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Pricing with Cookies: Behavior-Based Price Discrimination and Spatial Competition*

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Abstract:
We study a two-period model of spatial competition with two symmetric firms where firms learn customers' preferences from the first-period purchase, which they use for personalized pricing in the second period. With product choice exogenously fixed with maximal differentiation, we show that there exist two asymmetric equilibria and customer switching is only from one firm to the other unless firms discount future too much. Firms are worse off with such personalized pricing than when they use pricing at higher levels of aggregation. When product choice is also made optimally, there continue to exist two asymmetric equilibria given sufficiently small discounting, none of which features maximal differentiation. More customer information hurts firms, and more so when they make both product choice and pricing decisions.

Keywords: Spatial competition, behavior-based price discrimination, personalized pricing, endogenous product choice
JEL Classification Number: D43, L13

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1 Introduction

Firms can tightly target pricing, marketing and even product characteristics to individual consumers, using information technology and large datasets of customer-level information dubbed big data. For example, tracking tools such as cookies, web beacons, or Etags, allow individual sellers or data brokers to record consumers' browsing histories on the Internet.[1] This information can be used by websites to target their offerings based on consumers' purchasing history, location, referring sites or even computer operating systems ('On Orbitz, Mac users steered to pricier hotels', *The Wall Street Journal*, August 23, 2012). Driven by the growth of mobile devices and use of mobile apps, traditional retailers, such as supermarkets, use loyalty schemes to gather personal and shopping data and offer consumers personalized discounts on-line or through mobile apps ('Individualized coupons aid price discrimination', *Forbes*, August 21, 2012; 'Supermarkets offer personalized pricing', *Bloomberg Business*, November 16, 2013).

The ability of firms to use big data to price discriminate and raise profits has spurred active research in marketing and economics. For example, using 2006 data on customers' web-browsing behavior, Schiller (2014) estimates that Netflix could have raised its profit by 0.8% if it had used prices based on customer demographics alone but by as much as 12.2% if it had used personalized prices based on web-browsing explanatory variables. On the other hand, it has also led to concerns about privacy and equity. Hannak et al. (2014, p. 305) argue that personalization on e-commerce sites may also be used to the user's disadvantage by manipulating the products shown (price steering) or by customizing the prices of products (price discrimination). The US Council of Economic Advisers (2015, p. 17) note a similar concern that "[s]ome consumer advocates suggest that we should ... limit the use of personalized pricing to offline settings or require its disclosure to buyers". In addition, fairness concerns present challenges to firms in how best to utilize big data while not triggering customer backlash ('Different customers, different prices, thanks to big data', *Forbes*, April 14, 2014).[2]

Existing research shows that access to consumer information can intensify competition and hurt firm profitability. For example, Thisse and Vives (1988, p. 124) note, because of their access to consumer information, "firms may get trapped into a Prisoner's Dilemma-type situation and end up with lower profits due to the intense competition

[1] A cookie is a file placed on a browser's computer by a website to allow the website owner to track the browser's interactions with the site. See, for example, 'Little brother', *The Economist* (September 13, 2014) or 'How companies learn your secrets', *The New York Times* (February 16, 2012). Bergemann and Bonatti (2015), and Braulin and Valletti (2016) provide a formal analysis of data brokers.

[2] A growing body of academic research incorporates fairness concerns in firms' pricing policies. For example, Richards et al. (2016) argue that allowing customer participation in the price-formation process may be one way to make price discrimination sustainable. On the other hand, Li and Jain (2016) show fairness concerns can benefit firms by softening competition.

unleashed". But this result depends on the static analysis in which consumer information is exogenously given.[3] Subsequent studies consider dynamic games in which firms gather consumer information through the first-period purchase, which they use for price discrimination in the future, hence called behavior-based price discrimination. General findings from these studies continue to confirm the insight from the static analysis that customer information can intensify competition, relative to the non-discrimination benchmark.[4]

A key assumption in the above studies is that competing firms are endowed with or acquire *symmetric* information, albeit at different levels of aggregation. In Thisse and Vives (1988), firms are endowed with perfect information about all consumers, which allows them to exercise personalized pricing. In Fudenberg and Tirole (2000), firms learn where all consumers purchased in the first period, but not any finer details about their characteristics. As a result, they exercise third-degree price discrimination using symmetric information on market segmentation.[5] Information collection by firms, however, will often be asymmetric. For example, a cookie can provide highly personalized information about a consumer. But that information is only available to the firm that installs the cookie. Similarly, loyalty programs provide extensive histories about a customer's shopping preferences at a particular retailer. But this information is not available to other retailers. While 'minimal' information about a particular customer may be available to a firm that fails to sell to that customer, the successful seller may gather significantly more information about the same customer.

In this paper, we study behavior-based price discrimination with asymmetric information acquisition and personalized prices using the two-firm/two-period framework of Fudenberg and Tirole (2000), allowing general discount factors for firms (δ_f) and consumers (δ_c). A firm that sells to a particular customer in the first period learns the exact 'location' of that customer. However, the other firm only knows that the customer chose the rival seller, which creates asymmetric information whereby a firm knows more about its customers than its rival. Using two versions of our model (exogenous product choice and endogenous product choice), we show that this asymmetric information acquisition leads to multiple asymmetric equilibria if firms do not discount future completely, i.e., $\delta_f > 0$. This is in sharp contrast to afore-mentioned studies that all obtain a unique symmetric equilibrium. Our equilibria collapse to a unique symmetric equilibrium if and

[3]Other studies show that introducing some heterogeneity among firms (Shaffer and Zhang, 2002; Matsumura and Matsushima, 2015) or quality choice by firms (Choudhary et al., 2005; Ghose and Huang, 2009) can resolve the prisoner's dilemma. But they also assume exogenously given information.

[4]See for example Villas-Boas (1999), Fudenberg and Tirole (2000), and Esteves (2010). Chen (2005) and Fudenberg and Villas-Boas (2007) survey the early literature.

[5]Zhang (2011, p. 173) refers to this as the "minimum information assumption about consumer purchase histories; a consumer's product choice reveals her relative preference between the two firms but not the precise strength of her preference".

only if $\delta_f = 0$, i.e., in the absence of any dynamic consideration.

With product choice exogenously fixed at maximal differentiation, there are two asymmetric equilibria, each favoring a firm with more aggressive pricing in the first period.[6] There is one-way customer switching whereby the firm with a larger market share loses some customers to its rival in the second period but still retains a larger market share.[7] The reason for the asymmetric equilibria is the asymmetric information created in the first period and the use of personalized pricing in the second period. Asymmetric information is clearly irrelevant if firms cannot use personalized pricing. When firms use personalized pricing in the second period, the information advantage over own customers allows firms to protect their turf better, which intensifies competition for market share in the first period. But the marginal change in the second-period profits with respect to the change in market share is asymmetric around the equal market share of 1/2 each: starting from equal market share, the second-period gain to a more aggressive firm outweighs the second-period loss to its less aggressive rival. An increase in market share allows the more aggressive firm to charge higher personalized prices to all its loyal customers. Thus the more aggressive firm benefits from both a larger market share and higher personalized prices. Although the less aggressive firm loses its market share, it can charge a higher poaching price, which compensates for the loss in market share. Consequently one firm's incentive to undercut its rival is stronger than the rival's incentive to match. But both firms are worse off in both equilibria compared to when they use third-degree price discrimination. Moreover they are worse off *in each period* compared to when they do not price-discriminate.

We then examine the case where customer recognition is imperfect in that the firm learns its customer's exact location only with some probability, which depends on the firm's investment in consumer-level information technology. This extension incorporates our basic model as one polar case where both probabilities are equal to one and the minimum symmetric information case as another polar case where both probabilities are equal to zero. We find that there continue to exist multiple equilibria if the two firms have sufficiently similar information technologies in the sense that the probabilities are not very much apart from each other. Otherwise the equilibrium is unique with asymmetric market shares in the first period, where the firm with superior information is better off than its rival. But both firms are worse off than when both probabilities are equal to

[6]Esteves (2010, Section 6) notes that an asymmetric equilibrium may arise in her model where one firm gains the entire market in the first period. In this case, no information is created through first period sales. In contrast, in our model, both sellers are always active in the market and information is created. Unlike Esteves (2010), it is the asymmetry of the information that drives asymmetric behavior in the first period.

[7]This is different from the case with the minimum *symmetric* information assumption, as in Fudenberg and Tirole (2000) and Zhang (2011) where two-way poaching occurs in the second period with each firm stealing some of its rival's first-period customers.

zero. This is because the possibility of using personalized pricing in the second period always intensifies the first-period competition. In fact the cost of intensified competition in the first period negates any gains from consumer information for each firm, regardless of the other firm's information. This implies that, if firms have choice over the investment in consumer-level information technology, then it is a dominant strategy for each firm to choose zero investment.[8] Of course this is subject to the caveat that the sole use of information is for pricing in this case. Customer information has much wider use in practice; for example, it is crucial to effective customer relationship management that goes beyond pricing.

When firms make product choice endogenously in the first period, we provide several results on equilibrium characterization for general values of (δ_c, δ_f), while focusing on the case $\delta_c = 0$ and $\delta_f = 1$ for more detailed discussions.[9] First, if δ_f is sufficiently small, then maximal differentiation obtains in equilibrium regardless of δ_c, followed by the multiple pricing equilibria described above. The reason is as follows. As products are less differentiated, price competition becomes more intense in the first period, but the perceived benefits of using personalized prices in the second period may outweigh the costs of more intense competition. Small δ_f means such future benefits are largely discounted, hence maximal differentiation. Second, if δ_f is above a certain threshold, then there exist two asymmetric equilibria, one being a mirror image of the other, where one firm chooses extreme differentiation but the other firm chooses a more aggressive product choice, which reduces differentiation. This includes the case we focus on, namely $\delta_c = 0$ and $\delta_f = 1$.

The reason for the equilibria without maximal differentiation is as follows. The benefits of maximal differentiation are mainly through reduced competition in the first period. But the benefits of aggressive product choice come largely in the second period: using its aggressive positioning, the firm enjoys a more loyal customer base, from whom it can extract surplus while poaching rival's customers. Thus δ_f needs to be sufficiently large for the equilibria without maximal differentiation. Nonetheless the aggressive positioning intensifies competition in the first period sufficiently, leaving both firms worse off than when product choice is at maximal differentiation. But the firms cannot coordinate onto the equilibrium with maximal differentiation because of the multiplicity of equilibria: the firm that deviates to the aggressive product choice gains at the expense of the other that continues to choose extreme differentiation. In this sense, making an aggressive product choice is loss-minimizing in that it helps the firm to avoid the worst outcome: the firm

[8] Chen and Iyer (2002) ask a similar question but obtain a different answer. The difference between their results and ours is discussed in Section 3.5.

[9] Finding closed-form solutions for equilibrium for general values of (δ_c, δ_f) involves solving simultaneous quadratic equations, which is generally not possible.

is better off in the best equilibrium under endogenous product choice where it makes an aggressive choice than in the worst equilibrium under exogenous product choice where it concedes a larger market share to its rival.

Zhang (2011, Section 5.1) considers a situation with the same information assumptions as in our model. But she allows costless personalization of products as well as prices. This means that once one firm has customer-specific information, the other firm cannot effectively serve that customer. The result is a symmetric equilibrium with highly aggressive pricing in the first period and perfect price/product discrimination in the second period. Thus her results differ substantially from our own. Further, while her assumption of costless product personalization may be relevant in some settings, in many situations it is reasonable to expect some limits to product variety. In that sense, our analysis is complementary to hers. Our model takes the opposite product assumption to hers (each firm only chooses one product), leading to very different but, in our opinion, widely applicable results.

Our paper significantly extends the existing literature on behavior-based price discrimination in a number of ways. First, the asymmetric information structure we analyze captures key features of actual information gathering by firms. But it has not been widely considered in the literature. As a result, and in contrast to the existing literature, our analysis shows that multiple asymmetric equilibria can arise even when there are two *ex ante* symmetric firms. This asymmetry feeds into all elements of the competitive process: product choice where relevant; pricing in both the first and second periods of the game; and customer poaching in the second period. Second, we allow general discount factors for consumers and firms and show that the two discount factors have asymmetric effects on equilibria. Third, we explicitly compare our results with those from the case with third-degree price discrimination and show firms are increasingly worse off as their pricing strategies change from uniform, to third-degree discrimination, to personalized pricing. Finally, we incorporate the firm's product choice decision into the existing studies on behavior-based price discrimination that mostly start from given product choice, and demonstrate that, as firms have more choices, the situation worsens even further. Our findings suggest various strategies firms may need to consider to make their dynamic pricing strategies more viable and sustainable.

The rest of this paper proceeds as follows. Section 2 describes the model. Section 3 analyzes the case with exogenously fixed product choice while Section 4 studies the case when product choice is also endogenous. Section 5 offers some discussions on the implications for management. Section 6 concludes the paper. Appendix A contains proofs not provided in the main text while Appendix B presents additional discussions on the results from Section 4.

2 The Model

Consider a Hotelling linear city where consumers are distributed uniformly over $[0, 1]$. Consumer located at x will be simply called consumer x. Each consumer buys one unit of good in each period for two periods, and derives utility v from each unit. We assume v is sufficiently large so that the entire market is covered in equilibrium. There are two firms indexed by $i = A, B$. Both firms have constant marginal cost of production, which is normalized to zero. Consumers have quadratic transportation costs.[10] Thus if firm i is located at l and sets a price $P_i(x)$ for consumer x, then consumer x gets a payoff of $v - P_i(x) - t(x - l)^2$ if she buys from firm i.

There are two periods in the game, indexed by $\tau = 1, 2$. The $\tau = 1$ is the standard Hotelling model: firms simultaneously choose locations which are fixed over two periods, after which they compete in price. The prices set by each firm in $\tau = 1$ are non-discriminatory: each firm sets a single price and sells to all consumers who wish to purchase at that price. Consumers observe these prices and choose to buy from one firm. In $\tau = 1$, each firm also uses 'cookies' to track consumers. Let $\mathcal{A}$ be the set of consumers that choose firm A in $\tau = 1$ and $\mathcal{B}$ be the set of consumers that choose firm B in $\tau = 1$. By assumption, all consumers are members of only one of these sets. At the end of $\tau = 1$, firm A knows, for each consumer x: (i) whether $x \in \mathcal{A}$ or $x \in \mathcal{B}$; and (ii) if $x \in \mathcal{A}$, then the location x. Similarly, at the end of $\tau = 1$, firm B knows, for each consumer y: (i) whether $y \in \mathcal{A}$ or $y \in \mathcal{B}$; and (ii) if $y \in \mathcal{B}$, then the location y.

In $\tau = 2$, firms chooses two types of prices since they can now set prices to discriminate between consumers based on the information acquired in $\tau = 1$. Thus firm A can set individual prices $P_A(x)$ to each consumer $x \in \mathcal{A}$, to be called personalized pricing. For its rival's $\tau = 1$ customers, firm A chooses a uniform 'poaching' price $P_A(\mathcal{B})$. Similarly, firm B chooses individual prices $P_B(y)$ for each consumer $y \in \mathcal{B}$ and a uniform price, $P_B(\mathcal{A})$ for the set of consumers in $\mathcal{A}$. As is standard in the literature (Thisse and Vives, 1988; Choudhary et al., 2005), we assume firms simultaneously choose uniform prices first, after which they choose personalized prices. This timing allows us to narrow down equilibrium prices to those that are subgame-perfect. In addition, this two-stage structure reflects a commonly held view that a firm's choice of 'regular price' is a higher-level managerial decision and is relatively slower to adjust in practice than a firm's choice of personalized prices.

In $\tau = 2$, consumers make their purchase decisions after observing all the relevant prices. Each consumer observes a personalized price offered to her, and the two poaching

[10] We assume quadratic transportation costs so that the one period game with endogenous product choice has a unique pure strategy equilibrium where firms choose maximal differentiation (Anderson et al., 1992). This makes our results easily comparable to other standard results.

prices. We follow the standard literature and assume consumers care about only the price they pay. This assumption rules out the possibilities where consumers' purchase decisions can also depend on behavioral elements such as fairness concerns or inequity aversion. In reality, they are important aspects of consumers' purchase decisions as shown by plenty of experimental and anecdotal evidence.[11] We abstract away from these issues mainly because our aim is to clearly understand how changes in informational assumptions lead to different equilibria in otherwise the same model such as the one in Fudenberg and Tirole (2000). Adding behavioral elements to consumer choice is likely to change our results substantially. In the concluding section, we offer some discussions on this.

In making first-period decisions, firms discount the $\tau = 2$ profit by $\delta_f \in [0, 1]$ and consumers discount their $\tau = 2$ surplus by $\delta_c \in [0, 1]$. Thus if firm i's profit in period τ is π_i^τ, then its $\tau = 1$ problem is to maximize $\Pi_i \equiv \pi_i^1 + \delta_f \pi_i^2$. Likewise if consumer x's surplus in period τ is u_x^τ, then her optimal decision in $\tau = 1$ is to maximize $u_x^1 + \delta_c u_x^2$. If $\delta_c = 0$, then consumers are completely myopic in the sense that their $\tau = 1$ purchase decisions are based only on their $\tau = 1$ surplus. As δ_c increases, consumers become more forward-looking in that they discount future less in their $\tau = 1$ purchase decisions. We consider the general case when consumers and firms may use different discount factors. As we will see, these discount factors have asymmetric effects on equilibria. In particular, the existence of asymmetric equilibria hinges crucially on the forward-looking behavior by firms ($\delta_f > 0$). If $\delta_f = 0$, then the equilibrium is symmetric regardless of δ_c.

2.1 Benchmark results

For future reference, we discuss below the results from the three benchmark models adapted to our framework. The first two benchmark models are essentially static, bypassing the issue of where customer information comes from. The third model is dynamic where firms gather customer information in $\tau = 1$, which they use for pricing in $\tau = 2$.

First, in the standard Hotelling model with quadratic transportation cost, one firm chooses location 0 and serves $[0, 1/2]$ while the other chooses location 1 and serves $[1/2, 1]$. Firms charge the same price t, earn profit $t/2$, and the average distance traveled by a consumer is $1/4$. Since this outcome is replicated in each period, each firm earns a total discounted profit equal to $\Pi_H \equiv (1 + \delta_f)t/2$.

The second benchmark is Thisse and Vives (1988) where both firms employ personalized pricing for all potential customers. Since firms' locations are exogenously fixed in their model, we can assume that firm A is at 0 and firm B at 1. Then in equilibrium,

[11] For example, see the references in Weisstein et al. (2013), Li and Jain (2016), and Richards et al. (2016). Li and Jain provide a formal model of behavior-based pricing with third-degree price discrimination where they show consumers' fairness concerns can soften competition. Weisstein et al. (2013), and Richards et al. (2016) provide experimental evidence on how to alleviate such fairness concerns.

firm A serves $[0, 1/2]$ with prices $P_A(x) = (1 - 2x)t$ for all $x < 1/2$ and firm B serves $[1/2, 1]$ with prices $P_B(y) = (2y - 1)t$ for all $y > 1/2$. The average distance traveled by a consumer is again $1/4$. Since each firm earns $t/4$ each period, the total discounted profit over two periods is $\Pi_{TV} \equiv (1 + \delta_f)t/4$.

Third, in Fudenberg and Tirole (2000), firms exercise third-degree price discrimination in $\tau = 2$ based on customers' purchase behavior in $\tau = 1$. They assume the same discount factor for firms and consumers: $\delta_c = \delta_f = \delta$. With firm A located at 0 and firm B at 1, the equilibrium in $\tau = 1$ is symmetric with price equal to $(3 + \delta)t/3$, and firm A's market share is $[0, 1/2]$, hence each firm's $\tau = 1$ profit is $(3 + \delta)t/6$. In $\tau = 2$, both firms charge $2t/3$ to their $\tau = 1$ customers and $t/3$ to their rival's customers and, as a result, consumers in $[1/3, 1/2]$ switch from firm A to firm B, and those in $[1/2, 2/3]$ switch from firm B to firm A. The $\tau = 2$ profit for each firm is then $5t/18$. The average distance traveled by consumers in $\tau = 2$ is $11/36 > 1/4$, hence social welfare is lower in $\tau = 2$ compared to the two previous cases because of inefficient customer switching. The total discounted profit for each firm in this case is $\Pi_{FT} \equiv (9 + 8\delta)t/18$.

In sum, all three models lead to a symmetric equilibrium in which the two firms have the same market share each period. As can be checked easily, the total discounted profits are the smallest with personalized pricing and the largest with uniform pricing: $\Pi_H \geq \Pi_{FT} \geq \Pi_{TV}$. Thus the more customer information firms use to devise finer pricing strategies, the more intense competition becomes, which hurts firm profitability.

3 Exogenously Fixed Locations

We start by analyzing the case in which the firms' locations are fixed at 0 and 1, i.e., maximal differentiation. Without loss of generality, suppose firm A is located at 0. Analyzing this case helps us understand the second-period pricing game more clearly than when the location choice is also endogenous. This will in turn facilitate solving the whole game with endogenous location choice. Note that the equilibrium in the standard Hotelling model with quadratic transportation cost also has maximal differentiation.

3.1 Second Period

Let us begin with the pricing game in $\tau = 2$. Since a standard revealed preference argument shows that each of $\mathcal{A}$ and $\mathcal{B}$ is a connected interval, we can define a unique value $z \in [0, 1]$ such that $x \in \mathcal{A}$ iff $x \leq z$. Then the equilibrium prices in $\tau = 2$ can be derived as follows.

Suppose $z \leq 1/2$. First, consider the segment $\mathcal{A} = [0, z]$. Since $z \leq 1/2$, firm A has a location advantage over firm B on this segment. Moreover firm A can use personalized prices $P_A(x)$ while firm B can use only a uniform price $P_B(\mathcal{A})$. Consumer x chooses firm A if $P_A(x) + x^2 t \leq P_B(\mathcal{A}) + (1-x)^2 t$ or $P_A(x) \leq P_B(\mathcal{A}) + (1-2x)t$. Thus the Bertrand competition on this segment leads to $P_B(\mathcal{A}) = 0$ and $P_A(x) = (1-2x)t$. Note that $P_A(x) \geq 0$ since $x \leq z \leq 1/2$ on $\mathcal{A}$.

Next, consider the segment $\mathcal{B} = [z, 1]$ for which firm A chooses a uniform price $P_A(\mathcal{B})$ while firm B uses personalized prices $P_B(y)$. Consumer y will choose firm B so long as $P_A(\mathcal{B}) + y^2 t > P_B(y) + (1-y)^2 t$ or $P_B(y) < P_A(\mathcal{B}) + (2y-1)t$. However, firm B will not want to sell to consumer y if $P_B(y) < 0$. Thus for any $P_A(\mathcal{B})$, we can define a critical value of y, denoted by $\tilde{y}$ such that $P_A(\mathcal{B}) = (1-2\tilde{y})t$ or $\tilde{y} = (t - P_A(\mathcal{B}))/2t$. Then for any price $P_A(\mathcal{B})$, consumer $y \in [z, \tilde{y}]$ chooses firm A (if $\tilde{y} > z$). On the other hand, firm B can choose nonnegative prices to serve all consumers $y \in [\tilde{y}, 1]$.

