Optimal Managerial Contracts with Self-Esteem Concerns When Managers Can Hedge

Chongwoo Choe* Donald Lien† and Chia-Feng Yu‡

Abstract
We study an optimal contracting problem when the manager has self-esteem concerns and access to a hedging market. We show that the manager's equilibrium hedging position increases when he is more risk-averse, more uncertain about his own ability, or has stronger self-esteem concerns. Each element of managerial hedging and self-esteem concerns added to an otherwise standard agency model increases the equilibrium pay-performance sensitivity. The agency cost increases as the manager's self-esteem concerns become stronger, but the manager's access to hedging opportunities itself does not change the agency cost. We also provide conditions for the positive relationship between risk and incentives.

Keywords: Managerial Hedging; Executive Compensation; Self-Esteem.
JEL Classification Numbers: D86, G02, G32.

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1 Introduction

“He that is proud eats up himself: pride is his own glass, his own trumpet, his own chronicle; and whatever praises itself but in the deed, devours the deed in the praise.” - Shakespeare: Troilus and Cressida: II, iii.

The classical agency theory of the firm starts from a conflict of interest between managers and shareholders and suggests that managers’ pay be contingent on firm performance to align the interests of managers with those of shareholders. With contingent compensation schemes, managers, on one hand, have a stronger incentive to exert effort to improve firm performance. On the other hand, managers are faced with greater exposure to risk beyond their control. The optimal managerial compensation is thus the result of a trade-off between incentive provision and risk sharing. But the classical theory is developed in a highly simplified environment that abstracts away a number of factors that may be significant in reality. The purpose of this paper is to incorporate two such factors into an otherwise standard contracting environment and examine how the optimal managerial contract deviates from the one in the classical environment. First, we consider the possibility that managers can use hedging instruments to unilaterally alter the incentives and the risk in their compensation packages. Second, we incorporate managers’ concern for self-esteem. In particular, we are interested in how these two factors change the pay-performance sensitivity and agency cost implied by the optimal contract. We provide below the rationale for why it is important to consider these two factors in studying the managerial contract observed in practice.

Current contract and security laws put limited barriers on managerial hedging transactions, and therefore managers have some leeway regarding their hedging decisions. A volume of empirical evidence documents that hedging activities of managers have grown rapidly along with the vigorous development of financial derivatives (e.g., Bettis et al. 2001; Easterbrook 2002; Jin 2002; Garvey and Milbourn 2003; Gao 2010). The implications of managerial hedging for executive compensation design have earned much academic attention. Jin (2002), Garvey and Milbourn (2003), and Ozerturk (2006a) study the case where managers can trade market indexes to diversify away systematic risk. Jin (2002) and Garvey and Milbourn (2003) find that the pay-performance sensitivity of incentive contracts falls with the idiosyncratic risk of firms’ cash flows but is invariant to market risk. Ozerturk (2006a) shows that due to imperfect market liquidity, the manager’s optimal hedge is not complete, and equilibrium pay-performance sensitivity and firm value increase in market liquidity. Further, Ozerturk (2006b) and Gao (2010) study the case where managers can hedge their firm-specific risk exposure in their undiversified portfolios. Ozerturk (2006b) shows that if the manager can hedge up to a known fixed number of trading rounds, the manager will not hedge completely, and the ex ante optimal pay-performance sensitivity with hedging is strictly higher than that with no hedging opportunity. Gao (2010) shows that pay-performance sensitivity decreases with the manager’s hedging

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1See Jensen and Meckling (1976) or Bolton and Dewatripont (2005).
2For example, according to Section 16 (c) of the Securities and Exchange Act of 1934 and Rule 16c-4, it is legal for a manager to buy put options as long as the amount of securities underlying the put equivalent position does not exceed the amount of underlying securities otherwise owned. Schizer (2000) and Gao (2010) also find that contractual prohibitions on executive hedge transactions are quite rare.
cost, and shareholders impose a high sensitivity of the manager’s wealth to stock volatility and increase financial leverage to resolve the managerial hedging problem. Following these studies, this paper focuses on where managers can trade to alter firm-specific risk in their compensation. It can be rationalized because managers absorb most of their wealth from working within the firm and thus have an undiversified portfolio position. Moreover many managerial hedging instruments are designed to hedge firm-specific risk rather than systematic risk (Bettis et al. 2001).³

The second element in our paper is the role of managers’ self-esteem, which can be defined as a person’s overall emotional evaluation of his or her own worth (Rosenberg 1979). The manager, at the top of the corporate ladder, enjoys the spotlight and applause and has strong self-esteem. Psychological research has long established a positive relationship between the vertical level of position within management and the degree of perceived importance of oneself (Porter 1963; Wiesenfeld et al. 2000; Mourier 2012). Fluctuations in their self-esteem coincide with major successes and failures in job performance (Judge and Bono 2001). An increase or a decrease in self-esteem brings strong emotional reactions from managers, which in turn influences their financial decisions and firm performance.⁴ Ishida (2012) studies a theoretical model of self-esteem where the manager’s utility depends on the self-assessment of his own ability. His main finding is that the self-esteem concerns engender self-handicapping, an attempt to handicap the learning about himself by intentionally reducing effort with a view to remaining vague about his own ability. The implication is that more uncertainty can reduce agency cost and result in stronger incentives, hence the standard trade-off between risk and incentives may break down. Self-esteem concerns thus complicate executive compensation design problems and the associated self-handicapping constitutes a serious issue for agency relationship between the board and the manager.

Although the prior studies provide insight into managerial hedging, the role of managers’ self-esteem in their hedging decisions and the associated compensation design problems remains unexplored. This paper fills the void by exploring the link among executive compensation, managerial hedging, and managers’ self-esteem.⁵ To analyze the issues in a tractable manner and to facilitate comparison with the literature, this paper combines elements of managerial hedging models and learning-about-oneself models.⁶ Specifically we consider a model where the

³Acharya and Bisin (2009) consider the substitution between systematic and firm-specific risk factor. The total risk is fixed, and the manager is incentivized to pass up firm-specific projects in favor of projects that contain greater aggregate risk, which gives rise to excessive aggregate risk in stock markets.

⁴For example, Koszegi (2006) shows that self-esteem concerns (ego utility, in his terminology) can give rise to biased beliefs, which in turn distort the choice of task.

⁵The relevance of ‘behavioral economics’ to hedging decisions has received increasing attention. For example, Lien (2001a, b) discusses how firms’ hedging decisions may be affected by their loss aversion and disappointment aversion respectively, and Broll et al. (2010) and Kauffman et al. (2011) examine hedging decisions of a competitive firm under price uncertainty within the prospect theory. However, these papers do not consider self-esteem and avoid agency issues.

⁶In theory, an individual’s self-esteem is built on learning about oneself. There are two main strands of literature on learning about oneself. The first strand considers interpersonal situations, where the agent is uncertain about his own attributes and gains information about himself from the informed principal’s action (e.g., Benabou and Tirole 2003; Ishida 2006). The second strand investigates the intrapersonal situations, where the agent gains information about himself through his own actions, which is called ‘self-learning through experimentation’ (e.g., Benabou and Tirole 2002, 2004; Santos-Pinto and Sobel 2005; Ishida 2012). The present paper belongs to the second strand because the manager in the model exerts effort to improve firm performance and forms the self-assessment of his own ability based on the ex post firm performance.
risk-neutral board of directors designs a contract for the risk-averse manager to elicit a desired level of effort that maximizes firm value. Firm value depends on both the manager’s effort and ability, the latter being initially unknown to all parties. After the contract is signed but before the manager chooses effort, there is an opportunity for the manager to enter into a hedging contract with a risk-averse third party in the hedging market. We, following Ozerturk (2006b) and Gao (2010), assume that the manager can alter the firm-specific risk in his remuneration by trading part of risky pay in his compensation contract for a fixed payment from the third party. Moreover, the manager has self-esteem concerns. As in Ishida (2012), we assume that the manager is a Bayesian learner and updates assessment on his own ability by observing the final firm value. The manager derives utility from the improved assessment, or positive self-esteem. Our main goal is to analyze the board’s contract design problem when the manager has hedging opportunities and self-esteem concerns.

