The Fiscal Theory of the Price Level
When All Income is Taxed

Pedro Gomis-Porqueras†, Solmaz Moslehi‡ and Vivianne Vilar§

Abstract
In this paper we explore how the nature of the equilibria changes when the interest income from nominal bond holdings is also taxed in an fully flexible endowment economy. We find that the stability properties of this economy depend on the slope and the intercept of both monetary and fiscal policy rules. Thus, the parameter space consistent with locally determinate equilibria is much larger compared to that of Leeper (1991). For instance, deviations from the Taylor principle can still yield determinate equilibria even when fiscal policy does not aggressively respond to rises in debt levels. In addition, we show that if the government taxes all sources of income and the fiscal authority sets taxes taking into account the level of debt, then the economy exhibits a Laffer curve yielding multiple steady states. As we can see, ignoring the tax treatment of interest income generated by bond holdings is not as innocuous as it may seem.

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‡ Monash University. Email: pedro.gomis@monash.edu
§ Monash University. Email: solmaz.moslehi@monash.edu
§ University of Melbourne. Email: vvilar@unimelb.edu.au

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1 Introduction

The seminal work of Sargent and Wallace (1981) emphasized how the government’s budget constraint forces a deep interconnection between monetary and fiscal policies. In order to fully characterize the evolution of nominal prices it is important to determine the beliefs about future inflation. These beliefs are affected by current and future tax policies as well as the operating procedure for monetary policy which ultimately impact the demand for real balances, as shown by Leeper (1991). Thus, the resulting evolution of prices critically depends on the interactions between monetary and fiscal policies implied by the specific policy rules considered.

The environment that Sargent and Wallace (1981) consider is one where fiscal rules are independent of inflation and government debt, while the central bank follows a constant money growth rate rule. There the price level adjusts to clear the money market while the fiscal authority adjusts its future taxes in order to satisfy the government budget constraint. As a result, fiscal policy is inflationary only if the central bank monetizes deficits. On the other hand, Leeper (1991), Sims (1994) and Woodford (1994) explore the role of alternative fiscal rules in determining the equilibrium price sequence.\(^1\) The proponents of the Fiscal Theory of the Price Level (FTPL) highlight another mechanism through which the price level can be characterized. This alternative view relies on the fact that bonds are denominated in nominal terms. Higher nominal debt may be fully backed by real resources or backed only by nominal cash flows. When real resources fully back debt, we obtain the Sargent and Wallace (1981) equilibria. But when nominal debt is not backed by real resources, fiscal policy creates a direct link between current and expected deficits and inflation. When long-term debt exists, the government can trade current for future inflation through debt operations.\(^2\) In particular, the government can exogenously set its revenue plans while the price level adjusts. This in turn affects the real debt obligations and ensures government solvency. Thus changes in the price level revalues nominal government liabilities and households experience a wealth effect, changing the expected present-value of current and future income flows.\(^3\) The wealth effect generated by changes in debt obligations when backed by nominal cash flows crucially relies on the fiscal rules considered as well as the sources of incomes that are being taxed.\(^4\)

A component of the income tax typically not considered in the FTPL is the one corresponding to interest income from bond holdings, even though Leeper, Richter, and Walker

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\(^1\)Canzoneri, Cumby, and Diba (2011) provide a recent survey of the fiscal theory of the price level (FTPL) and Canzoneri, Cumby, and Diba (2001) its empirical implications. For a critical view of the theory we refer the reader to McCallum and Nelson (2005).

\(^2\)This trade off can not occur when the government rolls over short-term debt.

\(^3\)For more on wealth effects in the context of the FTPL, see Leeper and Yun (2006).

\(^4\)In this spirit Cochrane (2001) analyzes long-term debt and optimal policy in the fiscal theory and finds that the maturity structure of the debt matters.
(2012), among others, use the differential tax treatment of municipal and treasury bonds to identify news about tax changes. A notable exception is that of Eggertsson (2011) who considers a standard New Keynesian environment with explicit taxes on risk-free bonds, a sales tax, a payroll tax, and a tax on profits. The author finds that tax cuts can deepen a recession if the short-term nominal interest rate is zero. Taxes on risk-free debt then have a direct effect on households’ saving and consumption decisions.

The objective of this paper is to complement the work of Eggertsson (2011) and analyze the consequences of taxing interest income on bond holdings for the determinacy of equilibria when the short-term nominal interest rate is away from the lower bound, the government does not impose sales taxes and there are no nominal rigidities. To better isolate the income and substitution effects of taxing all sources of income, we employ a deterministic version of the endowment environment of Leeper (1991) so that government policies do not affect future resources in the economy. Following the literature we consider a Taylor rule so that the interest rate reacts to the inflation rate and a fiscal rule that links taxes to the value of debt.

Typically in the FTPL the demand for real balances depends on the real return on bond holdings. The impact of fiscal policy on the demand for real balances is only observed through general equilibrium effects. The resulting equilibrium is determinate under an active monetary policy and a passive fiscal policy or under a passive monetary policy and an active fiscal policy. This implies that the eigenvalues associated to the unique steady state just depend on either fiscal or monetary policy parameters. This is not the case once interest income from bonds is taxed. When all sources of income are taxed, the demand for government liabilities explicitly depends on fiscal policy. There is then a new direct effect through which fiscal policy can affect the price level. In this new environment fiscal policy changes the effective real return so that an additional substitution effect exists between current and future consumption. This new feature drastically changes the dynamic properties of the equilibria. In particular, we show that the eigenvalues dictating the stability of the economy now depend on both fiscal and monetary policy parameters as in Yun (2011) and Sims (2011). Moreover, in contrast to the FTPL, we show that the stability properties of this economy depend on the slope and the intercept of both monetary and fiscal policy rules. Thus, the parameter space consistent with

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5 Fortune (1996) exploits this tax asymmetry in bonds to identify future tax changes. We refer the reader to Ang, Bhansali, and Xing (2010) for details of the municipal bond market.

6 An important assumption in this environment is that the price the firm sets is exclusive of the sales tax. This means that if the government cuts sales taxes, then consumers face a lower store price of exactly the amount of the tax cuts for firms that have not reset their prices.

