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The Cantillon Effect of Money Injection through Deficit Spending

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Abstract

This paper develops a simple dynamic model to study some of the implications of Cantillon's insight that new money enters an economy at a specific point and that it takes time for the new money to permeate the economy. It applies a process analysis and uses numerical simulations to map out how the economy changes from one period to the next following a money injection. It finds that, within the region of stability, a money injection can generate oscillating changes in real variables for a considerably long period of time before converging back to the initial steady state. It also finds that a money injection benefits first recipients of the new money, but hurts later recipients and savers. Our simulation suggests that in our model savers can lose from a money injection even if they are first recipients of the new money.

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1. Introduction

The arithmetic of the government's budget constraint dictates that fiscal policy and monetary policy are inescapably interdependent (Sargent, 1999). With balanced budgets, fiscal policy redistributes money; with monetised deficit spending, it has an additional effect of increasing the money supply. Thus Boulding (1962) argues that monetary policy has to do with the regulation of financial markets, while fiscal policy has to do with the regulation of the quantity of money. The purpose of this paper is to study the effects on an economy of an increase in money supply as a result of deficit spending. We focus on the impact of the money injection rather than the spending per se, and investigate the dynamics of the economic system following the money injection. Specifically we attempt to answer the following questions: how do relative prices and the structural composition of the economy change in response to a money injection? Are "short-run" responses different from "long-run" ones? How does the economy go from the "short run" to the "long run"? How long is the "long run"? And what are the welfare implications of the money injection?

We approach these questions by building a simple dynamic model that captures Cantillon's (1755) insight, namely that new money enters an economy at a specific point and that it takes time for the new money to permeate the economy. Since new money does not reach everyone at the same time, the injection of money increases the purchasing power of those who receive the new money first, enabling them to bid resources away from those who receive that money at a later time. As a result, relative prices will change, resources will be reallocated and income will be redistributed during the time interval between money injection and its final permeation in the economy. These changes are referred to as the "Cantillon effect".

Our model studies the Cantillon effect of deficit spending. It has a number of distinctive features:

1. The model assumes that government spending is not funded by tax and takes the form of a handout given to one of two groups in the economy, savers and workers. It further assumes that following the handout, goods prices immediately adjusts but wage response is delayed by one period. This assumption allows us to capture the Cantillon effect as it implies that the new money does not reach the two groups at the same time.
2. There are three production sectors (two final-good sectors and one intermediate-good sector) and a banking sector in the economy. This setup allows us to trace how money flows through different sectors and to study how relative prices respond to a money injection.
3. Consistent to the assumption that it takes time for money to permeate the economy, we also assume that production takes time. This implies that intermediate goods used in production at time t are produced at time $t-1$, which in turn implies that any real adjustment in the final good sector is constrained by the availability of the intermediate good in that period.
4. We conduct a process analysis. Starting from the steady state, we look at how the economy responds to a money injection one period after another. We present analytical solutions for key variables of the economy and use numerical simulations to illustrate how the key variables behave over time.

5. In our numerical simulations we first identify parameter regions within which the dynamic system is stable, and then conduct simulations to illustrate features of the system dynamics within the stable region.

We find that following a government handout which injects money to the economy, the relative prices of the final goods to the intermediate good and the output of all goods will oscillate around their initial steady state values. The initial responses of these real variables will be different depending on whether the government handout is given to savers (case 1) or to workers (case 2). In both cases however, the amplitude of the oscillations decrease over time to zero, and real variables converge back to their initial steady state levels. In other words, a money injection as a result of deficit spending can generate oscillating changes in real variables in the short run, but is neutral in the long run. Our numerical simulations suggest that the "long run" can be quite long. For instance, in our base-case simulation for case 1, it takes 8-10 periods for real variables and 11-12 periods for nominal variables to converge. As the parameters move closer to the unstable region, it takes even longer for the system to converge. We also find that following a money injection, nominal prices and wages oscillate over time before reaching new, higher steady state levels. Moreover, a money injection benefits first recipients of new money but hurts late recipients and savers. Our simulation suggests that workers gain if they are the recipients of government handout (case 2), but lose if the handout is given to savers (case 1). However savers may lose even if they are the recipients of government handout (case 1).

This paper belongs to a broad literature that studies the Cantillon effect, or the implications of the simple fact that it takes time for new money to permeate an economy. The first illustration of the Cantillon effect is of course by Cantillon (1755):

If the increase of hard money comes from gold and silver mines within the state, the owner of these mines, the entrepreneurs, the smelters, refiners, and all the other workers will increase their expenses in proportion to their profits. …Their households will consume more meat, wine, or beer than before. ... Consequently, they will give employment to several artisans who did not have that much work before and who, for the same reason, will increase their expenditures. ... The bargaining process of the market, with the demand for meat, wine, wool, etc., being stronger than usual, will not fail to increase their prices. These high prices will encourage farmers to employ more land to produce the following year, and these same farmers will profit from the increased prices and will increase their expenditure on their families like the others. Those who will suffer from these higher prices and increased consumption will be, first of all, the property owners, during the term of their leases, then their domestic servants and all the workmen or fixed wage earners who support their families on a salary. (p.148-149)

The "Cantillon effect" is further elaborated by Hume (1752) [1], who, after noting that the discovery of gold and silver in America (in the 16th century) increased the amount of money in Europe and encouraged its industry, suggests that the reason behind this is that it takes time for new money to circulate through the economy. In Hume's (1752) words,

"… though the high price of commodities be a necessary consequence of the encrease of gold and silver, yet it follows not immediately upon that encrease; but some time is required before the money circulates through the whole state, and makes its effect be

[1] The insight that new money enters an economy at a specific point and its effects gradually ripple through the economy is attributed to Cantillon (1755) rather Hume (1752) because, according to Saucier and Thorton's introduction to Cantillon (1755), Cantillon's *An Essay on Economic Theory* was completed in 1730 and was circulated privately for more than two decades before its formal publication.

felt on all ranks of people. At first, no alteration is perceived; by degrees the price rises, first of one commodity, then of another; till the whole at last reaches a just proportion with the new quantity of specie which is in the kingdom. In my opinion, it is only in this interval or intermediate situation, between the acquisition of money and rise of prices, that the encreasing quantity of gold and silver is favourable to industry." (II.III.7)

Thus both Cantillon and Hume attribute the short-term real effects of money to the simple fact that new money does not reach everyone at the same time and that it takes time for prices to fully adjust.

Although Cantillon is seldom mentioned by macroeconomists of today, the essence of the Cantillon effect is recognized in the modern literature on asset market segmentation and monetary policy. Different authors have demonstrated the short-run non-neutrality of money due to the Cantillon effect, but have accounted for the fact that new money does not reach everyone at the same time in different ways. For instance, Grossman and Weiss (1983), Rotemberg (1984) and Alvarez and Atkeson (1997) assume that people do not go to the bank at the same time and so open market operations initially affect only the people who happen to be at the bank. Fuerst (1992) presents a model where only borrowers have direct access to newly injected cash. Alvarez et al. (2002) allows only active traders to participate in open market operations. Williamson (2008) conjectures that a money injection is initially received by "connected" household and passed on to "unconnected" households through goods market transactions.

Our paper complements the asset market segmentation literature and differs from that literature in two respects. First, the subject of our investigation is deficit spending, not open market operations; and we account for the fact that money does not reach everyone at the same time by assuming that a government handout is given to only one of two groups in the economy. Secondly, we focus on the impact of a injection on the structure of the economy, thus our model has two final goods sectors and an input sector which allow us to study the dynamics of relative prices and outputs following a money injection. In contrast, most models in the market segmention literature focus on the impact of money injection on interest rate and/or exchange rates, so they tend to simplfy the production side of the economy by assuming either there is only one final good sector or that consumption goods are endowed instead of produced.

To the extent that our paper models the Cantillon effect as a reason behind the sluggishness in price responses to external changes, it also complements the New Keynesian literature (see for example, Ball & Romer, 1991; Blinder, 1991; Gordon, 1990; Mankiw, 1990). Some of the reasons for price and wage stickiness highlighted in the New Keynesian literature are staggered nominal wage and price (Calvo, 1983; Fischer, 1977; Taylor, 1980), heterogeneous expectations (Lines & Westerhoff, 2010) and heterogeneous contract lengths and price setting practices (Dixon & Kara, 2010). Other sources of price inertia identified include slow dissemination of information (Mankiw & Reis, 2002) and imperfect information and gradual learning (Dellas, 2006).

