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Competitive Personalized Pricing*

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Abstract:
We study a duopoly model where each firm chooses personalized prices for its targeted consumers, who can be active or passive in identity management. Active consumers can bypass price discrimination and have access to the price offered to non-targeted consumers, which passive consumers cannot. When all consumers are passive, personalized pricing leads to intense competition and total industry profit lower than that under the Hotelling equilibrium. But market is always fully covered. Active consumers raise the firm's cost of serving non-targeted consumers, which softens competition. When firms have sufficiently large and non-overlapping target segments, active consumers enable firms to extract full surplus from their targeted consumers through perfect price discrimination. With active consumers, firms also choose not to serve the entire market when the commonly non-targeted market segment is small. Thus active identity management can lead to lower consumer surplus and lower social welfare. We also discuss the regulatory implications for the use of consumer information by firms as well as the implications for management.

Keywords: Personalized pricing, identity management, customer targeting
JEL Codes: D43, D8, L13, L5

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1 Introduction

Firms in various service industries can collect a huge volume of individual-level consumer information at unprecedented levels of variety and velocity, commonly dubbed big data, thanks to ubiquitous online search and transactions, interactions on various social networking websites, etc. Much of the relevant information is collected and distributed by data brokers ranging from established companies such as Acxiom and Bloomberg, to more recently established ones such as BlueKai (Oracle), and Teradata.[1] According to TRUSTe, a technology compliance and security company, the 100 most widely used websites are monitored by more than 1,300 firms ("Getting to know you", *The Economist*, September 11, 2014). Firms also collect information through their customers' past purchases, often utilizing various loyalty programs and payment records. The rapid development in information and communication technologies allows firms to collect big data at increasingly lower costs. It is now common for a typical firm to have a dedicated team for data analytics and, in many cases, a position titled as chief data officer ("Just using big data isn't enough any more", *Harvard Business Review*, February 9, 2016).

The availability of consumer information allows firms to divide the market into different segments and make a tailored offer to each segment. A classic example is discount coupons targeted at price-sensitive consumers (Shaffer and Zhang, 1995). As the quality and quantity of information improves, firms can afford a finer market segmentation and more personalized offers. For example, more information makes even coupons more personalized and targeted: Coupons.com, an online platform offering digital coupons, uses its proprietary data on consumer behavior to personalize and target coupons (Ezrachi and Stucke, 2016, p. 91). Other examples abound: Safeway's Just for U program uses complex algorithms to process customers' purchase data, based on which to send personalized offers online or through mobile apps ("Supermarkets offer personalized pricing", *Bloomberg*, November 16, 2013); Uber's "route-based pricing" charges customers based on their predicted willingness to pay ("Uber starts charging what it thinks you're willing to pay", *Bloomberg*, May 19, 2017). These are just a few examples that herald the possible arrival of even real-time personalization (Esteves and Resende, 2016).[2]

As firms gain access to more consumer information, consumers are becoming increasingly aware that their online activities are being monitored and their personal information being collected and possibly shared. With privacy concerns added to such awareness,

[1]These firms collect data from various sources, including their own data collection technologies, arrangements with website owners for installing tracking tools such as cookies, public information, data from public authorities and third-party companies such as banks. The industry is estimated to turn over $156 billion annually ("The dark market for personal data", *The New York Times*, October 16, 2014).

[2]There are many other examples of personalized pricing used by airlines, grocery chains, online travel portals, etc. For example, see "Dynamic pricing: which customers are worth the most? Amazon, Delta Airlines, and Staples weigh in", *Forbes*, April 17, 2015; "How retailers use personalized prices to test what you're willing to pay", *Harvard Business Review*, October 20, 2017. Ezrachi and Stucke (2016, Chapter 10) provide more discussions and examples of personalized pricing.

some consumers take proactive measures to protect themselves and thwart firms' attempt to use personalized pricing. Such actions can include using virtual private networks for online activities, erasing or blocking browser cookies, creating a new account for new transactions, maintaining several online identities, and using different credit cards for different transactions, etc. These actions are not confined to online transactions only. In industries such as retail banking and insurance, telecommunications, cable TV services, and power and utilities, firms often offer new-customer-only special discounts. In that case, an existing customer may search for a better offer, based on which to directly haggle with the firm.[3] She may also cancel the existing account and create a new one if doing so is not too costly. Such proactive measures by consumers, which we collectively call *identity management* (Acquisti 2008), incur various transactions costs such as time, effort, and even money.

In this paper, we study personalized pricing in an oligopolistic environment in which consumers may engage in identity management. It is well known that more consumer information can intensify price competition and hurt profitability unless there are sufficient heterogeneities at the firm level.[4] But this result is based on the assumption that consumers cannot engage in identity management. Several recent studies reviewed in Section 2 incorporate different types of identity management, but mostly for the case of monopoly. Thus our study fills the gap in the literature by incorporating identity management in a competitive setting. By doing so, we also address important policy issues that relate to the use of consumer information for price discrimination. For example, the US Council of Economic Advisers (2015) proposed giving consumers greater access to and control over their information, arguing that increased transparency into how companies use consumer information would promote more competition and better informed consumer choice. As we show in this paper, whether more transparency will promote competition hinges crucially on how it affects identity management. Specifically, if more transparency leads to more active identity management, then we show that competition is softened, which reduces consumer surplus and, in some cases, even reduces social welfare.

Formally, we consider a Hotelling linear city with two firms at each end. Each firm has a target segment on which it has complete information about consumer preferences and can offer personalized prices. For consumers outside its target segment, each firm can charge only a uniform price. In our static duopoly model, we assume firms are endowed

[3]According to a 2013 survey of 2,000 American adults conducted by the Consumer Reports National Research Center, 33% of customers negotiated with their existing mobile phone providers. Of these, 76% reported being successful at least once with the average annual saving of \$80. Similar findings are reported for banks, credit-cards, and cable TV services. See http://www.consumerreports.org/cro/magazine/2013/08/how-to-bargain/index.htm. In Appendix B, we provide more detailed evidence based on the data collected from various internet portals.

[4]This literature is reviewed in Section 2. For example, Choe et al. (2017) show that a symmetric duopoly results in increasingly smaller industry profit when competition shifts from that in uniform pricing to third degree price discrimination, and to personalized pricing.

with relevant consumer information, possibly through purchase from data brokers, and focus on pricing equilibria for all possible market segmentations. Our key modelling ingredient is the possibility of identity management by consumers. We consider two types of consumers. An *active* consumer can costlessly engage in identity management, implying that, in addition to the personalized price offered by the targeting firm, she can also choose the uniform price set by the firm for non-targeted consumers. Thus an active consumer can potentially choose from three prices: a personalized price from the targeting firm and two uniform prices. But a *passive* consumer faces prohibitively high costs in identity management and cannot choose the uniform price set by the targeting firm. Thus a passive consumer faces at most two prices, a personalized price from the targeting firm and the uniform price from the rival firm.

Before summarizing our main findings, we start with two observations. First, personalized pricing allows firms to defend their target segments more aggressively than when they can charge prices at higher levels of aggregation. Because personalized prices are offered privately to targeted consumers, a firm can reduce a personalized price for a targeted consumer without reducing personalized prices for other targeted consumers. Thus firms can defend their targeted consumers individually and, if necessary, cut the price for the marginal consumer down to zero. Second, firms become more aggressive in poaching the rival's targeted consumers when their own targeted consumers are passive than when they are active. It is because passive targeted consumers cannot choose the firm's uniform poaching price, which allows the firm to delink its personalized prices from the poaching price. But this is not possible when the firm's targeted consumers are active since a low poaching price will also attract the firm's targeted consumers. This raises the cost of poaching, which softens competition.

The above observations imply that both the poaching and defending sides will behave aggressively when they face passive consumers. This intensifies competition, leading to the following results when all consumers are passive. First, when each firm targets all its loyal consumers and there is some overlapping target segment, competition is the most intense in that the equilibrium outcome is that in Thisse and Vives (1988). Second, firms are collectively better off when there is a market segment that is not targeted by either firm. Firms compete à la Hotelling on the commonly non-targeted segment, which works as a cushion and softens competition on each firm's target segment. Needless to say, the Hotelling equilibrium obtains when the entire market is non-targeted by either firm. Third, in equilibrium for any given market segmentation, the total industry profit is bounded above by the Hotelling profit, and below by the profit from the Thisse and Vives outcome. Thus more consumer information unambiguously lowers industry profitability when it is used for price discrimination. Nonetheless all consumers are served in any equilibrium since firms can separate competition on the targeted and non-targeted segments. Given our model assumption, such full market coverage implies that there is no deadweight loss.

The results change drastically when consumers are active because firms become less aggressive in poaching in this case. Since personalized prices and the poaching price cannot be considered in separation, there are many different types of equilibria depending on market segmentation. One salient equilibrium is where each firm exercises perfect price discrimination and extracts full consumer surplus from all of its targeted consumers. This equilibrium is possible when the market is fully targeted and firms have sufficiently large and non-overlapping target segments. In this case, neither firm has incentives to poach the rival's targeted consumers and, as a result, both set high uniform prices that are not accepted on the equilibrium path. This allows them to exercise perfect price discrimination for all of their targeted consumers. Thus a collusive outcome is obtained in the one-shot game without implicit or explicit collusion. This is in stark contrast to the case with passive consumers: for the same market segmentation that supports perfect price discrimination when consumers are active, the equilibrium with passive consumers can result in the most intense competition as in Thisse and Vives. Another noteworthy equilibrium is when there is a small market segment that is not targeted by either firm. This can lead to firms exercising perfect price discrimination for their targeted consumers while choosing not to serve the non-targeted segment, which creates a deadweight loss. The insight that competition is softened when consumers are active holds more generally for all other types of equilibria. Indeed we show that, for any market segmentation, the total industry profit in equilibrium is larger when consumers are active than when they are passive.

Our analysis also reveals a prisoner's dilemma that consumers face in identity management. Each individual consumer can benefit from being active. But consumers are collectively worse off when more consumers become active because of the negative externality that each active consumer imposes on others. We show this by looking at the case where a fraction of consumers are active and analyze the equilibrium as the fraction increases. The comparative statics leads to a number of results: as more consumers become active, the equilibrium with deadweight loss obtains for a wider range of market segmentation; the equilibrium with perfect price discrimination is also more likely for a wider range of market segmentation; and more consumer information is required to intensify competition. The last point can be also related to privacy and consumer welfare. The equilibrium with the largest consumer surplus is the one with the most intense competition, i.e., the Thisse and Vives outcome. When all consumers are passive, any market segmentation in which each firm has information about only its loyal consumers leads to this outcome. When all consumers are active, however, the same outcome obtains if and only if when both firms have full information about all consumers. That is, no privacy can lead to the equilibrium with the largest consumer surplus when all consumers are active. We discuss this point further in Section 7.

Our paper extends the existing literature on competitive price discrimination in an important way and provides new insight towards understanding price discrimination when consumers can take measures to bypass firms' attempt to price discriminate. In the age of big data, behavioral targeting, and personalized pricing, more technologies and tools are becoming available for identity management by consumers. By incorporating identity management explicitly, we show that the results from the existing literature are completely overturned when all consumers are active in identity management. Even when only a fraction of consumers are active, our general insight carries through in that competitive price discrimination benefits firms as more consumers become active. We also contribute to the debate on the regulation of the collection and use of consumer information by firms. Strict privacy rules that prevent firms from using consumer information for price discrimination can soften competition and benefit firms when consumers are passive. In addition, policies that improve the transparency in and the disclosure of the use of consumer information may soften competition and benefit firms, if they also induce consumers to become more active in identity management.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the model. Section 4 analyzes the case where all consumers are passive and Section 5, the case where all consumers are active. In Section 6, we consider the hybrid case and provide comparative statics results with respect to the fraction of active consumers. Section 7 provides various discussions including the regulatory implications for collection/use of personal data and the implications for management. Section 8 concludes the paper. Appendix A contains all the proofs while Appendix B provides some evidence on identity management by consumers.

2 Related literature

There is a large literature on competitive price discrimination.[5] Two general findings from the literature can be summarized as follows. First, price discrimination can intensify competition and hurt firm profitability compared to the non-discrimination benchmark unless there are sufficient heterogeneities at the firm- or consumer-levels. This is true whether firms compete in third-degree price discrimination (Shaffer and Zhang, 1995; Chen, 1997; Villas-Boas, 1999; Fudenberg and Tirole, 2000; Pazgal and Soberman, 2008; Esteves, 2010) or in personalized pricing (Thisse and Vives, 1988; Zhang, 2011; Chen and Iyer, 2002; Choe et al., 2017). Second, competition becomes more intense when firms compete in personalized pricing than in third-degree price discrimination (Thisse and Vives, 1988; Fudenberg and Tirole, 2000; Choe et al., 2017). The intuition behind these results is that more customer information makes firms more aggressive in pricing. For example, when firms compete in personalized pricing, they compete for each consumer individually without having to change prices set for other consumers. This gives them

[5]See Stole (2007) for a comprehensive survey and, for surveys of the literature where competition is based on behavior-based price discrimination, see Chen (2005) or Fudenberg and Villas-Boas (2006, 2012).

a (false) sense of their capability to protect their turf, making competition more intense than when they compete with less customer information. But these results hold only when all consumers are passive in the way we have defined the term in this paper. With active consumers, we show both results are overturned.

Although our model is essentially static, it is nonetheless related to the literature on behavior-based price discrimination where consumers can take actions to remain anonymous, dubbed identity management (Acquisti, 2008). Our basic assumption is that active consumers can choose identity management strategies to prevent firms from using their information for price discrimination. We may group these studies based on *when* consumers choose to do so. First, Taylor (2004), Villas-Boas (2004), and Acquisti and Varian (2005) consider the case where consumers can prevent the firm from tracking their purchase record that can be used for price discrimination. For example, consumers can do so by delaying purchase or refusing to accept cookies. We may call this *ex ante* identity management as it prevents firms from gathering consumer information in the first place. Not surprisingly, they show that a monopolist is worse off under ex ante identity management. The second type, which we call *interim* identity management, refers to the case where consumers can anonymize themselves after the initial purchase but before firms choose prices for repeat purchase. This case was studied by Conitzer et al. (2012) and Belleflamme and Vergote (2016), both in the monopoly setting. They find that the firm is better off under interim identity management, as it renders the monopolist the commitment power not to price discriminate in repeat purchase. Third, consumers can act as if they are new customers after observing the prices firms choose for repeat purchase. We call this *ex post* identity management, which is the case studied in our paper. We think ex post identity management is more realistic than interim identity management since, if consumers can choose to become anonymous, it makes sense to do so only when it is in their interest to do so. In addition, all the above studies are couched in the monopoly setting whereas we consider the duopoly case.[6]

The fact that active consumers can choose the lowest of their firms' offers is reminiscent of a most-favored customer clause (MFCC), which is a contractual provision that a firm offers to its customer that no other customers will be offered a lower price. As the MFCC raises the cost of poaching, it signals a firm's intention not to poach the rival's customers. The competition-softening effect of the MFCC has been studied by many (e.g., Cooper, 1986; Schnitzer, 1994).[7] There are several important differences between our work and these studies. First, the MFCC is a strategic decision while we take consumer identity management as exogenous and focus on firms' pricing strategies given all possible types of market segmentation. Second, the MFCC is extended to all

[6]Acquisti and Varian (2005) briefly touch upon the duopoly case with two consumer types.

[7]In some cases, the MFCC can be ruled anti-competitive, Du Pont and Ethyl being the best-known example. More recently a similar decision was made in Germany against online hotel portals (Heinz, 2016).

existing customers while the poaching price in our model applies only to active, targeted consumers. Third, the MFCC is shown to facilitate tacit collusion in a dynamic setting whereas we show such outcome is possible in a one-shot game when consumers are active and the market is fully targeted without an overlap.