7 Data from FRED shows that the interest outlays by the Federal Government, which is subject to taxation, averaged 2% of GDP over the last 65 years.

8 We use the language of Leeper (1991) in characterizing policies as active or passive. The exact bounds for active and passive policies are model-specific.

9 In these environments, the standard classification of active and passive policies described in Leeper (1991) is less informative.
locally determinate equilibria is much larger compared to that of Leeper (1991) and others. For instance, we show that there exists a unique local equilibrium even when monetary policy does not follow the Taylor principle. In particular, deviations from the Taylor principle can still yield determinate equilibria even when fiscal policy does not aggressively respond to rises in debt levels.

Finally, we show that if the government taxes all sources of income and the fiscal authority sets taxes taking into account the level of debt, then the economy exhibits a Laffer curve even though tax policies do not change the amount of future resources. This is the case because the demand for nominal bonds explicitly depends on the income tax rate, and once bond income is taxed, part of the tax base changes in response to new taxes. As is typically the case when a Laffer curve exists, more than one steady state is possible. In this paper we find multiple stationary equilibria. One of the steady states is associated with positive tax rates and positive nominal interest rates. The government is then distorting both of its government liabilities, fiat money and nominal bonds. In contrast, the other steady state has a higher inflation rate and bond holdings are subsidized through seignorage.

The rest of the paper is organized as follows: We introduce the model in Section 2. In Section 3 we derive and characterize the equilibria and look at three special cases: constant taxes, pegged interest rates and hybrid monetary rules. Section 4 presents some quantitative examples that illustrate the different equilibria induced by the fiscal and monetary rules. Section 5 concludes.

2 Model

The environment is based on Leeper (1991). The economy is populated by a representative infinitely-lived household that receives a constant endowment of $y$ units of the consumption good every period and a government that needs to finance a constant stream of expenditures $g$. The next subsections describe the two agents in this economy.

2.1 Household

The household has preferences over consumption ($c_t$) and real money balances ($m_t$) and adjusts his portfolio of fiat currency ($M_t$) and nominal bonds ($B_t$) to maximize his utility within a fully flexible price environment. As in the U.S. and other OECD countries, all household income is taxed. The government extracts a constant flow of goods, $g < y$, that yield no
utility to consumers. The problem of the household can then be expressed as follows

$$\max_{\{c_t, m_t, b_t\}} \sum_{t=0}^{\infty} \beta^t [\log(c_t) + \log(m_t)]$$

s.t. $$c_t + \frac{M_t - M_{t-1}}{P_t} + \frac{B_t}{P_t} \leq (1 - \tau_t) y + \frac{B_{t-1}}{P_t} (1 + (1 - \tau_t) r_{t-1})$$

where $\beta$, $\tau_t$ and $P_t$ represent the discount rate, the income tax rate, and the price level, respectively. Upper case letters are used to identify nominal variables and lower case letters their real values, with the exception of $r_t$ which denotes the net before-tax nominal interest rate.

The first order conditions for the household yield the following Fisher and money demand equations

$$\frac{\Pi_{t+1}}{\beta} = 1 + (1 - \tau_{t+1}) r_t, \quad (2)$$

$$m_t = c \left[ \frac{1 + (1 - \tau_{t+1}) r_t}{(1 - \tau_{t+1}) r_t} \right], \quad (3)$$

where $\Pi_t = P_t/P_{t-1}$ denotes gross inflation. Since the endowment $y$ does not change over time and we have a constant level of government expenditure, households obtain a constant level of consumption, $c_t = c = y - g \forall t$.

As we can see, a proportional income tax only distorts the savings decision through changes in the rate of return that the household receives. Even when income is endogenous, this margin is not affected when lump sum taxes are considered, or when only labor and capital income are taxed as shown in Leeper and Yun (2006). Instead, in our environment the taxation of income from bond holdings affects the demand for government liabilities. In contrast with most papers in the literature, now the Fisher equation relates the after-tax interest on bonds to inflation which allows for a new channel for fiscal policy to affect the price level. In particular, now real balances are directly linked to both the monetary policy instrument, $r_t$, and the fiscal policy instrument, $\tau_t$. This new channel has important consequences for the local uniqueness of equilibrium.\(^{10}\) As in Schabert and Thadden (2009) we analyze how distortionary taxation, in our case through bond income, can drastically change the nature of equilibrium.\(^{11}\)

\(^{10}\)We would get similar qualitative results if a shopping-time setup was used instead. This is because in both cases (a) the Fisher equation relates the after-tax interest rate on bonds to inflation, and (b) the money demand depends on interest income from bond holdings.

\(^{11}\)Schabert and Thadden (2009) show that under passive fiscal policies, lump-sum taxes generate nominal indeterminacy, while with distortionary labor income taxes indeterminacy can be real and not just purely nominal.
2.2 Government

In order to finance its expenditures, the government relies on a proportional tax on endowments and interest income as well as the issuance of nominal bonds and fiat money. The resulting government budget constraint is then

\[
\frac{B_t}{P_t} + \frac{M_t}{P_t} + \tau_t y = g + \frac{M_{t-1}}{P_t} + (1 + (1 - \tau_t)r_{t-1})\frac{B_{t-1}}{P_t}.
\]

(4)

Following the FTPL literature, we specify the policy rules such that the central bank sets the interest rate following a Taylor rule and the fiscal authority takes the level of debt into account when deciding the tax rate. The policy rules are given by

\[
r_t = \alpha_0 + \alpha \Pi_t,
\]

(5)

\[
\tau_t = \gamma_0 + \gamma b_{t-1}.
\]

(6)

Note that the Taylor rule features net nominal interest rate rather than gross nominal interest rate. This implies that our constant \( \alpha_0 \) is equivalent to the intercept in Leeper (1991) minus one.