Different from the New Keynesian literature, our paper highlights Cantillon and Hume's insight that new money is first obtained by a subset of market participants and only gradually through *sequential* transactions will the new money reach other market participants. For example, when the government spends newly created money to buy certain goods from a firm, the firm is the first recipient of the new money. The owners (and perhaps workers as well) of the firm then spend some of the new money on other goods, and the suppliers of other goods in turn pass on the new money through their own purchases. As new money "travels through" the economy via multiple sequential transactions, the nominal income levels of different individuals also increase sequentially, and so do their levels of nominal demand and correspondingly the nominal prices of the goods that they demand. Viewed in this way, money has real effects in the short run not because prices are sticky (in the sense that sellers are reluctant to change them in response to changing demand), but rather because money itself is sticky (in the sense that new money does not arrive in all market participants' pockets instantaneously). In short, sticky money leads to a gradual response of nominal demand to a monetary shock, which gives rise to short-run real effects and the symptom of price stickiness (in the sense that prices do not adjust fully immediately after a monetary shock). This result holds even if there is no menu cost in price adjustments and all individuals have perfect information and rational expectations.

2. The Model and Its Steady State

2.1. Set-up of the model

Consider an economy with two groups of individuals: workers and savers. Workers (as a group) are endowed with one unit of labor for each period. They receive a wage for the labor they supply, and spend all their wage income on consumption. Savers (as a group) own an initial stock of capital good (K_0). Corresponding to the real capital endowment, savers have a monetary endowment[2], which they deposit with a bank. They receive an interest income, and

[2] The monetary endowment may be viewed as claims against the real capital goods.

have an infinite time horizon when making their consumption decisions. The savers are assumed to be owners of the production sector.

The bank receives deposits from savers and lends to producers. For simplicity, we assume that there is no cost in producing banking services and that the bank is a non-profit intermediary, therefore the lending interest rate is the same as the deposit interest rate.

There are three production sectors, producing two consumption goods X and Y, and an intermediate good, K. X is produced with labor only, Y is produced with labor and the intermediate good. The intermediate good is produced with labor only. The production technologies are characterised by the following functions:

$$X_t = a_x l_{xt} \tag{1}$$

$$Y_t = \min(a_y l_{yt}, k_t) \tag{2}$$

$$K_t = a_k l_{kt} \tag{3}$$

Transactions in the economy proceed as follows. At the beginning of each period *t*, savers deposit their money with the bank. The bank in turn lends producers the money which is then used to buy inputs. The loans the bank extends to the producers are, respectively:

X producer (to hire workers): $c_{xt} = w_t l_{xt}$

Y producer (to hire workers and buy K): $c_{yt} = w_t l_{yt} + p_{kt} k_t$

K producer (to hire workers at beginning of t=1): $c_{kt} = w_t l_{kt}$

where w_t is the wage rate in period *t*; l_{it} $(i = x, y, k)$ is labor demanded by sector *i* in period *t*; k_t is the quantity of good K demanded by the Y producer in period *t* and p_{kt} is the price of good K in period *t*.

We assume that the total amount of loans to producers is equal to the total deposit received, that is, $S_t = w_x(l_{xt} + l_{yt} + wl_{kt}) + p_{kt}k_t = w_x + p_{kt}k_t$, where S_t is the total deposit at the beginning of time *t*.

Once producers have purchased their inputs,[3] they commence production. The production of each good takes one period of time. At the end of period *t* (which is the same as the beginning of period *t+1*), workers spend all their wages on goods X and Y. Savers receive their interest income and have a balance of $(1+i_t)S_t$ with the bank. They use some of their deposits to buy goods X and Y, and leave the rest, S_{t+1}, in deposit with the bank for another period. After X and Y producers have sold their products, they repay their loans with interest to the bank, and borrow new loans to buy inputs for the next period. Some of the loans are used to buy good K, enabling K producers to repay their loans to the bank with interest.

It should be noted that before the producers repay the principal plus interest to the bank, the bank needs to pay savers interest on their deposits (say, in cheque) which they use to purchase goods. When X and Y producers repay the bank after selling their goods (some of which are bought by savers with a cheque on the bank), the cheque is returned to the bank. In effect, the bank, as an intermediary, "creates" money to pay interests to savers to facilitate their purchases. However this "created money" is backed by the producers' promise to pay interests on their loans, and is withdrawn when loans are repaid. This is different from the unbacked money injection we will model later in this paper.

[3] We assume that input owners cannot deposit their revenue with the bank for an interest as the bank cannot lend the cash out again in period *t*.

2.2. Decision problems

We describe the decision problems of the economic agents in the following[4].

(1) Workers

Workers sell labor in exchange for wages. They consume all their period t wage income in period t. Their decision problem is:

$$\max_{x_{wt}, y_{wt}} U_{wt} = \sqrt{x_{wt} y_{wt}}$$

subject to $$p_{xt} x_{wt} + p_{yt} y_{wt} = w_t l_t \tag{4}$$

where x_{wt} and y_{wt} are workers' consumption of good X and good Y in period t; p_{xt} and p_{yt} are the (nominal) prices of good X and good Y, respectively.

(2) Savers:

Savers do not work; they have a monetary endowment, which they deposit with the bank and earn an interest income. It is assumed that they have an infinite time horizon, and their decision problem is:

$$\max_{x_{st}, y_{st}} U_s = \sum_{t=1}^{\infty} \beta^{t-1} \sqrt{x_{st} y_{st}}$$

subject to $$p_{xt} x_t + p_{yt} y_t = (1 + i_t) S_t - S_{t+1} \tag{5}$$

where β ($0 < \beta < 1$) is the discount factor characterising savers' time preference; i_t is the interest rate; S_t and S_{t+1} are savings at the beginning of period *t* and period *t+1* respectively.

(3) Producers

[4] We do not model the bank's decision problem as it is assumed to be a not-for-profit intermediary.

Producers of goods X, Y and Z are assumed to operate in a perfectly competitive environment so that they are price takers in both input and output markets. Their decision problems are, respectively

(i) Producer X:

$$\max_{X_t} \ \pi_{xt} = p_{xt}X_t - (1+i_t)w_t l_{xt}$$

subject to $\quad X_t = a_x l_{xt} \qquad (6)$

(ii) Producer Y:

$$\max_{Y_t} \ \pi_{yt} = p_{yt}Y_t - (1+i_t)(p_{kt}k_t + w_t l_{yt})$$

subject to $\quad Y_t = \min(a_y l_{yt}, k_t) \qquad (7)$

(iii) Producer K:

$$\max_{K_t} \ \pi_{kt} = p_{k(t+1)}K_t - (1+i_t)w_t l_{kt}$$

s.t. $\quad K_t = a_k l_{kt} \qquad (8)$

In the above decision problems, X_t, Y_t, K_t are quantities of goods X, Y and K produced in period *t*; and k_t is the quantity demanded for good K in period *t*. Since input K is sold in period t+1, producer K's revenue is determined by the price of K in period t+1 ($p_{k(t+1)}$).

2.3. Steady state

The steady state of the model can be characterised by the following conditions.

(1) Prices and interest rate do not change over time:

$$p_{jt} = p_{j(t+1)}, \text{ where } j = x, y, k \qquad (9)$$

$$i_t = i_{t+1} \tag{10}$$

(2) Production and savings do not change over time:

$$J_t = J_{t+1}, \text{ where } J = X, Y, K \tag{11}$$

$$S_t = S_{t+1} = S_0 \tag{12}$$

(3) Consumers' (workers and savers) utility levels are maximised (*i.e.*, their consumption choices are determined by the solutions of their decision problems (4) and (5))

(4) Producers maximize profit and the profit of all producers is zero:

$$\pi_{xt} = p_{xt}X_t - (1+i_t)w_t l_{xt} = 0 \tag{13}$$

$$\pi_{yt} = p_{yt}Y_t - (1+i_t)(p_{kt}k_t + w_{yt}l_{yt}) = 0 \tag{14}$$

$$\pi_{kt} = p_{k(t+1)}K_t - (1+i_t)w_{kt}l_{kt} = 0 \tag{15}$$

(5) Goods markets clear:

$$x_{wt} + x_{st} = X_t \tag{16}$$

$$y_{wt} + y_{st} = Y_t \tag{17}$$

$$k_t = K_{t-1} = K_0 \tag{18}$$

(6) Labor market clears:

$$l_{xt} + l_{yt} + l_{kt} = 1 \tag{19}$$

(7) Loans market clears, that is, the supply of loans (which equals to total savings) is equal to the demand for loans (which equals to the total value of inputs at the beginning of each period):

$$S_t = w_t + p_{kt}k_t \tag{20}$$

(8) Total income in each period is equal to the total expenditure on final goods:

$$w_t + i_t S_t = p_{xt} x_t + p_{yt} y_t \quad (21)$$

From the solutions to the decision problems outlined in section 2.2 and equations (9)-(21), we can solve for the steady state of the model. The steady state values are presented in Table 1. Since the production of good Y uses Leontief technology, there is a constraint on the initial endowment of good K in order to obtain the steady state.

