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Market-Share Contracts, Exclusive Dealing, and the Integer Problem*

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Abstract:

Exclusionary contracts have long been a focus of antitrust law and the subject of much scholarly debate. This paper compares two types of exclusionary contracts, exclusive-dealing and market-share contracts, in a model of naked exclusion. We discuss the different mechanisms through which each works and identify a fundamental tradeoff that arises: market-share contracts are better at maximizing a seller's benefit from foreclosure (because they allow the seller to obtain any foreclosure level it desires) whereas exclusive-dealing contracts are better at minimizing a seller's cost of foreclosure (because, unlike with market-share contracts, the seller does not have to overpay for the units it forecloses). We identify settings in which each can be more profitable and show that welfare can be worse under market-share contracts.

Keywords: Exclusive dealing, Market-share contracts, Dominant Firm, Foreclosure

JEL Codes: L13, L41, L42, K21, D86

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1 Introduction

Contracts in which a seller puts restrictions on how much a buyer can purchase from other sellers have long been a focus of antitrust law and the subject of much scholarly debate.¹ Examples include contracts that restrict a rival’s market-share, and contracts that specify exclusive dealing. In an exclusive-dealing contract, the buyer agrees to purchase exclusively from the seller. In a market-share contract, the buyer agrees to purchase some minimum share of its requirements from the seller.² In both cases, the concern is over whether such contracts might significantly foreclose entry and/or leave existing rivals with insufficient alternatives to compete for sales.

Lost in this debate, however, is any consideration of why some sellers opt for exclusive dealing while other sellers opt for the less restrictive market-share requirements. Indeed, the use of market-share contracts by excluding sellers is puzzling. Under exclusive dealing, a seller leaves no room for other firms to sell to the buyer, whereas the door to sales is not fully closed to the other firms under a market-share contract. Exclusive dealing would thus seem to be the more effective of the two if the seller’s intent is to exclude a potential entrant or an existing rival. This view is also implicit in court proceedings, where plaintiffs often seek to bolster their cases by having the defendants’ market-share contracts declared to be “*de facto*” exclusive dealing.³ But this only begs the question, if exclusive dealing really is more effective at achieving exclusion, then why did the defendants in these cases not opt to go with exclusive dealing in the first place?

In posing the question “Why are market-share contracts sometimes observed when exclusive dealing would seem to be the better choice for achieving exclusion?” to antitrust authorities and policymakers, we have encountered many responses. One is that market-share contracts may simply be a “poor man’s” way of excluding – not as effective or as powerful as exclusive dealing in foreclosing rival sellers, but also not as costly to implement (in the sense that the compensation that would be needed to induce buyers to agree to such contracts would be less than it would be

¹See the discussion in Tom et al. (2000) and Whinston (2006). See also the transcripts of the FTC - DOJ Workshop on Conditional Pricing Practices: Economic Analysis & Policy Implications (June 23, 2014), available at www.ftc.gov/news-events/events-calendar/2014/06/conditional-pricing-practices-economic-analysis-legal-policy.

²Recent cases involving exclusive dealing include *Van den Bergh Foods v. Commission* [Case T-65/98 (2003)], *Conwood v. U. S. Tobacco* [290 F. 3d 768 (2002)], *U.S. v. Visa U.S.A.* [344 F. 3d 229 (2003)], *U.S. v. Dentsply* [399 F. 3d 181 (2005)], *FTC v. Transitions Optical* [File No. 091 0062 (2010)], and *FTC v. McWane, Inc* [2014]. Recent cases involving market-share contracts include *Masimo Corp. v. Tyco Healthcare Group*, [No. 02-CV-4770 (2005)], *Prokent v. Tomra* [Case COMP/E-1/38.113 (2006)], *FTC v. Intel* [File No. 061 0247 (2010)], *ZF Meritor v. Eaton* [696 F. 3d 254 (2012)], *Intel v. Commission* [Case T-286/09 (2014)], and *Eisai v. Sanofi Aventis* [2016].

³In *ZF Meritor v. Eaton* [2012], for example, *ZF Meritor* alleged that *Eaton*’s contract, which required high minimum percentage purchases from *Eaton*, was a *de facto* exclusive-dealing arrangement and the court agreed. And similarly, in *Eisai v. Sanofi Aventis* [2016], *Eisai* argued that *Sanofi Aventis*’ provision that buyers purchase at least 90% of their anticoagulant drug requirements from *Sanofi Aventis* was the same as exclusive dealing.

with exclusive dealing).⁴ According to this view, sellers who choose to use market-share contracts when their intent is to exclude are simply making a rational cost-benefit calculation (believing that the relatively lower cost of exclusion will more than offset the relatively lower effectiveness).

In this paper, we consider the claim that market-share contracts are a poor man's exclusive dealing. We use a framework that has the same timing and many of the same features as the "naked exclusion" literature put forward by Rasmusen, Ramseyer, and Wiley (1991) (hereafter RRW) and Segal and Whinston (2000) (hereafter SW). In this framework, there is an incumbent seller, an entrant, and N buyers. The incumbent can offer an exclusionary contract to one or more buyers. The buyers decide whether to accept or reject it. The entrant, on observing all offers and acceptance decisions, decides whether to enter the market. If it does so, it competes against the incumbent for any uncommitted purchases. As in SW, we allow buyers to coordinate their acceptance decisions, thereby ruling out equilibria in which the seller is able to obtain exclusion costlessly. Unlike in RRW-SW, however, we assume that the entrant's costs are unknown at the time the seller makes its offers. This implies that neither the seller nor the buyers know with certainty how many buyers the seller must sign up in order for the entrant to be deterred.⁵

Solving the model yields several insights regarding the seller's preferred choice of contract. First, we find that although both exclusive dealing and market-share contracts can be profitable for the seller, the mechanism through which the two types of contracts work differs. When a buyer signs one of these contracts, the attractiveness of the market for the entrant is reduced, which (weakly) reduces the probability of entry. This reduction in turn negatively impacts the buyers in two ways. First, it reduces the expected surplus of the buyers who have not signed the contract. This is due to the *inter-group externality* that signed buyers impose on unsigned buyers. Second, it reduces the expected surplus of the uncommitted purchases of each signed buyer. While each signed buyer must be compensated for the expected reduction in surplus that its signing causes for *its* own uncommitted purchases, the negative impact that its signing causes for the *other* signed buyers' uncommitted purchases is not compensated. Importantly, however, this *intra-group externality* arises only when market-share contracts are offered (because the buyers do not have any uncommitted purchases when they sign an exclusive-dealing contract).

⁴Another response is that sellers may believe that by using market-share contracts instead of exclusive-dealing contracts, they can fool authorities into believing that their intent is benign. It has also been claimed that the transactions costs of implementing exclusive dealing may be higher than the transactions costs of implementing market-share contracts (although one could also argue the opposite: it may be less costly to verify whether a buyer is purchasing something from a rival seller than it is to verify how much a buyer is purchasing from the rival).

⁵In contrast to RRW-SW, where the entrant's fixed costs are known at the time of contracting, and thus where it is known how many buyers are needed to deter the entrant, the seller and the buyers in our model only know that the probability of entry is (weakly) decreasing in the number of buyers who agree to the seller's contract.

Second, we find that market-share contracts potentially offer a huge advantage over exclusive dealing because they allow the seller to achieve any interior foreclosure level that it desires. The same cannot be said for exclusive dealing because of “the integer problem.” To illustrate this difference, suppose that there are four equally-sized buyers ($N = 4$), and that given the distribution of entry costs, the seller would ideally like to foreclose 60% of the market. This poses a problem for exclusive dealing because under exclusive dealing the closest the seller can come to its ideal is to sign either two buyers, which forecloses 50% of the market, or three buyers, which forecloses 75% of the market. By contrast, with market-share contracts, the seller can realize a foreclosure level of 60% simply by offering its contract to all four buyers and imposing a minimum-share requirement of 60% (alternatively, it can also achieve a foreclosure level of 60% by offering its contract to only three buyers and imposing a minimum-share requirement of 80%).

Third, we find that market-share contracts sometimes give rise to higher foreclosure levels than would have been the case under exclusive dealing. To continue with our example, recall that the seller’s two best options under exclusive dealing are to foreclose either 50% or 75% of the market. Suppose the former yields the higher profit. Then, under exclusive dealing, the seller would be observed to foreclose 50% of the market, whereas under the optimal market-share contract it would be observed to foreclose 60%. Market-share contracts in this instance would be worse for welfare.⁶ It follows that one cannot say in general that market-share contracts are necessarily a weaker version of exclusive dealing, and thus, by implication, less powerful. Indeed, depending on the setting, market-share contracts can be more powerful than exclusive dealing.

Last, we find that exclusive dealing does have some advantages. In particular, we find that the cost of foreclosure on a per-unit basis is higher when market-share contracts are used than when exclusive dealing is used. This is because the seller needs to compensate each signed buyer not only for its expected loss on the purchases it commits to the seller but also for the negative externality that its committed purchases imposes on its uncommitted purchases. It follows that with market-share contracts, the seller must over-compensate each signed buyer for the contribution of its committed purchases. By contrast, there is no such over-compensation under exclusive dealing because in this case the signed buyers have no uncommitted purchases.

This last finding, in combination with our second finding above, suggests that in the absence of an integer problem (e.g., when the seller can precisely achieve its target level of foreclosure with exclusive dealing), the optimal design of the exclusionary contract always favors exclusive dealing. In the presence of an integer problem, however, the seller’s choice is more complicated. It must

⁶Alternatively, if 75% yields higher profit than 50%, then market-share contracts would be better for welfare.

balance a delicate tradeoff. On the one hand, market-share contracts do better at maximizing the seller’s benefit from foreclosure (because unlike with exclusive dealing, they allow the seller to obtain any foreclosure level in the interior that it desires), whereas on the other hand, exclusive-dealing contracts do better at minimizing the seller’s cost of foreclosure (because, unlike with market-share contracts, the seller does not have to overpay for the units that it forecloses).

We show that depending on the distribution of entry costs, the resulting tradeoff can go either way. Market-share contracts can be more profitable than exclusive dealing in some settings, while in other settings, even with the integer problem, the opposite holds and exclusive dealing is more profitable. We also show that for a given distribution of entry costs, as the number of buyers in the market increases, market-share contracts are more likely to dominate exclusive-dealing contracts all else equal and thus are more likely to be the seller’s preferred means of exclusion.

Needless to say, our findings contrast sharply with the view that market-share contracts are a poor man’s exclusive dealing. Although there is indeed a cost-benefit calculation to be made, the seller must pay a *higher per-unit cost* of foreclosure under market-share contracts than it would pay under exclusive dealing for the same number of units, and in exchange, the seller is better able to fine tune its optimal level of foreclosure, implying *more effectiveness*, not less.

Literature review

The literature on exclusion has focused mostly on exclusive dealing.⁷ The two most prominent theories of harm are the aforementioned “naked exclusion” scenario put forward by RRW-SW, and the “rent-extraction” scenario of Aghion and Bolton (1987) (hereafter AB). RRW show that exclusive dealing imposes a negative externality on buyers, which the incumbent may be able to exploit when the buyers are unable to coordinate their acceptance decisions. SW show that by adopting a “divide-and-conquer” strategy in which it offers to fully compensate some buyers in order to earn monopoly profits on the remaining buyers, the incumbent may be able to exclude the entrant profitably even in the absence of a coordination failure. In contrast, AB show that an incumbent may be able to extract rents from a more efficient entrant by offering exclusive-dealing contracts that contain penalty clauses. In their model, exclusion may be induced inadvertently.

In addition to the work by RRW-SW, related literature includes: Landeo and Spier (2009, 2012), and Smith (2010), who look at naked exclusion using RRW-SW’s framework in experimental settings, and Miklós-Thal and Shaffer (2016), who consider the efficacy of divide-and

⁷With the exception of Chen and Shaffer (2014), discussed below, the literature on market-share contracts has focused on other motives. In particular, market-share contracts have been alleged to facilitate rent shifting (Marx and Shaffer, 2004), screen buyers when demand is private information (Majumdar and Shaffer, 2009; Calzolari and Denicolo, 2013), induce more service provision (Mills, 2010), and soften competition (Inderst and Shaffer, 2010).

conquer strategies when contracts are unobservable. The RRW-SW strand of literature has also been extended to settings in which buyers compete.⁸ In addition to AB’s work, related literature includes: Chen and Sappington (2011), who find that exclusive dealing generally reduces the entrant’s R&D and can reduce the incumbent’s R&D, and Ide, Montero, and Figueroa (2016), who find that exclusionary contracts cannot be anticompetitive in the absence of upfront payments.

Calzolari and Denicolo (2015) look at exclusive-dealing contracts in a setting with one buyer, where the buyer’s willingness to pay is private information. In their model, a dominant firm competes with an active set of fringe firms, and sells to a single buyer (as in the common-agency models of O’Brien and Shaffer, 1997, and Bernheim and Whinston, 1998). In contrast, in our model, the entrant is unable to contract with the buyers until after the incumbent has done so.

Our model also differs from the model in Chen and Shaffer (2014), who focus on a setting in which the incumbent is restricted to offering exclusionary contracts to all buyers. In their model, exclusive dealing is never profitable (because there is no inter-group externality), but market-share contracts are profitable (because there remains the intra-group externality). In contrast, our focus is on optimal contract design when the number of offers is endogenously determined.

The rest of the paper proceeds as follows. In Section 2, we set up the model and introduce notation. In Section 3, we discuss the two types of externalities that can arise. In Section 4, we identify the fundamental trade-off that determines the incumbent’s choice of contract. Section 5 obtains further results and characterizes the optimal contracts. Section 6 considers extensions. Section 7 concludes. The proofs of the lemmas and propositions can be found in the appendix.

2 The Model

2.1 The Setting

We consider a setting with an incumbent seller, a potential entrant, and a set of N homogeneous buyers. The seller(s) offer a single divisible good for sale. Each buyer demands a fixed amount of the good, which we normalize to one unit. The incumbent can produce at constant marginal cost c , while the potential entrant, if it decides to enter, can produce at constant marginal cost $c - \delta$, where $\delta \in (0, c)$. Each buyer is willing to pay up to a maximum of $v > c$ for its unit.