3 Equilibrium

After imposing market clearing conditions, the optimality conditions for the household, combining individual as well as government budget constraints and the policy rules, the monetary and fiscal equilibrium can be summarized by two intertemporal equations. These characterize the evolution of real government bonds, \( b_t \), and the inflation rate, \( \Pi_{t+1} \), which are given by

\[
\Pi_{t+1} = \beta + \beta(1 - \gamma_0 - \gamma b_t)(\alpha_0 + \alpha \Pi_t),
\]

(7)

\[
b_t = (1 - \gamma_0 - \gamma b_{t-1})y + (y - g) \left( \frac{1 + \beta - \Pi_t}{\Pi_t - \beta} \right) + \frac{b_{t-1}}{\beta} - (y - g) \left( \frac{\Pi_{t+1}}{\Pi_{t+1} - \beta} \right).
\]

(8)

As we can see, the equilibrium is characterized by a nonlinear system. In contrast to Leeper (1991), future inflation depends on both current inflation and current debt. As a result, the evolution of inflation and debt affect each other. This dynamic link has also been found in other environments. For instance, Yun (2011) shows how public debt plays a role in the intertemporal optimization problems of households in environments with sovereign risk. Davig and Leeper (2011) have an economy that randomly switches between active and passive policies through a Markov process and this delivers an environment where future inflation rates depend on past and current debt levels. Canzoneri and Diba (2005), on the other hand,
consider the liquidity services of bonds which predicts that future inflation rates depend both on past and current debt levels. In our setting it is the fact that income from bond holdings is taxed that generates this dynamic property. In contrast to the previous papers, the treatment of interest income can also yield multiple steady states which we explore in the next section.

3.1 Steady States

After imposing steady state conditions on equations (7) and (8), we can derive a quadratic equation characterizing the steady state income tax rate which is given by

\[-\Gamma_2 (1 - \tau)^2 + \Gamma_1 (1 - \tau) - \Gamma_0 = 0,\]

where \(\Gamma_2 = \left[\gamma y - \left(\frac{1}{\beta} - 1\right)\right] \Phi, \Gamma_1 = \left[(\alpha_0 + \alpha + \Phi) c\gamma - \Phi \left(\frac{1}{\beta} - 1\right) (1 - \gamma_0)\right]\) and \(\Gamma_0 = c\gamma \left(\frac{1}{\beta} - 1\right),\)

with \(\Phi \equiv (\alpha_0 + \alpha \beta) > 0.\) The steady state tax rates are then given by

\[\tau_i = 1 - \frac{-\Gamma_1 \pm \sqrt{(\Gamma_1)^2 - 4\Gamma_2\Gamma_0}}{-2\Gamma_2} \quad i = 1, 2.\] (9)

Even though fiscal and monetary policies in our environment do not change future resources, once bond income is taxed part of the tax base changes in response to new taxes. This is because the fiscal authority sets taxes taking into account debt and the demand for nominal bonds explicitly depends on the income tax rate. Thus it is not too surprising that our economy can potentially deliver multiple steady states. A Laffer curve is also observed when income is endogenous.

We now establish conditions that guarantee the existence of multiple steady states. In order to obtain an economically meaningful equilibrium, we restrict the tax rates so that \(\tau_i < 1.\) Notice that in principle a government can subsidize more than the income generated from bond holdings, \(\tau_i < -1,\) as long as it pays this subsidy through seignorage.

**Proposition 1** There exist two steady states with tax rates less than one when the conditions for Case (I) are satisfied. A unique steady state with a tax rate less than one exists when the conditions for Case (II) or Case (III) are satisfied.

**Proof.** See Appendix A. □

As in Leeper (1991), we impose no a priori restrictions on signs and magnitudes of the policy parameters. We simply assume that \(\alpha_0, \alpha, \gamma_0\) and \(\gamma\) are such that we obtain steady state values that are economically meaningful. Thus we focus on equilibria such that the tax rate is less than one and real debt, real money balances and nominal interest rate are positive.
\[
\begin{array}{ccc}
\Gamma_0 & \Gamma_1 & \Gamma_2 \\
\hline
\text{Case (I): } \tau_1 < 1, \tau_2 < 1 & + & + & + \\
\text{Case (II): } \tau_1 < 1, \tau_2 > 1 & - & - & + \\
\text{Case (III): } \tau_1 > 1, \tau_2 < 1 & + & + & - \\
\end{array}
\]

**Table 1:** The three cases giving at least one \( \tau_i < 1 \).

The table shows the conditions on \( \Gamma_0, \Gamma_1 \) and \( \Gamma_2 \) that will give at least one \( \tau_i < 1 \). In Case (I) both taxes are less than one which means that are two steady states. In cases (II) and (III) a unique steady state exists since it is not feasible to have a tax of over 100% of income.

### 3.2 Dynamic Properties

The dynamic monetary equilibrium of this economy is characterized by equations (7) and (8). In order to study the local stability of the equilibria, we evaluate the Jacobian of this system at steady state, which is given by

\[
J_{ss} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}
\]

where

\[
A = \frac{\partial \Pi_{t+1}}{\partial \Pi_t} \bigg|_{ss} = \frac{\alpha \beta (1 - \gamma_0 - \gamma b) + \theta}{[1 + \beta \theta]} \\
B = \frac{\partial \Pi_{t+1}}{\partial b_{t-1}} \bigg|_{ss} = -\beta \gamma (\alpha_0 + \alpha \Pi) D \\
C = \frac{\partial b_t}{\partial \Pi_t} \bigg|_{ss} = \frac{(y - g)}{(\Pi - \beta)^2} [\beta A - 1] \\
D = \frac{\partial b_t}{\partial b_{t-1}} \bigg|_{ss} = \frac{-\gamma y + \frac{1}{\beta}}{[1 + \beta \theta]}
\]

with \( \theta \equiv \beta \gamma (\alpha_0 + \alpha \Pi) \left[ \frac{(y - g)}{(\Pi - \beta)^2} \right] > 0 \).

We can then compute the eigenvalues associated with \( J_{ss} \) which are given by

\[
\lambda_i = \frac{\text{tr}(J_{ss}) \pm \sqrt{\Delta_j}}{2}, \quad i = 1, 2,
\]

where \( \Delta_j = (A - D)^2 + 4D \theta [1 - \beta A] \) and \( \text{tr}(J_{ss}) = \left( \alpha \beta (1 - \gamma_0 - \gamma b) + \theta - \gamma y + \frac{1}{\beta} \right) / (1 + \beta \theta) \).