Our main results are summarized as follows. First, the manager’s equilibrium hedging position increases when he is more risk-averse, more uncertain about his own ability, or has stronger self-esteem concerns, the latter two because both the increased uncertainty about ability and stronger self-esteem concerns increase the variance in the manager’s payoff. Second, each element of managerial hedging and self-esteem concerns added to an otherwise standard agency model increases the equilibrium pay-performance sensitivity. As a result, the optimal pay-performance sensitivity is the largest in the presence of managerial hedging and self-esteem concerns. Third, as the manager’s self-esteem concerns become stronger, the equilibrium pay-performance sensitivity increases more than the increase in the manager’s hedging position, thereby increasing the manager’s net hedging position. This increases the agency cost. Fourth, the manager’s access to a hedging market does not affect the agency cost since the rational board correctly anticipates and takes into account the manager’s hedging decision in the contract design. This implies that the increased agency cost is entirely due to self-esteem concerns. Finally we provide conditions for the positive relationship between risk and incentives.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 analyzes the hedging decision problem for the manager and the optimal contract design problem for the board of directors. Section 4 provides discussions and testable hypotheses. Section 5 concludes the paper. Omitted proofs are collected in the Appendix.

2 The Model

Technology and Preferences

A board of directors (she) represents shareholders of an all-equity firm and hires a manager (he) to run the firm. We assume that the manager is risk-averse and the board is risk neutral.\footnote{This assumption is standard in agency models. Stulz (1984), Smith and Stulz (1985), DeMarzo and Duffie (1995) and many subsequent studies use this assumption in analyzing managerial hedging. It can be justified since the manager absorbs most of his wealth from working within the firm and has an undiversified portfolio, which in turn makes him more sensitive to his wealth realization than the owners of the firm.}

The final firm value $x$ is determined by the following stochastic technology:

$$x = \eta e + \varepsilon,$$

(1)
where $\eta$ is the manager’s ability, $e$ is the level of the manager’s costly effort, and $\varepsilon \sim N(0, \sigma^2_{\varepsilon})$ is the production noise that is beyond the manager’s control. The manager’s ability is initially unknown to all players including the manager himself. The commonly held prior belief about the manager’s ability is represented by $\eta \sim N(\mu, \sigma^2_{\eta})$ where $\eta$ is independent of $\varepsilon$. This assumption is standard in managerial career concern models (Holmström 1999; Milbourne 2002). We interpret $\mu > 0$ as the manager’s initial self-esteem level and $\sigma^2_{\eta}$ as the fragility of the manager’s self-esteem.\footnote{We exclude the possibility that $\mu \leq 0$. Studies in psychology (e.g., Porter 1963; Wiesenfeld et al. 2000; Mourier 2012) find that managers have a positive level of self-esteem. Moreover, as we show later, $\mu < 0$ results in an optimal contract with negative pay-performance sensitivity, which is not plausible.}

The realization of $x$ is publicly observable and verifiable, whereas the choice of $e$ is the hidden action of the manager. For tractability, we follow Ishida (2012) and assume that effort choice is binary, $e \in \{0, 1\}$.\footnote{The assumption of binary effort choice is also used in studies on managerial hedging (e.g., Nan 2008, 2011). Continuous effort choice in our setup makes it technically difficult to derive the closed-form solution since the conditional variance of the manager’s payoff becomes a polynomial of degree 4 in $e$.}

The manager’s private cost of effort is denoted by $c(e)$ with $c(1) = c$ and $c(0) = 0$. We assume that $\mu$ is sufficiently large relative to $c$ so that the board’s problem is to design the manager’s contract to implement effort level $e = 1$.

The manager’s preferences are represented by an exponential function with the coefficient of constant absolute risk aversion denoted by $a$. The manager derives utility not only from material benefits but also from the improvement in his self-esteem. Specifically the manager updates the assessment of his own ability based on realized $x$. If the updated ability is $\tilde{\mu}$, then the manager derives utility from self-esteem concerns given by $d(\tilde{\mu} - \mu)$ where the parameter $d \geq 0$ measures the strength of his self-esteem.\footnote{Self-esteem concerns may arise directly when people like to consider themselves as able, attractive, and so on. The need for self-esteem may also arise indirectly when high self-esteem is instrumental in inducing better performance and expected to lead to higher payoff in the future (Benabou and Tirole 2002).}

Thus the manager’s utility can be written as:

$$u[w - c(e) + d(\tilde{\mu} - \mu)] = -\exp[-a (w - c(e) + d(\tilde{\mu} - \mu))],$$

where $w$ is the monetary payoff.

The board designs a contract to maximize the final firm value net of the remuneration for the manager. As in Ishida (2012), it is assumed that the board knows the manager’s self-esteem strength when designing the managerial compensation. This assumption can be justified because there are several practical methods that the firm can use to assess the manager’s self-esteem.\footnote{For instance, the firm can use the Rosenberg Self-Esteem Scale to have a self-report inventory yielding a score on a continuous scale from low to high self-esteem. See Rosenberg (1979) for details.}

We restrict attention to the class of linear contracts:\footnote{Bose, Pal, and Sappington (2011), for example, show that linear contracts are at least 95 percent approximation of unrestricted optimal contracts.}

$$w = f + \alpha x,$$

where $w$ is the wage for the manager, $f$ is the fixed salary, $\alpha$ is the pay-performance sensitivity of the contract. The contract between the board and the manager is publicly observable. In practice, the public disclosure of executive compensation has long been the norm. For instance, the US federal securities laws require clear, concise and understandable disclosure about compensation paid to managers and certain other high-ranking executive officers of public
companies. The implication of such observability is that third parties in the hedging market can infer the manager’s self-esteem strength by observing the contract signed by the manager. We turn to this below.

Trading with Third Parties

In order to alter risk in his remuneration, the manager can trade one round with third parties in a hedging market before making an effort choice. As in the prior studies (e.g., Ozerturk 2006a, 2006b; Gao 2010), we assume that the board cannot perfectly control the manager’s hedging decision, except that the manager is not allowed to short-sell the firm’s stock. This assumption is supported by empirical findings in Ofek and Yermack (2000) and Schizer (2000) and can be rationalized as follows. First, the rule that requires managers to disclose their personal portfolio to the public is at best lax. Second, the manager may have incentives not to attract too much of the board’s and the market’s attention, so he may try to keep his hedging transactions secret. Third, it may be too costly for the board to monitor the manager’s use of hedging instruments.

There is a diverse variety of financial derivatives in the market. Because multiple financial instruments can achieve the same risk management objective, this paper, as in Ozerturk (2006b), focuses on the use of executive equity swap - one of the most common hedge instruments used by managers (Bolster et al. 1996; Bettis, et al. 2001). In an equity swap transaction, the manager enters a bilateral agreement with a third party (e.g., an investment bank or a derivatives securities dealer). In this agreement, part of the manager’s shares in his firm are synthetically sold by depositing them with the third party. For a pre-specified period, the third party receives the return from the shares, which is uncertain at the trading stage, and the manager receives the return from an alternative investment, such as a fixed income security, which is certain at the trading stage. To be specific, the manager contracts with a third party to promise a portion \( \beta (\leq \alpha) \) of \( x \) in exchange for a fixed payment \( g \). With an equity swap transaction, the manager can unilaterally alter the link between his remuneration and the shareholder’s wealth and undo the incentives of the initial contract. For example, if the manager chooses \( \beta = \alpha \),
then the manager will receive a constant pay, irrespective of the final firm value. However, we show below that it is not optimal for the manager to choose full hedging.

Following Ozerturk (2006b), we assume that the third parties have preferences represented by an exponential function with the coefficient of constant absolute risk aversion denoted by $b$. The third parties are competitive so that they are willing to trade at zero expected profit. They have rational expectations about the manager’s subsequent effort choice, based on the observation of the manager’s initial contract and eventual holding of the firm’s shares.

**Sequence of Events and Information**

At stage 1, the board offers the manager a remuneration package $(f, \alpha)$. If the manager accepts the offer, then at stage 2, an equity swap transaction occurs between the manager and a third party in the hedging market. They engage in a binding agreement that the manager pays $\beta x$ in exchange for a fixed payment $g$ from the third party. At stage 3, the manager chooses his effort level $e$. At stage 4, the final firm value $x$ is realized and all players consume their wealth. Figure 1 summarizes the sequence of events, and Table 1 lists the notation used in this paper.