3 The Model and Preliminaries

3.1 The model

There are two firms, A and B, selling competing brands of a consumer good. The good is produced at a constant marginal cost that is normalized to zero. There is a continuum of consumers, each demanding one unit of the good. Consumers are heterogeneous in their preferences, or brand loyalty denoted by l. Following Fudenberg and Tirole (2000) and Shaffer and Zhang (2002), we define a consumer's brand loyalty as the minimum price differential necessary to induce that consumer to purchase her less preferred brand. Specifically a consumer with brand loyalty l derives a gross value $V_A(l) := v + l/2$ from buying brand A and derives a gross value $V_B(l) := v - l/2$ from purchasing brand B where $l \in [-l_B, l_A]$. Thus a consumer with brand loyalty l will buy brand B if and only if firm A's price exceeds firm B's price by more than l. To present our results in most clear and parsimonious way, we assume $v = 1$ and that l is distributed uniformly on $[-1/2, 1/2]$, hence the total population of consumers is normalized to one. As we show later, this simplification also ensures that the market is fully covered if it was served by a monopoly. Our main insight and the qualitative results from our analysis carry through for general values of v, l_A and l_B insofar as $l_A, l_B > 0$ and v is large enough to ensure full market coverage under monopoly.

Each firm has a target segment of the market, in which it has full information about the exact preferences of all consumers. For example, firms can gather customer information either by purchasing it from other firms such as data brokers or by collecting it on their own through the use of various tracking technologies.[8] Each firm can choose personalized prices for its targeted consumers. For non-targeted consumers, each firm charges a uniform price. We consider a natural case in which each firm's target segment is a connected set and includes consumers that are more loyal to the firm than to its rival. Let $[-1/2, b]$ be firm B's target segment and $[a, 1/2]$ be firm A's target segment. Let $p_A(x)$ be the personalized price firm A chooses for its targeted consumer $x \in [a, 1/2]$, and q_A be the uniform price firm A chooses for all its non-targeted consumers. Likewise, let $p_B(y)$ be the personalized price firm B chooses for its targeted consumer $y \in [-1/2, b]$, and q_B be the uniform price it chooses for the rest. We assume that the values of a and

[8]Gathering customer information necessary for market segmentation is costly. In this paper, we assume away such costs and focus on pricing equilibria taking various configurations of market segmentation as given. For studies that consider endogenous information gathering, see Chen and Iyer (2002) or Bergemann and Bonatti (2015).

b are publicly known.

The timing of the game is as follows, which is standard in the literature (Thisse and Vives, 1988; Choudhary et al. 2005; Matsumura and Matsushima, 2015; Choe et al. 2017). In the first stage, firms offer uniform prices to the non-targeted consumers, which are publicly observed. In the second stage, they charge personalized prices to their targeted consumers. By definition, each personalized price is observed only by the targeted consumer. Finally, each consumer makes a purchase decision after observing the uniform prices and the relevant personalized price. This two-stage pricing structure reflects the fact that the uniform prices are 'public' to the non-targeted consumers whereas the personalized prices are private. This is consistent with the commonly held view that a firm's choice of publicly observable prices is a higher-level managerial decision and is relatively slower to adjust in practice than a firm's choice of personalized prices that are offered privately.

Although uniform prices are publicly observed, firms may set some hurdles that prevent their targeted consumers from having access to the uniform prices, and bypassing these hurdles may incur various transactions costs. Our key modelling assumption is that some consumers can bypass the hurdles for price discrimination by utilizing various identity management strategies, as discussed in the introduction. We assume there are two types of consumers. A *passive* consumer faces prohibitively high transactions costs in identity management and cannot have access to the uniform price offered to non-targeted consumers by the firm targeting that consumer. Thus a passive consumer x targeted by firm A can choose between the personalized price $p_A(x)$ and the uniform price offered by the rival firm q_B. Likewise a passive consumer y targeted by firm B can choose between $p_B(y)$ and q_A. In contrast, an *active* consumer does not face any transactions costs in identity management and can thus choose an additional uniform price offered by the firm targeting that consumer. Consequently, active consumer x targeted by firm A can choose from $p_A(x)$, q_B, and q_A, and active consumer y targeted by firm B can choose from $p_B(y)$, q_A, and q_B.

As discussed in Section 2, we consider *ex post* identity management. This is different from *ex ante* identity management, which prevents firms from exercising price discrimination in the first place. It is also different from *interim* identity management, which does not have bite in our static model. For example, an interim active consumer x targeted by firm A can decide between $p_A(x)$ and q_A, but before observing these prices. That is, her only choice is to remain targeted or non-targeted. Our assumption of ex post identity management reflects the fact that it becomes increasingly difficult for consumers to hide personal information ex ante, due to the advances in technologies and the proliferation of data brokers. In addition, active consumers have no reason to manage their identity ex ante if they are confident they can do so ex post after having observed all the options available to them. In Appendix B, we provide evidence supporting this type of active consumers.

3.2 Benchmark results

For future reference, we document the results from four benchmark models adapted to our setup. In all these models, we keep the standard assumption that all consumers are passive.

First, the standard Hotelling model corresponds to the case $a = -b = 1/2$. Given q_A and q_B, the marginal consumer z is determined by $z = q_A - q_B$. Thus firm A's profit is equal to $\pi_A = q_A(1/2 - q_A + q_B)$, and firm B's profit is $\pi_B = q_B(q_A - q_B + 1/2)$. Solving for the Hotelling equilibrium then gives us $q_A = q_B = 1/2$, and each firm earns profit equal to $1/4$, hence the total industry profit of $1/2$.

Second, in Fudenberg and Tirole (2000), firms compete using third degree price discrimination where each firm targets all its loyal consumers: $a = b = 0$.[9] Let p_i be the price firm i chooses for its target segment and q_i be the price for the rival's target segment. As in Fudenberg and Tirole, we assume firms choose these prices simultaneously. Then on $[-1/2, 0]$, consumers face two prices, p_B and q_A, hence the marginal consumer is given by $z = q_A - p_B$. Thus profits from this segment are $\pi_A = q_A(p_B - q_A)$ and $\pi_B = p_B(q_A - p_B + 1/2)$. This leads to $p_B = 1/3, q_A = 1/6$ and $z = -1/6$. The other segment $[0, 1/2]$ is symmetric, leading to $p_A = 1/3, q_B = 1/6$ and the marginal consumer's location at $1/6$. As a result, firm A serves $[-1/6, 0] \cup [1/6, 1/2]$ and firm B serves $[-1/2, -1/6] \cup [0, 1/6]$, and each firm has profit equal to $5/36$, hence the total industry profit equal to $5/18$.

The third case is Thisse and Vives (1988) where all consumers are targeted by both firms ($a = -b = -1/2$) and firms compete in personalized pricing. Then firm A chooses personalized prices $p_A(x) = x$ serving all its loyal consumers on $[0, 1/2]$ and firm B chooses $p_B(y) = -y$ serving all its loyal consumers on $[-1/2, 0]$. Each firm's profit is then $1/8$ and so the total industry profit is $1/4$. Comparing these three cases shows that firms are worst off when they compete in personalized prices and best off in the Hotelling equilibrium. It is in this sense that more consumer information that can be used for price competition intensifies competition and hurts profitability.

Finally, consider the case with a non-discriminating monopolist. Without loss of generality, suppose the market is monopolized by firm A. Given the monopoly price q_A, the marginal consumer z is given by $1 + z/2 = q_A$. Thus the firm's profit is $\pi_A = (1 + z/2)(1/2 - z)$. Maximizing it leads to $z = -3/4 < -1/2$, hence the market is fully covered. The monopoly price is then $q_A = 5/8$, which is also equal to the firm's profit.

3.3 Non-contestable consumers

Personalized pricing allows firms to defend their target segment aggressively. Since personalized prices are offered privately to the targeted consumers, a firm can reduce a

[9] We only report the second-period outcome from Fudenberg and Tirole (2000) since competition is in third degree price discrimination only in the second period of their model.

personalized price for a particular consumer without changing the offers to other targeted consumers. In other words, a firm can defend each of its targeted consumers individually with personalized pricing and, if necessary, reduce the price for the marginal consumer down to zero.

To see this, suppose firm B chooses a uniform price q_B with a view to poaching some of firm A's targeted consumers. Then firm A's targeted consumer $x \in [a, 1/2]$ chooses between two prices: the personalized price $p_A(x)$ offered by firm A and the uniform price q_B posted by firm B. Thus consumer x will choose firm A if and only if $x \geq p_A(x) - q_B$, implying that firm A's optimal personalized price for consumer x is $p_A(x) = \max\{q_B + x, 0\}$.[10] If firm B aims to poach firm A's targeted consumers, the most aggressive price it can offer is $q_B = 0$. In this case, the personalized price firm A can choose to defend its targeted consumer x becomes $p_A(x) = \max\{x, 0\}$. Thus, for any $q_B \geq 0$, firm A can use personalized pricing to profitably defend all its targeted consumers as long as $a \geq 0$.

We say firm A's targeted consumer x is *non-contestable* by firm B if, for any $q_B \geq 0$, firm A can find $p_A(x) \geq 0$ that consumer x will choose over q_B. Firm A's non-contestable segment consists of all firm A's targeted consumers who are non-contestable by firm B. Firm B's non-contestable segment is defined in a similar way. Based on the argument above, it is straightforward to see that firm A's non-contestable segment is given by $\mathbf{N}_A \equiv [\max\{0, a\}, 1/2]$, and firm B's non-contestable segment is given by $\mathbf{N}_B \equiv [-1/2, \min\{b, 0\}]$. We summarize this in the following lemma.

Lemma 1 *When firms can charge personalized prices, firm A has a set of targeted consumers* $\mathbf{N}_A = [\max\{0, a\}, 1/2]$ *non-contestable by firm B and firm B has a set of targeted consumers* $\mathbf{N}_B = [-1/2, \min\{b, 0\}]$ *non-contestable by firm A.*

Lemma 1 has important implications. First, by definition, a firm will serve all targeted consumers in its non-contestable segment. Thus firm A does not benefit from trying to poach firm B's targeted consumers if $b \leq 0$ and, similarly, firm B can never gain in trying to poach firm A's targeted consumers when $a \geq 0$. Second, each firm has a maximum set of non-contestable consumers, $[0, 1/2]$ for firm A and $[-1/2, 0]$ for firm B. If a firm has a target segment beyond this maximum set, its targeted consumers with negative loyalty can be poached by its rival.

3.4 Equilibrium

We adopt subgame perfection as our basic equilibrium concept. But subgame perfection admits multiple equilibria when a firm chooses a uniform price in its rival's

[10] We make the following tie-breaking assumptions. First, a consumer chooses the firm that targets her when she is indifferent. Second, when a consumer is not targeted by either firm, she chooses firm A when indifferent.

non-contestable market segment. For example, suppose firm B chooses q_B in $\mathbf{N}_A$. Then for any $q_B \geq 0$, firm A can successfully defend $\mathbf{N}_A$ by choosing $p_A(x) = q_B + x$ for all $x \in \mathbf{N}_A$. Thus firm B is indifferent to any $q_B \geq 0$. In this case, we use a refinement in the spirit of Fudenberg et al. (1988). Specifically, for a given game without a commonly contestable market segment, we consider a perturbed game by allowing a small segment that is contestable by both firms. For each firm, the perturbed game has a unique uniform price in equilibrium. Then we select the equilibrium of our original game as the limit of the equilibrium of the perturbed game as the area of contestable segment converges to zero.[11] The following lemma shows that such a refinement allows us to select a unique uniform price in equilibrium.

Lemma 2 *Suppose $z_i \in \mathbf{N}_j$ for $i, j = A, B$. Consider an ϵ-neighborhood of z_i where both firms choose uniform price and let $q_i(\epsilon)$ be the equilibrium uniform price chosen by firm i. Then $\lim_{\epsilon \to 0} q_i(\epsilon) = 0$.*

Proof: See Appendix A. ∎

3.5 Preview of the key results

Before we proceed to the full analysis of our model in the next section, we offer a brief account of the two most striking results from the analysis. The purpose is to highlight the main differences in equilibrium that result when consumers are passive *vis-à-vis* active, and provide intuition behind the differences.

Our first result shows that, when consumers are active, a small proportion of non-targeted consumers may not be served in equilibrium, resulting in a deadweight loss. In contrast, when consumers are passive, the market is always fully covered since firms can delink their pricing policies for targeted and non-targeted consumers. As an illustration, suppose $a = -b = \delta$ where $\delta > 0$ is sufficiently small, hence $[-\delta, \delta]$ is not targeted by either firm. When consumers are passive, the two firms compete à la Hotelling on $[-\delta, \delta]$, and the resulting uniform prices are $q_A = q_B = \delta$. Equilibrium personalized prices are then $p_A(x) = x + \delta$ for all $x \in [\delta, 1/2]$ and $p_B(y) = -y + \delta$ for all $y \in [-1/2, -\delta]$, and each firm serves all its loyal consumers. With active consumers, however, serving consumers on $[-\delta, \delta]$ can be costly since the uniform price that can appeal to them should be sufficiently low because they have weak brand loyalty to both firms. But the low uniform price will also attract targeted consumers. Therefore, when δ is small enough, both firms would optimally choose not to serve the segment $[-\delta, \delta]$. In equilibrium,

[11] This is similar to the perturbation in ϵ-equilibria (Jackson et al., 2012). Firms consider by mistake that there is a small segment that is commonly contestable and compete using uniform price. We look at the equilibrium as such mistakes vanish in the limit. We need such a refinement only in the analysis when all consumers are passive. When some consumers are active, a firm's poaching price is uniquely determined in all cases.

firms choose sufficiently high uniform prices ($q_A, q_B \geq 5/4$) that are not accepted by any consumers, both targeted and non-targeted. This allows them to exercise perfect price discrimination for their targeted consumers: $p_A(x) = 1 + x/2$ for all $x \in [\delta, 1/2]$ and $p_B(y) = 1 - y/2$ for all $y \in [-1/2, -\delta]$. In sum, non-targeted consumers are not served by either firm, hence lower welfare compared to when all consumers are served.

Our second result shows that competition is softened when consumers are active, which can allow firms to extract full consumer surplus through perfect price discrimination. In contrast, passive consumers toughen competition by making firms aggressive in poaching, leading to an all-out competition à la Thisse and Vives (1988). This can be illustrated by the previous example, $a = -b = \delta > 0$. When consumers are passive, we showed that firms choose uniform prices, $q_A = q_B = \delta$. In the limit as $\delta \to 0$, we have $q_A = q_B = 0$, and the personalized prices converge to those in Thisse and Vives (1988): $p_A(x) = x$ and $q_B(y) = -y$. This outcome results in the lowest possible total industry profit, 1/4. When consumers are active, however, firms do not serve $[-\delta, \delta]$ while extracting full consumer surplus from all its targeted consumers. As $\delta \to 0$, each firm serves all its loyal consumers, resulting in the equilibrium with perfect price discrimination and the maximum possible total industry profit, 9/8.

The above two results highlight the downside when consumers are active or any policy intended in that direction. Namely, active identity management by consumers can soften competition and lead to lower consumer surplus and lower social welfare. In what follows, we show that the general intuition extends in other cases or even when there is only a small measure of active consumers.