As we can see, the eigenvalues in our environment are quite different from those found
when interest income from holding bonds is not taxed which are given by

\[ \lambda_1 = \alpha \beta \quad \text{and} \quad \lambda_2 = \frac{1}{\beta} - \gamma. \]

When bond income is taxed, the intercepts in the fiscal and monetary policy rules, \( \alpha_0 \) and \( \gamma_0 \), affect the dynamic properties of the equilibria. Moreover, the stability of the economy is a highly nonlinear function of the intercepts and slopes of fiscal and monetary policy rules. As a result, the traditional policy prescriptions in the FTPL regarding the uniqueness of equilibria are not going to hold anymore. This is the case as the demand for real balances are directly linked to both monetary and fiscal policy rules. As a result, the size of the wealth effect induced by changes in the initial price level are going to be rather different.

### 3.3 Special Cases

In this section we study some environments that allow us to obtain analytic solutions and that are widely studied in the literature. This can help us to shed some light on the key aspects of the model driving our results.

#### 3.3.1 Constant Taxes

To better understand what drives the multiplicity of steady states and the dynamic properties of our economy, we analyze a fiscal rule where the fiscal authority sets taxes independently of the level of debt as in Sims (2011). Under this new environment where \( \tau_t = \gamma_0 \), the resulting dynamical system is given by

\[
\Pi_{t+1} = \beta + \beta(1 - \gamma_0)(\alpha_0 + \alpha \Pi_t),
\]

\[
b_t = (1 - \gamma_0)y + (y - g)\left(1 + \frac{\beta - \Pi_t}{\Pi_t - \beta}\right) + \frac{b_{t-1}}{\beta} - (y - g)\left(\frac{\Pi_{t+1}}{\Pi_{t+1} - \beta}\right).
\]

As we can see, the evolution of inflation depends on the fiscal rule through \( \gamma_0 \). However, with this simpler fiscal policy rule, the evolution of the inflation rate is now independent of the level of debt, \( b_t \). As a result, the dynamic equations describing inflation are linear and independent of the and evolution of debt. Current debt depends linearly on past debt so at most a unique steady state exists. Thus, multiple steady states are not possible when income tax rates are independent of debt even when bond income is taxed. This is the case as the income tax rate can not adjust so that a Laffer curve does not exist. The steady state inflation
rate associated with this simpler fiscal rule is given by

\[ \Pi = \frac{\beta(1 + (1 - \gamma_0)\alpha_0)}{1 - \alpha \beta (1 - \gamma_0)}, \]

with associated eigenvalues given by

\[ \lambda_1 = \alpha \beta (1 - \gamma_0) \quad \text{and} \quad \lambda_2 = 1/\beta. \]

As in Sims (2011), in our economy there exist equilibria where interest rate rules that satisfy the Taylor principle can be consistent with a unique equilibrium in which inflation explodes. Such an equilibrium is consistent with \( \alpha > 1/\beta \) and when \( \gamma_0 \) is small. Moreover, our environment is also able to support a larger set of determinate equilibria that follow the Taylor principle and where \( \gamma_0 > 0 \) is sufficiently large. This example clearly illustrates the importance of the policy rules’ intercepts in determining the nature of the equilibrium, something which has not been emphasized by the literature. Thus the traditional policy prescriptions under this simpler environment are rather different from those found in Leeper (1991) as the intercept of the fiscal rule matters.

### 3.3.2 Pegged Nominal Interest Rates

Rather than simplifying the fiscal rule, we now consider a monetary policy rule that does not respond changes in the economic environment. When the monetary authority sets a pegged net nominal interest rate, \( \alpha = 0 \), the monetary policy rule simply becomes \( r_t = \alpha_0 > 0 \). In contrast to the previous case, the fiscal authority relates current taxes with the level of debt so that \( \tau_t = \gamma_0 + \gamma b_{t-1} \). The corresponding dynamical system describing this new environment is given by

\[ \Pi_{t+1} = \beta \left[ 1 + (1 - \gamma_0 - \gamma b_t)\alpha_0 \right], \]

\[ b_t = g + \frac{c}{\Pi_t - \beta} + \frac{b_{t-1}}{\beta} - c \left( \frac{\Pi_{t+1}}{\Pi_{t+1} - \beta} \right) - (\gamma_0 + \gamma b_{t-1})y. \]

As we can see, the evolution of the inflation rate now depends on the current level of debt. Moreover, there exists a nonlinear relationship between current and past debt as current inflation is linked with past debt. Thus this environment can, in principle, generate multiple steady states. The steady state income tax rates are given by

\[ \tau_i = 1 - \frac{-\Gamma_1 \pm \sqrt{(\Gamma_1)^2 - 4\Gamma_2 \Gamma_0}}{-2\Gamma_2} \quad i = 1, 2 \quad (11) \]
where \( \Gamma_2 = \left[ \gamma y - \left( \frac{1}{\beta} - 1 \right) \right] \alpha_0 \), \( \Gamma_1 = 2\alpha_0 c\gamma - \alpha_0 \left( \frac{1}{\beta} - 1 \right) (1 - \gamma_0) \) and \( \Gamma_0 = c\gamma \left( \frac{1}{\beta} - 1 \right) \). The condition that guarantees the existence of multiple equilibria is a special case of Proposition 1 and is given by:

**Proposition 2** There exist two steady states with tax rates less than one when the conditions for Case \((I_p)\) are satisfied. There exists a unique steady state with a tax rate smaller than one when the conditions for Case \((II_p)\) are satisfied.

**Proof.** See Appendix A. ■

<table>
<thead>
<tr>
<th>( )</th>
<th>( \Gamma_0 )</th>
<th>( \Gamma_1 )</th>
<th>( \Gamma_2 )</th>
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</thead>
<tbody>
<tr>
<td>Case ((I_p)): ( \tau_1 &lt; 1, \tau_2 &lt; 1 )</td>
<td>+</td>
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<td>+</td>
</tr>
<tr>
<td>Case ((II_p)): ( \tau_1 &gt; 1, \tau_2 &lt; 1 )</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
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Table 2: Description of the various cases for \( \tau_i \) when \( \alpha = 0 \).