Unlike the incumbent, who is already established in the market, we assume that the entrant must incur a fixed cost of entry, f , before it can produce anything. Moreover, we assume it is common knowledge among all players that f is distributed with positive density $g(\cdot)$ between

⁸See Fumagalli and Motta (2006), Simpson and Wickelgren (2007), Abito and Wright (2008), and Wright (2009). Other models of exclusion with competing buyers include Argenton (2010) and Asker and Bar-Isaac (2014).

zero and $\bar{N}\delta$, where $\bar{N} \leq N$. The lower bound ensures that attempts by the incumbent to reduce the entrant's flow payoff will succeed in deterring entry if the reduction is large enough. The upper bound ensures that in the absence of such attempts, entry will occur with probability one.⁹

As in RRW-SW, the incumbent has a first-mover advantage and can offer exclusionary, non-renegotiable contracts to buyers prior to the entrant's entry decision. Unlike in RRW-SW, who restrict attention to contracts with exclusive-dealing, however, we consider a class of contracts in which a signing buyer agrees only to purchase some minimum share $s \in [0, 1]$ of its total purchases from the incumbent, with exclusive-dealing emerging as a special case of this when $s = 1$. We also differ from RRW-SW in that we allow the incumbent not only to offer a lump sum payment to each buyer who signs its contract, but also to commit to a per-unit price.¹⁰

Timing of the game

The timing of the game proceeds as follows:

Period 1: The incumbent offers to each of K buyers an exclusionary contract $C = \{s, x, p\}$. In this contract, $s \in [0, 1]$ denotes the minimum share of the buyer's total purchases that must be purchased from the incumbent, x denotes the lump-sum payment to be paid, and p denotes the per-unit price at which the buyer can purchase the incumbent's good. For ease of exposition, it is convenient to restrict attention in the text to contracts in which the incumbent sets $p \in [c, v]$.¹¹

Period 2: Buyers simultaneously decide whether to accept or reject the incumbent's offers.

Period 3: The potential entrant learns the value of f and, after observing the incumbent's offers and the buyers' accept-or-reject decisions, decides whether or not to enter the market.

Period 4: The incumbent and the entrant (if active) compete for the uncommitted purchases of each buyer by posting prices in a spot market.¹² Buyers who have agreed to the incumbent's exclusionary offer have an option to buy as much as they want from the incumbent at the price p , but must buy at least s share of their purchases at this price. For the rest of their purchases,

⁹Since the entrant can always set a per-unit price slightly lower than the incumbent's marginal cost, we assume, for simplicity, that each buyer will purchase from the entrant if the incumbent and the entrant charge the same price. The claim then follows because in the event of entry, and assuming all buyers are free to purchase from either seller, the entrant can always earn a net profit of $N\delta - f > 0$ by charging a per-unit price of c to each buyer.

¹⁰Price commitments typically arise in settings in which the nature of the good to be delivered in future periods is known to the incumbent and the buyers at the time the contracts are written and signed. They may also arise in other settings, but may be less likely. See SW's justification for the absence of price commitments in RRW.

¹¹Offers of $p < c$ invite possible rent-shifting along the lines discussed in Aghion and Bolton (1987), and as such are likely to engender additional scrutiny from competition authorities on predatory pricing grounds. We rule out such offers in the spirit of RRW-SW in order to keep the focus on the relative foreclosure potential of exclusive dealing versus market-share contracts. We show in the online appendix that $p > v$ cannot arise in equilibrium.

¹²The uncommitted purchases consist of $1 - s$ share of the purchases of a buyer who has signed the incumbent's exclusionary contract and all the purchases of a buyer who has not signed the incumbent's exclusionary contract.

they can either buy from the incumbent at the contract price of p , or they can buy from either firm in the spot market. Buyers who have not agreed to the incumbent's exclusionary offer can purchase as much as they want from either the incumbent or the entrant in the spot market.

We will refer to contracts in which buyers agree to purchase only from the incumbent, $s = 1$, as *exclusive-dealing* contracts (*ED*). We will refer to contracts in which buyers must make at least s share of their purchases from the incumbent, with $s < 1$, as *market-share* contracts (*MS*).

2.2 Characterization of Equilibria

Let $n \in \{0, 1, \dots, K\}$ denote the number of buyers who accept the contract $C = \{s, x, p\}$.

Pricing decisions

We solve for the equilibrium of the game using backwards induction. Consider first the pricing game in Period 4. There are two cases to consider. If the entrant does not enter, it is optimal for the incumbent to charge the monopoly price v to all unsigned buyers. In this case each unsigned buyer will purchase one unit of the good and obtain a surplus of zero. Signed buyers on the other hand will exercise their right to fulfill their entire demand at the per-unit price p . Because $p \leq v$, each signed buyer will thus purchase one unit of the good and obtain a surplus of $v - p$.

If the entrant does enter, we assume that competition *a la* Bertrand for the uncommitted purchases of the signed and unsigned buyers will drive the entrant's price down to the incumbent's per-unit cost c . Unsigned buyers will thus purchase one unit of the good from the entrant and obtain a surplus of $v - c$. Signed buyers, however, can only purchase at most $1 - s$ share of their total purchases from the entrant at the entrant's price of c . The remaining s share of their total purchases must be purchased from the incumbent at the per-unit contract price of p . Each signed buyer thus faces an average price of $p_a = sp + (1 - s)c$ and obtains a surplus of $v - p_a$.¹³

Entrant's entry decision

If the entrant enters in Period 3, the entrant incurs a fixed cost of entry f and earns a flow payoff of $n(1 - s)\delta$ from the signed buyers and $(N - n)\delta$ from the unsigned buyers, for a total flow payoff of $\Pi_E(n, s) \equiv n(1 - s)\delta + (N - n)\delta = (N - ns)\delta$. In contrast, the entrant earns zero if it does not enter. Thus, it is profitable for the entrant to enter in Period 3 if and only if

$$f \leq \Pi_E(n, s). \quad (1)$$

¹³To see the role of our assumption that the incumbent can commit to a per-unit price, note that with such a commitment, buyers gain under MS when the entrant enters because $v - p_a > v - p$ for all $p > c$. In contrast, in the absence of such a commitment, buyers would not gain when the entrant enters. The reason is that the incumbent would optimally charge $p = v$ when the entrant does not enter, and $p = (v - (1 - s)c)/s$ (i.e., p such that $p_a = v$) when the entrant does enter, implying that in both cases the buyer would earn zero. Given this, the intra-group externality that we identify in the next section would not arise, and ED would always dominate MS.

Here we see that $\Pi_E(n, s)$ is decreasing in the number of units that have been foreclosed. This implies that the incumbent has two ways of decreasing the entrant's payoff. It can induce more buyers to sign its contract, or it can require a larger minimum share of each buyer's purchases.

Buyers' acceptance decisions

The value of f is unknown at the time the buyers must accept or reject the incumbent's offer. When making their decisions, therefore, buyers must form expectations of the likelihood of entry. Let $G(\cdot)$ denote the distribution of f . Then, using condition (1), the likelihood of entry is¹⁴

$$\alpha_n(s) \equiv G(\Pi_E(n, s)).$$

It follows that a buyer who accepts the incumbent's offer in Period 2 receives surplus $v - p_a + x$ with probability α_n and $v - p + x$ with probability $1 - \alpha_n$, whereas if it rejects the incumbent's offer, it receives surplus $v - c$ with probability α_{n-1} and no surplus with probability $1 - \alpha_{n-1}$.

Putting it all together, the expected surplus of a buyer who accepts the incumbent's offer in Period 2 when $n - 1$ other buyers are also accepting the incumbent's offer is given by

$$U_A(n) \equiv \alpha_n(v - p_a) + (1 - \alpha_n)(v - p) + x,$$

whereas its expected surplus if it rejects the incumbent's offer in Period 2 is given by

$$U_R(n - 1) \equiv \alpha_{n-1}(v - c).$$

It is thus optimal for the buyer to accept the incumbent's offer if and only if it receives a lump-sum payment of $x \geq x_n(s, p)$, where $x_n(s, p)$ is the value of x such that $U_A(n) = U_R(n - 1)$.¹⁵

$$x_n(s, p) \equiv \alpha_{n-1}(v - c) - \alpha_n(v - p_a) - (1 - \alpha_n)(v - p). \quad (2)$$

It should be clear from the definition of $x_n(s, p)$ that if $x \geq x_K(s, p)$, then all K buyers accepting the incumbent's offer is an equilibrium because $U_A(K) \geq U_R(K - 1)$ implies that no unilateral deviation is profitable. However, there may also be equilibria in which $k < K$ accept.¹⁶

To narrow the set of possible equilibria, we follow SW in allowing buyers to coordinate their decisions when choosing whether to accept or reject the incumbent's offer. The only restriction that SW impose is that the buyers' collective actions must be self-enforcing. This idea is captured

¹⁴Since $G(\cdot)$ is non-decreasing and $\Pi_E(n, s)$ is decreasing in n and s , $\alpha_n(s)$ will also be decreasing in n and s .

¹⁵Here we see that the sign of $x_n(s, p)$ depends on p . When p is sufficiently small (e.g. $p = c$, $x_n(s, c) = (\alpha_{n-1} - 1)(v - c) \leq 0$), the payment flows from the buyer to the incumbent. However, when p is sufficiently large (e.g., $p = v$, $x_n(s, v) = \alpha_{n-1}(v - c) - \alpha_n(v - p_a) > 0$), the payment flows from the incumbent to the buyer.

¹⁶This is so if $U_A(k) \geq U_R(k - 1)$ and $U_R(k) \geq U_A(k + 1)$. When $x = x_K(s, p)$, these inequalities can hold if the relationship among $x_K(s, p)$, $x_k(s, p)$, and $x_{k+1}(s, p)$ is such that $x_K(s, p) \geq x_k(s, p)$ and $x_K(s, p) \leq x_{k+1}(s, p)$.

by Bernheim et al.'s (1987) concept of a perfectly coalition-proof Nash equilibrium (PCPNE), which requires that all equilibria of the game be immune to self-enforcing coalitional deviations.¹⁷

In solving for PCPNE in our setting, note first that if the payment offered by the incumbent exceeds $x_{k+1}(s, p)$, then there cannot be an equilibrium in which only $k < K$ buyers accept the incumbent's offer because the payment would exceed the minimum payment that would be needed to induce a buyer to accept the incumbent's offer if k other buyers are also accepting the incumbent's offer. Since the same logic applies to any number of buyers, $k = 0, \dots, K - 1$, it follows that if the incumbent were to offer buyers a lump-sum payment of $x > x^*(s, p)$, where

$$x^*(s, p) \equiv \max_{n \leq K} \{x_n(s, p)\}, \quad (3)$$

all K buyers would accept the offer in the unique Nash equilibrium of the continuation game.

Note next that this unique Nash equilibrium of the continuation game is also coalition-proof because a lump-sum payment of $x > x^*(s, p)$ means that $U_A(n) > U_R(n - 1)$ for all $n \leq K$, from which it follows that there is no self-enforcing coalitional deviation that can benefit the buyers.¹⁸

Note finally that if the incumbent were to offer $x < x^*(s, p)$, which implies that there exists some number $m \leq K$ such that $x < x_m(s, p)$, then the condition $U_R(m - 1) > U_A(m)$ implies that a coalition of $K - m + 1$ buyers would be better off jointly rejecting the incumbent's offer.

These findings, and those for the case of $x = x^*(s, p)$, are summarized in the following lemma.

Lemma 1 *Suppose the incumbent offers the contract $C = \{s, x, p\}$ to K buyers. Then, if*

- $x > x^*(s, p)$, all K buyers accept the offer in the unique PCPNE in the continuation game;
- $x < x^*(s, p)$, there is no PCPNE in the continuation game in which all K buyers accept;
- $x = x^*(s, p)$, there exists a PCPNE in the continuation game in which all K buyers accept.

Lemma 1 highlights the role of $x^*(s, p)$ in inducing all K buyers to accept the incumbent's contract in a PCPNE. It implies that (i) offering $x > x^*(s, p)$ is sufficient to induce all buyers to sign; and (ii) offering at least $x = x^*(s, p)$ is necessary to induce all buyers to sign. Given the importance of $x^*(s, p)$, therefore, it is useful to consider some of its properties before proceeding.

¹⁷In other words, valid deviations are judged by the same criteria used to judge the candidate equilibrium. They must be self-enforcing in the sense that no proper sub-coalition can reach a mutually beneficial agreement to deviate from the deviation. Any potential deviation by a sub-coalition must also be self-enforcing, and so on.

¹⁸Suppose $k \geq 2$ buyers were to deviate and jointly reject the offer. Then, each buyer in the coalition would get $U_R(K - k)$. However, this would not be self-enforcing because a buyer in the coalition would be able to make itself better off by deviating from the coalition and accepting the incumbent's offer, earning $U_A(K - k + 1) > U_R(K - k)$.

2.3 Properties of $x^*(s, p)$

We begin by noting that $x_1(s, p)$, and thus $x^*(s, p)$, is weakly greater than $s(p - c)$. Intuitively, a buyer who signs the incumbent's contract is committing to buy s share of its purchases from the incumbent at a per-unit price of p . By not signing, the buyer reasons that it will be able to purchase these same units in the spot market at a per-unit price of c . The buyer will therefore need to be compensated with a lump-sum payment of *at least* $s(p - c)$ in order for it to sign.¹⁹

The payment may need to be higher because there is an extra factor that comes into play when MS is offered. To see this, note that our assumption that $f \leq \bar{N}\delta$ implies that the incumbent must sign up more than $\Omega \equiv N - \bar{N}$ buyers if its contract is to have any effect (i.e., if it is to have any chance of reducing the probability of entry). So, suppose the incumbent offers its contract to exactly $\Omega + 1$ buyers, and consider the payment $x_{\Omega+1}(s, p)$. Using condition (2), we have:²⁰

$$\begin{aligned} x_{\Omega+1}(s, p) &= \alpha_{\Omega}(v - c) - (1 - \alpha_{\Omega+1})(v - p) - \alpha_{\Omega+1}(v - p_a) \\ &= v - c - (1 - \alpha_{\Omega+1})(v - p) + (1 - \alpha_{\Omega+1})(v - p_a) - (v - p_a), \\ &= s(p - c) + (1 - \alpha_{\Omega+1})(1 - s)(p - c). \end{aligned} \tag{4}$$

The extra factor can be seen from the last line in (4), which implies that the incumbent must compensate a signing buyer not just for the surplus that it expects to lose on the share of the total purchases that it commits to the incumbent, i.e., $s(p - c)$, but also for the reduction in surplus that its signing causes for the $1 - s$ share of its purchases that are uncommitted, an amount that is equal to $(1 - \alpha_{\Omega+1})(1 - s)(p - c)$. This follows because given that it expects Ω other buyers to sign, the buyer will recognize that (i) its signing will reduce the likelihood of entry from one to $\alpha_{\Omega+1}$, and (ii) this matters because if the entrant enters, the per-unit price of the uncommitted purchases in the spot market will be c , whereas if the entrant does not enter, the same units will only be available from the incumbent at the incumbent's per-unit price of p .

We now characterize what can be said about the relation between $x^*(s, p)$ and $s(p - c)$.

Lemma 2 *Suppose the incumbent offers the contract $C = \{s, x, p\}$ to K buyers. Then, if*

- *the contract specifies exclusive dealing, $x^*(1, p) = p - c$;*
- *the contract specifies a market-share of $s < 1$ and $\alpha_K(s) = 1$, $x^*(s, p) = s(p - c)$;*

¹⁹If the lump-sum compensation were less than $s(p - c)$, the buyers would have an incentive to form a coalition and jointly reject the incumbent's offer, thereby ensuring that the entrant would be able to enter profitably.

²⁰The first line of the expression in (4) follows directly from the definition of $x_n(s, p)$. The second line uses the fact that $\alpha_{\Omega}(s) = 1$, for all s , and the third line uses the fact that $p_a - c = s(p - c)$ and $p - p_a = (1 - s)(p - c)$.