4 Passive Consumers

Suppose all consumers are passive. In this case, firms can separate their pricing policies for targeted and non-targeted consumers. At first glance, such flexibility benefits firms because they can use personalized prices to protect their targeted consumers from possible poaching. But the flip side is that competition for non-targeted consumers intensifies. Since the rival's poaching offer limits the extent to which the firm can set personalized prices for its targeted consumers, the intensified competition for non-targeted consumers prevents firms from extracting full surplus from targeted consumers even though they use personalized prices. As we will show below, firms are worse off as they have more targeted consumers: the best outcome for firms is the Hotelling outcome when neither has any targeted consumers.

We now characterize all possible equilibria when consumers are passive. These equilibria can be grouped into three types. Let us start with the case where all consumers are targeted by at least one firm, $a \leq b$. This may correspond to an established market where firms are fairly well-informed about their customers and some consumers patronize both firms.

□ *The equilibrium with one-way poaching.* In this equilibrium, only one firm poaches some consumers targeted by its rival. This equilibrium is likely when only one firm aggressively targets consumers loyal to its competitor, i.e., either $a \leq b \leq 0$ or $0 \leq a \leq b$. Suppose $a \leq b \leq 0$. Then by Lemma 1, firm B's target segment $[-1/2, b]$ is not contestable by firm A. In addition to $[-1/2, b]$, firm B can poach some consumers targeted by firm A, say $[a, z]$, since consumers on $[a, 0]$ are loyal to firm B. Firm A will serve all remaining consumers. Since only firm B poaches some consumers targeted by firm A, we call this the equilibrium with one-way poaching by firm B. We show in Proposition 1 below that this equilibrium can be described formally as follows. Firm B serves $[-1/2, b]$ with personalized prices $p_B(y) = -y$, and $[b, b/2]$ with $q_B = -b/2$. Firm A serves $[b/2, 1/2]$ with $p_A(x) = x - b/2$, and chooses $p_A(x) = 0$ for $[a, b/2]$ and $q_A = 0$. The total industry profit is then

$$\pi_A + \pi_B = \int_{b/2}^{1/2} \left(x - \frac{b}{2}\right) dx + \left(\frac{b}{2}\right)^2 + \int_{-1/2}^{b} (-y)dy = \frac{2 - 2b - b^2}{8}. \tag{1}$$

The other case $0 \leq a \leq b$ is a mirror image of the above case and the equilibrium is with one-way poaching by firm A. Figure 1 illustrates the equilibrium with one-way poaching by firm B.

— Figure 1 goes about here. —

□ *The equilibrium with two-way poaching.* Suppose $a \leq 0 \leq b$ so that each firm targets some of its rival's loyal consumers. In this case, competition on $[a, b]$ leads to both firms choosing zero personalized price for the rival's loyal consumers. Given that both firms also choose zero uniform price, each firm serves all its loyal consumers, including some of its rival's targeted consumers. Thus we call this the equilibrium with two-way poaching. In the equilibrium with two-way poaching, firm A serves $[0, 1/2]$ with $p_A(x) = x$, firm B serves $[-1/2, 0]$ with $p_B(y) = -y$, and both firms choose $q_A = q_B = 0$. The total industry profit is then

$$\pi_A + \pi_B = \int_{0}^{1/2} x dx + \int_{-1/2}^{0} (-y)dy = \frac{1}{4}. \tag{2}$$

It is easy to see that $a \leq 0 \leq b$ is a necessary and sufficient condition for two-way poaching to arise in equilibrium. Note that this equilibrium is independent of (a, b). This includes the case in Thisse and Vives (1988) where the entire market is targeted by both firms, i.e., $-a = b = 1/2$. Figure 2 illustrates the equilibrium with two-way poaching.

— Figure 2 goes about here. —

Suppose now $b < a$ so that consumers on $[b, a]$ are not targeted by either firm, for whom the two firms compete using uniform price. One possibility is that one firm serves all consumers on $[b, a]$ and possibly some of its rival's targeted consumers as well. This case was already covered above. So we only consider a new possibility in which $[b, a]$ is served by both firms as a result of Hotelling competition: there is $z \in [b, a]$ such that firm B chooses q_B to serve $[b, z]$ and firm A chooses q_A to serve $[z, a]$. This leads to the third type of equilibrium.

□ *The equilibrium with partial Hotelling outcome.* This equilibrium is likely when the commonly contestable segment $[b, a]$ is large enough so that firms are better off sharing this segment through Hotelling competition than one firm monopolizing it. If one firm tries to monopolize the commonly contestable segment, competition in uniform price intensifies, which in turn reduces personalized prices that each firm can choose for its targeted consumers. In Proposition 1, we show that this equilibrium can be characterized as follows: firm A serves $[a, 1/2]$ with $p_A(x) = x + q_B$ and $[z, a]$ with q_A; firm B serves $[-1/2, b]$ with $p_B(y) = -y + q_A$ and $[b, z]$ with q_B where $z = (a + b)/3, q_A = (2a - b)/3$, and $q_B = (a - 2b)/3$. The total industry profit is then

$$\begin{aligned} \pi_A + \pi_B &= (a - z)q_A + \int_a^{1/2} (x + q_B)dx + (z - b)q_B + \int_{-1/2}^{b} (-y + q_A)dy \\ &= \frac{1}{4} + \frac{8ab + 9(a - b) - 5(a^2 + b^2)}{18}. \end{aligned} \tag{3}$$

Figure 3 illustrates the equilibrium with partial Hotelling outcome.

— Figure 3 goes about here. —

We now provide our first main result, which describes possible equilibria for all configurations of market segmentation as represented by (a, b). For each pair of (a, b), there is a unique equilibrium, which is one of the following three types.

Proposition 1 *When all consumers are passive, there are three types of equilibria:*

- *The equilibrium with one-way poaching - (i) one-way poaching by firm A if $0 \leq a \leq 2b$ with total industry profit $\Pi = (2+2a-a^2)/8 \in [1/4, 11/32]$; (ii) one-way poaching by firm B if $2a \leq b \leq 0$ with total industry profit $\Pi = (2-2b-b^2)/8 \in [1/4, 11/32]$.*
- *The equilibrium with two-way poaching if $a \leq 0 \leq b$ with total industry profit $\Pi = 1/4$.*
- *The equilibrium with partial Hotelling outcome if $2b \leq a$ and $b \leq 2a$ with total industry profit $\Pi = 1/4 + \left(8ab + 9(a - b) - 5(a^2 + b^2)\right)/18 \in [1/4, 1/2]$.*

Proof: See Appendix A. ∎

Proposition 1 yields a number of observations that would be useful in comparison with the case in the next section where all consumers are active. First, in all equilibria firms serve all consumers in the market. It is because firms can delink their uniform price from personalized prices, which allows them to serve every non-targeted consumer using a non-negative uniform price. Since every consumer derives a positive gross value from either firm, efficiency requires full market coverage. Thus all equilibria are efficient and market segmentation is welfare-neutral when all consumers are passive.

Second, firms are better off in the equilibrium with partial Hotelling outcome than in the equilibrium with two-way poaching. This is because the Hotelling competition in the commonly contestable segment results in non-negative uniform prices, which in turn increase personalized prices above the level that would be set in the equilibrium with two-way poaching. For example, suppose $a = -b = \delta > 0$. Then the equilibrium uniform prices are $q_A = q_B = \delta$, personalized prices are adjusted upwards by δ relative to the equilibrium with two-way poaching, and the total industry profit is $\Pi = 1/4 + \delta(1 - \delta)$. The total industry profit increases in δ and is equal to the Hotelling profit when $\delta = 1/2$. The equilibrium collapses to the one with two-way poaching if $\delta = 0$.

Third, in all equilibria, total industry profit is bounded above by 1/2, total industry profit under the Hotelling equilibrium. In addition, the Hotelling equilibrium obtains if and only if $a = -b = 1/2$, i.e., neither firm has any targeted consumer. This implies that firms are collectively best off when they have no consumer information, and that more consumer information as represented by larger target segments intensifies competition and hurts firm profitability.

Fourth, two-way poaching is possible if and only if both firms target their rival's loyal consumers, leading to the smallest total industry profit of 1/4. Thus aggressive targeting results in the worst possible outcome for the firms. But this does not require full information, i.e., $-a = b = 1/2$ as in Thisse and Vives (1988). Minimal information needed for the Thisse-Vives outcome is $a = b = 0$. Any additional information beyond this has no additional effect on the equilibrium.

We summarize the above points in the next proposition.

Proposition 2 *The equilibrium with passive consumers has the following properties:*

- *The market is fully covered in all equilibria.*
- *In all equilibria, total industry profit is bounded above by 1/2, total industry profit under the Hotelling equilibrium, and below by 1/4, total industry profit in the equilibrium with two-way poaching.*
- *The Hotelling equilibrium obtains if and only if $a = -b = 1/2$.*

5 Active Consumers

We now consider the case where all consumers are active. Recall that an active consumer can choose from both uniform prices in addition to the relevant personalized price. Recall also that a firm's targeted consumers are more loyal to the firm than non-targeted consumers by assumption, implying that any uniform price that is accepted by some non-targeted consumers should necessarily be lower than personalized prices for the targeted consumers. Thus if a firm's uniform price is accepted by some non-targeted consumers, then it is also accepted by all of its targeted consumers. This implies that a firm can serve its targeted consumers with personalized prices if and only if it does not serve any non-targeted consumers. A firm can exclude all non-targeted consumers by choosing a sufficiently high uniform price. Given that $\max\{V_i(l) : i = A, B;\ l \in [-1/2, 1/2]\} = 5/4$, the lower bound on such uniform prices is $5/4$. In sum, there are only two possible pricing options for each firm, namely, serving only its targeted consumers with personalized prices or choosing a uniform price that is accepted by all consumers, both targeted and non-targeted.

5.1 Perfect price discrimination

If both firms choose uniform prices higher than $5/4$, then there is no poaching and each firm serves its targeted consumers only. In this case, each firm can exercise perfect price discrimination (PPD) by setting maximum personalized prices and extract full surplus from all of its targeted consumers. When each firm has a significantly large target segment, the PPD outcome yields the highest possible profit for each firm for the given market segmentation. When such an equilibrium exists, we call it a *PPD equilibrium* and use it as a benchmark outcome.[12]

We describe the PPD equilibrium formally below. Given a and b with $b \leq a$, firms choose uniform prices $q_A, q_B \geq 0$, which are followed by each firm choosing maximum personalized prices for all its targeted consumers: $p_A^{PPD}(x) = V_A(x) = 1 + x/2$ for all $x \in [a, 1/2]$ and $p_B^{PPD}(y) = V_B(y) = 1 - y/2$ for all $y \in [-1/2, b]$. When firms deviate to $q'_A, q'_B < 5/4$, subgame perfection requires that, for any deviation by its rival, each firm optimally defends its turf with a nonnegative price that does not exceed any consumer's maximum willingness to pay. Thus personalized prices for each firm in this case are $p_A(x) = \min\{1 + x/2, \max\{q'_B + x, 0\}\}$ and $p_B(y) = \min\{1 - y/2, \max\{q'_A - y, 0\}\}$. Given these off-the-path personalized prices, a firm cannot deviate to poach its rival's non-contestable, targeted consumers; the only reason for a deviation is to serve non-targeted consumers and/or poach the rival's contestable, targeted consumers. Each firm's profit in the PPD equilibrium is then

$$\pi_A^{PPD}(a) = \int_a^{1/2} p_A^{PPD}(x)\, dx = \frac{9 - 16a - 4a^2}{16}, \tag{4}$$

[12] Strictly speaking, a PPD equilibrium can exist only when $b \leq a$, i.e., there is no overlapping target segment. But we will also use the term when there is an overlapping target segment if firms can charge maximum personalized prices to a subset of its targeted consumers.

and

$$\pi_B^{PPD}(b) = \int_{-1/2}^{b} p_B^{PPD}(y)\,dy = \frac{9+16b-4b^2}{16}. \tag{5}$$

Thus the total industry profit is equal to

$$\Pi^{PPD} = \pi_A^{PPD}(a) + \pi_B^{PPD}(b) = \frac{9-8(a-b)-2(a^2+b^2)}{8}. \tag{6}$$

We analyze when the PPD equilibrium exists and, when it does not, find the alternative equilibrium. Following the argument similar to the case with passive consumers, we can classify all possible equilibria into several types. In doing so, we note that, whenever a firm exercises PPD for all or part of its targeted consumers, it chooses a uniform price above 5/4 and the off-the-path personalized prices described previously. Thus we omit the description of these prices when it is clear from the context.

With active consumers, firms can no longer delink their personalized prices from the uniform price. This makes possible deviations from the candidate equilibrium more complicated, leading to more types of possible equilibria than when consumers are passive. Thus we divide the analysis into two cases, first when the market is fully targeted and second, when the market is partially targeted.

5.2 Fully targeted market

When the market is fully targeted, i.e., $a \leq b$, there are three types of possible equilibria as described below.

☐ *The PPD equilibrium.* This equilibrium is likely when each firm has a considerable target segment so that neither has incentives to deviate by lowering the uniform price below 5/4. It is possible only when the market is fully targeted without an overlap, i.e., $a = b$. The PPD equilibrium can lead to the highest total industry profit when $a = b = 0$. In this case, (6) shows that each firm earns profit $\pi_A^{PPD} = \pi_B^{PPD} = 9/16$ and the total industry profit 9/8 is larger than the monopoly profit 5/8. As it was formally defined above, we omit the details. The PPD equilibrium is illustrated in Figure 4.

— Figure 4 goes about here. —

☐ *The PPD equilibrium with partial Thisse-Vives outcome.* This equilibrium exists when there is an overlapping target segment that includes consumers loyal to both firms, i.e., $a < 0 < b$. It can be described as follows. Given (a, b), firm A exercises PPD to serve $[b, 1/2]$ and chooses personalized prices $p_A(x) = x$ to serve $[0, b]$. Firm B exercises PPD to serve $[-1/2, a]$ and chooses personalized prices $p_B(y) = -y$ to serve $[a, 0]$. Thus this equilibrium involves the Thisse and Vives outcome on the commonly targeted segment while each firm exercises PPD for its targeted consumers who are not targeted by its rival. The total industry profit is then equal to

$$\pi_A^{PPD}(b) + \pi_B^{PPD}(a) + \int_a^0 (-y)dy + \int_0^b xdx = \frac{9 + 8(a-b) + 2(a^2+b^2)}{8}. \tag{7}$$

The equilibrium with partial Thisse-Vives outcome is illustrated in Figure 5. A variant of this equilibrium is when the commonly targeted segment is served by one firm only. This is possible when the commonly targeted segment includes consumers who are loyal to only one firm. For example, if $0 \leq a < b$, and a and b are not large enough, then firm A serves $[a, b]$ with personalized prices $p_A(x) = x$ while firm B chooses $p_B(y) = 0$ on this segment. In the other case with $a < b \leq 0$, firm B can serve all commonly targeted consumers insofar as a and b are not too close to $-1/2$.