When \( \alpha = 0 \) there are only two possible combinations of values of \( \tau_i \) where at least one of them is economically plausible, i.e. less than one. In Case \((I_p)\) both \( \tau_i < 1 \) and there are two steady states, whereas in Case \((II_p)\) only one of the taxes is less than one and thus a unique steady state exists.

Once bond income is taxed and the income tax rate depends on debt, part of the tax base changes in response to new taxes. This is the case as the demand for nominal bonds explicitly depends on the income tax rate. Thus, the new tax directly affects the demand for bonds which results in a Laffer curve.

Given that the evolution of debt and inflation rates are not independent of each other, we are not able to obtain an analytic expression for the eigenvalues in terms of just policy and preference parameters. The Jacobian corresponding to this economy is given by

\[
J_{ss} = \frac{1}{\left( \Pi - \beta \right)^2 + \gamma \alpha_0 c\beta^2} \begin{bmatrix}
-\alpha_0 \gamma \beta & -\gamma \alpha_0 \left( 1 - \beta \gamma y \right) \left( \Pi - \beta \right)^2 \\
\gamma \beta y \beta & 1 - \gamma \beta y \beta \left( \Pi - \beta \right)^2
\end{bmatrix}.
\]

The eigenvalues of this Jacobian,

\[
\lambda_1 = 0 \quad \text{and} \quad \lambda_2 = \frac{1}{\beta} \frac{(1 - \gamma \beta y) \left( \Pi - \beta \right)^2 - \alpha_0 \gamma \beta^2}{\left( \Pi - \beta \right)^2 + \gamma \alpha_0 c\beta^2},
\]

depend not only on fiscal policy parameters but also on the monetary policy parameter. Since \( \lambda_1 = 0 \), the unique saddle-path equilibrium requires \( |\lambda_2| > 1 \).
The eigenvalues associated to an interest rate peg when bond income is not taxed, as in Leeper (1991), are given by

\[ \lambda_1 = 0 \quad \text{and} \quad \lambda_2 = 1/\beta - \gamma. \]

So long as \(|1/\beta - \gamma| > 1\) there exists a unique saddle-path equilibrium where the pegged nominal interest rate fixes real money balances and makes inflation constant. Hence, tax should adjust to satisfy the government budget constraint. In contrast, when bond income is taxed real money balances and inflation are not constant and depend on the parameters of the fiscal policy rule.

As we can see, there exist multiple steady states even when the monetary policy rule is simple. This is the case as taxing interest from bond holdings generate a Laffer curve that induces a nonlinear dynamical system.

### 3.3.3 Hybrid Monetary Rules

In this section we study a hybrid monetary policy rule where fiscal and monetary instruments are combined in one of the policy rules. In contrast to Yun (2011) we analyze a monetary policy rule that explicitly takes into account fiscal observables. In particular, the monetary policy rule we consider features the after tax net nominal interest rate. This augmented monetary policy is given by

\[ (1 - \tau_{t+1})r_t = \alpha_0 + \alpha\Pi_t. \]

Under this new monetary policy rule, the Fisher and money demand equations do not change and the income tax rate explicitly appears in both of them. However, this new channel has an important effect on the dynamic characterization of the equilibria. The intertemporal equations that describe the evolution of inflation and real debt are given by

\[ \Pi_{t+1} = \beta(1 + \alpha_0 + \alpha\Pi_t), \]
\[ b_t = g + \left( \frac{c}{\Pi_t - \beta} \right) + \frac{b_{t-1}}{\beta} - c \left( \frac{\Pi_{t+1}}{\Pi_{t+1} - \beta} \right) - (\gamma_0 + \gamma b_{t-1})y. \]

As we can see, the evolution of the inflation rate is now independent of the level of debt as in Leeper (1991). Moreover, the relationship between current and past inflation is linear. Similarly, the evolution of current debt depends linearly on past debt. As a result, this economy has a unique steady state. The steady state inflation and real bond holdings are
given by

\[ \Pi = \frac{\beta (1 + \alpha_0)}{1 - \alpha \beta}, \]
\[ b = \frac{g - \gamma_0 y + c \left( \frac{1 - \Pi}{\Pi - \beta} \right)}{\left( 1 - \frac{1}{\beta} + \gamma y \right)}. \]

Since the evolution of the inflation rate is now independent of the level of debt, the associated Jacobian is greatly simplified yielding the following eigenvalues

\[ \lambda_1 = \beta \alpha \quad \text{and} \quad \lambda_2 = \frac{1}{\beta} - \gamma y. \]

As this example shows, having a hybrid monetary policy can rule out multiplicity of equilibria. This is the case as the monetary authority internalizes the fact that changes in the fiscal rule affect the demand for real balances.

The three special cases examined indicate that what drives the multiplicity of steady states is the fact that income from bond holdings are taxed and that tax rates depend on debt levels. It is difficult to derive further analytical results since a sufficient condition that guarantees the stability properties of this economy depends on the slope and the intercept of both monetary and fiscal policy rules. For this reason, in the next section we conduct a quantitative analysis to study the dynamic properties of our economy.

4 Quantitative Examples

Given the nonlinear nature of the dynamical system that describes our economy, we resort to numerical examples to have a deeper understanding of the resulting equilibrium. In this section we present two simple experiments. First we consider a range of values for the intercept and slope parameters of the policy rules while examining the dynamic properties of the model in that parameter space. In the second experiment, we fix the values of the intercepts so that they are consistent with the empirical estimates found in the literature.

In what follows we consider an economy with a discount factor equal to 0.96 and shares of consumption and government expenditure equal to 0.7 and 0.3, respectively. We restrict attention to equilibria with tax rates less than one, positive nominal interest rates and positive demands for money and bond holdings.

By comparing our equilibria to those of Leeper (1991) we are able to highlight the consequences of taxing interest income generated by bond holdings. For that reason we mark in our figures Leeper’s regions of policy parameter space. The determinate regions, those where one
of the eigenvalues is less than one while the other is greater than one, are the areas labelled I and II. Area I corresponds to the monetary region \(|\alpha / \beta| > 1\) and \(|1/\beta - \gamma| < 1\), whereas area II is the fiscal region \(|\alpha / \beta| < 1\) and \(|1/\beta - \gamma| > 1\). When both eigenvalues are inside the unit circle, we have the sunspot (indeterminate) region, represented in the figures with a III. Finally, area IV is the explosive region where both eigenvalues lie outside the unit circle.