- the contract specifies a market-share of $s < 1$ and $\alpha_K(s) < 1$, $x^*(s, p) > s(p - c)$.

The characterization in Lemma 2 depends on whether the incumbent offers ED or MS. If the incumbent offers ED, or if the likelihood of entry would not be reduced, the incumbent will only have to compensate buyers by the amount $s(p - c)$ to induce them to sign,²¹ whereas if the incumbent offers MS and the likelihood of entry would be reduced, the incumbent will have to compensate buyers by more than $s(p - c)$ to induce them to sign. In the first instance, buyers have no uncommitted purchases and thus no stake in whether entry subsequently occurs. In the second instance, entry occurs with probability one, implying that any price $p > c$ represents a certain loss of $s(p - c)$ for these buyers. In the third instance, each buyer must be compensated not only for the expected loss in surplus on its committed purchases, but also for the expected loss in surplus that its signing causes for the $1 - s$ share of its purchases that are uncommitted.

3 Two externalities

We will focus on answering two main questions in this section: (i) when are ED and MS profitable, and (ii) how do they work. To this end, it is useful to distinguish between the profit the incumbent can expect to earn from signed buyers and the profit it can expect to earn from unsigned buyers.

We know from Lemma 1 and the discussion thus far that if the incumbent is to induce K buyers to sign its contract, each must be offered a payment of at least $x \geq x^*(s, p)$. Since there is no reason to offer any more than this amount, and since the incumbent's payoff is strictly decreasing in x , it follows that the incumbent will offer exactly $x = x^*(s, p)$ in any equilibrium. The incumbent's problem in Period 1 is thus to choose K , $s \in (0, 1]$, and $p \in [c, v]$ to maximize:

$$\Pi(K, s, p) = K\Pi^S(K, s, p) + (N - K)\Pi^U(K, s), \quad (5)$$

where

$$\begin{aligned} \Pi^S(K, s, p) &= s(p - c) + (1 - \alpha_K(s))(1 - s)(p - c) - x^*(s, p), \\ \Pi^U(K, s) &= (1 - \alpha_K(s))(v - c). \end{aligned}$$

Here we see that the incumbent's expected profit is a weighted sum of its expected profit from each signed buyer, $\Pi^S(K, s, p)$, and its expected profit from each unsigned buyer, $\Pi^U(K, s)$, where the weights are N and $N - K$, respectively. Its actual profit, of course, will depend on whether the entrant is deterred. If it is not, the incumbent will earn $K(s(p - c) - x^*(s, p))$, which

²¹The first result follows from the fact that $x_n(1, p) = p - c$ for all $n \leq \Omega + 1$ and $x_n(1, p) = \alpha_{n-1}(v - c) - (v - p)$ is decreasing in n for all $n \geq \Omega + 1$. The second result follows from the fact that $\alpha_n(s) \geq \alpha_K(s)$ for all $n \leq K$.

is what it obtains from the committed purchases of the signed buyers net of what it pays them. If it is (which occurs with probability $1 - \alpha_K(s)$), the incumbent will earn the aforementioned amount plus a further $(1 - s)(p - c)$ from each signed buyer and $v - c$ from each unsigned buyer.

Consider first the case of ED. Previous literature has shown that ED can be profitable for the incumbent because of the *inter-group externality* that signed buyers impose on unsigned buyers. This externality can also be seen in our setting by substituting $s = 1$ and $x^*(1, p) = p - c$ (which comes from Lemma 2) into the expressions for Π^S and Π^U above and noting that while the incumbent's payoff from each signed buyer is always zero, its expected payoff from each unsigned buyer is strictly positive for all $K < N$ such that $\alpha_K(1) < 1$. It is thus straightforward to see from this that ED will be profitable if and only if there exists a $K < N$ such that $\alpha_K(1) < 1$.

Now consider the case of MS. The inter-group externality, which is so crucial for the profitability of ED, is also present, of course, under MS (when there is a $K < N$ and $s < 1$ such that $\alpha_K(s) < 1$), but things are more nuanced under MS because there is also an *intra-group externality* which the incumbent can exploit. This intra-group externality can be seen most easily by supposing for now that the distribution of f is such that the maximum payment needed to ensure that all K buyers sign is given by $x^*(s, p) = x_{\Omega+1}(s, p)$.²² In this case, PCPNE requires the incumbent to compensate each buyer as if the entrant would enter with probability one if the buyer did not sign. This is a stringent requirement because it means that the incumbent must compensate each buyer not only for the loss of surplus the buyer will incur on its *committed* purchases when it signs, which is equal to $s(p - c)$, but also for the loss of surplus that the buyer's signing causes for the $1 - s$ share of its purchases that are uncommitted, which is equal to $(1 - \alpha_{\Omega+1})(1 - s)(p - c)$, for a total payment of $x^*(s, p) = s(p - c) + (1 - \alpha_{\Omega+1})(1 - s)(p - c)$.

Substituting this into the expression for Π^S above, and canceling common terms, we obtain

$$\Pi^S(K, s, p) = (\alpha_{\Omega+1}(s) - \alpha_K(s))(1 - s)(p - c). \quad (6)$$

It follows that the incumbent will earn strictly positive expected profit from the signed buyers for all $p > c$ and $s < 1$, such that $\alpha_{\Omega+1}(s) > \alpha_K(s)$. When this condition holds, each signing buyer imposes a negative externality not only on every unsigned buyer, but also on every other signing buyer, and this is so despite the fairly steep cost required to induce each buyer to sign.²³

²²It is straightforward to show that a sufficient condition for $x_{\Omega+1}(s, p)$ to be the maximum payment among all x_n is that signing up the $\Omega+1$ 'st buyer results in the largest drop in the probability of entry (i.e., $\alpha_{\Omega} - \alpha_{\Omega+1} \geq \alpha_{n-1} - \alpha_n$ for all $n \leq K$). We show in Appendix C that this condition holds, for instance, when $G(\cdot)$ is weakly convex.

²³We can note here the role of our assumption that entry costs are stochastic. Suppose it were known instead that entry would be deterred if and only $k \geq 1$ or more buyers signed the incumbent's contract, so that the probability of entry would be one if less than k buyers signed the incumbent's contract and zero if k or more buyers signed the incumbent's contract. Then $\alpha_{\Omega+1} = \alpha_K = 0$, and the intra-group externality would not arise.

As we did for ED, we now characterize when MS will be profitable. Surprisingly, we find that despite the different mechanisms at work (two externalities with MS versus one externality with ED), there exist profitable contracts with MS *if and only if* there exist profitable contracts with ED. The reason for this, loosely speaking, is because the intra-group externality, which arises only under MS, is not operative unless the inter-group externality is also operative, and the conditions for the inter-group externality to be operative are the same for both ED and MS.

To see this, note that if there exists some $K < N$ such that $\alpha_K(1) < 1$, then by continuity, it must be that $\alpha_K(s) < 1$ for some s sufficiently close to 1. And if there does not exist some $K < N$ such that $\alpha_K(1) < 1$, then there will also not exist some $K < N$ and $s < 1$ such that $\alpha_K(s) < 1$. It follows that the necessary and sufficient condition for the inter-group externality to hold is thus the same for both ED and MS. Moreover, to see that the intra-group externality cannot arise independently of the inter-group externality, note that if $\alpha_K(s) = 1$ for all $K < N$, then $x^*(s, p) = s(p - c)$ from Lemma 2, and the intra-group externality vanishes (the second term in the expression for Π^S vanishes, and the first plus the third terms cancel each other out).

It remains to state the condition in terms of the primitives on the distribution of entry costs.

Proposition 1 *There exist profitable exclusionary contracts if and only if no one buyer is sufficient to support entry with probability one (i.e., if and only if $f > \delta$ with positive probability). The profitability of ED can be attributed to the inter-group externality that the signed buyers impose on the unsigned buyers. The profitability of MS can be attributed not only to this inter-group externality but also to the intra-group externality that the signed buyers impose on each other.*

Proposition 1 lays the groundwork for what follows. It implies that exclusion will be profitable for the incumbent if and only if no one buyer is sufficient to support entry with probability one. The sufficiency of this condition should be clear. The reason for the “only if” in the proposition is that if one buyer alone were sufficient to support entry, there would be no inter- or intra-group externalities for the incumbent to exploit. There would be no inter-group externalities because the incumbent would have to sign up all buyers. There would be no intra-group externalities because each signing buyer would have to be fully compensated for its expected loss. Any gain from the exclusion would thus be offset by an equal lump-sum cost to induce each buyer to sign.

Proposition 1 also implies that MS contracts work somewhat differently from ED contracts in the sense that MS imposes two types of externalities on buyers, whereas ED imposes only one.²⁴ Although the inter-group externality that signed buyers impose on unsigned buyers is

²⁴Alternatively, one can think of there being one externality, the externality that *committed purchases* impose

well known from SW, the intra-group externality that signed buyers impose on each other is not. Its existence may even invite skepticism because it is difficult to understand how the incumbent can induce buyers who are able to coordinate to sign and yet still be able to profit from them. The insight that emerges, however, is that the incumbent only has to compensate each signing buyer for the loss that its own signing has on its own expected surplus. This leads to a prisoners' dilemma. Even when all other buyers are expected to reject the incumbent's offer, any one buyer can be induced to accept the incumbent's offer as long as it is compensated not only for the expected loss on its committed purchases but also for the reduction in the probability of entry that its signing has on the expected surplus of its uncommitted purchases.²⁵ The gain to the incumbent from these same purchases, however, is larger because of the larger reduction in the probability of entry that occurs when *all* K buyers sign (not just the marginal buyer). There is thus potentially an additional source of profit arising under MS that is not present under ED.

This additional source of profit has strong implications for the relative profitability of ED and MS. In its absence, if the only source of profit was from the unsigned buyers, ED would always dominate MS because $\Pi^U(K, 1) \geq \Pi^U(K, s)$ for all $K \leq N$ and all $s \leq 1$. With it, however, the profit the incumbent receives under MS from the signed buyers may be more than enough to offset MS's relative disadvantage on the unsigned buyers. It depends on how many signed buyers there are relative to the total number of buyers in the market. When the number of signed buyers is fixed at $K = 1$, we know that ED always weakly dominates MS because $\Pi^U(1, 1) \geq \Pi^U(1, s)$ for all $s \leq 1$ and there is no offsetting intra-group externality (because there needs to be at least two signed buyers for this externality to arise), whereas when the number of signed buyers is fixed at $K = N$, we know that MS always weakly dominates ED (because there are no unsigned buyers in this case, and hence, the incumbent's actual profit under ED is therefore always zero).

The existence of the intra-group externality also has strong implications for the incumbent's optimal price p . It can explain, for instance, why this price might be expected to differ between ED and MS. Under ED, the incumbent's expected profit is independent of the particular price p that is specified in the contract because its profit comes solely from the unsigned buyers. Increases or decreases in p which would otherwise affect the signed buyers' surplus, and therefore the incumbent's profit, are simply offset one for one by corresponding increases or decreases in x . This is *not* the case under MS, however, where, because of the intra-group externality,

on *uncommitted purchases*. The difference between ED and MS is that under ED, buyers who sign have no uncommitted purchases, whereas under MS, buyers who sign have committed purchases and uncommitted purchases.

²⁵In our example with $x^*(s, p) = x_{\Omega+1}$, this amount is equal to $(1 - \alpha_{\Omega+1}(s))(1 - s)(p - c)$. In contrast, the incumbent's expected gain from each signed buyer's uncommitted purchases is equal to $(1 - \alpha_K(s))(1 - s)(p - c)$.

increases in p do not have to be offset one for one by decreases in x . This can be seen formally by differentiating $\Pi^S(K, s, p)$ with respect to p and noting that it is strictly positive for all $p \leq v$.²⁶ As a result, we would expect per-unit prices to be weakly higher under MS than under ED, all else being equal, and at the optimum, we would expect the incumbent to set $p = v$ under MS.

4 The fundamental trade-off

Our finding that it is weakly optimal for the incumbent to set $p = v$ under ED and MS (it is strictly optimal under MS) is useful because it allows us to rewrite the incumbent's profit in (5), after grouping common terms, as the sum of the payoff the incumbent gets from the buyers' "uncommitted" purchases and the payoff it gets from the buyers' "committed" purchases:

$$\Pi(K, s, v) = (N - Ks)(1 - \alpha_K(s))(v - c) + K(s(v - c) - x^*(s, v)). \quad (7)$$

The first term represents the incumbent's payoff from the buyers' uncommitted purchases (i.e., the $N - Ks$ units that are not contractually committed to it when the contracts are signed). It is weakly positive because the incumbent receives $v - c$ from each of these units with probability $(1 - \alpha_K(s))$. The second term in (7) represents the incumbent's payoff from the buyers' committed units (i.e., the Ks units that are foreclosed to the entrant when the contracts are signed). For these units, the incumbent pays $x^*(s, v)$ to each signed buyer and receives $s(v - c)$ in return. This term is weakly negative because Lemma 2 implies that $x^*(s, v) \geq s(v - c)$ for all $s \leq 1$.

It follows that (7) consists of a weakly positive term, which can be thought of as the incumbent's benefit from foreclosure, and a weakly negative term, which can be thought of as the incumbent's cost of foreclosure. Both terms depend on the total number of foreclosed units. Interestingly, however, only the second term depends on how the foreclosure is achieved (i.e., on whether ED or MS is offered). To see this, let $\theta \equiv Ks$ denote the total level of foreclosure and recall that $\alpha_K(s) = G((N - Ks)\delta)$. Then, the incumbent's profit in (7) can be re-written as:

$$\tilde{\Pi}(\theta, s) = \underbrace{(N - \theta)(1 - G((N - \theta)\delta))(v - c)}_{\text{benefit from foreclosure}} - \theta \underbrace{\frac{(x^*(s, v) - s(v - c))}{s}}_{\text{cost of foreclosure}}. \quad (8)$$

Maximizing $\tilde{\Pi}(\theta, s)$ is seemingly straightforward. The cost of foreclosure, which depends on both θ and s , is minimized for any given θ at $s = 1$ (this follows directly from Lemma 2),²⁷ and

²⁶See the online appendix for the proof that the incumbent's profit under MS is increasing in p for all $p \leq v$.

²⁷Recall from Lemma 2 that the compensation that is needed to induce buyers to give up their right to purchase from the entrant altogether is $x^*(s, v) = v - c$, whereas the compensation that is needed to induce buyers only to give up their right to purchase $s < 1$ share of their units from the entrant is $x^*(s, v) > s(v - c)$ for all $s < 1$.

the benefit from foreclosure, which depends only on θ , is then maximized at $\theta = K^{ED}$, where

$$K^{ED} \equiv \arg \max (N - K) (1 - G((N - K)\delta)) (v - c).$$

It follows that if the incumbent can directly choose its desired level of θ , ED will always dominate.

The problem is that the incumbent cannot choose θ directly. Under ED, the incumbent can only choose the number of buyers K that will receive its contract — and K^{ED} generically is not an integer. This implies that under ED, the incumbent will almost always miss its mark. If, for instance, $N = 4$ and $K^{ED} = 2.4$ is the unique maximizer of $\tilde{\Pi}(\theta, 1)$, the incumbent would optimally want to foreclose 60% of the market, but would be forced under ED to choose between signing up either two buyers or three buyers (i.e., foreclosing either 50% or 75% of the market).