For the segment $[z, \tilde{y}]$, profit for firm A is $\int_z^{\tilde{y}} P_A(\mathcal{B}) dy = (\tilde{y} - z) P_A(\mathcal{B})$. Substituting for $\tilde{y}$ and maximizing, the optimal value of $P_A(\mathcal{B})$ for firm A is $P_A(\mathcal{B}) = (1-2z)t/2$, which is the price charged to all the consumers in $\mathcal{B}$ since firm A cannot price-discriminate these consumers. Given $P_A(\mathcal{B}) = (1-2z)t/2$, we have $\tilde{y} = (1+2z)/4$. It is easy to verify $z \leq \tilde{y} \leq 1/2$. Given $P_A(\mathcal{B})$ derived above, firm B sets personalized prices for the segment $[\tilde{y}, 1]$. They are given by $P_B(y) = (2y-1)t + P_A(\mathcal{B}) = (4y - 2z - 1)t/2$.

For the case $z > 1/2$, the same argument can be applied. We summarize these results in the following lemma.

Lemma 1 *(i) If $z \leq 1/2$, then the unique equilibrium in $\tau = 2$ is given by*

$$P_A(x) = \begin{cases} (1-2x)t & \text{if } x \in [0, z], \\ \dfrac{(1-2z)t}{2} & \text{if } x \in [z, 1], \end{cases}$$

$$P_B(y) = \begin{cases} 0 & \text{if } y \in [0, (1+2z)/4], \\ \dfrac{(4y-2z-1)t}{2} & \text{if } y \in [(1+2z)/4, 1]. \end{cases}$$

(ii) If $z \geq 1/2$, then the unique equilibrium in $\tau = 2$ is given by

$$P_A(x) = \begin{cases} \dfrac{(2z-4x+1)t}{2} & \text{if } x \in [0, (1+2z)/4], \\ 0 & \text{if } x \in [(1+2z)/4, 1], \end{cases}$$

$$P_B(y) = \begin{cases} \dfrac{(2z-1)t}{2} & \text{if } y \in [0, z], \\ (2y-1)t & \text{if } y \in [z, 1]. \end{cases}$$

Figure 1 describes the equilibrium with $z \leq 1/2$ in Lemma 1. The thick solid lines represent firm A's prices while the thick dashed lines show firm B's prices. Firm A serves its $\tau = 1$ customers with personalized prices that decrease from t to $(1-2z)t$ on $[0, z]$, and charges a uniform price $(1-2z)t/2$ to all firm B's $\tau = 1$ customers, poaching those on $[z, \tilde{y}]$ where $\tilde{y} = (1+2z)/4$. Firm B charges a uniform price 0 to all customers on $[0, z]$, $P_B(y) = 0$ to customers on $[z, \tilde{y}]$, and personalized prices that increase from 0 to $(3-2z)t/2$ on $[\tilde{y}, 1]$. Figure 1 also indicates how consumers choose firms over the two periods: those on $[0, z]$ choose firm A in both periods; those on $[z, \tilde{y}]$ choose firm B in $\tau = 1$ but switch to firm A in $\tau = 2$; those on $[\tilde{y}, 1]$ stay with firm B in both periods.

— **Figure 1 goes about here.** —

Based on the above, we can calculate the $\tau = 2$ profit for each firm. Consider first the case $z \leq 1/2$. Then consumers in $[0, z]$ continue to purchase from firm A, consumers in $[z, (1+2z)/4]$ switch from firm B to firm A, and consumers in $[(1+2z)/4, 1]$ continue to purchase from firm B. Thus firm A's $\tau = 2$ profit from its repeat customers is $\int_0^z (1-2x)t dx = (1-z)zt$ and its profit from switching customers is $\frac{(1-2z)t}{2}\left(\frac{1+2z}{4} - z\right) = (1-2z)^2 t/8$. So if $z \leq 1/2$, firm A will make $\tau = 2$ profit equal to:

$$\pi_A^2 = (1-z)zt + \frac{(1-2z)^2 t}{8} = \frac{(1+4z-4z^2)t}{8}.$$

If $z > 1/2$, then consumers in $[(1+2z)/4, z]$ switch from firm A to firm B. Thus firm A serves only those in $[0, (1+2z)/4]$ by charging personalized prices $P_A(x) = (2z-4x+1)t/2$. So it makes profit:

$$\pi_A^2 = \int_0^{\frac{1+2z}{4}} \frac{(2z-4x+1)t}{2} dx = \frac{(1+2z)^2 t}{16}.$$

Due to symmetry, firm B's profit is the same as firm A's profit in the relevant region when z is replaced by $1-z$. Summarizing the above, we have

Lemma 2 *Equilibrium profits in $\tau = 2$ are given by*

$$\pi_A^2 = \begin{cases} \dfrac{(1+4z-4z^2)t}{8} & \text{if } z \leq 1/2, \\ \dfrac{(1+2z)^2 t}{16} & \text{if } z \geq 1/2, \end{cases}$$

$$\pi_B^2 = \begin{cases} \dfrac{(3-2z)^2 t}{16} & \text{if } z \leq 1/2, \\ \dfrac{(1+4z-4z^2)t}{8} & \text{if } z \geq 1/2. \end{cases}$$

It is easy to verify that both profit functions are continuous, firm A's profit increases in z, and firm B's profit decreases in z. At $z = 1/2$, the two firms' profits are the same and are equal to $\frac{t}{4}$. Thus firm A has incentives to increase z and firm B has incentives to decrease z, which would intensify price competition in the first period. However, a change in $\tau = 1$ market share affects each firm's $\tau = 2$ profit in an asymmetric way. The reason is as follows. As can be checked from Lemma 1(ii), an increase in z above $1/2$ increases firm A's personalized prices and firm B's poaching price. Thus when z increases, firm A benefits from both higher personalized prices and more loyal customers. When z increases, firm B loses its loyal customer base, but it can charge a higher poaching price, which compensates for the loss in loyal customer base. Consequently, as z increases from $1/2$, firm A's $\tau = 2$ profit increases more than (the absolute value in) the decrease in firm B's $\tau = 2$ profit: from Lemma 2, we have $d\pi_A^2/dz = (1+2z)t/4 > |d\pi_B^2/dz| = (2z-1)t/2$ for all $z \geq 1/2$. In this case, firm A's incentives to cut $\tau = 1$ price are stronger than firm B's incentives to undercut firm A. Similarly, if $z < 1/2$, firm B has stronger incentives to cut $\tau = 1$ price. As we will see below, this asymmetry leads to asymmetric equilibria in the $\tau = 1$ price game.

3.2 First period

We start with the location of marginal consumer z. Consider first the equilibrium with $z \leq 1/2$. Based on our discussion in the previous section, the marginal consumer z is indifferent between choosing firm A in both periods, and choosing firm B in $\tau = 1$ while switching to firm A in $\tau = 2$. Thus we have

$$P_A^1 + z^2 t + \delta_c \left(P_A(z) + z^2 t\right) = P_B^1 + (1-z)^2 t + \delta_c \left(P_A(\mathcal{B}) + z^2 t\right).$$

Substituting $P_A(z) = (1-2z)t$ and $P_A(\mathcal{B}) = (1-2z)t/2$ in Lemma 1 into the above, we obtain

$$z = \frac{(2-\delta_c)t - 2(P_A^1 - P_B^1)}{2(2-\delta_c)t},$$

and $z \leq 1/2$ if and only if $P_A^1 \geq P_B^1$. Similarly, in the equilibrium with $z \geq 1/2$, the marginal consumer z is indifferent between choosing firm B in both periods, and choosing firm A in $\tau = 1$ while switching to firm B in $\tau = 2$. Proceeding as before, we again obtain the same z.

In $\tau = 1$, firm A's profit is $\pi_A^1 = P_A^1 z$ while firm B's profit is $\pi_B^1 = P_B^1(1-z)$ where P_i^1 is firm i's $\tau = 1$ price, $i = A, B$. Firm i chooses P_i^1 to maximize its total discounted profit $\Pi_i = \pi_i^1 + \delta_f \pi_i^2$. Based on these, the equilibria of the $\tau = 1$ price game can de derived as follows.

Lemma 3 *The price game in $\tau = 1$ has two equilibria:*

(i) $P_A^1 = \dfrac{(4-2\delta_c+\delta_f)(6-3\delta_c-2\delta_f)t}{2(12-6\delta_c+\delta_f)}$, $P_B^1 = \dfrac{\left(6(2-\delta_c)^2 - 3(2-\delta_c)\delta_f - 2\delta_f^2\right)t}{2(12-6\delta_c+\delta_f)}$ *with* $z = \dfrac{12-6\delta_c-\delta_f}{2(12-6\delta_c+\delta_f)}$ $(\leq 1/2)$*;*

(ii) $P_A^1 = \frac{\left(6(2-\delta_c)^2 - 3(2-\delta_c)\delta_f - 2\delta_f^2\right)t}{2(12-6\delta_c+\delta_f)}$, $P_B^1 = \frac{(4-2\delta_c+\delta_f)(6-3\delta_c-2\delta_f)t}{2(12-6\delta_c+\delta_f)}$ *with* $z = \frac{3(4-2\delta_c+\delta_f)}{2(12-6\delta_c+\delta_f)}$ $(\geq 1/2)$.

Proof: See Appendix A.

It is worth noting that asymmetric equilibria obtain even though the two firms are symmetric and their $\tau = 1$ locations are fixed exogenously at a maximal distance. This is in contrast to the three benchmark results discussed in the previous section. In the static case, both the Hotelling and the Thisse and Vives (1988) outcomes are symmetric. In the dynamic model of Fudenberg and Tirole (2000) where firms use third-degree price discrimination in $\tau = 2$, the $\tau = 1$ equilibrium is also unique with equal market share for each firm.

The reason for the asymmetric equilibria in our case is the asymmetric information created at the end of $\tau = 1$ and the use of personalized pricing in $\tau = 2$. Firms start with the same prior information about customers in $\tau = 1$ but they learn more about their own customers, precise locations in our case, at the end of $\tau = 1$. This information advantage over own customers allows firms to employ personalized pricing in $\tau = 2$. Personalized pricing enables firms to tailor their pricing policy for their own customers, allowing them to protect their market in $\tau = 2$ more effectively than when they use a uniform price. As a result, each firm's $\tau = 2$ profit increases if it has a larger market share in $\tau = 1$.[12] As shown previously, however, the effect of a change in market share on firms' $\tau = 2$ profits is asymmetric, which breaks down the symmetric equilibrium. Starting from the Hotelling price t and $z = 1/2$, a small increase in z increases firm A's $\tau = 2$ profit more than it decreases firm B's $\tau = 2$ profit. Likewise, a small decrease in z decreases firm A's $\tau = 2$ profit less than it increases firm B's $\tau = 2$ profit. Such asymmetric effects of z on $\tau = 2$ profits feed back into firms' pricing decisions in $\tau = 1$, rendering the symmetric outcome untenable.

To appreciate the asymmetric incentives better, we draw both reaction functions in Figure 2 where, for simplicity, we set $\delta_c = 0$ and $\delta_f = 1$. In this case, one can check that the two equilibria in Lemma 3 become (i) $P_A^1 = 10t/13$, $P_B^1 = 8t/13$, $z = 11/26$; (ii) $P_A^1 = 8t/13$, $P_B^1 = 10t/13$, $z = 15/26$. Suppose now firm B chooses $P_B^1 = t$. Then firm A's best response is to undercut it to $5t/7$. But firm B does not gain by further undercutting firm A: its best response is to lower its $\tau = 1$ price to $29t/35 > 5t/7t$. It is followed by further price cuts by both firms, each time firm B's price remaining higher

[12]When firms use third-degree price discrimination as in Fudenberg and Tirole (2000), each firm's equilibrium profit in $\tau = 2$ is independent of its market share in $\tau = 1$. This is discussed in detail in Section 3.4.

than firm A's. But continued price cuts are not in firm A's interest since an increase in its $\tau = 2$ profit is eventually offset by a decrease in its $\tau = 1$ profit. This leads to the equilibrium with $z = 15/26$. The adjustment process can be understood with help of Figure 2 where thick dashed lines (solid lines, resp.) represent firm A's (firm B's, resp.) reaction function and the two equilibria are indicated at the intersection of the two reactions functions.

— Figure 2 goes about here. —

Needless to say, the asymmetric incentives for price cuts matter only when firms care about the $\tau = 2$ profits in their pricing decisions in $\tau = 1$. Thus we expect the unique, symmetric equilibrium to re-emerge when $\delta_f = 0$. Indeed it is easy to check from Lemma 3 that there is a unique, symmetric equilibrium with $z = 1/2$ if and only if $\delta_f = 0$. In sum, the asymmetric equilibria are due to the presence of the second period when firms can exercise personalized pricing for their own customers. If firms do not exercise personalized pricing in $\tau = 2$ or if they are myopic ($\delta_f = 0$), then we have a unique, symmetric equilibrium.

3.3 Equilibria and discussions

We now describe the equilibria for the whole game. By substituting the value of z from the $\tau = 1$ equilibrium back into the $\tau = 2$ prices in Lemma 1, we have:

Proposition 1 *The equilibrium prices for the two periods are given by:*

(i) $P_A^1 = \dfrac{(4-2\delta_c+\delta_f)(6-3\delta_c-2\delta_f)t}{2(12-6\delta_c+\delta_f)}$, $P_B^1 = \dfrac{\left(6(2-\delta_c)^2 - 3(2-\delta_c)\delta_f - 2\delta_f^2\right)t}{2(12-6\delta_c+\delta_f)}$ *with* $z = \dfrac{12-6\delta_c-\delta_f}{2(12-6\delta_c+\delta_f)}$,

$$P_A(x) = \begin{cases} (1-2x)t & \text{if } x \in \left[0,\ \dfrac{12-6\delta_c-\delta_f}{2(12-6\delta_c+\delta_f)}\right], \\ \dfrac{\delta_f t}{12-6\delta_c+\delta_f} & \text{if } x \in \left[\dfrac{12-6\delta_c-\delta_f}{2(12-6\delta_c+\delta_f)},\ 1\right], \end{cases}$$

$$P_B(y) = \begin{cases} 0 & \text{if } y \in \left[0,\ \dfrac{3(2-\delta_c)}{12-6\delta_c+\delta_f}\right], \\ \left(2y - \dfrac{6(2-\delta_c)}{12-6\delta_c+\delta_f}\right)t & \text{if } y \in \left[\dfrac{3(2-\delta_c)}{12-6\delta_c+\delta_f},\ 1\right]. \end{cases}$$

(ii) $P_A^1 = \dfrac{\left(6(2-\delta_c)^2 - 3(2-\delta_c)\delta_f - 2\delta_f^2\right)t}{2(12-6\delta_c+\delta_f)}$, $P_B^1 = \dfrac{(4-2\delta_c+\delta_f)(6-3\delta_c-2\delta_f)t}{2(12-6\delta_c+\delta_f)}$ *with* $z = \dfrac{3(4-2\delta_c+\delta_f)}{2(12-6\delta_c+\delta_f)}$,

$$P_A(x) = \begin{cases} \left(\dfrac{2(6-3\delta_c+\delta_f)}{12-6\delta_c+\delta_f} - 2x\right)t & \textit{if} \;\; x \in \left[0, \; \dfrac{6-3\delta_c+\delta_f}{12-6\delta_c+\delta_f}\right], \\ 0 & \textit{if} \;\; x \in \left[\dfrac{6-3\delta_c+\delta_f}{12-6\delta_c+\delta_f}, \; 1\right], \end{cases}$$

$$P_B(y) = \begin{cases} \dfrac{\delta_f t}{12-6\delta_c+\delta_f} & \textit{if} \;\; y \in \left[0, \; \dfrac{3(4-2\delta_c+\delta_f)}{2(12-6\delta_c+\delta_f)}\right], \\ (2y-1)t & \textit{if} \;\; y \in \left[\dfrac{3(4-2\delta_c+\delta_f)}{2(12-6\delta_c+\delta_f)}, \; 1\right]. \end{cases}$$

To put Proposition 1 in perspectives, we offer comparative static analyses with respect to the discount factors. Without loss of generality, we consider the case $z \leq 1/2$. First, larger δ_c unambiguously hurts firms because, as consumers become more forward-looking, firms need to lower prices more to lock in consumers in $\tau = 1$. It is easy to check that both π_A^1 and π_A^2 decrease as δ_c increases, implying Π_A also decreases in δ_c. For firm B, however, π_B^1 decreases while π_B^2 increases as δ_c increases, the latter because the firm with a larger $\tau = 1$ market share has more advantage in $\tau = 2$ as consumers become more forward-looking. But the decrease in the $\tau = 1$ profit due to more intense competition dominates the increase in the $\tau = 2$ profit, implying Π_B also decreases in δ_c. On the other hand, changes in δ_f have opposing effects on per-period profit and total discounted profit. Larger δ_f reduces the $\tau = 1$ profit by intensifying competition for market share but the adverse effect is offset as the firm cares more about its $\tau = 2$ profit. Thus the total discounted profit is the largest when $\delta_c = 0$ and $\delta_f = 1$, and the smallest when $\delta_c = 1$ and $\delta_f = 0$.[13] The following can be shown easily from Proposition 1.

Corollary *(i) As δ_c increases, P_A^1, P_B^1, Π_A and Π_B decrease. (ii) As δ_f increases, P_A^1 and P_B^1 decrease, but Π_A and Π_B increase. (ii) The total discounted profit for each firm is the largest when $\delta_c = 0$ and $\delta_f = 1$, and the smallest when $\delta_c = 1$ and $\delta_f = 0$. (iii) The equilibria in $\tau = 1$ collapse to a unique, symmetric equilibrium with $z = 1/2$ if and only if $\delta_f = 0$.*

In what follows, we provide more detailed discussions of Proposition 1. Since a concrete example helps us understand the main ideas better, we focus on the case when $\delta_c = 0$ and $\delta_f = 1$. But qualitatively the same discussions apply for other values of δ_c

[13]Since there are two equilibria for each pair (δ_c, δ_f) except when $\delta_f = 0$, a more precise statement is as follows. Given any pair (δ_c, δ_f), consider an equilibrium in which one firm, say firm B, has larger total discounted profit: $\Pi_A < \Pi_B$, which implies that $z < 1/2$. Then among all such equilibria, Π_B is the largest when $\delta_c = 0$ and $\delta_f = 1$. The same is true when comparing Π_A for all (δ_c, δ_f). This is based on the comparison of total discounted profit. We note that un-discounted total profit for each firm, i.e., $\pi_i^1 + \pi_i^2$ (i = A, B), is the largest and equal to $3t/4$ when $\delta_c = \delta_f = 0$, in which case the equilibrium is symmetric with $\tau = 1$ equilibrium equal to the Hotelling outcome and the $\tau = 2$ equilibrium same as in Thisse and Vives (1988). But total discounted profit in this case is $t/2$, smaller than when $\delta_c = 0$ and $\delta_f = 1$ for each firm

and δ_f except when $\delta_f = 0$. As pointed out earlier, the equilibrium is unique if and only if $\delta_f = 0$. In all other cases, there are two equilibria that are mirror images of each other. We continue to consider the equilibrium with $z < 1/2$, i.e., firm B has a larger market share in $\tau = 1$. Then $P_A^1 = 10t/13$, $P_B^1 = 8t/13$, and $z = 11/26$.[14] Calculating equilibrium profits in this case, we have $\pi_A^1 = 0.325t, \pi_A^2 = 0.247t, \pi_B^1 = 0.355t$, and $\pi_B^2 = 0.290t$.

First, although firm B secures a larger market share by pricing below firm A in $\tau = 1$, its market share shrinks in $\tau = 2$ since its $\tau = 1$ customers in $[11/26, 6/13]$ switch to firm A. But firm B is better off having switching customers than having a smaller $\tau = 1$ market share. It is because switching customers are closer to firm A and they help firm B fend off firm A's aggressive pricing in $\tau = 2$ and use personalized pricing for the remaining customers that continue to purchase from firm B. Indeed firm B's most loyal customers, i.e., $x \in (25/26, 1]$, are charged price higher than the Hotelling price t. But the maximum price firm A charges is t. Thus firm B has larger profit than firm A in both periods. This implies that, when firms' locations are exogenously fixed, the main strategic decision is to choose the $\tau = 1$ price to secure a larger market share.

Second, the dynamic consideration and the accompanied personalized pricing in $\tau = 2$ intensify price competition in $\tau = 1$. As a result, both firms choose their $\tau = 1$ prices below the Hotelling price. Consequently their $\tau = 1$ profits are smaller than $t/2$, the Hotelling profit. The dynamic consideration also differentiates our $\tau = 2$ equilibrium from that in Thisse and Vives (1988), where the unique equilibrium is symmetric and each firm earns profit equal to $t/4$. In contrast, we have an asymmetric equilibrium in $\tau = 2$ where firm B's market share is larger than firm A's even though some of firm B's customers switch to firm A. In addition, firm B's profit is larger than in Thisse and Vives (1988) while firm A's profit is smaller: $\pi_B^2 = 0.290t > t/4 > \pi_A^2 = 0.247t$. But for both firms, the $\tau = 2$ profits are smaller than the Hotelling profit. This is generally consistent with Thisse and Vives (1988) that the ability to price-discriminate harms profitability by intensifying competition, although in our setup firms use personalized pricing only for their repeat customers. As pointed out earlier, the only case we obtain the Hotelling outcome in $\tau = 1$ and the Thisse and Vives (1988) outcome in $\tau = 2$ in our model is in the absence of any dynamic consideration by both consumers and firms, i.e., $\delta_c = \delta_f = 0$.

So far, we have seen that both firms are strictly worse off in each period when they use personalized pricing in $\tau = 2$ than when they use uniform price in both periods. But are consumers better off under price discrimination? To answer this, recall that, in the Hotelling equilibrium, price is t, all consumers $x \leq 1/2$ choose firm A in both periods at total cost $2(t + x^2 t)$, and all consumers $y > 1/2$ choose firm B in both periods at total

[14] The explanations for the other case are the same with firm B replaced by firm A.

cost $2\left(t+(1-y)^2t\right)$.