[Insert Table 1 here]

3. Impact of Money Injection: Savers Receive the New Money First (Case 1)

3.1. Assumptions

The economy is assumed to be in a steady state in period 0. At the end of period 0, let the government print μS_0 of cash to give to savers, who immediately deposit the extra cash with the bank. We assume that the cash handout is given after wage contracts are made, so that the wage rate in period 1 (w_1) is the same as the steady state wage rate, that is,

$$w_1 = w_0 = \frac{[(2+i)a_y + 2a_k]S_0}{(3+2i)a_y + 2a_k} \quad (22)$$

We assume further that savers only consume interest income, and leave the principal intact. [5] This means that nominal interest rate remains at $i = \frac{1-\beta}{\beta}$ over time.

Entering period 1, workers have a one unit of labor endowment; savers have a real endowment of good K, which is equal to the steady state production of K, that is,

[5] This is what savers do in steady state. An infinite time horizon implies that it is reasonable to assume savers do not alter their behaviour rule following a one-off receipt from the government.

$$K_0 = \frac{a_y a_k}{a_y i + 2(a_k + a_y)} \tag{23}$$

Savers also have a monetary endowment of $S_1 = S_0 + \mu S_0$, with μS_0 being a government handout funded by money creation.

3.2. Short-run dynamics

At the beginning of period 1, producers borrow from the bank (the amount of $(1+\mu)S_0$) to purchase inputs. From the loan market clearance condition, we obtain the price for good K for this period.

$$(1+\mu)S_0 = w_0 + p_{k1}K_0 \Rightarrow p_{k1} = \frac{(1+\mu)S_0 - w_0}{K_0} \tag{24}$$

Production takes place during period 1.

Solving decision problems ((4) and (5)), we obtain workers' and savers' demand for good X and good Y in period 1. Then from the demand functions, zero profit conditions (equations (13)-(14)) and market clearing conditions (equations (16)-(17)), we obtain the quantities and prices of goods produced (including the quantity and price of good K to be sold at the beginning of period 2) and labor allocations to each sector in period 1.

At the end of period 1, all goods X and Y produced are sold and consumed. Good K produced is sold at the beginning of period 2. So entering period 2, the real endowment of good K is the amount of good K produced in period 1, which is:

$$K_1 = a_k \frac{[a_y + 2a_y i - 2(1+i)K_0]w_0 - a_y i(1+\mu)S_0}{2a_y(1+i)w_0} \tag{25}$$

The monetary endowment corresponding to the real endowment is still $S_2 = (1+\mu)S_0$, and workers are endowed with one unit of labor.

Same as in period 1, period 2 begins with producers borrowing from the bank (the amount of $S_2 = (1+\mu)S_0$) to purchase inputs. By assumption, wage is flexible from period 2 onwards. The wage rate in period 2 is determined by the loan market clearance condition:

$$(1+\mu)S_0 = w_2 + p_{k2}K_1 \Rightarrow w_2 = (1+\mu)S_0 - p_{k2}K_1 = (1+\mu)S_0 - \frac{(1+i)w_0K_1}{a_k} \qquad (26)$$

We can solve for period 2 prices and quantities by solving the consumers' decision problems and then applying the zero profit conditions and market clearing conditions. After the final goods are bought and consumed at the end of period 3, the economy enters into period 3 with a real endowment of good K which is the production of good K in period 2. Following a similar approach, we can solve for period 3, 4, …, n prices and quantities. The solutions are presented in Table 2.

[Insert Table 2 here]

Intuitively, if the government injects money to the economy by giving a handout to savers, the following sequence of events may take place. In the first period after the money injection, the increase in credit pushes up the price for good K, which in turn pushes up the price of good Y (that uses K as an input). As the wage rate is unchanged, the relative price of good Y to good K also increases, so does the relative price of good Y to good X. Production of good X increases in response to higher demand, but production of Y remains unchanged as the available input K is fixed. As more labor is devoted to producing good X, labor devoted to producing good K for use in period 2 falls ($K_1 < K_0$) which amounts to a fall in real savings.

In the second period after the money injection, wages rise ($w_2 > w_1$), driving up the prices for both good X and good Y. Since the capital good K available in period 2 is produced in period 1 and its cost of production determined by the wage rate in period 1, its price falls relative to the consumption goods. The production of Y falls, as the available input K is lower. The production of X is also lower (due to higher wages, as can be shown that $\frac{\partial l_{xt}}{\partial w_t} < 0$), but the production of good K increases ($K_2 > K_1$).

Entering into the third period, wages fall ($w_3 < w_2$) because the total amount of bank credit remains the same, but the amount needed to pay for input K increases. Production of good Y increases as more K was produced in period 2; production of good X also increases due to lower wages. As labor endowment is fixed, more production of consumption goods mean lower production of the capital good K ($K_3 < K_2$).

What we can see is an emerging pattern of oscillation in wages and production. From the results in Table 2, it can be shown that the wage rate remains unchanged in the first period after the money injection, then rises in period 2, falls in period 3, rises in period 4 and so on, that is,

$$w_1 = w_0; \qquad w_2 > w_1; \qquad w_3 < w_2; \qquad w_4 > w_3 \ldots \tag{27}$$

Production also exhibits an oscillating pattern. Following the money injection, the production of good X first rises, then fall, then rises again, and falls again. The production of good Y remains constant for the first period; then follows a similar pattern as X production.

The production of K first falls, then rises, then fall again, and rises again. This can be seen from the pattern of labor allocation in different sectors over time:

$$l_{x1} > l_{x0}; \qquad l_{x2} < l_{x1}; \qquad l_{x3} > l_{x2}; \qquad l_{x4} < l_{x3} \cdots$$

$$l_{y1} = l_{y0}; \qquad l_{y2} < l_{y1}; \qquad l_{y3} > l_{y2}; \qquad l_{y4} < l_{y3} \cdots$$

$$l_{k1} < l_{k0} \qquad l_{k2} > l_{k1}; \qquad l_{k3} < l_{k2}; \qquad l_{k4} > l_{k3} \cdots \tag{28}$$

3.3. Long-run effects

We have found that in the short run both nominal wage rate and real production oscillate following a money injection.Whether the oscillations will decay and the variables converge back to the initial steady state in the long run is not clear from the analytical solutions. We therefore conduct numerical simulations (using MATLAB) to investigate the long-term effect of a money injection. In particular, we ask (1) Does the dynamical system converge for reasonable areas of the parameter space, and if so, which parameters are particularly important for convergence? (2) Within the stable region, what factor affect the length of time it takes for the system to converge impacts on periods to convergence? (3) How much do the variables deviate from the steady state during the transient dynamics? In the following, these questions shall be addressed in turn.

3.3.1. Stability

Intuitively, the dynamic system could become unstable, or infeasible (negative values for X, Y or K) in situations where a mismatch develops between the demand and supply for the intermediate and final goods. As such, key parameters affecting stability might be expected to be the relative size of labour productivity in the co-dependent industries, i.e. the ratio of a_k to a_y. Nevertheless, we also check the influence of the extent of the money creation (μ) and time preference (β) on stability.

To investigate stability, we ran the system at many different points in the (a_k / a_y, β, μ) parameter space, sampling in a regular pattern, for 300 periods. We over-looked the first 100 periods, treating them as transient, and then measured the variance in the value of *K* over the final 200 periods: vanishingly small variance would imply a fixed point attractor (convergence), whilst an intermediate or high variance would imply a periodic or chaotic regime (non-convergence). Second, since we are interested in identifying stable *and* feasible parameter regions, we applied a non-negativity pass/fail test to the early, transient 100 periods of data in each case. All experiments were run with fixed values for (a_x,a_y,S_0) = (1,1,10). The results of these analyse are given in Figure 1 and show that for reasonable values of β (domain approx. (0.8,1.0]) and low values of a_k / a_y (domain approx. (0,0.6)) a stable and feasible region exists. We found that μ had no additional effect on stability.

[Insert Figure 1 here]

To illustrate the system's dynamical properties in more detail we present in Figure 2 and Figure 3 the values of *K* reached by the system over 900 periods after a 100 period induction. Figure 2 explores the β = 0.9 line (varying a_k / a_y) and shows that a classical bifurcation pattern emerges as β approaches 1. Figure 3 explores the a_k / a_y = 0.2 line (varying β) and reveals a disrupted regime shift pattern.

[Insert Figure 2 and Figure 3 here]

3.3.2. Convergence times

As might be expected, we find a close connection between stability and convergence times. Figure 4 gives example dynamics of deviations in *K* as the system evolves within the stable region for a_k / a_y = 0.2 and 0.8 (β = 0.9, μ = 0.1, S_0 = 10) (note, scales are different). We find that as the a_k / a_y ratio approaches the instability frontier, the perturbations to *K* are larger in amplitude, and take longer to decay.