All players share the same prior beliefs about the distributions of random variables in the model, and all parameters of the model and the sequence of events are common knowledge. The remuneration package and the realization of the final firm value are publicly observable. Throughout the agency, the manager’s ability is unknown to all, including the manager himself. All players apply Bayes’ rule to update their beliefs about the manager’s ability based on the realization of the final firm value. The manager’s hedging position and effort choice are not directly observed by the board or the third party; nevertheless, the board and the third party hold rational expectations concerning the manager’s hedging decision and effort choice.

**Figure 1: Timeline of events**

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>The board designs the manager’s contract $(f, \alpha)$.</td>
<td>The manager trades return $\beta x$ for a fixed payment $g$ from a third party.</td>
<td>The manager chooses effort level $e$.</td>
<td>Final firm value $x$ is realized, and players consume their wealth.</td>
<td></td>
</tr>
</tbody>
</table>

3 Analysis

The game is solved backwards. We first consider the hedging decision for the manager at stage 2 for an arbitrarily given contract $(f, \alpha)$. We then analyze the contract design problem for the board at stage 1. As mentioned previously, we focus on the board’s problem of implementing $e = 1$ so that the manager’s effort choice is fixed at $e = 1$ at stage 3.

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17 As we show below, the manager trading with risk-neutral third parties ($b = 0$) will choose full hedging ($\alpha = \beta$), in which case equilibrium does not exist at the contract design stage.
Table 1: Notation

<table>
<thead>
<tr>
<th>Notation</th>
<th>Meaning</th>
</tr>
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<tbody>
<tr>
<td>$\eta$</td>
<td>manager’s ability</td>
</tr>
<tr>
<td>$\mu$</td>
<td>prior mean of manager’s ability</td>
</tr>
<tr>
<td>$\bar{\mu}$</td>
<td>manager’s posterior belief on his own ability</td>
</tr>
<tr>
<td>$\sigma^2_\eta$</td>
<td>variance of manager’s ability</td>
</tr>
<tr>
<td>$x$</td>
<td>final firm value</td>
</tr>
<tr>
<td>$e$</td>
<td>manager’s effort choice</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>noise in production</td>
</tr>
<tr>
<td>$\sigma^2_\varepsilon$</td>
<td>variance of noise in production</td>
</tr>
<tr>
<td>$c$</td>
<td>manager’s private cost of effort</td>
</tr>
<tr>
<td>$w$</td>
<td>wage for the manager</td>
</tr>
<tr>
<td>$f$</td>
<td>fixed salary</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>pay-performance sensitivity of the contract</td>
</tr>
<tr>
<td>$\beta$</td>
<td>manager’s equity swap choice (hedging position)</td>
</tr>
<tr>
<td>$\alpha - \beta$</td>
<td>manager’s net exposure</td>
</tr>
<tr>
<td>$g$</td>
<td>fixed payment from the third party</td>
</tr>
<tr>
<td>$a$</td>
<td>manager’s absolute risk aversion</td>
</tr>
<tr>
<td>$b$</td>
<td>third parties’ absolute risk aversion</td>
</tr>
<tr>
<td>$d$</td>
<td>strength of manager’s self-esteem</td>
</tr>
</tbody>
</table>

Hedging Problem for the Manager

Given the contract $(f, \alpha)$, the manager chooses a hedging position $\beta$ to maximize his expected utility subject to the no short sales constraint, i.e., $\alpha \geq \beta \geq 0$. Denote the manager’s final payoff by $W_m(\beta, e; f, \alpha) \equiv (f + \alpha x) + (g - \beta x) - c(e) + d(\bar{\mu} - \mu)$. With the normality assumption on distributions, CARA preferences, and linear contracts, the manager’s expected utility at stage 2 can be written in the mean-variance form:

$$E[W_m(\beta, e; f, \alpha)] - \frac{a}{2} Var[W_m(\beta, e; f, \alpha)].$$

(4)

**Lemma 1** For $e \in \{0, 1\}$,

$$E[W_m(\beta, e; f, \alpha)] = f + g + (\alpha - \beta)\mu e - c(e)$$

(5)

$$Var[W_m(\beta, e; f, \alpha)] = \Gamma^2_e (c^2\sigma^2_\eta + \sigma^2_\varepsilon)$$

(6)

where $\Gamma^2_e \equiv (\alpha - \beta + \frac{de\sigma^2_\eta}{\sigma^2_\varepsilon + \sigma^2_\eta})^2$.

**Proof**: See the Appendix.

Note from (6) that, even if the manager chooses to hedge maximally, i.e., $\beta = \alpha$, he

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18 Without the no short sales constraint, it is possible that $\beta > \alpha$ for sufficiently high $d$: the manager’s self-esteem concerns are so strong that he chooses $\beta > \alpha$ to reduce the variance in his utility even though it adversely affects his mean utility.
cannot completely eliminate the risk in his final payoff as long as he has self-esteem concerns \((d > 0)\) and his effort choice is not zero. This points out the importance of self-esteem concerns in managerial hedging.

Third parties in the hedging market can observe the initial contract between the manager and the board, and they have rational expectations concerning the manager’s subsequent effort choice \(e = 1\). Due to the competition among third parties, the equilibrium payment \(g\) is determined by the following zero profit condition:

\[
\beta E[x | \alpha, \beta] - (b/2)\text{Var}[x | \alpha, \beta] - g(\alpha, \beta) = 0, \tag{7}
\]

which leads to:

\[
g(\alpha, \beta) = \beta\mu - (b/2)\beta^2 (\sigma_x^2 + \sigma_\eta^2). \tag{8}
\]

From (4), (5), (6), and (8), and given that third parties anticipate \(e = 1\) in stage 3 in equilibrium, the manager’s hedging choice solves the following optimization program:

\[
\max_{\beta \leq \alpha} f + (\alpha - \beta)\mu + \left[\beta\mu - (b/2)\beta^2 (\sigma_x^2 + \sigma_\eta^2)\right] - c - (a/2)\left(\alpha - \beta + \frac{d\sigma_\eta^2}{\sigma_x^2 + \sigma_\eta^2}\right)^2 (\sigma_x^2 + \sigma_\eta^2). \tag{9}
\]

Given the constraint that \(\beta \leq \alpha\), it is easy to show that the optimal hedging position for a given contract \((f, \alpha)\) is given by:

\[
\beta(f, \alpha) = \min \left\{ \frac{a}{a + b} \left(\alpha + \frac{d\sigma_\eta^2}{\sigma_x^2 + \sigma_\eta^2}\right), \alpha \right\}. \tag{10}
\]

From (10) and \(\alpha \geq 0\), we obtain:

**Proposition 1** For a given contract \((f, \alpha)\), the managerial hedging decision \(\beta(f, \alpha)\) is characterized by the following properties:

(i) \(\beta(f, \alpha)\) is non-decreasing in the manager’s risk aversion \((a)\), the volatility of the manager’s ability \((\sigma_\eta^2)\), and the strength of self-esteem \((d)\);

(ii) \(\beta(f, \alpha)\) is non-increasing in the third party’s risk aversion \((b)\) and the volatility of the noise in production \((\sigma_x^2)\).

(iii) If the manager is risk-neutral \((a = 0)\), then \(\beta(\alpha, f) = 0\); if the third party is risk-neutral \((b = 0)\), then \(\beta(\alpha, f) = \alpha\). If the manager does not have self-esteem concerns \((d = 0)\), or if the manager’s ability is not volatile \((\sigma_\eta^2 = 0)\), then \(\beta(\alpha, f) = \left(\frac{a}{a+b}\right)\alpha\).

We offer some intuition for the above proposition. The manager hedges in order to reduce the variation in his final payoff, the sum of the material payoff from the remuneration package and the psychological utility from self-esteem. Note also that the manager’s self-esteem concerns affect his utility only through the variance in the final payoff. When the manager is more risk-averse, more uncertain about his own ability, or more concerned about his self-image, his final payoff will be more sensitive to job performance, and therefore the manager will hedge more. When the third party becomes more risk-averse, they will require a higher stock return for
a given fixed payment, which in turn reduces the manager’s incentives to hedge. When the production noise increases, the final firm value is less informative of the manager’s ability, so the manager will be less inclined to hedge. If the manager is risk-neutral, then clearly he has no incentive to hedge since neither the variance in his monetary payoff nor his self-esteem concerns matter. If the third party is risk-neutral, on the other hand, the manager will hedge to the maximum amount allowed by the contract, i.e., $\beta = \alpha$. If the manager obtains no psychological utility from self-esteem, or if there is no uncertainty regarding the manager’s ability, then he will choose to hedge a proportion of the remuneration, which depends only on how risk-averse he is relative to the third party.