### 4.1 Benchmark Experiment

In this first experiment we consider parameter values for the intercept and slopes of policy rules \((\alpha_0, \gamma_0, \alpha, \gamma)\) that belong to the interval \([-2.6, 2.6]\). We only report combinations of policy parameters that deliver economically sensible steady states. Note that each point in any of the following figures may represent more than one combination of policy parameters as different equilibria that have the same slope, \(\alpha\) and \(\gamma\), may have different values for the intercepts, \(\alpha_0\) and \(\gamma_0\).

Figure 1 shows combinations of slopes of fiscal and policy rules, \(\alpha\) and \(\gamma\), that generate determinate equilibria. In Leeper (1991) this would correspond to Regions I and II, but in our case the area where determinate equilibria are found is much larger. This is because in our environment both the evolution of the inflation rate and debt levels are not independent of each other, and so the corresponding eigenvalues depend on both monetary and fiscal policy parameters. In other words, we can find combinations of parameter values that are outside Leeper’s regions I and II that still yield determinate equilibria in our economy.

As in Yun (2011), we can restore the local uniqueness of equilibrium even when monetary policy does not follow the Taylor principle, but we do that without assuming the risk of sovereign default. Instead we just simply impose that income generated by bond holdings is taxed.

Similarly to us, Canzoneri and Diba (2005) also find that deviations from the Taylor principle can yield determinate equilibria even when fiscal policy does not aggressively respond to rises in debt levels. The authors find that the resulting dynamics of debt are rather different from the standard model where bonds don’t provide liquidity services. In our economy the expansion of the determinate equilibria region is a robust feature as they are found in all of our parameter space.

Of particular interest is the case of pegged nominal interest rate. Under this simple monetary policy rule, the interest rate does not respond to changes in inflation so that \(\alpha\)

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12 A more rigorous quantitative approach requires calibrating the parameters in our policy rules. This is not possible since the number of parameters, five, is more than number of equations, three, in steady state. One possible approach to reduce the number of parameters is to set the intercepts \((\alpha_0\) and \(\gamma_0\)) to zero. This is not a good approach since in our model the intercepts affect the stability condition of system.

13 To simplify exposition we only report combination of parameter values found in the grid with a 0.2 step.
This figure shows the values of $\alpha$ and $\gamma$ for which determinate equilibria are found. While in Leeper (1991) the determinate region of the parameter space is restricted to the areas marked with I and II in the figure, when the interest income from bonds is taxed the set of possible parameter values resulting in a determinate equilibrium is greatly expanded as shown by the crosses.

is zero and $\alpha_0$ is positive. As can be seen from Figure 1, when $\alpha = 0$ we can still find determinate equilibria for a range of values of the fiscal policy parameter, $\gamma$. In contrast, when $\alpha = 0$ and $\gamma \in [1/\beta - 1, 1/\beta + 1]$, Leeper (1991) finds indeterminate equilibria. In other words, the determinate region in our setting is larger than Leeper’s even for the case of pegged interest rate. In our environment, the fact that income from bond holdings is taxed generates a new link for fiscal policy to affect the price level when monetary policy is passive. Other studies rely on other mechanisms through which fiscal policy can provide a nominal anchor in the case of interest rate peg.

Figure 1 clearly shows that the determinacy of equilibrium drastically changes once the government taxes interest income generated by bond holdings. This is the case as fiscal policy directly affects the demand for government liabilities. This feature, previously ignored by the literature, is of first order importance when analyzing the dynamic properties of the economy.

Combinations of slopes, $\alpha$ and $\gamma$, that deliver indeterminate equilibria are summarized in Figure 2. In Panel A we report equilibria with both eigenvalues inside the unit circle which for Leeper (1991) corresponds to region III. Panel B shows the explosive region where both eigenvalues lie outside the unit circle (region IV). In both panels equilibria with complex eigenvalues are represented by a dash while those with real eigenvalues are shown with a circle.

Figure 2 makes clear that introducing a tax on interest income considerably alters the steady state and the dynamic properties of the model relative to Leeper (1991). For instance, the explosive region can be consistent with equilibria following the Taylor principle and fiscal policy not aggressively adjusting to debt levels. Furthermore, sunspot and explosive equilibria can generate non-damped oscillation as their associated eigenvalues are complex. With repre-
sentative agent frameworks, major departures from the standard model are needed to generate complex eigenvalues. In contrast, in this paper we show that complex eigenvalues can exist in a complete market endowment economy with fully flexible prices and infinitely-lived agents. The only requirement is for interest income from bond holdings to be taxed which is a less controversial and more realistic feature.

Panel A in Figure 3 depicts the combinations $\alpha$ and $\gamma$ that only yield determinate equilibria, regardless of the values of the intercept. In other words, this graph contains the subset of steady states from Figure 1 which are not in Figure 2. It should be noted that the vertical axis reports the absolute value of $\alpha$ and hence the points in Figure 3 located in the south west corner which appear to also be in Figure 2 have different signs for $\alpha$. In particular, the determinate equilibria corresponds to $\alpha > 0$.

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14 Examples include large non-convexities due to monopolistic competition, as in Gali (1996), or technological increasing returns to scale, as in Jess and Farmer (1994). Samuelson (1939) also generate complex eigenvalues in a non-classical growth model with investment, gestation and production lags. Azariadis, Bullard, and Ohanian (2004) also show that complex eigenvalues exist in overlapping generations economies with pure exchange.
Panel A: Subset of Determinate Equilibria

Panel B: Subset of Determinate Equilibria, $\alpha_0,\gamma_0 = 0$

**Figure 3:** Parameter values which deliver determinate equilibria only.

Here we show the subset of values of $\alpha$ and $\gamma$ which yield only determinate equilibria. Panel A does that for all values of the intercept considered, $\alpha_0,\gamma_0 \in [-2.6,2.6]$, whereas Panel B restricts the intercept to be equal to zero. The point here is to see that even if we disregard the role of intercepts in expanding the set of determinate steady states, we still observe an expansion of the determinate region in relation to Leeper (1991) since in that paper we would be restricted to the areas marked with I or II only.

