final goods producer solves (9); (iv) taking all prices as given the oil producer solves (11); (v) the oil, final goods, and labour markets clear; and (vi) each consumer’s budget constraint is met.
2.4 Data and Calibration

Quarterly real data for the period 1984-2010 on U.S. GDP, consumption, investment, imported oil prices, and non-US oil production are used to assess the model. The first three data sets are taken from the Federal Reserve Economic Database (FRED) GDP and components section, the oil price is U.S. imported refiner acquisition costs, and this comes from the U.S. Energy Information Administration (EIA) along with the non-U.S. production data. Following Stock and Watson (1999), all time series are H-P filtered with a smoothing parameter of 1600, and are in logs.

The first three rows of Table 1 summarise standard business cycle statistics. Consumption ($C_t$) is less volatile than U.S. GDP ($Y_t$) over this time horizon, and the correlation between the two is over 87%. Investment ($I_t$) has a stronger correlation with output, at just under 90%, and is over five times more volatile than GDP.

The fourth row of Table 1 shows there has been a positive correlation between the imported oil price and U.S. GDP, at just under 40%. Unsurprisingly, the oil price is very volatile when compared with GDP, with a relative standard deviation over 18 times as large. The final row shows that non-U.S. oil production also has a positive correlation with U.S. GDP, at just under 35%. It has also been 1.73 times as volatile as GDP as well.

The model parameter values are chosen using a calibration procedure. The CRRA parameters ($\sigma_c$ and $\sigma_{c^*}$), discount factors ($\beta$ and $\beta^*$), and capital shares ($\psi$ and $\alpha$) in both regions are given standard values for a quarterly calibration of 2, 0.99, and 0.36. The depreciation rate in the oil producing region ($\delta^*$) is also given the standard value of 0.025. The depreciation rate in the oil consuming region ($\delta$) is chosen so that the steady-state capital to output ratio is 12 as in Dhawan and Jeske (2008). This capital to output ratio is consistent across all different versions considered in the simulations below.

The oil share parameter in capital services ($\gamma$) is chosen so that the share of oil in final goods output matches the average U.S. value of oil imports during the sample period of 0.015, consistent with Aguiar-Conraria and Wen (2007). The parameters $\xi$ and $\xi^*$, are set at 10, as in Bodenstein et al. (2011), which imply a Frisch elasticity of 0.2. The other parameters on labour, $\tau_0$ and $\tau_0^*$ are chosen so that labour supply in either region is 0.33 of available time. When capacity utilisation is added to the model, $\eta$ is chosen so that the steady state capacity utilisation rate is 0.80, which is the U.S. average over the sample period. In the extension with monopolistic competition, $\theta$ is

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Corr($X_t, Y_t$)</th>
<th>SD</th>
<th>$SD_x/SD_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_t$</td>
<td>GDP</td>
<td>1.00</td>
<td>0.011</td>
<td>1.00</td>
</tr>
<tr>
<td>$C_t$</td>
<td>Consumption</td>
<td>0.871</td>
<td>0.009</td>
<td>0.808</td>
</tr>
<tr>
<td>$I_t$</td>
<td>Investment</td>
<td>0.894</td>
<td>0.059</td>
<td>5.36</td>
</tr>
<tr>
<td>$p_{q,t}$</td>
<td>Oil Price</td>
<td>0.394</td>
<td>0.201</td>
<td>18.27</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>Non-US Oil Production</td>
<td>0.344</td>
<td>0.019</td>
<td>1.73</td>
</tr>
</tbody>
</table>

Table 1: Summary Statistics 1984-2010

Additional details on data collection and manipulation can be found in Appendix 1.

SD is standard deviation.
chosen to be 0.9, as in Chari et al. (2000).

A key parameter in the model is the elasticity of substitution between oil and capital ($\sigma_{qk}$) in the consuming region. The initial value taken for this parameter is 0.40, which comes from the estimation results in Bodenstein et al. (2011). This is slightly higher than is used in both Backus and Crucini (2000) and Balke et al. (2010), but in the range of estimates given by Miyazawa (2009). The persistence and volatility on the oil consuming region TFP process are standard U.S. values of 0.979 and 0.007, as in King and Rebelo (1999). In the oil producing region, the persistence used is also 0.979, but the volatility is chosen so that the relative volatility of oil production with respect to final goods output matches the relative volatility of non-U.S. oil production to U.S. GDP.