— Figure 5 goes about here. —

□ *The equilibrium with one-way poaching.* We noted above that a variant of the PPD equilibrium with partial Thisse-Vives outcome involves firm A serving all commonly targeted consumers as long as a and b are not too close to $1/2$. Otherwise firm A can profitably deviate by charging a uniform price q_A and serving more consumers on $[z, 1/2]$ for some $z < b$. We show in Proposition 3 that the optimal deviation involves $q_A = z = 1/4$. Given firm A's deviation, firm B's best response is as was described previously in the definition of PPD equilibrium. Specifically, this equilibrium can be described as follows: firm A charges $p_A(x) = 1/4$ on $[a, 1/2]$ and $q_A = 1/4$, serving $[1/4, 1/2]$; firm B charges $p_B(y) = -y + 1/4$ and serves $[-1/2, 1/4]$, chooses $p_B(y) = 0$ on $[1/4, b]$, and $q_B = 0$ on $[b, 1/2]$. As firm A serves some of firm B's targeted consumers, we call this the equilibrium with one-way poaching by firm A. This equilibrium is illustrated in Figure 6. The total industry profit in this case is

$$\pi_A + \pi_B = \frac{1}{16} + \int_{-1/2}^{1/4} \left(-y + \frac{1}{4}\right) dy = \frac{11}{32}. \tag{8}$$

When $a < b \leq 0$, there is an equilibrium with one-way poaching by firm B that mirrors the above equilibrium, hence $q_B = 1/4$ and $p_A(x) = x + 1/4$, and the same total industry profit as in (8). In any equilibria with one-way poaching, the poaching firm chooses a uniform price $1/4$ and serves the most loyal $1/4$ fraction of the market.

— Figure 6 goes about here. —

Proposition 3 *When all consumers are active and the market is fully targeted, there are three types of equilibria as follows:*

- *The PPD equilibrium if $a = b \in [2 - \sqrt{6}, -2 + \sqrt{6}]$ with total industry profit as given in (6).*

- *The PPD equilibrium with partial Thisse-Vives outcome - (i) firms sharing the commonly targeted segment if* $a \le 0 \le b$ *with total industry profit as given in (7); (ii) firm A serving the commonly targeted segment if* $0 \le a \le b \le -2 + \sqrt{2(1+a^2)}$ *with total industry profit* $\Pi = \left(9 + 8(a-b) - 6a^2 + 2b^2\right)/8$*; (iii) firm B serving the commonly targeted segment if* $-2 + \sqrt{2(1+b^2)} \le a \le b \le 0$ *with total industry profit* $\Pi = \left(9 + 8(a-b) + 2a^2 - 6b^2\right)/8$.

- *The equilibrium with one-way poaching - (i) one-way poaching by firm A if* $b \ge a \ge 0$ *and* $b \ge -2 + \sqrt{2(1+a^2)}$ *with total industry profit* $\Pi = 11/32$*; (ii) one-way poaching by firm B if* $a \le b \le 0$ *and* $a \le -2 + \sqrt{2(1+b^2)}$ *with total industry profit* $\Pi = 11/32$.

Proof: See Appendix A. ■

5.3 Partially targeted market

When $b < a$ so that some consumers are not targeted by either firm, it is clear that neither the PPD equilibrium nor the PPD equilibrium with partial Thisse-Vives outcome exists. But the third type, the equilibrium with one-way poaching, continues to exist: one-way poaching by firm A if b is close to $1/2$ and one-way poaching by firm B is a is close to $-1/2$. In addition, there are three new types of equilibrium when the market is not fully targeted.

□ *The PPD equilibrium without full market coverage.* Suppose the non-targeted segment is small and each firm has a considerable target segment. Then both firms can find it profitable to exercise PPD on their target segments and ignore the non-targeted segment by setting high enough uniform prices that are not accepted on the equilibrium path. This is a variant of the PPD equilibrium but the description is precisely the same with the only difference that this equilibrium is under the condition $b < a$. It is illustrated in Figure 7.

— Figure 7 goes about here. —

□ *The equilibrium with PPD and a uniform price.* In this equilibrium, one firm exercises PPD and serves its targeted consumers only while the other firm uses a uniform price to serve the rest. This equilibrium is possible when the firm exercising PPD has a large enough target segment but the other firm choosing a uniform price has only a small target segment. Thus the non-targeted segment is not significant enough for the firm with a large target segment to give up its PPD, but significant enough for the firm with a small target segment. Specifically, given (b, a) with b sufficiently far away from $-1/2$ and a close enough to $1/2$, firm B exercises PPD to serve $[-1/2, b]$ and firm A chooses $q_A = p_B(b) + b = 1 + b/2$ to serve $[b, 1/2]$. Given $q_A = 1 + b/2$, consumer b is indifferent between staying with firm B and switching to firm A. Needless to say, firm B chooses $q_B \geq 5/4$. This equilibrium is illustrated in Figure 8. The total industry profit in this case is

$$\pi_A + \pi_B = \left(1 + \frac{b}{2}\right)\left(\frac{1}{2} - b\right) + \pi_B^{PPD}(b) = \frac{17 + 4b - 12b^2}{16}. \tag{9}$$

This equilibrium is similar to the equilibrium with one-way poaching in that one firm uses a uniform price and the other firm uses personalized prices on the equilibrium path. But the difference is that, in the equilibrium with PPD and a uniform price, there is no poaching and the personalized prices are equal to PPD prices. Note also that a mirror equilibrium exists when firm A exercises PPD and firm B chooses a uniform price $q_B = 1 - a/2$.

— Figure 8 goes about here. —

☐ *The Hotelling equilibrium.* When both firms have relatively small target segments, neither firm has an incentive to exercise PPD while ignoring the large, non-targeted segment. As a result, both firms choose uniform price, in which case the equilibrium is the Hotelling outcome. Of course this is subject to the conditions that neither firm has an incentive to deviate by reverting to personalized prices given the rival's Hotelling price. In sum, the Hotelling equilibrium is likely when there is a large segment not targeted by either firm.

The following proposition shows the range of (a, b) that admits each type of equilibrium.

Proposition 4 *When all consumers are active and the market is partially targeted, there are four types of equilibria as follows:*

- *The equilibrium with one-way poaching - (i) one-way poaching by firm A if $a \geq -2 + \sqrt{6}$ and $a > b > 1/4$ with industry profit $\Pi = 11/32$; (ii) one-way poaching by firm B if $b \leq 2 - \sqrt{6}$ and $b < a < -1/4$ with total industry profit $\Pi = 11/32$.*
- *The PPD equilibrium without full market coverage if (i) $a > b \geq \max\{(-3 + \sqrt{7 + 32a + 8a^2})/4, (4 - \sqrt{17 - 12a + 8a^2})/2\}$ or (ii) $(3 - \sqrt{23})/4 \geq a > b \geq 2 - \sqrt{6}$ or (iii) $(-3 + \sqrt{23})/4 \leq a < b \leq -2 + \sqrt{6}$ with total industry profit as given in (6).*
- *The equilibrium with PPD and a uniform price - (i) PPD by firm B if $a > b$ and $(3 - 2a - \sqrt{10(1 - 2a + 2a^2)})/2 < b < (-3 + \sqrt{7 + 32a + 8a^2})/4$ with total industry profit as given in (9); (ii) PPD by firm A if $a > b$ and $(-1 + a + \sqrt{5a(2 + a)})/4 < b < (4 - \sqrt{17 - 12a + 8a^2})/2$ with total industry profit $\Pi = (17 - 4a - 12a^2)/16$.*
- *The Hotelling equilibrium if $b \leq (1 - \sqrt{2})/2$ and $a \geq (\sqrt{2} - 1)/2$ with total industry profit $\Pi = 1/2$.*

Proof: See Appendix A. ∎

5.4 Comparing equilibria with passive or active consumers

Comparing the equilibria depending on whether consumers are passive or active, we can formally show the results previewed in Section 3.3. Namely, active consumers can benefit firms by softening price competition and enabling them to exercise PPD; active consumers can also induce firms to ignore some market segment that is not commonly targeted, resulting in an inefficient outcome. Moreover, for any market segmentation (a, b), total industry profit is higher when consumer are active than when they are passive. We discuss each of these points below.

First, consider the case where each firm's target segment includes all its loyal consumers and there is some commonly targeted segment, i.e., $a < 0 < b$. Then the PPD equilibrium with partial Thisse-Vives outcome obtains when consumers are active whereas the equilibrium is with two-way poaching when consumers are passive. As the commonly targeted segment vanishes, i.e., $a = b = 0$, firms can enjoy the full PPD profits with active consumers, resulting in the highest possible industry profit 9/8. But the equilibrium does not change when consumers are passive. Thus each firm is strictly better off when consumers are active. This was illustrated in Section 3.3. In addition, for all (a, b) such that $a \leq 0 \leq b$, the equilibrium is with two-way poaching when consumers are passive, leading to the lowest possible total industry profit 1/4. When consumers are active, however, the total industry profit is 1/4 if and only if both firms have full information, i.e., $-a = b = 1/2$. Thus more information is required to intensify competition when consumers are active. The following results are immediate from comparing Propositions 1 and 3.

Proposition 5 *(i) Suppose* $a = b = 0$. *When consumers are active, the PPD equilibrium obtains with the highest possible total industry profit* $9/8$. *When consumers are passive, the equilibrium results in the Thisse-Vives outcome with the lowest possible total industry profit* $1/4$. *(ii) The Thisse-Vives outcome obtains for all* $a \leq 0 \leq b$ *when consumers are passive, but if and only if* $-a = b = 1/2$ *when consumers are active.*

Second, suppose there is a small, commonly non-targeted segment around $l = 0$. As discussed in Section 3.3, firms may choose not to serve this segment when consumers are active since a low uniform price that can appeal to the non-targeted consumers will be also chosen by active, targeted consumers. This leads to the PPD equilibrium without full market coverage. From Proposition 4, one can show that the values of a and b that lead to the PPD equilibrium without full market coverage should necessarily satisfy $a \leq (\sqrt{17} - 4)/2 \approx 0.06$ and $b \geq (-\sqrt{17} + 4)/2 \approx -0.06$. With passive consumers, however, the market is always fully covered as firms compete à la Hotelling on the commonly non-targeted segment, no matter how small it is. Given that the social optimum requires full market coverage in our model, active consumers can lead to an inefficient outcome. In contrast, all equilibria are efficient when consumers are passive.

Proposition 6 *When consumers are active, there is a pair* (a, b) *with* $(-\sqrt{17}+4)/2 \leq b < 0 < a \leq (\sqrt{17}-4)/2$ *such that consumers on the segment* (a, b) *are not served in equilibrium, hence an inefficient outcome. When consumers are passive, the equilibrium is with partial Hotelling competition for all* $b < 0 < a$ *and the market is fully covered, hence efficient.*

Third, when consumers are passive, total industry profit is maximized in the Hotelling equilibrium, which is possible if and only if neither firm has any consumer information, i.e., $a = -b = 1/2$. Whenever firms acquire some consumer information, equilibrium departs from the Hotelling outcome, leading to smaller total industry profit. This is consistent with the conventional result that consumer information intensifies price competition and hurts firm profitability. When consumers are active, however, the Hotelling equilibrium obtains even when both firms have some consumer information. From Proposition 4, the threshold values of (a, b) are given by $a = -b = (\sqrt{2}-1)/2 \approx 0.21$. Whenever both firms have smaller target segments than indicated by these threshold values, equilibrium results in the Hotelling outcome. Once again, this implies that active consumers dull the competition-intensifying effect of consumer information. From Propositions 2 and 4, we have the following.

Proposition 7 *The Hotelling equilibrium obtains if and only if* $a = -b = 1/2$ *when consumers are passive, but if and only if* $b \leq (1-\sqrt{2})/2$ *and* $a \geq (\sqrt{2}-1)/2$ *when consumers are active.*

The results so far show how active consumers can benefit firms by softening price competition, but only in some special cases. We show below that this holds more generally in any possible equilibria for any given market segmentation.

Proposition 8 *For any* (a, b)*, total industry profit in equilibrium with active consumers is larger than that with passive consumers.*

Proof: See Appendix A. ∎

6 Robustness

So far we have considered the two polar cases where all consumers are either active or passive. Our main findings are that consumers are worse off and, in some cases, social welfare is also lower when all consumers are active. We now examine whether this insight carries through to the more realistic case with a mix of active and passive consumers.

Specifically, we ask if our main findings are more likely to hold when the proportion of active consumers increases.

To this end, suppose now that a fraction $\alpha \in (0, 1)$ of consumers are active. We can also interpret α as the probability a randomly drawn consumer is active. With $\alpha < 1$, it is now possible for each firm to use both personalized prices and a uniform price that are accepted on the equilibrium path, as it expects only a proportion α of its targeted consumers to choose the uniform price. Since the additional parameter α complicates our analysis, we examine only the cases with either $a = b$ or $a = -b$. Obviously we do not consider market segmentation for all values of (a, b) because of this simplification. But we do cover all possible types of market segmentation including the fully targeted market with an overlapping target segment ($-a = b = \delta > 0$) or without one ($a = b$), and the partially targeted market ($a = -b = \delta > 0$).

6.1 Fully targeted market

Suppose $a = b = \delta \geq 0$ so that the market is fully targeted without an overlapping target segment. The case with $\delta < 0$ is symmetric so the same analysis can be applied. We analyze how α affects the conditions for the existence of PPD equilibrium and the likelihood firms may deviate from the PPD equilibrium. Recall that the equilibrium is with one-way poaching for all $a = b \neq 0$ when $\alpha = 0$. When $\alpha = 1$, we have either the PPD equilibrium or the equilibrium with one-way poaching.

Consider the PPD equilibrium corresponding to (a, b). From Lemma 1, all consumers targeted by firm A are non-contestable by firm B. Thus firm B cannot benefit from poaching them, implying that firm B does not have incentives to deviate from the PPD equilibrium. On the other hand, firm A may try to poach some consumers on $[0, \delta]$ targeted by firm B. Intuitively, firm A's gain from poaching increases with δ. But active consumers targeted by firm A can also take advantage of the low poaching price. The cost associated with such active consumers decreases as firm A's target segment becomes smaller, i.e., as δ increases. Thus firm A is more likely to deviate from the PPD equilibrium as δ increases, implying a cutoff value of δ such that the PPD equilibrium exists when δ is below the cutoff.

To characterize the cutoff value, suppose firm A deviates from the PPD equilibrium by choosing a uniform price $q_A \leq 5/4$. Since firm B's off-the-path personalized prices on $[0, \delta]$ are zero due to subgame perfection, the marginal consumer z upon firm A's deviation is given by $z = q_A$. Thus firm A can poach firm B's targeted consumers on $[z, \delta]$ for an extra profit $q_A(\delta - q_A)$.[13] On the other hand, since q_A is lower than $p_A^{PPD}(x)$ for all $x \in [\delta, 1/2]$, firm A expects to lose $p_A^{PPD}(x) - q_A$ when its targeted consumer x also chooses q_A instead of $p_A^{PPD}(x)$. Given that the fraction of active consumers is α, firm A's expected loss is $\alpha \left[\pi_A^{PPD}(\delta) - q_A(1/2 - \delta)\right]$. Thus firm A' net benefit from the

[13]We only focus on the case $\delta \geq q_A$. Obviously, firm A will not deviate if this is not the case.

deviation is given by

$$\Gamma\left(q_A;\alpha,\delta\right) \equiv q_A\left(\delta - q_A\right) + \alpha q_A\left(\frac{1}{2} - \delta\right) - \alpha\pi_A^{PPD}(\delta). \tag{10}$$

Choosing q_A to maximize the above net gain leads to the optimal deviation price

$$q_A^d = \frac{\alpha + 2\delta(1-\alpha)}{4}, \tag{11}$$

and the maximum net gain from the deviation is equal to

$$\Gamma(\delta;\alpha) = (q_A^d)^2 - \alpha\pi_A^{PPD}(\delta). \tag{12}$$

Firm A does not deviate from the PPD equilibrium if $\Gamma(\delta;\alpha) \leq 0$.