[Insert Figure 4 here]

To summarise this finding, we established a *convergence criterion*, namely, that three successive values of *K* should fall within 0.5% of the first value in the sequence and explored periods to convergence along the β = 0.9 line. A dramatic increase in convergence times is evident in Figure 5, with convergence not evident within the chaotic region (a_k / a_y > 0.9). Indeed, a variation in a_k / a_y from 0.50 to 0.85 had an approximately *order of magnitude* increase in the time taken for the model economy to return to the steady state.

[Insert Figure 5 here]

3.3.3. Impacts of money injection within the region of dynamic stability

Having established the region of dynamic stability of the model, we can now proceed to investigate the impacts of a money injection on the model economy within the stable region. First we present in Table 3 (top half) the base-case simulation results, with the following parameter values: μ=0.1, β=0.9, S=10, a_y = a_x =1, a_k=0.2, *t*=100. The time paths of the nominal and real variables are illustrated in Figure 6 and Figure 7, respectively.

[Insert Table 3 here]

[Insert Figure 6 and Figure 7 here]

Our base-case simulation suggests that the real variables (K , X , Y) converge to the initial steady state level; the nominal variables (w , p_x , p_y , p_k) converge to a new steady state with higher values, and the percentage increases in the values of nominal variables prices are approximately 10%, the same as the increase in money supply (μ =0.1). This result supports the key proposition of the quantity theory of money, namely that changes in money in circulation will have proportional effects on monetary prices, but no effects on real variables. Given our parameter values, the long run seems to be quite long compared to the production period – it takes between 4 to 10 periods for real variables to converge, and 11 or 12 periods for nominal variables to converge.

Our base-case simulation also suggests that for a 10% increase in money supply, the maximum deviations in real variables from their steady state values are relatively small (0.3% to 1.4% from above and 0.5% to 1.8% from below). The maximum deviations in nominal variables are larger (4% to 20% from above and 9.1% from below). Notably, the positive variation in the price of capital good K is considerably larger than the positive variations in consumer goods X and Y (20% compared to 4.8%).

To test how the results of our base-case simulation may change with different parameter values, we ran simulations with a variety of money injection sizes (μ) and values of a_k / a_y (holding β = 0.9, a_y=1, S_0 =10) and present in Figure 8 maximal percentage deviations from the steady state value of *K*. Unsurprisingly, large money injections (large μ)

lead to larger perturbations in the production of K and the largest perturbations occurred when combined with high values of a_k / a_y.

[**Insert Figure 8 here]**

Summarising the results of our analysis in this section, we have:

Proposition 1. *If savers are the first recipients of the new money created by government deficit spending, then the initial effects of the money injection will be that the price of the capital good will rise, so will the price of the consumption good that uses the capital good as an input. Moreover, consumption will increase, and real savings will fall. Afterwards, both nominal and real variables will oscillate. The model economy has a large region of dynamic stability within which the oscillation of variables following the money injection will decay over time and converge to a steady state. Nominal variables will converge to higher steady state values and real variables will converge to their initial steady state values.*

4. Impact of Money Injection: Workers Receive the New Money First (Case 2)

In this section, we look at how the impact of money injection may be different if money injection is through a government handout to workers instead of savers. We use the same model as in the last section, except now at the beginning of period 1, savers have a monetary endowment of only $S_1 = S_0$ which they deposit with the bank. Workers receive a wage from producers and also a subsidy μS_0 from the government (both wage the subsidy are held in cash). Accordingly, the decision problem of workers changes to:

$$\max_{x_{wt}, y_{wt}} U_{wt} = \sqrt{x_{wt} y_{wt}}$$

subject to $$p_{xt}x_{wt} + p_{yt}y_{wt} = w_t l_t + \mu S_0 \quad (29)$$

The decision problem of savers remains the same as specified in (5) with $S_1 = S_0$. Solving the decision problems gives us the demand for consumption goods X and Y in period 1. Since the subsidy to workers only increases the total income and has no effect on costs of production, once all workers' income is spent on consumption goods, the subsidy is passed on to final goods producers as profits. Given our utility function, half of the profits go to X producers and half to Y producers. From the demand functions for good X and Y, the profit functions for goods X and Y, the zero profit function for good K and market clearing conditions for all goods, we can solve for period 1 wage rate, prices and quantities of all goods.

Since savers are assumed to be owners of the production sector, the handout to workers in period 1 becomes part of the savers monetary endowment in period 2. Therefore from period 2 onwards, the dynamics of the system is similar to that of Case 1 except that the real endowment at the beginning of period 2 is different. The variable values describing the short-run impact of the money injection are presented in Table 4. To illustrate the dynamics of the system in comparison to that of case 1, we perform a numerical simulation using the same parameter values as used in the base-case simulation for case 1. The results are presented in Table 3 (bottom half). From Table 3, it is clear that in both cases nominal variables converge to the same higher steady state values, and real variables converge to the initial steady state values. It takes the same amount of time for the system to reach a new steady state in both cases. The only notable difference between the two cases is that in case 1, there is a sharp jump in the price of the capital good K (20%), whereas in case 2, the maximum positive deviation of the price of K from its new steady state is just above 4%.

This difference is due to the fact that in case 1, the handout to savers translates to a higher level of credit which sharply pushes up the price of the capital good (when the wage rate is fixed in period 1); whereas in case 2, by the time the injected money gets to savers, wage rate also increases, which relieves some of the upward price pressure on K.

[Insert Table 4 here]

Now focusing on the initial impact of money injection, we can see from Table 4 that following the money injection, production of all goods remains unchanged. The price for the intermediate good K remains unchanged; the prices of both good and good Y increase. In other words, the money injection has no immediate real effects; instead, the money injection simply raises the prices of consumption goods and increasing workers' share of total consumption in the first period following the money injection. To summarize, we have

Proposition 2. *If workers are the first recipients of the new money created by government deficit depending, then the initial impact of the money injection is that the prices of consumption goods will increase, so will the share of consumption for workers. However, there is no change in production. The real effects will set in after the injected money reaches savers, and from then on, the impact of the money injection is qualitatively the same as the case where savers are the first the recipients of the new money.*

5. Welfare Effects of Money Injection

We now attempt to answer the following two questions: (1) who are the winners and losers following a money injection? (2) what can we say about the welfare effects of a money injection?

Obviously the answers to these questions depend on who receive the money first. As a rule, those who receive the new money earlier would be better off. This is because not all prices go up at the same time and to the same degree, which means the purchasing power of the new money is higher in the hands of early recipients. Moreover, money injection tends to hurt savers if it artificially increases the supply of credit. These considerations suggest that workers would be winners in case 2, but it is unclear whether savers would win in case 1.

Since a money injection has no real effects in the long run, but creates volatilities in the short run, its net welfare effects on the economy would seem negative. However, because a money injection creates winners and losers, the net "social welfare" effects would depend on how "social welfare" is defined.[6] For our purposes, we focus on how the welfare of each group (savers or workers) may change with a money injection.

To answer our questions regarding welfare effects more concretely, we conduct numerical simulations for both case 1 and case 2. The results are illustrated in Figures 9 and 10. In our simulations, we define welfare as discounted utility over 100 periods. Accordingly, change of welfare due to money injection is measured by the difference between total discounted utility over 100 periods after money injection and total discounted steady state utility over 100 periods, i.e, $(\sum_{t=1}^{100}\beta^{t-1}\sqrt{x_t y_t}-\sum_{t=1}^{100}\beta^{t-1}\sqrt{x^* y^*})/\sum_{t=1}^{100}\beta^{t-1}\sqrt{x^* y^*})$. As shown in Figures 9 and 10, our simulations suggest that if money is injected through a handout to savers, both savers and workers will lose (case 1), so the "social welfare" is negative however it is defined. If money is injected through a handout to workers, workers

[6] For example, if we define social welfare as the sum of utilities of both groups (workers and savers), a redistribution of wealth from the "rich" group (one that has higher steady state consumption levels) to the "poor" group will increase social welfare given our utility specification. In other words, the social welfare effect will be highly sensitive to initial conditions.

will gain, but savers will lose (case 2). In both cases, savers are affected proportionally more than workers, and they are more affected in case 1 than in case 2. Unsurprisingly, in case 1, both savers and workers lose more from a larger money injection (a larger μ). In case 2, workers benefit more and savers lose more from a larger money injection.

To summarise, we have

Proposition 3. *If a money injection created by government deficit spending takes the form of a handout to savers, both savers and workers will lose (case 1). If the money injection takes the form of a handout to workers, workers will gain, but savers will lose (case 2). Savers are proportionally more affected by a money injection than workers, and they are more affected in case 1 than in case 2.*

5. Conclusion

In this paper, we have conducted a process analysis based on a simple model to study how a money injection created by government deficit spending affects an economy. Our analysis captures Cantillon's (1755) insight that it takes time for new money to permeate the economy. It is this "stickiness" of money that is behind the real short run effects of money injection in our model. The process analysis allows us to follow the "money trail" to find out how new money affects the economy one period after another. This type of analysis is potentially useful in other contexts where we care more about the process of reaching a steady state rather than the steady state itself.