Since both the intercept and slope parameters affect the eigenvalues in a nonlinear fashion we are able to expand the set of determinate equilibria. In order to isolate the effect of the slopes, in Panel B we set the policy rule intercepts to zero and plot the combinations of $\alpha$ and $\gamma$ which give determinate equilibria. As we can see, even when we ignore the role of the intercepts, the nonlinear interactions of the slopes still deliver a plenitude of determinate steady states. Thus we can conclude that the nonlinearity introduced by taxing income from bond holdings is key in expanding the set of determinate equilibria.

Thus far we just have reported combinations of parameter values that deliver a unique steady state. Nonetheless, as we have shown in Section 3.1, it is possible to generate economies with two steady states. This type of equilibrium is depicted in Figure 4. Panel A shows the combinations of slope coefficients where at least one of the steady states is associated with a determinate equilibrium. Among these equilibria, the roots that have at least one pair of complex eigenvalues are represented by a dash. Panel B, on the other hand, shows equilibria where both steady states are determinate.
Figure 4: Multiple steady states.

Here we show the combinations of slope parameters that deliver two steady states. In Panel A we have the cases where at least one of the steady states gives a determinate equilibrium, whereas Panel B shows the cases where both steady states are determinate.

For the case where $\alpha = 0$, i.e. pegged interest, we find multiple steady states. This is in sharp contrast to the typical predictions of the FTPL. The literature that investigates the issue of determinacy under interest rate peg finds a unique steady state when price is determinate.\(^\text{15}\)

As we have seen, once the government taxes interest income generated from bond holdings, the particular value of the intercept of the fiscal and monetary rules are crucial. Depending on the specific value that it takes, the economy can generate a large set of determinate equilibria where the Taylor principle does not apply and the fiscal authority aggressively responds to rises in debt.

### 4.2 Empirically Plausible Intercept Values

Results from Section 3.2 and the quantitative examples from the previous section highlight that both monetary and fiscal policy intercepts, $\alpha_0$ and $\gamma_0$, drastically affect the stability

\(^{15}\)As an example see Canzoneri and Diba (2005) that explores the determinacy of equilibrium when bonds provide liquidity services.
properties of the equilibria. In this section we restrict our attention to values of the intercepts that are consistent with the empirical literature on fiscal and Taylor rules.

Since the effect of intercepts on stability conditions in the FTPL has not been previously recognized, it is not too surprising then that the empirical literature estimating fiscal and monetary rules typically do not report estimates for these intercepts.\textsuperscript{16} An exception is that of Davig and Leeper (2011). These authors estimate similar fiscal and monetary policy rules and find the intercepts to be $\alpha_0 = 0.0058$ and $\gamma_0 = 0.004$, respectively.

In what follows we fix the value of the intercepts to the previous estimates and analyze the properties of equilibria when the slopes of the fiscal and monetary policy rules belong to the interval $[-2.6, 2.6]$. Figure 5 reports the unique determinate equilibria. We can conclude then that set of parameter values consistent with determinate equilibria is still larger than in Leeper (1991). Moreover, there exist equilibria that can accommodate policies that deviate from the Taylor principle while having an active or passive fiscal policy. Relative to the unrestricted case, Figure 1, the set of determinate equilibria belonging to the north west region is empty and the set belonging to the region on the south west corner is smaller. All of these results are robust a one standard deviation change from the intercept estimates.

Equilibria with multiplicity of steady states is also observed even when we restrict the intercepts to the parameter estimates in the empirical literature. In this case, all eigenvalues are real and we show the corresponding equilibria in Panel A of Figure 6. Interestingly, since two steady states only exist when $\alpha = 0.06$, it will always be the case that the Taylor principle does not hold and monetary policy barely reacts to inflation.

\textsuperscript{16}See for example Reicher (2009). Some papers also estimate the rules in log deviations from the mean and for that reason do not include the intercept. This presumes that the intercepts aren’t important, a belief that is made clear when Clarida, Gali, and Gertler (2000, p.169) mention the fact that they ignore “uninteresting constants”.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{Determinate Equilibria when $\alpha_0 = 0.0058$ and $\gamma_0 = 0.004$.}
\end{figure}
Figure 6: Multiple steady states when $\alpha_0 = 0.0058$ and $\gamma_0 = 0.004$.

Panel A shows the values of $\alpha$ and $\beta$ which result in multiplicity of steady states when the intercepts of the policy rules are restricted to match the empirical literature. In this case, the only value of the slope of the monetary policy rule consistent with multiple steady states is $\alpha = 0.06$. Panel B shows the values of $\gamma$ giving rise to two steady states and the corresponding values of taxes in each of them when $\alpha_0 = 0.0058$, $\gamma_0 = 0.04$ and $\alpha = 0.06$.

We can conclude then that under empirically plausible parameter estimates for the intercepts of the monetary and fiscal policy rules, multiplicity of equilibria is unlikely. The corresponding steady state values for the tax rate $\tau$ range from 0.07 to a maximum of 0.49. As we lower $\gamma$ the possibility of having multiple steady states declines.

5 Conclusion

This paper incorporates the taxation of interest income from bond holdings into the standard FTPL framework, a feature that is observed in the U.S. and many other industrialized economies. This minimal and realistic departure from the standard FTPL generates a new set of results. In this new environment, the demand for government liabilities explicitly depends on fiscal policy. There is then a new direct effect through which fiscal policy can affect the price level. Thus an additional substitution effect exists between current and future consumption. This new feature drastically changes the nature of the equilibrium.
We show that the stability properties of this economy depend on both the slope and the intercept of the monetary and fiscal rules. The eigenvalues are a highly nonlinear function of the parameters of the policy rules. Thus the properties of the equilibria are rather different from the literature, where the intercept plays no role. The introduction of a tax on bond income expands the set of policies that yield determinate equilibria relative to Leeper (1991). In particular, deviations from the Taylor principle can still yield determinate equilibria even when fiscal policy does not aggressively respond to rises in debt levels.