We now characterize the conditions for the existence of PPD equilibrium. Note that $\Gamma(\delta;\alpha)$ increases in δ since q_A^d increases in δ and $\pi_A^{PPD}(\delta)$ decreases in δ. Note also that $\Gamma(0;\alpha) < 0$ and $\Gamma(1/2;\alpha) > 0$ for any $\alpha \in (0,1)$. Thus there exists a cutoff value, $\bar{\delta}(\alpha)$ solving $\Gamma(\bar{\delta}(\alpha),\alpha) = 0$ such that $\Gamma(\delta;\alpha) \leq 0$ if and only if $\delta \leq \bar{\delta}(\alpha)$. It follows that PPD equilibrium can be sustained if $\delta \leq \bar{\delta}(\alpha)$. The same analysis applies to the case $\delta < 0$, where the PPD equilibrium can be sustained if $\delta \geq -\bar{\delta}(\alpha)$. Combining the two cases, we conclude that the PPD equilibrium exists if $\delta \in [-\bar{\delta}(\alpha), \bar{\delta}(\alpha)]$.

Then how does α affect the range of δ that sustains the PPD equilibrium? The main point from our analysis in the previous section is that active consumers benefit firms by softening competition. Thus we expect that the PPD equilibrium is more likely when α increases. That is, as α increases, we expect $\bar{\delta}(\alpha)$ to increase so that the PPD equilibrium exists for a wider range of δ. This is easy to verify: by totally differentiating $\Gamma(\delta;\alpha) = 0$, we obtain $[q_A^d(1-\alpha) + \alpha(1+\delta/2)]d\delta - [\pi_A^{PPD}(\delta) - q_A^d(1/2-\delta)]d\alpha = 0$. Since the terms inside the brackets are positive, we have $d\delta/d\alpha \geq 0$ for all $\alpha \in (0,1)$ and $\delta \in [-1/2, 1/2]$. For example, simple calculation shows $\bar{\delta}(1/10) \approx 0.294$ and $\bar{\delta}(1/2) \approx 0.415$. Thus as the fraction of active consumers increases from 0.1 to 0.5, the likelihood of PPD equilibrium increases from 60% to 83% of the parameter range. It is also easy to check $lim_{\alpha\to 1}\bar{\delta}(\alpha) = -2 + \sqrt{6}$, as is consistent with Proposition 3.

When $\delta > \bar{\delta}(\alpha)$ or $\delta < -\bar{\delta}(\alpha)$, one of the firms deviates from the PPD equilibrium. Thus we have an equilibrium with one-way poaching. But the difference from the case where all consumers are active is that the firm that chooses a uniform poaching price also sets personalized prices so that only its active, targeted consumers choose the uniform price instead of personalized prices. Suppose $\delta > \bar{\delta}(\alpha)$ so that the equilibrium is the one with one-way poaching by firm A. Then its profit is $\pi_A(\delta;\alpha) = (q_A^d)^2 + (1-\alpha)\pi_A^{PPD}(\delta)$. As α increases, firm A's uniform price in (11) increases since firm A needs to make up for the loss from not being able to charge personalized prices to its active consumers. But the higher uniform price means firm A can poach less from its rival. It is easy to check that $\pi_A(\delta;\alpha)$ decreases in α and is equal to 1/16 when $\alpha = 1$, as shown in (8). We summarize the above discussions below.

Proposition 9 *Suppose a fraction α of consumers are active and the market is fully targeted without commonly targeted segment, i.e., $a = b$. Then there exists a cutoff $\bar{\delta}(\alpha)$ that solves $\Gamma(\bar{\delta}(\alpha), \alpha) = 0$ where $\Gamma(\cdot, \cdot)$ is given in (12).*

- *For any $\delta \in (-\bar{\delta}(\alpha), \bar{\delta}(\alpha))$, a unique PPD equilibrium exists.*
- *If $\delta < -\bar{\delta}(\alpha)$ or $\delta > \bar{\delta}(\alpha)$, then the equilibrium is with one-way poaching where the poaching firm uses the uniform price that increases in α.*
- *$\bar{\delta}(\alpha)$ is increasing in α and converges to $-2 + \sqrt{6}$ as α increases to 1. Thus the PPD equilibrium is supported by a wider range of δ as α increases.*

Suppose now $-a = b = \delta > 0$ so that the market is fully targeted with an overlapping target segment $[-\delta, \delta]$. Then the equilibrium results in the Thisse-Vives outcome when $\alpha = 0$, and PPD with partial Thisse-Vives outcome when $\alpha = 1$. Importantly, neither firm has any incentives to use a uniform price to poach its rival's targeted consumers. Since active consumers matter only when a firm uses a uniform price on the equilibrium path, these equilibria are invariant to α when $-a = b = \delta > 0$.

6.2 Partially targeted market

Suppose $a = -b = \delta > 0$ so that consumers on $[-\delta, \delta]$ are not targeted by either firm. With passive consumers, we have the equilibrium with partial Hotelling outcome while, with active consumers, we have the PPD equilibrium without full market coverage when δ is sufficiently small.[14] We consider how changes in α affect the firm's incentives to deviate from the PPD equilibrium without full market coverage.

Suppose firm A deviates from the candidate equilibrium. Since $[-1/2, -\delta]$ is firm B's non-contestable segment, firm A cannot poach any consumers targeted by firm B. Thus firm A's deviation involves serving commonly non-targeted segment $[-\delta, \delta]$ by choosing a uniform price q_A. Since the market is fully covered when served by a monopoly, firm A will optimally serve all consumers on $[-\delta, \delta]$, implying that its optimal deviation price is $q_A = 1 - \delta/2$. Then firm A earns additional profit $2\delta q_A$ from $[-\delta, \delta]$. From its targeted consumers who switch to the uniform price, firm A earns profit $\alpha(1/2 - \delta)q_A$. On the other hand, firm A loses an α fraction of its PPD profit. Putting all together, firm A's net benefit from the deviation is given by

$$\Delta(\delta; \alpha) \equiv \left[2\delta + \alpha\left(\frac{1}{2} - \delta\right)\right]\left(1 - \frac{\delta}{2}\right) - \alpha\pi_A^{PPD}(\delta). \tag{13}$$

[14] By substituting $a = -b = \delta > 0$ into the conditions for the PPD equilibrium without full market coverage in Proposition 4, one can find that the threshold value of δ is $(7 - 4\sqrt{3})/2 \approx 0.036$.

It is easy to check $\Delta(0;\alpha) = -\alpha/16 < 0$, $\Delta(1/2;\alpha) = 3/4 > 0$, and $\Delta(\delta;\alpha)$ increases in δ. Thus there is $\hat{\delta}(\alpha) \in [0, 1/2]$ such that $\Delta(\hat{\delta}(\alpha), \alpha) = 0$ and $\Delta(\delta;\alpha) \geq 0$ if and only if $\delta \geq \hat{\delta}(\alpha)$, implying that firm A does not deviate if $\delta \leq \hat{\delta}(\alpha)$. By symmetry, firm B does not deviate if $\delta \geq -\hat{\delta}(\alpha)$. In sum, the PPD equilibrium without full market coverage is sustained by $\delta \in (-\hat{\delta}(\alpha), \hat{\delta}(\alpha))$.

To see how changes in α affect the range of δ that sustains the above equilibrium, totally differentiate $\Delta(\delta, \alpha) = 0$. This leads to $[q_A(2-\alpha) + (\alpha + \delta/2)]\, d\delta - \left[\pi_A^{PPD}(\delta) - q_A(1/2 - \delta)\right] d\alpha = 0$ where $q_A = 1 - \delta/2$. Again both terms inside the brackets are positive, hence $d\delta/d\alpha \geq 0$. Thus the PPD equilibrium without full market coverage is more likely when the fraction of active consumers increases. For instance, it is easy to verify $\hat{\delta}(1/2) \approx 0.017$ and $\hat{\delta}(1) \approx 0.036$. This implies that 3.4% of the market is not served when a half of consumers are active but it increases to 7.2% when all consumers are active. In sum, as more consumers become active, an inefficient equilibrium becomes more likely. This extends Proposition 6 to the case $\alpha \in (0, 1)$.

Proposition 10 *Suppose a fraction α of consumers are active and the segment $[-\delta, \delta]$ is not targeted by either firm. Then there exists a cutoff $\hat{\delta}(\alpha)$ that solves $\Delta(\hat{\delta}(\alpha);\alpha) = 0$ where $\Delta(\cdot,\cdot)$ is given in (13).*

- *For any $\delta \in (-\hat{\delta}(\alpha), \hat{\delta}(\alpha))$, a unique PPD equilibrium without full market coverage exists.*
- *$\hat{\delta}(\alpha)$ is increasing in α. Thus the PPD equilibrium without full market coverage is supported by a wider range of δ as α increases.*

The above PPD equilibrium without full market coverage cannot be sustained if δ is sufficiently large. When $\alpha = 1$, Proposition 4 shows that the Hotelling equilibrium obtains if $\delta \geq (\sqrt{2} - 1)/2 \approx 0.21$. When $\alpha = 1$, however, Hotelling competition is for the entire market, not just on the commonly non-targeted segment $[-\delta, \delta]$, because firms cannot use both the uniform and personalized prices on the equilibrium path. With $\alpha < 1$, firms can use both types of prices and they can compete à la Hotelling on $[-\delta, \delta]$. In this case, we show below that, as more consumers become active, total industry profit increases because the (symmetric) Hotelling price increases.

To see this, suppose firms choose uniform prices q_A and q_B. Then firm A can attract consumers on $[q_A - q_B, \delta]$, hence earns profit $(\delta - q_A + q_B)q_A$. In addition, firm A's profit from its active, targeted consumers who also choose q_A is $\alpha(1/2 - \delta)q_A$. For the rest of its targeted consumers, firm A's personalized prices are given by $p_A(x) = x + q_B$, hence profit equal to $(1-\alpha)\int_\delta^{1/2}(x + q_B)dx$. Thus firm A's total profit is

$$\pi_A(q_A; q_B, \alpha) = (\delta - q_A + q_B)q_A + \alpha(1/2 - \delta)q_A + (1-\alpha)\int_\delta^{1/2}(x + q_B)dx. \qquad (14)$$

Choosing q_A to maximize the above profit and solving for the symmetric equilibrium prices, we obtain

$$q_A = q_B = \delta + (1/2 - \delta)\alpha. \tag{15}$$

This equilibrium price is higher than the Hotelling price δ on $[-\delta, \delta]$, with the term $(1/2 - \delta)\alpha$ reflecting the marginal benefit firms enjoy as more consumers become active. We summarize this below.

Proposition 11 *Suppose a fraction α of consumers are active and the segment $[-\delta, \delta]$ is not targeted by either firm. If $\delta \geq (\sqrt{2} - 1)/2$, then there exists a symmetric equilibrium in which both firms exercise PPD and choose a uniform price $\hat{q} = \delta + (1/2 - \delta)\alpha$.*

- *$\hat{q}$ is higher than the Hotelling equilibrium price δ on $[-\delta, \delta]$.*
- *$\hat{q}$ increases in α and, when $\alpha = 1$, it is equal to the Hotelling equilibrium price* $1/2$ *on* $[-1/2, 1/2]$.

7 Discussions

7.1 Implications for policy and consumer welfare

The use of big data for price discrimination has triggered debate in policy circles. In the US, for example, a report by the Council of Economic Advisers (2015) raised concerns that some consumers can be made worse off without knowing why, and proposed giving consumers greater access to and control over their information. The report also argued that increased transparency into how companies use consumer information would promote competition and better informed consumer choice. On February 27, 2015, President Obama announced the Consumer Privacy Bill of Rights Act, a draft bill intended to govern the collection and dissemination of consumer data, and in October, 2016, the Federal Communications Commission (FCC) adopted new rules that require ISPs to get consumers' opt-in consent before collecting, selling or sharing their information.[15] Opponents argued that the rules are duplicative regulation and would place an undue burden on ISPs and that all actors in the online space should be subject to the same rules. For instance, the Federal Trade Commission's acting chair, Maureen Ohlhausen, said that the industry should largely be left to regulate itself. She also defended the practices of personalized pricing by saying, "Information can be used to target some consumers with a higher price, but the same information can be used to target some consumers with a better deal."[16] On March 28, 2017, the US House of Representatives voted to repeal the

[15] More stringent rules that go beyond ISPs are in place or being proposed elsewhere. See, for example, the General Data Protection Regulation in the European Union (https://www.eugdpr.org/) or a report by the Productivity Commission (2017) in Australia. Indeed the EU regulation is the default option that many countries around the world follow with the US being an exceptional case ("Customer data: designing for transparency and trust", *Harvard Business Review*, May 2015).

[16] https://www.theguardian.com/technology/2017/mar/14/federal-trade-commission-internet-things-regulation

FCC's online privacy rules, which was signed by President Trump the following Monday ("Trump just killed Obama's internet-privacy rules - here's what that means for you", *Business Insider*, April 4, 2017).

Our analysis sheds light on the two related issues that are central to the above debate, namely, privacy rules and consumer empowerment. First, to the extent that stricter privacy rules make personalized pricing more difficult to implement, they have the effect of softening competition. Although many questions about privacy go beyond economic considerations, it is worth noting that laxer privacy rules are more likely to benefit consumers by intensifying competition. This is true whether consumers are passive or active. In the extreme case of no privacy where firms can freely use all consumer information, hence $a = -b = -1/2$, consumers benefit the most as competition becomes the most intense, as shown in Proposition 5-(ii). Second, consumer empowerment relates to giving consumers more control over their information and enabling them to have access to various tools that can help them with price search and comparison. If empowered consumers are more likely to be active in identity management, then consumer empowerment is more likely to benefit firms, as our analysis shows. Both considerations seem to favor a more liberal approach to the use of consumer information by firms when the focus is on improving consumer welfare. Of course this is subject to the caveat that our analysis is based on the assumption that consumer information is used solely for pricing purposes in the competitive setting. Either when consumer information can be used to provide value added thereby softening competition (Acquisti and Varian, 2005) or when it is used by a dominant firm, more liberal use of consumer information can benefit firms.

7.2 Implications for management

The general conclusion from our study is that more consumer information can intensify competition if the information is used solely for pricing purposes. But this effect can be mitigated when more consumers are active in identity management. In view of this conclusion, we discuss below some implications for management in regards to customer information and pricing strategies.

First, firms can rely on a set of strategies that can credibly signal their intent to soften price competition, such as price matching guarantees or most-favored customer clauses. By adopting a price matching guarantee, a firm commits to its own targeted consumers to match the rival's poaching price. For example, if firm A has a price matching guarantee in place, its targeted consumer x now faces a personalized price that cannot be higher than firm B's poaching price, i.e., $p_A(x) \leq q_B$. When both firms adopt price matching guarantees, the end result is softened competition and higher poaching price.[17] A most-favored customer clause is a firm's promise to a customer that no other customers will

[17] It is well known that price matching guarantees facilitate tacit collusion in a concentrated industry with low consumer search costs, although the opposite may be the case with enough heterogeneity in consumer search costs and brand loyalty. See, for example, Chen et al. (2001).

be offered a lower price. Suppose firm A issues a most-favored customer clause to all its targeted consumers. Then consumer x faces a personalized price $p_A(x) \leq q_A$. This has the same effect as when consumer x is active in identity management. Once again, this softens competition as shown in our paper.

Second, the sole use of customer information in our paper was to extract consumer surplus through personalized pricing, which is why more information can intensify competition. A natural implication is then customer information needs to be used not only for pricing purposes but also for broader customer relationship management, of which ultimate aim is to drive value by building one-to-one relationships with customers. For example, a firm can provide enhanced services to its targeted consumers such as personalized product recommendations or lowered transactions costs (Acquisti and Varian, 2005). It can also build a loyal customer base through various reward programs, personalized discounts, customer experience management, and so on (Kumar and Shah, 2004; Verhoef et al. 2009). Loyal customers are less price-sensitive, which increases the cost of poaching by the rival, which in turn softens competition. In addition, reward programs can also soften competition by facilitating tacit collusion (Fong and Liu, 2011).