As an initial attempt to model the Cantillon effects, we have made some strong assumptions to simplify calculations. Future research may try to relax some of the assumptions. For example, savers may not follow the rule of consuming only interest income.

Relaxing this assumption implies that nominal interest rate will vary following a money injection, which has ramifications on consumption and prices. Also the model may be extended to include heterogeneous preferences (that is, different people consume different goods). This extension is likely to strength the Cantillon effect as the retribution of income as a result of a money injection will lead to a greater shift in consumption patterns.

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Table 1. Steady State

Consumption	$x^*_w = \frac{w^*}{2p_x^*}$; $y^*_w = \frac{w^*}{2p_y^*}$; $x_s^* = \frac{iS_0}{2p_x^*}$; $y_s^* = \frac{i^* S_0}{2p_y^*}$
Labor allocations	$l_x^* = \frac{w^* + i^* S_0}{2(1+i^*)w^*}$; $l_y^* = \frac{K_0}{a_y}$; $l_k^* = \frac{K_0}{a_k}$
Interest rate	$i^* = \frac{1-\beta}{\beta}$
Wage rate	$w^* = \frac{[(2+i)a_y + 2a_k]S_0}{(3+2i)a_y + 2a_k}$
Prices	$p_x^* = \frac{(1+i^*)w^*}{a_x}$; $p_y^* = \frac{w^* + iS_0}{2K_0}$; $p_k^* = \frac{(1+i)w^*}{a_k}$
Constraint on parameter values	$K_0 = \frac{a_y a_k}{a_y i + 2(a_k + a_y)}$

Table 2. Impact of Money Injection (Case 1)

	Wage and prices	**Labor allocations**
t=1	$w_1 = w_0 = \frac{[(2+i)a_y + 2a_k]S_0}{(3+2i)a_y + 2a_k}$ $p_{x1} = \frac{(1+i)w_1}{a_x}$ $p_{y1} = \frac{w_1 + i(1+\mu)S_0}{2K_0}$ $p_{k1} = \frac{(1+\mu)S_0 - w_0}{K_0}$	$l_{x1} = \frac{w_1 + i(1+\mu)S_0}{2(1+i)w_1}$ $l_{y1} = \frac{K_0}{a_y}$ $l_{k1} = 1 - l_{x1} - l_{y1}$
t=2	$w_2 = (1+\mu)S_0 - \frac{(1+i)w_1K_1}{a_k}$ $p_{x2} = \frac{(1+i)w_2}{a_x}$ $p_{y2} = \frac{w_2 + i(1+\mu)S_0}{2K_1}$ $p_{k2} = \frac{(1+i)w_1}{a_k}$	$l_{x2} = \frac{w_2 + i(1+\mu)S_0}{2(1+i)w_2}$ $l_{y2} = \frac{K_1}{a_y}$ $l_{k2} = 1 - l_{x2} - l_{y2}$
t=3	$w_3 = (1+\mu)S_0 - \frac{(1+i)w_2K_2}{a_k}$ $p_{x3} = \frac{(1+i)w_3}{a_x}$ $p_{y3} = \frac{w_3 + i(1+\mu)S_0}{2K_2}$ $p_{k3} = \frac{(1+i)w_2}{a_k}$	$l_{x3} = \frac{w_3 + i(1+\mu)S_0}{2(1+i)w_3}$ $l_{y3} = \frac{K_2}{a_y}$ $l_{k3} = 1 - l_{x3} - l_{y3}$
...		
t=n n 2	$w_n = (1+\mu)S_0 - \frac{(1+i)w_{(n-1)}K_{(n-1)}}{a_k}$ $p_{xn} = \frac{(1+i)w_n}{a_x}$ $p_{yn} = \frac{w_n + i(1+\mu)S_0}{2K_{(n-1)}}$ $p_{kn} = \frac{(1+i)w_{(n-1)}}{a_k}$	$l_{xn} = \frac{w_n + i(1+\mu)S_0}{2(1+i)w_n}$ $l_{yn} = \frac{K_{n-1}}{a_y}$ $l_{kn} = 1 - l_{xn} - l_{yn}$

Table 3. Base-Case Simulation
(Parameter values: μ =0.1, β =0.90, S_0 =10, a_x=1, a_y =1, a_k =0.2, T=100)

Variable	Initial Steady State Value	Convergence Value (T=60)	Deviation at convergence from steady state (%)	Periods to convergence	Largest positive deviation from convergence value (%)	Largest negative deviation from convergence value (%)
Case 1						
w	6.93	7.63	10	11	4.75	-9.09
p_k	38.5	42.4	10	12	20.1	-9.09
p_x	7.7	8.5	10	11	4.75	-9.09
p_y	50.5	55.6	10	12	6.01	-9.09
K	0.08	0.08	0	9	1.18	-1.81
X	0.52	0.52	0	8	1.38	-0.63
Y	0.08	0.08	0	10	1.18	-1.81
Case 2						
w	6.93	7.63	10	11	4.02	-9.09
p_k	38.5	42.4	10	12	4.02	-9.09
p_x	7.7	8.5	10	11	4.02	-9.09
p_y	50.5	55.6	10	11	3.47	-9.09
K	0.08	0.08	0	9	0.7	-0.53
X	0.52	0.52	0	7	0.3	-0.53
Y	0.08	0.08	0	4	0.7	-0.53

Table 4. Impact of Money Injection (Case 2)

	Wage and prices	Labor allocations
t=1	$w_1 = w_0 = \frac{[(2+i)a_y + 2a_k]S_0}{(3+2i)a_y + 2a_k}$ $p_{x1} = \frac{(1+i)w_1 + \frac{\mu S_0}{2l_{x1}}}{a_x}$ $p_{y1} = \frac{w_1 + (\mu + i)S_0}{2K_0}$ $p_{k1} = \frac{S_0 - w_1}{K_0}$	$l_{x1} = \frac{w_1 + iS_0}{2(1+i)w_1}$ $l_{y1} = \frac{K_0}{a_y}$ $l_{k1} = 1 - l_{x1} - l_{y1}$
t=2	$w_2 = (1+\mu)S_0 - \frac{(1+i)w_1K_1}{a_k}$ $p_{x2} = \frac{(1+i)w_2}{a_x}$ $p_{y2} = \frac{w_2 + i(1+\mu)S_0}{2K_1}$ $p_{k2} = \frac{(1+i)w_1}{a_k}$	$l_{x2} = \frac{w_2 + i(1+\mu)S_0}{2(1+i)w_2}$ $l_{y2} = \frac{K_1}{a_y}$ $l_{k2} = 1 - l_{x2} - l_{y2}$
...		
t=n n 2	$w_n = (1+\mu)S_0 - \frac{(1+i)w_{(n-1)}K_{(n-1)}}{a_k}$ $p_{xn} = \frac{(1+i)w_n}{a_x}$ $p_{yn} = \frac{w_n + i(1+\mu)S_0}{2K_{(n-1)}}$ $p_{kn} = \frac{(1+i)w_{(n-1)}}{a_k}$	$l_{xn} = \frac{w_n + i(1+\mu)S_0}{2(1+i)w_n}$ $l_{yn} = \frac{K_{n-1}}{a_y}$ $l_{kn} = 1 - l_{xn} - l_{yn}$

FIGURE 1 Stability of the model: **(a)** variance in K resulting from 200 periods after a 100 period induction phase (log scale, the area in the middle has high variance (instability), the two areas on the outside has low variance (convergence)), cross-sections explored at $\beta = 0.9$ and $a_k = 0.2$ are given in Figures 2 and 3 respectively indicated by dashed lines; and **(b)** black indicates regions where K in the model remains positive during the first 100 update induction phase.