Finally, we show that having a fiscal rule that depends on previous debt and taxing interest income are key in generating multiplicity of steady states. A sufficient condition that guarantees that the stability properties of the economy depend on the slope and the intercept of both monetary and fiscal policy rules is for bond income to be taxed. The traditional policy prescriptions of the FTPL no longer hold once bond income is taxed. Thus ignoring the tax treatment of the interest income from bond holdings is not as innocuous as it may seem.

References


A Appendix

Proof of Proposition 1. The proof is trivial and we just show it for four cases. Recall that

\[ \tau_1 = 1 - \frac{-\Gamma_1 - \sqrt{\Delta}}{-2\Gamma_2} \quad \text{and} \quad \tau_2 = 1 - \frac{-\Gamma_1 + \sqrt{\Delta}}{-2\Gamma_2} \]

In all cases we assume that \( \Delta \equiv (\Gamma_1)^2 - 4\Gamma_2 \Gamma_0 \) is greater than zero.

1) Suppose \( \Gamma_0, \Gamma_1, \Gamma_2 > 0 \). In this case, \(-4\Gamma_2 \Gamma_0, -\Gamma_1, -2\Gamma_2 < 0 \). In addition \( |\sqrt{\Delta}| < |\Gamma_1| \). This implies \(( -\Gamma_1 \pm \sqrt{\Delta} ) / ( -2\Gamma_2 ) > 0 \) and therefore \( \tau_1 \) and \( \tau_2 \) are less than one.

2) Suppose \( \Gamma_0, \Gamma_1, \Gamma_2 < 0 \). Then \(-\Gamma_1, -2\Gamma_2 > 0 \) while \(-4\Gamma_2 \Gamma_0 < 0 \) and consequently \( |\sqrt{\Delta}| < |\Gamma_1| \). This implies \(( -\Gamma_1 \pm \sqrt{\Delta} ) / ( -2\Gamma_2 ) > 0 \) and thus \( \tau_1 \) and \( \tau_2 \) are less than one.

3) Suppose \( \Gamma_0 < 0 \) and \( \Gamma_1, \Gamma_2 > 0 \). In this case \(-4\Gamma_2 \Gamma_0 > 0 \) and hence \( |\sqrt{\Delta}| > |\Gamma_1| \). This implies \(( -\Gamma_1 - \sqrt{\Delta} ) / ( -2\Gamma_2 ) > 0 \) while \(( -\Gamma_1 + \sqrt{\Delta} ) / ( -2\Gamma_2 ) < 0 \). Therefore, \( \tau_1 \) (\( \tau_2 \)) is less (greater) than one. The proof for the case where \( \Gamma_0, \Gamma_1 < 0 \) and \( \Gamma_2 > 0 \) is similar to this one.

4) Suppose \( \Gamma_0 > 0 \) and \( \Gamma_1, \Gamma_2 < 0 \). Then \(-4\Gamma_2 \Gamma_0 > 0 \) and hence \( |\sqrt{\Delta}| > |\Gamma_1| \). This implies \(( -\Gamma_1 - \sqrt{\Delta} ) / ( -2\Gamma_2 ) < 0 \) while \(( -\Gamma_1 + \sqrt{\Delta} ) / ( -2\Gamma_2 ) > 0 \). Hence, \( \tau_2 \) (\( \tau_1 \)) is less (greater) than one. The proof for the case where \( \Gamma_0, \Gamma_1 > 0 \) while \( \Gamma_2 < 0 \) is analogous to this one. \( \blacksquare \)

Proof of Proposition 2. The proof is based on the Proof of Proposition 1. Steady state tax rates, in the case of pegged interest rate, are given by

\[ \tau_1 = 1 - \frac{-\Gamma_1 - \sqrt{\Delta}}{-2\Gamma_2} \quad \text{and} \quad \tau_2 = 1 - \frac{-\Gamma_1 + \sqrt{\Delta}}{-2\Gamma_2} \]

where \( \Delta \equiv (\Gamma_1)^2 - 4\Gamma_2 \Gamma_0 \), \( \Gamma_2 = \left[ \gamma y - \left( \frac{1}{\beta} - 1 \right) \right] \alpha_0 \), \( \Gamma_1 = 2\alpha_0 c\gamma - \alpha_0 \left( \frac{1}{\beta} - 1 \right) (1 - \gamma_0) \) and \( \Gamma_0 = c\gamma \left( \frac{1}{\beta} - 1 \right) \) and in all cases we assume that \( \Delta > 0 \). Recall that \( \alpha_0 \) should also be greater than zero in this case since the net interest rate is bounded below by zero.

First, suppose \( \gamma < 0 \). Then \( \Gamma_2 \) and \( \Gamma_0 \) are negative while \( \Gamma_1 \) can be positive or negative. When \( \Gamma_2 < 0 \), \( \Gamma_0 < 0 \) and \( \Gamma_1 < 0 \) we end up in case (2) of the Proof of Proposition 1 in which \( \tau_1 \) and \( \tau_2 \) are less than one. If \( \Gamma_2 < 0 \), \( \Gamma_0 < 0 \) and \( \Gamma_1 > 0 \) we end up in the case where \( \tau_1 \) and \( \tau_2 \) are both greater than one and we have an explosive equilibrium.

Second, suppose \( \gamma > 0 \) and therefore \( \Gamma_0 > 0 \). In this case \( \Gamma_1 \) and \( \Gamma_2 \) can be positive or negative. When \( \Gamma_0, \Gamma_1, \Gamma_2 > 0 \) we will be in case (1) of the Proof of Proposition 1 in which two steady states exist and both tax rates are less than 1. When \( \Gamma_0, \Gamma_1 > 0 \) and \( \Gamma_2 < 0 \) or when \( \Gamma_0 > 0 \) and \( \Gamma_1, \Gamma_2 < 0 \) we end up in case (4) of the Proof of Proposition 1 and we have
a determinate equilibrium with one steady state. Both tax rates are greater than one when $\Gamma_0, \Gamma_2 > 0$ and $\Gamma_1 < 0$. ■