Finally, our analysis shows that firms can benefit from consumer empowerment if it leads to more active identity management by consumers. In addition, firms can gain invaluable customer trust by being transparent about the collection and use of customer information, by giving customers control over their personal information, and by delivering value in return ("Customer data: designing for transparency and trust", *Harvard Business Review*, May 2015). But this requires coordinated effort by all competing firms. A firm that unilaterally adopts measures that empower its customers is likely to lose out at least in the short term since it cannot rely on effective personalized prices while its rivals can. A better approach would be collective and coordinated support for privacy rules that would lead to more, not less, consumer empowerment. At first glance, laxer privacy rules may appear pro-business by reducing red tape and increasing the benefits of data use. But this is only one side of the story. If stricter privacy rules lead to more active identity management by consumers, then both softened competition thanks to active identity management and additional value of customer trust can benefit firms.

8 Conclusion

This paper has studied a duopoly model where each firm has a target segment on which it can exercise personalized pricing, and consumers can engage in identity management. We find that more consumer information can intensify price competition but the effect can be mitigated when more consumers become active in identity management. When consumers are passive in identity management, firms are collectively best off by competing without consumer information, which leads to the Hotelling equilibrium. When consumers are active, firms are best off with information about their loyal consumers only, which allows them to exercise perfect price discrimination.

These results are driven by different mechanisms whereby firms can soften competition by refraining from choosing aggressive uniform prices. With passive consumers, firms cannot credibly commit not to use consumer information for personalized pricing for their targeted consumers. It is because they can choose personalized prices independent of the uniform price. This leads to Bertrand-type competition in uniform price. Thus having no information solves the commitment problem. On the other hand, active consumers increase the cost of choosing aggressive uniform prices, which decreases the value of personalized pricing. To make the most of personalized pricing, firms therefore have incentives to set high enough uniform prices that will not be chosen by their active, targeted consumers.

Our analysis shows that established results from the literature on competitive price discrimination change drastically when consumers are active in identity management. Indeed the conventional wisdom that consumer information intensifies competition no longer holds when consumers are active in identity management. Consumer information can benefit competing firms and, in some case, it allows them to achieve a collusive outcome without explicit or implicit collusion. In the age of big data and rapid advances in technology, more consumers have access to various tools to manage their personal data and search for better deals and lower price. This suggests that more consumers are likely to be active in identity management, undermining firms' attempt to price discriminate. This aspect of consumer reaction is an important element to take into account in a more realistic model of competitive price discrimination. The analytical results from the enriched model can be very different from those from the conventional model, often leading to widely varying implications for policy and management.

Among a number of important issues we left out in the current paper, we briefly discuss two. First, another limit to a firm's ability to price discriminate is a behavioral element such as consumers' concerns for fairness, as experienced by Amazon ("Test of "dynamic pricing" angers Amazon customers", *Washington Post*, October 7, 2000). Such fairness concerns have a similar effect as active identity management. For example, if consumers feel disutility when their personalized prices are higher than the firm's public price, this effectively limits the firm's ability to separate personalized prices from the public price. This in turn will soften competition, as shown by Li and Jain (2016) albeit in the two-period model as in Fudenberg and Tirole (2000). The key insight is that what limits an individual firm's ability to use consumer information for price discrimination can be good for firms collectively, as it can soften competition.

The second issue relates to a firm's targeting strategy. In our paper, each firm is endowed with the exogenously given target segment. We chose this approach since our purpose was to examine various pricing equilibria for all possible market segmentations. But the choice of target segment, or investment in consumer addressability in general, is an important management decision (Chen and Iyer, 2002). A natural extension of our model to endogenize target segments is to add another layer of game in the beginning where firms choose investment in consumer information. To start with, the investment can be identified with the choice of a and b. Then one can consider more general targeting strategies that allow any types of target segment. For example, a firm's target segment may not necessarily be a connected interval starting from the firm's location. It would be interesting to examine how the firm's targeting strategy is affected by identity management by consumers, and whether the main thrust of the current paper will continue to hold.

Appendix A: Proofs

Proof of Lemma 2. Consider the case $z_B \in \mathbf{N}_A$. First, suppose $z_B > 0$ and consider an ϵ-neighborhood of z_B given by $(z, z+\epsilon)$ where $z = z_B - \epsilon/2$. Firms compete on $(z, z+\epsilon)$ by choosing q_A and q_B. Let $\hat{z} \in (z, z+\epsilon)$ be such that $q_A = q_B + \hat{z}$, hence $\hat{z} = q_A - q_B$. Then firm A chooses $q_A \geq 0$ to maximize $(z+\epsilon-\hat{z})q_A$ and firm B chooses q_B to maximize $(\hat{z}-z)q_B$. Solving the first-order conditions leads to $q_B = (\epsilon - z)/3 < 0$. Thus $q_B = 0$ and $q_A = (z+\epsilon)/2 \to z_B/2$ as $\epsilon \to 0$. Second, suppose $z_B = 0$. Repeating the same step, we find $q_A = q_B = \epsilon/2 \to 0$ as $\epsilon \to 0$. The other case $z_A \in \mathbf{N}_B$ is symmetric, hence the proof is omitted. ∎

Proof of Proposition 1.

Fully targeted market ($a \leq b$). Suppose $a = b > 0$. First, consider $[b, 1/2]$. Consumers on this segment are not contestable by firm B. Thus firm B chooses $q_B = 0$ by Lemma 2, hence $p_A(x) = x$ for all $x \in [b, 1/2]$. Second, consider the segment $[a, b]$ where the two firms compete using personalized prices. Since this segment is non-contestable by firm B, firm A can serve the entire segment by choosing $p_A(x) = p_B(x) + x$. Bertrand competition then results in $p_B(x) = 0$ for all $x \in [a, b]$, leading to $p_A(x) = x$. Third, consider the segment $[0, a]$. Suppose there is $z \in [0, a]$ who is indifferent between choosing either firm. Given any q_A, firm B's optimal personalized price for z is $p_B(z) = q_A - z$. Anticipating this, firm A can lower q_A until the competition for z leads to $p_B(z) = 0$. Then firm A can choose $q_A = z$ to serve consumers on $[z, a]$. Firm A chooses q_A to maximize $(a - q_A)q_A$, leading to $q_A = z = a/2$. On $[0, a]$, firm A thus serves $[a/2, a]$ with $q_A = a/2$ and firm B chooses $p_B(x) = 0$ for all $x \in [a/2, a]$. Finally, firm B serves the remaining consumers using personalized prices $p_B(y) = q_A - y = a/2 - y$ for all $y \in [-1/2, a/2]$. Then firm A's profit is $\pi_A \equiv q_A(a - a/2) + \int_a^{1/2} x dx = 1/8 - a^2/4$ and firm B's profit is $\pi_B \equiv \int_{-1/2}^{a/2}(-y + a/2)dy = (1 + 2a + a^2)/8$. Thus the total industry profit is $\Pi \equiv (2 + 2a - a^2)/8$. On the range $a \in [0, b]$, the industry profit attains the minimum of $1/4$ when $a = 0$ and the maximum of $11/32$ when $a = b = 1/2$. This shows that the equilibrium is with one-way poaching by firm A if $a = b > 0$

Using the same argument, one can show that the case $a = b < 0$ leads to the equilibrium with one-way poaching by firm B and the total industry profit $\Pi \in [1/2, 11/32]$.

Consider now the remaining case $a \leq 0 \leq b$. Firm A serves the segment $[b, 1/2]$ by choosing $p_A(x) = x$ since Lemma 2 implies $q_B = 0$. Similarly, firm B serves the segment $[-1/2, a]$ using $p_B(y) = -y$ since $q_A = 0$. On the overlapping segment $[a, b]$, both firms use personalized prices. For each consumer, each firm's lowest personalized price is zero. This implies that firm A can never profitably serve $[a, 0]$ and firm B can never profitably serve $[0, b]$. Thus the marginal consumer's location is at zero. Consequently, firm A serves $[0, b]$ with $p_A(x) = x$ and firm B serves $[a, 0]$ with $p_B(y) = -y$. Then each firm earn profit equal to $1/8$, hence the total industry profit of $1/4$. This shows that the equilibrium is with two-way poaching if $a \leq 0 \leq b$.

Partially targeted market ($b < a$). Consider first the segment $[a, 1/2]$. Given q_B, firm A can choose $p_A(x) = x + q_B$ and serve all consumers on this segment as long as $p_A(x) \geq 0$ for all $x \in [a, 1/2]$. Since $p_A(x)$ is increasing in x, the sufficient condition is $p_A(a) = a + q_B \geq 0$. Similarly firm B can choose $p_B(y) = -y + q_A$ and serve all consumers on $[-1/2, b]$ as long as $p_B(b) = -b + q_A \geq 0$. Consider next $[b, a]$. It is easy to find the Hotelling equilibrium on $[b, a]$: $q_A = (2a - b)/3$, $q_B = (a - 2b)/3$, $z = (a + b)/3$, and $z \in [b, a]$ if and only if $q_A, q_B \geq 0$, or $2a \geq b$ and $a \geq 2b$. Given these conditions, we can check $p_A(a) = 2q_A \geq 0$ and $p_B(b) = 2q_B \geq 0$. Since firm A's profit is $\pi_A = (a - z)q_A + \int_a^{1/2}(x + q_B)dx$ and firm B's profit is $\pi_B = (z - b)q_B + \int_{-1/2}^{a}(-y + q_A)dy$, total industry profit is $\Pi = \pi_A + \pi_B = 1/4 + \left(8ab + 9(a - b) - 5(a^2 + b^2)\right)/18$. It is easy to check that Π attains a minimum $1/4$ when $a = b = 0$ and a maximum $1/2$ when $a = -b = 1/2$. This shows that the equilibrium is with partial Hotelling competition if $2a \geq b$ and $a \geq 2b$.

Suppose $0 \leq a \leq 2b$. Then we have $q_B \leq 0$ from the Hotelling outcome. Thus firm B chooses $q_B = 0$. In addition, firm B chooses $p_B(z) = 0$ if $z < b$, hence $z = q_A$. Thus firm A chooses q_A to maximize $(a - q_A)q_A$, leading to $q_A = z = a/2 \leq b$. In this case, firm A chooses q_A to serve all consumers on $[b, a]$ and additional consumers on $[a/2, b]$ targeted by firm B. So this case leads to the equilibrium with one-way poaching by firm A. Using the same argument, we can show the case $2a \leq b \leq 0$ leads to the equilibrium with one-way poaching by firm B.

Combining all the above cases, we have Proposition 1 ∎

Proof of Proposition 3. First, suppose $a = b = \delta > 0$. If firm A deviates from the PPD equilibrium by choosing $q_A^d < 5/4$, then it serves consumers on $[z, 1/2]$ where $z = q_A^d - \max\{q_A^d - z, 0\} = q_A^d$. Maximizing firm A's deviation profit $\pi_A^d = (1/2 - q_A^d)q_A^d$ leads to $q_A^d = z = 1/4$ and $\pi_A^d = 1/16$. Comparing π_A^d with $\pi_A^{PPD}(\delta)$, we find $\pi_A^{PPD}(\delta) < 1/16$ iff $\delta > -2 + \sqrt{6}$. A symmetric argument shows $\pi_B^{PPD}(\delta) < 1/16$ iff $\delta < 2 - \sqrt{6}$. Thus the PPD equilibrium exists iff $\delta \in [2 - \sqrt{6}, -2 + \sqrt{6}]$.

If $\delta > -2 + \sqrt{6}$, then firm A deviates from the PPD equilibrium by setting $q_A^d = 1/4$, serving those on $[1/4, 1/2]$. To this, firm B's best response is to choose personalized prices $p_B(y) = q_A^d - y$ for all $y \in [-1/2, 1/4]$ and $q_B \geq \max\{q_A - y : y \in [-1/2, 1/4]\} = 3/4$. The other case $\delta < -2 + \sqrt{6}$ is symmetric. Summarizing the case $a = b$, the PPD equilibrium exists for all $a = b \in [2 - \sqrt{6}, -2 + \sqrt{6}]$; the equilibrium is with one-way poaching by firm A if $a = b > -2 + \sqrt{6}$; the equilibrium is with one-way poaching by firm B if $a = b < 2 - \sqrt{6}$.

Consider now the case with an overlapping target zone, i.e., $a < b$. First, suppose $a < 0 < b$. Then on $[a, b]$, competition in personalized prices leads to the Thisse and Vives outcome: $p_B(y) = y$ for all $y \in [a, 0]$, $= 0$ for all $y \in [0, b]$, and similarly for $p_A(x)$. Thus the candidate equilibrium is the PPD equilibrium with partial Thisse-Vives outcome. If one firm deviates from this, then its deviation profit is $1/16$ as shown above. Since the equilibrium profit in the candidate equilibrium is bounded below by $1/8$, the profit from the Thisse and Vives outcome (when $-a = b = 1/2$), neither firm has an incentive to deviate. Thus if $a \leq 0 \leq b$, the unique equilibrium is the PPD equilibrium with partial Thisse-Vives outcome where the two firms share the commonly targeted segment.

Second, suppose $0 \leq a < b$. Then on $[a, b]$, firm B charges $p_B(y) = 0$, hence $p_A(x) = x$. Suppose firm A exercises PPD on $[b, 1/2]$ and firm B exercises PPD on $[-1/2, a]$. For this to be an equilibrium outcome, we only need to consider firm A's possible deviation since firm B does not benefit from any deviation. In the candidate equilibrium, firm A's profit is $\int_a^b x dx + \pi_A^{PPD}(b) = (4b^2 - 16b + 9 - 8a^2)/16$. There are two possible ways firm A can deviate from the candidate equilibrium. First, it can choose $q_A^d = 1 + b/2$ and serve $[b, 1/2]$. But this is clearly worse than the candidate equilibrium where firm A exercises PPD on $[b, 1/2]$. Thus we only need to consider the case where firm A chooses $q_A^d = 1/4$ and serves $[1/4, 1/2]$ with the deviation profit $1/16$. If $(4b^2 - 16b - 8a^2 + 9)/16 \geq 1/16$ or $b \leq 2 - \sqrt{2(1 + a^2)}$, then firm A does not deviate. In sum, if $0 \leq a < b \leq 2 - \sqrt{2(1 + a^2)}$, then we have the PPD equilibrium with partial Thisse-Vives outcome where firm A serves the commonly targeted segment.

If $b \geq 2 - \sqrt{2(1 + a^2)}$, then firm A deviates to $q_A^d = 1/4$ serving $[1/4, 1/2]$ and firm B chooses $p_B(y) = q_A - y$ serving $y \in [-1/2, 1/4]$. This leads to the equilibrium with one-way poaching by firm A. Thus the range of (a, b) supporting this equilibrium is $(a, b) \in \{(a, b) | 0 \leq a < b, b \geq 2 - \sqrt{2(1 + a^2)}\}$. Note that this covers the case $a = b \geq -2 + \sqrt{6}$ as a limiting case when $a = b$.

The other case $a < b \leq 0$ is symmetric. So we omit the details. ∎

Proof of Proposition 4. We analyze the case $b < 0 < a$ and identify the conditions for the existence of different types of equilibria. After that, we discuss any difference in the conditions for the other cases, i.e., $b < a \leq 0$ and $0 \leq b < a$.