3 Simulations

We first consider a situation where either the oil price or production are exogenous and compare the results with data. This includes a benchmark perfectly competitive case, and two common additions: variable capacity utilisation and monopolistic competition. These two particular extensions are chosen because they are frequently used in papers which model oil (or energy) as an input to a production process. Rotemberg and Woodford (1996) showed that in the case of market power, the current framework can replicate some of the movements in U.S. GDP and wages seen after an oil price shock. Finn (2000) was able to obtain similar responses by using variable capacity utilisation instead of market power.

We then analyse the case where both the oil price and production are endogenously determined in the model and compare the results to data. The importance of endogenously modeling oil has been emphasised by Kilian (2009) and Kilian and Lewis (2009). Perfect competition, variable capacity utilisation, and monopolistic competition in the oil consuming region are also considered here. This allows us to gauge any impact these extensions may have on the fit of the model.

3.1 Exogenous Prices

In order to allow for exogenous oil prices, we assume that the oil price follows an AR1 process and that the oil produced is exactly what the oil consuming region demands. This structure is standard in the literature (see e.g. Leduc and Sill (2004)). In what follows we take the persistence and standard deviation of the innovations from the imported oil price series referenced above (0.678 and 0.145 respectively). Table 2 compares all three variants of the model with exogenous oil prices to the relevant U.S. statistics. In the table, PC denotes perfect competition, VCU stands for variable capacity utilisation, and MC represents monopolistic competition.

Consider first the perfectly competitive case. The business cycle variables ($C_t$ and $I_t$) fit the data in-line with standard real business cycle models (King and Rebelo, 1999). The problem is in the simulated correlation between the oil price and final goods output, which has the wrong sign. The correlation between oil production and final goods output is also too high. The relative volatility of oil production, however, accounts for the data much better. But the relative volatility of the oil price is far too low.

These general observations hold in both the variable capacity utilisation and monopolistic competition cases. Monopolistic competition is very close to the perfectly competitive case, indicating
that the markup is not having much of an impact in these simulations. Variable capacity utilisation fares even worse than the perfectly competitive case with respect to the correlation between the oil price and final goods output. It also has a slightly lower relative volatility of the oil price to final goods production.

The primary concern with these results is that the correlation between the oil price and final goods output has the wrong sign. Why is this the case? Consider a rise in the oil price. Equation (18) can be rearranged to show that oil demand must fall:

\[
Q_t = \left( \frac{p_{q,t} J_t}{\theta \psi \gamma Y_t} \right)^{-\frac{1}{\rho-1}}
\]  

(23)

It follows that capital services will decrease (capital and oil are complements), so that output will fall. Of course this negative correlation between oil prices and final goods output may be offset somewhat by follow-on impacts, but this can only work through \( Y_t \) because \( p_{q,t} \) is exogenous. In other words, the reason why the oil price rose in the first place is not captured here, and it cannot offset the negative impact of the oil price shock on demand to better match the data.

3.2 Exogenous Production

In this new environment oil demand (or non-U.S. production) is assumed to follow an AR1 process. As in the sample data, the persistence of this process is 0.587 and the innovations have a standard deviation of 0.016. The oil price becomes endogenous here, and adjusts to the appropriate level given the exogenously specified demand. Table 3 reports all three variants of the model with exogenous oil production.

<table>
<thead>
<tr>
<th></th>
<th>( C_t )</th>
<th>( I_t )</th>
<th>( p_{q,t} )</th>
<th>( Q_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.871</td>
<td>0.894</td>
<td>0.394</td>
<td>0.344</td>
</tr>
<tr>
<td>PC</td>
<td>0.973</td>
<td>0.932</td>
<td>-0.031</td>
<td>0.529</td>
</tr>
<tr>
<td>VCU</td>
<td>0.907</td>
<td>0.878</td>
<td>-0.071</td>
<td>0.571</td>
</tr>
<tr>
<td>MC</td>
<td>0.973</td>
<td>0.911</td>
<td>-0.035</td>
<td>0.536</td>
</tr>
</tbody>
</table>

(a) Correlations with \( Y_t \)