The situation is markedly different under MS. Under MS, the incumbent can achieve whatever foreclosure level it wants merely by offering its contract to more than K^{ED} buyers and choosing its market-share requirements accordingly. For example, the incumbent can hit the target by offering MS either to three buyers and setting $s = .80$ or all four buyers and setting $s = .60$. However, this embarrassment of riches comes at the cost of having to overpay for the buyers' committed units: unlike with ED, contracts with MS do not minimize the cost of foreclosure.

This highlights a fundamental trade-off between ED and MS, which we can express as follows:

Proposition 2 *ED and MS have different strengths and weaknesses. ED's strength is that it minimizes the per-unit cost of foreclosure. Its weakness is that it does not allow the incumbent to fine-tune the level. MS's strength is that it allows the incumbent to achieve whatever foreclosure level it wants. Its weakness is that the incumbent will have to overpay for the foreclosed units.*

Proposition 2 sheds light on aspects of ED and MS that have not been well understood. Policy makers tend to think of MS as a “poor man’s ED” in the sense that ED is thought to be the more effective of the two at deterring entry — but also more costly. It is thought to be the more effective of the two at deterring entry because the entrant is completely foreclosed from selling to the buyer under ED, whereas only $s < 1$ share of the buyer’s demand is foreclosed under MS. It is thought to be more costly because the buyer will almost surely require a higher compensation under ED than it will under MS.²⁸ According to this logic, if one observes a seller using MS for exclusionary purposes, then it must be that the seller believes that the relatively lower cost of exclusion under MS more than makes up for the relatively lower effectiveness.

²⁸This can be verified in our setting by noting that under ED, the incumbent must compensate each buyer by the amount $p - c$, whereas under MS, $x^*(s, p) = s(p - c) + (1 - \alpha_{\Omega+1})(1 - s)(p - c)$, which is strictly less than $p - c$.

However, our findings suggest that the opposite may be true. Relative to ED, there is reason to believe that foreclosed units will be more costly to obtain under MS (i.e., will be purchased at a premium, not at a discount), and that because buyers are not divisible, but share requirements are, MS will be more effective than ED at achieving foreclosure, not less. This in turn suggests that the reason why we would expect to observe sellers choosing MS over ED for exclusionary purposes is because having the ability to fine tune the level of foreclosure gives MS a significant advantage over ED, which in many settings can more than offset its relative cost disadvantage.²⁹

5 Characterizing optimal contracts

It remains to characterize the optimal ED and MS contracts and to compare foreclosure levels. To set the stage, consider Figure 1 below, which illustrates possible outcomes when the incumbent is unable to achieve its optimal foreclosure level under ED, and thus must settle for second best.

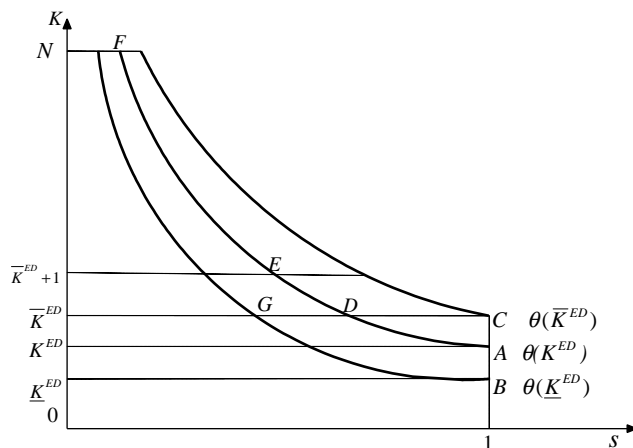


Figure 1: Illustrative iso-foreclosure loci

Depicted in Figure 1 are three iso-foreclosure loci. Each locus represents combinations of K and s that yield the same foreclosure level. The first-best solution occurs at Point A, which corresponds to $\theta = K^{ED}$ and $s = 1$. If this point is not attainable, and the incumbent wants to continue with ED to minimize costs, it can opt for a point such as Point B, which has a foreclosure level of $\theta = \underline{K}^{ED}$, or for a point such as Point C, which has a foreclosure level of $\theta = \overline{K}^{ED}$, where \underline{K}^{ED} and \overline{K}^{ED} are the two closest integers to K^{ED} . Both points are clearly compromises, which

²⁹We illustrate the fundamental trade-off between ED and MS in appendix E, where we work through two examples. In the first example, MS dominates ED. In the second example, ED dominates MS.

may result in a substantial loss in profit relative to the first best. Alternatively, the incumbent can use MS to obtain any foreclosure level it wants, albeit at a higher cost per-unit of foreclosure. To achieve the same foreclosure level as in Point A, for example, the incumbent can offer MS to (i) $K = \bar{K}^{ED}$ buyers and set $s = \frac{K^{ED}}{\bar{K}^{ED}}$ (Point D), (ii) $K = \bar{K}^{ED} + 1$ buyers and set $s = \frac{K^{ED}}{(\bar{K}^{ED} + 1)}$ (Point E), or (iii) all N buyers and set $s = \frac{K^{ED}}{N}$ (Point F). And similarly, it can achieve other foreclosure levels as well (for example, it can implement Point G by offering MS to \bar{K}^{ED} buyers). Once again, however, the losses in profits relative to the first best at Point A can be significant.

Whether the losses relative to the first best are higher under ED or MS depend on the distribution of f . What we can say in general is that for all levels of foreclosure that are attainable under ED, ED dominates MS (this is because for any given θ , ED minimizes the total cost of foreclosure). Thus, for example, we know that Point B dominates all other points on the iso-foreclosure curve with total foreclosure $\theta = \underline{K}^{ED}$, and we know that Point C dominates all other points on the iso-foreclosure curve with total foreclosure $\theta = \bar{K}^{ED}$. What are not as easy to compare are the MS combinations that lie on iso-foreclosure curves that are *unattainable* under ED. Thus, for example, it is not possible to say in general whether the incumbent's profit under MS at point F is higher or lower than its profit under ED at points B and C, nor is it possible to compare among points D, E, and F because although all three points lie on the same iso-foreclosure curve, one cannot say in general which will be associated with the lowest cost.

To gain further insights, we will assume in what follows that (i) $G(\cdot)$ is differentiable, and (ii) $zG''(z) > -2G'(z), \forall z \in [0, \bar{N}\delta]$. The latter is necessary and sufficient to ensure that the incumbent's profit under ED is strictly concave in θ for all θ . For example, it holds if $G(\cdot)$ is weakly convex, and may hold if $G(\cdot)$ is weakly concave, as long as it is not too concave. Condition (ii) is also quite useful because it implies that the per-unit cost of foreclosure is decreasing in s :

Lemma 3

- *The incumbent's profit under ED, $\tilde{\Pi}(\theta, 1)$, is concave in θ for all θ .*
- *The incumbent's per-unit cost of foreclosure is decreasing in s for all θ .*

Using the result that $\tilde{\Pi}(\theta, 1)$ is concave in θ , and that the incumbent's cost of foreclosure is uniquely minimized at $s = 1$, it follows from the definition of K^{ED} as the argmax of $\tilde{\Pi}(\theta, 1)$ that

$$\tilde{\Pi}(\underline{K}^{ED}, 1) > \tilde{\Pi}(\theta, 1) > \tilde{\Pi}(\theta, s), \quad \text{for all } \theta < \underline{K}^{ED} \text{ and } s < 1,$$

and

$$\tilde{\Pi}(\bar{K}^{ED}, 1) > \tilde{\Pi}(\theta, 1) > \tilde{\Pi}(\theta, s), \quad \text{for all } \theta > \bar{K}^{ED} \text{ and } s < 1.$$

These conditions in turn imply that the optimal foreclosure level under ED will either be at $\theta = \underline{K}^{ED}$ (the integer just below K^{ED}) or $\theta = \overline{K}^{ED}$ (the integer just above K^{ED}), as any other foreclosure level will be farther away from the unconstrained maximum at $\theta = K^{ED}$. They also imply that in any setting in which MS is more profitable than ED, the optimal foreclosure level under MS must be such that $\underline{K}^{ED} < \theta < \overline{K}^{ED}$ (otherwise, ED would do better on both maximizing the benefits and minimizing the costs). In terms of Figure 1, it follows that the optimal foreclosure level under ED will either be at Point B or Point C. And it follows that the optimal foreclosure level under MS, whenever MS is more profitable than ED, will be above the iso-foreclosure locus ending in Point B and below the iso-foreclosure locus ending in Point C.

Using the result that the benefit from foreclosure is independent of s , and that, for any given θ , the cost of foreclosure is decreasing in s , it follows that, for any given θ , an increase in s (equivalently, a decrease in K) will increase the incumbent's profit. This means that the incumbent's profit will be increasing from left to right along the iso-foreclosure loci in Figure 1 (Points E and F will be dominated by Point D). It also means that since $\theta \in (\underline{K}^{ED}, \overline{K}^{ED})$ when MS is more profitable than ED, the incumbent's profit will be maximized when MS is given to \overline{K}^{ED} buyers. If MS is given to fewer buyers, then $\theta \in (\underline{K}^{ED}, \overline{K}^{ED})$ cannot be achieved. If MS is given to more buyers, then $\theta \in (\underline{K}^{ED}, \overline{K}^{ED})$ can be achieved, but not in the least cost way.

Summarizing the discussion in this section thus leads to the following proposition:

Proposition 3 *There exist settings in which ED is more profitable than MS, and settings in which MS is more profitable than ED. When ED is more profitable than MS, the optimal contract has $s = 1$ and is offered to either \underline{K}^{ED} or \overline{K}^{ED} buyers. When MS is more profitable than ED, the optimal contract has $s \in \left(\frac{\underline{K}^{ED}}{\overline{K}^{ED}}, 1 \right)$ and is offered to exactly \overline{K}^{ED} buyers. In the former case, θ is either \underline{K}^{ED} or \overline{K}^{ED} . In the latter case, θ lies between $\theta = \underline{K}^{ED}$ and $\theta = \overline{K}^{ED}$. The profit-maximizing foreclosure level under MS can thus sometimes be higher than under ED.*

In characterizing the optimal contracts under both ED and MS, we are now able to address some outstanding questions. One of the more basic questions is how should the incumbent use MS to achieve its desired foreclosure level when there is more than one way to do it? For example, in foreclosing 70% of an eight-buyer market, is it better for the incumbent to offer $s = .933$ to six buyers, $s = .80$ to seven buyers, or $s = .70$ to eight buyers? Does it even matter for the incumbent's profits? The answer, according to Proposition 3 is that it does matter, for cost-minimizing reasons, and that the least costly way for the incumbent to do it is to offer MS to the minimum number of buyers. It follows that the incumbent should offer $s = .933$ to six buyers.

We can also get a sense from Proposition 3 as to how large one would expect the share requirements to be when MS is offered. The prescription that MS should be offered to the minimum number of buyers necessary to achieve a given foreclosure level suggests that s will be on the high side, and this is made more precise by noting that the optimal s is indeed such that

$$s > \frac{\underline{K}^{ED}}{\overline{K}^{ED}} = \frac{\underline{K}^{ED}}{\underline{K}^{ED} + 1}.$$

This provides a lower bound on s , and it implies, for example, that $s > 3/4$ for all $\underline{K}^{ED} \geq 3$.³⁰

Notice finally that the optimal number of signed buyers under MS is weakly higher than the optimal number of signed buyers under ED. This is an important finding because if it were not true, ED would always give rise to higher foreclosure levels and thus would always be worse for welfare. As it turns out, however, Proposition 3 implies that MS can sometimes be worse. In particular, it shows that simple rules of thumb such as “ED is likely to be more powerful than MS because it forecloses a greater fraction of the market to the entrant” are not only misleading, but can often be wrong. When MS is more profitable than ED, the optimal exclusive contract is given to \overline{K}^{ED} buyers, and the optimal foreclosure level is strictly greater than \underline{K}^{ED} but less than \overline{K}^{ED} . By contrast, under ED, the optimal exclusive contract is given to either \underline{K}^{ED} or \overline{K}^{ED} buyers, with corresponding foreclosure levels of \underline{K}^{ED} and \overline{K}^{ED} , respectively. MS thus results in a higher foreclosure level relative to the former case, and a lower foreclosure level relative to the latter case. This suggests that welfare under MS can indeed be worse than welfare under ED.

6 Extensions

In this section, we consider three extensions to the model. First, we extend the model to allow for downward-sloping demands. Second, we consider the implications for the relative profitability of ED and MS of increasing the number of buyers N . Third, we examine whether the incumbent can do better if, instead of offering MS to all \overline{K}^{ED} buyers, it offers ED to the first \underline{K}^{ED} buyers and MS only to the last buyer. Details of these extensions can be found in the online appendix.

6.1 Elastic demand

We have assumed that each buyer demands a fixed amount of the good. This assumption simplifies the analysis because it means that the incumbent only needs to care about how many

³⁰This accords with minimum-share requirements that one observes in practice. For example, in the Intel case, *NEC* was required to purchase no less than 80% of its CPU needs from Intel, and *HP* was required to purchase no less than 95% of its needs from Intel. See the European Commission’s Decision C(2009) 3726, (May 13, 2009).

buyers to sign up and what type of contract to give them. Nevertheless, it is interesting to ask what would happen if each buyer faces a downward-sloping demand curve. One conjecture is that when buyers have downward-sloping demands, the incumbent may be able to solve the integer problem almost entirely without MS. The underlying idea in this case is that with elastic demand, price adjustments may increase the incumbent's flexibility to fine tune the level of foreclosure.

To consider this, let $q(p)$ denote each buyer's demand, $\pi(p) = (p - c)q(p)$ denote the incumbent's profit per buyer gross of any fixed payments, and p_m denote the monopoly price. Suppose the incumbent offers ED to K buyers and gives them a payment, $\hat{x}^*(1, p)$, that is just sufficient to induce them to accept ($\hat{x}^*(1, p)$ is analogous to $x^*(1, p)$ but with downward-sloping demands). Then, the incumbent's problem in Period 1 under ED is to choose K and $p \in [c, v]$, to maximize:

$$\hat{\Pi}(K, 1, p) = K\hat{\Pi}^S(K, 1, p) + (N - K)\hat{\Pi}^U(K, 1), \quad (9)$$

where

$$\begin{aligned} \hat{\Pi}^U(K, 1) &= (1 - G(N - K)\delta q(c))\pi(p_m), \\ \hat{\Pi}^S(K, 1, p) &= \pi(p) - \hat{x}^*(1, p). \end{aligned}$$

In this case, we note that the profit from the unsigned buyers makes use of the fact that the incumbent will charge them p_m in the event the entrant does not enter, and that the profit from the signed buyers will necessarily be weakly negative for all $p \geq c$, with equality if and only if $p = c$ (where $\pi(p) - \hat{x}^*(1, p)$ is the deadweight loss arising from the downward-sloping demand).