Let us again consider the equilibrium with $z = 11/26$. We have shown previously that both firms charge price strictly below t in $\tau = 1$. In $\tau = 2$, only consumers in $(25/26, 1]$ pay price higher than t. For all other consumers, price is strictly lower than t in both periods. In addition, these consumers have an option to choose the firms in the Hotelling equilibrium and pay the same transportation costs. Thus all consumers in $[0, 25/26]$ are strictly better off under price discrimination. Consider now $y \in (25/26, 1]$, who chooses firm B in both periods, pays the $\tau = 1$ price $P_B^1 = 8t/13 < t$ and the $\tau = 2$ price $P_B(y) = (2y - 12/13)\,t > t$. But $P_B^1 + P_B(y) < 22t/13 < 2t$ for all $y \in (25/26, 1]$, hence total cost for y is $P_B^1 + P_B(y) + 2(1-y)^2t < 2(t + (1-y)^2t)$. Thus all consumers $(25/26, 1]$ are better off under price discrimination.[15]

But welfare is lower in both periods than in the Hotelling equilibrium since firms have asymmetric market shares in both periods except when $\delta_f = 0$. Given the two firms' locations at each end, the average distance traveled is minimized when $z = 1/2$. On the other hand, welfare in $\tau = 2$ is higher than in Fudenberg and Tirole (2000) since there is only one-way customer switching in our case.[16] The following proposition summarizes the above discussions.

Proposition 2 *In equilibrium where firms use personalized pricing in $\tau = 2$, firms are worse off in each period, all consumers are better off, but social welfare is lower compared to when firms do not exercise price discrimination.*

3.4 Personalized pricing vs. third-degree price discrimination

The current case with exogenously fixed locations is identical to the model in Fudenberg and Tirole (2000), to be called FT henceforth, except two differences. First, firms use personalized pricing in $\tau = 2$ in our model while they use third-degree price discrimination in FT. Second, we consider general discount factors whereas firms and consumers use the same discount factor in FT. We discuss below why equilibrium changes drastically given these two differences.

Let us note first that when the FT model allows general discount factors δ_c and δ_f, there continues to exist a unique symmetric equilibrium described previously with the only change that the $\tau = 1$ price becomes $(3 + \delta_c)t/3$.[17] Thus the total discounted profit is now $\Pi_{FT} = (3 + \delta_c)t/6 + \delta_f(5t/18)$. Because prices in $\tau = 1$ increase in δ_c, firms are

[15] One can show that this argument applies for all values of δ_c and δ_f. That is, $P_B^1 + (1-y)^2t + \delta_c(P_B(y) + (1-y)^2t) < (1+\delta_c)(t + (1-y)^2t)$ for all $y \in \mathcal{B}$ with equality if and only if $\delta_c = \delta_f = 0$.

[16] In $\tau = 2$, the average distance traveled is $(7/13)^2 \times 1/2 + (6/13)^2 \times 1/2 = 85/338$. It is smaller than $11/36$, the average distance traveled in $\tau = 2$ in Fudenberg and Tirole (2000).

[17] That firms' discount factor does not matter follows from the fact that each firm's $\tau = 2$ profit is independent of its $\tau = 1$ market share thanks to the two-way customer switching.

better off when consumers are more forward-looking. In particular, the $\tau = 1$ price is higher than the Hotelling price for all $\delta_c > 0$. The higher $\tau = 1$ price is offset by lower prices in $\tau = 2$.

When firms use third-degree price discrimination, one can verify that the equilibrium profit in $\tau = 2$ is the same for both firms *regardless* of firm A's market share $[0, z]$ in $\tau = 1$. This is because customer switching is two-way and, as firm A's market share in $\tau = 1$ increases, more customers switch to firm B in $\tau = 2$. Specifically, the fraction of customers switching from firm A to firm B and the fraction of those switching from firm B to firm A are exactly the same if and only if $z = 1/2$, and the former (latter) is larger if $z > (<)1/2$.[18] Such two-way customer switching is due to the assumption that firms use third-degree price discrimination. Because each firm has to charge the same price to all its $\tau = 1$ customers, protecting its turf in $\tau = 2$ can be too costly if the firm has a large market share. If a firm wants to continue to serve its marginal customer, it has to reduce price for its most loyal customers as well. Likewise, firms cannot price too aggressively to poach their rival's customers. The end result is that both firms poach some of their rival's customers.

In contrast, the ability to use personalized pricing in our model allows firms to protect their turf better. Specifically, if $z \leq 1/2$, then firm A can continue to serve all its $\tau = 1$ customers while poaching some customers from firm B. As discussed previously, however, such customer switching benefits firm B since it allows firm B to use personalized pricing for its remaining customers and extract larger surplus than when firm B has a smaller $\tau = 1$ market share. Thus firm B has a larger profit than firm A in each period even though it loses some of its customers to firm A in $\tau = 2$. Similarly if $z \geq 1/2$, customer switching is only from firm A to firm B but firm A has a larger profit than firm B. The flip side of the ability to use personalized pricing is that firms choose more aggressive poaching offers than in FT. It is easy to check that, in all equilibria in our model, each firm's successful poaching offer is lower than $t/3$, the unique poaching offer in FT.

The two-way customer switching in FT implies that a larger $\tau = 1$ market share does not lead to a larger profit in $\tau = 2$. In addition, the firm with a larger market share in $\tau = 1$ loses more customers to its rival in $\tau = 2$, which softens the $\tau = 1$ price competition. As a result, the $\tau = 1$ equilibrium price in FT is not lower than the Hotelling price. In our model, a larger market share implies a larger profit in each period, which makes price competition in $\tau = 1$ tougher than in the static Hotelling model. As a result, both firms charge their $\tau = 1$ price below the Hotelling price.

As mentioned previously, firms in FT are better off when consumers are more forward-

[18]This argument applies as long as $z \in [1/4, 3/4]$, which is true in equilibrium. More precisely, one can show that the fraction of customers switching from firm A to B is $(4z - 1)/6$ and the fraction switching from firm B to A is $(3 - 4z)/6$ with $(4z - 1)/6 \geq (3 - 4z)/6$ if and only if $z \geq 1/2$.

looking. This is because forward-looking consumers anticipate favorable poaching offers in $\tau = 2$, which makes them less sensitive to prices charged in $\tau = 1$. But these poaching offers, equal to $t/3$, are independent of consumers' discount factor because of symmetric two-way customer switching. Given that there is a unique symmetric equilibrium in FT, the only effect forward-looking consumers have on the firm's behavior is thus to soften competition in $\tau = 1$. In our results, customer switching is one-way and there are two asymmetric equilibria, one favoring one firm over the other. This intensifies first-period competition relative to FT, and more so when consumers are more forward-looking, as shown in the corollary to Proposition 1. The discussions so far suggest that competition in personalized pricing makes firms worse off compared to when they compete in third-degree price discrimination.

Proposition 3 *For all values of (δ_c, δ_f), each firm's total discounted profit in equilibrium is smaller when competition in $\tau = 2$ is in personalized pricing than when it is in third-degree price discrimination.*

Proof: See Appendix A.

In the end, the feasibility of different pricing strategies would depend on various factors pertaining to industries, products, or types of consumers and sellers. We finish this section with some discussions on when personalized pricing is more likely to be feasible.

Personalized pricing is a limiting case of price discrimination as the number of identifiable consumer segments increases. As in other types of price discrimination, the feasibility of personalized pricing depends on the availability of consumer information and the firm's ability to prevent consumers from discovering price differentials and exploiting arbitrage opportunities. The information in itself is only necessary for successful implementation of personalized pricing since the information needs to yield usable insights that can be operationalized (Arora et al., 2008). Thus personalized pricing is more likely to work for products that can be more easily customized, or that can be combined with quality differentiation. Examples in the first category include large enterprise-level business software, for which transactions are often based on negotiated customized pricing, while computer servers or storage devices are examples in the second category (Choudhary et al., 2005). In addition, personalized pricing can work better for products whose pricing is too complex and variable for consumers to compare price differentials easily such as insurance, financial products, or mobile phone plans ('Caveat emptor.com; Online prices', *The Economist*, June 30, 2012).

The above discussions suggest that it may be difficult to use personalized pricing for standardized products such as grocery items or DVDs. An example is Amazon's ill-fated attempt to exercise price discrimination for DVD titles.[19] But even for standardized products, firms can operationalize personalized pricing through using various price-framing tactics (Weisstein et al., 2013), or targeted advertising and promotions such as Safeway's Just for U program (http://m.safeway.com/just-for-u/) or Target's Cartwheel mobile couponing application (https://cartwheel.target.com/). For example, Safeway's Just for U program uses complex algorithms to process customers' purchase data, based on which the retailer can send personalized offers on-line or through mobile apps ('Supermarkets offer personalized pricing', *Bloomberg Business*, November 16, 2013). As mobile technologies advance rapidly, this type of personalization becomes more prevalent and even real-time personalized offers may become eventually possible (Esteves and Resende, 2016). These are more effective than traditional paper coupons in making direct price comparison harder, which can render personalized pricing feasible.

From the above discussions, we can identify several factors that would make personalized pricing more likely to be feasible such as the quality of customer information, the extent to which products can be customized and can be combined with added value, the degree of complexity in pricing, the technologies employed in operationalizing personalized pricing, etc. But the difficulty in clearly identifying the number of market segments used for pricing limits rigorous empirical research on the use of personalized pricing in practice; the evidence is anecdotal at best or based only on isolated examples.

3.5 Imperfect customer recognition

So far we have assumed that firms observe their customers' locations perfectly at the end of $\tau = 1$. We now discuss a more plausible case when such customer recognition is imperfect. To this end, we consider an extension of our model in which firm i learns the exact locations of all its $\tau = 1$ customers with probability ϕ_i, $i = A, B$. With probability $1 - \phi_i$, firm i learns only who its $\tau = 1$ customers are, but not their exact locations.[20] Then firm i can use personalized prices for all its $\tau = 1$ customers with probability ϕ_i, but can charge only a uniform price to all its $\tau = 1$ customers with probability $1 - \phi_i$. A natural interpretation is that ϕ_i reflects firm i's investment in its consumer-level information technology, or consumer addressability. Given such interpretation, it

[19] The main reason was that Amazon failed to operationalize personalized pricing since price differentials were easy to find and led to consumer backlash. In one case, one customer ordered a DVD for \$24.49, only to find out that the price had jumped to \$26.24 next time he went back. But when he used the computer removing the tags that identified him as an Amazon customer, the price fell to \$22.74 ('Test of 'dynamic pricing' angers Amazon customers', *The Washington Post*, October 7, 2000).

[20] An alternative to model imperfect customer recognition is to introduce some noise. For example, firm A observes $s(x) = x + \epsilon$ for $x \in \mathcal{A}$ where ϵ is a white noise. But in this case, the meaning of personalized prices becomes unclear since the personalized price for customer x depends on $s(x)$, which could also be for all other customers if ϵ has a full support.

is also natural to assume ϕ_A and ϕ_B are independent.[21] To simplify notation, we also assume $\delta_c = \delta_f = 1$.

The above change in information structure leads to four possibilities in competition in $t = 2$ depending on what pricing strategies firms can choose for their repeat customers. First, with probability $\phi_A \phi_B$, both firms use personalized prices for their $\tau = 1$ customers. This is the case studied in our main model. Second, with probability $\phi_A(1 - \phi_B)$, firm A uses personalized prices while firm B uses a uniform price. Third, with probability $(1 - \phi_A)\phi_B$, firm A uses a uniform price while firm B uses personalized prices. Finally, with probability $(1 - \phi_A)(1 - \phi_B)$, both firms use uniform prices, which corresponds to the case analyzed in Fudenberg and Tirole (2000). Since we already know the results from the first and the fourth cases, the additional cases to analyze are when only one firm uses personalized prices in $\tau = 2$. The following lemma summarizes the results when only firm A uses personalized prices.[22]

Lemma 4 *Suppose $\phi_A = 1$ and $\phi_B = 0$. Then there is a unique equilibrium such that:*
(i) The location of marginal consumer in $\tau = 1$ is $z = 6/17$.
(ii) Prices in $\tau = 1$ are $P_A = 15t/17$ and $P_B = 6t/17$.
(iii) Prices in $\tau = 2$ are

$$P_A^2 = \begin{cases} P_A(x) = (1 - 2x)t & \text{if } x \in [0, 6/17], \\ P_A(\mathcal{B}) = 9t/17, \end{cases} \qquad P_B^2 = \begin{cases} P_B(\mathcal{A}) = 0, \\ P_B(\mathcal{B}) = 13t/17. \end{cases}$$

(iv) Consumers in $[0, 6/17]$ choose firm A in both periods, those in $[6/17, 21/34]$ choose firm B in $\tau = 1$ but switch to firm A in $\tau = 2$, and those in $[21/34, 1]$ choose firm B in both periods.
(v) The total profits are $\Pi_A = 393t/578$ and $\Pi_B = 301t/578$.

Proof: See Appendix A.

The above lemma shows an important difference between the case when both firms can use personalized prices and the case when only one firm can use personalized prices. In the latter case, Lemma 4 shows that there is a unique equilibrium with asymmetric market share in $\tau = 1$ accompanied by one-way customer switching. For firm A that can use personalized prices, the market share is $6/17$ in $\tau = 1$ but it increases to $21/34$ in

[21] This setup is similar to Chen and Iyer (2002) whose main focus is on the firm's decision to invest in consumer addressability. But there are two important differences between our model and theirs. First, we consider dynamic pricing over two periods; the firm's investment in addressability allows the firm to gather consumer-level information only through the first-period interaction. In contrast, Chen and Iyer (2002) study a static model of pricing game in which the information comes directly from the investment. Second, they assume each firm can target only those consumers that are addressable whereas firms in our model use personalized prices for addressable consumers and a uniform price for others. These differences lead to different equilibria in the investment game, as we will discuss later.

[22] The other case when only firm B uses personalized prices is a mirror image of this case.

$\tau = 2$. Unlike the case when both firms use personalized prices, however, the equilibrium with $z > 1/2$ does not exist in this case. The reason is as follows. When $z > 1/2$, there is two-way customer switching in $\tau = 2$ as shown in the proof of Lemma 4, which implies that firm B does not benefit from having a smaller market share in $\tau = 1$. Thus firm B has incentives to reduce z by decreasing its $\tau = 1$ price, which breaks down the equilibrium with $z > 1/2$. Clearly this is because firm B cannot continue to serve all its repeat customers when it cannot use personalized prices.[23]

In the above equilibrium, firm A has larger profits in both periods: in $\tau = 1$, it has a smaller market share but charges higher price and, in $\tau = 2$, it serves all its repeat customers using personalized prices and poaches some of firm B's $\tau = 1$ customers. Nonetheless, firm A is better off if it uses a uniform price instead of personalized prices for its $\tau = 1$ customers. This is because firm A's sole ability to use personalized prices in $\tau = 2$ makes firm B more aggressive, which intensifies competition in $\tau = 1$. Setting $t = 1$, quick calculation shows that firm A's total profit is broken down to $\pi_A^1 + \pi_A^2 = 90/289 + 213/578 = 393/578$. When both firms use uniform prices in $\tau = 2$ for their repeat customers, we have the outcome from Fudenberg and Tirole (2000): with $\delta_c = \delta_f = 1$, firm A's total profit is $\pi_A^1 + \pi_A^2 = 2/3 + 5/18 = 17/18 > 393/578$. Thus firm A's $\tau = 2$ profit is larger when it is the only firm using personalized prices, but its $\tau = 1$ profit is significantly smaller than when both use uniform prices. It is easy to check that the above relation also holds when $\delta_c < 1$.

We can summarize the results so far in the following observations. First, when $\phi_A = \phi_B = 0$, the equilibrium is unique with $z = 1/2$. Second, when $\phi_A = \phi_B = 1$, there are two equilibria with asymmetric market shares in $\tau = 1$, one being a mirror image of the other. Third, when $\phi_A = 1$ and $\phi_B = 0$, the equilibrium is unique with $z < 1/2$. Fourth, when $\phi_A = 0$ and $\phi_B = 1$, the equilibrium is unique with $z > 1/2$. These observations suggest the possibility of a unique equilibrium with asymmetric market share in $\tau = 1$ when ϕ_A and ϕ_B are sufficiently different from each other. The intuition is as follows. Starting from the case with two asymmetric equilibria, sufficient differences in pricing capabilities between the two firms enable the firm with more sophisticated information technology to set a higher $\tau = 1$ price. By doing so, it can increase its $\tau = 1$ profit while poaching some customers from its rival that is more likely to use a uniform price and hence less likely to protect its customer base. Thus the firm that is more likely to use personalized prices will choose an equilibrium with a smaller market share in $\tau = 1$.

The following proposition characterizes all possible equilibria for all values of ϕ_A and

[23]When firm B also uses personalized prices as in our main model, customer switching is only from firm A to firm B if $z > 1/2$. Thus firm B does not have incentives to undercut firm A in $\tau = 1$. This is why there are two asymmetric equilibria when both firms use personalized prices.

ϕ_B. For each pair (ϕ_A, ϕ_B), equilibrium can be found in several steps. First, given the $\tau = 1$ marginal consumer z, we solve for $\tau = 2$ equilibria in each case of $z \leq 1/2$ and $z \geq 1/2$. Second, based on the $\tau = 2$ equilibria, we express z in terms of $\tau = 1$ prices and (ϕ_A, ϕ_B). Third, we find equilibrium prices in $\tau = 1$ by solving the two firms' best response functions in each case of $z \leq 1/2$ and $z \geq 1/2$. In the last step, we verify that these prices indeed constitute equilibrium by deriving the conditions that guarantee each equilibrium is consistent with the restriction on z. These conditions involve polynomial inequalities in terms of ϕ_A and ϕ_B. We define the following:

$$\begin{aligned}
F(\phi_A, \phi_B) \equiv\ & (64 + 4\phi_A - 5\phi_B)\phi_B\sqrt{2(28 - 2\phi_A + \phi_B)(56 - 13\phi_A - 16\phi_B)} \\
& +2(28 - 2\phi_A + \phi_B)(10\phi_A^2 - (64 - 15\phi_A)\phi_B + 3\phi_B^2), \\
G(\phi_A, \phi_B) \equiv\ & (64 + 4\phi_B - 5\phi_A)\phi_A\sqrt{2(28 - 2\phi_B + \phi_A)(56 - 13\phi_B - 16\phi_A)} \\
& +2(28 - 2\phi_B + \phi_A)(10\phi_B^2 - (64 - 15\phi_B)\phi_A + 3\phi_A^2).
\end{aligned}$$

As shown in the proof, the condition $F(\phi_A, \phi_B) > 0$ guarantees the consistency of equilibrium with $z < 1/2$, and the condition $G(\phi_A, \phi_B) > 0$ guarantees the consistency of equilibrium with $z > 1/2$.

Proposition 4 *(i) If $F(\phi_A, \phi_B) > 0$ and $G(\phi_A, \phi_B) < 0$, then the equilibrium is unique with the $\tau = 1$ marginal consumer's location $z < 1/2$.*

(ii) If $F(\phi_B, \phi_A) < 0$ and $G(\phi_B, \phi_A) > 0$, then the equilibrium is unique with $z > 1/2$.

(iii) If $F(\phi_A, \phi_B) > 0$ and $G(\phi_A, \phi_B) > 0$, then there are two equilibria with asymmetric market shares in $\tau = 1$.

(iv) The equilibrium is unique with $z = 1/2$ if and only if $\phi_A = \phi_B = 0$.

(v) For firm i, total equilibrium profit is the largest when $\phi_i = 0$ regardless of ϕ_j, $j \neq i$.

Proof: See Appendix A.

As the conditions in the above proposition are somewhat complicated, we provide Figure 3 to illustrate them where each shaded area indicates the range of (ϕ_A, ϕ_B) for the existence of each type of equilibrium. As shown in Figure 3, the conditions for the equilibrium with $z < 1/2$ require ϕ_A be larger than ϕ_B, and include the case analyzed in Lemma 4. As we have also argued above, the conditions for the existence of both equilibria are that ϕ_A and ϕ_B are not very much apart from each other, and include the case studied in our main model, i.e., $\phi_A = \phi_B = 1$.

— Figure 3 goes about here. —

The last part of Proposition 4 has important implications on the firm's decision to invest in consumer addressability. It implies that, if firms have choice over ϕ_i's, it is a dominant strategy for each firm to choose $\phi_i = 0$. Thus even if there is no cost in choosing ϕ_i, both firms choose $\phi_i = 0$ in dominant strategy equilibrium. This is in contrast to Chen and Iyer (2002) who find that, even when the cost of investment is zero, the equilibrium is asymmetric with one firm choosing full addressability, i.e., $\phi_i = 1$. The main difference is dynamics. As shown previously, use of personalized prices in $\tau = 2$ intensifies competition in $\tau = 1$, and more so when both firms use personalized prices. The intensified competition in $\tau = 1$ negates the benefits of consumer addressability and more sophisticated pricing capabilities in $\tau = 2$. In addition, Chen and Iyer (2002) assume firms do not target the consumers who are not addressable, which further softens competition and increases the value of addressability.

4 Endogenous Location Choice

We now turn to the full game where firms optimally choose their locations. The first period is the standard Hotelling game in which each firm chooses location and a uniform price. As before, the locations are fixed over two periods. Denote firm A's location by a and firm B's location by b with $0 \le a \le b \le 1$. Once again it is easy to see that each firm's $\tau = 1$ market segment is a connected set. Without loss of generality, we denote firm A's $\tau = 1$ market segment by $\mathcal{A} = [0, z]$ with $a \le z \le b$.[24] In the second period, firms compete by using personalized prices whenever possible. An equilibrium consists of each firm's location and prices over the two periods.

For general values of δ_c and δ_f, finding closed-form solutions for equilibrium locations is not possible as it involves solving quadratic equations simultaneously. In addition, there are only mixed-strategy equilibria for some values of the discount factors, making it hard to derive clear intuition. Thus we focus on the case when $\delta_c = 0$ and $\delta_f = 1$, which is the same case we discussed in detail in the previous section. This allows us a clear comparison of the equilibria with or without location choice. Nonetheless we provide in Appendix B general analysis for all possible values of the discount factors and derive several observations on the properties of equilibria.[25] For example, in case $\delta_c = 1$, the analysis shows: for small values of δ_f, maximal differentiation obtains in equilibrium and the pricing equilibria for the subgame are identical to those in the previous section; as δ_f increases, there are two pure-strategy equilibria with one firm choosing an interior location; when $\delta_f = 1$, pure-strategy equilibria do not exist. The analysis also shows

[24]To see that $a \le z \le b$ holds in equilibrium, we note that firm A's (firm B's, resp.) $\tau = 2$ profit increases (decreases, resp.) in z. In addition, if $z < a$, then firm A can increase its $\tau = 1$ profit by lowering its $\tau = 1$ price, hence increasing z. Similarly if $z > b$, then firm B can increase its $\tau = 1$ profit by lowering its $\tau = 1$ price, thereby decreasing z.