**Contract Design Problem for the Board**

Let us now turn to the contract design problem for the board at stage 1. Recall that the board takes into account the manager’s self-esteem concerns and chooses a contract $(f, \alpha)$ to implement $e = 1$. Thus the optimal contract design problem for the board is:

$$
\max_{f, \alpha} (1 - \alpha) E(x) - f
$$

subject to

$$
E[W_m | \beta(f, \alpha), e = 1; f, \alpha] - (a/2) \text{Var}[W_m | \beta(f, \alpha), e = 1; f, \alpha] 
\geq E[W_m | \beta(f, \alpha), e = 0; f, \alpha] - (a/2) \text{Var}[W_m | \beta(f, \alpha), e = 0; f, \alpha],
$$

$$
\beta(f, \alpha) \in \arg \max_\beta E[W_m | \beta, e = 1; f, \alpha] - (a/2) \text{Var}[W_m | \beta, e = 1; f, \alpha],
$$

$$
E[W_m | \beta(f, \alpha), e = 1; f, \alpha] - (a/2) \text{Var}[W_m | \beta(f, \alpha), e = 1; f, \alpha] \geq 0.
$$

Equations (12) and (13) are the incentive compatibility constraints for the manager’s effort choice and hedging position, respectively, and (14) is the manager’s participation constraint, where the reservation utility is normalized to zero. Denote the solution to the above problem by $(f^*, \alpha^*)$ and the associated hedging position $\beta^* = \beta(f^*, \alpha^*)$. Given the optimal contract, the associated agency cost in implementing $e = 1$ can be defined as the material compensation paid by the board for the manager’s effort cost and risk bearing: $AC^* \equiv c + (a/2) \text{Var}[W_m | e = 1]$. The following proposition is our main result, where we impose a regularity condition on $\mu$:

$$
\mu \geq \bar{\mu} \equiv ada_\eta^2 + \sqrt{a^2 d^2 \sigma_\eta^4 \left(\frac{\sigma^2}{\sigma_\eta^2 + \sigma^2}\right) + 2ac\sigma_\eta^2}.
$$

This condition is sufficient to ensure that the solution to the optimal contracting problem is not complex-valued.

**Proposition 2** Suppose $\mu$ is large enough in the sense $\mu \geq \bar{\mu}$. Then,

(i) The optimal contract $(f^*, \alpha^*)$ and the associated hedging position for the manager $\beta^*$ are
\[
given by:
\]
\[
\alpha^* = -d \left[ 1 + \frac{a\sigma^2}{b(\sigma^2_\varepsilon + \sigma^2_\eta)} \right] + \frac{(a + b) \left[ \mu - \sqrt{(\mu - ad\sigma^2_\eta)^2 - a\sigma^2_\eta \left( \frac{ad^2\sigma^4_{\eta}}{\sigma^2_\varepsilon + \sigma^2_\eta} + 2c \right)} \right]}{abo^2_\eta},
\]
\[
\beta^* = \frac{a}{a + b} \left( \alpha^* + \frac{da_\eta^2}{\sigma^2_\varepsilon + \sigma^2_\eta} \right) < \alpha^*,
\]
\[
f^* = -\alpha^* \mu + \frac{b}{2}(\sigma^2_\varepsilon + \sigma^2_\eta)(\beta^*)^2 + c + \frac{a}{2}(\sigma^2_\varepsilon + \sigma^2_\eta) \left( \alpha^* - \beta^* + \frac{da_\eta^2}{\sigma^2_\varepsilon + \sigma^2_\eta} \right)^2.
\]

(ii) The associated agency cost is:
\[
AC^* = c + \frac{a}{2}(\sigma^2_\varepsilon + \sigma^2_\eta) \left( \alpha^* - \beta^* + \frac{da_\eta^2}{\sigma^2_\varepsilon + \sigma^2_\eta} \right)^2.
\]

**Proof:** See the Appendix.

The above proposition shows that, when the board takes into account the manager’s access to hedging opportunities and self-esteem concerns, she sets the optimal pay-performance sensitivity in such a way that the manager does not fully hedge his position, i.e., \( \alpha^* > \beta^* \). To see this, suppose the manager can hedge but does not have self-esteem concerns \( (d = 0) \). In this case, the manager’s expected payoff from choosing \( e = 1 \) is \( f^* + g(\alpha^*, \beta^*) - c \) if he chooses \( \alpha^* = \beta^* \). This is smaller than his expected payoff when he chooses \( e = 0 \). In order to implement \( e = 1 \), the board should then increase \( \alpha^* \) to induce \( \alpha^* > \beta^* \), which is necessary to compensate for the cost of effort. The manager’s self-esteem concerns add an additional dimension to this. Suppose now that the manager has self-esteem concerns and chooses \( \alpha^* = \beta^* \). Then his expected payoff from choosing \( e = 1 \) decreases further by \( (a/2)(\sigma^2_\varepsilon + \sigma^2_\eta) \left( \frac{da_\eta^2}{\sigma^2_\varepsilon + \sigma^2_\eta} \right)^2 \). This requires a further upward adjustment in \( \alpha^* \). These discussions suggest that both the optimal pay-performance sensitivity and the agency cost are likely to increase when the manager has self-esteem concerns than when he does not. We discuss these issues in the next section and provide empirical implications that can be derived from our analysis.

4 Discussions and Empirical Implications

4.1 Optimal Pay-Performance Sensitivity

One of the central issues in contract design is the power of a contract, which relates to how the manager’s pay should change when performance changes. Such pay-performance sensitivity (PPS) is captured by \( \alpha \) in our model. In the standard principal-agent setup without hedging nor self-esteem concerns, the optimal PPS hinges primarily on the manager’s risk aversion and the noise in the performance measurement. Then how does the optimal PPS change when the manager has access to hedging opportunities and/or self-esteem concerns? To address this question, we compare in this section the PPS under four alternative scenarios.
The first scenario is the baseline case where the manager has neither hedging opportunity nor self-esteem concerns. This is the much-studied, standard principal-agent framework. The manager’s final payoff in this case is \( W_m = f + \alpha x - c \). The binding IC is therefore:

\[
f + \alpha \mu - c - \frac{a}{2} (\sigma_e^2 + \sigma_\eta^2) \alpha^2 = f - \frac{a}{2} \sigma_e^2 \alpha^2.
\]

The optimal PPS, denoted by \( \alpha_B \), can be found by solving the above binding IC

\[
\alpha_B = \frac{\mu - \sqrt{\mu^2 - 2ac\sigma_\eta^2}}{a\sigma_\eta^2}.
\] (19)

In the second case, there are no hedging opportunities but the manager has self-esteem concerns. In this case, the manager’s final payoff is \( W_m = f + \alpha x - c + d(\tilde{\mu} - \mu) \), leading to the binding IC:

\[
f + \alpha \mu - c - \frac{a}{2} (\sigma_e^2 + \sigma_\eta^2) \left( \alpha + \frac{d\sigma_\eta^2}{\sigma_e^2 + \sigma_\eta^2} \right)^2 = f - \frac{a}{2} \sigma_e^2 \alpha^2.
\]

Solving the above yields the optimal PPS denoted by \( \alpha_S \):

\[
\alpha_S = \frac{(\mu - ada\sigma_\eta^2) - \sqrt{(\mu - ada\sigma_\eta^2)^2 - ada\sigma_\eta^2 (\frac{ada\sigma_\eta^2}{\sigma_e^2 + \sigma_\eta^2} + 2c)}}{ada\sigma_\eta^2}.
\] (20)

Notice that \( \alpha_S \) coincides with \( \alpha_B \) when there are no self-esteem concerns, i.e., \( d = 0 \).