From here, it should be clear that because p appears only in $\hat{\Pi}^S(K, 1, p)$, maximizing $\hat{\Pi}(K, 1, p)$ entails setting $p = c$ (to eliminate the signed buyers' deadweight loss) and choosing K to solve $\max_K (N - K)(1 - G(N - K)\delta q(c))\pi(p_m)$. The optimal K in this case is generically not an integer. It follows that the conjecture is not correct. Normalizing the number of units to make $q(c) = 1$, we can see that if the optimal number of buyers in the fixed demand case was not an integer, then the optimal number of buyers in the elastic demand case will also not be an integer.

One way to think about this is to note that what matters for the incumbent's success in entry deterrence is not how many units the incumbent sells to the signed buyers, but rather how many units are potentially available to the entrant. This number is independent of the incumbent's price p under ED. Another way to think about it is to note that K^{ED} does not depend on p .

6.2 Increasing N

We have focused on a small number of buyers in our examples because the ability of the buyers to coordinate with each other is more plausible when N is small. Nevertheless, one might think

that this focus biases the outcomes in favor of MS. More specifically, one might think that the integer problem would be decreasing in importance as the number of buyers increases, and that as a result, increases in N would tend to favor ED over MS because there would then be less of a need for the incumbent to fine tune the level of foreclosure. This reasoning is not correct.

The problem with the reasoning is that it implicitly assumes that the percentage of the market that the incumbent would like to foreclose is fixed. What matters, however, is not what percentage of the market to foreclose, but rather how many units to foreclose and thus how many units to leave uncommitted. To see this, note that the incumbent's profit under ED is given by

$$\begin{aligned}\tilde{\Pi}(\theta, 1) &= (N - \theta)(1 - G((N - \theta)\delta))(v - c) \\ &= (N - K)(1 - G((N - K)\delta))(v - c),\end{aligned}\tag{10}$$

which, for a given δ and $v - c$, depends only on $N - K$. It follows that maximizing $\tilde{\Pi}(\theta, 1)$ with respect to K is equivalent to maximizing it with respect to $N - K$, and thus, from the definition of K^{ED} , it is optimal for the incumbent to leave $N - K^{ED}$ units uncommitted. It also follows that this optimum does not change when N changes. If, for example, N were to increase by ΔN , the unconstrained optimum would simply increase from K^{ED} to $K^{ED} + \Delta N$ and the incumbent would realize the same profit as before. It cannot do better because if it could increase its profit by making a different number of offers, then K^{ED} would not have been optimal in the first place.

It follows that increasing N does not affect the incumbent's maximized profit under ED. As we show in the appendix, however, its maximized profit under MS will be affected, and positively so. To understand this, note that with the optimal number of signed buyers increasing one for one with an increase in N , the profit-maximizing share under MS will necessarily also be increasing, which means, per Lemma 3, that the per-unit cost of foreclosure will be decreasing.³¹ The incumbent will thus be able to obtain the same level of foreclosure more cheaply. This means that increases in N favor MS over ED, not the other way around, as one might have thought.

6.3 Mixed ED with MS

We have assumed that the incumbent must offer the same contract terms to all buyers. This assumption greatly simplifies the analysis and allows us to present the main ideas in the most transparent manner. It can be justified in the presence of regulatory constraints, and/or in settings in which contract terms are not easily observable and seller opportunism is a concern.

³¹To illustrate the main ideas, suppose that $N = 4$, that $K^{ED} = 2.4$, and that under the optimal MS, the incumbent signs up three buyers and set $s = .80$. Now let N increase to $N = 7$, Then the new K^{ED} will be 5.4, and under the optimal MS, the incumbent will need to sign up six buyers and set with $s = 5.4/6 = .90$.

Nevertheless, one might conjecture that if the incumbent could offer different contracts to different buyers, then the optimal way to foreclose would be to offer ED to \underline{K}^{ED} buyers and give MS to the last buyer with $s = K^{ED} - \underline{K}^{ED}$. The idea is that this would still allow MS to fine tune the level of foreclosure, but would have the advantage of allowing the incumbent to obtain the ‘interior’ units more cheaply (since ED is better at minimizing costs on a unit per unit basis).

However, what this argument misses is that the overpay to the last buyer (for obtaining its foreclosed units) will be higher, perhaps significantly higher, than the overpay would have been if all \overline{K}^{ED} buyers had received MS. In both cases, the same number of units are foreclosed, implying that the benefit from foreclosure will be the same, but whether the cost of foreclosure will in general have increased or decreased is ambiguous. The per-unit cost of foreclosing the first \underline{K}^{ED} buyers will have decreased, as these buyers will now have 100% of their demand foreclosed (and we know that ED minimizes the cost of foreclosure), but the incumbent’s per-unit cost to obtain the last buyer’s fixed number of units will have increased (see Lemma 3), given that the share s to this buyer will have decreased. On balance, it is not possible to say whether the sum of the cost changes will be positive or negative (in the online appendix, we construct a simple example to show that the incumbent’s profit may be substantially less under the proposed alternative).

7 Conclusion

This paper uses the naked-exclusion framework of RRW-SW to study the optimal design of exclusionary contracts for an incumbent seller when both market-share contracts (MS) and exclusive-dealing contracts (ED) are feasible. We ask whether it is better for the incumbent to require some of the buyers to buy all of their purchases from the incumbent, or some or all of the buyers to buy only a fraction of their purchases from the incumbent. Consistent with observing both ED and MS in practice, we have shown that sometimes the former is better, and sometimes the latter is better. The tradeoff is that whereas MS has an advantage in that it can fine tune the level of foreclosure to achieve foreclosure levels that ED cannot, ED has an advantage in that the per-unit cost of foreclosure under ED is relatively cheaper than it is under MS. Depending on the distribution of entry costs, we have shown that either of these two effects can dominate.

We have also shown that the profitability of exclusionary contracts comes from two sources. There is an inter-group externality that signed buyers impose on unsigned buyers, and an intra-group externality that signed buyers impose on each other. Whereas the profitability of ED arises solely from the former, the profitability of MS stems from both types of externalities.

This difference in how the contracts work gives rise to some useful comparative-static im-

plications. Because the incumbent does not earn profit from the signed buyers under ED, the per-unit price p given to these buyers plays no role. Increases or decreases in p are simply offset by corresponding increases or decreases in lump-sum compensation. However, under MS, the incumbent does care, and in general it would prefer that p be as high as possible. This suggests that MS would be expected to be associated with relatively higher per-unit prices all else equal.

Another comparative-static implication relates to the effect of an increase in the number of buyers on the incumbent's profitability. Under ED, there is no effect. Increases in the number of buyers are offset by increases in the number of signed buyers, with no change in profit (albeit the share of the market that is foreclosed is higher). In contrast, increases in the number of buyers increase the profitability of MS. With more signed buyers, the amount that has to be paid to any one buyer for the reduction it causes in the probability of entry decreases, and hence, an increase in the number of buyers reduces the amount by which each buyer has to be overcompensated.

We believe that our findings have important implications for how public policy should view MS. Our experience has been that policy makers tend to focus on the adverse effects of foreclosure on a buyer per-buyer basis, giving rise to the view that whenever MS is observed, it will not be as effective, or as powerful, at foreclosing an entrant as ED would have been (hence, the perceived need by plaintiffs to argue that the minimum-share required by the incumbent in their given setting was effectively the same as exclusive dealing). We have argued instead that this view is misleading and ignores the endogeneity of the incumbent's decisions. What matters for the incumbent's decisions are the level of foreclosure (which determines the likelihood of entry) relative to the cost of foreclosure (i.e., the cost of compensating the buyers for accepting the seller's exclusionary terms), and that what when viewed through this lens, market-share contracts are not the poor-man's exclusive dealing that they have been made out to be. Moreover, we have shown that, because the incumbent will sign up weakly more buyers under MS than would have been the case under ED, the level of foreclosure may well be higher under MS than under ED. This suggests that MS, not ED, may well be the more powerful of the two at inducing exclusion.

Appendix

A: Proof of Lemma 1

(1): Suppose the incumbent offers contract $C = \{s, x, p\}$ to K buyers with $x > x^*(s, p)$. Notice that $x^*(s, p) \equiv \max_{n \leq K} \{x_n(s, p)\}$ is the maximum inducement among all x_n for all $n \leq K$. Then, for all $n \in \{1, 2, \dots, K\}$, $x > x^*(s, p)$ implies $U_A(n) > U_R(n - 1)$. To show that all K buyers accept the incumbent's contract in the unique PCPNE of the continuation game, we first show that all buyers accepting the incumbent's contract forms a PCPNE. Suppose all buyers accept the incumbent's contract. Then, $U_A(K) > U_R(K - 1)$ implies that a unilateral deviation is not profitable. Suppose now a coalition of $k \geq 2$ buyers deviates and jointly rejects the incumbent's contract. Then each buyer in the coalition earns payoff $U_R(K - k)$. However, this joint deviation is not self-enforcing given that a buyer in this coalition can make itself better off by deviating from the coalitional action and accepting the incumbent's contract, since, in this case, $U_A(n) > U_R(n - 1)$ for any $n \in \{1, 2, \dots, K\}$ implies $U_A(K - k + 1) > U_R(K - k)$. Therefore, in this case, there is no self-enforcing coalitional deviation that can benefit buyers.

Secondly, we must show that no other PCPNE of the continuation game exists. Suppose to the contrary that there exists a PCPNE where k buyers, $k \neq K$, accept the contract while the other $K - k$ buyers reject the contract. Then each buyer in the coalition $K - k$ who jointly rejects the contract can benefit from a unilateral deviation to accept the contract; doing so will bring more surplus, $U_A(k + 1) > U_R(k)$, and therefore this cannot form an equilibrium.

(2): Suppose the incumbent offers a contract to K buyers with $x < x^*(s, p)$, and K buyers accept the contract. We show this cannot form a PCPNE. Let $x_m(s, p) = x^*(s, p)$, for some $1 \leq m \leq K$, then $x < x_m(s, p)$ implies that $U_A(m) < U_R(m - 1)$. Since $U_A(K) \leq U_A(K - 1) \leq \dots \leq U_A(m) < U_R(m - 1)$, this implies that $K - m + 1$ buyers could jointly deviate by rejecting the contract offer and make themselves better (each buyer now earns $U_R(m - 1)$). To show that such coalitional deviation is self-enforcing, first note that unilateral deviation from this coalition is not profitable since $U_R(m - 1) > U_A(m)$. Suppose now there exist $j \geq 2$ buyers in this coalition that want to deviate and jointly accept the incumbent's offer, by which each member can earn payoff $U_A(m - 1 + j)$. However, this joint deviation from the coalition is not profitable since $U_A(m - 1 + j) \leq U_A(m) < U_R(m - 1)$.

(3): Suppose the incumbent offers a contract to K buyers with $x = x^*(s, p) = x_m(s, p)$, and K buyers accept the contract, then each signed buyer gets $U_A(K)$. Consider two cases:

Case A: $x = x^*(s, p) = x_m(s, p)$ and $p > c$. As $x = x^*(s, p) \geq x_K(s, p)$ implies $U_A(K) \geq U_R(K - 1)$, the unilateral deviation is not profitable. Suppose now a subset of k buyers jointly

reject the offer, in which each deviated buyer obtains $U_R(K - k)$. But $x = x^*(s, p) \geq x_{K-k+1}$ implies $U_R(K - k) \leq U_A(K - k + 1)$, and thus such coalition deviation is not self-enforcing.

Case B: $x = x^*(s, c) = 0$ and $p = c$. Then $p_a = p = c$ implies $U_A(n) = \alpha_n(v - p_a) + (1 - \alpha_n)(v - p) + x = v - c$ for all $n \leq K$, that is, each buyer is secured a payoff $v - c$ in this case. On the other hand, $U_R(n) = \alpha_n(v - c) \leq v - c$. To show that K buyers accepting the incumbent's contract forms a PCPNE, note that $U_A(K) \geq U_R(K - 1)$ implies that a unilateral deviation is not profitable. Moreover, there is no coalitional deviation of $k \geq 2$ buyers that is profitable because $U_R(0) = U_A(n) = v - c$ for all n , and $U_A(n) \geq U_R(n - 1)$ implies $U_A(K) \geq U_R(K - k)$.

Q.E.D.

B. Proof of Lemma 2

Suppose the incumbent offers contract $C = \{s, x, p\}$ to K buyers. Consider three cases:

- (1) If the contract specifies $s = 1$, then, $p_a = p$, and

$$x_n(1, p) = \alpha_{n-1}(v - c) - (v - p).$$

It follows that $x_n(1, p)$ decreases with the number of signed buyers n and thus $x^*(1, p) = x_{\Omega+1}(1, p) = p - c$ (note that $\alpha_\Omega = 1$).

(2) If the contract specifies $s < 1$, but signing up K buyers does not reduce the probability of entry, i.e., $\alpha_K(s) = 1$, then $\alpha_n(s) = 1$ for all $n \leq K$. This implies $x_n(s, p) = p_a - c = s(p - c)$, for all $n \leq K$, and thus $x^*(s, p) = s(p - c)$.

(3) If the contract specifies $s < 1$, and signing up K buyers reduces effectively the probability of entry, i.e., $\alpha_K(s) < 1$. There exists a cut-off number \hat{n} with $\Omega < \hat{n} < K$ such that $\alpha_n(s) = 1$ for all $n \leq \hat{n}$ and $\alpha_n(s) < 1$ for all $n > \hat{n}$. Then

$$\begin{aligned} x^*(s, p) &\geq x_{\hat{n}+1}(s, p) = s(p - c) + (1 - \alpha_{\hat{n}+1}(s))(1 - s)(p - c) - (1 - \alpha_{\hat{n}}(s))(v - c) \\ &= s(p - c) + (1 - \alpha_{\hat{n}+1}(s))(1 - s)(p - c) > s(p - c), \end{aligned}$$

where we have used the fact that $\alpha_{\hat{n}}(s) = 1$ and $\alpha_{\hat{n}+1}(s) < 1$. **Q.E.D.**

C. Characterization of $x^*(s, p)$

We now characterize sufficient conditions for $x^*(s, p) = x_{\Omega+1}(s, p)$. Using (2), we obtain

$$x_{\Omega+1}(s, p) - x_n(s, p) = (\alpha_\Omega(s) - \alpha_{n-1}(s))(v - c) - (\alpha_{\Omega+1}(s) - \alpha_n(s))(p - p_a).$$

Consider two cases:

(i) If $s = 1$, $p = p_a$, then $x_{\Omega+1}(s, p) \geq x_n(s, p)$ for all $n \geq \Omega + 1$, thus $x^*(1, p) = x_{\Omega+1}(1, p)$ under ED.

(ii) If $s < 1$ and $\alpha_\Omega(s) - \alpha_{\Omega+1}(s) \geq \alpha_{n-1}(s) - \alpha_n(s)$ for all $n \leq K$, then $\alpha_\Omega(s) - \alpha_{n-1}(s) \geq \alpha_{\Omega+1}(s) - \alpha_n(s)$ and $v - c \geq p - p_a$ imply $x_{\Omega+1}(s, p) \geq x_n(s, p)$ for all $n \leq K$. Note that this condition holds when $G(\cdot)$ is weakly convex, as $\alpha_\Omega(s) - \alpha_{\Omega+1}(s) \geq \alpha_{\Omega+1}(s) - \alpha_{\Omega+2}(s) \geq \dots \geq \alpha_n(s) - \alpha_{n+1}(s)$ for all n .