<table>
<thead>
<tr>
<th></th>
<th>( C_t )</th>
<th>( I_t )</th>
<th>( p_{q,t} )</th>
<th>( Q_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.808</td>
<td>5.36</td>
<td>18.27</td>
<td>1.73</td>
</tr>
<tr>
<td>PC</td>
<td>0.801</td>
<td>1.88</td>
<td>4.16</td>
<td>2.04</td>
</tr>
<tr>
<td>VCU</td>
<td>0.757</td>
<td>2.37</td>
<td>3.92</td>
<td>2.00</td>
</tr>
<tr>
<td>MC</td>
<td>0.808</td>
<td>2.10</td>
<td>4.14</td>
<td>2.05</td>
</tr>
</tbody>
</table>

(b) Standard Deviations Relative to \( Y_t \)

Table 3: Simulations with Exogenous Oil Production

Begin again with the perfectly competitive case. As before, the standard business cycle variables account for the data reasonably well. But now the correlation between the oil price and final goods...
output is more than twice the value observed over the sample period. In addition, oil production and final goods output have effectively no correlation in the simulations. The relative volatilities seem to be worse than when prices are exogenous. The relative volatility of the oil price is 2.76, accounting for only 15% of the data. The relative volatility of oil production overshoots substantially.

As before, the general results hold in both variants. Monopolistic competition has results quite close to the perfectly competitive case. Variable capacity utilisation is also close, but has a slightly higher correlation between the oil price and final goods output, and gives a slightly lower relative volatility. What is driving these results? Consider this time a rise in demand (or production). Equation (18) shows that the oil price must fall:

\[ p_{q,t} = \frac{\theta \omega \gamma Q_t^{\rho-1} Y_t}{J_t^\rho} \]  

(24)

But the rise in oil demand will translate to higher capital services and higher output. Equation (18) shows that both of these rises will push up the price of oil, and this can explain the strong correlation between the price and final goods output. But because demand is unable to adjust, it has little correlation with final goods output, as the rise in the oil price does not change demand.

### 3.3 All Endogenous

In this new environment we consider a situation where the oil producing region takes into account the decisions of households and firms in the oil consuming region. The structure of the both regions is as described above, and the calibration is slightly changed. The innovations of the TFP process in the oil producing region are chosen so that the relative volatility of oil production to final goods production matches the data. Table 4 compares all three variants of the model with both the oil price and production endogenous.

<table>
<thead>
<tr>
<th></th>
<th>C_t</th>
<th>I_t</th>
<th>p_{q,t}</th>
<th>Q_t</th>
<th>Correlations with Y_t</th>
<th>Standard Deviations Relative to Y_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.871</td>
<td>0.894</td>
<td>0.394</td>
<td>0.344</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PC</td>
<td>0.973</td>
<td>0.935</td>
<td>0.271</td>
<td>0.331</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VCU</td>
<td>0.909</td>
<td>0.880</td>
<td>0.241</td>
<td>0.364</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MC</td>
<td>0.972</td>
<td>0.919</td>
<td>0.274</td>
<td>0.331</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C_t</td>
<td>I_t</td>
<td>p_{q,t}</td>
<td>Q_t</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>0.808</td>
<td>5.36</td>
<td>18.27</td>
<td>1.73</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PC</td>
<td>0.795</td>
<td>1.85</td>
<td>3.98</td>
<td>1.73</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VCU</td>
<td>0.756</td>
<td>2.30</td>
<td>3.90</td>
<td>1.73</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MC</td>
<td>0.801</td>
<td>2.03</td>
<td>3.98</td>
<td>1.73</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Simulations with Both Endogenous

The perfectly competitive case shows that the relevant statistics for consumption and investment are no worse than they were with either the oil price or production exogenous. But the correlations of the oil-related variables improve dramatically. The correlation between the oil price and final goods output now accounts for almost 70% of the data, and that between oil production and final goods output accounts for over 96% of the data. The relative volatility between the oil price and final goods output is still far too low. In this case it can only account for just under 22% of the data.

As before both variants give similar results to the perfectly competitive case. For the statistics of interest, monopolistic competition is virtually identical to perfect competition, indicating that
the mark-up is not very important for these particular statistics. Variable capacity utilisation has a slightly lower correlation between the oil price and final goods output, and a slightly higher correlation between oil production and final goods output. The relative volatility of the oil price in this case is too low as well at 3.90.