Let us start with the PPD equilibrium without full market coverage. If firm A deviates from this equilibrium, then it chooses a uniform price q_A^d to serve $[z, 1/2]$ where $z = 2(q_A^d - 1)$ if $z \in [b, a]$, or $z = b$ if $q_A^d = 1 + b/2$. It is because, in the PPD equilibrium, consumers on (b, a) are not being served while consumer b receives the PPD price $1 - b/2$ from firm B. Maximizing the deviation profit leads to $q_A^d = 1 + b/2$, hence $\pi_A^d = (1/2 - b)(1 + b/2) = (2 - 3b - 2b^2)/4$. Thus firm A does not deviate if $\pi_A^{PPD}(a) \geq \pi_A^d$, or $b \geq (-3 + \sqrt{7 + 32a + 8a^2})/4$. Similarly, firm B's optimal deviation is given by $q_B^d = 1 - a/2$ and the deviation profit $\pi_B^d = (2 + 3a - 2a^2)/4$. Firm B does not deviate if $\pi_B^d > \pi_B^{PPD}(b)$, or $b \geq (4 - \sqrt{17 - 12a + 8a^2})/2$. Thus the PPD equilibrium without full market coverage exists if both of these conditions are satisfied along with $b < 0 < a$.

Consider next the PPD equilibrium with a uniform price where firm B exercises

PPD and firm A chooses $q_A = 1 + b/2$, serving $[b, 1/2]$. Firm A's profit is equal to the deviation profit from the previous case. Thus firm A does not deviate to PPD if $b \leq (-3 + \sqrt{7 + 32a + 8a^2})/4$. On the other hand, firm B's deviation involves choosing a uniform price $q_B^d = q_A - a = (2 + b - 2a)/2$, with which firm B can serve $[-1/2, a]$. Its deviation profit is then $\pi_B^d = (a + 1/2)q_B^d = \left(2 + b + 2(1 + b)a - 4a^2\right)/4$. Thus firm B does not deviate if $\pi_B^{PPD}(b) \geq \pi_B^d$ or $b \geq (3 - 2a - \sqrt{10(1 - 2a + 2a^2)})/2$.

Consider next the PPD equilibrium with a uniform price where firm A exercises PPD and firm B chooses $q_B = 1 - a/2$. Proceeding similarly as in the second case, we can show that neither firm deviates if $(-1 + a + \sqrt{5a(2 + a)})/4 < b < (4 - \sqrt{17 - 12a + 8a^2})/2$. The calculation of total industry profit is straightforward and is omitted.

Consider now the Hotelling equilibrium: $q_A = q_B = 1/2$ and $\pi_A = \pi_B = 1/4$. Firm A's deviation involves serving only its targeted consumers only using personalized prices $p_A^d(x) = q_B + x$ for all $x \in [a, 1/2]$. Its profit is then $\pi_A^d = \int_a^{1/2}(x + 1/2)dx = (3 - 4a - 4a^2)/8$. Thus firm A does not deviate if $1/4 \geq \pi_A^d$ or $a \geq (\sqrt{2} - 1)/2$. Similarly firm B does not deviate if $b \leq (1 - \sqrt{2})/2$. Thus the Hotelling equilibrium exists if and only if $a \geq (\sqrt{2} - 1)/2$ and $b \leq (1 - \sqrt{2})/2$.

Finally, the equilibrium one-way poaching does not exist in the current case $b < 0 < a$ since this equilibrium is possible only if $b < a < -1/4$ or $1/4 < b < a$.

Let us now turn to the case $b < a \leq 0$. First, this case does not admit the Hotelling equilibrium, nor the equilibrium with one-way poaching by firm A. Second, the equilibrium with one-way poaching by firm B exists only if $a < -1/4$. In this case, firm A has no incentives to deviate. Firm B's deviation involves exercising PPD on $[-1/2, b]$, leading to $\pi_B^d = \pi_B^{PPD}(b)$. Thus firm B does not deviate either if $\pi_B = 1/16 \geq \pi_B^{PPD}(b)$ or $b \leq 2 - \sqrt{6}$. This implies that the equilibrium with one-way poaching by firm B exists if $b < a < -1/4$ and $b \leq 2 - \sqrt{6}$. Third, for the PPD equilibrium without full market coverage, we also need to consider the case where firm B's optimal deviation involves $q_B^d = 1/4$, which is possible if $1/16 \geq (2 + 3a - 2a^2)/4$ or $a \leq (3 - \sqrt{23})/4$. In this case, firm B does not deviate from PPD if $\pi_B^{PPD}(b) \geq 1/6$ or $b \geq 2 - \sqrt{6}$. Summarizing, the PPD equilibrium without full market coverage can also exist if $(3 - \sqrt{23})/4 \geq a > b \geq 2 - \sqrt{6}$.

The other case $0 \leq b < a$ is analogous to the case $b < a \leq 0$. First, neither the Hotelling equilibrium nor the equilibrium with one-way poaching by firm B exists. Second, the equilibrium with one-way poaching by firm A exists if $1/4 < b < a$ and $a \geq -1 + \sqrt{6}$. Third, the PPD equilibrium without full market coverage can also exist if $(-3 + \sqrt{23})/4 \leq b < a \leq -2 + \sqrt{6}$. ∎

Proof of Proposition 8. We start with the fully targeted market. First, consider the case $a < 0 < b$. Then with passive consumers, the equilibrium is with two-way poaching and total industry profit is equal to 1/4. With active consumers, we have the PPD equilibrium with partial Thisse-Vives outcome where total industry profit is bounded below by 1/4 and is equal to 1/4 if and only if $-a = b = 1/2$, as can be verified from

(7). Consider next the case $0 \le a < b$ or $a < b \le 0$. With passive consumers, the equilibrium is with one-way poaching where total industry profit is bounded above by 11/32. With active consumers, we have either the equilibrium with partial Thisse-Vives outcome or the equilibrium with one-way poaching. In the former case, total industry profit is bounded below by 9/16 since one firm exercises PPD for all its loyal consumers, hence its profit is not smaller than $\int_{-1/2}^{0}(1-y/2)dy = \int_{0}^{1/2}(1+x/2)dx = 9/16$. In the latter case, total industry profit is 11/32 as shown in (8).

We now turn to the partially targeted market. First, consider the case $b < 0 < a$. With passive consumers, the equilibrium is with partial Hotelling outcome and total industry profit is bounded above by 1/2. With active consumers, there are three possibilities. First, in the PPD equilibrium, it can be verified that the conditions in Proposition 4 imply that the range of (a, b) admitting this equilibrium is such that $a < (\sqrt{17}-4)/2 < 1/16$ and $b > (-\sqrt{17}+4)/2 > -1/16$. It is easy to see $\pi_A^{PPD}(1/16)+\pi_B^{PPD}(-1/16) > 1/2$. Second, in the equilibrium with PPD and a uniform price, $\pi_A + \pi_B \ge 3/4$, as can be verified by (9). Third, in the Hotelling equilibrium, total industry profit is 1/2. Next consider the case $b < a \le 0$. With passive consumers, the equilibrium is either with one-way poaching or with partial Hotelling outcome. Total industry profit is at most 11/32 in the former and 1/2 in the latter. With active consumers, we have the PPD equilibrium without full market coverage or the equilibrium with PPD and a uniform price, or the equilibrium with one-way poaching. In the first two cases, total industry profit is always greater than 1/2. In the third case, total industry profit is 11/32 and the range of (a, b) supporting this equilibrium is $b \le 2-\sqrt{6}$ and $b < a < -1/4$. It is easy to see that this range of (a, b) admits only the equilibrium with one-way poaching when consumers are passive. The same argument applies to the remaining case $0 \le b < a$. ∎

Appendix B: Evidence on active consumers

In this appendix, we offer some evidence in support of our assumption that some consumers are active in the way described in this paper. We note first that, firms in various service industries offer new customer-only special deals at considerable discount compared to what existing customers pay. Examples of these industries include retail banking, information and communications, and cable TV services. Table 1 shows the savings received by new customers relative to existing customers.

— Table 1 goes about here. —

The information on whether existing customers haggle and also receive these new customer-only offers - active consumers in our terminology - is not publicly available. But one can search inside various Internet forums specifically designed to share experiences that consumers have in dealing with their service providers. For example, our search captures instances where posters indicate they are existing customers, where they mentioned the new customer offer, and where they reported to have made savings. Although our evidence is based on unverifiable cheap talk and is plagued by issues such as selection bias, it nonetheless tells us that a large number of consumers proactively react to firms' strategies to price-discriminate. The method we used to gain some insight from such data is as follows:

(1) We start with the search engine Google within the Internet forum to identify messages that contain a set of keywords that are likely to indicate that the messages are related to existing customers negotiating with their service providers.

(2) With the number of hits from (1) as a base, we further add keywords to the search that indicate some level of success in the negotiation.

(3) If reported, we record the dollar value of savings from the negotiation, although this relied on individual scraping of a subset of these results.

Table 2 shows the sources of our search, the sample size, and the keywords used for the search. As for the keywords that indicate a post from an existing customer, we used identifiers such as 'loyal', 'existing' or 'current'. We chose identifiers such as 'saved', 'reduced' or 'discounted' as the keywords indicating successful negotiation. We started our search with Reddit (https://www.reddit.com/), one of the largest websites for social news aggregation, web content rating, and discussions. Our search covered over 205 million posts and returned hits in services such as cable TV, credit cards, broadband Internet, and cell phone plans. Next, we extended our search to all Internet forums including any indexed page from Google that is categorized as a 'forum'. The search covered over 20 billion posts.

— Table 2 goes about here. —

Table 3 shows the results from each search as well as average savings from a successful negotiation. For example, the results from Reddit forum indicate that around 0.22 million customers reportedly negotiated with their Internet service providers with 70% being successful for average savings of \$138. The search over all forums obviously returned more hits but with varying degrees of success across different types of services. It is interesting that success rates are lower in this case than those reported in Reddit forum, especially for Internet and cell phone plans. Nonetheless, a significant number of consumers report to have haggled with their current service providers and succeeded in securing a discount. Unfortunately we cannot tell whether these savings are equivalent to what the existing customers could have secured from new customer-only deals. Judging from the information given in Table 1, the savings could be a fraction of new customer savings. But the presence of such active consumers and the savings they secure through negotiation undoubtedly undermine the effectiveness of discriminatory pricing, which in turn softens price competition.

— Table 3 goes about here. —

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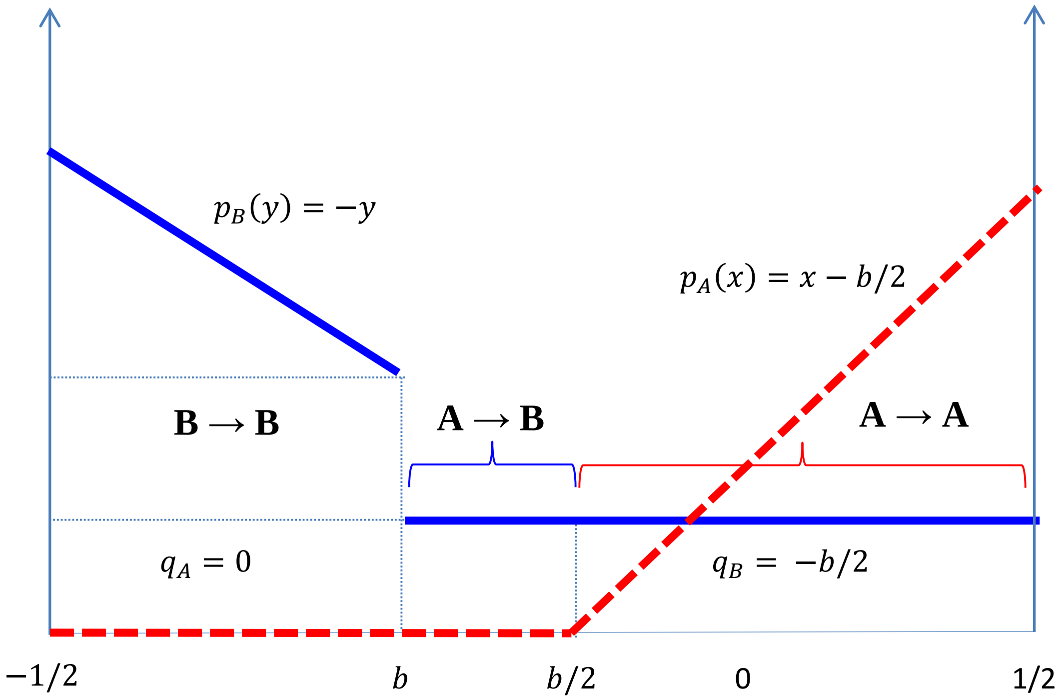