Thus, $x^*(s, p) = x_{\Omega+1}(s, p)$ if ED is offered, or if $G(\cdot)$ is weakly convex under MS. **Q.E.D.**

D. Proof of Proposition 1

Suppose the incumbent offers contract $C = \{s, x, p\}$ to K buyers and all K buyers have accepted the contract in the PCPNE of the continuation game. We show first that $f > \delta$ with positive probability is a necessary condition for profitable exclusionary contracts. Suppose instead that $f \leq \delta$ with probability one. Then, the entrant's profit when facing only one free buyer is sufficiently high to cover the entry cost, that is, $\Pi_E(N - 1, s) = (N - 1)(1 - s)\delta + \delta \geq \delta$, and the entrant will enter with probability one as long as there is one unsigned buyer. This implies that, for all $K < N$ and $s \leq 1$, $\alpha_K(s) = 1$, and by Lemma 2 $x^*(s, p) = s(p - c)$, thus

$$\Pi^S(K, s, p) = s(p - c) + (1 - \alpha_K(s))(1 - s)(p - c) - x^*(s, p) = 0.$$

Hence, offering exclusionary contracts to $K < N$, whatever ED or MS, is not profitable.

Obviously, signing up all buyers with ED is not profitable. If instead the incumbent signs up $K = N$ buyers with MS, it must pay each signed buyer at least $x^*(s, p) \geq x_N(s, p) = s(p - c) + (1 - \alpha_N(s))(1 - s)(p - c)$, and the incumbent ends up with non-positive profit since

$$\begin{aligned} \Pi^S(N, s, p) &= s(p - c) + (1 - \alpha_N)(1 - s)(p - c) - x^*(s, p) \\ &\leq s(p - c) + (1 - \alpha_N)(1 - s)(p - c) - x_N(s, p) = 0. \end{aligned}$$

We show second that $f > \delta$ with positive probability is also a sufficient condition. Suppose that $f > \delta$ with positive probability. There exists some $\gamma > 0$ such that $\Pr\{f \geq \delta + \varepsilon\} = \gamma$, where ε can be an arbitrarily small number. Then, signing up $N - 1$ buyers with ED reduces effectively the probability of entry, that is, $\alpha_K(1) = \alpha_{N-1}(1) = \Pr\{f < \delta\} \leq 1 - \gamma < 1$, and thus $\Pi^U(N - 1, 1) > 0$.

To see that the incumbent can make a positive profit by offering a market share contract, it suffices to show that there exists some $s < 1$ such that $\Pi^U(N - 1, s) > 0$, since the profit from signed buyers is always non-negative. Note that the entrant's profit when $N - 1$ buyers have

signed the contract is given by $\Pi_E(N-1, s) = (N-1)(1-s)\delta + \delta$, then choosing any market share such that

$$s > \hat{s} \equiv 1 - \frac{\varepsilon}{(N-1)\delta}$$

ensures that $\Pi_E < \delta + \varepsilon$, which reduces effectively the probability of entry to $\alpha_{N-1}(s) < 1 - \gamma < 1$. Thus, $\Pi^U(N-1, s) > 0$ for all $s > \hat{s}$, and the incumbent earns a positive profit. **Q.E.D.**

E. Numerical Examples

We will now work through two illustrative examples. In both examples, there are $N = 3$ buyers, $v - c = 1000$, and $\delta = 100$. Only the distribution of entry costs is different.

Example 1: MS dominates ED

Suppose that f can take on one of five values, with the listed probabilities:

$$\begin{array}{l} f : \quad 50, \quad 100, \quad 150, \quad 225, \quad 275 \\ g : \quad 0.1, \quad 0.1, \quad 0.6, \quad 0.1, \quad 0.1. \end{array}$$

In the absence of ED and MS, the entrant can earn a flow profit of 300 if it enters. Since this exceeds the maximum value of f , we would expect the entrant to enter with probability one in the absence of any exclusionary contracts. On the other hand, if, for instance, the incumbent can foreclose 1.5 units, then only 1.5 units will be available to the entrant, in which case the entrant's profit will be reduced to 150 and the probability of entry will be reduced to $\Pr\{f < 150\} = 0.2$.³²

Consider ED first. By signing up one buyer, the incumbent can reduce the probability of entry to $\alpha_1(1) = 0.8$, and earn an expected profit of $(3-1)(1-.8)1000 = 400$. Signing up two buyers, however, does even better. The probability of entry is reduced to $\alpha_2(1) = 0.1$, and the incumbent earns an expected profit of $(3-2)(1-.1)1000 = 900$. Signing up all three buyers yields zero. The incumbent will therefore sign up two buyers and earn an expected profit of 900.

Notice that the incumbent could have done even better if it could have signed up 1.5 buyers because then the probability of entry would have been reduced 0.2, and its expected profit would have been $(3-1.5)(1-.2)1000 = 1200$. But this is not possible under ED. With MS, on the other hand, achieving this level of foreclosure is feasible. The incumbent can, for example, offer $s = .75$ to two buyers and achieve the same reduction in the probability of entry, and thus the same maximum benefit from foreclosure. In order to induce the two buyers to accept MS with $s = .75$, however, the incumbent will have to pay each buyer an amount equal to

³²Assume, for simplicity, that the entrant will not enter unless its profit exceeds its fixed cost of entry.

$(1 - \alpha_1(s))(1 - s)(v - c) = 50$. Nevertheless, on balance, this still leaves the incumbent with a profit of $1200 - 100 = 1100$, which is more than 20% higher than what it can earn under ED.³³

Example 2: ED dominates MS

Now suppose that the distribution of f is modified as follows:

$$\begin{aligned} f : & 50, & 100, & 150, & 225, & 275 \\ g : & 0.1, & 0.25, & 0.45, & 0.1, & 0.1. \end{aligned}$$

Notice that this modification does not affect the incumbent's maximum profit under exclusive dealing. It is still 900. The modification also does not affect the over pay under MS when $s = .75$. It is still 100. However, it does change the probability of entry when 1.5 units are foreclosed. Instead of 0.2, it is now 0.35, which implies that the incumbent's maximum benefit from foreclosure is now equal to $(3 - 1.5)(1 - .35)1000 = 975$. The incumbent's maximum profit under MS is therefore only $975 - 100 = 875$ in this case, which is less than it can earn under ED.

The examples illustrate that whether ED or MS dominates depends on whether the gain from being able to realize the optimal foreclosure level under MS is enough to offset the loss from having to overpay for the buyers' committed units. In the first example, where the benefit from realizing the foreclosure level of 1.5 is relatively greater, MS dominates. In the second example, where the benefit from realizing the foreclosure level of 1.5 is relatively less, ED dominates.

F. Proof of Lemma 3

We show first that the incumbent's profit under ED, $\tilde{\Pi}(\theta, 1)$, is concave in θ for all θ if and only if $zG''(z) > -2G'(z), \forall z \in [0, \bar{N}\delta]$. Recall that

$$\tilde{\Pi}(\theta, 1) = (1 - G((N - \theta)\delta))(N - \theta)(v - c).$$

Differentiating $\tilde{\Pi}(\theta, 1)$ with respect to θ , we obtain the first-order and the second-order derivatives respectively

$$\begin{aligned} \frac{\partial \tilde{\Pi}(\theta, 1)}{\partial \theta} &= [(N - \theta)\delta g'((N - \theta)\delta) - (1 - G((N - \theta)\delta))](v - c), \\ \frac{\partial^2 \tilde{\Pi}(\theta, 1)}{\partial \theta^2} &= -[2g'((N - \theta)\delta) + (N - \theta)\delta g''((N - \theta)\delta)]\delta(v - c). \end{aligned}$$

Thus, the profit function is concave if and only if

$$2g'((N - \theta)\delta) + (N - \theta)\delta g''((N - \theta)\delta) > 0.$$

³³It is easy to check that $x^*(.75, v) = x_1(.75, v)$ in this case, where $x_1(.75, v) = .75(v - c) + (1 - \alpha_1(.75)).25(v - c)$.

The above condition holds if and only if $zG''(z) > -2G'(z)$ for $z = (N - \theta)\delta$.

We show second that the incumbent's per-unit cost of foreclosure under MS is decreasing in s . For this purpose, we denote by $C(s) \equiv x^*(s) - s(v - c)$ the per-buyer cost and $AC(s) = C(s)/s$ the per-unit cost of foreclosure. Since

$$\frac{dAC(s)}{ds} = \frac{sC'(s) - C(s)}{s^2},$$

then $AC(s)$ is decreasing in s if and only if

$$\Phi(s) \equiv C(s) - sC'(s) > 0.$$

Consider two cases as follows:

Case (1): Suppose $G(\cdot)$ is weakly convex. From Appendix B, we know that $x^*(s) = x_{\Omega+1}(s)$, which implies

$$C(s) = (1 - s)(1 - \alpha_{\Omega+1}(s))(v - c).$$

Differentiating $C(s)$ with respect to s , we have

$$C'(s) = [-(1 - \alpha_{\Omega+1}(s)) - (1 - s)\alpha'_{\Omega+1}(s)](v - c),$$

and thus

$$\Phi(s) = C(s) - sC'(s) = [1 - \alpha_{\Omega+1}(s) + s(1 - s)\alpha'_{\Omega+1}(s)](v - c).$$

Note that when $s = 0$, $\Phi(0) = 0$, and when $s = 1$, $\Phi(1) = (1 - \alpha_{\Omega+1}(1))(v - c) > 0$. Moreover, $\Phi(s)$ is increasing in s since

$$\begin{aligned} \Phi'(s) &= -\alpha'_{\Omega+1}(s) + (1 - 2s)\alpha'_{\Omega+1}(s) + s(1 - s)\alpha''_{\Omega+1}(s) \\ &= -2s\alpha'_{\Omega+1}(s) + s(1 - s)\alpha''_{\Omega+1}(s) \\ &= 2s(\Omega + 1)\delta g((N - (\Omega + 1)s)\delta) + s(1 - s)(\Omega + 1)^2\delta^2 g'((N - (\Omega + 1)s)\delta) \\ &> 0, \end{aligned}$$

where the inequality comes from the fact that $G(\cdot)$ is weakly convex. It thus follows that $\Phi(s) > 0$ for any $s \in (0, 1]$.

Case (2). Suppose $G(\cdot)$ is weakly concave such that $zG''(z) > -2G'(z)$. Let $x^*(s) \equiv x_m(s) = (\alpha_{m-1}(s) - (1 - s)\alpha_m(s))(v - c)$, then

$$C(s) = x^*(s) - s(v - c) = (\alpha_{m-1}(s) - (1 - s)\alpha_m(s) - s)(v - c),$$

and

$$C'(s) = [\alpha'_{m-1}(s) + \alpha_m(s) - (1 - s)\alpha'_m(s) - 1](v - c),$$

it thus follows that

$$\Phi(s) = C(s) - sC'(s) = [\alpha_{m-1}(s) - \alpha_m(s) - s\alpha'_{m-1}(s) + s(1-s)\alpha'_m(s)](v-c).$$

Note that $\Phi(0) = 0$ when $s = 0$. Note also that $x^*(s) = x_{\Omega+1}(s)$ when $s = 1$ (i.e., $m = \Omega + 1$), thus $\Phi(1) = (1 - \alpha_{\Omega+1}(1))(v-c) > 0$. We show now $\Phi(s)$ is increasing in s , and thus $\Phi(s) > 0$ for all $s \in [0, 1]$.

To see this, differentiating $\Phi(s)$ with respect to s , we obtain

$$\begin{aligned} \Phi'(s) &= \alpha'_{m-1}(s) - \alpha'_m(s) - \alpha'_{m-1}(s) - s\alpha''_{m-1}(s) + s(1-s)\alpha''_m(s) + (1-2s)\alpha'_m(s) \\ &= s(1-s)\alpha''_m(s) - s\alpha''_{m-1}(s) - 2s\alpha'_m(s) \\ &= s \left[2m\delta g((N-ms)\delta) + (1-s)m^2\delta^2 g'((N-ms)\delta) - (m-1)^2\delta^2 g'((N-(m-1)s)\delta) \right] \\ &> s \left[-m\delta^2 g'((N-ms)\delta)(N-ms - (1-s)m) - (m-1)^2\delta^2 g'((N-(m-1)s)\delta) \right] \\ &= -s \left[m\delta^2 g'((N-ms)\delta)(N-m) + (m-1)^2\delta^2 g'((N-(m-1)s)\delta) \right] \\ &> 0, \end{aligned}$$

where we have used the assumption $2g(z) > -zg'(z)$ for $z = (N-ms)\delta$ to derive the first inequality, and the second inequality comes from the fact that $G(\cdot)$ is weakly concave. **Q.E.D.**

G. Proof of Proposition 3

We show that ED can be more profitable than MS and *vice versa* with two illustrative examples.

Example 3: ED is more profitable than MS

Consider a market with three identical buyers. Each buyer desires a fixed amount of the good and the gains from trade are given by $v - c = 1000$. For simplicity, we assume here that the entry cost is distributed between 0 and 3δ , according to the following distribution function

$$\begin{aligned} G(x) &= \frac{0.4x}{\delta}, \text{ for } 0 < x < \delta, \\ &= 0.4 + \frac{0.3(x-\delta)}{\delta}, \text{ for } \delta < x < 3\delta. \end{aligned}$$

That is, with probability 0.4, f is uniformly distributed between 0 and δ , and with probability 0.3, f is uniformly distributed between δ and 3δ . It follows that the probability of entry is given by $G((3-\theta)\delta) = 0.4(3-\theta)$ for $\theta \geq 2$ and $G((3-\theta)\delta) = 0.4 + 0.3(2-\theta)$ for $\theta \leq 2$.

Suppose the incumbent offers a contract $C = \{s, v, x\}$ to K buyers and all these buyers accepted the contract in the equilibrium. Using (8) and noting that $x^*(s) = x_1(s) = s(v-c) +$

$(1-s)(1-\alpha_1(s))(v-c)$,³⁴ we can rewrite the incumbent's profit as

$$\tilde{\Pi}(\theta, s) = 1000(3-\theta)(1-G((3-\theta)\delta)) - \theta \frac{(x^*(s, v) - 1000s)}{s} \quad (\text{A.1})$$

$$= 1000(3-\theta)(1-G((3-\theta)\delta)) - 1000\theta \frac{(1-s)(1-G((3-s)\delta))}{s}, \quad (\text{A.2})$$

where the first term is the benefit from foreclosure whereas the second term is the cost of foreclosure. Maximizing the first term yields the optimal level of foreclosure $\theta^* = K^{ED} = 1.75$,³⁵ and the associated expected benefit is equal to 625. Suppose the incumbent offers ED contract, it is optimal to offer to $\bar{K}^{ED} = 2$ buyers which reduces the cost of foreclosure to zero, and the incumbent's profit is $\tilde{\Pi}(2, 1) = 600$.