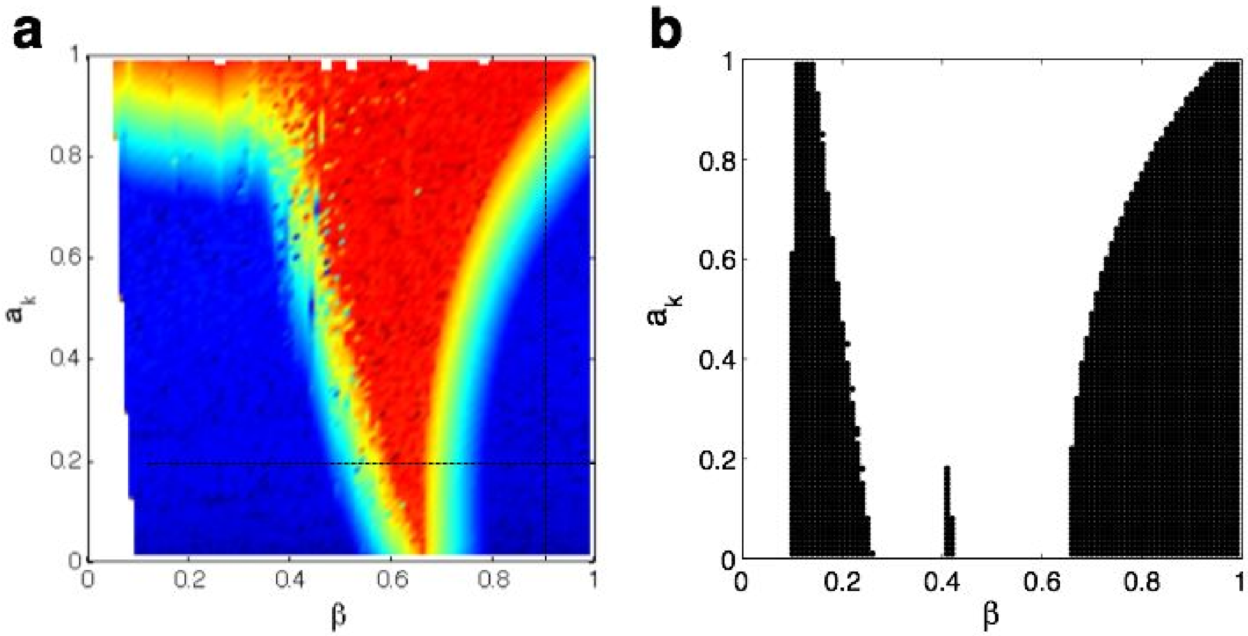

FIGURE 2 Example bifurcation diagram along the a_k dimension holding β = 0.9 showing: **(a)** convergence in a broad region (a_k < 0.85); and then **(b, inset)** a chaotic regime beyond a_k > 0.9. Data are given for K over 900 periods after a 100 period induction period. Refer caption to Figure 1.

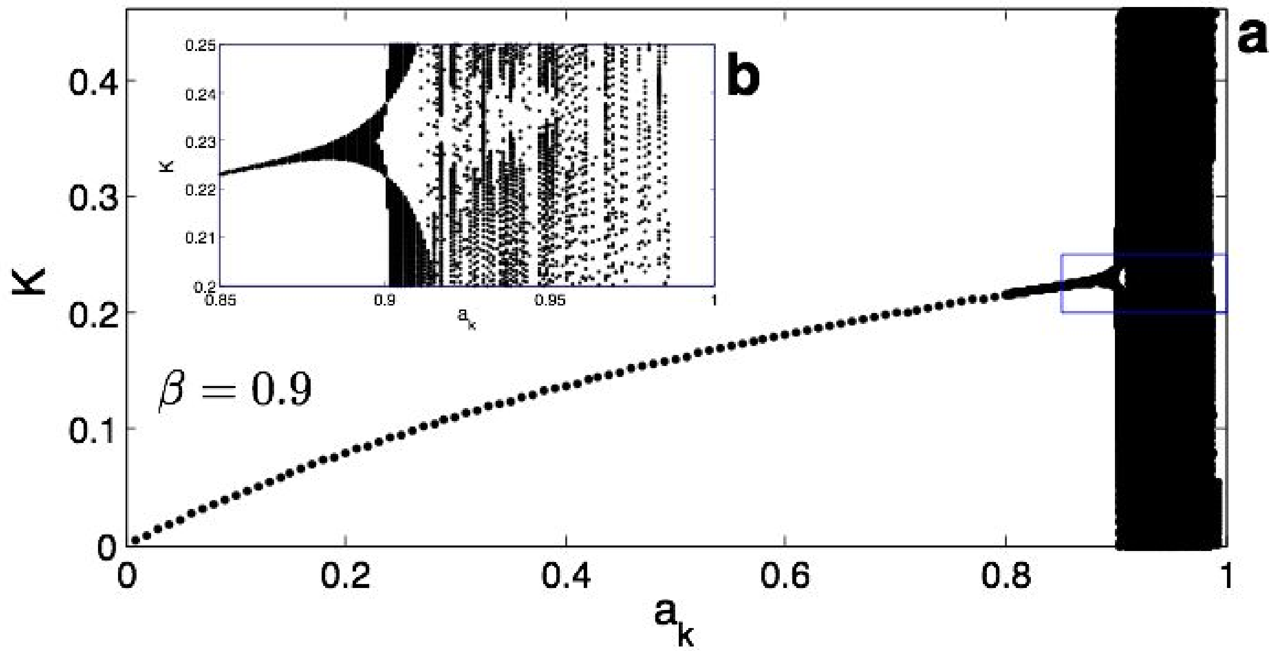

FIGURE 3 Example bifurcation diagram for values of K along the β dimension holding a_k = 0.2 showing system convergence either side of a chaotic regime around a_k ~ 0.62. Refer caption to Figure 1.

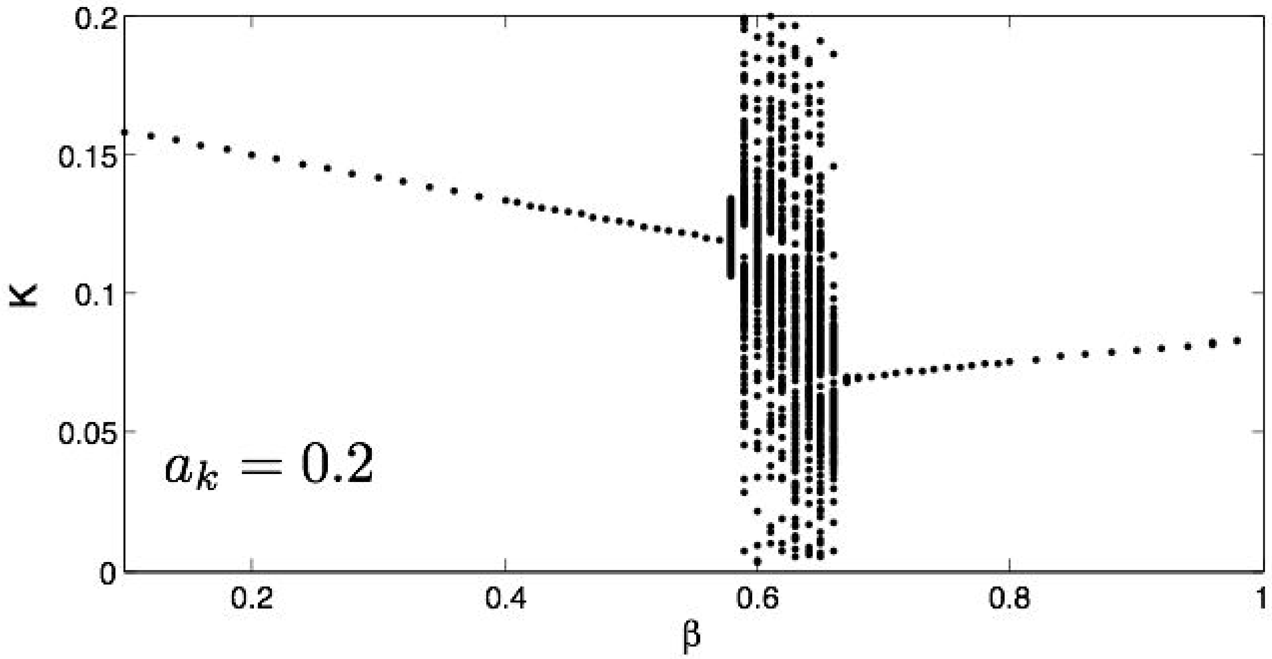

FIGURE 4 Example runs of the model within the stable region: **(a)** $a_k = 0.2$; and **(b)** $a_k = 0.8$. NB: Scales are different.