[25]First, we derive necessary and sufficient conditions for maximal differentiation ($a = 0$, $b = 1$) to become an equilibrium outcome. Second, we derive sufficient conditions for the existence of pure-strategy equilibria. Third, we show that there does not exist a pure-strategy equilibrium if $\delta_c = \delta_f = 1$.

that, when $\delta_c = 0$, equilibria exist in pure strategies with maximal differentiation for small values of δ_f, and asymmetric location choice for large values of δ_f. Thus the case we focus on in this section, $\delta_c = 0$ and $\delta_f = 1$, leads to asymmetric location choice in equilibrium.

We solve the game backwards in three steps. First, given (a, b, z), we find the equilibrium of the $\tau = 2$ pricing game. Second, given (a, b), we solve for the $\tau = 1$ prices. Given our simplification $\delta_f = 1$, each firm chooses its $\tau = 1$ price to maximize un-discounted total profit. This leads to both $\tau = 1$ prices and z expressed in terms of (a, b). Finally we solve for the equilibrium location choice.

4.1 Second period

Analogous to the previous case with maximal differentiation, we divide analysis into two cases depending on where consumers lie relative to the midpoint between the two firms: $z \leq (a+b)/2$ and $z \geq (a+b)/2$. Following the argument used previously, we can show the following.

Lemma 5 *Suppose $z \leq (a+b)/2$. Then the unique equilibrium in $\tau = 2$ is given by*

$$P_A(x) = \begin{cases} (a+b-2x)(b-a)t & \text{if } x \in [0, z], \\ \dfrac{(a+b-2z)(b-a)t}{2} & \text{if } x \in [z, 1], \end{cases}$$

$$P_B(y) = \begin{cases} 0 & \text{if } y \in [0, \dfrac{a+b+2z}{4}], \\ \dfrac{(4y-2z-a-b)(b-a)t}{2} & \text{if } y \in [\dfrac{a+b+2z}{4}, 1]. \end{cases}$$

The corresponding profits are

$$\pi_A^2 = \frac{t}{8}(b-a)\left((a+b)^2 + 4(a+b)z - 4z^2\right),$$
$$\pi_B^2 = \frac{t}{16}(b-a)(4-a-b-2z)^2.$$

Proof: See Appendix A.

Lemma 6 *Suppose $z \geq (a+b)/2$. Then the unique equilibrium in $\tau = 2$ is given by*

$$P_A(x) = \begin{cases} \dfrac{(2z-4x+a+b)(b-a)t}{2} & \text{if } x \in [0, \dfrac{a+b+2z}{4}], \\ 0 & \text{if } x \in [\dfrac{a+b+2z}{4}, 1], \end{cases}$$

$$P_B(y) = \begin{cases} \dfrac{(2z-a-b)(b-a)t}{2} & \text{if } y \in [0, z], \\ (2y-a-b)(b-a)t & \text{if } y \in [z, 1]. \end{cases}$$

The corresponding profits are

$$\begin{aligned}
\pi_A^2 &= \frac{t}{16}(b-a)(a+b+2z)^2, \\
\pi_B^2 &= \frac{t}{8}(b-a)\left(8(1-a-b)+(a+b)^2+4(a+b)z-4z^2\right).
\end{aligned}$$

Proof: See Appendix A.

As before, one can verify that both firms' $\tau = 2$ profit functions are continuous in z, π_A^2 increases in z, and π_B^2 decreases in z. In equilibrium with $z \le (a+b)/2$, all consumers in $[0, z]$ choose firm A in both periods, those in $[z, \frac{a+b+2z}{4}]$ choose firm B in $\tau = 1$ but switch to firm A in $\tau = 2$, and the rest choose firm B in both periods. In the other equilibrium, some consumers switch from firm A to firm B.

4.2 First period: price

Next we solve for the equilibrium prices in $\tau = 1$ given locations fixed at a and b. Given $\delta_c = 0$, the marginal consumer z satisfies $P_A^1 + (z-a)^2 t = P_B^1 + (b-z)^2 t$, hence

$$z = \frac{a+b}{2} + \frac{P_B^1 - P_A^1}{2(b-a)t}$$

where $z \le (a+b)/2$ if and only if $P_A^1 \ge P_B^1$. Proceeding similarly as before, we can show the following.

Lemma 7 *Given fixed locations a and b with $a \le b$, the price game in $\tau = 1$ has two equilibria:*

(i) $P_A^1 = \dfrac{2(3+2a+2b)(b-a)t}{13}$ *and* $P_B^1 = \dfrac{2(5-a-b)(b-a)t}{13}$ *with* $z = \dfrac{4+7a+7b}{26}$;

(ii) $P_A^1 = \dfrac{2(3+a+b)(b-a)t}{13}$ *and* $P_B^1 = \dfrac{2(7-2a-2b)(b-a)t}{13}$ *with* $z = \dfrac{8+7a+7b}{26}$.

1. If $a+b > \frac{84}{13\sqrt{70}-28} \simeq 1.04$, then only the first equilibrium exists. 2. If $\frac{84}{13\sqrt{70}-28} \ge a+b \ge \frac{2(13\sqrt{70}-70)}{13\sqrt{70}-28} \simeq 0.96$, then both equilibria exist. 3. If $\frac{2(13\sqrt{70}-70)}{13\sqrt{70}-28} > a+b$, then only the second equilibrium exists.

Proof: See Appendix A.

Lemma 7 shows that the $\tau = 1$ pricing game has different equilibria depending on the range of $a+b$. In particular, there are multiple equilibria when $a+b$ is close to 1. When $a+b = 1$, the two firms' locations are symmetric in that they are located exactly in the opposite position from the center. This includes the case we studied in Section 3, namely $a = 0$ and $b = 1$. As discussed previously, symmetric locations lead to asymmetric incentives for price cut whereby one firm has more incentives to price aggressively in $\tau = 1$ than the other firm. This results in multiple equilibria in the continuation game. As we show below, the multiplicity of pricing equilibria in turn makes symmetric locations untenable as an equilibrium outcome since one firm can always deviate to its preferred equilibrium in the continuation game.

4.3 First period: location

Let us now turn to the equilibrium location choice. Lemma 7 shows that the continuation game has multiple equilibria in the intermediate range of $a + b$. Thus each firm's location choice depends on which of these equilibria each firm anticipates in the subgame following its location choice. The equilibrium location choice in turn should be consistent with the anticipated equilibrium of the pricing subgame.

Lemma 8 *The location game in $\tau = 1$ has two equilibria:*

(i) $a = \dfrac{2\sqrt{56029} - 347}{621} \simeq 0.2$ *and* $b = 1$*, which is followed by the equilibrium of the pricing subgame where* $z = \dfrac{4 + 7a + 7b}{26}$*;*

(ii) $a = 0$ *and* $b = \dfrac{968 - 2\sqrt{56029}}{621} \simeq 0.8$*, which is followed by the equilibrium of the pricing subgame where* $z = \dfrac{8 + 7a + 7b}{26}$.

Proof: See Appendix A.

It is worth noting that equilibrium product choice does not lead to maximal differentiation in either of the two equilibria. We offer detailed discussions in the next section where, for clarity of exposition, we round equilibrium locations to the first decimal point, i.e., $a = 0.2$ in the first equilibrium and $b = 0.8$ in the second. This simplification does not change our qualitative results and discussions in any meaningful way.

4.4 Equilibria and discussions

Collecting the results from Lemmas 5 - 8, we have

Proposition 5 *With endogenous location choice, there are two equilibria given by:*

(i) $a = 0.2,\ b = 1;\ P_A^1 = 216t/325,\ P_B^1 = 152t/325$ *with* $z = 31/65$*;*

$$P_A(x) = \begin{cases} \dfrac{8(3-5x)t}{25} & \text{if } x \in [0, \dfrac{31}{65}], \\ \dfrac{32}{325}t & \text{if } x \in [\dfrac{31}{65}, 1], \end{cases}$$

$$P_B(y) = \begin{cases} 0 & \text{if } y \in [0, \dfrac{7}{13}], \\ \dfrac{8(13y-7)t}{65} & \text{if } y \in [\dfrac{7}{13}, 1]. \end{cases}$$

(ii) $a = 0,\ b = 0.8;\ P_A^1 = 152t/325,\ P_B^1 = 216t/325$ *with* $z = 34/65$*;*

$$P_A(x) = \begin{cases} \dfrac{8(7-13x)t}{65} & \text{if } x \in [0, \dfrac{7}{13}], \\ 0 & \text{if } x \in [\dfrac{7}{13}, 1], \end{cases}$$

$$P_B(y) = \begin{cases} \dfrac{32t}{325} & \text{if } y \in [0, \dfrac{34}{65}], \\ \dfrac{8(5y-2)t}{25} & \text{if } y \in [\dfrac{34}{65}, 1]. \end{cases}$$

In Section 3 where locations were exogenously fixed at $a = 0$ and $b = 1$, we found that there are two equilibria in the $\tau = 1$ pricing game. When locations are chosen endogenously, the above proposition shows that there are two equilibria in the $\tau = 1$ location game, each associated with a unique equilibrium in the continuation pricing game. In particular, there cannot be a location equilibrium followed by multiple equilibria in the continuation game. The intuition is as follows. Fix firm B's location at b. Suppose firm A's best response is a such that the continuation game admits two equilibria, one favoring firm A and the other favoring firm B. If the continuation game is played in firm A's favor, then firm B is better off deviating from b. If the continuation game is played in firm B's favor instead, then firm A is better off deviating from a. This argument also implies that there cannot be equilibrium locations such that $a + b = 1$, since they lead to multiple pricing equilibria as shown in Lemma 7. Thus the multiplicity of equilibrium prices changes to the multiplicity of equilibrium locations when location choice is endogenous.

Neither of these equilibria feature maximal differentiation, i.e., $a = 0$, $b = 1$. We have already argued above that the case $a + b = 1$ leads to multiple equilibria in the continuation game, which creates opportunities for profitable deviation. An alternative explanation can be offered based on dynamic consideration and the use of personalized pricing in $\tau = 2$. The former compels firms to make forward-looking decisions in $\tau = 1$ while the latter allows firms to protect their market more effectively than when they choose prices at higher levels of aggregation. Suppose firm B chooses $b = 1$. If firm A

chooses $a = 0$, it can benefit from softened competition, hence larger $\tau = 1$ profits than when it chooses an interior location. If firm A chooses an interior location instead, it can improve its $\tau = 2$ profits due to its aggressive positioning, which allows firm A to protect its turf better and poach some of firm B's customers in $\tau = 2$. Thus choosing an interior location can be optimal if firm A does not discount its $\tau = 2$ profits too much, or δ_f is not too small.

To see this more clearly, suppose firm A chooses $a = 0$ in response to $b = 1$. Then there are two possibilities in the continuation game based on the equilibria found in Lemma 7 (also Proposition 1). First, firm A can price less aggressively and have a smaller market share in $\tau = 1$, i.e., $z = 11/26$ (Lemma 7(i)). In this case, firm A's profits are $\pi_A^1 = 0.325t$ and $\pi_A^2 = 0.247t$. But if firm A deviates by choosing $a = 0.2$, its profits change to $\pi_A^1 = 0.317t$ and $\pi_A^2 = 0.282t$ (Proposition 3(i)). Firm A's $\tau = 1$ profit decreases since it is now closer to firm B, which intensifies competition. But its $\tau = 2$ profit increases thanks to its position closer to the center. If δ_f is large enough, then the discounted profit from the latter can be larger than that from the former, whence $a = 2$ can be firm A's profitable deviation. Second, firm A can price more aggressively with a view to securing a larger market share, $z = 15/26$ (Lemma 7(ii)). But in this case, firm B can deviate by choosing $b = 0.8$ and increase its profits insofar as firm B does not discount its $\tau = 2$ profits too much.

The above argument suggests that maximal differentiation can be an equilibrium outcome if δ_f is small. Consider the case $\delta_f = 0$ for example. Then firms do not care about their $\tau = 2$ profit in their $\tau = 1$ location decisions. Softening the $\tau = 1$ competition becomes the primary concern in this case, which would lead to maximal differentiation. Thus $\delta_f = 0$ is one sufficient condition for the equilibrium with maximal differentiation. In Appendix B, we provide the complete necessary and sufficient conditions for maximal differentiation to be an equilibrium outcome. Roughly speaking, the conditions require δ_f to be bounded above such that, for small values of δ_f, δ_c is irrelevant, and for larger values of δ_f, δ_c needs to be bounded above as well with the bound decreasing as δ_f increases. The bound on δ_c can be understood since larger δ_c intensifies competition, which leads one firm to choose an interior location.

Comparing the above equilibria with those from Section 3 leads to several observations. We discuss them below based on the equilibrium with $a = 0.2$, $b = 1$ and $z = 31/65$.[26] In this equilibrium, firm A's $\tau = 1$ market share is $[0, z]$ but its $\tau = 2$ market share increases to $[0, \tilde{y}]$ where $\tilde{y} = 7/13$. In $\tau = 2$, firm A continues to serve all its $\tau = 1$ customers with personalized price $P_A(x)$ that decreases from $24t/25$ to $64t/325$ on $x \in [0, z]$, and poaches firm B's $\tau = 1$ customers in $[z, \tilde{y}]$ with a uniform

[26] For the other case, we can simply swap firm A and firm B and the same explanations apply.

price $P_A(\mathcal{B}) = 32t/325$. In $\tau = 2$, firm B charges a uniform price 0 to all firm A's $\tau = 1$ customers as well as its $\tau = 1$ customers in $[z, \tilde{y}]$. But the latter switch to firm A. For its remaining $\tau = 1$ customers, firm B uses personalized price $P_B(y)$ that increases from 0 to $48t/65$ on $y \in [\tilde{y}, 1]$. Calculating equilibrium profits in this case, we have $\pi_A^1 = 0.317t, \pi_A^2 = 0.282t, \pi_B^1 = 0.245t$ and $\pi_B^2 = 0.170t$. Thus firm A has larger profit than firm B in both periods. Figure 4 shows each firm's pricing strategies in $\tau = 2$ and how market shares change over time in this equilibrium, where thick solid lines (dashed lines, resp.) represent firm A's (B's, resp.) prices.

— Figure 4 goes about here. —

First, firm A has a smaller market share in $\tau = 1$ although its location is closer to the center compared to firm B. But firm A secures a larger market share in $\tau = 2$. This is due to different pricing strategies available in each period. In $\tau = 1$, firm A has to charge the same price to all its customers including those to the left of firm A who are in its backyard. For these customers, firm A has significant location advantage over firm B. Thus firm A can extract large surplus from these customers by charging high price. But firm A has to charge the same high price to customers to its right and, as a result, firm A concedes a larger market share to its rival. In $\tau = 2$, however, firm A can leverage its location to protect its turf through personalized prices and poach firm B's customers. Thus one can interpret firm A's strategy in this case as using its strategic location to extract large surplus from its loyal customers in $\tau = 1$ when it cannot price-discriminate, and expanding by poaching in $\tau = 2$ when it can use personalized prices. In this equilibrium, firm A has larger profit than firm B in both periods. This is in contrast to the case when locations were fixed at 0 and 1: in that case, the firm with a larger market share in $\tau = 1$ continues to have a larger market share in $\tau = 2$ and obtains larger profit in both periods although its market share decreases in $\tau = 2$ due to customer switching.

Second, the $\tau = 1$ prices are lower than the Hotelling price, once again confirming the intuition that the competition in personalized pricing in $\tau = 2$ intensifies competition in $\tau = 1$. Compared to the case when locations are fixed at 0 and 1, one firm charges a higher price while the other charges a lower price. Of course we need to be more precise in the comparison since there are two equilibria given fixed locations. Since the above equilibrium is the one that favors firm A, a meaningful comparison would be with the equilibrium given fixed locations that also favors firm A, i.e., the second equilibrium in Proposition 1 with $\delta_c = 0$ and $\delta_f = 1$. In the latter, the first-period prices are $P_A^1 = 8t/13 < 216t/325$ and $P_B^1 = 10t/13 > 152t/325$. Although one firm charges a higher price and the other charges a lower price, the average price is lower under endogenous location choice: the average $\tau = 1$ price is $zP_A^1 + (1-z)P_B^1$, which equals $0.68t$ under fixed locations and $0.562t$ under endogenous location choice. In this sense, endogenous location choice intensifies competition in $\tau = 1$.

Third, profits are smaller for both firms in each period when location choice is endogenous. Comparing the same pair of equilibria as before, firm A's profit changes from $0.355t$ to $0.317t$ in $\tau = 1$ and from $0.290t$ to $0.282t$ in $\tau = 2$. Firm B's profit changes from $0.325t$ to $0.245t$ in $\tau = 1$ and from $0.247t$ to $0.170t$ in $\tau = 2$. The decrease in profits is primarily due to the fact that, given endogenous location choice, the two products are less than maximally differentiated, which intensifies competition in both periods. It is easy to verify that there is more customer switching with endogenous location choice. In the equilibrium with $a = 0.2$ and $b = 1$, the fraction of customers who switch from firm B to firm A is $7/13 - 31/65 \approx 0.061$. In the equilibrium with fixed locations that favors firm A, the fraction of customers who switch from firm A to firm B is $15/26 - 7/13 \approx 0.038$. Thus firms are worse off when they choose locations than when locations are fixed exogenously at 0 and 1.

Finally, some consumers are better off and some worse off when firms choose locations optimally compared to when locations are fixed at 0 and 1. For example, in the equilibrium with $a = 0.2$ and $b = 1$, it is easy to verify that consumer $x = 0$ is worse off. It is because this consumer is in the deepest territory of firm A when $a = 0.2$ and has to incur transportation costs in both periods while not benefiting from firm B's poaching offer. On the other hand, consumer $y = 1$ is better off in equilibrium with $a = 0.2$ and $b = 1$. But welfare is higher with endogenous location choice because the average distance traveled by a consumer is smaller. Specifically, in equilibrium with $a = 0.2$ and $b = 1$, the average distance traveled by a consumer is around 0.195 in $\tau = 1$ and 0.184 in $\tau = 2$.[27] When locations are fixed at 0 and 1, the minimum average distance traveled by a consumer is 0.25.

We can summarize the main difference between the case with fixed locations and endogenous location choice as follows. With locations fixed at maximal distance, each firm's main strategy is to price aggressively and secure a larger market share in $\tau = 1$. Although the firm with a larger market share will inevitably lose some customers to its rival in $\tau = 2$, personalized pricing enables the firm to minimize such poaching. When location choice is endogenous, firms cannot commit to maximal differentiation. Thus their main strategy is to choose aggressive positioning and extract surplus from loyal customers in $\tau = 1$, and leverage its position to poach rival's customers in $\tau = 2$. But

[27] In equilibrium with $a = 0.2$ and $b = 1$, consumers in $[0, 0.477]$ purchase from firm A in $\tau = 1$. Thus the average distance traveled is $(0.2^2 + (0.477 - 0.2)^2 + (1 - 0.477)^2)/2 \approx 0.195$. In $\tau = 2$, consumers in $[0, 0.538]$ purchase from firm A. Thus the average distance traveled is $(0.2^2 + (0.538 - 0.2)^2 + (1 - 0.538)^2)/2 \approx 0.184$.

such aggressive positioning intensifies competition, making both firms worse off relative to the case with maximal differentiation. Nevertheless aggressive positioning is loss-minimizing in the sense that it helps the firm to avoid the worst outcome: the firm is better off in the best equilibrium under endogenous location choice where it chooses an interior location than in the worst equilibrium under fixed locations where it concedes a larger market share to its rival. Summarizing, we have

Proposition 6 *Suppose $\delta_c = 0$ and $\delta_f = 1$. In equilibrium where firms choose locations in $\tau = 1$, firms are worse off in each period, but social welfare is higher compared to when locations are fixed at maximal distance.*

5 Implications for Management

The central message from our study is that more customer information is bad for competing firms insofar as the sole use of customer information is for pricing. The main driver of these results is the asymmetric information whereby firms know more about their own customers, allowing them to protect their turf better in the second period. This in turn intensifies competition in the first period when information is gathered. Unless firms discount future too much, such asymmetric information leads to multiple equilibria, each favoring one firm over the other. When product choice is fixed at maximal differentiation, the multiplicity of equilibria results in a contest for market share since a larger market share renders the firm strategic edge in the second period. When product choice is also endogenous, the contest is for more aggressive positioning. In the end, the more aggressive firm, whether through pricing or positioning, can force the game to be played to its advantage. But both firms end up worse off compared to when they can credibly make any of the commitments that include use of simpler pricing strategies, substantial product differentiation, or pricing strategies that depend on short-term profits. In view of these findings, we discuss below some implications for management decisions in regards to customer information and pricing.

Customer information is crucial to effective customer relationship management, which aims at building one-to-one relationships with customers that can ultimately drive value for the firm. Consumer heterogeneity implies that the firm should rely on diverse marketing strategies to target different customers. For example, some customers may value customer service more than lower price, or prefer interacting through live chats to phone conversations. Likewise a firm's decision to invest in consumer addressability also depends on diverse factors, improved pricing capabilities being only one of them. But if competitors interpret such investment primarily as a signal to employ more aggressive pricing policies, competition can be intensified. To counter this, the firm can rely on a number of strategies that can credibly signal its intent to soften competition.

The first set of such strategies directly relate to the firm's pricing policy such as price matching guarantees or most-favored customer clauses. By adopting a price matching guarantee, the firm commits to its own $\tau = 1$ customers to match the rival's poaching price. For example, if firm A has a price matching guarantee in place, consumer $x \in \mathcal{A}$ now faces a personalized price that cannot be higher than firm B's poaching price, i.e., $P_A(x) \leq P_B(\mathcal{A})$. This breaks down the strategy to extract surplus from loyal customers by charging them more and poach the rival's customers by charging them less. This in turn implies that the firm does benefit from a larger market share, thereby softening competition in $\tau = 1$. When both firms adopt price matching guarantees in our duopoly case, the end result is softened competition and higher price.[28] A most-favored customer clause is a firm's promise to a customer that no other customers will be offered a lower price. Suppose firm A issues a most-favored customer clause to all its $\tau = 1$ customers. Then consumer $x \in \mathcal{A}$ faces in $\tau = 2$ a personalized price $P_A(x) \leq P_A(\mathcal{B})$. Thus firm A cannot be too aggressive in offering a poaching price to new customers since the low poaching price also applies to all its $\tau = 1$ customers. Similar to price matching guarantees, the end result is also softened competition when both firms issue most-favored customer clauses to their $\tau = 1$ customers (Cooper, 1986; Schnitzer, 1994).[29]

The second set of strategies can soften competition through creating various switching costs for a firm's repeat customers. First, a firm can utilize customer information and offer new enhanced services to its repeat customers. For example, repeat customers at Amazon can save the cost of entering information on repeat purchase, receive 'reminder services' or other recommendations. To the extent that consumers value this type of services, the resulting switching costs make it harder for firms to poach rival's customers. As shown by Acquisti and Varian (2005), these additional enhanced services can make behavior-based price discrimination profitable. In a similar vein, a firm can build a loyal customer base through various reward programs, personalized discounts, customer experience management, and so on (Kumar and Shah, 2004; Verhoef et al. 2009). Loyal customers are less price-sensitive, which allows firms to use dynamic pricing to screen out the more price-sensitive segment (Chen and Zhang, 2009). Loyal customers may also have higher-volume demand compared to one-off buyers (Shin and Sudhir, 2010). In addition, reward programs such as loyalty rewards can soften competition by facilitating tacit collusion (Fong and Liu, 2011). Thus the factors mentioned above can also lead to

[28] It is well known that price matching guarantees facilitate tacit collusion in a concentrated industry with low consumer search costs, although the opposite may be the case with enough heterogeneity in consumer search costs and brand loyalty. See, for example, Chen et al. (2001).