Endogenising both the oil price and production improves the correlations dramatically because the cause of the change in either variable is captured. Consider again a rise in the oil price. In this case, however, we must also specify why the oil price rises. In the model it can be due to a positive TFP shock in the oil consuming region or a positive TFP shock in the producing region. Suppose this rise is driven by the positive TFP shock in the consuming region. As described above, this will reduce demand for oil, capital services, and final goods production, all else equal. But all else is not equal. In particular, the rise in TFP has increased the returns to each factor of production, and therefore income in both regions is higher. This leads to greater demand for final goods, which can lead to a positive correlation between oil and final goods output. This is the point made by Kilian (2009).

The same logic holds when there is a positive TFP shock in the oil producing region. This raises the oil price because the marginal costs of the producer have risen. We would expect oil demand to fall, along with capital services and final goods production. There is an off-setting income effect in this case as well. The oil producer now has greater income and will demand more final goods from the oil consumer, raising final goods production. This point has been surveyed by Brown and Yucel (2002). In both cases the eventual direction of correlation depends on the specific calibration, but the reason why the oil price changes has important consequences for simulation statistics.

The puzzle of why the relative volatilities of the oil price with respect to final goods output are so low compared with the data remains. This conclusion holds over a variety of different models in addition to the ones reported here. This question has been addressed by Arora (2011), who shows that modeling the asset characteristics of oil in the ground can lead to improvements in matching observed oil price volatilities. This occurs because the oil producer now responds to changes in interest rates (the asset channel) in addition to changes in demand (the production channel). This can lead to a higher volatility in the price of oil.

A similar approach to the one taken by Arora (2011) is put forth in Balke et al. (2010). They also model oil’s asset characteristics, but incorporate additions as well as extraction of oil reserves. This allows them to find a deterministic steady state for the model and approximate a solution using standard linear techniques. An alternative approach to these supply-side responses is taken by Nakov and Pescatori (2009) and Nakov and Pescatori (2010), where they model the oil producer as a dominant supplier among many other fringe producers. This technique may generate better responses than the monopolistic competition used as a robustness check in this paper.

4 Conclusion

This paper explicitly considers the dynamics of the oil price, oil production, and their macroeconomic determinants. We first take a standard real business cycle model with both the oil price

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14This includes the addition of non-contingent bonds in both regions, habit persistence in consumption in both regions, direct consumption of oil, variable capacity utilisation in the oil producing region, monopolistic competition in the oil producing region, and varying the calibration of the model.
and oil production exogenous, and compare its business cycle properties against data. Oil is an intermediate input in final goods production, and is not demanded directly by consumers. Keeping either the oil price or production exogenous, we then consider variable capacity utilisation and monopolistic competition. We then extend the benchmark real business cycle model to include an oil exporting region so that both the oil price and production are endogenous. Oil is still an intermediate good into final production, but it must be imported from the other region. We then re-evaluate the model while considering perfect competition, variable capacity utilisation, and market power in the oil consuming region.

We show that when oil prices are exogenous, the model delivers counterfactual oil predictions at business cycle frequencies. Once oil is endogenous, the model can account for most of oil’s business cycle properties while being consistent with the standard business cycle facts. These results are robust across commonly used model variations in the literature. Our findings suggest that any framework used for providing policy analysis that takes into account movements in oil prices should endogenise both its price and production as the relative price dynamics behave quite differently when oil is treated exogenously.

5 References


Appendix 1: Data and Model Parameter Values

Data

Data on real GDP, real consumption, and real investment are taken from the Federal Reserve Bank of St. Louis’s Federal Reserve Economic Data (FRED) database at a quarterly frequency. Each of these series are in billions of chained 2005 dollars. Consumption data is personal consumption expenditures and investment data is real fixed private investment. The logarithm of these real series are H-P filtered, with a smoothing parameter of 1600, to extract the cyclical components. The reported standard deviations and correlations are based on these cyclical series.

Oil price data is taken from the U.S. Energy Information Administration (EIA) (imported) refiner acquisition costs at a monthly frequency. This nominal series is deflated by the U.S. Consumer Price Index (CPI) less energy, which is also taken from FRED. The level of the oil price is taken to be the logarithm of these values. The monthly values are aggregated to quarterly by taking an average, these series are H-P filtered with a smoothing parameter of 1600, and the reported
statistics are based on the cyclical series. Non-U.S. oil production data is taken from the EIA’s international energy statistics section on petroleum production at a monthly frequency. An average aggregates these values to quarterly, the logarithm of this series is H-P filtered with a smoothing parameter of 1600, and the reported statistics are based on the cyclical series.