Suppose instead the incumbent offers MS to two buyers, its profit is then given by

$$\begin{aligned} \Pi(3, s) &= 1000(3-2s)(1-G((3-2s)\delta)) - 2000(1-s)(1-G((3-s)\delta)) \\ &= 600(3-2s)s - 600(1-s)s, \end{aligned}$$

where the second line is derived by substituting the probability of entry $G((3-\theta)\delta) = 0.4 + 0.3(2-\theta)$. Suppose the incumbent offers $s^* = K^{ED}/2$ to maximize the benefit (which is equal to 625), then the cost of foreclosure is equal to 65.6, and the incumbent's profit is equal to 559.4, which is 7% less than the profit under ED. It appears that the incentive of minimizing the cost dominates the incentive of maximizing the benefit, thus offering ED is optimal in this case.³⁶

Example 4: MS is more profitable than ED

Consider the same setup as above, but let the distribution function now be given by

$$\begin{aligned} G(x) &= \frac{0.8x}{\delta}, \text{ for } 0 < x < \delta, \\ &= 0.8 + \frac{0.1(x-\delta)}{\delta}, \text{ for } \delta < x < 3\delta. \end{aligned}$$

That is, with probability 0.8, f is uniformly distributed between 0 and δ , and with probability 0.1, f is uniformly distributed between δ and 3δ . The incumbent's expected profit is still given by (A.1), but the probability of entry is different: $G((3-\theta)\delta) = 0.8(3-\theta)$ for $\theta \geq 2$ and $G((3-\theta)\delta) = 0.8 + 0.1(2-\theta)$ for $\theta \leq 2$. Maximizing the benefit then yields $\theta^* = K^{ED} = 2.375$ and the associate profit is equal to 312.5.

³⁴It is easy to check that $x_1(s) \geq x_2(s) \geq x_3(s)$ in this example.

³⁵We need to compare the benefit in two cases regarding on $\theta \geq 2$ and $\theta \leq 2$ respectively with different probability of entry. It is straightforward to see that the benefit function is maximized in the second case at $\theta^* = 1.75$.

³⁶Indeed, the profit function can be reduced to $\Pi(3, s) = 600s(2-s)$, which is increasing in s , and it is thus optimal to set $s = 1$.

Suppose the incumbent offers ED, it is optimal to offer ED to $\underline{K}^{ED} = 2$ buyers (giving ED to all buyers yields zero profit), which yields an expected profit

$$\Pi^{ED}(2, 1) = 1000(3 - 2)(1 - G(2\delta)) = 200.$$

While offering ED reduces the cost of foreclosure to zero, the resulting loss of the benefit is equal to 112.5, which accounts for more than 56% of its actual profit.

This suggests that offering MS that maximizes the benefit from foreclosure could do better. Suppose the incumbent offers $\hat{s} = K^{ED}/3$ to three buyers, which achieves the maximum benefit of 312.5. The resulting cost of foreclosure is equal to (using $G((3 - \hat{s})\delta) = 0.8 + 0.1(2 - \hat{s})$)

$$3000(1 - \hat{s})(1 - G((3 - \hat{s})\delta)) = 300(1 - \hat{s})\hat{s} \simeq 49.5,$$

and the incumbent earns a net profit of 263. Thus, the incentive of maximizing benefit from foreclosure dominates the incentive of minimizing the cost. Note that offering MS can increase the incumbent's profit by more than 30% in this case.

Indeed, in this example the optimal MS contract leads to higher foreclosure than θ^* . To see this, we can rewrite the incumbent's expected payoff as (given $s > 2/3$)

$$\begin{aligned} \Pi(3, s) &= 1000[(3 - 3s)(1 - G((3 - 3s)\delta)) - 3(1 - s)G((3 - s)\delta)] \\ &= 3000(1 - s)(2.3s - 1.4). \end{aligned}$$

Maximizing the above profit leads to the optimal market share

$$s^* = \frac{37}{46} \simeq 0.8,$$

and the corresponding expected profit is equal to 264.1.³⁷ Thus, the equilibrium foreclosure, $3s^*$, is greater than θ^* , and thus higher than the foreclosure under ED.

The property that $\tilde{\Pi}(\theta, 1)$ is concave implies that the exclusive dealing contracts must be offered to either \overline{K}^{ED} or \underline{K}^{ED} buyers. We now characterize the optimal market-share contract when it is more profitable than ED. The property that $\tilde{\Pi}(\theta, 1)$ is concave also implies that the optimal foreclosure θ^* is an interior solution satisfying $\Omega\delta < \theta^* < N\delta$, and that the profit function $\tilde{\Pi}(\theta, 1)$ increases in θ for all $\theta < \theta^*$ and decreases in θ for all $\theta > \theta^*$. Thus, the optimal foreclosure under MS must satisfy $\theta \in (\underline{K}^{ED}, \overline{K}^{ED})$. If a market-share contract results in $\theta \leq \underline{K}^{ED}$ (respectively $\theta \geq \overline{K}^{ED}$), then it is strictly dominated by the exclusive dealing contract

³⁷Notice that the inducement is given by $x_n(s) = (\alpha_{n-1}(s) - (1 - s)\alpha_n(s))(v - c)$. Evaluating at s^* , we have $x_1(s^*) = 820$, $x_2(s^*) = 755$, and $x_3(s^*) = 747$. It thus follows that $x_1(s^*)$ is the largest compensation among all.

giving to \underline{K}^{ED} buyers (respectively \overline{K}^{ED} buyers), as the latter results in foreclosure closer to the optimum and at the same time minimizes the cost of foreclosure as $x^*(1) - (v - c) = 0$.

The optimal market-share contract must give to $K \geq \overline{K}^{ED}$ buyers, and by Lemma 3, the cost of foreclosure is decreasing in s , which ensures that offering MS to exactly \overline{K}^{ED} buyers is optimal. Thus, the optimal market share, $s^* = \theta/\overline{K}^{ED}$, must satisfy $s^* \in \left(\frac{\underline{K}^{ED}}{\overline{K}^{ED}}, 1\right)$. **Q.E.D.**

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Online Appendix

“Market-Share Contracts, Exclusive Dealing, and the Integer Problem”

by Zhijun Chen and Greg Shaffer

A. Optimal Pricing under MS

We show that the optimal per-unit price under MS is equal to v . Suppose the incumbent offers $C = \{s, x, p\}$ to K buyers with $s < 1$ and all K buyers accept the contract in the PCPNE of the continuation game. We restrict to the scenario with $p \geq c$, as any price below its unit cost is allegable for predatory pricing and could be prohibited by antitrust laws. Consider two cases:

Case (1): The incumbent charges a price $p \leq v$. In this case, the signed buyers will purchase all of their demands from the incumbent when the entrant does not enter. Note that the profit from unsigned buyers is independent of the committed price p , whereas the profit from signed buyers depends on p . To see further the impact of price change on the incumbent’s profit, let $x^*(s, p) = x_m(s, p) = \alpha_{m-1}(s)(v - c) - (1 - \alpha_m(s))(v - p) - \alpha_m(s)(v - p_a)$ and substitute into the profit from each signed buyer, we have

$$\begin{aligned}\Pi^S(K, s, p) &= s(p - c) + (1 - \alpha_K(s))(1 - s)(p - c) - x^*(s, p) \\ &= (1 - \alpha_{m-1}(s))(v - c) + (1 - s)(\alpha_m(s) - \alpha_K(s))(p - c)\end{aligned}$$

Differentiating $\Pi^S(K, s, p)$ with respect to p , we obtain

$$\frac{\partial \Pi^S(K, s, p)}{\partial p} = (1 - s)(\alpha_m(s) - \alpha_K(s)).$$

The above derivative is strictly positive for any $s < 1$ and $\alpha_m > \alpha_K$, thus, the profit from signed buyers is always increasing in p , and the optimal price under MS is equal to v . If $\alpha_m = \alpha_K$, then $\Pi^S(K, s, p) = (1 - \alpha_{m-1}(s))(v - c)$ and the incumbent’s profit is independent of p .

Case (2): Suppose the incumbent charges a price $p > v$. Then each signed buyer will only purchase s units regardless whether the entrant enters or not, as buying more units incurs more loss to the signed buyer. When the entrant enters, each signed buyer will purchase the residual demand of $1 - s$ units from the entrant at a per unit price c , and obtains a surplus $v - p_a$. When the entrant stays out, each signed buyer will only purchase s units from the incumbent and obtains a (negative) surplus $s(v - p)$. Thus, the expected surplus of a buyer who agrees to

the incumbent's offer when $n - 1$ other buyers are also agreeing to the incumbent's offer is

$$\begin{aligned} U_A(n) &= (1 - \alpha_n) s (v - p) + \alpha_n (v - p_a) + x \\ &= s (v - p) + \alpha_n (1 - s) (v - c) + x. \end{aligned}$$

The expected surplus of this same buyer if it rejects the incumbent's offer is

$$U_R(n - 1) = \alpha_{n-1} (v - c).$$

Thus, the compensation that just makes this buyer indifferent between accepting or rejecting is

$$\tilde{x}_n(s, p) = \alpha_{n-1} (v - c) - s (v - p) - \alpha_n (1 - s) (v - c).$$

Let $\tilde{x}^*(s, p) = \tilde{x}_m(s, p)$ be the maximum payment. Then, the incumbent's profit from each signed buyer is

$$\begin{aligned} \tilde{\Pi}^S(K, s, p) &= s(p - c) - \tilde{x}^*(s, p) \\ &= s(p - c) - \alpha_{m-1} (v - c) + s(v - p) + \alpha_m (1 - s) (v - c) \\ &= (s + \alpha_m (1 - s) - \alpha_{m-1}) (v - c). \end{aligned}$$

It follows that the incumbent's profit is independent of p for any $p > v$. Since $s + \alpha_m (1 - s) - \alpha_{m-1} < 1 - \alpha_{m-1}$, we have

$$\tilde{\Pi}^S(K, s, p) < (1 - \alpha_{m-1}) (v - c) + (1 - s) (\alpha_m - \alpha_K) (v - c) = \Pi^S(K, s, v).$$

Therefore, charging $p > v$ is strictly dominated by setting $p = v$. **Q.E.D.**

B. Elastic demand

Consider now that each buyer faces a downward sloping demand $q(p)$. Let $\pi(p) = (p - c)q(p)$ denote the incumbent's profit gross of any fixed payments, and let p_m denote its argmax.

Proposition A.1 *When the incumbent offers contract $C = \{1, x, p\}$ to a subset of buyers and each buyer faces a downward-sloping demand curve, it is optimal to charge $p = c$. Moreover, the optimal number of signed buyers, K^{ED} , is independent of p , and is not an integer in generic.*

Proof: Suppose the incumbent offers ED to K buyers. If a buyer accepts the contract, it will have to purchase all of its demand from the incumbent at the price p , regardless whether the entrant

enters or not. The signed buyer then gains a surplus of $S(p) + x$, where $S(p) \equiv \int_p^{+\infty} q(v)dv$ denotes the buyer's surplus at price p . If, instead, this buyer rejects the offer jointly with other $K - 1$ buyers, the entrant will enter for sure and it gains a surplus of $S(c)$. Thus, all K buyers will accept the ED contract in PCPNE if and only if the incumbent's inducement x satisfies

$$x \geq \hat{x}^*(p) = S(c) - S(p).$$

When all K buyers accept the incumbent's contract, the entrant's profit is reduced to $\Pi_E(K) = (N - K) \delta q(c)$. Thus, the probability of entry is reduced to $G((N - K) \delta q(c))$. Note that the entrant's profit is not affected by p , because the entrant can only contest for the unsigned buyers, and competition for the unsigned buyers drives the spot-market price down to c .

The incumbent earns $\pi(p)$ from each signed buyer, but it has to offer each a payment of $\hat{x}^*(p) = S(c) - S(p)$. The incumbent's net profit from each signed buyer is thus equal to

$$\begin{aligned} \hat{\Pi}^S(K, 1, p) &= \pi(p) - \hat{x}^*(p) \\ &= \pi(p) + S(p) - S(c), \end{aligned}$$

which is weakly negative for all $p \geq c$, and equal to zero if and only if $p = c$.

On the other hand, the incumbent can exploit from the unsigned buyers due to the inter-group externality that the signed buyers impose on the unsigned buyers. The incumbent charges the monopoly price p_m to each unsigned buyer when the entrant does not enter, and its expected payoff from each unsigned buyer is given by

$$\hat{\Pi}^U(K, 1) = (1 - G((N - K) \delta q(c))) \pi(p_m).$$

Therefore, the incumbent's problem in Period 1 under ED is to choose K and $p \in [c, v]$ to maximize the expected payoff

$$\begin{aligned} \hat{\Pi}(K, 1, p) &= K \hat{\Pi}^S(K, 1, p) + (N - K) \hat{\Pi}^U(K, 1) \\ &= K [\pi(p) + S(p) - S(c)] + (N - K) (1 - G((N - K) \delta q(c))) \pi(p_m). \end{aligned}$$

Since the incumbent's profit from the signed buyers is maximized at $p = c$ (which is equal to zero) while the incumbent's profit from the unsigned buyers is independent of p , it follows that the incumbent's optimal price must be $p = c$ under ED, and the incumbent's profit is given by

$$\hat{\Pi}(K, 1, c) = (N - K) (1 - G((N - K) \delta q(c))) \pi(p_m).$$

Maximizing this with respect to K , it follows that the optimal number of signed buyers, \hat{K}^{ED} , is independent of p . Moreover, when we normalize demand to make $q(c) = 1$, we can easily see that the solution \hat{K}^{ED} is exactly the same as K^{ED} was in the case of inelastic demand.³⁸

Q.E.D.

C. Increasing N

We have shown that increasing N does not affect the incumbent's maximized profit under ED. This implies that if \tilde{K} denotes the optimal number of signed buyers before the increase in N , then $\tilde{K} + \Delta N$ will be the optimal number of signed buyers after the increase in N , and thus $N - \tilde{K}$ and the incumbent's constrained maximized profit under ED will also be unaffected. The intuition for this is simple. Under ED, the incumbent only earns profit from the unsigned buyers. After K optimally adjusts to the increase in N , the number of unsigned buyers does not change, nor is there any change in the probability of entry given that the number of uncommitted units is also unchanged. It follows that the incumbent will not benefit (or lose) from an increase in N .

Proposition A.2 *For a given δ , $v - c$, and distribution of entry costs, the incumbent's profit under ED depends only on the number of uncommitted units. Increases in N therefore have no effect on the incumbent's maximized profit under ED. As N increases, the incumbent simply adjusts the number of offers it makes in order to leave the number of uncommitted units unchanged. Although this increases the number of signed buyers, and thus the percentage of the market that is foreclosed to the entrant, it does not change the probability that the entrant will be deterred.*

Several implications follow from Proposition A.2. First, and most importantly, Proposition A.2 implies that the constraint that arises from K having to be an integer does *not* decrease in importance as the number of buyers increases, contrary to what one might have thought. To see this, suppose N increases from $N = 4$ to $N = 10$. If the flawed reasoning were correct (i.e., if the foreclosure percentage were fixed at 60%), the incumbent would offer ED to exactly six buyers and the integer constraint would no longer be binding. This is, however, clearly not optimal. Proposition A.2 implies that the incumbent should instead optimally foreclose 8.4 units in order to keep the number of uncommitted units at 1.6 units. The gap between the desired foreclosure level and the obtainable foreclosure level is therefore exactly the same as before. That is, it is equal to 0.4 units if ED is offered to $K = 8$ buyers, or 0.6 units if ED is offered to $K = 9$ buyers.