[29] In some cases, most-favored customer clauses can be ruled anticompetitive, Du Pont and Ethyl being the best-known example. More recently a similar decision was made in Germany against online hotel portals (Heinz, 2016).

more profitable behavior-based pricing than what was analyzed in our model.

Finally, firms may benefit from sharing customer information.[30] The multiplicity of equilibria in our model stems from the asymmetric information in $\tau = 2$, which intensifies competition in $\tau = 1$ as each firm tries to play its preferred equilibrium. Therefore if firms have symmetric information in $\tau = 2$ leading to symmetric equilibrium, then competition in $\tau = 1$ can be softened even though it may intensify competition in $\tau = 2$. To see this, suppose the two firms share the information about their $\tau = 1$ customers and, as a result, both firms have perfect information on all consumers in $\tau = 2$. Then the equilibrium in $\tau = 2$ will be the same as the Thisse and Vives outcome and so the equilibrium in $\tau = 1$ will be the standard Hotelling outcome. The resulting profit for each firm is $t/2$ in $\tau = 1$ and $t/4$ in $\tau = 2$. It is easy to see the total discounted profit in this case is larger for both firms than in both equilibria in our main model for all (δ_c, δ_f).[31]

6 Conclusion

This paper has studied a two-period model of differentiated duopoly where firms compete à la Hotelling in the first period, and compete using personalized pricing for their repeat customers in the second period. A key departure in our paper from most of the existing studies is that information is asymmetric and personalized. The asymmetric information significantly alters the competitive outcomes. In contrast to the existing literature, gathering consumer information results in multiple asymmetric equilibria in pricing and (when endogenous) in product choice unless firms discount future too much. Firms are worse off when they have more customer information that can be used for pricing, and more so when they also make product choice. Thus an important management question is how best to utilize customer information while avoiding destructive price competition. This paper has provided some discussions in this direction.

There are a number of important issues we have left out in the current paper. We briefly discuss two of them. First, consumers can behave strategically to interfere with information gathering by firms, or to take advantage of favorable poaching offers. For example, consumers can delay purchasing with a view to receiving a lower price offer in

[30] Firms can and do share customer information in various ways. Examples include data co-ops, data-marketing firms such as the DMA (Data & Marketing Association - https://thedma.org) or the ADMA (Association for Data-Driven Marketing and Advertising - https://www.adma.com.au), or Computer Reservation Systems in various travel industries. See Liu and Serfes (2006) or Jentzsch et al. (2013) for more details.

[31] Although not directly comparable to our model, several other studies discuss the benefits and costs of sharing customer information for pricing purposes. Chen et al. (2001a) analyze beneficial information sharing when customer segmentation is imperfect. Shaffer and Zhang (2002) and Liu and Serfes (2006) discuss beneficial one-way information sharing given enough firm heterogeneity. Jentzsch et al. (2013) show information sharing can be beneficial even when firms are symmetric if customer information is multi-dimensional.

the future (Villas-Boas, 2004; Chen and Zhang, 2009). They can also behave more proactively by, for example, deleting cookies, creating new accounts, or adopting anonymous payments (Acquisti and Varian, 2005). It may appear at first glance that such strategic behavior by consumers would undermine personalized pricing. But we suspect different types of strategic behavior work through different mechanisms, leading to different outcomes. For example, delay in purchase could signal consumers' price sensitivity, which sellers can take advantage of to screen different types of consumers (Chen and Zhang, 2009). But this argument rests on sufficient pre-existing consumer heterogeneity. On the other hand, the second type of strategic behavior, namely 'deleting cookies', is likely to have more robust implications. Acquisti and Varian (2005) briefly touch on this case, showing that competing sellers can benefit even when consumers can delete cookies if they can offer enhanced services to repeat buyers. Our preliminary work in this direction suggests that sellers benefit from this type of strategic consumers even in the absence of enhanced services. The reasoning is as follows. Suppose consumer $x \in \mathcal{A}$ deletes cookies. Then in $\tau = 2$ she faces three prices, $P_A(x), P_B(\mathcal{A}), P_A(\mathcal{B})$ instead of the first two as in our current paper. This makes it harder for firm A to poach firm B's customers as the low poaching price $P_A(\mathcal{B})$ also applies to all its loyal customers who delete cookies. The end result is softened competition since poaching becomes more costly facing strategic consumers.

Second, we have assumed away behavioral elements in consumers' purchase decisions such as fairness concerns. But they have real economic consequences if consumers eschew buying from a business that engages in personalized pricing as the Amazon example shows. Li and Jain (2016) incorporate fairness concerns to the model in Fudenberg and Tirole (2000) where fairness concerns are assumed to generate negative utility to consumers in proportion to the difference in the two prices their firm offers in the second period. Clearly such fairness concerns make it harder for firms to offer an aggressive poaching price. As a result, competition is softened in the second period and, given this, the first-period competition is also softened. Although Li and Jain (2016) focus on third-degree price discrimination, adding fairness concerns to our model with personalized pricing is also likely to lead to similar results. To see this, suppose consumer $x \in \mathcal{A}$ derives disutility in $\tau = 2$ equal to $\lambda|P_A(x) - P_A(\mathcal{B})|$ where $\lambda > 0$ indicates the magnitude of disutility.[32] Then consumer x will choose firm A in $\tau = 2$ if $P_A(x) + x^2 t + \lambda|P_A(x) - P_A(\mathcal{B})| \leq P_B(\mathcal{A}) + (1-x)^2 t$. The third term on the left-hand side means that whenever firm A cuts $P_A(\mathcal{B})$ to poach firm B's customers, firm A should also cut $P_A(x)$ to continue to serve its loyal customers. This increases the cost of poaching,

[32] By the definition of personalized prices, we assume that consumer $x \in \mathcal{A}$ cannot observe $P_A(x')$ for all $x' \in \mathcal{A}, x' \neq x$.

which should soften competition. Interestingly, in the extreme case where $\lambda \to \infty$, this model will become equivalent to the case where consumers can delete cookies because any meaningful poaching price is also what loyal customers would choose to pay when they can delete cookies. That is, firm A should offer $P_A(x) = P_A(\mathcal{B})$ to keep customer x. The types of responses by consumers to personalized pricing discussed above, whether strategic or behavioral, remain areas for future research.

Appendix A: Proofs

Proof of Lemma 3: Using π_A^2 in Lemma 2 and the location of marginal consumer z, we can express firm A's profit function as

$$\Pi_A = \begin{cases} P_A^1 \dfrac{(2-\delta_c)t - 2(P_A^1 - P_B^1)}{2(2-\delta_c)t} + \delta_f \dfrac{(2-\delta_c)^2 t^2 - 2(P_B^1 - P_A^1)^2}{4(2-\delta_c)^2 t} & \text{if } P_A^1 \geq P_B^1, \\ P_A^1 \dfrac{(2-\delta_c)t - 2(P_A^1 - P_B^1)}{2(2-\delta_c)t} + \delta_f \dfrac{[(2-\delta_c)t + P_B^1 - P_A^1]^2}{4(2-\delta_c)^2 t} & \text{if } P_A^1 \leq P_B^1. \end{cases}$$

The first-order conditions for profit maximization are then

$$\frac{\partial \Pi_A}{\partial P_A^1} = \begin{cases} \dfrac{2(2-\delta_c+\delta_f)P_B^1 - 2(4-2\delta_c+\delta_f)P_A^1 + (2-\delta_c)^2 t}{2(2-\delta_c)^2 t} = 0 & \text{if } P_A^1 \geq P_B^1, \\ \dfrac{(4-2\delta_c-\delta_f)P_B^1 - (8-4\delta_c-\delta_f)P_A^1 + (2-\delta_c)(2-\delta_c-\delta_f)t}{2(2-\delta_c)^2 t} = 0 & \text{if } P_A^1 \leq P_B^1. \end{cases}$$

From the first-order conditions, we have the following reaction function and the respective profit for firm A:

$$P_A^1(P_B^1) = \begin{cases} \tilde{P}_A^1(P_B^1) \equiv \dfrac{2(2-\delta_c+\delta_f)P_B^1 + (2-\delta_c)^2 t}{2(4-2\delta_c+\delta_f)} & \text{if } P_B^1 \leq \frac{(2-\delta_c)t}{2}, \\ \hat{P}_A^1(P_B^1) \equiv \dfrac{(4-2\delta_c-\delta_f)P_B^1 + (2-\delta_c)(2-\delta_c-\delta_f)t}{(8-4\delta_c-\delta_f)} & \text{if } P_B^1 \geq \frac{(2-\delta_c-\delta_f)t}{2}. \end{cases}$$

$$\Pi_A(P_B^1) = \begin{cases} \dfrac{4(P_B^1)^2 + 4(2-\delta_c+\delta_f)t(P_B^1) + [(2-\delta_c)^2 + 4(2-\delta_c)\delta_f + 2\delta_f^2]t^2}{8(4-2\delta_c+\delta_f)t} \\ \qquad\qquad\qquad\qquad \text{when } P_A^1(P_B^1) = \tilde{P}_A^1(P_B^1), \\ \dfrac{4(P_B^1)^2 + 2(4-2\delta_c+\delta_f)t(P_B^1) + (2-\delta_c)(2-\delta_c+2\delta_f)t^2}{4(8-4\delta_c-\delta_f)t} \\ \qquad\qquad\qquad\qquad \text{when } P_A^1(P_B^1) = \hat{P}_A^1(P_B^1). \end{cases}$$

For $P_B^1 \in [\frac{(2-\delta_c-\delta_f)t}{2}, \frac{(2-\delta_c)t}{2}]$, firm A has two local optimal prices, $\tilde{P}_A^1(P_B^1)$ and

$\hat{P}^1_A(P^1_B)$. Comparing firm A's profits for each case, we have

$$\begin{aligned}
& \frac{4(P^1_B)^2 + 4(2-\delta_c+\delta_f)t(P^1_B) + [(2-\delta_c)^2 + 4(2-\delta_c)\delta_f + 2\delta_f^2]t^2}{8(4-2\delta_c+\delta_f)t} \\
\geq\ & \frac{4(P^1_B)^2 + 2(4-2\delta_c+\delta_f)t(P^1_B) + (2-\delta_c)(2-\delta_c+2\delta_f)t^2}{4(8-4\delta_c-\delta_f)t} \\
\Leftrightarrow\ & P^1_B \leq \frac{\left(\sqrt{2[8(2-\delta)^2 + 2(2-\delta_c)\delta_f - \delta_f^2]} - (2-\delta_c+2\delta_f)\right)t}{6} \equiv \bar{P}^1.
\end{aligned}$$

One can verify that $\bar{P}^1 \in [\frac{(2-\delta_c-\delta_f)t}{2}, \frac{(2-\delta_c)t}{2}]$. Thus firm A's reaction function is

$$P^1_A(P^1_B) = \begin{cases} \tilde{P}^1_A(P^1_B) \equiv \dfrac{2(2-\delta_c+\delta_f)P^1_B + (2-\delta_c)^2 t}{2(4-2\delta_c+\delta_f)} & \text{if } P^1_B \leq \bar{P}^1, \\[2ex] \hat{P}^1_A(P^1_B) \equiv \dfrac{(4-2\delta_c-\delta_f)P^1_B + (2-\delta_c)(2-\delta_c-\delta_f)t}{(8-4\delta_c-\delta_f)} & \text{if } P^1_B \geq \bar{P}^1. \end{cases}$$

Applying the same argument to firm B's best response problem, we can derive its reaction function as:

$$P^1_B(P^1_A) = \begin{cases} \tilde{P}^1_B(P^1_A) \equiv \dfrac{2(2-\delta_c+\delta_f)P^1_A + (2-\delta_c)^2 t}{2(4-2\delta_c+\delta_f)} & \text{if } P^1_A \leq \bar{P}^1, \\[2ex] \hat{P}^1_B(P^1_A) \equiv \dfrac{(4-2\delta_c-\delta_f)P^1_A + (2-\delta_c)(2-\delta_c-\delta_f)t}{8-4\delta_c-\delta_f} & \text{if } P^1_A \geq \bar{P}^1. \end{cases}$$

Solving these reaction functions simultaneously, we can derive the equilibrium prices given in Lemma 3. □

Proof of Proposition 3: The total discounted profit given third-degree price discrimination is $\Pi_{FT} = (3+\delta_c)t/6 + \delta_f(5t/18)$. With personalized pricing, one firm has larger total discounted profit than the other. Denote the larger discounted profit by $\Pi_{PP} = \pi^1 + \delta_f \pi^2$. We will show $\Pi_{FT} \geq \Pi_{PP}$ for all (δ_c, δ_f).

First, consider the case $\delta_f = 0$. Then $\Pi_{FT} = (3+\delta_c)t/6 \geq t/2$ for all $\delta_c \geq 0$. From Proposition 1, we have $\pi^1 = (2-\delta_c)/4$, hence $\pi^1 \leq 1/2$ for all δ_c with equality when $\delta_c = 0$. Thus $\Pi_{FT} \geq \Pi_{PP}$ for all δ_c. Consider next the case $\delta_f \neq 0$. For each δ_f, Π_{FT} attains the minimum value when $\delta_c = 0$, which is denoted by $\Pi^{min}_{FT} = t/2 + \delta_f(5t/18)$. On the other hand, Π_{PP} attains the maximum value when $\delta_c = 0$, which we denote by Π^{max}_{PP}. It suffices to show $\Pi^{min}_{FT} \geq \Pi^{max}_{PP}$ for all δ_f. From Proposition 1, one can show that $\Pi^{min}_{FT} - \Pi^{max}_{PP}$ is strictly increasing in δ_f with $\Pi^{min}_{FT} - \Pi^{max}_{PP} = 0$ when $\delta_f = 0$. Thus $\Pi^{min}_{FT} > \Pi^{max}_{PP}$ for all $\delta_f \neq 0$. Combining the two cases, we have shown $\Pi_{FT} \geq \Pi_{PP}$ for all (δ_c, δ_f). □

Proof of Lemma 4: This is a special case of Proposition 4 when $\phi_A = 1$ and $\phi_B = 0$.

So the proof is provided at the end of the proof of Proposition 4. □

Proof of Proposition 4: We start with $\tau = 2$. In what follows, we denote personalized pricing by P and uniform price by U.

Suppose $z \leq 1/2$. Consider the segment $[0, z]$ first. If firm A uses P, then the prices are given by $P_A(x) = (1 - 2x)t$ and $P_B(\mathcal{A}) = 0$ as in our main model. If firm A uses U, then the indifferent consumer is $x = (t + P_B(\mathcal{A}) - P_A(\mathcal{A}))/2t$. The profits from this segment are $P_A(\mathcal{A})x$ for firm A and $P_B(\mathcal{A})(z-x)$ for firm B. Thus the prices are given by $P_A(\mathcal{A}) = (1+2z)t/3$ and $P_B(\mathcal{A}) = (4z-1)t/3$ (we ignore the case $z \leq 1/4$, which never happens on the equilibrium path). Consider next the segment $[z, 1]$. If firm B uses P, the prices are given by $P_A(\mathcal{B}) = (1-2z)t/2$ and $P_B(y) = (4y-2z-1)t/2$ as in our main model. If firm B uses U, then the indifferent consumer is $x = (t + P_B(\mathcal{B}) - P_A(\mathcal{B}))/2t$ and the profits from this segment are $P_A(\mathcal{B})(x - z)$ and $P_B(\mathcal{B})(1 - x)$. Thus the prices are $P_A(\mathcal{B}) = (3 - 4z)t/3$ and $P_B(\mathcal{B}) = (3 - 2z)t/3$.

Suppose next $z \geq 1/2$. On the segment $[0, z]$, if firm A uses P, the prices are given by $P_A(x) = (1 + 2z - 4x)t/2$ and $P_B(\mathcal{A}) = (2z - 1)t/2$ as in our main model. If firm A uses U, then the outcome was already derived above for the case $z \leq 1/2$, so the prices are $P_A(\mathcal{A}) = (1+2z)t/3$ and $P_B(\mathcal{A}) = (4z-1)t/3$. On the segment $[z, 1]$, if firm B uses P, the prices are given by $P_A(\mathcal{B}) = 0$ and $P_B(y) = (2y - 1)t$ as in our main model. If firm B uses U, then the outcome is the same as in the case $z \leq 1/2$, and the prices are $P_A(\mathcal{B}) = (3-4z)t/3$ and $P_B(\mathcal{B}) = (3-2z)t/3$ (again we ignore the case of $z \geq 3/4$ which never happens on the equilibrium path).

Based on the results, we derive below the location of the $\tau = 1$ marginal consumer z.

Suppose first $z \leq 1/2$. If z chooses firm A in $\tau = 1$, then in $\tau = 2$, z stays with firm A if and only if firm A uses P, regardless of firm B's pricing policy, and pays price $P_A(z) = (1-2z)t$. But z switches to firm B if and only if firm A uses U, again regardless of firm B's pricing policy, and pays the poaching price $P_B(\mathcal{A}) = (4z-1)t/3$. On the other hand, if z chooses firm B in $\tau = 1$, then in $\tau = 2$, z switches to firm A at the poaching price $P_A(\mathcal{B}) = (1 - 2z)t/2$ if and only if firm B uses P, and at $P_A(\mathcal{B}) = (3 - 4z)t/3$ if and only if firm B uses U. The indifference condition for z is then given by

$$\begin{aligned} & P_A^1 + tz^2 + \phi_A\left((1-2z)t + tz^2\right) + (1-\phi_A)\left(\frac{(4z-1)t}{3} + t(1-z)^2\right) \\ = \quad & P_B^1 + t(1-z)^2 + \phi_B\left(\frac{(1-2z)t}{2} + tz^2\right) + (1-\phi_B)\left(\frac{(3-4z)t}{3} + tz^2\right). \end{aligned}$$

The solution to the above, denoted by $\underline{z}$ is

$$\underline{z} = \frac{1}{2} + \frac{-3P_A^1 + 3P_B^1 + (\phi_A - \phi_B)t}{(8 - 4\phi_A - \phi_B)t}.$$

Note that $\underline{z} \leq 1/2$ if and only if $3P_B^1 \leq 3P_A^1 + (\phi_B - \phi_A)t$.

Suppose next $z \geq 1/2$. First, if z chooses firm B in $\tau = 1$, then z stays with firm B if and only if firm B uses P, and pays price $P_B(z) = (2z-1)t$. But z switches to firm A if and only if firm B uses U, and pays the poaching price $P_A(\mathcal{B}) = (3-4z)t/3$. On the other hand, if z chooses firm A in $\tau = 1$, then in $\tau = 2$, z switches to firm B at the poaching price $P_B(\mathcal{A}) = (2z-1)t/2$ if and only if firm A uses P, and at $P_B(\mathcal{A}) = (4z-1)t/3$ if and only if firm A uses U. Thus the indifference condition becomes

$$\begin{aligned}
& P_A^1 + tz^2 + \phi_A\left(\frac{(2z-1)t}{2} + t(1-z)^2\right) + (1-\phi_A)\left(\frac{(4z-1)t}{3} + t(1-z)^2\right) \\
= \;\; & P_B^1 + t(1-z)^2 + \phi_B\left((2z-1)t + t(1-z)^2\right) + (1-\phi_B)\left(\frac{(3-4z)t}{3} + tz^2\right).
\end{aligned}$$

The solution to the above, denoted by $\bar{z}$ is

$$\bar{z} = \frac{1}{2} + \frac{-3P_A^1 + 3P_B^1 + (\phi_A - \phi_B)t}{(8 - 4\phi_B - \phi_A)t}.$$

Note that $\bar{z} \geq 1/2$ if and only if $3P_B^1 \geq 3P_A^1 + (\phi_B - \phi_A)t$.