Parameter Values

Table 5 summarises parameter values for the full simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Oil Consumer</th>
<th>Oil Producer</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>CRRA</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation</td>
<td>varies</td>
<td>0.025</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Labour supply parameter</td>
<td>10.0</td>
<td>10.0</td>
</tr>
<tr>
<td>$\xi_0$</td>
<td>Labour supply parameter</td>
<td>varies</td>
<td>varies</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share of oil prod</td>
<td>-</td>
<td>0.36</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Capital services share of final goods prod</td>
<td>0.36</td>
<td>-</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Oil share of final goods prod</td>
<td>varies</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_{qk}$</td>
<td>Elasticity of substitution, oil and capital</td>
<td>0.40</td>
<td>-</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Capital utilisation parameter</td>
<td>varies</td>
<td>-</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Persistence of technology shock</td>
<td>0.979</td>
<td>0.979</td>
</tr>
<tr>
<td>$\rho_{po}$</td>
<td>Persistence of exogenous oil process</td>
<td>-</td>
<td>0.678</td>
</tr>
<tr>
<td>$\rho_q$</td>
<td>Persistence of exogenous oil prod</td>
<td>-</td>
<td>0.579</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>Volatility of technology shock</td>
<td>0.007</td>
<td>varies</td>
</tr>
<tr>
<td>$\sigma_{v,po}$</td>
<td>Volatility of exogenous oil price shock</td>
<td>0.145</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_{v,q}$</td>
<td>Volatility of exogenous oil production shock</td>
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<td>-</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$\frac{1}{\text{markup}}$</td>
<td>0.90</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 5: Model Parameter Values

Appendix 2: Sensitivity

Each of the parameters in the model, and the capital-to-output ratio in the oil consuming region, were varied to gauge the sensitivity of results. The most important parameter is the elasticity of substitution between oil and capital in final goods production, $\sigma_{qk}$.$^{15}$ Table 6 shows simulations results for a sensitivity analysis on the value of $\sigma_{qk}$.

These results highlight a trade-off in the model based on $\sigma_{qk}$. The less substitutable are oil and capital (lower $\sigma_{qk}$), the higher the relative volatility of the oil price with respect to final goods output. But a lower value of $\sigma_{qk}$ also leads to less correlation between the oil price and final goods output. As $\sigma_{qk}$ falls, the correlation between oil production and final goods output also increases.

$^{15}$The volatility of the innovation on TFP in the oil producing region is another important input. When this is altered, the changes in results depend on the value of $\sigma_{qk}$. 

16
<table>
<thead>
<tr>
<th>( \sigma_{qk} )</th>
<th>( C_t )</th>
<th>( I_t )</th>
<th>( p_{q,t} )</th>
<th>( Q_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{qk}=0.20 )</td>
<td>0.973</td>
<td>0.922</td>
<td>0.177</td>
<td>0.435</td>
</tr>
<tr>
<td>( \sigma_{qk}=0.40 )</td>
<td>0.973</td>
<td>0.935</td>
<td>0.271</td>
<td>0.331</td>
</tr>
<tr>
<td>( \sigma_{qk}=0.60 )</td>
<td>0.973</td>
<td>0.938</td>
<td>0.314</td>
<td>0.273</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \sigma_{qk} )</th>
<th>( C_t )</th>
<th>( I_t )</th>
<th>( p_{q,t} )</th>
<th>( Q_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{qk}=0.20 )</td>
<td>0.793</td>
<td>1.86</td>
<td>7.03</td>
<td>1.73</td>
</tr>
<tr>
<td>( \sigma_{qk}=0.40 )</td>
<td>0.795</td>
<td>1.85</td>
<td>3.98</td>
<td>1.73</td>
</tr>
<tr>
<td>( \sigma_{qk}=0.60 )</td>
<td>0.797</td>
<td>1.86</td>
<td>2.82</td>
<td>1.73</td>
</tr>
</tbody>
</table>

(a) Correlations with \( Y_t \)
(b) Standard Deviations Relative to \( Y_t \)

Table 6: Simulation Results: Vary \( \sigma_{qk} \), 1984-2010