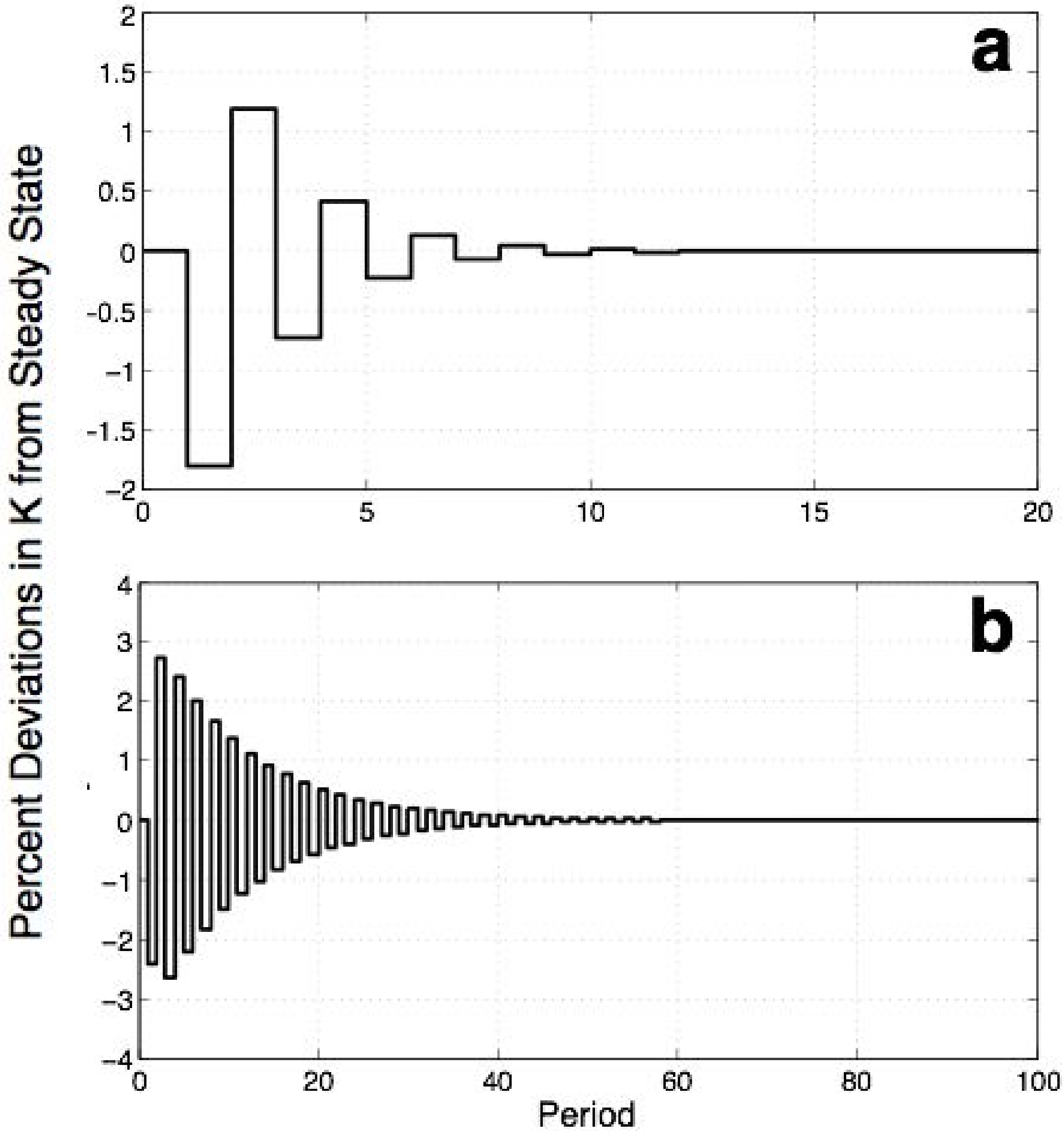

FIGURE 5 A dramatic increase in periods to convergence as the system approaches the chaotic regime (β = 0.9, refer Figs. 1 & 2). Convergence required three successive values of K within 0.5% of the first. Note log scale for y-axis. NB: apparently converging trials above $a_k > 0.9$ are due to phantom convergence events within the chaotic dynamics.

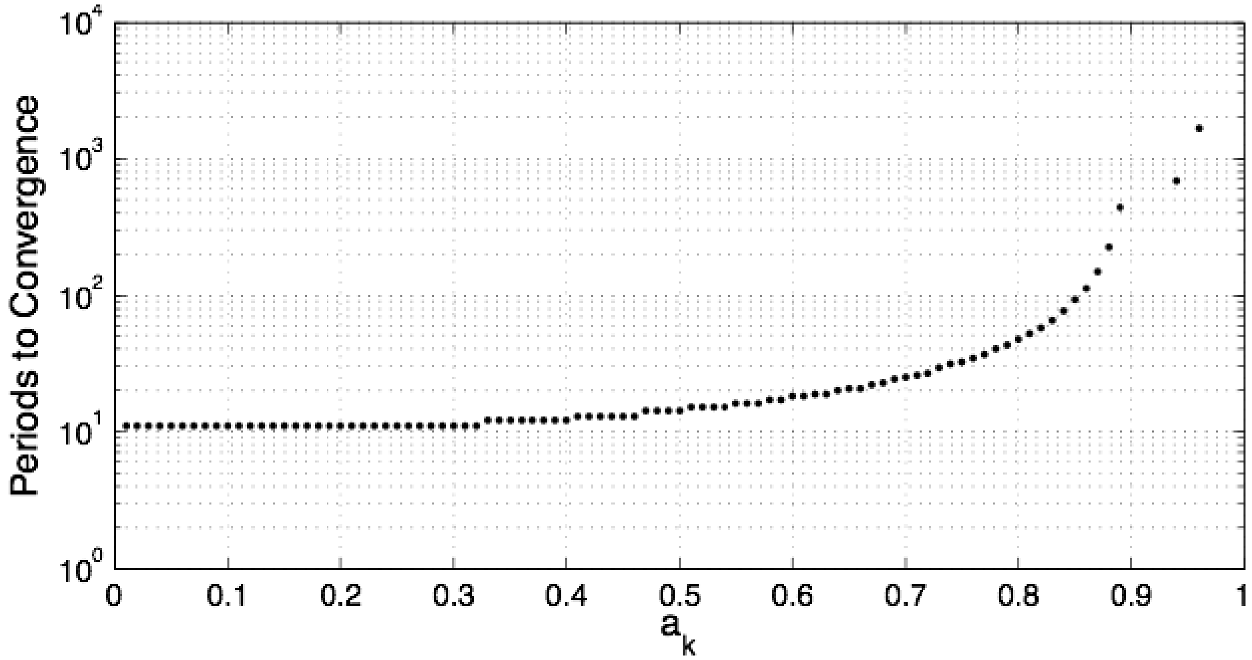

FIGURE 6 Maximum percent deviations in K away from steady state value versus size of the money injection (μ) at given values of a_k.