Given our assumption $\delta_f = 1$, each firm's total profit is then

$$\Pi_A = \begin{cases}
P_A^1\underline{z} + \Big\{\phi_A\phi_B\frac{(1+4\underline{z}-4\underline{z}^2)t}{8} + \phi_A(1-\phi_B)\left((1-\underline{z})\underline{z}t + \frac{(3-4\underline{z})^2t}{18}\right) & \\
\qquad + (1-\phi_A)\phi_B\left(\frac{(1+2\underline{z})^2t}{18} + \frac{(1-2\underline{z})^2t}{8}\right) & \text{if } \underline{z} \leq 1/2, \\
\qquad + (1-\phi_A)(1-\phi_B)\left(\frac{(1+2\underline{z})^2t}{18} + \frac{(3-4\underline{z})^2t}{18}\right)\Big\}, & \\
P_A^1\bar{z} + \Big\{\phi_A\phi_B\frac{(1+2\bar{z})^2t}{16} + \phi_A(1-\phi_B)\left(\frac{(1+2\bar{z})^2t}{16} + \frac{(3-4\bar{z})^2t}{18}\right) & \\
\qquad + (1-\phi_A)\phi_B\frac{(1+2\bar{z})^2t}{18} & \text{if } \bar{z} \geq 1/2, \\
\qquad + (1-\phi_A)(1-\phi_B)\left(\frac{(1+2\bar{z})^2t}{18} + \frac{(3-4\bar{z})^2t}{18}\right)\Big\}, &
\end{cases}$$

$$\Pi_B = \begin{cases}
P_B^1(1-\underline{z}) + \Big\{\phi_A\phi_B\frac{(3-2\underline{z})^2t}{16} + \phi_A(1-\phi_B)\frac{(3-2\underline{z})^2t}{18} & \\
\qquad + (1-\phi_A)\phi_B\left(\frac{(4\underline{z}-1)^2t}{18} + \frac{(3-2\underline{z})^2t}{16}\right) & \text{if } \underline{z} \leq 1/2, \\
\qquad + (1-\phi_A)(1-\phi_B)\left(\frac{(4\underline{z}-1)^2t}{18} + \frac{(3-2\underline{z})^2t}{18}\right)\Big\}, & \\
P_B^1(1-\bar{z}) + \Big\{\phi_A\phi_B\frac{(1+4\bar{z}-4\bar{z}^2)t}{8} + \phi_A(1-\phi_B)\left(\frac{(2\bar{z}-1)^2t}{8} + \frac{(3-2\bar{z})^2t}{18}\right) & \\
\qquad + (1-\phi_A)\phi_B\left(\frac{(4\bar{z}-1)^2t}{18} + \bar{z}(1-\bar{z})t\right) & \text{if } \bar{z} \geq 1/2. \\
\qquad + (1-\phi_A)(1-\phi_B)\left(\frac{(4\bar{z}-1)^2t}{18} + \frac{(3-2\bar{z})^2t}{18}\right)\Big\}, &
\end{cases}$$

We now solve for the equilibrium prices in $\tau = 1$. From the above profit functions, we can derive firm A's reaction function as follows:

$$P_A^1(P_B^1) = \begin{cases} \dfrac{t(64-40\phi_B+\phi_B^2-4(10-3\phi_B)\phi_A+12\phi_A^2)+4(2+5\phi_A+2\phi_B)P_B^1}{2(28-2\phi_A+\phi_B)}, \\ \qquad\qquad \text{if } P_B^1 \leq \dfrac{(24-10\phi_A-5\phi_B)t}{18}, \\ \dfrac{t(64-16\phi_A-\phi_A^2-(88-17\phi_A)\phi_B+24\phi_B^2)+(8+8\phi_B-7\phi_A)P_B^1}{(56-16\phi_B-13\phi_A)} \\ \qquad\qquad \text{if } P_B^1 \geq \dfrac{(12-5\phi_A-7\phi_B)t}{9}. \end{cases}$$

Expressing firm A's profit in terms of P_B^1 that reflects firm A's best response, we have

$$\Pi_A(P_B^1) = \begin{cases} \dfrac{t^2(1136-668\phi_B+9\phi_B^2-2(64-73\phi_B)\phi_A+72\phi_A^2)}{72(28-2\phi_A+\phi_B)t} \\ \qquad +\dfrac{72t(2+4\phi_A+3\phi_B)P_B^1+324(P_B^1)^2}{72(28-2\phi_A+\phi_B)t} \\ \qquad\qquad \text{if } P_B^1 \leq \dfrac{(24-10\phi_A-5\phi_B)t}{18}, \\ \dfrac{t^2(4(284-320\phi_B+81\phi_B^2)-2(109-136\phi_B)\phi_A+9\phi_A^2)}{36(56-16\phi_B-13\phi_A)t} \\ \qquad +\dfrac{18t(8+7\phi_A+12\phi_B)P_B^1+324(P_B^1)^2}{36(56-16\phi_B-13\phi_A)t} \\ \qquad\qquad \text{if } P_B^1 \geq \dfrac{(12-5\phi_A-7\phi_B)t}{9}. \end{cases}$$

For $P_B^1 \in [\frac{(12-5\phi_A-7\phi_B)t}{9}, \frac{(24-10\phi_A-\phi_B)t}{18}]$, firm A has two local optimal prices. Comparing firm A's profits from each case, we find that the profit when $P_A^1 \geq P_B^1+(\phi_A-\phi_B))t/3$ (hence $\underline{z} \leq 1/2$) is larger than or equal to that when $P_A^1 \leq P_B^1+(\phi_A-\phi_B))t/3$ (hence $\bar{z} \geq 1/2$) if and only if

$$P_B^1 \leq \left(\frac{\phi_B\sqrt{2(28-2\phi_A+\phi_B)(56-13\phi_A-16\phi_B)}}{18(\phi_A+2\phi_B)} - \frac{(10\phi_A^2-3(8-7\phi_B)\phi_A+4\phi_B(2+3\phi_B))}{18(\phi_A+2\phi_B)}\right) t \equiv P_{B\phi}.$$

One can verify that $P_{B\phi} \in [\frac{(12-5\phi_A-7\phi_B)t}{9}, \frac{(24-10\phi_A-\phi_B)t}{18}]$. Thus firm A's reaction function is

$$P_A^1(P_B^1) = \begin{cases} \dfrac{t(64-40\phi_B+\phi_B^2-4(10-3\phi_B)\phi_A+12\phi_A^2)+4(2+5\phi_A+2\phi_B)P_B^1}{2(28-2\phi_A+\phi_B)}, \\ \qquad\qquad \text{if } P_B^1 \leq P_{B\phi}, \\ \dfrac{t(64-16\phi_A-\phi_A^2-(88-17\phi_A)\phi_B+24\phi_B^2)+(8+8\phi_B-7\phi_A)P_B^1}{(56-16\phi_B-13\phi_A)} \\ \qquad\qquad \text{if } P_B^1 \geq P_{B\phi}. \end{cases}$$

Proceeding similarly, we can derive firm B's the reaction function:

$$P_B^1(P_A^1) = \begin{cases} \dfrac{t(64-16\phi_B-\phi_B^2-(88-17\phi_B)\phi_A+24\phi_A^2)+(8+8\phi_A-7\phi_B)P_A^1}{(56-16\phi_A-13\phi_B)}, \\ \hfill \text{if } P_A^1 \geq P_{A\phi}, \\ \dfrac{t(64-40\phi_A+\phi_A^2-4(10-3\phi_A)\phi_B+12\phi_B^2)+4(2+5\phi_B+2\phi_A)P_A^1}{2(28-2\phi_B+\phi_A)}, \\ \hfill \text{if } P_A^1 \leq P_{A\phi}, \end{cases}$$

where

$$\begin{aligned} P_{A\phi} \quad \equiv \quad & \Bigg(\frac{\phi_B\sqrt{2(28-2\phi_B+\phi_A)(56-13\phi_B-16\phi_A)}}{18(\phi_B+2\phi_A)} \\ & -\frac{(10\phi_B^2-3(8-7\phi_A)\phi_B+4\phi_A(2+3\phi_A))}{18(\phi_B+2\phi_A)}\Bigg)\, t. \end{aligned}$$

For $\underline{z} \leq 1/2$, solving the simultaneous equations, $P_A^1(P_B^1) = 0$ when $P_B^1 \leq P_{B\phi}$ and $P_B^1(P_A^1) = 0$ when $P_A^1 \geq P_{A\phi}$, we obtain

$$\begin{aligned} P_A^1 &= \frac{(512-272\phi_B+21\phi_B^2-4(20+7\phi_B)\phi_A+21\phi_B^2)t}{6(64+4\phi_A-5\phi_B)}, \\ P_B^1 &= \frac{(512-128\phi_B+9\phi_B^2-2(184-\phi_B)\phi_A)t}{6(64+4\phi_A-5\phi_B)}. \end{aligned}$$

If these prices satisfy $P_B^1 \leq P_{B\phi}$ and $P_A^1 \geq P_{A\phi}$, the price pair indeed constitutes equilibrium and $\underline{z} \leq 1/2$. The condition is

$$\begin{aligned} F(\phi_A,\phi_B) \quad \equiv \quad & (64+4\phi_A-5\phi_B)\phi_B\sqrt{2(28-2\phi_A+\phi_B)(56-13\phi_A-16\phi_B)} \\ & +2(28-2\phi_A+\phi_B)(10\phi_A^2-(64-15\phi_A)\phi_B+3\phi_B^2) \geq 0. \end{aligned}$$

The location of the $\tau = 1$ marginal consumer is then

$$\underline{z} = \frac{64-16\phi_A-3\phi_B}{2(64+4\phi_A-5\phi_B)}$$

and $\underline{z} \leq 1/2$ if and only if $10\phi_A \geq \phi_B$, which is satisfied if $F(\phi_A,\phi_B) \geq 0$.

For $\bar{z} \geq 1/2$, solving the simultaneous equations, $P_A^1(P_B^1) = 0$ when $P_B^1 \geq P_{B\phi}$ and $P_B^1(P_A^1) = 0$ when $P_A^1 \leq P_{A\phi}$, we obtain

$$\begin{aligned} P_A^1 &= \frac{(512-128\phi_A+9\phi_A^2-2(184-\phi_A)\phi_B)t}{6(64+4\phi_B-5\phi_A)}, \\ P_B^1 &= \frac{(512-272\phi_A+21\phi_A^2-4(20+7\phi_A)\phi_B-72\phi_B^2)t}{6(64+4\phi_B-5\phi_A)}. \end{aligned}$$

If these prices satisfy $P_B^1 \geq P_{B\phi}$ and $P_A^1 \leq P_{A\phi}$, the price pair constitutes equilibrium and $\bar{z} \geq 1/2$. The condition is

$$\begin{aligned} G(\phi_A, \phi_B) \equiv & \ (64 + 4\phi_B - 5\phi_A)\phi_A\sqrt{2(28 - 2\phi_B + \phi_A)(56 - 13\phi_B - 16\phi_A)} \\ & +2(28 - 2\phi_B + \phi_A)(10\phi_B^2 - (64 - 15\phi_B)\phi_A + 3\phi_A^2) \geq 0. \end{aligned}$$

The location of the $\tau = 1$ marginal consumer is then

$$\bar{z} = \frac{64 + 24\phi_B - 7\phi_A}{2(64 + 4\phi_B - 5\phi_A)}$$

and $\bar{z} \geq 1/2$ if and only if $10\phi_B \geq \phi_A$ which is satisfied if $G(\phi_A, \phi_B) \geq 0$.

From the above follows (i)—(iii). For (iv), it is easy to see that none of the above conditions are satisfied when $\phi_A = \phi_B = 0$. Therefore there is only a unique, symmetric equilibrium, which was shown in Section 3.4. For (v), let $\Pi_i(\phi_A, \phi_B)$ be firm i's total profit in equilibrium given (ϕ_A, ϕ_B). We have already shown $\Pi_A(0,0) > \Pi_A(1,0) > \Pi_B(1,0)$. One can check the same relation holds for any ϕ_B, hence $\partial\Pi_A/\partial\phi_A < 0$ for all ϕ_B, and similarly for firm B. Thus Π_i decreases monotonically in ϕ_i for all ϕ_j, $i, j = A, B$, which gives us (v).

For the proof of Lemma 4, we substitute $\phi_A = 1$ and $\phi_B = 0$ into $\underline{z}$, P_A, and P_B in the case of $\underline{z} \leq 1/2$, and obtain $\underline{z} = 6/17$, $P_A = 15t/17$, and $P_B = 6t/17$. From the first discussion in the proof of this proposition, we can obtain prices in $\tau = 2$ under $z \leq 1/2$. On the segment $[0, z]$, the prices are $P_A(x) = (1 - 2x)t$ and $P_B(\mathcal{A}) = 0$. On the segment $[z, 1]$, the prices are $P_A(\mathcal{B}) = (3 - 4z)t/3 = 9t/17$ and $P_B(\mathcal{B}) = (3 - 2z)t/3 = 13t/17$, and the indifferent consumer is $x = (t + P_B(\mathcal{B}) - P_A(\mathcal{B}))/2t = 21/34$. Thus consumers in $[6/17, 21/34]$ switch to firm A in $\tau = 2$ and those in $[21/34, 1]$ choose firm B in both periods. Finally, substituting $\phi_A = 1$, $\phi_B = 0$, and $\underline{z} = 6/17$ into Π_A and Π_B in the case of $\underline{z} \leq 1/2$, we obtain $\Pi_A = 393t/578$ and $\Pi_B = 301t/578$. □

Proof of Lemma 5: First, $x \in [0, z]$ chooses firm A if $P_A(x) + (x - a)^2t \leq P_B(\mathcal{A}) + (x - b)^2t$ or $P_A(x) \leq P_B(\mathcal{A}) + (a + b - 2x)(b - a)t$. Noting that $a + b \geq 2x$, the Bertrand competition on this segment leads to $P_A(x) = (a + b - 2x)(b - a)t$ and $P_B(\mathcal{A}) = 0$. Second, since $z \leq \frac{a+b}{2}$, firm A can serve additional customers on the segment $[z, \tilde{y}]$ where $\tilde{y}$ satisfies $P_A(\mathcal{B}) + (2\tilde{y} - a - b)(b - a)t = 0$ or $\tilde{y} = \frac{a+b}{2} - \frac{P_A(\mathcal{B})}{2(b-a)t}$. Firm A chooses $P_A(\mathcal{B})$ to maximize profit from this segment given by $(\tilde{y} - z)P_A(\mathcal{B})$. This leads to $P_A(\mathcal{B}) = \frac{(a+b-2z)(b-a)t}{2}$. Substituting $P_A(\mathcal{B})$ back into $\tilde{y}$, we obtain $\tilde{y} = \frac{a+b+2z}{4}$. On $[z, \tilde{y}]$, firm B's best response to $P_A(\mathcal{B})$ is $P_B(y) = 0$. Finally $y \in [\tilde{y}, 1]$ chooses firm B if $P_B(y) + (y - b)^2t \leq P_A(\mathcal{B}) + (y - a)^2t$. Thus firm B's optimal pricing on this segment is $P_B(y) = P_A(\mathcal{B}) + (2y - a - b)(b - a)t = \frac{(4y-2z-a-b)(b-a)t}{2}$.

Firm A's second-period profit in this equilibrium is $\pi_A^2 = \int_0^z (a + b - 2x)(b - a)tdx +$

$(\tilde{y}-z)P_A(\mathcal{B})$. Firm B's profit is $\int_{\tilde{y}}^{1} \frac{(4y-2z-a-b)(b-a)t}{2} dy$. Straightforward calculation leads to the desired results. □

Proof of Lemma 6: The same argument used in Lemma 5 applies. □

Proof of Lemma 7: Substituting $z = ((b^2-a^2)t + P_B^1 - P_A^1)/(2(b-a)t)$ into firm A's profit function, we have

$$\Pi_A = \begin{cases} P_A^1 \dfrac{P_B^1 - P_A^1 + (b^2-a^2)t}{2(b-a)t} + \dfrac{(b-a)t((b+a)(a+b+4z)-4z^2)}{8} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } P_A^1 \le P_B^1, \\ P_A^1 \dfrac{P_B^1 - P_A^1 + (b^2-a^2)t}{2(b-a)t} + \dfrac{(b-a)t(a+b+2z)^2}{16} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } P_A^1 \ge P_B^1. \end{cases}$$

The first-order conditions for profit maximization are

$$\frac{\partial \Pi_A}{\partial P_A^1} = \begin{cases} \dfrac{3P_B^1 - 7P_A^1 + 2(b^2-a^2)t}{8(b-a)t} & \text{if } P_A^1 \le P_B^1, \\ \dfrac{3P_B^1 - 5P_A^1 + 2(b^2-a^2)t}{4(b-a)t} & \text{if } P_A^1 \ge P_B^1. \end{cases}$$

From the above first-order conditions, we have the following local optimal prices and respective profit of firm A:

$$\begin{aligned} P_A^1(P_B^1) &= \begin{cases} \tilde{P}_A^1(P_B^1) \equiv \dfrac{3P_B^1 + 2(b^2-a^2)t}{5} & \text{if } P_B^1 \le (b-a)(b+a)t, \\ \hat{P}_A^1(P_B^1) \equiv \dfrac{3P_B^1 + 2(b^2-a^2)t}{7} & \text{if } P_B^1 \ge (b-a)(b+a)t/2. \end{cases} \\ \Pi_A(P_B^1) &= \begin{cases} \dfrac{2(P_B^1)^2 + 6(b^2-a^2)tP_B^1 + 7(b^2-a^2)^2t^2}{20(b-a)t} & \text{when } P_A^1(P_B^1) = \tilde{P}_A^1(P_B^1), \\ \dfrac{2(P_B^1)^2 + 5(b^2-a^2)tP_B^1 + 4(b^2-a^2)^2t^2}{14(b-a)t} & \text{when } P_A^1(P_B^1) = \hat{P}_A^1(P_B^1). \end{cases} \end{aligned}$$

For $P_B^1 \in [(b-a)(b+a)t/2, (b-a)(b+a)t]$, firm A has two local optimal prices, $\tilde{P}_A^1(P_B^1)$ and $\hat{P}_A^1(P_B^1)$. Comparing the profits from each case, we have

$$\begin{aligned} &\frac{2(P_B^1)^2 + 6(b^2-a^2)tP_B^1 + 7(b^2-a^2)^2t^2}{20(b-a)t} \ge \frac{2(P_B^1)^2 + 5(b^2-a^2)tP_B^1 + 4(b^2-a^2)^2t^2}{14(b-a)t} \\ \Leftrightarrow\ & P_B^1 \le \frac{(\sqrt{70}-4)(b^2-a^2)t}{6}. \end{aligned}$$

Thus firm A's reaction function is given by

$$P_A^1(P_B^1) = \begin{cases} \tilde{P}_A^1(P_B^1) = \dfrac{3P_B^1 + 2(b^2 - a^2)t}{5} & \text{if } P_B^1 \leq \dfrac{(\sqrt{70} - 4)(b^2 - a^2)t}{6}, \\ \hat{P}_A^1(P_B^1) = \dfrac{3P_B^1 + 2(b^2 - a^2)t}{7} & \text{if } P_B^1 \geq \dfrac{(\sqrt{70} - 4)(b^2 - a^2)t}{6}. \end{cases}$$

Applying the same argument to firm B's optimization problem, we can derive firm B's reaction function:

$$P_B^1(P_A^1) = \begin{cases} \tilde{P}_B^1(P_A^1) \equiv \dfrac{3P_A^1 + 2(b-a)(2-(a+b))t}{5} \\ \qquad\qquad \text{if } P_A^1 \leq \dfrac{(\sqrt{70} - 4)(b-a)(2-(a+b))t}{6}, \\ \hat{P}_B^1(P_A^1) \equiv \dfrac{3P_A^1 + 2(b-a)(2-(a+b))t}{7} \\ \qquad\qquad \text{if } P_A^1 \geq \dfrac{(\sqrt{70} - 4)(b-a)(2-(a+b))t}{6}. \end{cases}$$

Using the above reaction functions, we derive the equilibrium prices. First, we solve for the intersection of the following two reaction functions:

$$\begin{cases} P_A^1(P_B^1) = \dfrac{3P_B^1 + 2(b^2 - a^2)t}{5} & \text{if } \ P_B^1 \leq \dfrac{(\sqrt{70} - 4)(b^2 - a^2)t}{6}, \\ P_B^1(P_A^1) = \dfrac{3P_A^1 + 2(b-a)(2-(a+b))t}{7} & \text{if } \ P_A^1 \geq \dfrac{(\sqrt{70} - 4)(b-a)(2-(a+b))t}{6}. \end{cases}$$

The resulting prices and the value of z are given as

$$(P_A^{1*}, P_B^{1*}) = \left(\frac{2(b-a)(3+2(a+b))t}{13}, \frac{2(b-a)(5-(a+b))t}{13}\right), \quad z^* = \frac{4+7(a+b)}{26}.$$

The prices satisfy the above two inequalities if and only if

$$a + b \geq \frac{2(13\sqrt{70} - 70)}{13\sqrt{70} - 28} \simeq 0.96.$$

Second, we solve for the intersection of the following two reaction functions:

$$\begin{cases} P_A^1(P_B^1) = \dfrac{3P_B^1 + (b^2 - a^2)2t}{7} & \text{if } \ P_B^1 \geq \dfrac{(\sqrt{70} - 4)(b^2 - a^2)t}{6}, \\ P_B^1(P_A^1) = \dfrac{3P_A^1 + 2(b-a)(2-(a+b))t}{5} & \text{if } \ P_A^1 \leq \dfrac{(\sqrt{70} - 4)(b-a)(2-(a+b))t}{6}. \end{cases}$$

The resulting prices and the realized value of z are

$$(P_A^{1*}, P_B^{1*}) = \left(\frac{2(b-a)(3+(a+b))t}{13}, \frac{2(b-a)(7-2(a+b))t}{13}\right), \ z^* = \frac{8+7(a+b)}{26}.$$

The prices satisfy the above inequalities if and only if

$$a + b \leq \frac{84}{13\sqrt{70} - 28} \simeq 1.04.$$

Combining these two cases gives us Lemma 7. $\square$

Proof of Lemma 8: Firm A's profit is $\Pi_A = P_A^1 z + \pi_A^2$ and firm B's profit is $\Pi_B = P_B^1(1-z) + \pi_B^2$. If $a + b > 84/(13\sqrt{70} - 28)$, then Lemma 6 shows that the $\tau = 1$ pricing subgame has a unique equilibrium with $z = \frac{4+7a+7b}{26} < \frac{a+b}{2}$, which we call E1. If $a + b < 2(13\sqrt{70} - 70)/(13\sqrt{70} - 28)$, then the $\tau = 1$ pricing game has a unique equilibrium with $z = \frac{8+7a+7b}{26} > \frac{a+b}{2}$, which we call E2. If $84/(13\sqrt{70} - 28) \geq a + b \geq 2(13\sqrt{70} - 70)/(13\sqrt{70} - 28)$, both E1 and E2 are possible. In this case, each firm's location choice depends on which of the two pricing equilibria they expect in the subgame. Given that E1 follows when $a + b > 84/(13\sqrt{70} - 28)$ and E2 follows when $a + b < 2(13\sqrt{70} - 70)/(13\sqrt{70} - 28)$, by continuity we assume that both firms expect E1 if and only if $a + b > k$ for some $k \in (2(13\sqrt{70} - 70)/(13\sqrt{70} - 28), 84/(13\sqrt{70} - 28))$. In what follows, we assume $k = 1$. But it is easy to verify that our argument applies for any $k \in (2(13\sqrt{70} - 70)/(13\sqrt{70} - 28), 84/(13\sqrt{70} - 28))$. Given $k = 1$ and the stipulated expectation, each firm's profit function can be written as

$$\Pi_A = \begin{cases} \dfrac{(b-a)\left[207(a+b)^2 + 140(a+b) + 40\right]}{676} & \text{if } a+b > 1, \\ \dfrac{(b-a)\left[32(a+b)^2 + 49(a+b) + 28\right]}{169} & \text{if } a+b \leq 1, \end{cases}$$

$$\Pi_B = \begin{cases} \dfrac{(b-a)\left[32(a+b)^2 - 177(a+b) + 254\right]}{169} & \text{if } a+b > 1, \\ \dfrac{(b-a)\left[207(a+b)^2 - 968(a+b) + 1148\right]}{676} & \text{if } a+b \leq 1, \end{cases}$$

with the corresponding derivatives

$$\frac{\partial \Pi_A}{\partial a} = \begin{cases} \dfrac{-621a^2 - (280 + 414b)a - (40 - 207b^2)}{676} & \text{if } a+b > 1, \\ \dfrac{-2\left[14 - 16b^2 + (49 + 32b)a + 48a^2\right]}{169} & \text{if } a+b \leq 1, \end{cases}$$

$$\frac{\partial \Pi_B}{\partial b} = \begin{cases} \dfrac{2\left[127 - 16a^2 - (177 - 32a)b + 48b^2\right]}{169} & \text{if } a+b > 1, \\ \dfrac{1148 - 207a^2 - (1936 - 414a)b + 621b^2}{676} & \text{if } a+b \leq 1. \end{cases}$$

Solving the above, we have the following candidate equilibria:

$$\begin{cases} a = \dfrac{2\sqrt{56029} - 347}{621} \simeq 0.2, \ b = 1 & \text{if } a + b > 1, \\ a = 0, \ b = \dfrac{968 - 2\sqrt{56029}}{621} \simeq 0.8 & \text{if } a + b \leq 1. \end{cases}$$

The first equilibrium is consistent with E1 since $a + b > 84/(13\sqrt{70} - 28)$ in E1. Given $b = 1$, firm A's best response problem is over the entire range of $[0, 1]$. Thus firm A does not have an incentive to deviate from $a \simeq 0.2$. On the other hand, firm B may choose to deviate by locating at b such that E2 is realized in the pricing subgame. However we can easily show that firm B does not have an incentive to change its location: plotting Π_B given $a = 0.2$ shows that Π_B is indeed maximized when $b = 1$. Thus $a = 0.2, b = 1$ constitute an equilibrium. Similarly one can verify that $a = 0, b = 0.8$ also constitute an equilibrium, which is followed by E2 in the pricing subgame. $\square$

Appendix B: Endogenous location choice with general discount factors

This appendix provides analysis of equilibria under endogenous location choice and general discount factors. Since full characterization of equilibria for all values of (δ_c, δ_f) is not analytically possible, we establish partial characterization of the conditions under which (i) maximal differentiation is an equilibrium outcome and (ii) pure-strategy equilibria exist. We also show that pure-strategy equilibria do not exist when $\delta_c = \delta_f = 1$.