In the third case, there are hedging opportunities but no self-esteem concerns, when the optimal PPS is denoted by \( \alpha_H \). Then the the manager’s final payoff is \( W_m = (f + ax) + (g - \beta x) - c \). By the zero profit condition, the fixed payment from the third party is \( g = \beta \mu - \frac{b}{2} (\sigma_e^2 + \sigma_\eta^2) \beta^2 \). Since the manager’s hedging choice is given by \( \beta = \left( \frac{a}{a + b} \right) \alpha \), the binding IC becomes:

\[
f + g + (\alpha - \beta) \mu - c - \frac{a}{2} (\sigma_e^2 + \sigma_\eta^2)(\alpha - \beta)^2 = f + g - \frac{a}{2} \sigma_e^2 (\alpha - \beta)^2.
\]

Solving the above leads to

\[
\alpha_H = \frac{(a + b) \left( \mu - \sqrt{\mu^2 - 2ac\sigma_\eta^2} \right)}{ab\sigma_\eta^2}.
\] (21)

Note that \( \alpha_H = \left( \frac{a+b}{b} \right) \alpha_B = (a/b)\alpha_B + \alpha_B > \alpha_B \). Thus when the manager can hedge, his optimal hedging choice would undo part of the PPS, which the board needs to take into account in contact design. As a result, the optimal PPS is larger when the manager can hedge.

The fourth case is what we have analyzed in the previous section where both hedging and self-esteem concerns are present. For comparison with other cases, we denote the optimal PPS in this case by \( \alpha_{SH} \), which is given in (15).

**Proposition 3** The optimal PPS is the smallest in the baseline case where there are no hedging opportunities nor self-esteem concerns, and the largest when there are both hedging opportunities and self-esteem concerns. That is, \( \alpha_B < \min\{\alpha_S, \alpha_H\} \leq \max\{\alpha_S, \alpha_H\} < \alpha_{SH} \).
The above proposition highlights the vital role played by hedging opportunities and self-esteem concerns in the optimal design of managerial contract. Each element added to the standard principal-agent framework calls for an increase in the optimal PPS. Consider first the manager’s access to hedging opportunities. Since the manager’s hedging position fully takes into account the risk-incentive trade-off, the optimal strategy for the board is then to set the PPS as if the manager is risk-neutral. The board completely ignores any risk sharing considerations in the design of the contract when the manager can hedge his risk exposure. Since incentive considerations call for higher-powered contracts, the optimal PPS will then be higher when the manager can hedge. When the manager has self-esteem concerns, he has incentives to intentionally reduce his effort to handicap the learning about himself (self-handicapping). This is because, by not exerting effort, the manager compresses the distribution of outcomes and could blame poor performance on the lack of effort while maintaining confidence in his own ability in case of failure. Thus the PPS needs to be increased further to provide sufficient incentives to elicit the desired effort level.

4.2 Agency Cost

This section discusses how the manager’s self-esteem concerns affect the agency cost of inducing the desired level of effort. The literature in social psychology often finds a positive association between high self-esteem and outcomes such as academic achievement and exercise behavior, although the causal relation is not clearly established.\footnote{See, for example, Rosenberg et al. (1989) and the references therein.} A relevant question is then whether the board should try to boost the manager’s self-esteem. For instance, the board could demonstrate her trust in the manager’s ability by minimizing her intervention, or could award bonus for a job well done to let him know that the firm appreciates his effort. As has been alluded to in the previous section, however, the higher self-esteem the manager has, the more costly it becomes to induce effort, which increases the agency cost. This is because higher self-esteem leads the manager to hedge more, undoing the incentives provided through $\alpha$, which in turn requires a higher pay-performance sensitivity. More formally, we have:

**Proposition 4** Suppose $\mu \geq \hat{\mu}$ so that the optimal contract is as given in Proposition 2. Then as the manager’s self-esteem strength increases, (i) The manager’s net exposure increases $\left(\frac{\partial(\alpha^* - \beta^*)}{\partial d} > 0\right)$, and (ii) the agency cost in inducing effort increases $\left(\frac{\partial AC^*}{\partial d} > 0\right)$.

**Proof:** See the Appendix.

An increase in the self-esteem strength has direct and indirect effects on the agency cost. The direct effect is that when the manager’s self-esteem concerns become stronger, the incentive for self-handicapping increases. As a result, he will be less inclined to exert effort and face the truth about his own ability. The agency cost to induce effort is thus higher. The indirect effect stems from the change in the corresponding pay-performance sensitivity and hedging position. When the manager is more concerned about his self-esteem, he would choose a higher
hedging position to reduce his risk exposure, which renders him less incentive to exert effort. The board needs to take this into consideration and set a higher pay-performance sensitivity to maintain enough incentive provision for the effort choice. The indirect effect depends on how the manager’s net exposure changes in the strength of his self-esteem. As the above proposition shows, the manager’s net exposure increases in the self-esteem strength, so the sum of the direct and indirect effects is positive. The agency cost thus increases in the manager’s self-esteem strength.

As shown in Proposition 4, the agency cost increases when the manager’s self-esteem concerns become stronger. Since hedging can reduce the uncertainty faced by the manager, it may appear that allowing the manager to hedge could mitigate self-handicapping issues and reduce the agency cost. As the next proposition shows, however, managerial hedging does not mitigate self-handicapping issues and the agency cost remains the same with or without managerial hedging. To show this, we compare the agency cost given in (18) with those in the three alternative scenarios discussed in the previous section. Recall that, in each scenario, the associated agency cost in implementing \( e = 1 \) is given by \( AC = c + (a/2)\text{Var}[W_m | e = 1] \).

First, in the baseline case where the manager has neither hedging opportunity nor self-esteem concerns, the agency cost, denoted by \( AC_B^* \) is given by

\[
AC_B^* = c + \frac{a}{2}(\sigma_e^2 + \sigma_\eta^2)\alpha_B^2
\]  

(22)

where \( \alpha_B \) is as in (19). Second, when the manager has self-esteem concerns but cannot hedge, the agency cost, denoted by \( AC_S^* \), is

\[
AC_S^* = c + \frac{a}{2}(\sigma_e^2 + \sigma_\eta^2)\left(\alpha_S + \frac{d\sigma_\eta^2}{\sigma_e^2 + \sigma_\eta^2}\right)^2
\]  

(23)

where \( \alpha_S \) is given in (20). Third, when there are hedging opportunities but no self-esteem concerns, the agency cost, denoted by \( AC_H^* \) is

\[
AC_H^* = c + \frac{a}{2}(\sigma_e^2 + \sigma_\eta^2)(\alpha_H - \beta_H)^2
\]  

(24)

where \( \alpha_H \) is given in (21) and \( \beta_H = \left(\frac{a}{a+b}\right)\alpha_H \). Comparing the above agency costs yields the following proposition:

**Proposition 5** The increase in the agency cost given managerial hedging and self-esteem concerns is driven entirely by self-esteem concerns: \( AC^* = AC_S^* > AC_B^* = AC_H^* \).

**Proof:** See the Appendix.

The above proposition shows that managerial hedging is irrelevant for the agency cost. This is because the rational board will correctly anticipate the manager’s subsequent hedging choice, and balance the benefits and costs of managerial hedging when designing the contract. The costs are that, when the manager has access to a hedging market, he can unilaterally alter his risk exposure and undo the incentives provided in the initial contract. This requires more incentive provision and increases the agency cost. On the other hand, allowing the manager to hedge
generates two types of benefits. First, since the manager can swap his uncertain remuneration for a fixed pay through hedging, he would require less risk premium, which reduces the agency cost. Second and more interestingly, hedging has an additional benefit when the manager has self-esteem concerns. When the manager’s self-esteem concerns increase, the manager is more reluctant to exert effort with a view to remaining vague about his own ability. But hedging can reduce the uncertainty about his future payoff. As hedging occurs before effort choice, this gives the manager some buffer against job performance shock to his self-esteem, which, in turn, makes it less costly to encourage the manager to exert effort than when he cannot hedge. In sum, allowing the manager to hedge is thus both a curse and a blessing. Nevertheless, its net effect on the agency cost is zero. As shown in the proof of Proposition 5, the board sets the pay-performance sensitivity so that the manager’s net exposure is the same with or without hedging, i.e., \( \alpha^* - \beta^* = \alpha_S \) and \( \alpha_H - \beta_H = \alpha_B \). Therefore the board can offset any potential increase in the agency cost by suitably adjusting the contract in anticipation of managerial hedging.

4.3 The Relationship between Risk and Incentives

The risk-incentives trade-off is a central result of the agency theory. The canonical agency models (e.g., Holmström 1979) predict a negative relationship between risk and incentives. This is because when the output generated by effort is more uncertain, the incentives for the manager should be set lower to reduce the manager’s risk exposure and the associated agency cost to induce effort. Ishida (2012), on the other hand, contends that uncertainty may yield a positive incentive effect when the manager has self-esteem concerns. This is because the uncertainty associated with the task obscures the manager’s true worth, which reduces the need for self-handicapping. As the need for self-handicapping decreases, the agency cost to induce effort decreases as well. Consequently an increase in uncertainty may actually decrease the agency cost and thus result in stronger incentives.