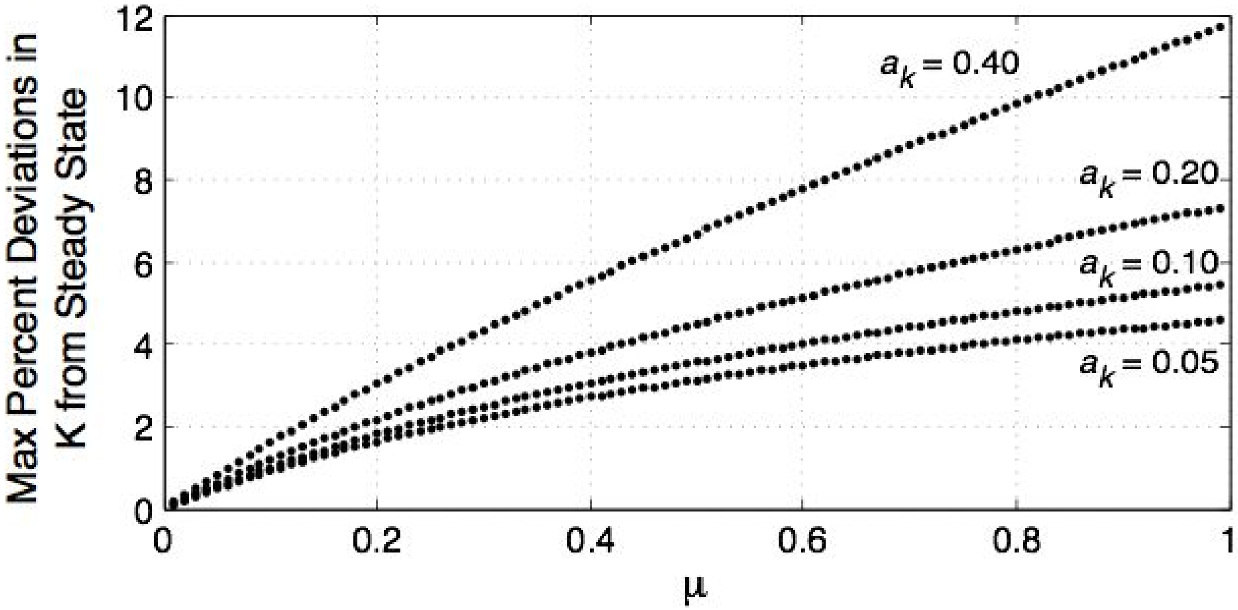

FIGURE 7 Case 1: Base-Case Time Paths of Nominal Prices and Wage Rate

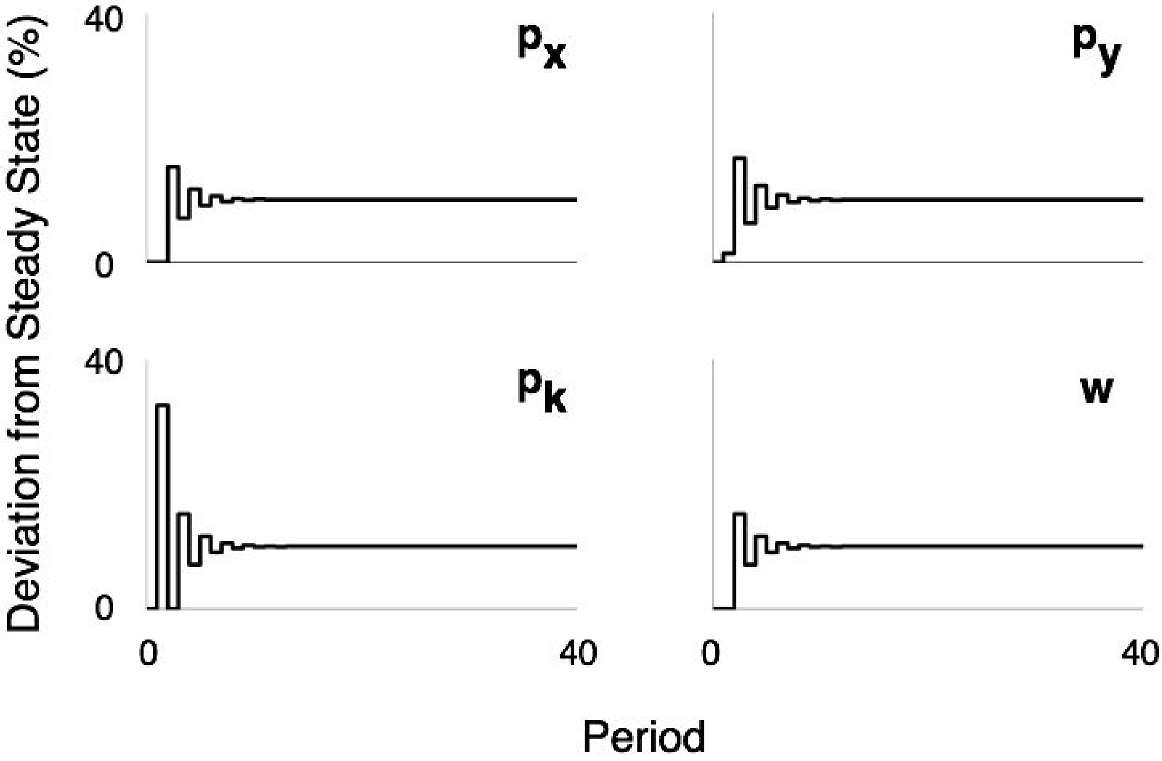

FIGURE 8 Case 1: Base-Case simulation: Time Paths of Production

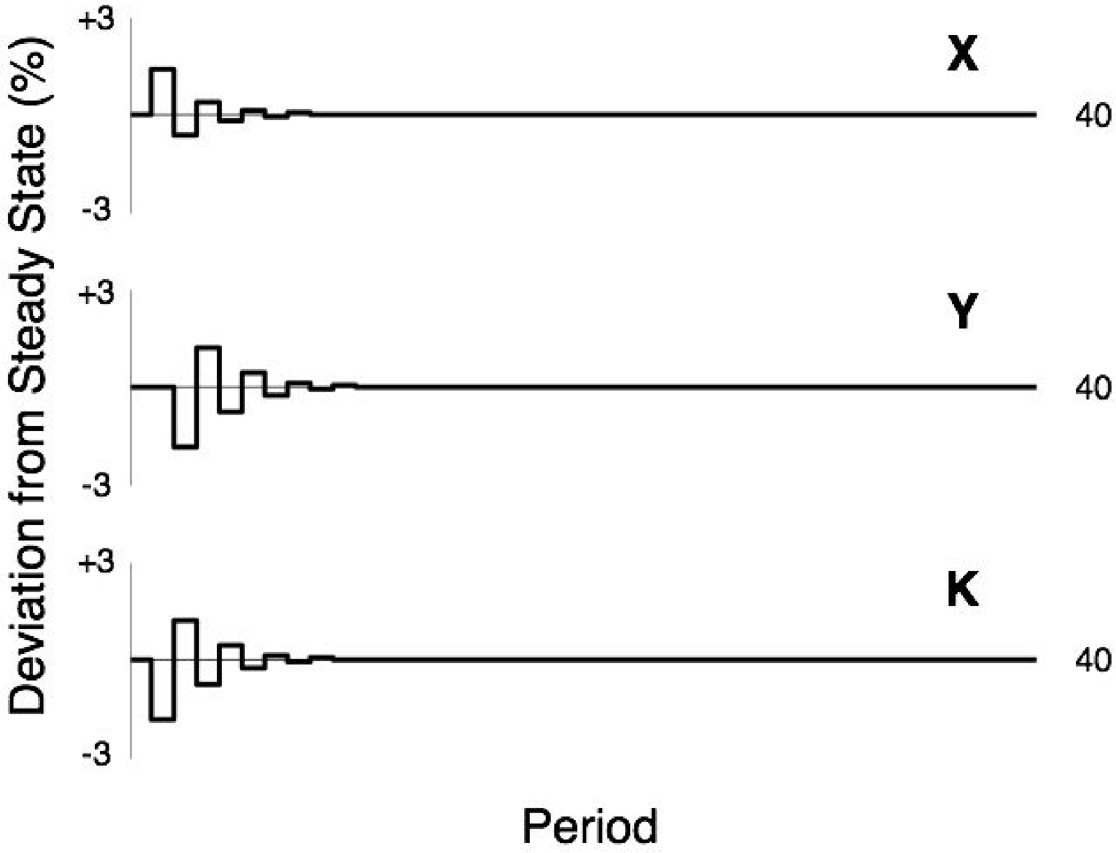

FIGURE 9 Case 1: Relative welfare loss for each group over 100 periods for a range of money injection values, μ. [$\beta = 0.90$, and $a_y = a_x = 1.0$, $a_K = 0.2$ and $S_0 = 10$].

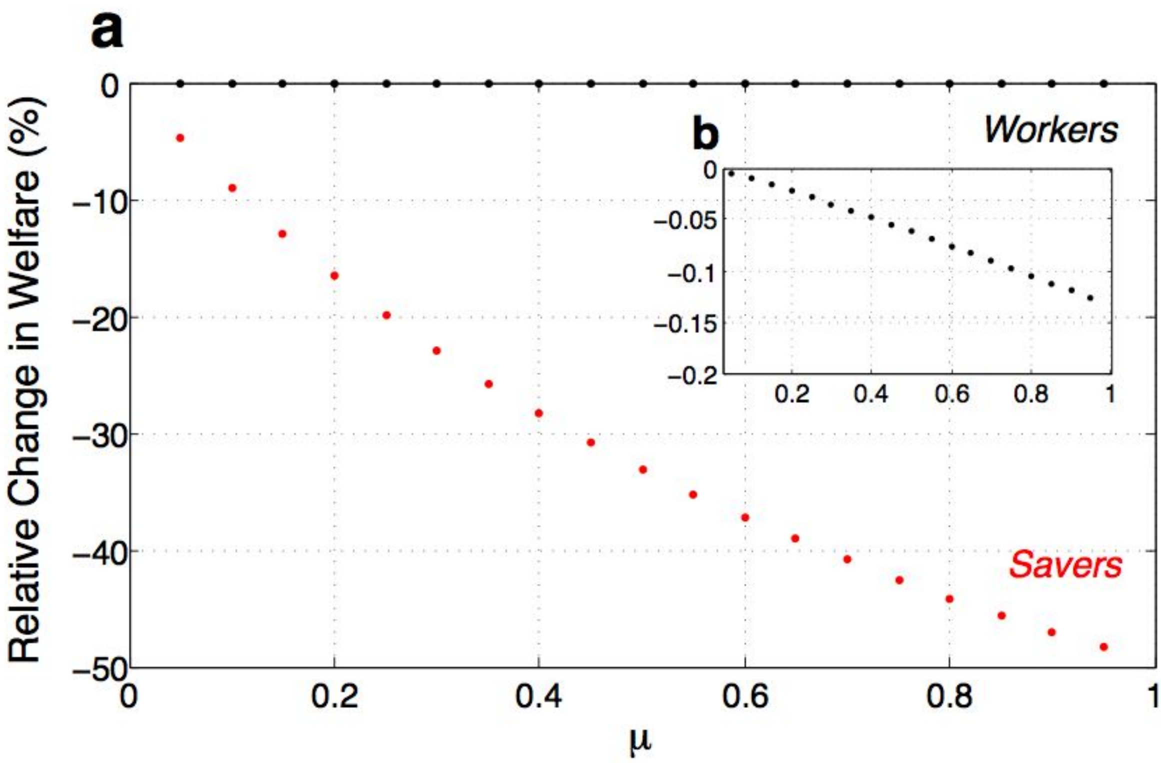

FIGURE 10 Case 2: Relative welfare loss for each group over 100 periods for a range of money injection values, μ. [$\beta = 0.90$, and $a_y = a_x = 1.0$, $a_K = 0.2$ and $S_0 = 10$].