Figure 1: Equilibrium with one-way poaching by firm B

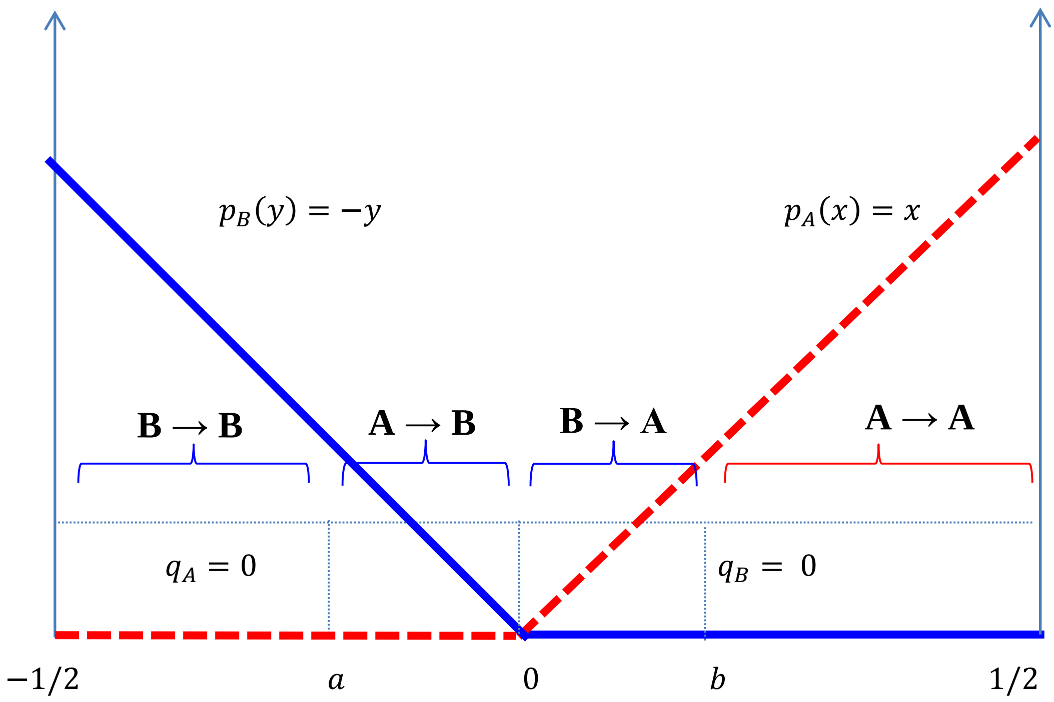

Figure 2: Equilibrium with two-way poaching

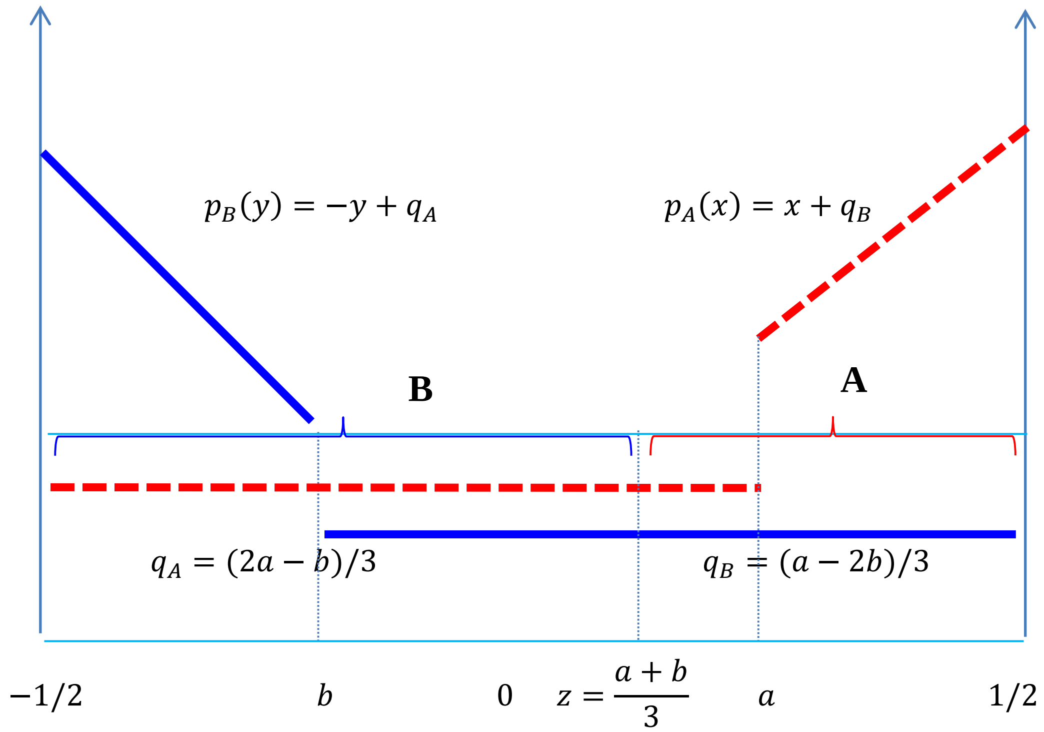

Figure 3: Equilibrium with partial Hotelling outcome

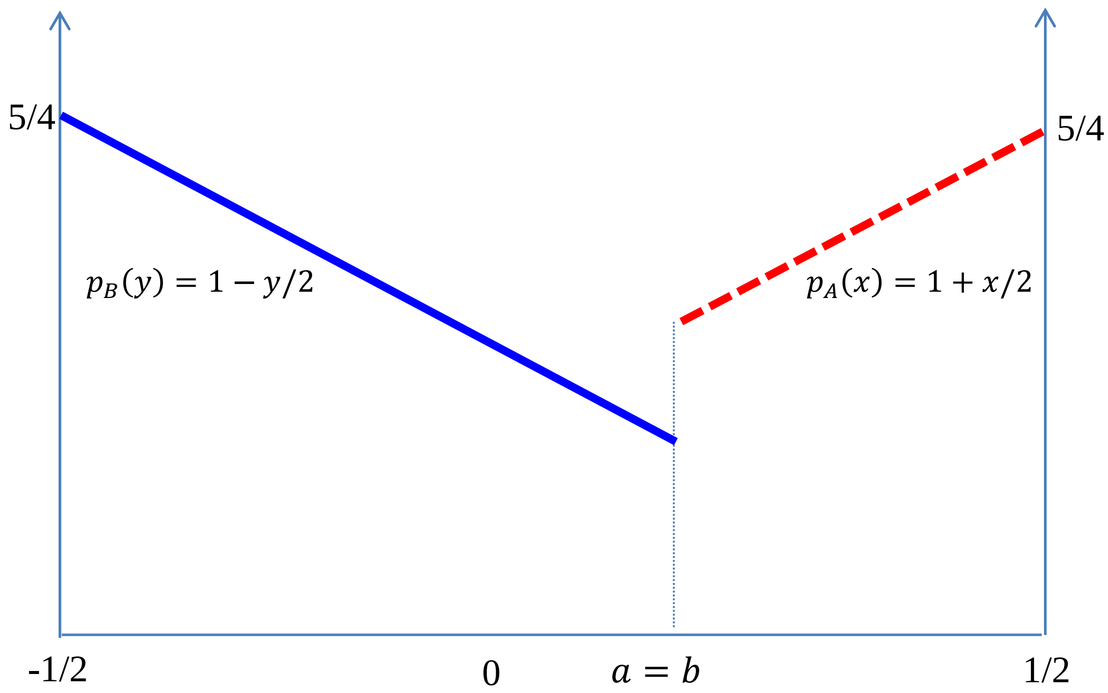

Figure 4: PPD equilibrium

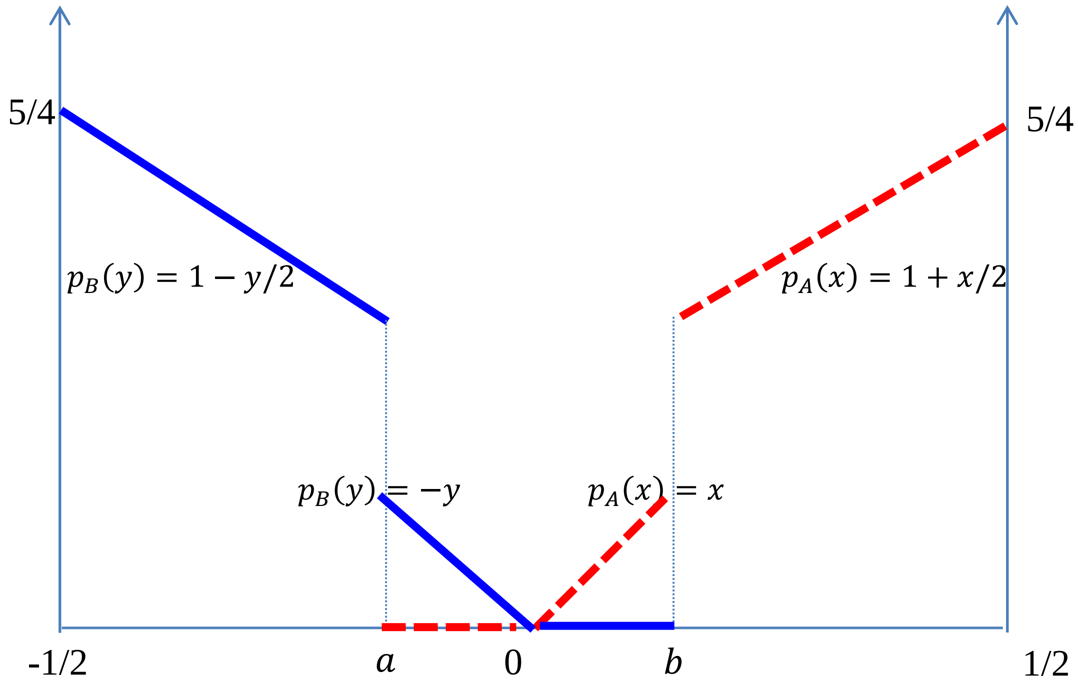

Figure 5: PPD Equilibrium with partial Thisse-Vives outcome

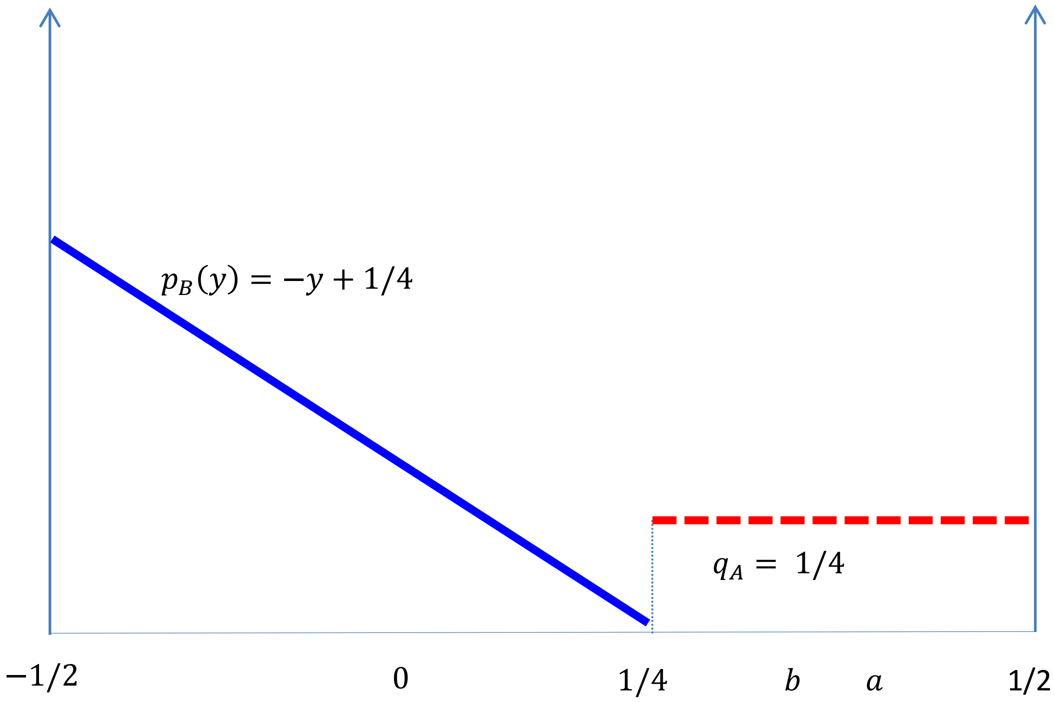

Figure 6: Equilibrium with one-way poaching by firm A

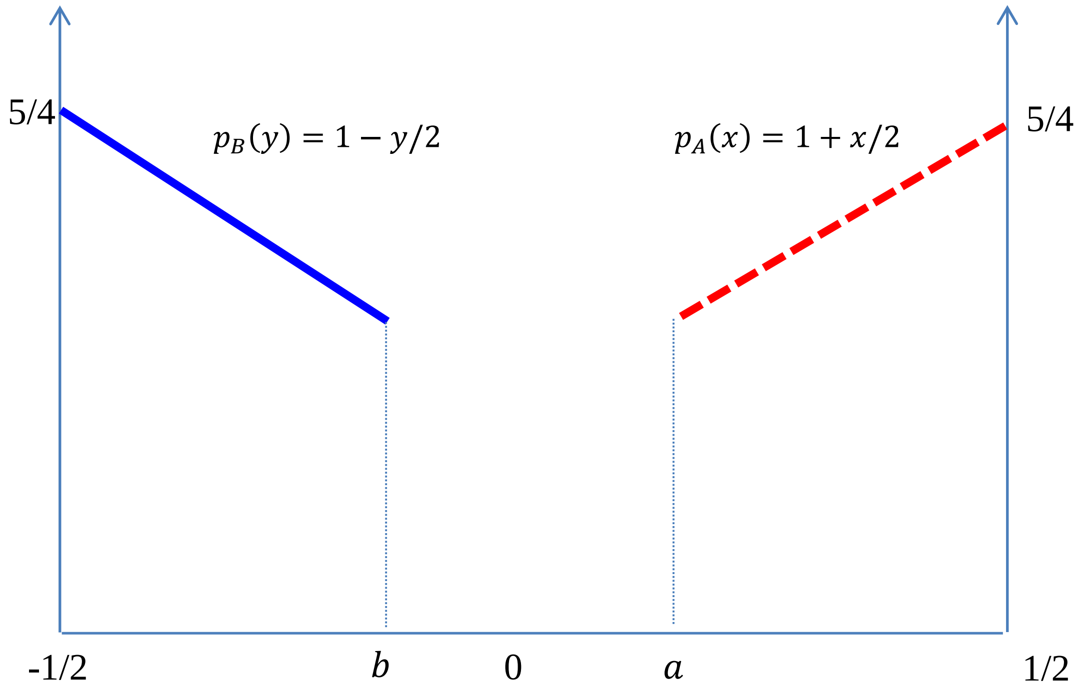

Figure 7: PPD equilibrium without full market coverage

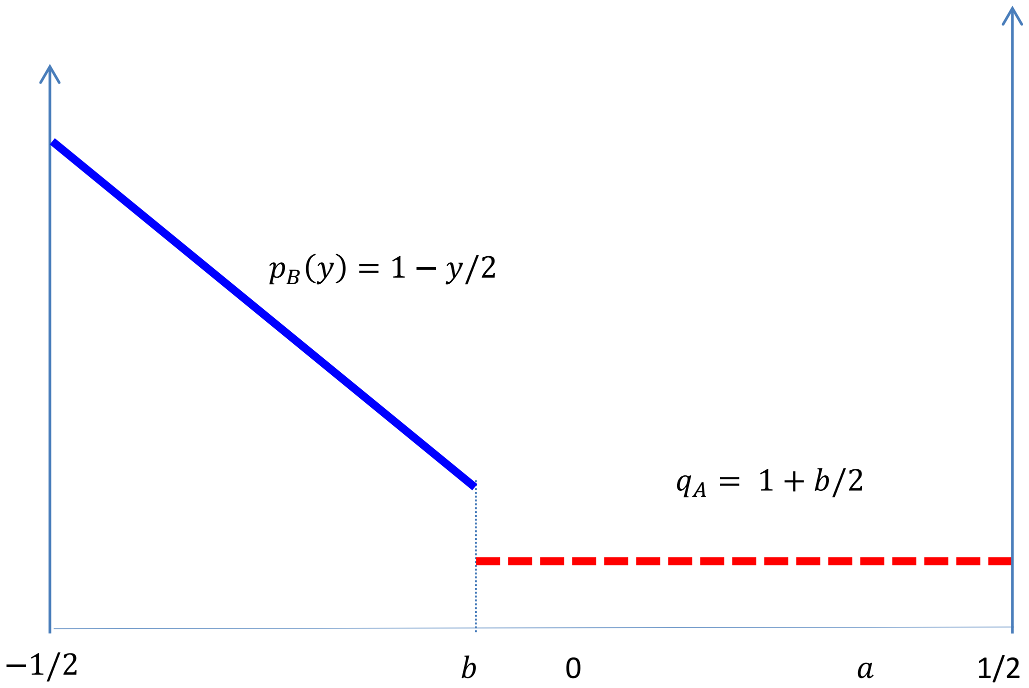

Figure 8: Equilibrium with PPD and a uniform price

Table 1: Savings for New Customers

Category	New Customer Savings (annualised, USD)
Cell phone	$360
Bank or credit card fees	$230
TV/Broadband/Homephone	$410
All	$375

Source: Offers collected from Whistleout.com, choice.com, whirlpool.net.au, comparethemarket.com during May – June, 2017. HTML pages and aggregate data were cleaned by web scraping methods using grep.

Table 2: Sample Source and Sentiment Operators in Google Search

Sample	Google search operator	Unqualified sample size
Reddit	https://www.reddit.com	205,000,000
All Forums	Inurl:forum	20,900,000,000
Sentiment	Words and operators	
Successfully saved money	(Saved OR reduced OR discounted OR gave me OR discounted OR rebate) AND (quantity OR percent OR free)	
Mentioned existing	(time OR duration) OR loyal OR long-time OR long+time OR existing+customer OR loyal+customer OR current+customer OR existing+user OR loyal+user OR current+user	

Table 3: Evidence of Consumer Sophistication from Internet Forums

Category	# of results	# of results with successful savings	Average saved (USD)	Source
Cable	143,094	102,056	$187.50	Reddit
Credit card	74,723	62,376	$119.00	Reddit
Internet	223,017	176,085	$138.00	Reddit
Cell phone	53,887	51,114	$262.33	Reddit
Cable	1,210,084	705,002	$231.21	All forums
Credit card	915,047	632,067	$85.35	All forums
Internet	1,420,083	476,021	$202.25	All forums
Cell phone	899,054	361,043	$147.50	All forums