The positive relationship between risk and incentives in our setup is given by \( \partial AC^*/\partial \sigma^2 < 0 \). Ishida (2012) shows that a sufficient condition for this is for \( \mu \) to be sufficiently large but does not mention explicitly how large \( \mu \) should be: his result relies on \( \mu \) tending to infinity. In contrast, we establish below an explicit lower bound on \( \mu \) that is necessary for \( \partial AC^*/\partial \sigma^2 < 0 \), and an explicit upper bound on \( c \) that is sufficient for \( \partial AC^*/\partial \sigma^2 < 0 \).

Proposition 6 (i) If \( \partial AC^*/\partial \sigma^2 < 0 \), then \( \mu \geq \hat{\mu} = ad\sigma^2_\eta + \sqrt{a^2d^2\sigma^4_\eta \left( \frac{\sigma^2_\eta}{\sigma^2_\eta + \sigma^2_\xi} \right) + 2ac\sigma^2_\eta} \).

(ii) If \( c < \tilde{c} \equiv \frac{ad^2 \sigma^4_\eta (2\sigma^2_\eta + \sigma^2_\xi)}{2(\sigma^2_\eta + \sigma^2_\xi)^2}, \) then \( \partial AC^*/\partial \sigma^2 < 0 \).

Proof: See the Appendix.

Note that the necessary condition is the same as the condition used in Proposition 2. This implies that if there is a solution to the board’s contract design problem, then increased uncertainty in production necessarily decreases the agency cost. The main reason is that the presence of managerial hedging relaxes the risk-incentives trade-off compared to when the manager’s self-esteem concerns are the only additional element to otherwise standard agency models.
To understand this better, we compare the sufficient condition in the above proposition with that when the manager cannot hedge, when the relevant agency cost $AC_{\varepsilon}^*$ is as given in (23). Using a similar line of argument used in the proof of Proposition 6, one can show that a sufficient condition for $\partial AC_{\varepsilon}^*/\partial \sigma_{\varepsilon} < 0$ is given by

$$c < \hat{c} \equiv \frac{ad^2 \sigma_{\eta}^4 (\sigma_{\varepsilon}^2 + \sigma_{\eta}^2 + \sigma_{\varepsilon}^6)}{2(\sigma_{\eta}^2 + \sigma_{\varepsilon}^2)^2}.$$ 

Simple algebra yields $\bar{c} > \hat{c}$ if and only if $\sigma_{\eta}^8 + \sigma_{\eta}^4 < 2 \sigma_{\eta}^4$. That is, hedging opportunities can slacken the sufficient condition when production noise is relatively larger than the noise in the manager’s ability. Finally we also note that our sufficient condition that $c$ is sufficiently small is consistent with the presumption that it is optimal for the board to implement $e = 1$. Provided that the necessary condition is met, no further restriction on $\mu$ is required for our results.

5 Conclusion

Extensive research in management and psychology provides evidence on the importance of self-esteem up the corporate hierarchy and its real effect on job performance and financial decisions. Numerous studies also offer ample evidence on how managers use hedging instruments to alter the incentives and the risk in their remuneration packages. Both the manager’s self-esteem concerns and hedging are important elements that can enrich our understanding of the managerial contract observed in practice. But we are not aware of a study that incorporates both into an optimal contracting framework.

This paper has examined how the manager’s self-esteem concerns affect managerial hedging and the design of optimal managerial compensation. Our model combines the elements of managerial hedging and learning-about-oneself in a moral hazard setting. Our findings show that the manager’s self-esteem concerns impose additional costs on the board compared to when managerial hedging is the only additional element in an otherwise standard contracting environment. This is because, in the latter, the board can anticipate the manager’s equilibrium hedging choice and factor it into contract design by adjusting the equilibrium pay-performance sensitivity. In contrast, the manager’s self-esteem concerns are embedded in his utility, which the board cannot fully undo through contract design. The only contracting tool the board has is the pay-performance sensitivity. Thus the manager’s self-esteem concerns increase both the equilibrium pay-performance sensitivity and the agency cost compared to when the manager can hedge but does not have self-esteem concerns.

Our main conclusion that the manager’s self-esteem concerns raise the agency cost and require a higher pay-performance sensitivity echoes the empirical evidence provided by Chatterjee and Hambrick (2011). They define more narcissistic CEOs as those who are more confident about their ability level measured by higher positive capability cues, which parallels our notion of self-esteem strength. They find that more narcissistic CEOs are less responsive to objective performance. This implies that, for the same level of incentives, more narcissistic CEOs require a higher pay-performance sensitivity compared to less narcissistic CEOs. They also find that more narcissistic CEOs are more likely to undertake risky investment, and more likely to pay
higher acquisition premiums. To the extent that these risky investments are not driven by firm value maximization, this evidence is consistent with higher agency cost. Given that the link between managerial contracts and managerial hedging is relatively well understood, our findings and the empirical evidence together suggest that the board should pay as much attention to deflate the manager’s self-esteem concerns as to the design of optimal contract.

Appendix

Proof of Lemma 1:

Let \( \hat{\eta} = x/e = \eta + \varepsilon/e \) be the best estimate of \( \eta \) from the observation of \( x \). Since \( \eta \sim N(\mu, \sigma^2_\eta) \) and \( \hat{\eta} \mid \eta \sim N\left(\frac{\sigma^2_\eta \mu + \varepsilon^2 \sigma^2_{\hat{\eta}}}{\sigma^2_\varepsilon + \varepsilon^2 \sigma^2_{\eta}}, \frac{1}{\sigma^2_\eta} + \frac{\varepsilon^2}{\sigma^2_\varepsilon}\right) \), the well-known results for conjugate distributions (DeGroot 1970, p. 167) give us \( \eta \mid \hat{\eta} \sim N\left(\frac{\sigma^2_\eta \mu + \varepsilon^2 \sigma^2_{\hat{\eta}}}{\sigma^2_\varepsilon + \varepsilon^2 \sigma^2_{\eta}}, \frac{1}{\sigma^2_\eta} + \frac{\varepsilon^2}{\sigma^2_\varepsilon}\right) \). Thus the posterior belief of the manager’s ability is \( \tilde{\mu} = E[\eta \mid \hat{\eta}] = \frac{\sigma^2_\eta \mu + \varepsilon^2 \sigma^2_{\hat{\eta}}}{\sigma^2_\varepsilon + \varepsilon^2 \sigma^2_{\eta}} \). Then the manager’s final payoff can be rewritten as

\[
W_m = f + g + (\alpha - \beta)(\eta e + \varepsilon) - c(e) + d \left( \frac{e^2 \sigma^2_{\eta} (\eta - \mu) + e \sigma^2_{\varepsilon} \varepsilon}{\sigma^2_\varepsilon + e^2 \sigma^2_{\eta}} \right).
\]

Taking the expectation of \( W_m \) yields

\[
E[W_m(\beta, e; \alpha, f)] = f + g + (\alpha - \beta)\mu e - c(e)
\]

since \( E(\eta) = \mu \) and \( E(\varepsilon) = 0 \). Also, taking the variance of \( W_m \) yields

\[
Var[W_m(\beta, e; \alpha, f)] = \left(\alpha - \beta\right)e + \frac{de^2 \sigma^2_{\eta}}{\sigma^2_\varepsilon + e^2 \sigma^2_{\eta}} \sigma^2_{\eta} + \left(\alpha - \beta\right) + \frac{de \sigma^2_{\eta}}{\sigma^2_\varepsilon + e^2 \sigma^2_{\eta}} \right)^2 \sigma^2_{\varepsilon}.
\]

Proof of Proposition 2:

For (i), note that the incentive compatibility constraint (12) holds as an equality at optimum:

\[
f + (\alpha - \beta)\mu + g - c - \frac{a}{2} \left(\sigma^2_\varepsilon + \sigma^2_{\eta}\right) \left(\alpha - \beta + \frac{d \sigma^2_{\eta}}{\sigma^2_\varepsilon + \sigma^2_{\eta}}\right)^2 = f + g - \frac{a}{2} (\alpha - \beta)^2 \sigma^2_{\varepsilon},
\]

\[
\iff \alpha = \beta + \frac{c}{\mu} + \frac{a}{2\mu} \left(\sigma^2_\varepsilon + \sigma^2_{\eta}\right) \left(\alpha - \beta + \frac{d \sigma^2_{\eta}}{\sigma^2_\varepsilon + \sigma^2_{\eta}}\right)^2 - (\alpha - \beta)^2 \sigma^2_{\varepsilon} \right). \tag{25}
\]

Since the RHS of (25) is positive, the roots of this equation, if they exist, must be real and positive, i.e., \( \alpha^* > 0 \). Also, the terms inside the large brackets on the RHS are positive. It means that if the solutions to (25) exist, we must have \( \alpha^* > \beta^* \), which, by (10), implies \( \beta^* = \frac{a}{a+b} \left(\alpha^* + \frac{d \sigma^2_{\eta}}{\sigma^2_\varepsilon + \sigma^2_{\eta}}\right) \).