³⁸Note that the constant term $\pi(p_m)$ does not affect \hat{K}^{ED} .

Second, the fact that the gap between the incumbent's desired foreclosure level and its obtainable foreclosure level under ED is independent of N implies that there is just as much of a need to fine tune the level of foreclosure after an increase in N as there was before the increase. This implies that from the benefit side of foreclosure, increases in N favor neither ED nor MS.

Third, from a policy perspective, Proposition A.2 implies that judging the relative harm from exclusion in any given case by focusing on the percentage of the market that is foreclosed, as courts are often inclined to do, is at best misleading. In our stylized examples, the incumbent would ideally like to foreclose 60% of market when $N = 4$, leaving the entrant with only 1.6 uncommitted units, whereas, when $N = 8$, the incumbent would ideally like to foreclose 80% of market (because then $K^{ED} = 6.4$). Although the latter percentage is significantly higher than the former percentage, the number of uncommitted units is the same in the two cases, and therefore it follows that the entrant will neither be better or worse off in one case or the other.

Cost savings and the dilution effect

Our finding that increases in N need not affect the number of uncommitted units under ED also applies to MS. Under MS, the incumbent controls both K and s , and it can always adjust them in such a way as to keep the benefit from foreclosure the same. To see this, note that if $N - \hat{K}\hat{s}$ is the initial number of uncommitted units under MS before the increase in N , and N increases by ΔN , then the incumbent can always choose $K = \hat{K} + \Delta N$ and $s \equiv s^* = \frac{\hat{K}\hat{s} + \Delta N}{\hat{K} + \Delta N} > \hat{s}$ after the increase to achieve the same number of uncommitted units as before.³⁹ Since the benefit from foreclosure under MS, as it was under ED, depends only on the number of uncommitted units, it follows that the benefit from foreclosure need not change under MS when N increases.

Under ED, this finding, along with showing that the incumbent cannot do better, was sufficient to establish that the incumbent's maximized profit under ED is independent of N . It is not the end of the story under MS, however, because an increase in N impacts MS differently from ED. In addition to the benefit from foreclosure, there is also the cost of foreclosure to consider. Under ED, this cost is always zero. Under MS, however, this cost depends on both K and s .

Continuing with our example, where $K = \hat{K}$, $s = \hat{s}$, and f is distributed such that $x^*(\hat{s}, v) = x_{\Omega+1}(\hat{s}, v)$, the incumbent's cost of foreclosure under MS before the increase in N is

$$\begin{aligned} & \hat{K} (x^*(\hat{s}, v) - s(v - c)) \\ &= \hat{K} (1 - \hat{s}) (1 - \alpha_{\Omega+1}(\hat{s})) (v - c). \end{aligned} \tag{A.1}$$

³⁹Note that $N + \Delta N - (\hat{K} + \Delta N) s^* = N + \Delta N - (\hat{K}\hat{s} + \Delta N) = N - \hat{K}\hat{s}$.

After the increase in N , and after K and s adjust to keep the benefit from foreclosure the same (i.e., $K = \hat{K} + \Delta N$ and $s = s^*$), the incumbent's cost of foreclosure under MS is

$$(\hat{K} + \Delta N)(1 - s^*)(1 - \hat{\alpha}_{\Omega+1+\Delta N}(s^*))(v - c),$$

where $\hat{\alpha}_n(s) \equiv G((N + \Delta N - ns)\delta)$ denotes the probability of entry when n out of $N + \Delta N$ buyers sign the incumbent's contract. Substituting in for s^* , and rearranging terms, yields

$$\hat{K}(1 - \hat{s})(1 - \hat{\alpha}_{\Omega+1+\Delta N}(s^*))(v - c). \quad (\text{A.2})$$

Comparing the cost of foreclosure in (A.2) with the cost of foreclosure in (A.1), we can see that the cost of foreclosure in (A.2) will be lower if and only if the probability of entry when the incumbent signs up the first effective buyer is higher after the increase in N than it was before. That is, the incumbent's cost of foreclosure will be lower in (A.2) than in (A.1) if and only if

$$\hat{\alpha}_{\Omega+1+\Delta N}(s^*) > \alpha_{\Omega+1}(\hat{s}). \quad (\text{A.3})$$

Fortunately, this relationship turns out to be relatively easy to establish because the same adjustments in K and s that keep the benefit from foreclosure the same imply that the actual probability of entry before and after the increase in N will be the same, and thus we know that

$$\hat{\alpha}_{\hat{K}+\Delta N}(s^*) = \alpha_{\hat{K}}(\hat{s}). \quad (\text{A.4})$$

Going from the equality in (A.4) to establishing that the inequality in (A.3) holds then follows straightforwardly once it is recognized that after the increase in N , the difference between the actual number of uncommitted units available to the entrant and the number of uncommitted units that are available to the entrant after the first effective buyer is signed is greater than the corresponding difference before the increase in N . After the increase in N , the difference is $s^*(\hat{K} - (\Omega + 1))$ units, whereas before the increase, the difference is only $\hat{s}(\hat{K} - (\Omega + 1))$ units.

The intuition is that the incumbent must sign at least $\Omega + 1$ buyers before the increase in N if it is to have any effect on lowering the probability of entry, whereas after the increase in N , it must sign this many plus an additional ΔN more buyers if it is to have any effect. Each signing buyer's contribution to the initial reduction in the probability of entry is thereby effectively diluted when the number of buyers increases. We call this the *dilution effect*, and it implies that signing buyers do not have to be compensated as much for their contribution toward exclusion.

We can illustrate this effect with the help of our example in which $N = 4$ and $K^{ED} = 2.4$. Under MS, the incumbent can realize the optimal foreclosure level of 2.4 units by offering MS to

three buyers with $\hat{s} = 0.8$. Signing the first buyer in this case reduces the number of uncommitted units to $4 - 0.8 = 3.2$ units. Suppose now that $N = 7$. Here, the incumbent can realize the optimal foreclosure level of 5.4 units by offering MS to six buyers with $s^* = 0.9$ (so that the number of uncommitted units remains at 1.6). Signing the fourth buyer reduces the number of uncommitted units to $7 - 4 \times 0.9 = 3.4$, which is greater than it was before the increase in N .

We have just shown that while an increase in the number of buyers need not affect the incumbent's benefit from foreclosure, it would be expected to reduce its cost of foreclosure. It follows that we would expect the incumbent to strictly gain from an increase in N under MS.

Proposition A.3 *Because of the dilution effect, the incumbent need not compensate each signing buyer as much for its contribution toward exclusion after an increase in N as it did before the increase in N . The incumbent's maximized profit under MS is thus strictly increasing in N .*

Proof: We need to show that $\alpha_{\Omega+1+\Delta N}(s^*) > \alpha_{\Omega+1}(\hat{s})$, and thus the cost of foreclosure is decreasing when N increases. The rest of the proposition has already been shown. Note that

$$\begin{aligned}\alpha_{\Omega+1+\Delta N}(s^*) &= G((N + \Delta N - (\Omega + 1 + \Delta N)s^*)\delta), \\ \alpha_{\Omega+1}(\hat{s}) &= G((N - (\Omega + 1)\hat{s})\delta),\end{aligned}$$

then $\alpha_{\Omega+1+\Delta N}(s^*) > \alpha_{\Omega+1}(\hat{s})$ if and only if $N + \Delta N - (\Omega + 1 + \Delta N)s^* > N - (\Omega + 1)\hat{s}$. To see this, note that the incumbent chooses s^* such that the optimal amount of uncommitted purchases is exactly the same as before N increases, that is,

$$N + \Delta N - (\hat{K} + \Delta N)s^* = N - \hat{K}\hat{s}. \quad (\text{A.5})$$

When N increases by ΔN , the incumbent must sign up $(\Omega + 1 + \Delta N)s^*$ units to reduce the probability of entry effectively, and sign up further $(\hat{K} - (\Omega + 1))s^*$ units to reduce the uncommitted purchases to the optimal level. Thus,

$$\begin{aligned}& N + \Delta N - (\Omega + 1 + \Delta N)s^* \\ &= N + \Delta N - (\hat{K} + \Delta N)s^* + (\hat{K} - (\Omega + 1))s^* \\ &= N - \hat{K}\hat{s} + (\hat{K} - (\Omega + 1))s^* \\ &> N - \hat{K}\hat{s} + (\hat{K} - (\Omega + 1))\hat{s} \\ &= N - (\Omega + 1)\hat{s},\end{aligned}$$

where we have used (A.5) to get the third line and the inequality comes from the fact that $s^* > \hat{s}$.

Q.E.D.

Proposition A.3 in conjunction with our earlier finding that the incumbent's maximized profit under ED is independent of N , implies that an increase in N expands the number of settings in which the incumbent will choose MS over ED. This is the exact opposite of what one might have expected, and it implies that an increase in N benefits MS relative to ED in the sense that (i) if initial conditions are such that MS is more profitable than ED, then MS will continue to be more profitable when the number of buyers increases; and (ii) if initial conditions are such that ED is more profitable than MS, then for a given increase in the number of buyers, the gap will narrow, and it is possible that MS could even overtake ED ex-post and become more profitable.

As we have seen, one way to think about why increases in N favor MS over ED is that, for a given benefit from foreclosure, the cost of foreclosure under ED is independent of N , whereas the cost of foreclosure under MS is decreasing in N . However, another way to think about the relative effects of an increase in N is to note that while an increase in N leads to a corresponding increase in the number of signed buyers under both ED and MS, this increase does not help the incumbent under ED because full compensation must be offered to all signed buyers and therefore the incumbent cannot profit from them. But the increase in the number of signed buyers does help the incumbent under MS because of the intra-group externality that signed buyers impose on each other. In fact, one can show that this externality only gets stronger as N increases, which is what generates the cost savings and thus allows increases in N to favor MS.

Numerical Example

We now construct an example in which ED dominates MS when there are only three buyers, but in which MS dominates ED when the number of buyers increases to five or more.

Consider the same set-up as in Example 2, but suppose now that $N = 5$. In this case, the incumbent earns its maximized expected profit of 900 under ED by increasing the number of signed buyers from two to four. Under MS, the incumbent can achieve its desired foreclosure level of 3.5 units by offering $s^* = .875$ to four buyers. This reduces the actual probability of entry to $\hat{\alpha}_4(s^*) = .35$, but increases the probability of entry when the $\Omega + 1$ 'st buyer signs to⁴⁰

$$\hat{\alpha}_3(s^*) = G((5 - 3 \times .875) 100) = G(237.5) = 0.9.$$

⁴⁰In contrast, when $N = 3$, this probability was $\alpha_1(.75) = .8$. It is easy to check that $x_3(s^*) > x_4(s^*)$.

The cost of foreclosure when N increases to $N = 5$ is thus reduced from 100 when $N = 3$ to

$$4(1 - s^*)(1 - \hat{\alpha}_3(s^*))(v - c) = 500 \times 0.1 = 50.$$

Thus, the profit under MS is now equal to $975 - 50 = 925$, which is higher than that under ED.

D. Mixed ED with MS

The following example shows that offering ED to the first \underline{K}^{ED} buyers and MS only to the last buyer can be worse than offering MS to all \overline{K}^{ED} buyers. The example has the same set-up as in Example 1 in the main appendix, except that there are five buyers instead of three buyers.

Assume that $v - c = 1000$, $\delta = 100$, and $N = 5$. As in Example 1, suppose that f can take on one of five values, with the listed probabilities:

$$\begin{aligned} f : & \quad 50, \quad 100, \quad 150, \quad 225, \quad 275 \\ g : & \quad 0.1, \quad 0.1, \quad 0.6, \quad 0.1, \quad 0.1. \end{aligned}$$

In the absence of ED and MS, the entrant can earn a flow profit of 500 if it enters. Since this exceeds the maximum value of f , we would expect the entrant to enter with probability one in the absence of any exclusionary contracts. On the other hand, if, for instance, the incumbent can foreclose 3.5 units, then only 1.5 units will be available to the entrant, in which case the entrant's profit will be reduced to 150 and the probability of entry will be reduced to $\Pr\{f < 150\} = 0.2$.

If the incumbent offers ED only, it will be optimal give it to four buyers, in which case the incumbent can reduce the probability of entry to $\alpha_4(1) = G(100) = 0.1$, and the incumbent earns an expected profit of $(5 - 4)(1 - 0.1)1000 = 900$.

Notice that the incumbent could have done even better if it could have signed up 3.5 buyers because then the probability of entry would have been reduced 0.2, and its expected profit would have been $(5 - 3.5)(1 - 0.2)1000 = 1200$. This maximum benefit from foreclosure can be achieved with MS, by offering $s = 0.875$ to four buyers. In order to induce all four buyers to accept MS with $s = 0.875$, however, the incumbent will have to give each buyer an inducement (here $\Omega = 2$) of $x^*(s) = x_{\Omega+1}(s) = (\alpha_2 - \alpha_3(s)(1 - s))(v - c)$. The total cost of foreclosure is then given by

$$C = 4[x^*(s) - s(v - c)] = 4[(\alpha_2 - \alpha_3(s)(1 - s))(v - c) - s(v - c)].$$

Note that signing up only two buyers does not reduce the probability of entry, that is, $\alpha_2 = 1$. On the other hand, signing up three buyers reduces the probability of entry effectively, with

$\alpha_3(s) = G((5 - 3 \times .875) \delta) = G(237.5) = 0.9$. Thus, the cost of foreclosure is given by

$$C = 4[1 - 0.9(1 - 0.875) - 0.875](v - c) = 0.05(v - c) = 50,$$

and the resulting profit is equal to $1200 - 50 = 1150$.

Suppose now the incumbent offers ED to three buyers and MS to the fourth buyer, with $s = 0.5$. The incumbent fully compensates the first three buyer with inducement $x^*(1) = v - c$, and the cost of foreclosure from these three buyers is zero. On the other hand, to sign up the fourth buyer, the incumbent will have to offer

$$x = x_4(s) = (\alpha_3 - \alpha_4(1 - s))(v - c).$$

Note that signing up the first three buyers with ED reduces the probability of entry to $\alpha_3 = G(200) = 0.8$, while signing up the fourth buyer further reduces the probability of entry to $\alpha_4 = G((5 - 3.5) \delta) = G(150) = 0.2$. Thus, the cost of signing up the fourth buyer is equal to

$$\begin{aligned} C &= x_4(s) - s(v - c) = (\alpha_3 - \alpha_4(1 - s) - s)(v - c) \\ &= (0.8 - 0.2 \times 0.5 - 0.5)(v - c) = 0.2(v - c) = 200, \end{aligned}$$

which is much higher than the cost of signing up four buyers with the uniform MS contract. As a result, the incumbent's profit, which is equal to $1200 - 200 = 1000$, is much lower. **Q.E.D.**