Since the $\tau = 2$ equilibria do not depend on the discount factors, our results in the main text continue to hold. To analyze the $\tau = 1$ equilibria, we start with the marginal consumer's location in $\tau = 1$. First, consider the equilibrium with $z \leq \frac{a+b}{2}$. Then consumer z is indifferent between choosing firm A in both periods, and choosing firm B in $\tau = 1$ but switching to firm A in $\tau = 2$. Thus we have

$$P_A^1 + (z-a)^2 t + \delta_c(P_A(z) + (z-a)^2 t) = P_B^1 + (b-z)^2 t + \delta_c(P_A(\mathcal{B}) + (z-a)^2 t).$$

Substituting $P_A(z) = (a+b-2z)(b-a)t$ and $P_A(\mathcal{B}) = \frac{(a+b-2z)(b-a)t}{2}$ in Lemma 5 into the above, we obtain

$$z = \frac{a+b}{2} + \frac{P_B^1 - P_A^1}{(b-a)(2-\delta_c)t}$$

where $z \leq \frac{a+b}{2}$ if and only if $P_A^1 \geq P_B^1$. Similarly, in the equilibrium with $z \geq \frac{a+b}{2}$, the marginal consumer z is indifferent between choosing firm B in both periods, and choosing firm A in $\tau = 1$ while switching to firm B in $\tau = 2$. Proceeding as before, we again obtain the same z derived above.

Substituting z into firm A's profit function, $\Pi_A = P_A^1 z + \delta_f \pi_A^2$, we obtain

$$\Pi_A = \begin{cases} P_A^1\left(\dfrac{a+b}{2} + \dfrac{P_B^1 - P_A^1}{(b-a)(2-\delta_c)t}\right) + \delta_f \dfrac{(b^2-a^2)^2(2-\delta_c)^2t^2 - 2(P_B^1 - P_A^1)^2}{4(b-a)(2-\delta_c)^2 t} & \text{if } P_A^1 \geq P_B^1, \\ P_A^1\left(\dfrac{a+b}{2} + \dfrac{P_B^1 - P_A^1}{(b-a)(2-\delta_c)t}\right) + \delta_f \dfrac{((b^2-a^2)(2-\delta_c)t + P_B^1 - P_A^1)^2}{4(b-a)(2-\delta_c)^2 t} & \text{if } P_A^1 \leq P_B^1. \end{cases}$$

The first-order conditions for profit maximization are then

$$\frac{\partial \Pi_A}{\partial P_A^1} = \begin{cases} \dfrac{2(2-\delta_c+\delta_f)P_B^1 - 2(4-2\delta_c+\delta_f)P_A^1 + (b^2-a^2)(2-\delta_c)^2 t}{2(b-a)(2-\delta_c)^2 t} = 0 & \text{if } P_A^1 \geq P_B^1, \\ \dfrac{(4-2\delta_c-\delta_f)P_B^1 - (8-4\delta_c-\delta_f)P_A^1 + (b^2-a^2)(2-\delta_c)(2-\delta_c-\delta_f)t}{2(b-a)(2-\delta_c)^2 t} = 0 & \text{if } P_A^1 \leq P_B^1. \end{cases}$$

Using the first-order conditions, we can derive the following reaction function and the respective profit for firm A:

$$P_A^1(P_B^1) = \begin{cases} \tilde{P}_A^1(P_B^1) \equiv \dfrac{2(2-\delta_c+\delta_f)P_B^1 + (b^2-a^2)(2-\delta_c)^2 t}{2(4-2\delta_c+\delta_f)} \\ \hfill \text{if } P_B^1 \leq \frac{(b^2-a^2)(2-\delta_c)t}{2}, \\ \hat{P}_A^1(P_B^1) \equiv \dfrac{(4-2\delta_c-\delta_f)P_B^1 + (b^2-a^2)(2-\delta_c)(2-\delta_c-\delta_f)t}{(8-4\delta_c-\delta_f)} \\ \hfill \text{if } P_B^1 \geq \frac{(b^2-a^2)(2-\delta_c-\delta_f)t}{2}. \end{cases}$$

$$\Pi_A(P_B^1) = \begin{cases} \dfrac{4(P_B^1)^2 + 4(b^2-a^2)(2-\delta_c+\delta_f)t(P_B^1)}{8(4-2\delta_c+\delta_f)t} \\ \quad + \dfrac{(b^2-a^2)^2((2-\delta_c)^2 + 4(2-\delta_c)\delta_f + 2\delta_f^2)t^2}{8(4-2\delta_c+\delta_f)t} & \text{when } P_A^1(P_B^1) = \tilde{P}_A^1(P_B^1), \\ \dfrac{4(P_B^1)^2 + 2(b^2-a^2)(4-2\delta_c+\delta_f)t(P_B^1)}{4(b-a)(8-4\delta_c-\delta_f)t} \\ \quad + \dfrac{(b^2-a^2)^2(2-\delta_c)(2-\delta_c+2\delta_f)t^2}{4(b-a)(8-4\delta_c-\delta_f)t} & \text{when } P_A^1(P_B^1) = \hat{P}_A^1(P_B^1). \end{cases}$$

For $P_B^1 \in [\frac{(b^2-a^2)(2-\delta_c-\delta_f)t}{2}, \frac{(b^2-a^2)(2-\delta_c)t}{2}]$, firm A has two local optimal prices, $\tilde{P}_A^1(P_B^1)$ and $\hat{P}_A^1(P_B^1)$. Comparing firm A's profits for each case, we find that the former is larger than the latter if and only if

$$P_B^1 \leq \frac{(b^2-a^2)\left(\sqrt{2(8(2-\delta_c)^2 + 2(2-\delta_c)\delta_f - \delta_f^2)} - (2-\delta_c+2\delta_f)\right)t}{6} \equiv \bar{P}_E^1.$$

One can easily verify $\bar{P}_E^1 \in [\frac{(b^2-a^2)(2-\delta_c-\delta_f)t}{2}, \frac{(b^2-a^2)(2-\delta_c)t}{2}]$. Thus, firm A's reaction function is

$$P_A^1(P_B^1) = \begin{cases} \tilde{P}_A^1(P_B^1) \equiv \dfrac{2(2-\delta_c+\delta_f)P_B^1 + (b^2-a^2)(2-\delta_c)^2 t}{2(4-2\delta_c+\delta_f)} \\ \hfill \text{if } P_B^1 \leq \bar{P}_E^1, \\ \hat{P}_A^1(P_B^1) \equiv \dfrac{(4-2\delta_c-\delta_f)P_B^1 + (b^2-a^2)(2-\delta_c)(2-\delta_c-\delta_f)t}{(8-4\delta_c-\delta_f)} \\ \hfill \text{if } P_B^1 \geq \bar{P}_E^1. \end{cases}$$

Applying the same argument to firm B's best response problem, we can derive its reaction function as:

$$P_B^1(P_A^1) \quad = \quad \begin{cases} \tilde{P}_B^1(P_A^1) \equiv \dfrac{2(2-\delta_c+\delta_f)P_A^1+(b-a)(2-a-b)(2-\delta_c)^2 t}{2(4-2\delta_c+\delta_f)} \\ \hfill \text{if } P_A^1 \leq \bar{\bar{P}}_E^1, \\ \hat{P}_B^1(P_A^1) \equiv \dfrac{(4-2\delta_c-\delta_f)P_A^1+(b-a)(2-a-b)(2-\delta_c)(2-\delta_c-\delta_f)t}{8-4\delta_c-\delta_f} \\ \hfill \text{if } P_A^1 \geq \bar{\bar{P}}_E^1, \end{cases}$$

where

$$\bar{\bar{P}}_E^1 \equiv \frac{(b-a)(2-a-b)\left(\sqrt{2(8(2-\delta_c)^2+2(2-\delta_c)\delta_f-\delta_f^2)}-(2-\delta_c+2\delta_f)\right)t}{6}.$$

Solving the above reaction functions simultaneously, we can derive the equilibrium prices

$$\begin{aligned}
(i) \qquad & P_A^1 = \frac{(b-a)\left(2(2+a+b)(2-\delta_c)^2-(a+b)(2-\delta_c)\delta_f-2(2-a-b)\delta_f^2\right)t}{2(12-6\delta_c+\delta_f)}, \\
& P_B^1 = \frac{(b-a)\left(2(4-a-b)(2-\delta_c)^2-(4-a-b)(2-\delta_c)\delta_f-2(2-a-b)\delta_f^2\right)t}{2(12-6\delta_c+\delta_f)}, \\
& \text{with } z = \frac{2(2+a+b)(2-\delta_c)-(4-3a-3b)\delta_f}{2(12-6\delta_c+\delta_f)}; \\
(ii) \qquad & P_A^1 = \frac{(b-a)\left(2(2+a+b)(2-\delta_c)^2-(2+a+b)(2-\delta_c)\delta_f-2(a+b)\delta_f^2\right)t}{2(12-6\delta_c+\delta_f)}, \\
& P_B^1 = \frac{(b-a)\left(2(4-a-b)(2-\delta_c)^2-(2-a-b)(2-\delta_c)\delta_f-2(a+b)\delta_f^2\right)t}{2(12-6\delta_c+\delta_f)}, \\
& \text{with } z = \frac{2(2+a+b)(2-\delta_c)+3(a+b)\delta_f}{2(12-6\delta_c+\delta_f)}.
\end{aligned}$$

As in Lemma 6, we need to specify the condition under which the above equilibria can exist. First, if $P_A^1 \geq \bar{\bar{P}}_E^1$ and $P_B^1 \leq \bar{P}_E^1$, then the equilibrium in (i) exists. This condition can be written as

$$\begin{aligned}
a+b \geq & \frac{2\left(4\delta_f^2-13(2-\delta_c)\delta_f-12(2-\delta_c)^2\right)}{4\delta_f^2-16(2-\delta_c)\delta_f+(6(2-\delta_c)+\delta_f)\sqrt{2(8(2-\delta_c)^2+2(2-\delta_c)\delta_f-\delta_f^2)}} \\
& + \frac{2\left((6(2-\delta_c)+\delta_f)\sqrt{2(8(2-\delta_c)^2+2(2-\delta_c)\delta_f-\delta_f^2)}\right)}{4\delta_f^2-16(2-\delta_c)\delta_f+(6(2-\delta_c)+\delta_f)\sqrt{2(8(2-\delta_c)^2+2(2-\delta_c)\delta_f-\delta_f^2)}} \equiv \underline{a+b}.
\end{aligned}$$

Second, if $P_A^1 \leq \bar{\bar{P}}_E^1$ and $P_B^1 \geq \bar{P}_E^1$, then the equilibrium in (ii) exists, of which condition can be expressed as

$$a+b \leq \frac{6(2-\delta_c)(4(2-\delta_c)-\delta_f)}{4\delta_f^2-16(2-\delta_c)\delta_f+(6(2-\delta_c)+\delta_f)\sqrt{2(8(2-\delta_c)^2+2(2-\delta_c)\delta_f-\delta_f^2)}} \equiv \overline{a+b}.$$

Finally if $a+b \in [\underline{a+b}, \overline{a+b}]$, then both equilibria exist. When $\delta_f = 1$, the two threshold values can be shown to be equal to $\underline{a+b} \approx 0.96$ and $\overline{a+b} \approx 1.04$, as in Lemma 6.

Substituting the value of z from the $\tau = 1$ equilibrium back into the $\tau = 2$ prices in Lemma 5, we find the $\tau = 2$ prices in Case (i):

$$P_A(x) = \begin{cases} (a+b-2x)(b-a)t \\ \qquad \text{if } x \in \left[0, \dfrac{2(2+a+b)(2-\delta_c)-(4-3a-3b)\delta_f}{2(12-6\delta_c+\delta_f)}\right], \\ \dfrac{(b-a)((2-a-b)\delta_f-2(1-a-b)(2-\delta_c))t}{12-6\delta_c+\delta_f} \\ \qquad \text{if } x \in \left[\dfrac{2(2+a+b)(2-\delta_c)-(4-3a-3b)\delta_f}{2(12-6\delta_c+\delta_f)}, 1\right], \end{cases}$$

$$P_B(y) = \begin{cases} 0 \qquad \text{if } y \in \left[0, \dfrac{(1+2a+2b)(2-\delta_c)-(1-a-b)\delta_f}{12-6\delta_c+\delta_f}\right], \\ 2(b-a)\left(y - \dfrac{(1+2a+2b)(2-\delta_c)-(1-a-b)\delta_f}{12-6\delta_c+\delta_f}\right)t \\ \qquad \text{if } y \in \left[\dfrac{(1+2a+2b)(2-\delta_c)-(1-a-b)\delta_f}{12-6\delta_c+\delta_f}, 1\right]. \end{cases}$$

Similarly, by substituting the value of z from the $\tau = 1$ equilibrium back into the $\tau = 2$ prices in Lemma 5, we find the $\tau = 2$ prices in Case (ii):

$$P_A(x) = \begin{cases} 2(b-a)\left(\dfrac{(1+2a+2b)(2-\delta_c)+(a+b)\delta_f}{12-6\delta_c+\delta_f} - x\right)t \\ \qquad \text{if } x \in \left[0, \dfrac{(1+2a+2b)(2-\delta_c)+(a+b)\delta_f}{12-6\delta_c+\delta_f}\right], \\ 0 \qquad \text{if } x \in \left[\dfrac{(1+2a+2b)(2-\delta_c)+(a+b)\delta_f}{12-6\delta_c+\delta_f}, 1\right], \end{cases}$$

$$P_B(y) = \begin{cases} \dfrac{(b-a)((a+b)\delta_f+2(1-a-b)(2-\delta_c))t}{12-6\delta_c+\delta_f} \\ \qquad \text{if } y \in \left[0, \dfrac{2(2+a+b)(2-\delta_c)+3(a+b)\delta_f}{2(12-6\delta_c+\delta_f)}\right], \\ (2y-a-b)(b-a)t \\ \qquad \text{if } y \in \left[\dfrac{2(2+a+b)(2-\delta_c)+3(a+b)\delta_f}{2(12-6\delta_c+\delta_f)}, 1\right]. \end{cases}$$

The profits are then

$$\Pi_A = \begin{cases} \dfrac{(b-a)t}{4(6(2-\delta_c)+\delta_f)^2} \\ \quad \times\{4(2+a+b)^2(2-\delta_c)^3 + 8(4(a+b)^2+2(a+b)-3)(2-\delta_c)^2\delta_f \\ \qquad +(a+b)(21(a+b)-20)(2-\delta_c)\delta_f^2 + (5(a+b)^2-12(a+b)+8)\delta_f^3\} \\ \hfill \text{in Case } (i), \\ \dfrac{(b-a)t}{4(6(2-\delta_c)+\delta_f)^2} \\ \quad \times\{4(2+a+b)^2(2-\delta_c)^3 + 4(5(a+b)^2+5(a+b)-1)(2-\delta_c)^2\delta_f \\ \qquad +3(a+b)(3(a+b)-2)(2-\delta_c)\delta_f^2 - 2(a+b)^2\delta_f^3\} \\ \hfill \text{in Case } (ii), \end{cases}$$

$$\Pi_B = \begin{cases} \dfrac{(b-a)t}{4(6(2-\delta_c)+\delta_f)^2} \\ \quad \times\{4(4-a-b)^2(2-\delta_c)^3 + 4(5(a+b)^2-25(a+b)+29)(2-\delta_c)^2\delta_f \\ \qquad +3(2-a-b)(4-3(a+b))(2-\delta_c)\delta_f^2 - 2(2-a-b)^2\delta_f^3\} \\ \hfill \text{in Case } (i), \\ \dfrac{(b-a)t}{4(6(2-\delta_c)+\delta_f)^2} \\ \quad \times\{4(4-a-b)(2-\delta_c)^3 + 8(4(a+b)^2-18(a+b)+17)(2-\delta_c)^2\delta_f \\ \qquad +(2-a-b)(22-21(a+b))(2-\delta_c)\delta_f^2 + (5(a+b)^2-8(a+b)+4)\delta_f^3\} \\ \hfill \text{in Case } (ii). \end{cases}$$

Based on the above profit functions, we now check the conditions for maximal differentiation to be an equilibrium outcome. Let us focus on Case (i). The conditions for Case (ii) are the same since these two cases are a mirror image of each other. By differentiating the above profit functions, we obtain

$$\begin{aligned} \frac{\partial \Pi_A}{\partial a} = & \frac{t}{4(6(2-\delta_c)+\delta_f)^2} \\ & \times\{4(2+a+b)(2+3a-b)(2-\delta_c)^3 - 8(12a^2+4(1+2b)a-(3+4b^2))(2-\delta_c)^2\delta_f \\ & -(63a^2-2(20-21b)a-21b^2)(2-\delta_c)\delta_f^2 - (15a^2-2(12-5b)a+(8-5b^2))\delta_f^3\}, \end{aligned}$$

$$\begin{aligned} \frac{\partial \Pi_B}{\partial b} = & \frac{t}{4(6(2-\delta_c)+\delta_f)^2} \\ & \times\{4(4-a-b)(4+a-3b)(2-\delta_c)^3 - 4(15b^2-10(5-a)b+29-5a^2)(2-\delta_c)^2\delta_f \\ & +3(9b^2-2(10-3a)b+8-3a^2)(2-\delta_c)\delta_f^2 + 2(2-a-b)(3b-2-a)\delta_f^3\}, \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 \Pi_A}{\partial a^2} = & \frac{-t}{2(6(2-\delta_c)+\delta_f)^2} \\ & \times\{(4(2-\delta_c)^3 + 32(2-\delta_c)^2\delta_f + 21(2-\delta_c)\delta_f^2 + 5\delta_f^3)(3a+b) \\ & +4(4-2\delta_c+3\delta_f)(4-2\delta_c+\delta_f)(2-\delta_c-\delta_f)\}, \end{aligned}$$

$$\frac{\partial^2 \Pi_B}{\partial b^2} = \frac{t}{4(6(2-\delta_c)+\delta_f)^2}$$
$$\times\{2(4(2-\delta_c)^3 + 20(2-\delta_c)^2\delta_f + 9(2-\delta_c)\delta_f^2 - 2\delta_f^3)(a+3b)$$
$$-(64(2-\delta_c)^3 + 200(2-\delta_c)^2\delta_f + 60(2-\delta_c)\delta_f^2 - 16\delta_f^3)\}.$$

Notice that $\partial^2\Pi_A/\partial a^2 < 0$ for all (δ_c, δ_f). It is also easy to see $\partial^2\Pi_B/\partial b^2 < 0$ for all (δ_c, δ_f) since $\partial^2\Pi_B/\partial b^2$ increases in a and b, and $\partial^2\Pi_B/\partial b^2|_{a=b=1} < 0$. Thus each profit function is strictly concave with respect to the relevant location choice so the first-order condition is necessary and sufficient.

Suppose now firm B's location choice is $b = 1$. We will find the conditions under which $a = 0$ and $b = 1$ constitute a Nash equilibrium. Firm A's best response to $b = 1$, denoted by $a(b = 1)$, is given by

$$a(b=1) = \max\{0, \tilde{a}\}$$
$$\text{where } \tilde{a} \equiv \frac{2\sqrt{H} - (20(2-\delta_c)^3 + 48(2-\delta_c)^2\delta_f + (2-\delta_c)\delta_f^2 - 7\delta_f^3)}{12(2-\delta_c)^3 + 96(2-\delta_c)^2\delta_f + 63(2-\delta_c)\delta_f^2 + 15\delta_f^3}$$

and $H \equiv 64(2-\delta_c)^6 + 360(2-\delta_c)^5\delta_f + 1804(2-\delta_c)^4\delta_f^2 + 1286(2-\delta_c)^3\delta_f^3 + 301(2-\delta_c)^2\delta_f^4 + 28(2-\delta_c)\delta_f^5 + \delta_f^6$. In the figures below, we describe firm A's best response problem.