---

Of the two roots to (25), the smaller is the solution for our purpose, which is the optimal pay-performance sensitivity:

\[ \alpha^* = -d \left[ 1 + \frac{a \sigma^2}{b(\sigma^2 + \sigma^2)} \right] + \frac{(a + b)\mu(\sigma^2 + \sigma^2)}{ab\sigma^2(\sigma^2 + \sigma^2)} \]  

(26)

where \( A \equiv (a + b)^2(\sigma^2 + \sigma^2)L(\mu) \) and \( L(\mu) \equiv (\mu - a\sigma^2)2 - a\sigma^2 \left( \frac{ad^2\sigma^4}{\sigma^2 + \sigma^2} \right) + 2c \). It is evident that \( A \geq 0 \) iff \( L(\mu) \geq 0 \). Since \(-a\sigma^2 \left( \frac{ad^2\sigma^4}{\sigma^2 + \sigma^2} \right) + 2c < 0\), there exists an open interval \((\mu_1, \mu_2)\) such that \( L(\mu) \geq 0 \) for \( \mu /in (\mu_1, \mu_2) \). Solving \( L(\mu) = 0 \) yields \( \mu_1 = ad\sigma^2 - \sqrt{s} \) and \( \mu_2 = ad\sigma^2 + \sqrt{s} = \hat{\mu} \), where \( s \equiv d^2\sigma^4 \left( \frac{\sigma^2}{\sigma^2 + \sigma^2} \right) + 2ac\sigma^2 \). The condition \( \mu \geq \hat{\mu} \) ensures \( A \geq 0 \), hence \( \alpha^* \) is real-valued. Substituting \( A \) into (26) leads to (15). Since the participation constraint is also binding at optimum, the optimal fixed salary \( f^* \) can be found by substituting \( \alpha^* \) and \( \beta^* \) into (14) set as an equality, which leads to (17).

For (ii), applying Lemma 1 to the definition \( AC^* = c + (a/2)Var[W_m|e = 1] \) yields the result. \( \square \)

**Proof of Proposition 3:**

First, we show \( \alpha_S > \alpha_B \). Comparing the numerators of (19), (20), we have

\[
\begin{align*}
\mu - a\sigma_n^2 - \sqrt{(\mu - a\sigma_n^2)^2 - a\sigma_n^2 \left( \frac{ad^2\sigma^4}{\sigma^2 + \sigma^2} + 2c \right)} \\
> \mu - a\sigma_n^2 - \sqrt{(\mu - a\sigma_n^2)^2 - 2ac\sigma_n^2} \\
> \mu - \sqrt{\mu^2 - 2ac\sigma_n^2}
\end{align*}
\]

where the second inequality follows from the fact that \( \frac{d}{dx} (x - \sqrt{x^2 - y}) < 0 \) for all \( x > 0 \) and \( x^2 - y > 0 \). Thus we have \( \alpha_S > \alpha_B \).

Second, we show \( \alpha_{SH} > \alpha_S \). Let \( x \equiv (\mu - a\sigma_n^2)^2 - a\sigma_n^2 \left( \frac{ad^2\sigma^4}{\sigma^2 + \sigma^2} + 2c \right) \). Then \( \alpha_S \) and \( \alpha_{SH} \) can be rewritten as:

\[
\alpha_S = \frac{\mu - \sqrt{x}}{a\sigma_n^2} - d \quad \text{and} \quad \alpha_{SH} = -d \left[ 1 + \frac{a\sigma^2}{b(\sigma^2 + \sigma^2)} \right] + \frac{(a + b)(\mu - \sqrt{x})}{ab\sigma^2}.
\]
Thus we have:

\[\alpha_{SH} - \alpha_S = -d \left[ 1 + \frac{a\sigma^2_x}{b(\sigma^2_x + \sigma^2_\eta)} \right] + \mu - \sqrt{\frac{x}{b\sigma^2_\eta}} + d\]

\[= \frac{\mu - \sqrt{x}}{b\sigma^2_\eta} - \frac{ad}{b} \left( \frac{\sigma^2_x}{\sigma^2_x + \sigma^2_\eta} \right)\]

\[> \frac{\mu - \sqrt{x} - ad\sigma^2_\eta}{b\sigma^2_\eta}\]

\[= \frac{a}{b} \alpha_S\]

\[> 0.\]

Since we have already shown \(\alpha_H = \left(\frac{a+b}{b}\right) \alpha_B > \alpha_B\), it only remains to show \(\alpha_{SH} > \alpha_H\). Let

\[z \equiv \frac{(a + b) \left[ (\mu - ad\sigma^2_\eta) - \sqrt{(\mu - ad\sigma^2_\eta)^2 - a\sigma^2_\eta} \left( \frac{ad^2\sigma^4_\eta}{\sigma^2_x + \sigma^2_\eta} + 2c \right) \right]}{ab\sigma^2_\eta}\]

\[> \frac{a + b}{ab\sigma^2_\eta} \left[ \mu - \sqrt{\mu^2 - a\sigma^2_\eta} \left( \frac{ad^2\sigma^4_\eta}{\sigma^2_x + \sigma^2_\eta} + 2c \right) \right]\]

\[> \frac{a + b}{ab\sigma^2_\eta} \left[ \mu - \sqrt{\mu^2 - 2a\sigma^2_\eta} \right]\]

\[= \alpha_H\]

where the first inequality is again due to the fact \(\frac{d}{dx} (x - \sqrt{x^2 - y}) < 0\) for all \(x > 0\) and \(x^2 - y > 0\). Then

\[\alpha_{SH} = \frac{(a + b)d}{b} - d \left[ 1 + \frac{a\sigma^2_x}{b(\sigma^2_x + \sigma^2_\eta)} \right] + z.\]

Since \(d \left[ 1 + \frac{a\sigma^2_x}{b(\sigma^2_x + \sigma^2_\eta)} \right] < d \left( 1 + \frac{a}{b} \right) = \frac{(a+b)d}{b}\) and \(z > \alpha_H\), we have \(\alpha_{SH} > \alpha_H\).

**Proof of Proposition 4:**

(i) From (15) and (16), we have:

\[\frac{\partial (\alpha^* - \beta^*)}{\partial d} = \frac{(a + b) \left[ -\mu\sigma^2_\eta + \sigma^2_x(-\mu + ad\sigma^2_\eta) \right]}{\sqrt{A}} - 1\]

where we recall from the proof of Proposition 2 that \(A = (a + b)^2(\sigma^2_x + \sigma^2_\eta)L(\mu)\) and \(L(\mu) = \)
\[(\mu - ad\sigma^2)\mu - a\sigma^2 (\frac{ad^2\sigma^4}{\sigma^2 + \sigma^2} + 2c)\]. Then,

\[
\frac{\partial (\alpha^* - \beta^*)}{\partial d} > 0
\]

\[
\iff (a + b) [\mu\sigma^2 + \sigma^2 (\mu - ad\sigma^2)] > \sqrt{A},
\]

\[
\iff \mu - \frac{ad\sigma^2\sigma^2}{\sigma^2 + \sigma^2} > \sqrt{L(\mu)} \text{ by the definition of } A,
\]

\[
\iff \mu^2 - 2ad\sigma^2\sigma^2\mu + \frac{a^2d^2\sigma^4\sigma^4}{(\sigma^2 + \sigma^2)^2} > \mu^2 - 2ad\sigma^2\mu + \left(\frac{a^2d^2\sigma^4\sigma^4}{\sigma^2 + \sigma^2} - 2ac\sigma^2\right),
\]

\[
\iff 2ad\sigma^2\left(1 - \frac{\sigma^2}{\sigma^2 + \sigma^2}\right) \mu > \frac{a^2d^2\sigma^4\sigma^4}{\sigma^2 + \sigma^2} \left(1 - \frac{\sigma^2}{\sigma^2 + \sigma^2}\right) - 2ac\sigma^2,
\]

\[
\iff \mu > \frac{ad\sigma^2\sigma^2}{2(\sigma^2 + \sigma^2)} - c(\sigma^2 + \sigma^2)\frac{\sigma^2 + \sigma^2}{d\sigma^2}.
\]

where the last inequality is implied by \(\mu \geq \hat{\mu}\).