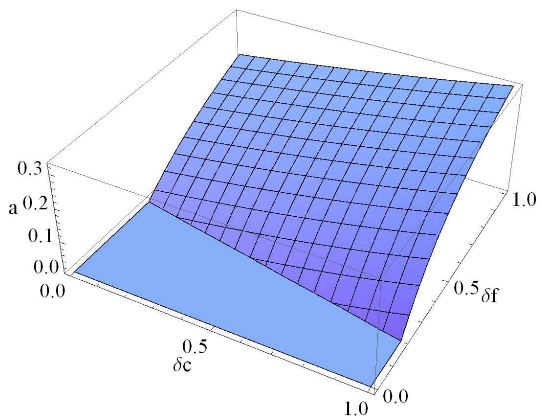

Fig. 1: $a(b = 1)$

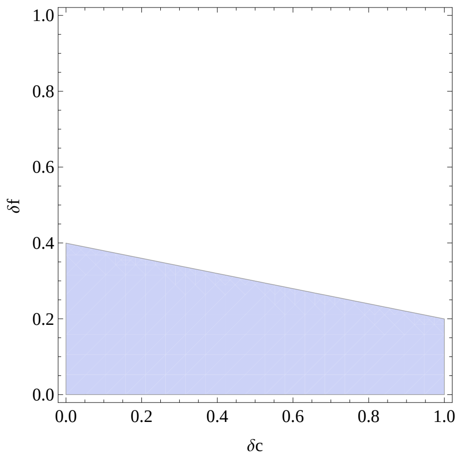

Fig. 2: The region where $a(b = 1) = 0$

Fig. 1 plots $a(b = 1)$ against (δ_c, δ_f) and shows that firm A's best response to $b = 1$ varies within the range of 0 and 0.3 as δ_c and δ_f change. Fig. 2 corresponds to the flat part of Fig. 1 and shows the range of (δ_c, δ_f) in which $\tilde{a} \le 0$, hence $a = 0$ is firm A's best response to $b = 1$. The threshold value of δ_f at which $\delta_c = 0$ ($\delta_c = 1$, resp.) is roughly 0.4 (0.2, resp.). Thus $a = 0$ is the best response to $b = 1$ (i) for all values of δ_c if $\delta_f \le 0.2$ and (ii) for δ_c bounded above with the bound decreasing in δ_f if $0.2 \le \delta_f \le 0.4$. The upper bound of δ_c, denoted by $\bar{\delta}_c(\delta_f)$, is defined implicitly by the equation $\tilde{a} = 0$.

Next we check the conditions under which $b = 1$ is firm B's best response to $a(b = 1)$.

Since Π_B is strictly concave in b, the conditions are $\frac{\partial \Pi_B}{\partial b}|_{b=1} \geq 0$, which lead to

$$\frac{H_1 + 8H_2\sqrt{H}}{9(6(2-\delta_c)+\delta_f)^2(4(2-\delta_c)^3 + 32(2-\delta_c)^2\delta_f + 21(2-\delta_c)\delta_f^2 + 5\delta_f^3)^2} \geq 0$$

where $H_1 \equiv -704(2-\delta_c)^9 - 7984(2-\delta_c)^8\delta_f - 47600(2-\delta_c)^7\delta_f^2 - 168568(2-\delta_c)^6\delta_f^3 - 179884(2-\delta_c)^5\delta_f^4 - 76619(2-\delta_c)^4\delta_f^5 - 11803(2-\delta_c)^3\delta_f^6 + 681(2-\delta_c)^2\delta_f^7 + 415(2-\delta_c)\delta_f^8 + 34\delta_f^9$ and $H_2 \equiv 16(2-\delta_c)^6 + 152(2-\delta_c)^5\delta_f + 428(2-\delta_c)^4\delta_f^2 + 318(2-\delta_c)^3\delta_f^3 + 56(2-\delta_c)^2\delta_f^4 - 7(2-\delta_c)\delta_f^5 - 2\delta_f^6$. The following figure shows the range of (δ_c, δ_f) satisfying the above inequality.[33]

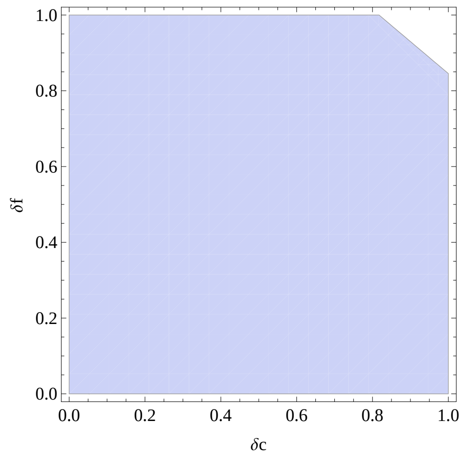

Fig. 3: The region where $b = 1$ is firm B's best response to $a(b = 1)$

It is clear that the region in Fig. 2 is a proper subset of the region in Fig. 3. Thus the region in Fig. 2 describes the necessary and sufficient conditions for maximal differentiation to be an equilibrium outcome. In addition, the region in Fig. 3 shows sufficient conditions under which location equilibria exist in pure strategies. For example, the case considered in the main text, namely $\delta_c = 0$ and $\delta_f = 1$, is outside the region in Fig. 2, hence non-maximal differentiation in equilibrium. Nonetheless it is within the region in Fig. 3, leading to the pure-strategy equilibria shown in Proposition 4. We summarize these results below.

Observation 1: Equilibrium locations are $a = 0$ and $b = 1$ if and only if $\delta_f \leq 0.4$ and $\delta_c \leq \bar{\delta}_c(\delta_f)$ where $\bar{\delta}_c(\delta_f)$ is implicitly defined by $\tilde{a} = 0$, as shown in Fig. 2.

Observation 2: Location equilibria exist in pure strategies for all values of δ_c and δ_f that satisfy $\frac{\partial \Pi_B}{\partial b}|_{b=1} \geq 0$, as shown in Fig. 3.

[33] We also need to show that firm B does not benefit by deviating to some $\hat{b}$, at which the subgame from Case (ii) becomes relevant, i.e., $a(b = 1) + \hat{b} \leq \overline{a + b}$. It can be shown that firm B's deviation in this case leads to smaller profit than when $b = 1$. Thus $b = 1$ is indeed firm B's global best response to $a(b = 1)$.

The contrapositive of Observation 2 is that when location equilibria do not exist in pure strategies, the range of (δ_c, δ_f) is necessarily outside what was shown in Fig. 3. Characterizing all such equilibria in this region is not possible as there could be pure- or mixed-strategy equilibria depending on different values of (δ_c, δ_f). Instead, we show below that equilibria in pure strategies do not exist if $\delta_c = \delta_f = 1$.

Consider Case (i). If a pure-strategy equilibrium exists in this case, it is unique and determined by the intersection of the relevant reaction functions. Given $\delta_c = \delta_f = 1$, the first-order conditions become

$$\begin{aligned}\frac{\partial \Pi_A}{\partial a} &= \frac{31(b-3a)(a+b)t}{98} = 0,\\ \frac{\partial \Pi_B}{\partial b} &= \frac{(196-31a^2-(308-62a)b+93b^2)t}{196} = 0.\end{aligned}$$

Similarly we can derive the first-order conditions in Case (ii). Fig. 4 shows the reaction functions where $a(b)$ is firm A's reaction function and $b(a)$ is firm B's. The blue lines are the reaction functions in Case (i) and the red lines are the reaction functions in Case (ii). The values of a and b at the intersection of the blue lines are

$$a^* = \frac{7(33-\sqrt{97})}{496} \simeq 0.327,\ b^* = \frac{21(33-\sqrt{97})}{496} \simeq 0.980,$$

and the values of a and b at the intersection of the red lines are

$$a^{**} = \frac{21\sqrt{97}-197}{496} \simeq 0.020,\ b^{**} = \frac{265+7\sqrt{97}}{496} \simeq 0.673.$$

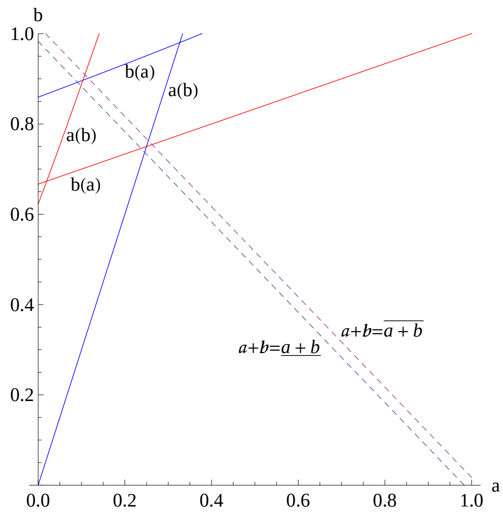

Fig. 4: Reaction functions ($\delta_c = \delta_f = 1$)

We now show that the intersection of the blue lines cannot be equilibrium in Case (i). Notice first that Case (i) requires $a+b \geq \underline{a+b} = \frac{2(-21+21\sqrt{2})}{-12+21\sqrt{2}} \simeq 0.98296$. Firm A's best response to b^* is a^* insofar as Case (i) is the subsequent subgame. Consider now firm A's deviation to a' such that $a'+b^* < \underline{a+b}$. This is possible since $b^* < \underline{a+b}$. Given the deviation, the subsequent subgame shifts to Case (ii) and firm A's profit changes accordingly. Below we draw firm A's profit from the deviation.

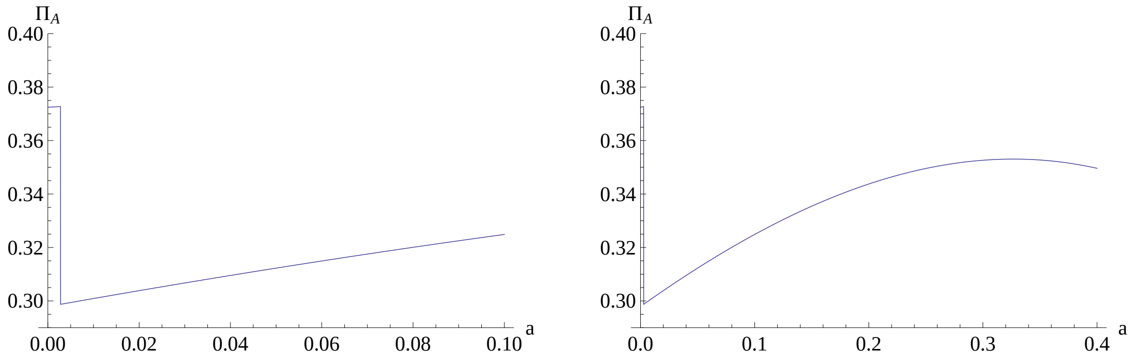

Fig. 5: Firm A's profit when $b = b^*$

The figure on the right in Fig. 5 shows firm A's profit when $b = b^*$. As long as $a + b^* \geq \underline{a+b}$, firm A's best response to $b = b^*$ is $a = a^* \simeq 0.327$, which is a local best response. However when firm A deviates to a' such that $a' + b^* < \underline{a+b}$, there is a discontinuous jump in firm A's profit due to the subgame shifting from Case (i) to Case (ii). The figure on the left shows the jump in profit more clearly. This shows that a^* is not a global best response to b^*. Thus (a^*, b^*) cannot be equilibrium in Case (i). By symmetry, one can show that (a^{**}, b^{**}) cannot be equilibrium in Case (ii). Summarizing, we have

Observation 3: Pure-strategy equilibria do not exist if $\delta_c = \delta_f = 1$.

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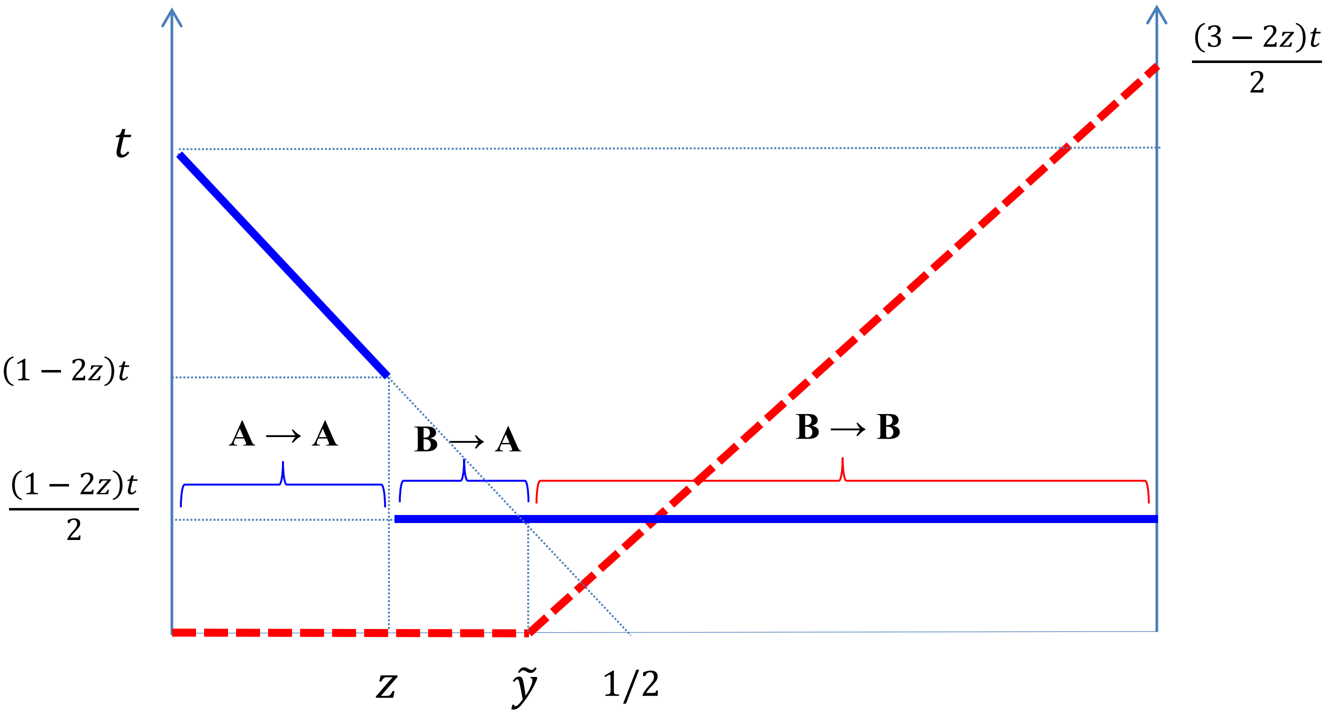


Figure 1: Equilibrium of the second-period pricing game with $z \leq 1/2$

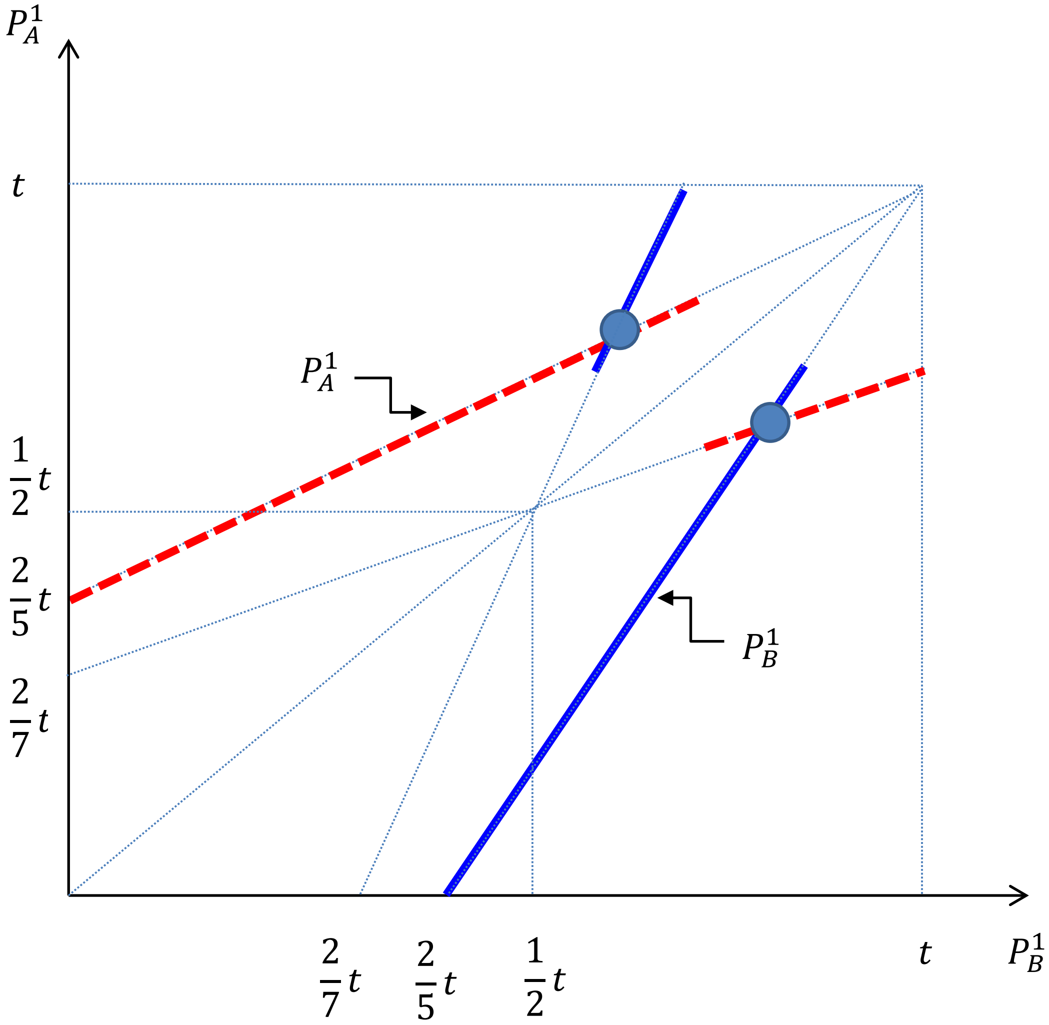
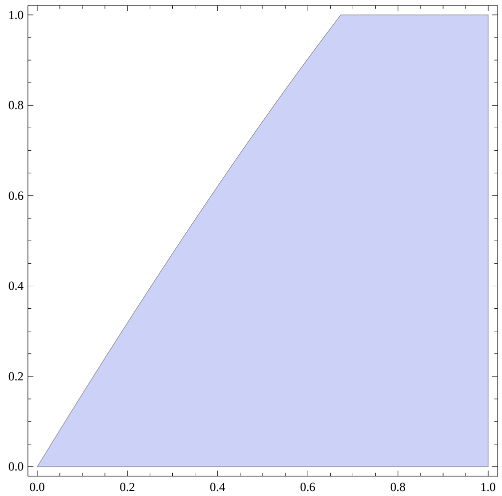
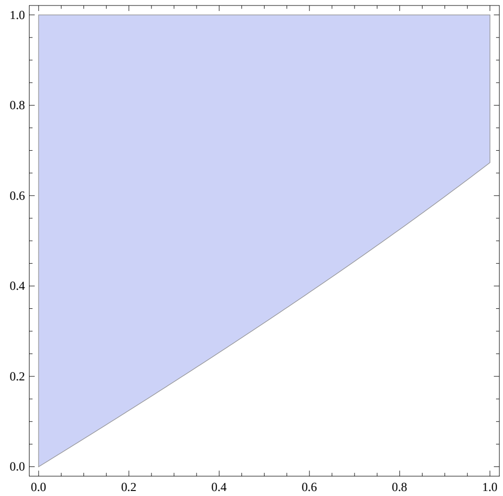


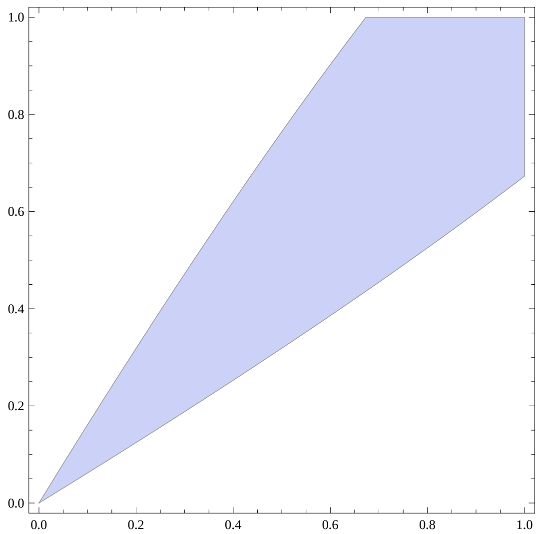
Figure 2: Equilibria of the first-period pricing game



Conditions for $z < 1/2$ equilibrium



Conditions for $z > 1/2$ equilibrium



Conditions for both equilibria

Note: Horizontal-axis ϕ_A, Vertical-axis ϕ_B

Figure 3: Conditions for various equilibria

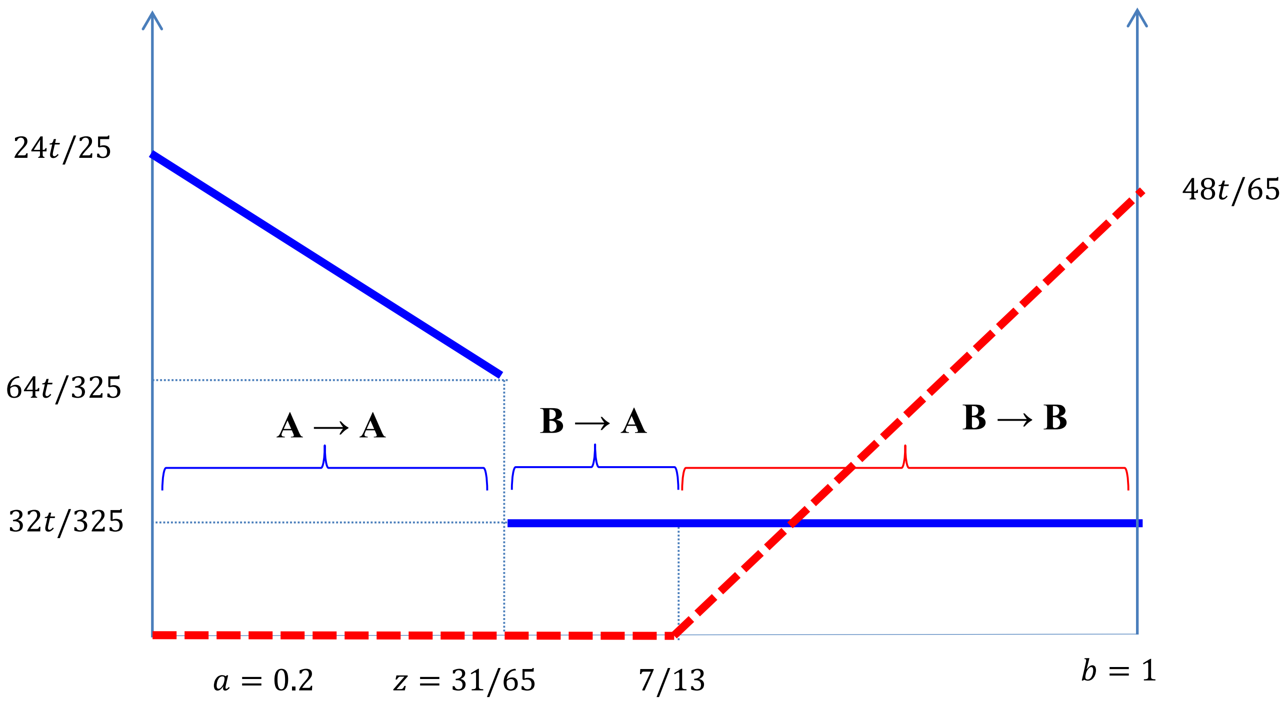


Figure 4: Equilibrium with endogenous location choice (a = 0.2)