(ii) Differentiating (18) with respect to \(d\) yields:

\[
\frac{\partial AC^*}{\partial d} = a(\sigma^2 + \sigma^2) \left(\alpha^* - \beta^* + \frac{d\sigma^2}{\sigma^2 + \sigma^2}\right) \left[\frac{\partial (\alpha^* - \beta^*)}{\partial d} + \frac{\sigma^2}{\sigma^2 + \sigma^2}\right].
\]

Since \(\alpha^* > \beta^*\) by Proposition 2 and \(\frac{\partial (\alpha^* - \beta^*)}{\partial d} > 0\) from (i), we have \(\frac{\partial AC^*}{\partial d} > 0\). ■

**Proof of Proposition 5:**

From (15) and (20), we have

\[
\alpha^* = \alpha_S + \frac{\mu - \sqrt{L(\mu)}}{b\sigma^2} - \frac{ad\sigma^2}{b(\sigma^2 + \sigma^2)}
\]

where \(L(\mu)\) is as defined in the proof of Proposition 2. Substituting the above into (16) leads to \((\alpha^* - \beta^*) - \alpha_S = 0\). Then from (18) and (23), \(AC^* = AC^*_S\) follows. Since \(\alpha_S > \alpha_B\) by Proposition 3, we have \(AC^*_S > AC^*_B\). Finally, it is easy to see \(\alpha_H - \beta_H = \alpha_B\), hence \(AC^*_B = AC^*_H\). ■

**Proof of Proposition 6:**

First, we show that condition (i) is necessary for \(\partial AC^*/\partial \sigma^2 < 0\). From (18), we have:

\[
\frac{\partial AC^*}{\partial \sigma^2} = \frac{a}{2} \left\{ K^2 + 2K(\sigma^2 + \sigma^2) \left[\frac{\partial (\alpha^* - \beta^*)}{\partial \sigma^2} - \frac{d\sigma^2}{(\sigma^2 + \sigma^2)^2}\right]\right\}
\]

where \(K \equiv \alpha^* - \beta^* + \frac{d\sigma^2}{\sigma^2 + \sigma^2}\). Since \(K > 0\), we have:
\frac{\partial AC^*}{\partial \sigma^2_{\pi}} < 0 \iff \frac{1}{2} K + (\sigma^2_{\pi} + \sigma^2_{\eta}) \frac{\partial (\alpha^* - \beta^*)}{\partial \sigma^2_{\pi}} - \frac{d\sigma^2_{\eta}}{\sigma^2_{\pi} + \sigma^2_{\eta}} < 0,

\iff \frac{\partial (\sigma^2_{\pi} + \sigma^2_{\eta})(\mu - ad\sigma^2_{\eta}) - \sqrt{A/(a + b)}}{aa^2_{\eta} (\sigma^2_{\pi} + \sigma^2_{\eta})} - \frac{ad^2\sigma^4_{\eta}}{\sqrt{A/(a + b)}(\sigma^2_{\pi} + \sigma^2_{\eta})} - \frac{d\sigma^2_{\eta}}{\sigma^2_{\pi} + \sigma^2_{\eta}} < 0,

\iff \frac{\mu - ad\sigma^2_{\eta}}{aa^2_{\eta}} - \sqrt{L(\mu)} - \frac{ad^2\sigma^4_{\eta}}{(\sigma^2_{\pi} + \sigma^2_{\eta})\sqrt{L(\mu)}} - \frac{d\sigma^2_{\eta}}{\sigma^2_{\pi} + \sigma^2_{\eta}} < 0 \tag{27}

where \( A \) and \( L(\mu) \) are as defined in the proof of Proposition 2. Let \( \phi \equiv \frac{d^2\sigma^2_{\eta}}{\sigma^2_{\pi} + \sigma^2_{\eta}}, \ v \equiv \mu - ad\sigma^2_{\eta}, \) and \( s \equiv a^2d^2\sigma^4_{\eta}\left(\frac{\sigma^2_{\pi}}{\sigma^2_{\pi} + \sigma^2_{\eta}}\right) + 2ad\sigma^2_{\eta}. \) Then, (27) can be equivalently expressed as:

\[ \frac{v}{aa^2_{\eta}} - \frac{\sqrt{v^2 - s}}{\sqrt{v^2 - s}} - \frac{aa^2_{\eta}\phi}{d} < 0 \iff \frac{v}{\sqrt{v^2 - s}} - \frac{aa^2_{\eta}}{\phi a^2_{\eta}} - \frac{1}{d} < 0. \tag{28} \]

Next, let \( t \equiv \sqrt{v^2 - s}. \) For \( t \) to be well-defined, it is necessary that \( v^2 - s \geq 0, \) hence

\[ \mu \geq \hat{\mu} = ad\sigma^2_{\eta} + \sqrt{s}, \ \text{or} \ \mu \leq ad\sigma^2_{\eta} - \sqrt{s}. \tag{29} \]

Since \( ad\sigma^2_{\eta} - \sqrt{s} \) can be negative for some range of parameter values, contrary to our assumption that \( \mu > 0, \) the first inequality in (29) is a necessary condition for \( \partial AC^*/\partial \sigma^2_{\pi} < 0. \)

Next, we show that condition (ii) is sufficient for \( \partial AC^*/\partial \sigma^2_{\pi} < 0. \) By the definition of \( t, \) (28) is equivalent to:

\[ \frac{\sqrt{t^2 + s} - t}{\phi a\sigma^2_{\eta}} - \frac{aa^2_{\eta}}{t} - \frac{1}{d} < 0 \]

\[ \iff \frac{\sqrt{t^2 + s}}{\phi a\sigma^2_{\eta}} < \frac{t}{\phi a\sigma^2_{\eta}} + \frac{aa^2_{\eta}}{t} + \frac{1}{d}, \]

\[ \iff \frac{s}{\phi^2 a^2\sigma^4_{\eta}} < \frac{a^2\sigma^4_{\eta}}{t^2} + \frac{1}{\phi} + \frac{2t}{\phi a\sigma^2_{\eta}} + \frac{2a\sigma^2_{\eta}}{d} \text{ by squaring both sides,} \]

\[ \iff \frac{st^2}{\phi^2 a^2\sigma^4_{\eta}} < a^2\sigma^4_{\eta} + \left(\frac{1}{\phi^2} + \frac{2}{\phi}\right) t^2 + \frac{2}{\phi a\sigma^2_{\eta}} t^3 + \frac{2a\sigma^2_{\eta}}{d} t. \tag{30} \]

After rearranging terms and some simplification, (30) can be equivalently written as:

\[ \frac{2}{ad\phi a\sigma^2_{\eta}} t^3 + \left(\frac{1}{\phi^2} + \frac{2}{\phi}\right) t^2 + \frac{2a\sigma^2_{\eta}}{d} t + a^2\sigma^4_{\eta} \equiv \Phi(t) > 0. \]

Note that \( \Phi(t) \) is a polynomial of degree 3 in \( t \) and, given \( \mu \geq \hat{\mu}, \) we have \( t \geq 0. \) Condition (ii) implies that the coefficient to \( t^2 \) is strictly positive. Thus if condition (ii) holds, then all the coefficients in \( \Phi(t) \) are strictly positive. Consequently we have \( \Phi(0) > 0 \) and \( \Phi'(t) > 0 \) for all \( t \geq 0. \) This implies \( \Phi(t) > 0 \) for all \( t \geq 0, \) hence \( \partial AC^*/\partial \sigma^2_{\pi} < 0. \) \( \blacksquare \)
References


